SYSTEM OF PARTICLES AND ROTATIONAL MOTION

MAIN POINTS

1. A rigid body is one for which the distance between different particles of the body do not change, even though there is a force acting on it.
2. A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure rotation or a combination of translation and rotation.
3. In pure translation every particle of the body moves with the same velocity at any instant of time.
4. In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every point in the rotating rigid body has the same angular velocity at any instant of time.
5. Angular velocity is a vector. Its magnitude is $\omega = d\Theta/dt$ and it is directed along the axis of rotation. For rotation about a fixed axis, this vector ω has a direction.
6. The vector or a cross product of two vectors A and B is a vector written as A X B. The magnitude of this vector is $AB \sin \Theta$ and its direction is given by right handed screw or the right hand rule.
7. The linear velocity of a particle of rigid body rotating about a fixed axis is given by $v = \omega \times r$ wherre r is the position vector of the particle with respect to an origin along the fixed axis. The relation applies even to more general rotation of a rigid body with one point fixed. In that case r is the position vector of the particle with respect to the fixed point taken as the origin.
8. The centre of mass of the system particles is defined as the point whose position vector *Is* $\boldsymbol{R = (\sum m_i r_i)/M}$
9. Velocity of the centre of mass of a system of particles is given by $\mathbf{V = p/M}$, **where P is the linear momentum of the system.** The centre of mass moves as if all the mass of the system is concentrated at this point and all the external forces act at it .If the total external force is zero, then the total linear momentum of the system is constant.
10. The angular momentum of a system of n particles about the origin is

$$\mathbf{L = \sum_{i=1}^{n} (r_i \times p_i)}$$

11. The Torque or moment of force on a system of n particles about the origin is given by

$$\tau = \sum_{i=1}^{n} (r_i \times F_i)$$

The force F_i acting on the i^{th}particle includes the external as well as the internal forces.

Assuming Newton's third law and that forces between any two particles act along the

line joining the particles, we can show that $\tau_{int} = 0$ and $dL/dt = \tau_{ext}$.

12. A rigid body is in mechanical equilibrium if
    (i)It is I n translational equilibrium, i.e.' the total external force on it is zero:
    $\sum F_i = 0$and
    (ii) It is in rotational equilibrium, i.e., the total external torque on it is zero:

$$\sum \tau_i = \sum r_i \times F_i = 0$$

13. The centre of gravity of an extended body is that point where the total gravitational torque on the body is zero.
14. The moment of inertia of a rigid body is defined by the formula $\mathbf{I = \sum m_i r_i^2}$
    Where ri Is the perpendicular distance of the i^{th} point of the body from the axis. The kinetic energy of rotation is $K = \frac{1}{2} I \omega^2$
15. The theorem of parallel axes :$I' = I_G + M a^2$ where $I_{G\ moment}$ of inertia bout the axis passing through centre of gravity.

Allows us to determine the moment of inertia of a rigid body about an axis as the sum

of the moment of inertia of the rigid body about a parallel axis passing through its centre of

mass and the product of its mass and the square of the perpendicular distance between the

two parallel axes.

16. Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics

and dynamics.

17. For a rigid body rotating about a fixed axis of rotation $L_z = I_z\omega$whereIz is the moment of inertia about z-axis. In general , the angular momentum about the axis of rotation , L is along the axis of rotation. I n that case $|L| = L_z = I\omega$. The angular acceleration of a rigid body rotating about a fixed axis is given by $I\alpha = \tau$. If the external torque acting on the body $\tau = 0$, the component of angular momentum about the fixed axis of such a rotating body is constant,
18. For rolling motion without slipping $v_{cm} = r\omega$, where vcmis the velocity of translation.' r ' is he radius and m is the mass of the body. The kinetic energy of such rolling motion of the body is the sum of kinetic energies of translation and rotation.

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

19. To determine the motion of the centre of mass of a system, we need to know external forces acting on the body.
20. The time rate of change of angular momentum is the Torque acting on the body,
21. The total torque on a system is independent of the origin, if the total external force is zero.
22. The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one part of the body to the other part of the body.

23. Principle of conservation of angular momentum:

It states that if there is no external torque acting on the system the total angular momentum of the system remains constant.

i.e. If τ ext = 0, dL/dt =0, Hence L= constant.

24. Kepler's laws:

(i) All planets revolve around the sun in elliptical orbits with sun at one of its foci.

(ii) The line joining the sun to the planet sweeps out equal areas in equal intervals of time

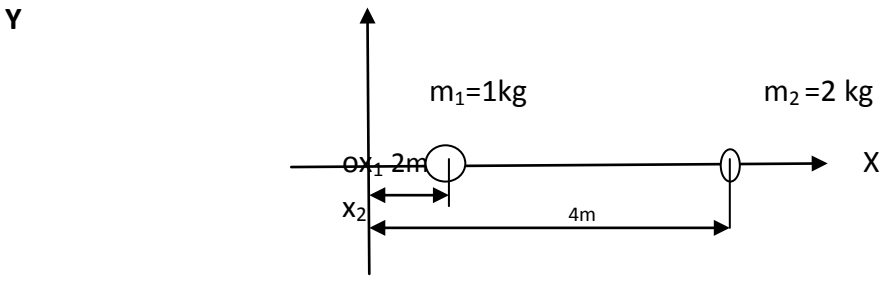
That is the areal velocity of a planet remains constant.

(iii)The square of the timeperiod of revolution of the planet is proportional to the cube of the semi major axis.

$$T^2 \propto R^3$$

Answer the following questions.Each question carries one mark

1. Define the term 'Centre of mass of a system of particles.
2. What will be the centre of mass of the pair of particles described belowin fig on the x-axis?



3. If two masses are equal where does their centre of mass lie?
4. Define a rigid body?

5. Is centre of mass same as the centre of gravity of a body? How can a rigid body be balanced?
6. Write an expression for the the velocity of the centre of mass of particles.
7. Does the total momentum of a system of particles depend upon the velocity of the centre of mass?
8. Write the expression for the acceleration of the centre of mass of particles. A projectile fired into the air suddenly explodes into several fragments. What can you say about motion of the fragments after collision?
9. Briefly explain about the centre of mass of the earth- moon system.
10. Define one radian.
11. Convert one radian into degrees.
12. Define angular velocity? What is its SI unit?
13. Define angular acceleration. What are its SI units?
14. Write the dimensional formula of angular acceleration.
15. Write the dimensional formula of angular velocity.
16. Considering rotational motion about some fixed axis, write equations corresponding to
    (i) $x(t) = x(0) + v(0)\, t + \frac{1}{2} a\, t^2$
    (ii) $v^2(t) = v^2(0) + 2a\,[x(t) - x(0)$
    (iii) $v(t) = v(0) + at$
    (iv) $v(t) = \frac{[\, v(t) + v(0)\,]}{2}$
17. Define angular momentum. Write the SI unit of angular momentum.
18. Name the dimensional constant whose dimensions are same as that of angular momentum.
19. Does the magnitude and direction of angular momentum $\bar{L}$ depend on the choice of the origin?
20. Express torque in terms of the rate of change of linear momentum.
21. Define moment of inertia of a body.
22. Is moment of inertia scalar or vector physical quantity? Write the SI unit of moment of inertia
23. Why is the most of the mass of a fly wheel placed on the rim?
24. Why are the spokes fitted in a cycle wheel?
25. The cap of pen can be opened with help of two fingers than with one finger. Explain why?
26. State the Work-Energy theorem for rotational motion.
27. State the law of conservation of anguar momentum.
28. For an isolated system plot a graph between moment of inertia (I) and angular velocity (ω)
29. What is the law of rotation?
30. State the theorem of parallel axes
31. State the theorem of perpendicular axes.

Answer the following questions.Each question carries 2 marks.

1. Write an expression for the moment of inertia of a ring of mass M and radius R,

   (i) About an axis passing through the centre , and perpendicular to its plane
   (ii) About a diameter
   (iii) About a tangent to its plane
   (iv) About a tangent perpendicular to the plane of the ring.

2. Write an expression for the moment of inertia of a circular disc of mass M and radius R,

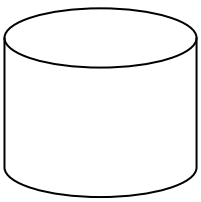
   (i) About an axis passing through the centre , and perpendicular to its plane
   (ii) About a diameter
   (iii) About a tangent to its plane
   (iv) About a tangent perpendicular to the plane of the disc.

3.Calculate the angular momentum of the earth rotating about its own axis.

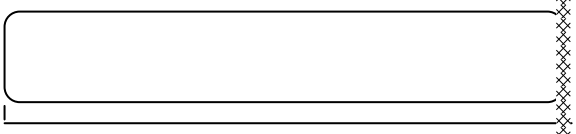
Mass of the earth = 5.98×10^{27} kg, radius of the earth = 6.37×10^{6} m.

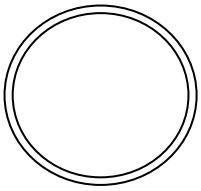
4. A thin metal hoop of radius 0.25 m and mass 2 kg starts from rest, and rolls downan inclined plane. Its linear velocity on reaching the foot of the plane is $4ms^{-1}$. What is the rotational kinetic energy when it reaches the foot of the inclined plane?
5. Three mass points m_1, m2, m3 are located at the vertices of an equilateral triangle of length 'a'. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1?
6. If the angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved?
7. A sphere of radius 10 cm weighs 1 kg. Calculate the moment of inertia
   (i) About the diameter
   (ii) about the tangent
8. A wheel rotates with a constant angular acceleration of 3.6 rad/s2. If the angular velocity of the wheel is 4.0 rad/s at t= 0. What angle does the wheel rotate in 1 s? What will be its angular velocity at t= 1s?
9. Mark the centre of mass of the following figures.

(i) Right circular cylinder

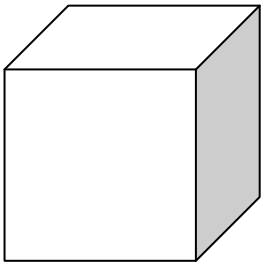


(ii) Cylindrical rod



(iii) 

Circular ring

(iv) 

Symmetrical cube.

10. To maintain a rotor at a uniform angular speed of 1oo s^{-1} an engine needs to transmit a torque of 200 Nm. What is the power of the engine required?
11. Two cars are going around two concentric circular paths at the same angular speed. Does the inner or the outer car have the larger speed?

Answer the following questions. Each question carries 3 marks.

1. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 A° ($1A^{\circ} = 10^{-10}$m). Find the approximate location of the centre of mass of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom, and nearly all the mass of an atom is concentrated in its nucleus.
2. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
3. A Long playing record revolves with a speed of 33⅓ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0.15, which of the two coins will revolve with the record.
4. State and prove the law of conservation of angular momentum.
5. Derive an expression for Torque acting on a body.
6. Explain the motion of centre of mass of a body with examples.
7. Find the torque of a force 7i^ + 3j^ -5k^ about the origin. The force acts on a particle whose position vector is I^ – j^+ K^?
8. What constant torque should be applied to a disc of mass 16 kg and diameter 0.5m; so that it acquires an angular velocity of 4π rad/s in 8 s? The disc is initially at rest, and rotates about an axis through the centre of the disc in a plane perpendicular to the disc.
9. A uniform ring of radius 0.5 m has a mass of 10 kg. A uniform circular disc of same radius has a mass of 10 kg. Which body will have the greater?

moment of inertia?Justify your answer.

10. Obtain a relation between torque applied to a body and angular acceleration produced. Hence define moment of inertia.
11. If the earth were to suddenly contract to half of its present size without change in its mass, what will be the duration of the new day.

12. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground? with maximum velocity?