Mathematics

Class VIII

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Foreword

The Changes in scenario of the Society and the nation entail the changes in the system of education which determine and accelerate the process of development in them. Education, beside other factors, is an important factor, responsible for the development of the society and the nation. To make School education, effective, useful and interesting, the changes in the curriculum from time to time is an essential step. The national curriculum framework 2005 and right to Free and Compulsory Child Education Act, 2009 in the present time make it evident that a child occupies a pivotal place in all the teaching-learning activities, conducted in any educational institution. Keeping this view in mind, our process of causing learning amongst the students should be such that they construct knowledge on their own on the basis of the knowledge acquired through their experiences. A child should be allowed maximum freedom in the process of learning and for that – teacher should act as a guide and helper rather than a preacher to make the curriculum easily accessible to children/students, a text book is an important means. That is why the government of Rajasthan has got the new text book written by making necessary changes in them in the light of the changes made in the curriculum.

While writing a text book it has been kept in view that the text book should be easy and comprehensible, with the help of simple language and interesting and attractive with the inclusion of pictures and varied activities through which the learners may not only imbibe the knowledge and information, contained in them but also associate themselves with the social, neighbourhood and local environment along with the development of and adherence to the knowledge about the historical, cultural glory and democratic values of the country so as to establish themselves as sincere, good and worthy citizens of our country, India.

I very humbly request the teachers that they should not only confine themselves to the completion of the teaching of the text book but also to present it in such a manner that a child gets ample opportunities of learning and accomplishing the objectives of teaching-learning on the basis of the curriculum and his/her experiences.

The state Institute of Educational Research and Training (SIERT), Udaipur acknowledges its thankfulness to all those government and private institutions viz. National Council of Educational Research and Training, New Delhi, State and National Census Departments, Ahad Museum, Udaipur. Directorate of Public Relations, Jaipur, Rajasthan, Rajasthan Text Book Board, Jaipur, Vidya Bharati, All India Educational Institute, Jaipur, Vidya Bhawan Reference Library, Udaipur, different writers, newspapers and magazines, publishers and websites that have cooperated with us in choosing and making the required material available for writing and developing the text book.

Inspite of best efforts, if the name of any writer, publisher, institution, organization and website has not been included here, we apologize for that and extend our thankfulness to them. In this connection, their names will be incorporated in the next editions of this book in future. It (SIERT) also extends thanks to Mr. Damodar Lal Kabra, Retd. Principal, Chittorgarh for cooperation with us in the translation work of this book.

To enhance the quality of the text books, we have received timely guidance and precious suggestions from Shri Kunji Lal Meena Secretary, Elementary Education, Govt. of Rajasthan, Shri Naresh Pal Gangwar Secretary, Secondary Education Govt. of Rajasthan, and Commissioner National Secondary Education Council, Shri Suwa Lal Meena, Director Secondary Education, Govt. of Rajasthan, Shri Babulal Meena, Director Elementary Education, Govt. of Rajasthan and Shri B.L. Jatawat, Commissioner Elementary Educati[illegible] Govt. of Rajasthan Jaipur, and as such the institute (SIERT) expresse[illegible]eartiest gratefulness to all of them.

This book has been prepared with the financial and the[illegible]pport of UNICEF. In this connection we are grateful to Mr. Sam[illegible], UNICEF Jaipur, Sulgana Roy, Education Specialist and all the re[illegible]s of UNICEF for their timely support and cooperation. Besides them[illegible]e appreciates the efforts of all those officers and other members of t[illegible]ho have directly or indirectly cooperated with us in accomplishin[illegible] of book writing and publishing it.

I am highly delighted to submit this b[illegible]ou all with this belief in mind that it will not only prove beneficial to the[illegible]s and the students but also serve as an effective link in the teachin[illegible]ing process and the personality development of the students.

To value thoughts and sug[illegible]ions is a specific feature of a democracy; therefore the SIERT, Udaipur w[illegible] always welcome your precious thoughts and suggestions for improving the quality of this book and thus make it better in every respect.

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SIERT, Udaipur

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For the Teachers

The new curriculum and the text book(s) in Mathematics have been prepared with a view to developing the teachers' compentencies in the teaching process and methodogy in this subject (Mathematics) in the light of the present changing global scenario.

This curriculum and text book in Mathematics have been prepared with a view to developing the child's understanding regarding the world of education along with the development of his/her latent capacities, enhancement of humane and moral values in dedicated, sincere citizen as well a patriot.

It is expected of a teacher of Mathematics to imbibe the main guiding principles of teaching. Stated in N.C.F., 2005 and in the light of them make the learners not only understand the subject matter of the book but also to imbibe it for their benefit in future.

The text book contains the following main features in it viz. the students have been made aware of the subject matter of the lessons the help of examples from their neighbourhood. In doing so it has been kept in mind that the teaching-learning material is available to the learners at low cost in their surroundings so that the teachers may use it in their day-today teaching by conducting different activities in the class room with a view to ensuring the learners maximum participation in the teaching-learning process and thereby making his/her teaching effective, useful and purposeful.

Considering the child as the center of the teaching-learning process, the teachers of learning by doing and correcting their mistakes an their own so as to develop insight in them for grasping and imbibing the subject matter of the mathematical lessons properly.

In the light of the provision of the Act of the Right to free and Compulsory Education, 2009, the subject matter has been prepared according to the spirit of the 'Continuous and Comprehensive Evaluation'. Therefore students should be imparted instructions dividing them into groups, according to their standards for inculcating the mathematical competencies in them.

The concepts of Mathematics have been delineated in detail along with pertinent pictures and diagrams for them. Examples and exercise have been combined so that the learners may understand the concepts and thus there by develop the capacity to solve the mathematical problems with maximum participation.

Under the heading 'Learning by Doing' enough activities for the development of the skills of mathematical thinking. Researching of the mathematical facts drawing, lining and measuring have been given for practice. All these activities are to be accomplished by the students with the spirit of cooperation, tolerance and responsibility.

The topics of national concerns viz - Environmental protection. Road safety, Gender Sensitivity, Beti Bachao; Bati Padhao and uprooting of Social evils, etc – have been included at proper places in the text book which the teachers should pay heed to and the same should be conveyed to the learners through the mathematical problems and mathematical solution and other glossary. The learners should be brought home to these national concerns along with the development of the sense of understanding them.

The teacher should judiciously divide the class into groups and sub-groups in order to generate the skill of self learning amongst the learners through various activities given in the text book of Mathematics. At the end of every lesson in the text book the mathematical concepts, definitions and results have been given under the title 'We have Learnt' according to learners capacities and maturity of minds.

At proper places the life history of Indian mathematicians and their contribution to Mathematics have been given in order to make the learners understand and appreciate such great personalities.

The curriculum and the book of Mathematics have been prepared, keeping the child at the center of the teaching-learning process reposing great faith in the teacher who with their great devotion and sincere efforts will work with children to make them understand the mathematical problems, definition, concepts and solutions well with this very belief in mind. The group of writers presents this book of Mathematics to the teachers of Rajasthan.

In India Mathematics has had rich tradition(s). Since times immemorial Indian Scholars and mathematicians have done excellent work in this area. In order to use the old knowledge in modern Mathematics and to establish its harmony with a view to enriching it (modern Mathematics), the Indian numerical system (Devnagari) and Vedic Mathematics have been incorporated in this text book - Efforts have been made to make calculations easier through Vedic Mathematics.

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Chapter 1 Rational Numbers

1.1 Introduction

In previous class, experiencing the necessity of rational numbers, we defined them in the form of $\frac{p}{q}$, where p and q are integers and q $\neq$ 0. We have studied and practised in detail, about the representation of rational numbers on number line, their equivalency, their simplified (standard) form, their comparison and to find rational numbers between two given rational numbers etc. In this section, continuing the previous class work, we shall study and practise the operations on rational numbers, multiplicative inverse, properties of these rational numbers and insertion of rational numbers between two rational numbers by mean method.

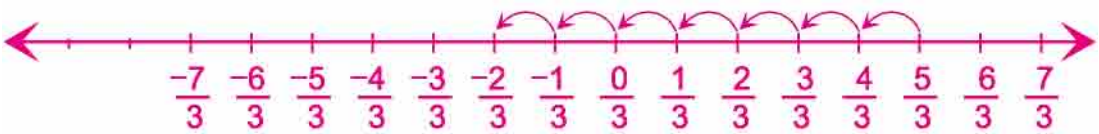
1.1 Operations on Rational Numbers

We know that how the integers and fractions are added, subtracted, multiplied and divided. Let us study these basic operations on rational numbers.

1.2.1 Addition

Vimala added two rational numbers $\frac{5}{3}$ and $-\frac{7}{3}$ which have the same denominators on number line in such a way

$$\frac{5}{3} + \left(-\frac{7}{3}\right)$$



On the above number line, distance between two successive points is $\frac{1}{3}$.

If we add $-\frac{7}{3}$ in to $\frac{5}{3}$ it means we have to move seven steps towards the left of $\frac{5}{3}$ (due to negative sign of $\frac{7}{3}$) Where we reach?

Now we reach on $\frac{-2}{3}$

Therefore, $\frac{5}{3} + \left(-\frac{7}{3}\right) = \frac{-2}{3}$

It can also be done like that

$$\frac{5}{3} + \left(-\frac{7}{3}\right)$$

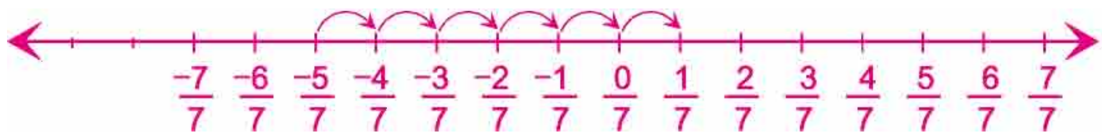
$$= \frac{5+(-7)}{3}$$

$$= \frac{5-7}{3}$$

$$= \frac{-2}{3}$$

Damoder also added two rational numbers $\frac{-5}{7}$ and $\frac{6}{7}$.

$$\frac{-5}{7} + \frac{6}{7}$$



Distance between two successive points on number line is $\frac{1}{7}$.

If we add $\frac{-5}{7}$ in to $\frac{6}{7}$, it means we move six steps towards the right of $\frac{-5}{7}$ $\left(\text{due to positive sign of } \frac{6}{7}\right)$.

We reach on $\frac{1}{7}$ Therefore, $\frac{-5}{7} + \frac{6}{7} = \frac{1}{7}$

It can also be solved as $\frac{-5}{7} + \frac{6}{7}$

$$= \frac{-5+6}{7}$$

$$= \frac{1}{7}$$

Do and Learn

- Find the value

  (i) $\frac{-11}{7} + \frac{4}{7}$ (ii) $\frac{3}{5} + \left(\frac{-2}{5}\right)$ (iii) $\frac{-3}{4} + \left(-\frac{5}{4}\right)$

Let us see some other examples-

$$\frac{4}{3} + \frac{1}{3} = \frac{4+1}{3} = \frac{5}{3}$$

$$\frac{-7}{5} + \frac{9}{5} = \frac{-7+9}{5} = \frac{2}{5}$$

$$\frac{4}{7} + \left(\frac{-9}{7}\right) = \frac{4-9}{7} = \frac{-5}{7}$$

$$\frac{-1}{4} + \left(\frac{-2}{4}\right) = \frac{-1-2}{4} = \frac{-3}{4}$$

Thus, we see that while adding the rational numbers having same denominators, numerators are added keeping the denominator same.

Vimala asked Damoder, "How we will add two rational numbers having distinct denominators ?"

Damoder - "Do you remember, we added two fractions with distinct denominators?"

1. Like fractions, first we obtain LCM of these denominators.
2. After that, the equivalent rational numbers are determined with the same LCM as obtained in step -1.
3. Then, both rational numbers are added (with common denominators).

Example 1 Add rational numbers $\frac{-4}{3}$ and $\frac{2}{5}$.

Solution $\frac{-4}{3} + \frac{2}{5}$

LCM of 3 and 5 is 15.

$$-\frac{4}{3} = -\frac{4 \times 5}{3 \times 5} = -\frac{20}{15}$$

and $$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

$$\frac{-4}{3} + \frac{2}{5} = -\frac{20}{15} + \frac{6}{15}$$

$$= \frac{-20+6}{15} = \frac{-14}{15}$$

Do and Learn

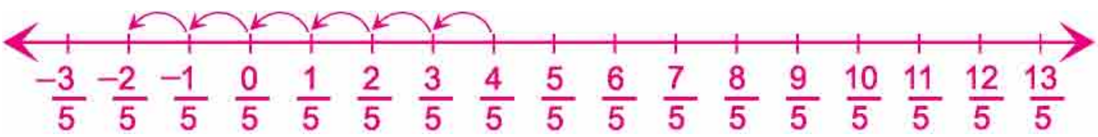
- Find the value

(i) $\frac{2}{5}+\frac{1}{6}$ (ii) $\frac{3}{8}+\left(\frac{-5}{2}\right)$ (iii) $\frac{-7}{20}+\frac{7}{3}$ (iv) $-\frac{5}{7}+\left(\frac{-2}{4}\right)$

1.2.2 Subtraction

Manish subtracts two rational numbers having common denominators $\frac{4}{5}$ and $\frac{6}{5}$ on number line like this:

$$\frac{4}{5}-\frac{6}{5}$$



On number line, distance between two successive points is $\frac{1}{5}$.
If we subtract $\frac{6}{5}$ from $\frac{4}{5}$, it means we move six steps towards the left of $\frac{4}{5}$
Where do we reach? We reach on $-\frac{2}{5}$.

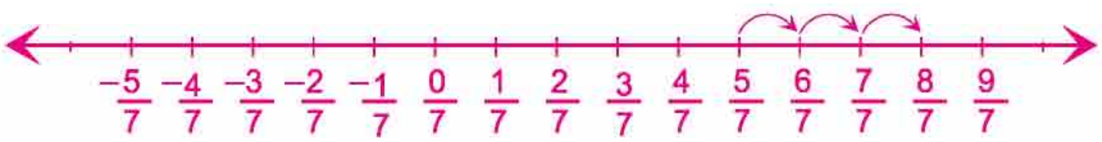
$$\text{So, } \frac{4}{5}-\frac{6}{5}=-\frac{2}{5}$$

We can also do like this, $\frac{4}{5}-\frac{6}{5}=\frac{4-6}{5}=-\frac{2}{5}$

Praneeti also subtracted two rational numbers $\frac{5}{7}$ and $\left(-\frac{3}{7}\right)$ like this way,

$$\frac{5}{7}-\left(-\frac{3}{7}\right)$$

.



On number line, distance between two successive points is $\frac{1}{7}$.

If we subtract $\left(-\frac{3}{7}\right)$ from $\frac{5}{7}$, it means we move three steps towards the right of $\frac{5}{7}$ $\left(\text{by subtracting } -\frac{3}{7}\right)$

Now, we reach on $\frac{8}{7}$.

Thus $\frac{5}{7} - \left(\frac{-3}{7}\right) = \frac{8}{7}$

It can also be written like this,

$$\frac{5}{7} - \left(\frac{-3}{7}\right) = \frac{5-(-3)}{7}$$
$$= \frac{5+3}{7}$$
$$= \frac{8}{7}$$

By this practice we can say that while subtracting the two rational numbers having common denominators, numerators are subtracted keeping the denominator the same.

Similarly, $\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

Other Method

Example 2 Subtract $\frac{-7}{8}$ from $\frac{5}{8}$.

Solution $\frac{5}{8} - \left(\frac{-7}{8}\right) = \frac{5}{8} + \left(\frac{7}{8}\right)$

$$= \frac{5}{8} + \frac{7}{8}$$
$$= \frac{5+7}{8}$$
$$= \frac{12}{8}$$
$$= \frac{3}{2}$$

Do and Learn

- Find the value-

(i) $\frac{10}{7} - \frac{4}{7}$ (ii) $\frac{-4}{5} - \left(\frac{-2}{5}\right)$ (iii) $\frac{7}{9} - \left(\frac{-4}{9}\right)$

Manish asked Praneeti to tell how to subtract two rational numbers having distinct denominators?

Praneet – Same as we did in addition but at final stage we subtract in place of adding.

Example 3 Subtract $-\frac{3}{8}$ from $-\frac{5}{4}$.

Solution $-\frac{5}{4} - \left(\frac{-3}{8}\right)$

8 is LCM of 4 and 8.

$$-\frac{5}{4} = -\frac{5\times 2}{4\times 2} = -\frac{10}{8}$$

$$-\frac{5}{4} - \left(\frac{-3}{8}\right) = -\frac{10}{8} - \left(\frac{-3}{8}\right)$$

$$= \frac{-10-(-3)}{8}$$

$$= \frac{-10+3}{8}$$

$$= -\frac{7}{8}$$

Do and Learn

- Find the value-

(i) $\frac{4}{3} - \frac{3}{8}$ (ii) $\left(\frac{-3}{7}\right) - \frac{2}{14}$ (iii) $\frac{5}{9} - \left(\frac{-2}{11}\right)$ (iv) $\left(\frac{-2}{9}\right) - \frac{7}{6}$

1.2.3 Multiplication

We have learnt multiplication of fractions.

Now consider the multiplication of rational numbers $\left(2 \times -\frac{5}{7}\right)$.

Method-1 (Repetitive addition)

$2 \times \left(-\frac{5}{7}\right)$ that is, two time addition of $-\frac{5}{7}$.

i.e. $\left(-\frac{5}{7}\right) + \left(-\frac{5}{7}\right) = \frac{-5}{7} - \frac{5}{7}$

$$= \frac{-5-5}{7}$$

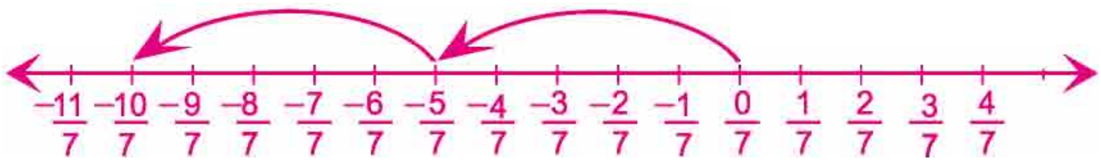
$$= \frac{-10}{7}$$

Method – 2 (By Number Line)

On number line, distance between two successive points is $\frac{1}{7}$.

Moving towards left of Zero ($\frac{5}{7}$ is a negative number).

Take two long jumps of $\frac{5}{7}$ distance (because of twice $\frac{-5}{7}$) . Where do we reach?



We reach on $-\frac{10}{7}$.

Therefore $2\times\left(-\frac{5}{7}\right)=-\frac{10}{7}$

Method-3 (By Multiplication)

$$2\times\left(-\frac{5}{7}\right)=\frac{2}{1}\times\left(-\frac{5}{7}\right)$$
$$=\frac{2\times(-5)}{1\times 7}$$
$$=-\frac{10}{7}$$

Do and Learn

- Find the Value-

(i) $4\times\left(-\frac{1}{3}\right)$ (ii) $\left(-\frac{3}{5}\right)\times 7$ (iii) $\left(-\frac{4}{5}\right)\times(-3)$

(iv) $\left(-\frac{3}{7}\right)\times\frac{2}{5}$ (v) $\frac{2}{3}-\left(\frac{-1}{4}\right)$ (vi) $\left(\frac{-3}{2}\right)\times\left(\frac{-9}{7}\right)$

Just think, which procedure you adopted to multiply these numbers.

When we multiply two rational numbers, the product of their numerators is written as a numerator and product of their denominators is written as denominator.

1.2.4 Division

We know the reciprocal of fractions.
Reciprocal (or inverse) of $\frac{3}{7}$ is $\frac{7}{3}$.

This concept is also applied on reciprocal of rational numbers.

Similarly, reciprocal of $-\frac{3}{4}$ is $\frac{4}{-3}$ or $\frac{-4}{3}$ and reciprocal of $-\frac{7}{9}$ is $-\frac{9}{7}$.
We know that, 10 × 5 = 50
In form of division, it can be written in the following two ways:

$50 \div 10 = 5$	$50 \div 5 = 10$
$\frac{50}{10} = 5$	$\frac{50}{5} = 10$
$50 \times \frac{1}{10} = 5$	$50 \times \frac{1}{5} = 10$

By this it is clear that, when dividend is divided by the divisor, the resultant is quotient and if dividend is multiplied by inverse of divisor then resultant number is equal to the quotient. Now it is clear that action of division can be transformed in form of multiplication.

Example 4 Solve $\frac{-21}{8} \div \frac{8}{3}$

Solution $\frac{-21}{8} \div \frac{8}{3} = \frac{-21}{8} \times \frac{3}{8}$

$$= \frac{-21 \times 3}{8 \times 8} = \frac{-63}{64}$$

By the above example, it is clear that to divide any rational number by another rational number, we multiply that rational number by an inverse of another rational number.

Do and Learn

- Solve-

(i) $-\frac{7}{2} \div 4$ (ii) $\frac{-12}{7} \div \frac{3}{4}$ (iii) $\frac{5}{9} \div \left(-\frac{4}{5}\right)$

Division of rational number by the same rational number

$$\frac{3}{7} \div \frac{3}{7} = \frac{3}{7} \times \left(\text{reciprocal of } \frac{3}{7}\right) = \frac{3}{7} \times \frac{7}{3} = 1$$

$$-\frac{4}{5} \div \left(\frac{-4}{5}\right) = \frac{-4}{5} \times \left(\text{reciprocal of} -\frac{4}{5}\right) = \frac{-4}{5} \times \left(-\frac{5}{4}\right) = 1$$

You also consider this type of example.
By the above discussion it is clear that if rational number is divided by the same rational number, result of the division is quotient 1..

In other words, product of any given rational number with its reciprocal is always 1.

Do and Learn

• Solve-

(i) $\frac{5}{7} \div \frac{5}{7}$ (ii) $\frac{-9}{4} \div \frac{-9}{4}$ (iii) $\frac{-7}{11} \div \frac{-7}{11}$

1.3 Properties of Rational Numbers

1.3.1 Closure Property:

(i) Addition Let us consider on sum of any two rational numbers.

$\frac{3}{4} + \frac{5}{3} = \frac{9}{12} + \frac{20}{12} = \frac{9+20}{12} = \frac{29}{12}$ is a rational number.

$\frac{2}{7} + \left(\frac{-6}{11}\right) = \frac{22}{77} + \left(\frac{-42}{77}\right) = \frac{22-42}{77} = \frac{-20}{77}$ is a rational number.

$\frac{5}{11} + \frac{6}{11} = \frac{5+6}{11} = \frac{11}{11} = 1$ is a rational number.

Verify this property on other rational numbers.

It is clear that the sum of any two rational numbers is always a rational number. Thus, **rational numbers are closure under addition** i.e., for any two rational numbers *x* and *y*, (*x*+*y*) is also a rational number.

(ii) Subtraction

Let us consider the subtraction of any two rational numbers

$\frac{5}{7} - \frac{3}{8} = \frac{40}{56} - \frac{21}{56} = \frac{40-21}{56} = \frac{19}{56}$ is a rational number.

$\frac{7}{8} - \frac{8}{9} = \frac{63}{72} - \frac{64}{72} = \frac{63-64}{72} = \frac{-1}{72}$ is a rational number.

$\frac{1}{4} - \frac{1}{4} = \frac{1-1}{4} = \frac{0}{4} = 0$ is a rational number.

Verify this property on other rational numbers.

It is clear that the subtraction of any two rational numbers is always a rational number. Thus, **rational numbers are closure under subtraction** i.e., for any two rational numbers *x* and *y*, (*x*-*y*) is also a rational number.

(iii) Multiplication

Let us consider on product of any two rational numbers.

$$\frac{-4}{5} \times \frac{3}{7} = \frac{(-4) \times 3}{5 \times 7} = \frac{-12}{35}$$ is a rational number.

$$\frac{2}{3} + \left(\frac{-4}{9}\right) = \frac{2 \times (-4)}{3 \times 9} = \frac{-8}{27}$$ is a rational number.

$$\left(\frac{-2}{7}\right) + \left(\frac{-1}{3}\right) = \frac{(-2) \times (-1)}{7 \times 3} = \frac{2}{21}$$ is a rational number.

Verify this property on other rational numbers.

It is clear that the product of any two rational numbers is always a rational number. Thus, **rational numbers are closure under multiplication** i.e., for any two rational numbers x and y, product $(x \times y)$ is also a rational number.

(iv) Division

Let us consider the division of any two rational numbers.

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$$ is a rational number.

$$\frac{-7}{2} \div \frac{3}{5} = \frac{-7}{2} \times \frac{5}{3} = \frac{-7 \times 5}{2 \times 3} = \frac{-35}{6}$$ is a rational number.

$$0 \div \frac{1}{2} = 0 \times \frac{2}{1} = 0$$ is a rational number.

$$5 \div 0 = 5 \times \frac{1}{0} = \frac{5}{0}$$ is not a rational number. (indefinite number).

Thus, we can say that it is not compulsory that division of any two rational numbers is always a rational number. Therefore, **rational numbers are not closure under division.**

Do and Learn Fill in the blanks-

Numbers	Closure under Operations			
	Addition	Subtraction	Multiplication	Division
Natural Number	Yes	-------	-------	-------
Whole Number	-------	-------	-------	No
Integers	-------	Yes	-------	-------
Rational Number	-------	-------	Yes	-------

1.3.2 Commutative Property

(i) Addition

Consider the sum of two rational numbers $\frac{3}{7}$ and $\frac{-1}{4}$.

$$\frac{3}{7} + \left(\frac{-1}{4}\right) = \frac{12}{28} + \left(\frac{-7}{28}\right) = \frac{12-7}{28} = \frac{5}{28}$$

and $$\left(\frac{-1}{4}\right) + \frac{3}{7} = \left(\frac{-7}{28}\right) + \frac{12}{28} = \frac{-7+12}{28} = \frac{5}{28}$$

So $$\frac{3}{7} + \left(\frac{-1}{4}\right) = \left(\frac{-1}{4}\right) + \frac{3}{7}$$

Therefore $$\frac{-4}{5} + \left(\frac{-2}{3}\right) = \frac{-12}{15} + \left(\frac{-10}{15}\right) = \frac{-12-10}{15} = \frac{-22}{15}$$

$$\frac{-2}{3} + \left(\frac{-4}{5}\right) = \frac{-10}{15} + \left(\frac{-12}{15}\right) = \frac{-10-12}{15} = \frac{-22}{15}$$

So $$\frac{-2}{3} + \left(\frac{-4}{5}\right) = \frac{-4}{5} + \left(\frac{-2}{3}\right)$$

You also verify this commutative property on any other rational numbers.

We can say even if we change the order of rational numbers, sum of any two rational numbers is always remain the same. Therefore, **rational numbers are commutative under the addition** i.e., for any two given rational numbers a and b.

$$\boxed{a + b = b + a}$$

(ii) Subtraction

Consider the subtraction of two rational numbers $\frac{2}{5}$ and $\frac{4}{7}$.

$$\frac{2}{5} - \frac{4}{7} = \frac{14}{35} - \frac{20}{35} = \frac{14-20}{35} = \frac{-6}{35}$$

$$\frac{4}{7} - \frac{2}{5} = \frac{20}{35} - \frac{14}{35} = \frac{20-14}{35} = \frac{6}{35}$$

$$\frac{2}{5} - \frac{4}{7} \neq \frac{4}{7} - \frac{2}{5}$$

Thus, we can say that, **rational numbers are not commutative under the subtraction** i.e., for any two given rational numbers a and b

$$\boxed{a - b \neq b - a}$$

(iii) Multiplication

Consider the multiplication of two rational numbers $\frac{-4}{5}$ and $\frac{3}{7}$.

$$\left(\frac{-4}{5}\right)\times\frac{3}{7} = \frac{(-4)\times 3}{5\times 7} = \frac{-12}{35}$$

and $$\frac{3}{7}\times\left(\frac{-4}{5}\right) = \frac{3\times(-4)}{7\times 5} = \frac{-12}{35}$$

So $$\left(\frac{-4}{5}\right)\times\frac{3}{7} = \frac{3}{7}\times\left(\frac{-4}{5}\right)$$

You also verify this commutative property on any other rational numbers.

We can say that the product of any two rational numbers is always the same even we change the order of multiples. Therefore, **rational numbers are commutative under the multiplication** i.e., for any two given rational numbers a and b

$$\boxed{a\times b = b\times a}$$

(iv) Division

Consider the division of two rational numbers $\frac{7}{3}$ and $\frac{14}{5}$.

$$\frac{7}{3}\div\frac{14}{5} = \frac{7}{3}\times\frac{5}{14} = \frac{35}{42}$$

and $$\frac{14}{5}\div\frac{7}{3} = \frac{14}{5}\times\frac{3}{7} = \frac{42}{35}$$

So $$\frac{7}{3}\div\frac{14}{5} \neq \frac{14}{5}\div\frac{7}{3}$$

Thus, we can say that , **rational numbers are not commutative under the division** i.e., for any two given rational numbers a and b

$$\boxed{a\div b \neq b\div a}$$

Do and Learn Fill in the blanks-

Numbers	Commutativity			
	Addition	Subtraction	Multiplication	Division
Natural Number	Yes	No	Yes	No
Whole Number	--------	--------	--------	--------
Integers	--------	--------	--------	--------
Rational Number	--------	--------	--------	--------

1.3.3 Associative Property

(i) Addition

Check this property with three rational numbers $\frac{-5}{4}$, $\frac{3}{8}$ and $\frac{-7}{6}$.

$$\frac{-5}{4} + \left(\frac{3}{8} + \frac{-7}{6}\right) = \frac{-5}{4} + \left(\frac{9-28}{24}\right) = \frac{-5}{4} + \left(\frac{-19}{24}\right) = \frac{-5}{4} - \frac{19}{24} = \frac{-30-19}{24} = \frac{-49}{24}$$

$$\left(\frac{-5}{4} + \frac{3}{8}\right) + \frac{-7}{6} = \left(\frac{-10+3}{8}\right) + \frac{-7}{6} = \left(\frac{-7}{8}\right) + \left(\frac{-7}{6}\right) = \frac{-21+(-28)}{24} = \frac{-21-28}{24} = \frac{-49}{24}$$

So $\frac{-5}{4} + \left(\frac{3}{8} + \frac{-7}{6}\right) = \left(\frac{-5}{4} + \frac{3}{8}\right) + \frac{-7}{6}$

Do and Learn

Are the addition on both sides the same?

(i) $\frac{-3}{5} + \left(\frac{2}{3} + \frac{4}{7}\right) = \left(\frac{-3}{5} + \frac{2}{3}\right) + \frac{4}{7}$

(ii) $\frac{1}{2} + \left(\frac{-3}{4} + \frac{-5}{8}\right) = \left(\frac{1}{2} + \frac{-3}{4}\right) + \frac{-5}{8}$

You also verify this associative property on any other rational numbers. We find that **rational numbers are associative under the addition** i.e., for any three given rational numbers a,b and c

$$\boxed{a + (b + c) = (a + b) + c}$$

(ii) Subtraction

Verify this on three rational numbers $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{-5}{4}$.

$$\frac{1}{2} - \left[\frac{3}{4} - \left(\frac{-5}{4}\right)\right] \qquad \left(\frac{1}{2} - \frac{3}{4}\right) - \left(\frac{-5}{4}\right)$$

$$= \frac{1}{2} - \left(\frac{3+5}{4}\right) \qquad = \left(\frac{2-3}{4}\right) - \left(\frac{-5}{4}\right)$$

$$= \frac{1}{2} - \frac{8}{4} \qquad = \frac{-1}{4} - \left(\frac{-5}{4}\right)$$

$$= \frac{2-8}{4} \qquad = \frac{-1}{4} + \frac{5}{4}$$

$$= \frac{-6}{4} \qquad = \frac{4}{4}$$

$$= \frac{-3}{2} \qquad = 1$$

$$\frac{1}{2} - \left[\left(\frac{3}{4} - \frac{-5}{4}\right)\right] \neq \left(\frac{1}{2} - \frac{3}{4}\right) - \left(\frac{-5}{4}\right)$$

We find that **subtraction is not associative for rational numbers** i.e., for any three rational numbers a, b and c

$$\boxed{a-(b-c) \neq (a-b)-c}$$

(iii) Multiplication Consider the three rational numbers $\frac{2}{3}$, $\frac{4}{7}$ and $\frac{-5}{7}$.

$$\frac{2}{3} \times \left[\frac{4}{7} \times \left(\frac{-5}{7}\right)\right] \qquad \left(\frac{2}{3} \times \frac{4}{7}\right) \times \left(\frac{-5}{7}\right)$$

$$= \frac{2}{3} \times \left[\frac{4 \times (-5)}{7 \times 7}\right] \qquad = \left(\frac{2 \times 4}{3 \times 7}\right) \times \left(\frac{-5}{7}\right)$$

$$= \frac{2}{3} \times \left(\frac{-20}{49}\right) \qquad = \frac{8}{21} \times \left(\frac{-5}{7}\right)$$

$$= \frac{2 \times (-20)}{3 \times 49} \qquad = \frac{8 \times (-5)}{21 \times 7}$$

$$= \frac{2 \times -20}{3 \times 49} \qquad = \frac{8 \times -5}{21 \times 7}$$

$$= \frac{-40}{147} \qquad = \frac{-40}{147}$$

So $$\frac{2}{3} \times \left(\frac{4}{7} \times \frac{-5}{7}\right) = \left(\frac{2}{3} \times \frac{4}{7}\right) \times \left(\frac{-5}{7}\right)$$

You also verify this property by taking any other rational numbers we find that **rational numbers are associative under multiplication** i.e., for any three rational numbers a, b and c

$$\boxed{a \times (b \times c) = (a \times b) \times c}$$

Do and Learn

Verify –

(i) $\frac{-4}{3}\times\left[\left(\frac{-2}{5}\right)\times\frac{1}{7}\right]=\left[\frac{4}{3}\times\left(\frac{-2}{5}\right)\right]\times\frac{1}{7}$

(ii) $\frac{-3}{5}\times\left[\frac{4}{11}\times\left(\frac{-3}{22}\right)\right]=\left[\left(\frac{-3}{5}\right)\times\frac{4}{11}\right]\times\frac{-3}{22}$

(iv) Division

Consider any given three rational numbers $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{-2}{7}$.

$$\frac{2}{3}\div\left[\frac{3}{4}\div\left(\frac{-2}{7}\right)\right]$$
$$=\frac{2}{3}\div\left[\frac{3}{4}\times\left(-\frac{7}{2}\right)\right]$$
$$=\frac{2}{3}\div\left[\frac{3\times(-7)}{4\times 2}\right]$$
$$=\frac{2}{3}\div\left(\frac{-21}{8}\right)$$
$$=\frac{2}{3}\times\frac{8}{-21}$$
$$=\frac{-16}{63}$$

$$\left(\frac{2}{3}\div\frac{3}{4}\right)\div\left(\frac{-2}{7}\right)$$
$$=\left(\frac{2}{3}\times\frac{4}{3}\right)\div\left(\frac{-2}{7}\right)$$
$$=\left(\frac{2\times 4}{3\times 3}\right)\div\left(\frac{-2}{7}\right)$$
$$=\frac{8}{9}\div\frac{-2}{7}$$
$$=\frac{8}{9}\times\left(\frac{7}{-2}\right)$$
$$=\frac{-56}{18}$$

$$\frac{2}{3}\div\left[\frac{3}{4}\div\left(\frac{-2}{7}\right)\right]\neq\left[\left(\frac{2}{3}\div\frac{3}{4}\right)\right]\div\left(\frac{-2}{7}\right)$$

We find that **rational numbers are not associative for division** i.e., for any three rational numbers a, b and c

$$\boxed{a\div(b\div c)\neq(a\div b)\div c}$$

Do and Learn Fill in the blanks -

Numbers	Associativity			
	Addition	Subtraction	Multiplication	Division
Natural Number	Yes	--------	--------	--------
Whole Number	--------	--------	--------	No
Integers	--------	--------	Yes	--------
Rational Number	--------	--------	--------	--------

1.3.4 Operations of Zero with rational numbers

Can you suggest any number which can be added with any number and obtain the same number? When zero (0) is added with any other rational number, then resultant is the same rational number.

$$5 + 0 = 0 + 5 = 5$$

$$(-3) + 0 = 0 + (-3) = -3$$

$$\left(\frac{-5}{7}\right) + 0 = 0 + \left(\frac{-5}{7}\right) = \frac{-5}{7}$$

Therefore, **Zero is called additive identity**, i.e., for any rational number a,

$$\boxed{a + 0 = 0 + a = a}$$

Consider , do natural numbers contain additive identity?

Do and Learn

Fill in the blanks:-

(i) $3 + \square = 3$ (ii) $\square + 0 = -7$ (iii) $\frac{-4}{9} + \square = \frac{-4}{9}$

(iv) $\square + \frac{9}{13} = \frac{9}{13}$ (v) $\frac{-5}{11} + 0 = \square$

1.3.5 Multiplicative Identity:

Fill in the blanks :

$8 \times \square = 8$ and $\square \times 8 = 8$

$(-5) \times \square = -5$ and $\square \times (-5) = -5$

$\left(\frac{2}{3}\right) \times \square = \frac{2}{3}$ and $\square \times \frac{2}{3} = \frac{2}{3}$

$\left(\frac{-4}{7}\right) \times \square = \frac{-4}{7}$ and $\square \times \left(\frac{-4}{7}\right) = \frac{-4}{7}$

By this exercise we can say that if any rational number is multiplied by 1, then product is the same as rational Number, i.e., **1 is called multiplicative identity** for rational numbers. For any rational number a,

$$\boxed{a \times 1 = 1 \times a = a}$$

Consider , what is the multiplicative identity of integers and whole numbers?

1.3.6 Additive Inverse: Fill in the blanks-

$2 + \square = 0$ and $\square + 2 = 0$

$-3 + \square = 0$ and $\square + (-3) = 0$

$\frac{3}{4} + \square = 0$ and $\square + \frac{3}{4} = 0$

$-\frac{5}{7} + \square = 0$ and $\square + \left(\frac{-5}{7}\right) = 0$

By this exercise we can say that when sum of two numbers is zero (additive identity) then these two numbers are additive inverse to each other. E.g. -1 is additive inverse of 1 and 1 is additive inverse of -1. Similarly, we can say that $\frac{-a}{b}$ is additive inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is additive inverse of $\frac{-a}{b}$.

Do and Learn

Write the additive inverse of the following rational numbers-

(i) 4 (ii) $\frac{-1}{3}$ (iii) $\frac{7}{2}$ (iv) $\frac{-3}{5}$ (v) $\frac{9}{2}$

1.3.7. Multiplicative Inverse:

Fil in the blanks -

$5 \times \square = 1$ and $\square \times 5 = 1$

$-7 \times \square = 1$ and $\square \times (-7) = 1$

$\frac{2}{3} \times \square = 1$ and $\square \times \frac{2}{3} = 1$

$-\frac{2}{3} \times \square = 1$ and $\square \times \left(\frac{-2}{3}\right) = 1$

By this exercise we can say that when product of two rational numbers is 1 (multiplicative identity) then these two numbers are multiplicative inverse (reciprocal) to each other. E.g. $\frac{4}{3}$ is the inverse of $\frac{3}{4}$ and $\frac{3}{4}$ is inverse of $\frac{4}{3}$. For any rational number $\frac{a}{b}$, if $\frac{a}{b} \times \frac{b}{a} = 1 = \frac{b}{a} \times \frac{a}{b}$ then we can say that the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ and the reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.

Can you tell the multiplicative inverse of zero?

Do and Learn

Write the multiplicative inverse of 3, $\frac{1}{5}$ $\frac{-3}{7}$ $\frac{2}{3}$ and $\frac{-5}{6}$ rational numbers.

1.3.8 Distributivity of Multiplication over Addition for Rational Numbers

Consider the following-

$$\frac{5}{4}\times\left[\left(\frac{-2}{8}\right)+\left(\frac{-3}{5}\right)\right]$$
$$=\frac{5}{4}\times\left(\frac{-10-24}{40}\right)$$
$$=\frac{5}{4}\times\left(\frac{-34}{40}\right)$$
$$=\frac{-170}{160}$$
$$=\frac{-17}{16}$$

$$\frac{5}{4}\times\left[\left(\frac{-2}{8}\right)+\left(\frac{-3}{5}\right)\right]$$
$$=\frac{5}{4}\times\left(\frac{-2}{8}\right)+\frac{5}{4}\times\left(\frac{-3}{5}\right)$$
$$=\frac{-10}{32}+\left(\frac{-15}{20}\right)$$
$$=\frac{-50-120}{160}$$
$$=\frac{-170}{160}$$
$$=\frac{-17}{16}$$

So $\frac{5}{4}\times\left[\left(\frac{-2}{8}\right)+\left(\frac{-3}{5}\right)\right]=\frac{5}{4}\times\left(\frac{-2}{8}\right)+\frac{5}{4}\times\left(\frac{-3}{5}\right)$

is $\frac{2}{5}\times\left[\left(\frac{1}{2}\right)+\left(\frac{-3}{4}\right)\right]=\frac{2}{5}\times\frac{1}{2}+\frac{2}{5}\times\left(\frac{-3}{4}\right)$?

This property is called the distributivity of multiplication over addition. Is in rational number disrtibutivity of multiplication is true over subtraction ? If a, b and c are three rational numbers, then

$$a\times(b+c)=a\times b+a\times c$$
$$a\times(b-c)=a\times b-a\times c$$

Do and Learn

(i) $\frac{5}{8}\times\left(\frac{-3}{7}\right)+\frac{5}{8}\times\left(\frac{-7}{6}\right)$ (ii) $\frac{2}{5}\times\left(\frac{-1}{9}\right)+\frac{2}{5}\times\left(\frac{-3}{7}\right)$

(iii) $\left(\frac{-4}{5}\right)\times\frac{2}{9}+\left(\frac{-4}{5}\right)\times\frac{7}{11}$ (iv) $\frac{3}{5}\times\left(\frac{-1}{3}\right)+\frac{3}{5}\times\frac{3}{5}$

1.3.9. Determine the Rational Number between Two Given Rational Numbers:

In previous class, we learnt about finding rational number between two given rational numbers. As we know the Mean method. So, in this section we shall study to find the rational number between two given rational numbers by average (mean) method.

We know that

4,3,2 are natural numbers between 5 and 1.

Is there any natural number between 2 and 3

-2,-1,0,1,2 are integers between -3 and 3.

Is there any integer between two consecutive integers?

There is no integer between two consecutive integers. But we can find rational numbers between two consecutive integers.

Example 5 Find a rational number between 2 and 3.

Solution $\frac{2+3}{2}=\frac{5}{2}$

So $2<\frac{5}{2}<3$

Example 6 Find a rational number between $\frac{3}{5}$ and $\frac{7}{2}$.

Solution

$$\frac{\frac{3}{5}+\frac{7}{2}}{2}$$

$$=\frac{\frac{6+35}{10}}{2}$$

$$=\frac{\frac{41}{10}}{2}$$

$$=\frac{41}{20}$$

By the above examples it is clear that to find the rational numbers between two rational numbers a and b, sum of rational numbers a and b is divided by 2.

$$\text{Mean} = \frac{a+b}{2}$$

Example 7 Find three rational numbers between 3 and 4.

Solution Rational numbers between 3 and 4 is $= \frac{3+4}{2}$

$= \frac{7}{2}$

Therefore $3 < \frac{7}{2} < 4$

Rational numbers between 3 and $\frac{7}{2}$ is $= \frac{3+\frac{7}{2}}{2} = \frac{\frac{6+7}{2}}{2} = \frac{\frac{13}{2}}{2} = \frac{13}{4}$

So $3 < \frac{13}{4} < \frac{7}{2} < 4$

Rational numbers between $\frac{7}{2}$ and 4 is $= \frac{\frac{7}{2}+4}{2} = \frac{\frac{7+8}{2}}{2} = \frac{\frac{15}{2}}{2} = \frac{15}{4}$

So $3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$

Thus, rational numbers between 3 and 4 are $\frac{13}{4}$, $\frac{7}{2}$ and $\frac{15}{4}$.

In this manner, we can obtain as many (infinite) rational numbers between two given rational numbers.

Do and Learn

1. Find rational number between -1 and 2.
2. Find rational number between $\frac{2}{3}$ and $\frac{3}{4}$.
3. Find three rational numbers between 2 and 3.

Exercise 1

1. Add the following rational numbers (solve any two on number line)-

(i) $\frac{5}{2}+\left(\frac{-3}{4}\right)$ (ii) $\frac{-2}{3}+\left(\frac{-4}{5}\right)+\frac{5}{6}$ (iii) $0+\frac{-2}{3}$

(iv) $-2\frac{1}{3}+4\frac{3}{5}$ (v) $\frac{-6}{5}+\left(\frac{-13}{7}\right)$ (vi) $\frac{-8}{19}+\frac{(-4)}{57}$

2. Find the value (solve any two on number line)-

(i) $\frac{2}{3}+\frac{5}{4}$ (ii) $-2\frac{1}{9}+7$ (iii) $\frac{-7}{16}+\frac{(-3)}{48}$

(iv) $\frac{-7}{63}+\left(\frac{-5}{21}\right)$ (v) $\frac{-2}{13}+\frac{(-1)}{7}$ (vi) $4\frac{3}{5}-\left(-2\frac{1}{3}\right)$

3. Multiply the following rational numbers-

(i) $\frac{13}{15}\times 5$ (ii) $\frac{4}{-5}\times\frac{-5}{4}$ (iii) $\frac{-2}{5}\times\left(\frac{-3}{7}\right)$

(iv) $\frac{15}{18}\times\frac{5}{6}\times\frac{21}{5}$ (v) $\frac{9}{4}\times\left(\frac{-7}{5}\right)\times\left(\frac{-6}{21}\right)$ (vi) $2\frac{1}{9}\times\left(-3\frac{1}{2}\right)$

4. Find the value-

(i) $(-6)\div\frac{3}{5}$ (ii) $\frac{-27}{5}\div\left(\frac{-54}{10}\right)$ (iii) $\frac{21}{36}\div\left(\frac{-7}{18}\right)$

(iv) $\frac{-7}{12}\div\left(\frac{-3}{13}\right)$ (v) $-2\frac{1}{9}\div 6\frac{1}{9}$ (vi) $\frac{2}{15}\div\left(\frac{-8}{45}\right)$

5. Find the value-

(i) $\frac{3}{5}+\frac{7}{10}+\left(\frac{-8}{12}\right)+\frac{4}{3}$ (ii) $2\frac{1}{2}+\left(-3\frac{1}{2}\right)+\left(-2\frac{1}{3}\right)+\left(2\frac{1}{9}\right)$

(iii) $\left(\frac{-7}{5}\right)\times\frac{2}{3}\times\frac{15}{16}\times\left(\frac{-8}{9}\right)$ (iv) $\frac{1}{2}\div\left[\left(\frac{-1}{3}\right)\div\frac{2}{7}\right]$

6. Find the value of the following using the appropriate properties-

(i) $\frac{3}{5}\times\left(\frac{-3}{7}\right)-\frac{2}{7}\times\frac{3}{2}+\frac{3}{15}\times\frac{5}{9}$ (ii) $\frac{5}{2}-\frac{3}{5}\times\frac{7}{2}+\frac{3}{5}\times\left(\frac{-2}{3}\right)$

7. Find the additive inverse of the following rational numbers –

(i) $\frac{7}{19}$ (ii) $\frac{-9}{5}$ (iii) $\frac{-3}{-7}$

(iv) $\frac{5}{-9}$ (v) $\frac{-13}{-17}$ (vi) $\frac{21}{-31}$

8. Find the multiplicative inverse of the following rational numbers-

(i) -17 (ii) $\frac{-11}{17}$ (iii) $-1\times\frac{-3}{5}$ (iv) $\frac{13}{-19}$

9. Multiply the rational number $\frac{5}{7}$ to inverse of $\frac{-7}{15}$.

10. Fill in the blanks-

(i) Product of two rational numbers is always...............(rational/integers).

(ii) Additive inverse of any negative rational number is .... (negative/positive).

(iii) Inverse of zero is...(zero/ indetermined).

(iv) Additive identity of rational number is.......................(zero/one).

(v) Multiplicative identity for rational number is....................(zero/one).

(vi) Reciprocal of rational number is.............of that (inverse/same).

(vii) Negative rational number on number line is always lies on .... of zero (right/left).

(viii) Positive rational number on number line is always lies on .... of zero (right/left).

(ix) When rational number is added with its additive inverse then result is always...............(zero/same).

(x) When rational number is divided by same rational number then result is always...............(zero/one).

11.By mean method-

(i) Write any five rational numbers between -3 and 0.

(ii) Write any four rational numbers larger than 0 and smaller than $\frac{5}{6}$.

(iii) Find any three rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$.

We Learnt

1. When two rational numbers having common denominators are added or subtracted then their numerators are added while keeping the denominator same.
2. To add or subtract rational numbers with distinct denominators , LCM are taken of their denominators.
3. For product of rational numbers, numerators are multiplied by numerators and denominators are multiplied by numerators.
4. To divide rational number by any other rational number, that rational number is multiplied by inverse of any other rational number.
5. Product of any rational number with its inverse is always 1.
6. Rational numbers are closure under addition, subtraction and multiplication operations.
7. Addition and multiplication operations are commutative and associative for rational numbers.
8. For rational numbers, 0(zero) is additive identity and 1(one) is multiplicative identity.
9. Additive inverse of rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa. Similarly, multiplicative inverse of rational number $\frac{a}{b}$ is $\frac{b}{a}$ and vice-versa.
10. Distributive law-
    If a, b and c are rational numbers then-

    $$a \times (b + c) = a \times b + a \times c$$
    $$a \times (b - c) = a \times b + a \times c$$

11. There are so many rational numbers between any two rational numbers. Rational numbers between two rational numbers can be determined by the mean method.

Chapter 2

Cube and Cube root

2.1 Cube and Cube root

Take some cubes from your mathematics kit and pay attention, you will find that all the cubes have equal length, breadth and height. By adding or arranging these cubic blocks, bigger cubes can also be made. But remember this; these cubes should also have equal lengths, breadth and height.

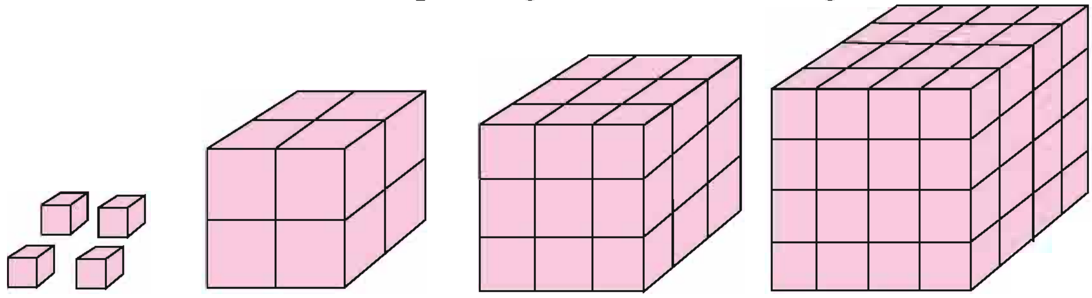


Figure 2.1

Sr. No.	Number of unit cubes in one side of large cube	Number of unit cubes in making of large cubes
1	1	1
2	2	8
3	3	27
4	4	---------------
5	5	---------------

Table 2.1

Now consider the numbers 1,8,27............. These numbers are called perfect cube or cube numbers. Can you tell, why this name is given to these numbers? These numbers are obtained only when this number is multiplied three times by the same number. Let us see

$$1 = 1 \times 1 \times 1 = 1^3$$
$$8 = 2 \times 2 \times 2 = 2^3$$
$$27 = 3 \times 3 \times 3 = 3^3$$

Since $5^3 = 5 \times 5 \times 5 = 125$, that is why 125 is a perfect cube. 9 is not a cube number because $9 \neq 3 \times 3 \times 3$ and there is no such natural number by which we can obtain 9 by multiplying itself three times. We know that $2 \times 2 \times 2 = 8$ and $3 \times 3 \times 3 = 27$.This clarifies that 9 is not a perfect cube. Cubes of 1 to 10 numbers are given below. Also, find the cube and fill in the blanks:-

Cube numbers	Numbers
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 = 27$
4	$4^3 = 4 \times 4 \times 4 = 64$
5	$5^3 = 5 \times 5 \times 5 =$ .....
6	$6^3 = 6 \times 6 \times 6 =$ .....
7	$7^3 = 7 \times 7 \times 7 =$ .....
8	$8^3 = 8 \times 8 \times 8 =$ .....
9	$9^3 = 9 \times 9 \times 9 =$ .....
10	$10^3 = 10 \times 10 \times 10 =$ .....

Table 2.2

We know that $2^2 = 4$, where $4 = 2 \times 2$ or $2 + 2$. Similarly, $2^3 = 8$, where $8 = 2 \times 2 \times 2$. Is it equal to $(2 + 2 + 2)$?

2.1.1 Cube of even and odd numbers

It is clear that there is only 10 cube between 1 to 1000. Look at the cube of even numbers and odd numbers in table 2.2. You will find that cube of even number is always even number and cube of odd number is always odd number.

2.1.2 Unit digit of cube numbers

Consider the unit digits of cubes in the above table. How many numbers have unit digit 1 in their cubes? Which numbers have same unit digit of cube that contain in base numbers. You will find that when any number has digit 0,1,4,5,6 at their unit place then the unit of cube also contain the same digit .

Do and Learn

1. Find the unit digit of cubes given below:

(i) 1331 (ii) 4444 (iii) 159 (iv) 1005

2. The cube of 46 will be even or odd ?

2.2 Some patterns related to cube Numbers

2.2.1 To add the successive odd numbers:

Look at the format of an addition of odd numbers

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
1
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$

By seeing this pattern, tell that how many successive odd numbers will be required to obtain the 10^3 by addition?

Do and Learn

• According to the above pattern, find the below in form of addition of odd numbers.

(i) 7^3 (ii) 8^3

2.2.2 Cube and their prime factors

Consider on the prime factors of some numbers and their cubes.

Number	Cube Number
4 = 2 x 2	4^3 = 64 = 2 x 2 x 2 x 2 x 2 x 2 = $2^3 \times 2^3$
6 = 2 x 3	6^3 = 216 = 2 x 2 x 2 x 3 x 3 x 3 = $2^3 \times 3^3$
10= 2 x 5	10^3 = 1000 = 2 x 2 x 2 x 5 x 5 x 5 = $2^3 \times 5^3$
12= 2 x 2 x 3	12^3 = 1728 =..............................

On the basis of the above pattern, it is clear that all the prime factors of cube numbers can be presented in the group of three similar factors.

Example 1 Is 729 a perfect cube number? Think.

Solution By prime factorisation of 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$$729 = \underbrace{3 \times 3 \times 3}_{} \times \underbrace{3 \times 3 \times 3}_{}$$

You can see that in prime factor two groups of triples can be made. Therefore 729 is a perfect cube.

Example 2 Is 432 a perfect cube?

Solution Resolving 432 into prime factor, we have

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$$432 = \underbrace{2 \times 2 \times 2}_{} \times 2 \times \underbrace{3 \times 3 \times 3}_{}$$

Clearly, in grouping the same factor in triples factor 2 is left over. Thus, 432 is not a perfect cube.

Example 3 Is 5400 a perfect cube? If not, then find the smallest natural number by which 5400 must be multiplied so that the product is a perfect cube.

Solution Resolving 5400 into prime factor, we get

$$5400 = \underbrace{2 \times 2 \times 2}_{} \times \underbrace{3 \times 3 \times 3}_{} \times 5 \times 5$$

We find that the prime factor of 5400 can be grouped into triples of 2 and 3 but 5 occurs as a prime factor only twice. Therefore, 5400 is not a perfect cube.

If we multiply 5400 by 5, 5 will also occur as a prime factor thrice and the product will be perfect cube. Thus, 5 is the smallest natural number.

Example 4 Is 1188 a perfect cube? If not, then find the smallest natural number by which 1188 must be divided so that the quotient is a perfect cube.

Solution Resolving 1188 into prime factor, we get

$$1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$$

Prime factor 2 and 11 are not in a group of triples. Therefore, 1188 is not a perfect cube. In the above factorization of 1188, 2 occur twice and 11occurs single. So, if we divide 1188 by 2 x 2 x 11 then 2 and 11 will not occur in prime factor of quotient. Thus, 44 is the smallest natural number by which 1188 must be divided for the perfect cube number.

Hence, Resultant perfect cube = $1188 \div 44 = 27 = 3^3$

i.e., after grouping the prime factors in triple, product of remaining prime factors are divided to obtain the perfect cube number.

Do and Learn

- Check the perfect cube in following numbers

(i) 2700	(ii) 16000	(iii) 64000	(iv) 900
(v) 125000	(vi) 36000	(vii) 21600	(viii) 10000
(ix) 27000000	(x) 11000		

Exercise 2.1

1. Which of the following numbers are not perfect cube?

   (i) 512 (ii) 243 (iii) 1000 (iv) 100 (v) 2700

2. Find out the smallest number multiplied by the following numbers to get the perfect cube?

   (i) 108 (ii) 500 (iii) 5400 (iv) 10584

3. Find out the smallest number by which following numbers must be divided to get the perfect cube?

   (i) 108 (ii) 500 (iii) 5400 (iv) 10584

4. Rehan works in soap factory. He is playing with arranging cubic soap by making cubes. If he has to arrange 216 soap then how many soaps will occur in first line of cube.

2.3 Cube Root

In the beginning of this chapter, we made large cubes by arranging cubic blocks of mathematic kit. Recall this action and tell that how many blocks are there on one side of cube made of 125 blocks? As we can find the side length by making cubes but, this work can also be done by using cube root.

To determine the square root is the reverse process of finding square similarly, to find out cube root is also a reverse process of finding cube.

As we know that $2^3 = 8$ that is why we can say that 2 is the cube root of 8. It is represented by $\sqrt[3]{8}$ Symbol of cube root is $\sqrt[3]{\ }$

Consider the following table:

Cube	Cube root	Cube	Cube root
$1^3 = 1$	$\sqrt[3]{1}$	$6^3 = 216$	$\sqrt[3]{216}$
$2^3 = 8$	$\sqrt[3]{8}$	$7^3 = 343$	$\sqrt[3]{343}$
$3^3 = 27$	$\sqrt[3]{27}$	$8^3 = 512$	$\sqrt[3]{512}$
$4^3 = 64$	$\sqrt[3]{64}$	$9^3 = 729$	$\sqrt[3]{729}$
$5^3 = 125$	$\sqrt[3]{125}$	$10^3 = 1000$	$\sqrt[3]{1000}$

Table 2.3

2.3.1 Determine cube root by prime factor method:

To determine the cube root of 1728 by prime factor method

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Resolving the given number into prime factor

$$1728 = \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times \underbrace{3 \times 3 \times 3}$$

Grouping the factors in triples of equal factors, we get

$1728 = 2^3 \times 2^3 \times 3^3 = (2 \times 2 \times 3)^3$

By taking cube root on both side, we get

$$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$$

Example 5 Find the cube root of 17576.

Solution Prime factors of 17576

$17576 = 2 \times 2 \times 2 \times 13 \times 13 \times 13$

$\sqrt[3]{17576} = 2 \times 13 = 26$

Example 6 Find the cube root of 9261 by prime factor method.

Solution Prime factors of $9261 = \underbrace{3 \times 3 \times 3} \times \underbrace{7 \times 7 \times 7}$

So $\sqrt[3]{9261} = 3 \times 7 = 21$

2.3.2 Cube root of a perfect cube(by observation method)

If you know that the given number is a perfect cube then the following steps are used to find the cube root of this number:

Step-1 Suppose 175616 is a cube, starting from the right side make, a group of 3-3 digits. We get.

$$\underset{\text{Second group}}{\underline{175}} \quad \underset{\text{First group}}{\underline{616}}$$

Cube roots of any given cube can be estimated step by step process. Here we get two groups 616 and 175 of three digits.

Step -2 By first group 616, we obtain 6 as the unit digit of cube root because the last digit of 616 is 6. We know that 6 occur at unit place when cube root has 6 as an unit digit ($6^3 = 216$).

Step-3 Now consider the second group 175. 175 occurs between $5^3 = 125$ and $6^3 = 216$.

i.e., $5^3 < 175 < 6^3$

So the digit on second place will be 5

Thus, $\sqrt[3]{175616} = 56$

Example 7 Find the cube root of 13824 by estimation method.

Solution Given 13824

Step-1 Starting from the right create groups of 3-3 digits we get.

$\underline{13} \quad \underline{824}$

Step-2 First group is 824 and its first digit is 4 which can only obtain by cube of unit digit 4 ($4^3 = 64$). Therefore, 4 will be the unit digit.

Step-3 Consider the second group 13 which occurs between

$2^3 = 8$ and $3^3 = 27$.

i.e., $2^3 < 13 < 3^3$

So the digit on second place will be 2.

Thus, the cube root of given number will be 24.

$\sqrt[3]{13824} = 24$

Exercise 2.2

1. Determine True/ False in the given statements.
   (i) Every even number has even cube.
   (ii) A perfect cube does not end with double zero(00).
   (iii) No one perfect cube end with 8.
   (iv) If square of any number is ending with 5 then its cube is end with 25.
   (v) Cube of single digit is also a single digits number.
   (vi) Cube of double digit number is of 4 to 6 digits.
2. Find the cube roots of the following numbers by estimate and prime factor method. Verify your answer-

   (i) 64 (ii) 343 (iii) 5832 (iv) 74088
   (v) 3375 (vi) 10648 (vii) 46656 (viii) 91125

We Learnt

1. Cube of a number is that number raised to the power 3., That means when a number multiplies thrice to itself.
2. Power 3 of any number is equal to the cube of that number. E.g., $2^3 = 2 \times 2 \times 2 = 8$
3. Cube of even number is always even number and cube of odd number is always odd number.
4. Numbers which ends with 0,1,4,5,6, cube of that number also has same unit digit.
5. In Cube number some prime factor occurs in triples, so the group of factors in triples can be created.
6. To find out cube root is an inverse operation of finding out cube.
7. Cube root of any perfect cube can be determined by the prime factor method.
8. Cube root of any large number can be calculated by estimate method by making group of triples from the right side.

Chapter 3

Powers and Exponents

3.1 In earlier class, we have learnt about exponential form of numbers. Let us recall these numbers:

$$10^3, 2^{10}, 5^5$$

How to express these numbers in extended form? Let us try to do this.

$10^3 = 10 \times 10 \times 10$

$2^{10} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$5^5 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

With this, we also learned that $10^2 \times 10^4 = 10^{2+5} = 10^7$

and $\dfrac{2^5}{2^3} = 2^{5-3} = 2^2$

i.e., when two numbers, having the common base, are multiplied then powers of their bases are added and on dividing, powers are subtracted.

$$a^m \times a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \text{ when } m > n$$

and $(a^m)^n = a^{mn}$

In this chapter, we shall study about the other problems related to the exponents.

3.2 Exponent (Integers), Base (Rational numbers $\neq 0$)

Consider the following exponent of rational numbers.

1. $\left(\frac{5}{7}\right)^4 = \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}$

   $\frac{5 \times 5 \times 5 \times 5}{7 \times 7 \times 7 \times 7} = \frac{5^4}{7^4}$

2. $\left(\frac{-3}{11}\right)^5 = \left\{(-1) \times \left(\frac{3}{11}\right)\right\}^5 = (-1)^5 \times \left(\frac{3}{11}\right)^5$

   $= (-1) \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \qquad [\because (-1)^5 = -1]$

   $= -\frac{3^5}{11^5}$

3. $\left(\frac{-4}{3}\right)^6 = (-1)^6 \times \left(\frac{4}{3}\right)^6$

$= \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3}$ $\quad [\because (-1)^6 = 1]$

$= \frac{4^6}{3^6}$

So, if we have any rational number $\left(\frac{5}{4}\right)^m$, then

$$\left(\frac{5}{4}\right)^m = \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \text{----------- (m times)}$$

$$= \frac{5 \times 5 \times 5 \times \text{----------- m times}}{4 \times 4 \times 4 \times \text{----------- m times}} = \frac{5^m}{4^m}$$

Do and Learn: Extend the following

$$\left(\frac{3}{2}\right)^3, \left(\frac{9}{4}\right)^5, \left(-\frac{4}{7}\right)^6, \left(-\frac{2}{5}\right)^3, \left(\frac{2}{3}\right)^p$$

If $\left(\frac{p}{q}\right)$ is any rational number raised to the power m, where $q \neq 0$, then

$$\left(\frac{p}{q}\right)^m = \frac{p}{q} \times \frac{p}{q} \times \frac{p}{q} \times \text{------ (m times)}$$

$$= \frac{p \times p \times p \times \text{----- (m times)}}{q \times q \times q \times \text{----- (m times)}} = \frac{p^m}{q^m}$$

i.e., $\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$ where, p and q are integers and $q \neq 0$

Now, if the power of rational number is negative then what would be the condition? Let us consider the following examples:

(i) $\left(\frac{5}{4}\right)^{-2}$

$= \frac{5^{-2}}{4^{-2}}$

$= \frac{\frac{1}{5^2}}{\frac{1}{4^2}}$

$= \frac{4^2}{5^2}$

$= \left(\frac{4}{5}\right)^2$

(ii) $\left(\frac{3}{7}\right)^{-4}$

$= \frac{3^{-4}}{7^{-4}}$

$= \frac{\frac{1}{3^4}}{\frac{1}{7^4}}$

$= \frac{7^4}{3^4}$

$= \left(\frac{7}{3}\right)^4$

(iii) $\left(\frac{2}{5}\right)^{-m}$

$= \frac{2^{-m}}{5^{-m}}$

$= \frac{\frac{1}{2^m}}{\frac{1}{5^m}}$

$= \frac{5^m}{2^m}$

$= \left(\frac{5}{2}\right)^m$

$\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ and } a^{-m} = \frac{1}{a^m}\right]$

Do and Learn: Express the following with positive exponent-

$$\left(\frac{7}{5}\right)^{-5}, \left(\frac{14}{13}\right)^{-9}, \left(\frac{15}{6}\right)^{-4}, \left(\frac{113}{53}\right)^{-11}, \left(\frac{5}{7}\right)^{-7}$$

Rethink on $\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$

Similarly, it is clear that

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

Here a and b are integers. And

$$a \neq 0 \text{ , } b \neq 0$$

Look at the following actions

$$5^4 \div 5^4 = 5^{4-4} = 5^0$$

$$5^4 \div 5^4 = \frac{5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 1$$

so $5^0 = 1$

Thus, any base with power 0 (zero) the result is always 1. For example,

i.e. (i) $(3)^4 \div (3)^4 = 3^{4-4} = 3^0 = 1$

(ii) $(-5)^6 \div (-5)^6 = (-5)^{6-6} = (-5)^0 = 1$

(iii) $\left(\frac{2}{5}\right)^3 \div \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^{3-3} = \left(\frac{2}{5}\right)^0 = 1$

From the above, it is clear that any number (except 0) with raised to the power 0, result is always 1.

If a is any rational number, then $a^0 = 1, (a \neq 0)$

Do and Learn:

Simplify the following-

(i) $\left(\frac{2}{7}\right)^{-3}$ (ii) $\left(\frac{3}{10}\right)^{-2}$ (iii) $\left(\frac{5}{12}\right)^{-3}$

(iv) $(3)^2 \div (3)^2$ (v) $(2)^5 \div (2)^5$

Ex. 1 : Find the value of $7^8 \div 7^8$

Sol. : $7^{8-8} = 7^0 = 1$

Ex. 2 : Find the value of $\left(\frac{4}{7}\right)^5 \div \left(\frac{4}{7}\right)^5$

Sol.

$\left(\frac{4}{7}\right)^5 \div \left(\frac{4}{7}\right)^5$

$\left(\frac{4}{7}\right)^{5-5} = \left(\frac{4}{7}\right)^0 = 1$

Ex. 3 : Find the value of $(2^3)^2$

Sol.

$(2^3)^2$

$= 2^{3\text{x}2}$

$= 2^6$

$(2^3)^2$	$(2^3)^2$
$= 2^3 \times 2^3 = 2^{3+3} = 2^6$ $= 64$	$= 2^{3 \text{ x } 3}$ $= 2^9 = 512$

Ex. 4: Simplify the following:

1. $\left(\frac{5}{7}\right)^4 \times \left(\frac{7}{5}\right)^2$

Sol.

$= \left(\frac{5}{7}\right)^4 \times \left(\frac{7}{5}\right)^2$

$= \left(\frac{5}{7}\right)^4 \times \left(\frac{5}{7}\right)^{-2}$

$= \left(\frac{5}{7}\right)^{4+(-2)}$

$= \left(\frac{5}{7}\right)^2$

$= \frac{5^2}{7^2}$

$= \frac{25}{49}$

2. $\left(-\frac{2}{9}\right)^{-4} \times \left(\frac{9}{2}\right)^2$

Sol.

$= \left(-\frac{2}{9}\right)^{-4} \times \left(\frac{9}{2}\right)^2$

$= \left(-\frac{9}{2}\right)^4 \times \left(\frac{9}{2}\right)^2$

$= (-1)^4 \times \left(\frac{9}{2}\right)^4 \times \left(\frac{9}{2}\right)^2$

$= 1 \times \left(\frac{9}{2}\right)^{4+2}$

$= \left(\frac{9}{2}\right)^6$

$= \frac{531441}{64}$

Exercise 3.1

1.Simplify the following:

(i) $\left(\frac{2}{7}\right)^3 \times \left(\frac{1}{2}\right)^3$ (ii) $\left(\frac{4}{5}\right)^4 \times \left(\frac{5}{4}\right)^2$

(iii) $(-5)^3 \times \left(-\frac{1}{5}\right)^2$ (iv) $\left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^{-5}$

2. Find the value.

(i) $(-5)^3$ (ii) $\left(\frac{1}{2}\right)^3$ (iii) $\left(-\frac{2}{3}\right)^4$

3. With the help of prime factor, change the following into exponent form.

(i) $\frac{1}{64}$ (ii) $\frac{16}{125}$ (iii) $-\frac{8}{27}$

(iv) $-\frac{1}{8}$ (v) $-\frac{25}{49}$

4. Find the value.

(i) $3^2 \times 3^3$ (ii) $\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$ (iii) $\left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^3$

(iv) $\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{2}\right)^4$ (v) $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{2}{5}\right)^3$

5. Answer in exponent form.

(i) $4^5 \div 4^2$ (ii) $(-5)^7 \div (-5)^4$

(iii) $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^4$ (iv) $\left(-\frac{1}{5}\right)^{11} \div \left(-\frac{1}{5}\right)^6$

6. Find the value.

(i) $(3^2)^3$ (ii) $(2^3)^2$ (iii) $(5^2)^2$

(iv) $(-2^4)^2$ (v) $\left[\left(\frac{1}{2}\right)^2\right]^4$ (vi) $\left[\left(-\frac{1}{3}\right)^3\right]^2$

7. Find the value.

(i) 3^0 (ii) 7^{5-5} (iii) $(-2)^{3-3}$

(iv) $\left(\frac{2}{5}\right)^{2+3-5}$ (v) $2^0 \times 3^0$ (vi) $2^0 + 5^0$

(vii) $\left(\frac{7}{15}\right)^0 + \left(\frac{1}{7}\right)^{3-3}$

8. Change into positive exponent numbers.

(i) 2^{-3} (ii) 3^{-5} (iii) a^{-4} (iv) $(-2)^{-5}$

(v) $(-x)^{-3}$ (vi) $\frac{1}{5^{-3}}$ (vii) $\frac{1}{y^{-3}}$ (viii) $\frac{1}{\left(\frac{2}{3}\right)^{-3}}$

9. Simplify the following in form of exponent.

(i) $(2^2 \times 3^3)^2$ (ii) $\left(\frac{15}{16}\right)^3 \div \left(\frac{9}{8}\right)^2$ (iii) $\left(\frac{4}{9}\right)^2 \div \left(\frac{28}{27}\right)^3$

(iv) $\left(\frac{2}{3}\right)^2 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^2$ (v) $\left(\frac{5^2}{3^2}\right)^2$ (vi) $\left[\frac{2^2 \times 3^2}{2^3 \times 6^2}\right]^2$

10. Find the value of $\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

3.3 Questions of more than one operations:

Ex. 5 Solve $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

Sol. $\left\{\left(\frac{3}{1}\right)^{2} - \left(\frac{2}{1}\right)^{3}\right\} \div \left(\frac{4}{1}\right)^{2}$

$= (3^2 - 2^3) \div 4^2$

$= (9 - 8) \div 16$

$= \frac{1}{16}$

Ex. 6 Solve $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Sol. $= \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \left(\frac{2+1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \frac{\cancel{3}^{1}}{\cancel{8}_{4}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = \frac{1}{4}$

Ex. 7 If $(-2)^{x+1} \times (-2)^3 = (-2)^5$ then find the value of x.

Sol. $(-2)^{x+1} \times (-2)^3 = (-2)^5$

Or $(-2)^{x+1+3} = (-2)^5$

Or $(-2)^{x+4} = (-2)^5$

Since, the bases are same. Therefore, powers are put equivalent.

$x + 4 = 5$

Or $x = 5 - 4 = 1$

Ex. 8 Solve $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Sol.

$$\frac{3^{-5} \times (2 \times 5)^{-5} \times (5 \times 5 \times 5)}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$= \frac{5^{-5} \times 5^3}{5^{-7}}$$

$$= \frac{5^{-5+3}}{5^{-7}} = \frac{5^{-2}}{5^{-7}} = 5^{-2+7}$$

$$= 5^5 = 3125$$

Ex. 9 Find the value of $\left(\frac{9}{8}\right)^{-3} \times \left(\frac{8}{9}\right)^{-2}$

Sol.

$$\left(\frac{9}{8}\right)^{-3} \times \left(\frac{8}{9}\right)^{-2}$$

$$= \left(\frac{8}{9}\right)^{3} \times \left(\frac{9}{8}\right)^{2}$$

$$= \frac{8^3}{9^3} \times \frac{9^2}{8^2}$$

$$= \frac{8^3}{8^2} \times \frac{9^2}{9^3}$$

$$= \frac{8^{3-2}}{9^{3-2}} = \frac{8}{9}$$

One more method

$$\left(\frac{8}{9}\right)^{3} \times \left(\frac{8}{9}\right)^{-2}$$

$$(a^m \times a^n = a^{m+n})$$

$$= \left(\frac{8}{9}\right)^{3-2} = \left(\frac{8}{9}\right)^{1} = \frac{8}{9}$$

Exercise 3.2

1. Find the value

(i) $(5^{-1} \times 2^{-1}) \div 6^{-1}$ (ii) $\left(\frac{5}{6}\right)^{6} \times \left(\frac{5}{6}\right)^{-4}$ (iii) $\left(\frac{5}{8}\right)^{-2} \times \left(\frac{8}{5}\right)^{-5}$ (iv) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{0}$

2. Simplify

(i) $\dfrac{16^{-1} \times 5^3}{2^{-4}}$ (ii) $\dfrac{25 \times t^{-4}}{5^{-3} \times 5 \times t^{-8}}$, $(t \neq 0)$

(iii) $\dfrac{6^3 \times 7^4 \times 8^5}{4^3 \times 9^2 \times 16}$ (iv) $\dfrac{15^3 \times 18^2}{3^5 \times 5^4 \times 12^2}$ (v) $\left(\dfrac{6}{15}\right)^3 \div \left(\dfrac{25}{32}\right)^2 \times \left(\dfrac{45}{16}\right)^3$

3. Find the value of x

(i) $\left(\dfrac{4}{3}\right)^{-4} \times \left(\dfrac{4}{3}\right)^{-5} = \left(\dfrac{4}{3}\right)^{-3x}$ (ii) $7^x \div 7^3 = 7^5$ (iii) $4^{2x+1} \div 16 = 64$

4. Find the value

(i) $\dfrac{3125 \times 1296}{6561 \times 1875}$ (ii) $\dfrac{1536 \times 972}{486 \times 1152}$

$\left[\text{Hint } \dfrac{3125 \times 1296}{6561 \times 1875} = \dfrac{5^5 \times 2^4 \times 3^4}{3^8 \times 3 \times 5^4}\right]$

3.4.1 Scientific Notation

In the previous classes, we have studied how large numbers are written in its standard form.

(i) $3,00,000 = 3 \times 1,00,000 = 3 \times 10^5$

(ii) $15,00,00,000 = 15 \times 1,00,00,000 = 1.5 \times 10^8$

(iii) $78,00,00,00,000 = 78 \times 1,00,00,000 = 7.8 \times 10^{10}$

Similarly,

(iv) $0.1 = \dfrac{1}{10} = \dfrac{1}{10^1} = 10^{-1}$

(v) $0.01 = \dfrac{1}{100} = \dfrac{1}{10^2} = 10^{-2}$

(vi) $0.0001 = \dfrac{1}{10000} = \dfrac{1}{10^4} = 10^{-4}$

As in previous class, we represented large numbers in standard form easily, can small numbers be written in standard form like this?

Ex. -Diameter of Red Blood Cell = 0.0000007 metre

Diameter of Wire of computer chip = 0.0000003 metre

In above example we have seen $0.0001 = \dfrac{1}{10000} = \dfrac{1}{10^4} = 10^{-4}$

Like this $0.0000007 = \dfrac{7}{10000000} = \dfrac{7}{10^7} = 7 \times 10^{-7}$

$0.0000003 = \dfrac{3}{10000000} = \dfrac{3}{10^7} = 3 \times 10^{-7}$

Other example $0.0000058 = \frac{58}{10000000} = \frac{58}{10^7} = \frac{5.8 \times 10}{10^7}$

$= 5.8 \times 10^1 \times 10^{-7} = 5.8 \times 10^{-6}$

Similarly, small numbers can easily be expressed in standard form.

Ex. 10 Express 150000000 in standard form.

Sol. $15 \times 10^7 = \frac{15}{10} \times 10^1 \times 10^7$

$= 1.5 \times 10^8$

Note:
In large number
1.50000000 decimal shift to 8 place on left side.

In small number
0.0000005 decimal shift to 7 place on right side.

Ex. 11: Express the measurement of virus 0.0000005 metre in standard form.

$= \frac{5}{10000000} = \frac{5}{10^7}$

$= 5 \times 10^{-7}$

Ex. 12 Express the following numbers in normal form :-

(i) 2.43×10^6 (ii) 9.3×10^{-5} (iii) 3×10^{-6}

Sol. (i) $2.43 \times 10^6 = 2.43 \times 10{,}00{,}000 = 2430000$

(ii) $9.3 \times 10^{-5} = \frac{9.3}{10^5} = \frac{9.3}{100000} = 0.000093$

(iii) $3 \times 10^{-6} = \frac{3}{10^6} = \frac{3}{1000000} = 0.000003$

Do and Learn

1. Express the following numbers in standard form

(i) 20700000 (ii) 0.000000154 (iii) 0.000095
(iv) 28400000 (v) 0.00002459

2. Express the following numbers in normal form

(i) 1.5×10^5 (ii) 2.78×10^3 (iii) 3.9×10^{-5}

3.4.2 Comparison of largest numbers with smallest numbers

If the mass of Earth and the Moon is 5.97×10^{24} kg and 7.35×10^{22} kg respectively, then how much mass of the earth is more than the moon.

On subtracting $= (5.97 \times 10^{24}\,\text{kg}) - (7.35 \times 10^{22}\,\text{kg})$

$= (5.97 \times 100 \times 10^{22}\,\text{kg}) - (7.35 \times 10^{22}\,\text{kg})$

$= 10^{22}(597 - 7.35)\,\text{kg}$ (by taking 10^{22} as common factor)

$= 10^{22} \times 589.65\,\text{kg}$

The Earth has $10^{22} \times 589.65$ kg mass more.

Similarly, if the distance between the Sun and the Earth is 1.496×10^{11} meter and the distance between the Earth and the Moon is 3.84×10^{8} meter, then the difference between these distances is

$= (1.496 \times 10^{11}\,\text{m}) - (3.84 \times 10^{8}\,\text{m})$

$= (1.496 \times 1000 \times 10^{8}\,\text{m}) - (3.84 \times 10^{8}\,\text{m})$

$= (1.496 \times 1000 - 3.84)\,10^{8}\,\text{m}$

$= (1496 - 3.84)\,10^{8}\text{m}$

$= 1492.16 \times 10^{8}\,\text{m}$

Note: When we subtract the numbers written in standard form then we change these in equal power of 10.

Comparison of the small numbers:

Size of a red blood cell $= 0.0000007\,\text{m} = 7 \times 10^{-6}\,\text{m}$

Size of a plant cell $= 0.00001275\,\text{m} = 1.1275 \times 10^{-5}\,\text{m}$

Difference between these $= (1.275 \times 10^{-5} - 7 \times 10^{-6})\,\text{m}$

$= (1.275 \times 10^{-5} - 7^{-1} \times 10^{-5})\,\text{m}$

$= (1.275 \times 0.7) \times 10^{-5}\,\text{m}$

$= 0.575 \times 10^{-5}\,\text{m} = 5.75 \times 10^{-6}\,\text{m}$

Compare it by dividing =

$$\frac{\text{Size of a red blood cell}}{\text{Size of a plant cell}} = \frac{1.275 \times 10^{-5}\,\text{m}}{7 \times 10^{-6}\,\text{m}}$$

$$\frac{1.275 \times 10^{-5-(-6)}}{7} = \frac{1.275 \times 10^{1}}{7} = \frac{12.75}{7} \cong 2 \text{ (approx. less than 2)}$$

Comparison of large number by division.

Diameter of the sun is (1.4×10^9)m and the diameter of the earth is (1.2756×10^7) m. Let us compare their diameters

$$\frac{\text{Diameter of the sun}}{\text{Diameter of the earth}} = \frac{(1.4 \times 10^9)\text{ m}}{(1.2756 \times 10^7)\text{ m}} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 10^2}{1.2756}$$

$$= \frac{1.4 \times 100}{1.2756} \text{ (which is approx 100 times)}$$

Exercise 3.3

1. Change in standard form
   (i) 128000000 (ii) 1680000000 (iii) 0.0005
   (iv) 0.00000017 (v) 0.000000000397 (vi) 0.00000004358

2. Express the following numbers in normal form
   (i) 4×10^9 (ii) 245×10^7 (iii) 5.61729×10^7
   (iv) 8.5×10^{-6} (v) 3.02×10^{-6} (vi) 7×10^{-4}
3. Change the numbers in standard form of the following statements -
   (i) Diameter of the thickness of human hair is 0.0002cm approx.
   (ii) Charge on an electron is 0.000,000,000,000,000,00016 coulomb.
   (iii) 1 Micron $= \frac{1}{100000}$ meter.

   (iv) Thickness of a paper = 0.0016 cm

We Learnt

1. $(-1)^{\text{even no}} = 1$ and $(-1)^{\text{odd no}} = -1$.

2. If $\frac{p}{q}$ is any rational number, then $\boxed{\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}}$

3. If $\frac{a}{b}$ is any rational number, then $\boxed{\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m}$

4. If a is a rational number other than 0, then $a^0 = 1$
5. By using the negative exponent, the smallest numbers can be expressed in standard form.

Chapter

4 Mental Exercise

4.1 Numbers came into the existence since man started counting. Human tendency has created a huge world of numbers by playing with numbers . We have studied about the numbers, their operations and their nature like even, odd, divisible and prime etc. We have also learned about the place value E.g. 5 is used three times in 555 but every 5 has specific value due to its respective place. You would also have played many calculative games like cross word, Sudoku etc. In this chapter we shall play some new games with numbers and try to know the hidden Mathematics behind it.

4.2 Expanded Form of Numbers

We consider the two digit numbers

$$27 = 20 + 7$$
$$= 2 \times 10 + 7 \times 1$$
$$69 = 60 + 9$$
$$= 6 \times 10 + 9 \times 1$$
$$90 = 90 + 0$$
$$= 9 \times 10 + 0 \times 1$$

Note: Numbers are converted in decimal system by multiplying units digit to 1 and tens digit to 10.

This is the Decimal system of numbers.

It is clear that if algebraic digit a is at tens place and b is at units place then the value will be-

$$10 \times a + b = 10a + b$$

$ab = 10 \times a + b = 10a+b$

$56 = 5 \times 10 + 6 = 50 + 6$

In two digits number ab, face value of 'a' lies between 1 to 9 and face value of 'b' lies between 0 to 9.

In numbers $ab \neq a \times b$

e.g. 56 cannot be written like 5×6.

Now, let us consider the three digit numbers. These can be written like this-

$$345 = 300 + 40 + 5 = 3 \times 100 + 4 \times 10 + 5 \times 1$$
$$467 = 400 + 60 + 7 = 4 \times 100 + 6 \times 10 + 7 \times 1$$
$$600 = 600 + 0 + 0 = 6 \times 100 + 0 \times 10 + 0 \times 1$$

In expanded form, number abc made of any three digits a, b and c can be written in this way

$$abc = a \times 100 + b \times 10 + c \times 1$$
$$= 100a + 10b + c$$

Here a, b and c are algebraic digits. a is non-zero digit, b and c can be any whole number from 0 to 9.

(non-zero $\rightarrow$ any digit between 1 to 9 ,except 0)

If 'a' becomes zero then the generalised form will be 10b +c

Do and Learn:

1. Fill in the blanks:
   (i) $42 = \square \times 10 + 2$
   (ii) $60 = \square \times 10 + \square$
   (iii) $99 = \square \times \square + \square$
   (iv) $\square = 7 \times 100 + 1 \times 10 + 8$
2. Write the following numbers in generalised form:
   (i) $10 \times 5 + 6$
   (ii) $8 \times 100 + 0 \times 10 + 5$
   (iii) $9 \times 100 + 9 \times 10 + 9$

4.3 Playing with Numbers

4.3.1 Interchange the Digits- two digit number

(i) Conversation of class 7 students, Cheeku and Chhotu.

Cheeku	Chhotu
Think any two digit number	(69) thought of
Interchange its digits	(96) interchanged
Add both numbers	Yes, (69+96 = 165) done
Divide the obtained number by 11	$\left(\frac{165}{11}\right) = 15$
Now, remainder is 0	Wow, but how did you know?

Now understand the cleverness of Cheeku. Assume Chhotu think the number ab. In decimal system we can write it 10a + b. Interchanging the digits 10b + a.

On addition of both- $10a + b + 10b + a$

$= 11a + 11b = 11(a+b)$

Obtained number is always multiple of 11. Remainder is always 0 and quotient is always (a+b).E.g.,

$69 + 96 = 165$

$a = 6$

$b = 9$

$a + b = 6 + 9 = 15$

Here, $\frac{165}{11} = 15$

So $11 \times 15 = 165$

You can also play this game with your friends.

(ii) Cheeku, play another game with Chhotu.

Again think about another two digit number (digits would not be same).

Cheeku	Chhotu
Think any two digit number	68 thought of
Interchange its digits	86 interchanged
Subtract the first discussed number	Yes, 86-68 = 18 done
Divide obtain number by 9	$\frac{18}{9} = 2$
Now, remainder is 0	Wow, but how did you know?

In general form, $ab = 10a + b$

Reversing $= 10b + a$

Subtracting both $= 10a + b - (10b + a)$

$= 10a - b - 10b - a$

$= 9a - 9b$

$= 9(a-b)$

Quotient $= 8 - 6 = 2$ (larger number - smaller number)

Do and Learn:

What will be the result if you think the following numbers

(i) 27 (ii) 67 (iii) 94

4.3.2. Interchanging the digits- three digit number

(i) Chhotu says Ramu to think about three digit numbers.

Chhotu	Ramu
Think any three digit number	I thought 149
Interchange its digits and make a new number	Ok I interchanged 941
Subtract the small number from large number	Yes, 941-149=792 done
Now divide the obtain number by 99	$\frac{792}{99}=8$
Now, remainder is 0	Yes, ofcourse.

How Chhotu's cleverness works? Let us see.

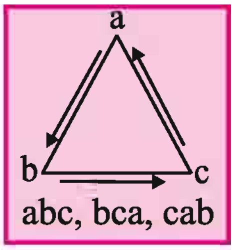
Three digit number $= abc = 100a + 10b + c$

Reversing digits $= cba = 100c + 10b + a$

Subtracting $= 100a + 10b + c - (100a + 10b + c)$

$= 100a + 10b + c - 100c - 10b - a$

$= 99a - 99c = 99(a-c)$



If $a = c$ then difference is 0.

Therefore, units digit and hundreds digit must not be equal.

(ii) Making three digit number by the given three digits.

Let us play a game.

Chintu - Chhotu think about any 3-digit number.

Chhotu - Ok. Done.

Chintu - Now by these digits of this number make another two 3-digit number in a such way-

Like - If you select abc then, first number cab

(i.e., units digit is now on hundreds place)

and second number bca (i.e., in cab units digit is on hundreds place).

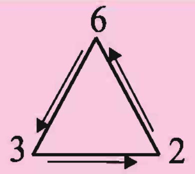
Now add these numbers and divide the resultant by 37.

Chhotu - Remainder will be 0.

Chintu - Yes, you are right.

E.g.

$$\begin{array}{r} 632 \\ 263 \\ +\ 326 \\ \hline 1221 \\ \hline \end{array}$$



632, 326, 263

How this cleverness works?

abc = 100a + 10 b + c
cab = 100 c + 10a + b
bca = 100 b + 10 c + a
abc + cab + bca
= 111 (a+b+c)
= 37×3 (a+b+c)

Remainder will be 0. $\frac{1221}{37} = 33$. Quotient = 33 and remainder = 0.

Do and Learn:

Test if Chhotu thinks the following numbers then what would be the result?

(i) 237 (ii) 119 (iii) 397 (iv) 435

4.4 Rules of Divisibility (In Algebraic Reference)

Previously, we learned about the divisibility in arithmetic reference now we discuss it in algebraic reference.

(i) **Divisibility by 2:** In class-VI we studied about the divisibility rule of 2.

A number is divisible by 2 if its units digit is even i.e., 0,2,4,6 or 8 .

Clearly, we can write any number.........cba in form of ....+100c +10b +a all next term will be divisible by 2 because they are coefficient of 10,100,1000.......i.e, the units digit in these numbers is always 0. If a number is divided by 2, its completely depend on numbers unit place that is 'a' in.... cba. given number is divisible by 2 if a= 0,2,4,6, or 8

Do and Learn:

If N is any number

		Remainder	Units Digit	
			Even	Odd
(1)	N ÷ 2	1	-	✓
(2)	N ÷ 2	0		
(3)	N ÷ 3	1		
(4)	N ÷ 3	0		

Divisibility by 2, 5 and 10 of any number can be find out by its units digit. i.e., only units digit is used other digits does not effect the divisibility. Unit digit is important in place value system.

(ii) **Divisibility by 3 and 9:** If sum of the digits of a natural number is divisible by 3, then the number is divisible by 3.

For ex. Consider the number 3576

The sum of the digits is = 3 + 5 + 7 + 6 = 21

21 is divisible by 3. Thus, 3576 is divisible by 3. Its extended form

$$= 3 \times 1000 + 5 \times 100 + 7 \times 10 + 6 \times 1$$
$$= 3 \times (999+1) + 5 \times (99+1) + 7 \times (9+1) + 6 \times 1$$
$$= 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 6)$$

Since, $3 + 5 + 7 + 6 = 21$ is not divisible by 9 but divisible by 3, therefore 3576 is not divisible by 9 but divisible by 3.

(i) A number is divisible by 9 if the sum of the digits of the number is divisible by 9.

(ii) A number is divisible by 3 if the sum of the digits of the number is divisible by 3.

If cba is any number then

$$100c + 10b + a = (99+1)c + (9+1)b + a$$
$$= 99c + 9b + (a+b+c)$$
$$= 9(11c+b) + (a+b+c)$$

Divisibility by 9 and 3 is only possible when (a+b+c) is divisible by 9 and 3 respectively.

Ex. 1 Test the divisibility of 3148569 by 9.

Sol: Sum of the digits of 3148569 is

$$= 3+1+4+8+5+6+9 = 36$$

which is divided by 9.

Is 36 divided by 3?

Test

$$= \frac{3148569}{9}$$
$$= 349841$$

completely divisible

Ex. 2 If three digit number 34A is divisible by 9 then what will be the value of A?

Sol: Sum of the digits

$= 3+4+A$

$= 7+A$ which should be divisible by 9.

This is possible only when 7+A is 9 or 18.

Since, A is a digit, therefore, $7+A = 9$,

$A = 9 - 7 = 2$

Do and Learn:

If 79y is divisible by 9, then is it possible more than one value of y?

(iii) **Divisibility by 11:** To test the divisibility of 11 in any number then multiply the units, tens, hundreds, thousands, ten thousand, lakhs, ten lakhs.....digits of that number by 1,-1,1,-1,1,-1,1......respectively. Then sum of these multiplication is determined. If this sum is completely divisible by 11 it means that number is divisible by 11.

Ex. 3 Test the divisibility of 11 in number 89,62, 426.

Sol: As per the above rule,

$= 6 \times 1 + 2\times(-1) + 4 \times 1 + 2\times(-1) + 6 \times 1 + 9\times(-1) + 8 \times 1$

$= 6 - 2 + 4 - 2 + 6 - 9 + 8$

$= 11$ which is divisible by 11.

Hence, 89, 62, 426 is divisible by 11.

Now taking general form of 4-digit number abcd

$M = 1000a + 100b + 10c + d$

As per the rule,

$N = d\times1 + c\times(-1) + b\times1 + a\times(-1)$

$= -a+b-c+d$

$M - N = (1000a+100b+10c+d) -(-a+b-c+d)$

$= 1001a + 99b + 11c$

$= 11(99a + 9b + c)$, which is divisible by 11.

If M is divisible by 11 then N will also be divisible by 11.

Reversible, if N is divisible by 11 then M will also be divisible by 11.

Do and Learn:

Test the divisibility of 59, 29, 003 by 11.

Exercise 4.1

1. If 3-digit number 24x is divisible by 9 then find the value of x. Where x is a digit.
2. If 3-digit number 89y is divisible by 9 then find the value of y.
3. 31M5 is a multiple of 9 and two values are obtained by M. Why? Where M is a digit.
4. If 3-digit number 24y is a multiple of 3, then find the value of y.
5. Test the divisibility of following numbers by 3,9 and 11

   (i) 294 (ii) 4455 (iii) 1041966
6. If R = 4 in number 31R1 then by the rule of divisibility find that this number is divisible by 11.
7. If 31P5 is a multiple of 3 then find the value of P, where P is a digit.

4.5 To Determine eliminated digits in Four Basic Operations (+, -, x, and ÷)

(i) Observe the following addition operation:

$$\begin{array}{r} 2\,* \\ 4\,6 \\ +\,9\,7 \\ \hline 166 \\ \hline \end{array}$$

Note that in the above addition operation, symbol * shows that digit is missing which we have to find out. Now the addition of units digits

$= * + 6 + 7$

$= * + 13$

$= * + 10 + 3$

$= 10 + (* + 3)$

Thus, in addition the units digit $= * + 3 = 6$

$* = 6 - 3$

$= 3$

See again-

$$\begin{array}{r} 45 \\ + \ *8 \\ + \ 78 \\ \hline 171 \\ \hline \end{array}$$

Here the sum of units digit is = 5+8+8 = 21

Where, 2 is tens digit. Therefore, in addition operation, it will be transferred on tens place left side in form of carry. Thus, sum of tens digit-

$= 2+4+*+7$

$= * + 13$

Which is equivalent of 17

$* + 13 = 17$

$* = 17 - 13$

$= 4$

Do and Learn : Find the value of * in following addition-operations:

(i)
$$\begin{array}{r} 3* \\ + \ 57 \\ + \ 34 \\ \hline 127 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 56 \\ + \ 77 \\ + \ *3 \\ \hline 216 \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} 443 \\ +*57 \\ +128 \\ \hline 928 \\ \hline \end{array}$$

(iv)
$$\begin{array}{r} 82 \\ + \ 55 \\ + \ 99 \\ \hline *36 \\ \hline \end{array}$$

(ii) Consider the following subtraction- operation

$$\begin{array}{r} 83 \\ - \ *2 \\ \hline 51 \\ \hline \end{array}$$

In this subtract-operation, on tens place-

$8 - * = 5$

Where * shows the number

So $* = 8 - 5 = 3$, Thus $* = 3$

See again

$$\begin{array}{r} 83 \\ - \ 2* \\ \hline 55 \\ \hline \end{array}$$

Here * is at units place and by subtracting * from 3 we get 5. So probably value of * is more than 3. Therefore, to complete the subtraction- process we will have to transfer tens on right side from tens place to units place.

So $(10+3) - * = 5$

$\therefore \quad * = (10+3) - 5 = 8$

It is clear that after transferring tens from tens place to units place, remaining tens is 8 - 1 = 7

Thus, subtracting 7 - 2 = 5 which is given.

Do and learn

(i) $\begin{array}{r} 76 \\ -\ 5* \\ \hline 25 \end{array}$ (ii) $\begin{array}{r} 54 \\ -\ 2* \\ \hline 28 \end{array}$ (iii) $\begin{array}{r} 84 \\ -\ *8 \\ \hline 16 \end{array}$ (iv) $\begin{array}{r} 803 \\ -2*6 \\ \hline 567 \end{array}$ (v) $\begin{array}{r} 782 \\ -\ *73 \\ \hline 209 \end{array}$

(iii) Observe the following multiplication operation

Note that here $46 \times 2\boxed{x} = 1104$

In multiplication, here x is on units place and we have to find out x.

$$46 \times 2\boxed{x} = 46 \times (20+x)$$
$$= 920 + 46x = 1104$$

So,
$$920 + 46x = 1104$$
$$46x = 1104-920$$
$$46x = 184$$
$$x = \frac{184}{46} \quad \text{or} \quad x = 4$$

Again look at the following example.

$$\begin{array}{r} \boxed{x}5 \\ \times\ 37 \\ \hline 3145 \end{array}$$

Here $\boxed{x}$ is on tens place and we have to find out the value of x

$$x\ 5 \times 37 = (10x + 5) \times 37$$
$$= 370x + 185$$

So,
$$370x + 185 = 3145$$
$$370x = 3145 - 185$$
$$370x = 2960$$
$$x = \frac{2960}{370} \quad \text{or} \quad x = 8$$

Do and Learn:

Find the value of Algebraic expression in the following multiplication- operation:

(i) $\begin{array}{r} 56 \\ \times\ \boxed{x}5 \\ \hline 1400 \end{array}$ (ii) $\begin{array}{r} 4\boxed{x} \\ \times\ 37 \\ \hline 1554 \end{array}$ (iii) $\begin{array}{r} 23 \\ \times\ 3\boxed{x} \\ \hline 736 \end{array}$

(iv) Observe the following division- operation

$x3)\,78\,(6$
$--$
0

In this, x is at tens place in divisor and we have to find the value of x

As we know by the division-operation

Divisible = Divisor × Quotient + Remainder

$78 = x\,3 \times 6 + 0$

$78 = (10x + 3) \times 6 \quad [\because\ x\,3 = 10x + 3\,]$

$78 = 60x + 18$

$78 - 18 = 60x$

$60 = 60x$

$x = \frac{60}{60}$ or $x = 1$

Thus 13 is a divisor.

Observe again $2\,\boxed{x})\,229\,(9$
$---$
4

Where x is at units place in divisor and we have to find the value of x. By the division - operation, $2\,x \times 9 + 4 = 229$

$(20 + x) \times 9 = 229 - 4$

$180 + 9x = 225$

$9x = 225 - 180$

$9x = 45$

$x = \frac{45}{9}$ or $x = 5$

Do and Learn: Find the value of x in following division- operation.

(i) $27)\,217\,(x$ / $---$ / 1 (ii) $x\,6)\,100\,(6$ / $---$ / 4 (iii) $1x)\,120\,(9$ / $---$ / 3

4.6 Some other ways:

(i) Alphabets for Numerals

Here are some puzzles where in arithmetic questions, alphabets are used in place of digits and problem is to find that which alphabet represent which digits? Generally, to solve these kind of puzzles, we use the following guidelines:

1. In puzzle, every alphabet should represent one digit.
2. First digit of any number should not be zero. For example, number fifty six should not written as 56 not 056 or 0056 .

[A] Find the value of P in following addition;

$$\begin{array}{r} 52P \\ +\ 1P3 \\ \hline 711 \\ \hline \end{array}$$

Sol: Here we have to find the value of P

In unit column, we get 1 from unit column P+3 i.e., we have to add a number in 3 so that we can get unit number 1. For this, value of P must be 8 because while adding 3 in 8 we get unit number 1.

$$\begin{array}{r} 528 \\ +\ 183 \\ \hline 711 \\ \hline \end{array}$$

$\therefore$ P = 8

[B] Find the value of P and Q.

$$\begin{array}{r} QP \\ \times\ Q6 \\ \hline 62P \\ \hline \end{array}$$

Sol: Here P and Q are alphabets for which we have to find the value. Since in 6 x P unit digit is P.

So, P = 4 (why?)

If we put Q = 1, i.e., 14 × 16 = 224 which is less.

If we put Q = 3, i.e., 34 × 36 = 1224 which is much greater.

If we put Q = 2, i.e., 24 × 26 = 624 which is appropriate.

Hence,

$$\begin{array}{r} 24 \\ \times\ 26 \\ \hline 624 \\ \hline \end{array}$$

$\therefore$ P = 4 and Q = 2

Exercise 4.2

1. Find the value of alphabets and show the causes of that process.

(i) $\begin{array}{r} 5A \\ +\ 34 \\ \hline B2 \\ \hline \end{array}$ (ii) $\begin{array}{r} 5A \\ +\ 79 \\ \hline CB3 \\ \hline \end{array}$ (iii) $\begin{array}{r} AB \\ +\ 37 \\ \hline 6A \\ \hline \end{array}$ (iv) $\begin{array}{r} 5AB \\ +AB1 \\ \hline B98 \\ \hline \end{array}$

(v) $\begin{array}{r} 12A \\ +6AB \\ \hline A09 \\ \hline \end{array}$ (vi) $\begin{array}{r} 1\,A \\ \times\ A \\ \hline 9\,A \\ \hline \end{array}$ (vii) $\begin{array}{r} AB \\ \times\ B \\ \hline CAB \\ \hline \end{array}$ (viii) $\begin{array}{r} AB \\ \times\ 6 \\ \hline BBB \\ \hline \end{array}$

2. Find the value of x or (*) in following questions

(i) $\begin{array}{r} 2\,* \\ +\ *\,8 \\ +\ 9\,5 \\ \hline 1\,6\,7 \\ \hline \end{array}$ (ii) $\begin{array}{r} 905 \\ +*12 \\ +88* \\ \hline 2100 \\ \hline \end{array}$ (iii) $\begin{array}{r} 7*3 \\ -281 \\ \hline 432 \\ \hline \end{array}$

(iv) $\begin{array}{r} 5\,7 \\ -\ 3\,* \\ \hline 1\,8 \\ \hline \end{array}$ (v) $\begin{array}{r} 68 \\ \times\ \boxed{x} \\ \hline 408 \\ \hline \end{array}$ (vi) $\begin{array}{r} 7\,6\,3 \\ \times\ 3\boxed{x} \\ \hline 25942 \\ \hline \end{array}$

(vii) $2x\,)216(\,8$
$\begin{array}{r} --- \\ \hline 0 \\ \hline \end{array}$

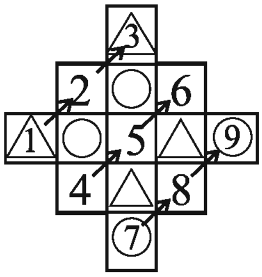
(viii) $x7\,)907(\,24$
$\begin{array}{r} --- \\ \hline 19 \\ \hline \end{array}$

4.7 Crossword

[A] Let us consider some crosswords. First we take 3×3 crossword. Which is filled by digits 1 to 9.

2	7	6
9	5	1
4	3	8

How much is the sum of horizontal and vertical squares?
Are these values equal?
How numbers are filled in these squares?
Let us find out. Draw a square above the middle square as shown in figure and fill the number according to the direction of an arrow(→→). Now added squares are filled like digit of Δ in Δ and ◯ in ◯ in the reverse direction.



[B] Let us consider another crossword. Here we change the digits diagonally like △ by △ , ○ by ○ , ◇ by ◇ and ⌂ by ⌂ . Then what will be the sum of horizontal and vertical squares? Find out.

This is 4×4 crossword.

△1	2	3	○4
5	⌂6	◇7	8
9	◇10	⌂11	12
○13	14	15	△16

If 4×4 crossword is filled by digit 2 to 17 then what will be the sum?

Do and Learn

1. Fill the 3×3 crossword by following digits in such a way that the sum of the horizontal, vertical and diagonal squares remain same.
   (i) 2 to 10 (ii) 5 to 13 (iii) 11 to 19
2. Fill the 4×4 crossword by following digits in such a way that the sum of the horizontal, vertical and diagonal squares remain same.
   (i) 11 to 26 (ii) 5 to 20 (iii) 2 to 17

We Learnt

1. To write and understand the generalise form of numbers. To write and understand two digit numbers ab in form of $10a + b$ and abc in form of $100a + 10b + c$ where a, b and c are numerals between 0 to 9 and $a \neq 0$.
2. Generally, for 2 and 3 digit numbers 2,3,5,9,10, 11 derive divisibility rule.
3. To know the logic related to the divisibility rules of numbers 2,3,5,9,10, 11.
4. Numbers related puzzles and games.

Chapter 5

Vedic Mathematics

5.1 In this chapter, you will be able to learn the multiplication of numbers of two and three digits by the Urdhwtiryak formula, multiplication of three numbers under the Nikhilam base and sub-base formula, and cubage and division by Dhawajank method.

5.1.1. Multiplication Operation (Urdhwtirgbhyaam Formula)

Product can be easily done by any method based on Urdhwtiryak formula. In this word Urdhwtiryak, the meaning of first word Urdhw is the just above i.e. the product of upper and below digits and the meaning of last word Tiryak is skew i.e. product of diagonal digits.

Example 1: Let us multiply 32 and 14.

Step-1: Multiplicand 32 and multiplier 14 can be written for multiplication in this way,

$$\begin{array}{r} 32 \\ \times 14 \\ \hline \end{array}$$

Step-2: Create Group-There will be three groups in multiplication of two digits by two digits which is shown by III, II and I.

III	II	I
3	3 2	2
↑	⤫	↑
1	1 4	4

Note: While multiplying, number of groups can be estimated by the (2n - 1) formula. Where n is the maximum digit number in multiplicand and multiplier. In this 32 and 14, the maximum digit is 2. So 2 × 2-1= 3 groups.

Step-3: Multiplication action-

Step-4: Product $3\times1 \,/\, 3\times4+1\times2 \,/\, 2\times4$

Step-5: $3 \,/\, 12+2 \,/\, 8$

Step-6: $3 \,/\, 14 \,/\, 8$

Step-7: Addition

$$\begin{array}{lccc} \text{Line 1} & 3 & 4 & 8 \\ \text{Line 2} & 1 & & \\ \hline & 4 & 4 & 8 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{ccc} 3 \,/ & 4 \,/ & 8 \\ +1 & & \\ \hline 4 & 4 & 8 \\ \hline \end{array} \quad (3+1=4)$$

Thus, the exact product of 32 and 14 is 448.

Note: Each group contains single digit. Group II contains two digit number 14 so tens digit 1 of 14 will be added in

Example 2: Multiplication of 123 and 45 by Urdhwtirgbhyaam method.

Sol:

Step-1: 123 is multiplicand and 45 is multiplier. For making multiplier of three digits it will be written 045.

$$= \begin{array}{r} 123 \\ \times 045 \\ \hline \end{array}$$

Step-2: Number of groups will be 5 in three digit numbers which is shown by V, IV, III, II and I.

V	IV	III	II	I
1	1 2	1 2 3	2 3	3
0	0 4	0 4 5	4 5	5

Step-3: Multiplication action and Product:

Multiplication action $1\times0 / 1\times4 + 2\times0 / 1\times5 + 3\times0 + 2\times4 / 2\times5 + 3\times4 / 3\times5$

Product $0 \;/\; 4+0 \;/\; 5+0+8 \;/\; 10+12 \;/\; 15$

Addition $0 / 4 / 13 / 22 / 15$

Step-4: There are 2 digits in first, second and third part but due to the base 10, only one digit is kept in each part. Therefore, the unit digit of first, second and third part is written in first row and tens digit is written in second row (a one place ahead). After adjusting the each row, sum of these digits are made.

Step-5: Addition:

$$\begin{array}{lcccc} \text{Line 1} & 4 & 3 & 2 & 5 \\ \text{Line 2} & 1 & 2 & 1 & - \\ \hline & 5 & 5 & 3 & 5 \\ \hline \end{array}$$

So exact product of 123 × 45 = 5535

Example 3: Multiply 57 × 68.

Number of group = 3

$$\begin{array}{c} 57 \\ 68 \\ \hline 5\times6 / 5\times8 + 6\times7 / 7\times8 \\ 30 \;/\; 40+42 \;/\; 56 \\ 30 \;/\; 82 \;/\; 56 \\ +8 \quad +5 \\ 3876 \end{array}$$

(Where in 82+ 5=87, 7 is add on tens and 8 is further add on 30 and resultant 38)

So the exact product is 3876.

Example 4: Multiply 349 × 986.

Sol: number of group = 5

$$\begin{array}{r} 3\ 4\ 9 \\ \times\ 9\ 8\ 6 \\ \hline \end{array}$$

$$3\times9 / 3\times8 + 4\times9 / 3\times6 + 9\times9 + 4\times8 / 4\times6 + 9\times8 / 9\times6$$

$$27 / 24 + 36 / 18 + 81 + 32 / 24 + 72 / 54$$

$$27 / 60 / 131 / 96 / 54$$

By arranging and adding the numbers in queues.

1	7	0	1	6	4	Line-1
2	6	3	9	5		Line-2
	1					Line-3
3	4	4	1	1	4	

Thus, the exact multiplication of number is 349 × 986 = 344114

Do and Learn:

1.Multiplying the following by Urdhwtirgbhyaam method.

(i) 15 × 12 (ii) 60 × 18

(iii) 71 × 8 (iv) 122 × 4

(v) 706 × 56 (vi) 497 × 173

5.1.2 Multiplication by NIkhilam Formula (sub-base)

By Nikhilam formula, large deviation can be obtained of any problem but the multiplication of them is very difficult. Suppose in 32 × 34, if we take base as a 10 then the product of its deviation 22 and 24 is larger.

Therefore, selection of sub-base is done on the basis of approximate tens so that deviation can be obtained smaller than sub-base. For example, in 68 × 66 we take sub-base 7 × 10 = 70 and in 32 × 27 we take sub-base 3×10 =30 and remaining method is like previous method.

Example 5: find the value of 32 × 34. Hint:

Step-1: Number Deviation

Number	Deviation
32	+2
34	+4
/	

(i) Base = 10, Sub base = 3 × 10 = 30

(ii)Sub base digit = Sub base ÷ Base = 30 ÷ 10 = 3

(iii)Deviation from sub base = number - base

= 32 - 30 = +2

= 34 - 30 = +4

Step-2: Solution will be in two parts

32	+2
34	+4
/	8

(Multiplication of deviation in right side = 2×4 = 8)

Step-3: In left side adding any number and the deviation of remaining numbers and multiplying the sum by subbase digit 3.

$$\begin{array}{lr} 32 & +2 \\ 34 & +4 \\ \hline 108\ / & \end{array}$$

(where $32+4=36$ or $34+2=36$ and $3\times 36=108$)

Step-4: Consolidating from step 1 to 3 and arranging by operating

$$\begin{array}{lr} 32 & +2 \\ 34 & +4 \\ \hline \end{array}$$

$= 108 / 8$

$= 1088$ (Removing skew line)

Thus the exact product of 32×34 is 1088.

Example 6: Solve 54×57.

Sol: Let us use step - 4 from the previous method

Number	Deviation
54	+4
57	+7

$5\{(54+7)$ or $(57+4)\} / 4\times 7$

$= 5\times 61 / 28$

$= 305 /_{2} 8$

$= 3078$

Hint:
(I) Base = 10, Sub base = $5\times 10=50$
(ii) Sub base digit = 5
(iii) Deviation from sub base = +4 and +7
(iv) In base 10, single 0 is there so there will be single digit in right side.
(v) Adding the tens digit in left side = $305+2=307$

Example 7: Solve 78×76.

Sol:

Number	Deviation
78	-2
76	-4

$=8\times(78-4) / -2\times -4$

$=8\times 74 / 8$

$=592 / 8$

$=5928$

Hint:
(I) Base 10, Sub base = $8\times 10=80$
(ii) Sub base digit = 8
(iii) Deviation from sub base = -2 and -4

Example 8: Solve 63 × 58.

Sol: Number Solution

$$\begin{array}{cc} 63 & +3 \\ \underline{58} & \underline{-2} \end{array}$$

$= 6\times(63-2) / +3\times -2$

$= 6\times(61) / -6$

$= 366 / -6$

$= 365+1 / -6$

$= 365 / 10-6$

$= 365 / 4$

$= 3654$

Hint:

(i) Base = 10 and Sub base= 6 × 10= 60

(ii) Sub base digit = 6

(iii) Deviation from sub base = (63- 60 = +3 and 58 – 60 = -2) = +3 and -2

(iv) Multiplication of deviation in right side = 3 × -2 = -6 which is negative therefore convert it into positive, taking 1 tens = 10 from left side in unit side, we get 10 – 6 = 4.

(v) Thus, product = 3654

Do and Learn: ◆ Find the product of the following:

(i) 11 × 15 (ii) 12 × 18 (iii) 19 × 17

(iv) 28 × 22 (v) 51 × 49 (vi) 99 × 96

5.2 Multiplication of Three Numbers

5.2.1 Formula Nikhilam (Base)

By Nikhilam formula, product of three numbers can be easily calculated in which deviation is same as base 10 or relative the power of 10.
Let us try to understand it.

Example 9: Multiply 12 × 13 × 17.

Step 1: Base of 12 × 13 × 17 = 10 and take deviation +2, +3, +7

Number	Deviation
12	+2
13	+3
17	+7

Step 2: Solution will be in three parts which will be known by right part, middle part and left part respectively. These three deviations are multiplied in right part.

Number	Deviation
12	+2
13	+3
17	+7

$/ \; 2\times 3\times 7$

Step 3: In middle part, products of two deviations and their addition are made. In deviation step, 2×3, 3×7 and 7×2.

$$\begin{array}{cc} 12 & +2 \\ 13 & +3 \\ 17 & +7 \\ \hline \multicolumn{2}{c}{/\ 2\times3+3\times7+2\times7\ /} \end{array}$$

Step 4: In left part, any single number and deviation of remaining two numbers i.e., we take any one out of 12 +3+7 or 13+2+7 or 17+3+2.

Number	Deviation
12	+ 2
13	+ 3
17	+ 7

$12 + 3 + 7 = 22$ /
or
$13 + 2 + 7 = 22$
or
$17 + 3 + 2 = 22$

Step 5: Consolidate step 1 to 4 and arranging by operation.

Number	Deviation
12	+2
13	+3
17	+7

(If base is 10 then middle part is multiplied by 1 and left part is multiplied by 1^2.)

$$1^2\,(12+3+7) / 1\,(2\times3+3\times7+2\times7) / 2\times3\times7$$

$$= 22 / 6 + 21 + 14 / 42$$

$$= 22 / 41 / 42$$

$$= 22 \underset{+4}{/} 1 \underset{+4}{/} 2$$

$$= 2\ 6\ 5\ 2$$

(There is only zero in base 10 therefore there will be single digit in right part and middle part. Second will be carried in next term. For example, in 42, 4 will be in middle and 2 will be right side. 4 will be added in 41 and it will be 45 in which 5 will be in middle and 4 will be added in 22 in left side.)

Thus the exact product of $12\times13\times17 = 2652$

Example 10: Find out multiplication of 9×8×15 by Nikhilam (base 10) formula.

Sol:

Number	Deviation
9	-1
8	-2
15	+5

Same base = 10
Base = 10, Deviation = -1
Base = 10, Deviation = -2
Base = 10, Deviation = +5

Direct use of previous step-5

Number	Deviation
9	-1
8	-2
15	+5

$$1^2\,(15-2-1)/1\{5\times(-2)+(-2)\times(-1)+(5)\times(-1)\}/\,(-1)\times(-2)\times(5)$$

$$12\,/-10\ +2-5/\,10$$

(Due to base 10, middle term is multiplied by sub base 1 and left term is multiplied by sub base 1^2).

$$12\,/-15+2/\ 10$$

$$10+2/-13/\,10$$

(12 = 10 + 2)

$$10/20\ -\ 13/10$$

Taking 2 from left side, in middle we get 2 × 10 = 20 and write 20 -13 = 7.

$$10/7/_{1}0$$

$$1080$$

Example-11: Multiply 22×23×24 by Nikhilam formula (sub base).

22	+2
23	+3
24	+4

$$2^2\,(22+3+4)/\,2\{2\times3+3\times4+4\times2\}/\ 2\times3\times4$$

$$=\ 2^2\,(29)\,/2\,(6+12+8)/\,24$$

$$=\ 4\,(29)/\,2\,(26)/\,24$$

$$=\ 116/_{5}2/_{2}4$$

$$=\ 12144$$

Base = 10, Sub base = 10 × 2 = 20, sub base digit = 2
Deviation from sub base = +2, +3, +4
Due to sub base 2 ×10, multiply middle term to sub base digit 2 and multiply left term to sub base digit 2^2.

Thus the exact multiplication of 22×23×24 will be = 12144.

Example 12: Multiply 101 ×102 ×103 by formula Nikhilam.

Sol:

101	+01	Same base = 100, (Deviation = +01)
102	+02	(Deviation = +02)
103	+03	(Deviation = +03)

$$101+2+3 \,/\, 2\times3+1\times2+3\times1 \,/\, 06$$
$$= 101+2+3 \,/\, 6+2+3 \,/\, 06$$
$$= 106/11/06$$
$$= 1061106$$

If base is 100 then there will be two digits in right and middle term because there are two zero in 100 so it will be written right side 06 in place of 6.

Example 13: Multiply 99×98×97 by formula Nikhilam.

Sol:

99	- 01	Same base = 100, (Deviation = -01)
98	- 02	(Deviation = -02)
97	- 03	(Deviation = -03)

$$99-02-03 \,/\, (-02)\times(-03)+(-01)\times(-02)+(-03)\times(-01) \,/\, (-01)\times(-02)\times(-03)$$
$$= 94 \,/\, 6+2+3 \,/\, \overline{06}$$
$$= 94/11/\overline{06}$$
$$= 94/10\,/\overset{1}{\overline{06}}$$

From the middle part, 1 will be taken in right side in form of 1 × 100 = 100.

$$= 94/10/100-6$$
$$= 941094$$

Do and Learn:

Multiply the following three numbers by formula Nikhilam:

(i) 11 × 12 × 13 (ii) 8 × 9 × 10

(iii) 6 × 7 × 8 (iv) 27 × 28 × 29

(v) 98 × 99 × 99 (vi) 51 × 52 × 53

5.3 Cube

1, 8, 27, 64..... are cube numbers which are obtained only multiplying by the three times with the same number. E.g.

$$1 \times 1 \times 1 = 1$$
$$2 \times 2 \times 2 = 8$$
$$3 \times 3 \times 3 = 27$$
- - - - - - - - -

Cube numbers are represented by raising to the power 3. E.g. cube of 2 will be 2^3 and 3 will be 3^3. We have studied the multiplication of three numbers by formula Nikhilam. To find out cube, that process will be repeated. Here all three numbers are same.

Example-14: Multiply 11×11×11 by formula Nikhilam.

Sol:

Base = 10

11	+1	(Deviation = +1)
11	+1	(Deviation = +1)
11	+1	(Deviation = +1)

$$11+1+1 \;/\; (1)\times(1)+(1)\times(1)+(1)\times(1) \;/\; (1)\times(1)\times(1)$$

i.e., number + 2 × (deviation) / 3 (deviation)^2 / (deviation)^3 is written in standard form. Now we will solve by using standard form.

Example 15: Find the cube of 15.

Sol: In 15, number is 15 and deviation is 5 then,

$$(15)^3 = 15 + 2 \times 5 / 3 \times 5^2 / 5^3$$
$$= 15 + 10 / 75 / 125$$
$$= 25 / 75 / 125$$

Arranging numbers line wise and adding them,

I	2	5	5	5
II		7	2	-
III		1	-	-
	3	3	7	5

or

$$25/75/5$$
$$25/8\,7/5$$
$$= 33/7/5$$
$$= 3375$$

Thus the multiplication of number 15×15×15 will be =3375.

Example 16: Find the cube of 103.

Sol: Base = 100, Deviation = +03

$103^3 = 103 + 2 \text{ x } 3 / 3 \times (03)^2 / (03)^3$

$= 103 + 6 / 27 / 27$

$= 1092727$

Example 17: Find the cubage of 96.

Sol: Base = 100, Deviation = -04

$96^3 = 96 + 2 \times (-04) / 3\times(-04)^2 / (-04)^3$

$= 96 - 08 / 3\times16 / -64$

$= 88 \quad 48 \quad -64$

$= 88 / 48 / 1\text{-}64$ (1 is in place of hundreds so the mean of 1 is 100 unit)

$= 88 / 47 / 100\text{-}64$

$= 88 / 47 / 36$

$= 884736$

5.4 Dhwajank Method:

This application is based on formula Urdhwtiryak and formula Dhwajank. By this method, every problem of division operation can be solved easily. Following points are kept in mind while writing the problem before the operation start:

(i) First the divisor is divided in two parts. Unit part of divisor is called Dhwajank and remaining part is called Cardinal or modified divisor. In dhwajank, there can be so many unit digits.

(ii) As similar the previous methods of division operation, in this method also, a set location is divided in to three parts.

(a) In first part, we take both parts of divisor. Cardinal is written below i.e. in place of base and dhwajank is written its above i.e., in place of exponent.

(b) Digits of dhwajank are same as final digit in third part of dividend and remaining digit will be written in middle part.

(iii) For dhwajank, 529 ÷23 can be written as the following:

First part	Middle part		Last part
2 ^{3}	5	2	9
	Quotient		Remainder

Method:

(i) For middle part, dividing the just left digit of dividend by cardinal a first digit which we get is written below the horizontal line at set place of quotient.

(ii) Obtained remainder is written from the left side below and before the second digit which is now new dividend.

(iii) From new dividend we get modified dividend by the following formula:
Modified dividend = New dividend - Quotient digit × Dhwajank

(iv) Previous actions are repeated if cardinal is divided into modified dividend. Completing the division process, we get remainder and quotient.

This method can be explained by the following examples.

Example 18: Divide 23 in to 552 by Dhwajank method.

Sol:

2 ^{3}	5 $_{1}$5	$_{1}$2
	2 4	0

Hint:

(i) Middle part 5 is divided by 2.

(ii) First digit of quotient 2 is written below the horizontal line.

(iii) Remainder = 1 written before the middle part 5 and obtained new dividend = 15.

(iv) Modified dividend = New dividend - First quotient × Dhwajank

$$= 15 - 2 \times 3 = 9$$

(v) 9 divided by cardinal 2. Second digit 4 of quotient is written below the horizontal line.

(vi) Remainder = 1, written before the third part 2 and obtained new dividend = 12.

(vii) Modified dividend = New dividend - Second quotient × Dhwajank

$$= 12 - 4 \times 3 = 0$$

(viii) Thus, quotient = 24, remainder = 0

Example 19: Divide 4096 by 64 (Dhwajank method).

Sol:

	4	40 $_{4}9$	$_{1}6$
6			
		6 4	

(i) Dividing 6 in dividend = 40, then quotient = 6 and remainder = 4.

(ii) Modified dividend = 49 - 6 × 4 = 25.

(iii) 25 is divided by cardinal 6. Second digit 4 of quotient will be written below the horizontal line.

(iv) Remainder = 1 is written before the third part 6 and get new dividend 16.

(v) Again modified dividend = 16 - 4 × 4 = 0

(vi) Thus, quotient = 64, remainder = 0.

Example 20: Divide 87653 by 53 (use Dhwajank method).

Sol:

	3	8 $_{3}7$ $_{4}6$ $_{3}5$	$_{5}3$
5			
		1 6 5 $\cancel{4}$ 3	53-3×3= 44

Hint:

(i) 5 is divided in to middle part 8

(ii) First digit 1 of quotient is written below the horizontal line.

(iii) Remainder 3 is written below and before the middle part 7.

(iv) New dividend = 37

(v) Modified dividend = 37 - 1 × 3 = 34

(vi) 34 is divided by cardinal 5. Second digit 6 of quotient will be written below the horizontal line.

(vii) Remainder 4 will be written in middle part before and below 6.

(viii) New dividend = 46

(ix) Modified dividend = 46 - 6 × 3 = 28.

(x) 28 is divided by cardinal 5. Third digit 5 of quotient will be written below the horizontal line.

(xi) Remainder 3 will be written in middle part before and below 5.

(xii) New dividend = 35

(xiii) Modified dividend = 35 - 5 × 3 = 20.

(xiv) 20 is divided by cardinal 5. Now quotient is 4 and remainder is 0.

(xv) Modified dividend = 3 -3 × 4 = -9. So, not giving the quotient in negative number 3 times quotient is given in place of 4 earlier.

(xvi) Remainder = 05 which is written in last part below 3. Remainder is 0.

(xvii) Remainder = 53 - 3 × 3 = 44.

Do and Learn

Division operation using Dhwajank method

(1) 1737 ÷ 21 (2) 37941 ÷ 47 (3) 23754 ÷ 74

(4) 3257 ÷ 74 (5) 7453 ÷ 79 (6) 59241 ÷ 82

Exercise 5

1. Multiply by using Urdhwtirgbhyaam formula-
   (i) 101×105 (ii) 11×15
   (iii) 18×81 (iv) 121×129

2. Multiply by using Nikhilam formula-
   (i) 48×51 (ii) 27×29 (iii) 36×34
   (iv) 18×21 (v) $21 \times 22 \times 23$ (vi) $31 \times 28 \times 27$
   (vii) $96 \times 97 \times 95$ (viii) $18 \times 18 \times 18$ (ix) $99 \times 99 \times 99$

3. Divide by using Dhwajank formula
   (i) 3987 ÷ 28 (ii) 5786 ÷ 78 (iii) 7396 ÷ 82

We Learnt

1. Urdhwtirygbhyaam contains two words Urdhw and Tiryak. Meaning of Urdhw is the product of two just written upper or lower digit and the meaning of Tiryak is the product of two diagonal digits.
2. There are grouping of numbers while multiplying by Urdhwtirygbhyaam formula. When two digits are multiplied by two digit then three groups are made and when three digits are multiplied by three digits then five groups are made.
3. Cubage can be determine in short by Nikhilam formula like this:
   $z^2(x+2y)/3y^2z/y^3$
   where number x, deviation y and sub base digit is z.
4. To find out the sub base digit, sub base is divided by base.

Chapter 6

Polygons

6.1 Leela made some shapes on paper with the help of pencil and scale.

Figure 6.1

Leela asked to Kapil , can you identify these shapes?

Kapil - In previous classes, we learned that on the basis of number of sides we can identify the different shapes. Just like in figure 6.1,(I) a shape made of three sides is triangle,(ii) a shape made of four sides is quadrilateral, (iii) it is pentagon.

Sushila - Identify (iv) and (v) shape.

Kapil - It is hexagon and octagon as it contains 6 and 8 sides.

These closed shapes are made of many sides. A closed shape made up of three or more than three sides is called a polygon.

6.2 Diagonal of Polygon

Diagonal of polygon is made of by joining each vertices to the other vertices other than the adjacent vertices. See the figure 6.2, in figure(i) vertices A is joined with C and D other than E and B and get diagonal AC and AD respectively. Similarly, other two diagonal can be drawn by the various vertices.

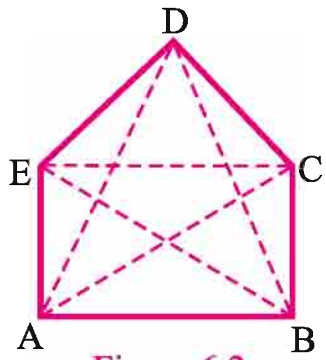


Figure 6.2

Do and Learn

See the diagonal in figure 6.3 and name them.

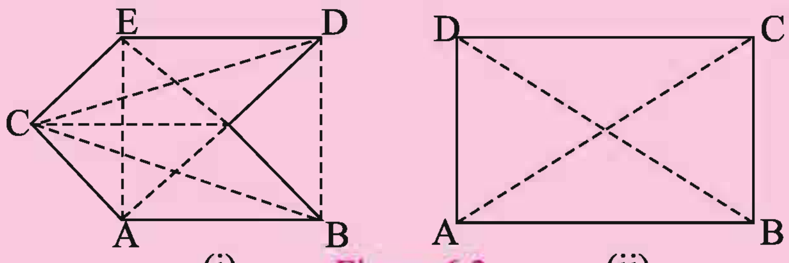


Figure 6.3

6.3 Concave and Convex Polygon

Given below some polygon shapes in two groups;

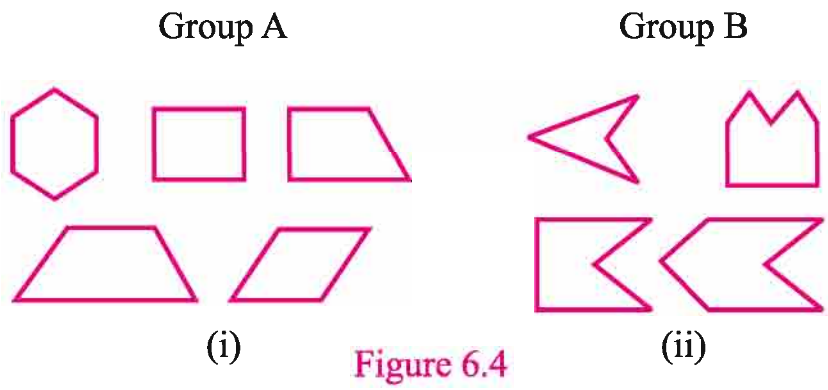


Figure 6.4

Give the name A, B, C, D, E to vertices and draw the diagonal from each vertices of the polygons.

Are all the diagonals of polygons of group A interior?

Are all the diagonals of polygons of group B interior?

Does any polygon of the group has the diagonal exterior of the polygon?

After drawing the diagonals of polygons of an each group, you will find that all the diagonals of group A's polygons are internal and in group B's polygons some of the diagonals are external.

Convex polygon does not have diagonal outside the polygon i.e, they are inside the polygons while in concave polygon, diagonals are inside and outside.

Note: In convex polygon, each angle is less than 180^0 and in concave polygon, at least one angle is more than 180^0.

6.4 Regular and Irregular Polygon

See the following polygon figures:

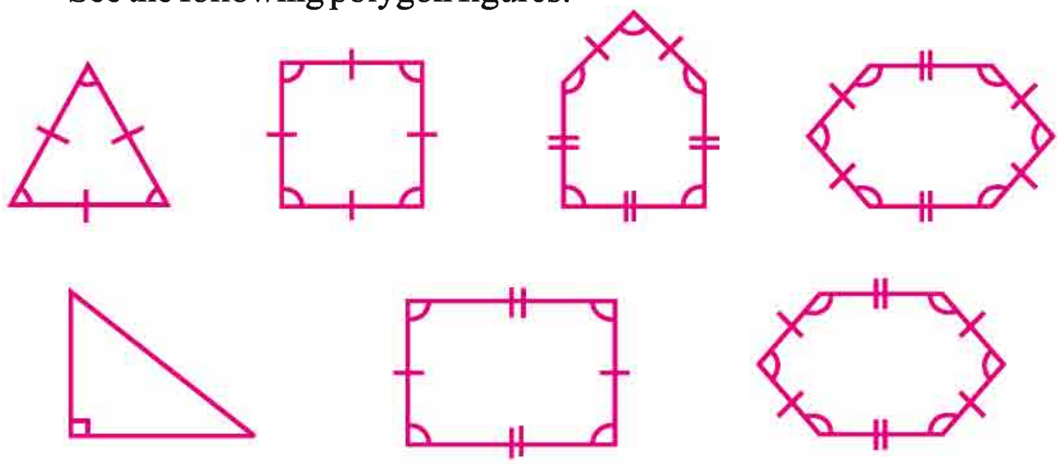


Figure 6.5

In these figures, symbol is used for equal side and symbol is used for equal angles.

A equiangular polygon has all angles equal while a regular polygon has all sides equal. In the above figure, which polygons are equiangular regular polygons? Are all regular polygons? Are all regular polygons equiangular also?

You will find that all the angles of every equiangular polygons are equal but it is not necessary that equiangular polygon is regular polygon since rectangular has equal angles but not equal sides.

Sum of the interior angles of polygon:

All the students draw a polygon in their note book and make a point inside this polygon.

1. Join the insider point to all vertices of the polygon. How many triangles are made? Discuss on it. What you conclude?
2. For example, a pentagon is drawn which has 5 angles.
3. There are 5 triangle in this figure.

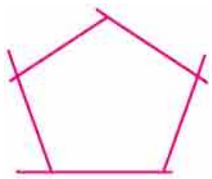


Figure 6.6

4. n-side polygon has – angles.
5. To join n-vertices with its center, – triangles are obtain.


Figure 6.7

The addition of total angles of all the triangles are equal to the number of triangles × 180^0 (since, sum of all the angles of a triangle is 180^0)

Thus, sum of all the angles of triangles made up of pentagon $= 5 \times 180^0$

Similarly, for hexagon $= 6 \times 180^0$

Thus, sum of all the angles of triangles made up of n-sides polygon $= n \times 180^0$

We know that the sum of all angles made on center $= 360^0$

1. So the sum of interior angles of pentagon is

$= 5 \times 180^0 - 360^0$

$= 5 \times 180^0 - 2 \times 180^0$

$= 180^0 (5 - 2)$

2. Sum of interior angles of hexagon is

$= 6 \times 180^0 - 360^0$

$= 6 \times 180^0 - 2 \times 180^0$

$= 180^0 (6 - 2)$

Similarly, the sum of interior angles of n-sides polygon is $= n \times 180^0 - 360^0$

$= n \times 180^0 - 2 \times 180^0$

$= 180^0 (n - 2)$

$= 180^0$ (no. of sides of polygon -2)

$= 180^0(n-2)$

we can get the sum of all interior angles of the polygon by subtracting 2 from the number of sides and multiplying it with 180^0,

For example, if $n = 7$, then the sum of interior angles $= (7-2)180^0$

$= 5 \times 180^0 = 900^0$

if $n = 4$, then the sum of interior angles $= (4-2)180^0 = 2 \times 180^0 = 360^0$

Thus, if we have to find out an interior angle of any regular polygon then divided the total sides n by the number of sum of all interior angles.

So ,each interior angle of n-sided regular polygon will be $= \dfrac{(n-2)\,180^0}{n}$

6.5 Sum of the measure of the exterior angle of a polygon

Exterior angle of a Polygon:

By increasing the sides in one direction (clockwise or anti-clockwise) outer of a polygon, the resultant angle (which is complimentary of interior angles) of that is exterior angles of a polygon.

In figure 6.8, angle $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ are exterior angles of a polygon.
In figure, sum of exterior and interior angle on each vertices is a linear angle-pair.
So, sum of angles made on vertices =

$$\text{number of sides} \times 180^0$$
$$= n \times 180^0 \text{..............(i)}$$

Interior angles of a polygon $= (n-2) \times 180^0$..........(ii)

Sum of exterior angles $= (i) - (ii)$

$$= n \times 180^0 - (n-2) \times 180^0$$
$$= 180^0 \times \{n - (n-2)\}$$
$$= 180^0 \times \{(n - n + 2)\}$$
$$= 180^0 \times 2$$

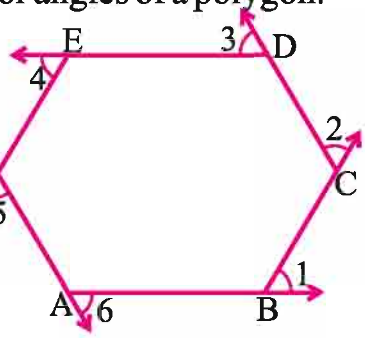


Figure 6.8

Thus, the sum of all exterior angles of a convex polygon is $= 360^0$

If there are n- sides in any regular polygon, then the value of each exterior angle will be $= \frac{360^0}{n}$

Ex. 1: Find the value of each exterior angle of a regular pentagon.

Sol: Value of all exterior angle of a polygon $= 360^0$

Number of sides of a polygon = 5

Value of an exterior angle $= \frac{360^0}{5}$

$= 72^0$

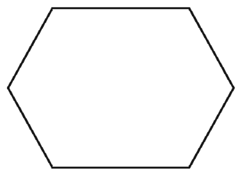
Ex. 2: If value of every exterior angle of a regular polygon is 60^0 then find the number of sides.

Sol: value of exterior angles = number of sides × value of an exterior angle

$$360^0 = n \times 60^0$$
$$n = \frac{360}{60}$$
$$n = 6 \text{ sides}$$

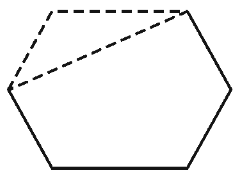
Activity:

Draw any polygon on piece of paper
(hexagon is shown in figure).



(i)

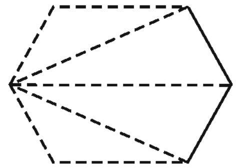
Adjoining the adjacent vertices,
cut by the scissors and separate the triangle.



(ii)

Similarly, make more triangles.

How many triangles you have made?
Think and discuss. Is there any relationship between number of sides of polygon and number of triangles?


(iii)

Is number of triangle less by two then number of sides? Figure 6.9

Hence, the number of triangle made up of six-sides polygon is = 4

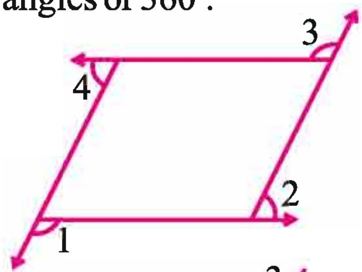
Sum of interior angles of a triangle is = 180^0

Sum of interior angles will be = 4×180^0

Activity:

Sum of exterior angles of a polygon is 4 right angles or 360^0.

1. Draw a polygon on a paper and make its exterior angles.



2. Now separate the exterior angles with the help of a scissors.

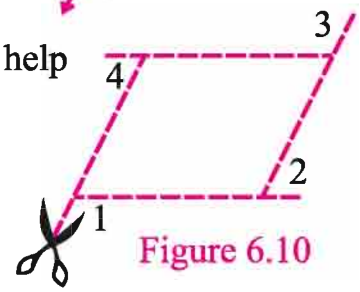
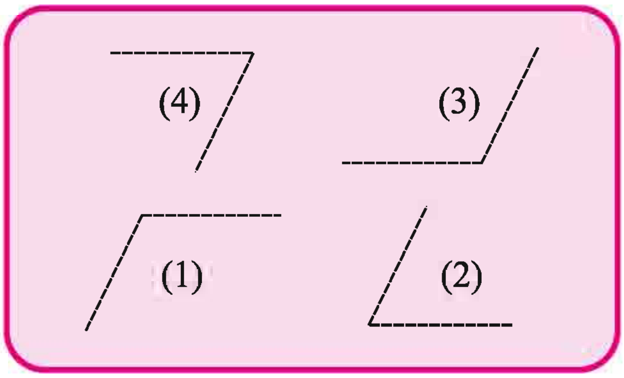
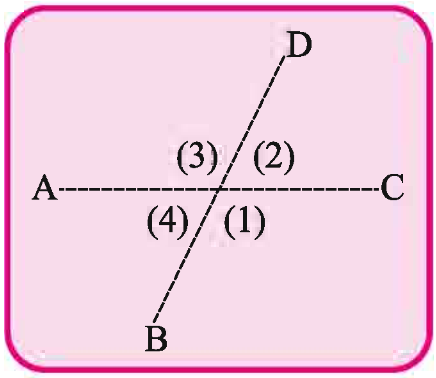


Figure 6.10

3. Now join the cutting parts.





In figure, by moving from part(1) and to reach the same, one round is complete. So $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^0$

Exercise 6.1

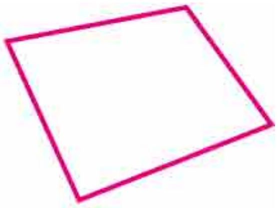
1. Draw the diagonal with the help of a pencil in the following figure and tell-

(i)

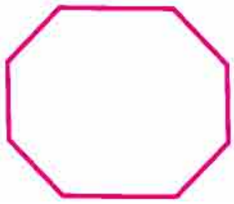
(ii)

(iii)

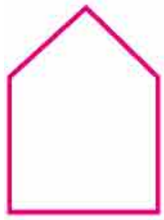
(iv)



(v)



(vi)



(i) In which shapes, diagonal will be inside?

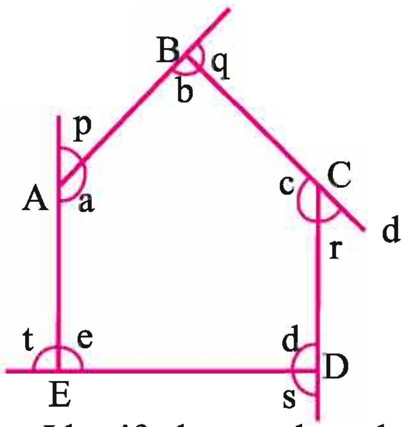
(ii) In which shapes, diagonal will be outside?

(iii) Identify the type of polygon (concave or convex)?

2. In the given polygon ABCDE

(i) Write the name of interior angles.

(ii) Write the name of exterior angles.



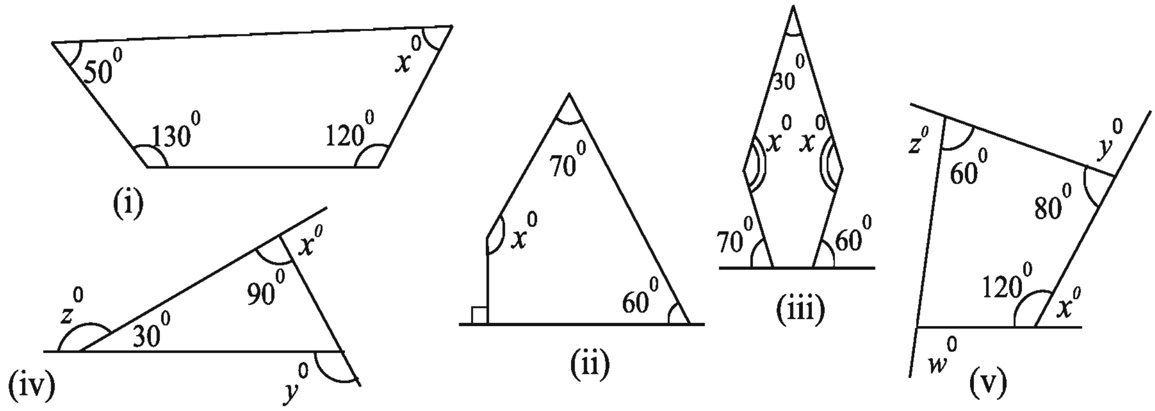
3. Define the regular polygon. Identify that regular polygon in which there are:

(i) 5 sides

(ii) 6 sides

(iii) 8 sides

4. Find the values of unknown angles (w, x, y, z) in the following figure:



5. Find the number of sides of a regular polygon whose measure of an each exterior angle is 45^0.
6. Find the number of sides of a regular polygon if its each interior angle is 165^0.
7. Find the number of sides of that regular polygon whose each exterior angle is 24^0.
8. Find the value of every interior angle of that regular polygon which has 10 sides.
9. If interior angles of any polygon is 115^0 then will it be regular polygon?
10. One interior angle of a hexagon is 165^0 and the measure of remaining interior angle is x^0 then find out the measure of all the angles.
11. By increasing the sides of a triangle in a single direction, obtained exterior angles are 110^0, 115^0 and x^0, then find the value of x.
12. Find the sum of all interior angles of a regular heptagon.

6.6 Characteristic of Quadrilateral

You know that the closed shape made up of four line segments is called quadrilateral. There are four angles, four vertices and two diagonals in any quadrilateral.

In quadrilateral ABCD,

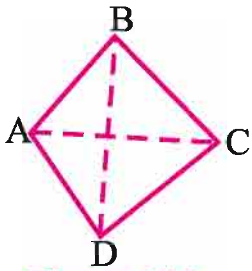


Figure 6.11

Vertices are A,B,C and D.
Sides are AB, BC, CD and DA.
Angles are $\angle$ABC, $\angle$ BCD, $\angle$CDA, and $\angle$ DAB.
Diagonals are AC and BD.

6.7 Different Types of Quadrilaterals

We have divided the triangle on the basis of their sides like equilateral triangle, isosceles triangles and scalene triangle and on the basis of their angles like acute angle triangle, right angle triangle and obtuse angle triangle.

Likewise some quadrilaterals name specific due to its sides and angles. Some such types of quadrilaterals are known as square, rectangle, parallelogram, trapezium and kite.

6.7.1 Trapezium

A quadrilateral having exactly one pair of parallel sides , is called a trapezium.

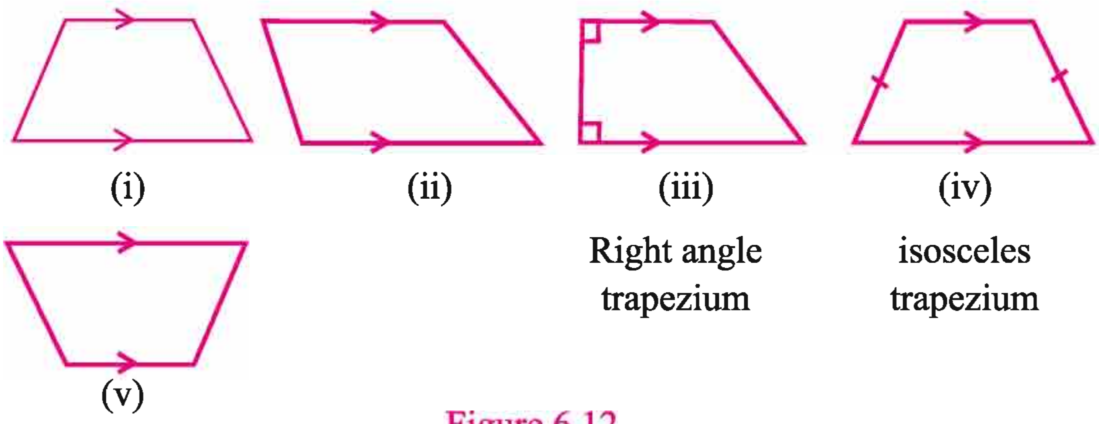


Figure 6.12

(arrow shows the parallel sides)

Above figure shows the trapezium but (iii) and (iv) are special kind of trapezium.

- A trapezium is said to be a right angle trapezium if its two angles are right angles as shown in figure(iii).
- A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal as shown in figure(iv).

Activity

Take set squares from your friends geometry box and arrange them in the following manner.

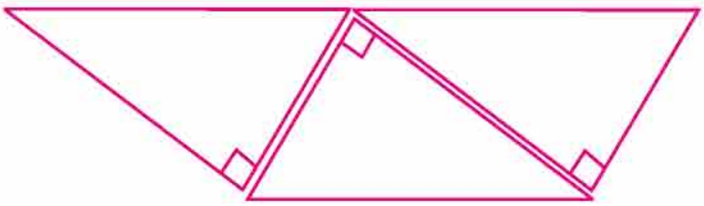


Figure 6.13

The obtained figure is trapezium.

Now by the use of set square, get the different shapes of trapezium.

6.7.2. Kite

A quadrilateral is a kite, if it has two pairs of adjacent sides equal.

As shown in figure, AB, BC and CD, DA are two equal pair of adjacent sides respectively.

so AB = BC and AD = CD

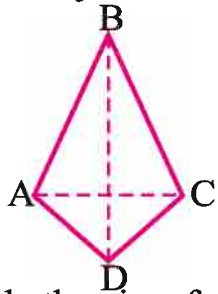


Figure 6.14

6.7.3 Parallelogram

A quadrilateral is a parallelogram if its both pairs of opposite sides are equal and parallel. Following are the characteristics of parallelogram:

1. Each pair of opposite sides is equal.
2. Each pair of opposite angles is equal.
3. Diagonal bisects each other.

Activity

You with your friends take two similar set of set squares into the geometry box and arrange them in the following way:

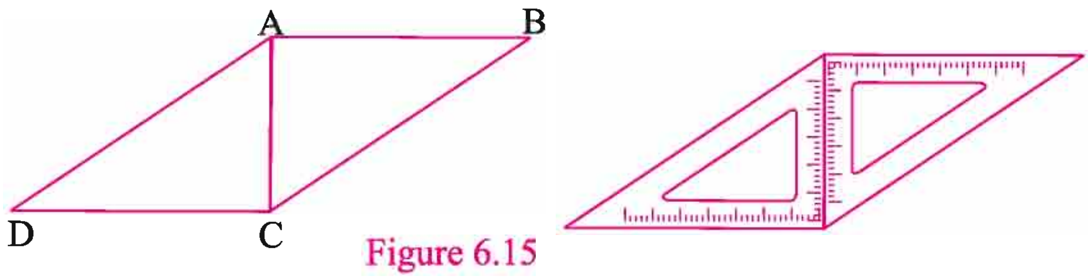


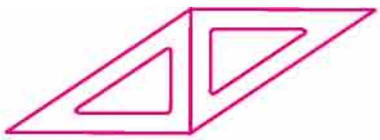
Figure 6.15

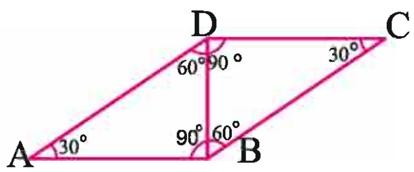
In figure, $AB \parallel DC, AD \parallel BC$ and $AB = DC, AD = BC$

Thus,

1. In a parallelogram both pairs of opposite sides are equal and parallel.
2. Each pair of opposite angles is equal.

Take a set of set- square and arrange them in the following way and find that the pairs of opposite angles are equal or not.





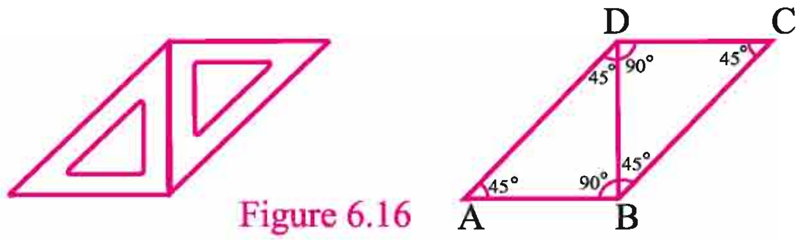


Figure 6.16

By the above figure it is clear that pairs of opposite angles are equal. The set-square of 30^0-60^0-90^0, the opposite pairs are 30^0-30^0 and 150^0-150^0.

3. Diagonal bisects each other. Draw parallelogram ABCD and measure AO, OB, OC and OD. On which conclusion you reach? Is OA = OC and OB = OD?

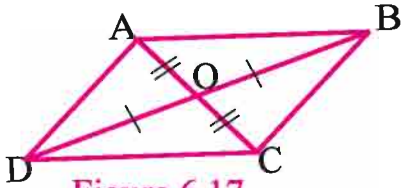


Figure 6.17

It is clear that diagonal AC and BD bisect each other at point O. So OA= OC and OB = OD.

6.7.4 Special Cases of Parallelogram

(i) Rectangle : A parallelogram in which each angle is a right angle is called a rectangle.

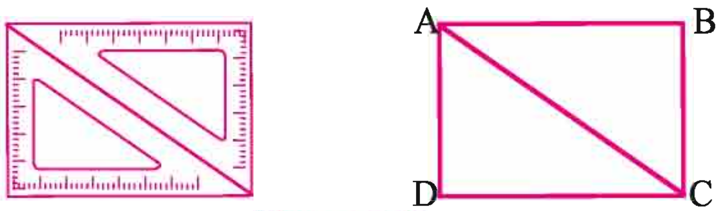


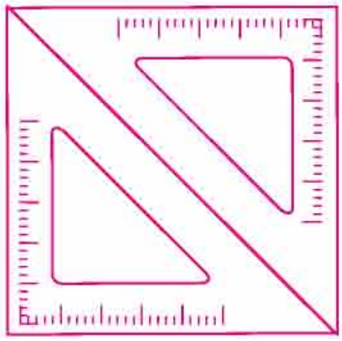
Figure 6.18

Arrange the two set square of 30^0-60^0-90^0 according to the figure and check that the opposite angles are right angle. $\angle A = \angle C = \angle B = \angle D$ are right angles and measure of AC and BD i.e., AC= BD.

Qualities of Rectangle

1. Opposite sides are equal.
2. Each angle is right angle.
3. Diagonals are same and bisect each other.

(ii) **Square:** A Rectangle in which all sides are equal is called square. Take two set of set-square of angle. Set them according to the figure and check that all the angles are right angle. Diagonals are perpendiculars to each other and $\angle A = \angle C = \angle B = \angle D$



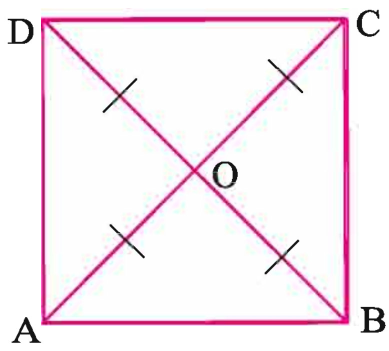
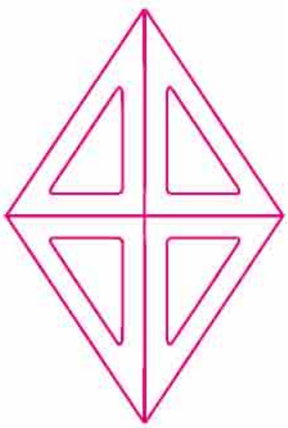


Figure 6.19

Qualities of Square:

1. Each side is same.
2. Diagonal bisects each other.
3. Diagonals are perpendicular.

6.7.5 Rhombus: A parallelogram having all sides equal and diagonal bisects perpendicular to each other. Take four set -square of 30^0-60^0-90^0 and arrange them according to the following figure. You will see that the diagonal bisects each other on right angle and all sides are equal.



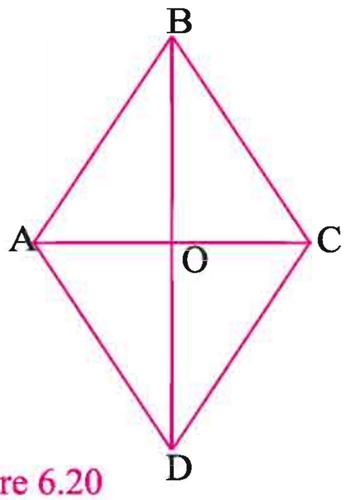


Figure 6.20

In figure, AC and BD bisect each other on right angle. OA=OC and OB = OD.

Do and Learn

On the basis of characteristics of quadrilateral tick the ✓ or ✗ on the given place.

	Parallelogram	Rectangle	Rhombus	Square	Trapezium whose non-parallel sides are equal	Trapezium	Kite
Opposite sides are parallel	✓	✓	✓	✓	✗	✗	✗
Opposite sides are equal.							
Opposite angles are equal.							
Diagonals make congruent triangle							
Diagonals bisect each other.							
Diagonals are perpendicular on each other.							
Diagonals are equal.							
All angles are right angle.							
All sides are equal.							

Exercise 6.2

1. Fill in the blanks choosing the right option-

(i) Adjacent angles of a parallelogram are.............(equal/ supplement).

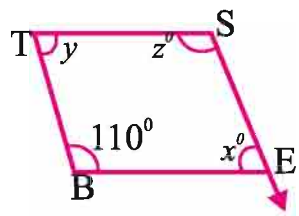
(ii) Diagonals of a rectangle are................(equal/ perpendicular bisect).

(iii) In any trapezium AB||CD, if A= 100^0 then the value of D will be........... (100^0/80^0)

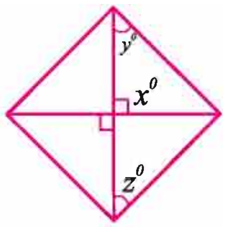
(iv) If in any quadrilateral, diagonal bisect each other on right angle then, it is called.................(parallelogram/ rhombus).

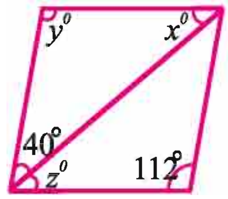
(v) All squares are ....(congruent/ similar).

2. In figure, BEST is a parallelogram.
   Find the value of x,y,z,



3. In following parallelograms, find the unknown value of x,y,z.

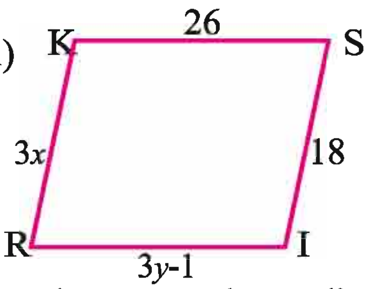




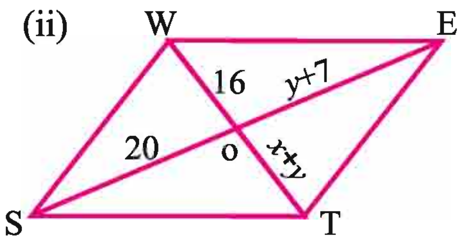
4. In any parallelogram, the ratio of two adjacent angles is 1: 5. Find the value of all angles of parallelogram.

5. Following figures RISK and STEW are parallelogram. Find the value of x and y(length in cm).

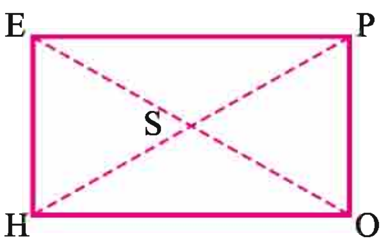
(i)



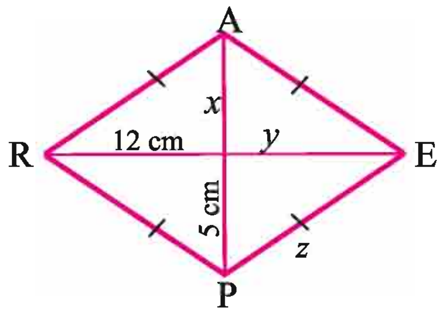
(ii)



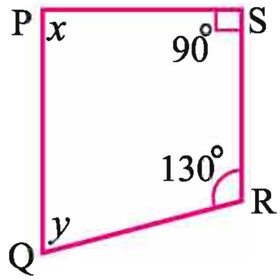
6. HOPE is a rectangle. Its diagonals intersect each other at point S. Find the value of x. If SH = $2x$+4 and SE = $3x$+1.



7. PEAR is a rhombus. Find the value of x,y and z, also write the causes.



8. In trapezium PQRS, PQ || SR find the value of $\angle x$ and $\angle y$.



We Learnt

1. A simple closed curve made up of three or more than three line segments is called polygon.
2. In any triangle, number of diagonal is less than two the number of sides. The sum of interior angles of a polygon is $(n-2)180^0$.
3. Those polygons whose all diagonals are the interior, are called convex polygon.
4. Those polygons in which at least one diagonal is exterior, are called concave polygon.
5. A polygon having all sides equal measure, is called a regular polygon.
6. Sum of all exterior angles of a polygon is 360^0.
7. A closed curve made up of four simple line segment is called quadrilateral. There are four angles, four vertices, four sides and two diagonals in any quadrilateral.
8. A quadrilateral having exactly one pair of parallel sides, is called a trapezium.
9. A quadrilateral is a kite, if it has two pairs of equal adjacent sides.

Chapter 7

Construction of Quadrilaterals

7.1 In previous class, we have learned about the construction of triangles. In this chapter, we shall discuss about the construction of the quadrilaterals. In triangle, construction of it is only possible when these three specific elements (SSS, SAS,ASA) out of 6 are available.

There are total 10 elements viz. four sides, four angles and two diagonals of a quadrilateral. Instead of this, there are some special features of quadrilaterals which we have learned in chapter-6.

For the construction of unique quadrilateral, we shall discuss the following cases:

7.1.1 When four sides and one diagonal are given.

7.1.2 When three sides and two diagonals are given.

7.1.3 When four sides and one angle are given.

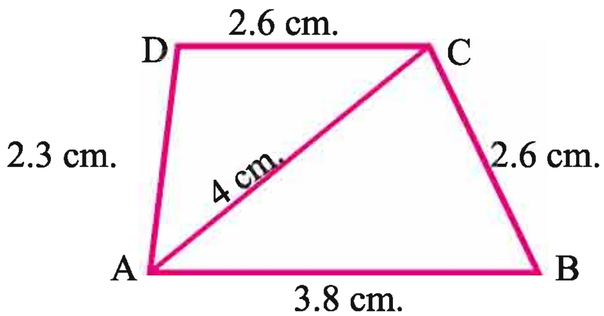
7.1.4 When three sides and its two angles between them are given.

7.1.5 When two adjacent sides and three angles are given.

7.1.1 To Construct a Quadrilateral When 4 Sides and One Diagonal are Given

Ex. 1: Construct a quadrilateral ABCD in which AB=3.8cm, BC= 2.6cm, CD= 2.6cm, AD = 2.3cm and diagonal AC = 4.0cm.

Sol: First we draw a rough sketch of quadrilateral ABCD and draw a diagonal AC in this. Write down its dimension in rough figure.

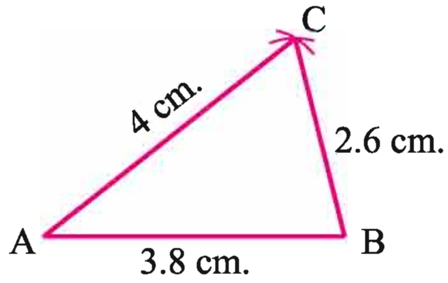


It is clear by the rough sketch that this quadrilateral is made up of triangle △ ADC and △ ABC.

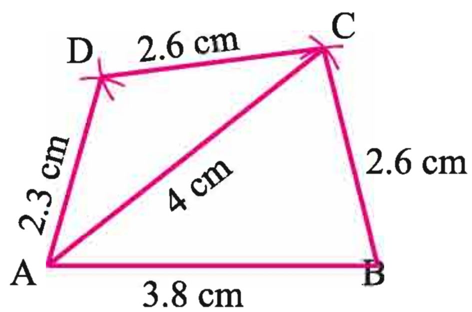
Step of Construction:

Step-1: First of all, draw AB line segment of 3.8cm.

Step 2: With B as centre draw an arc of 2.6cm and with A draw an arc of 4cm. Write the name C at the intersection point of both arc. Join AC and BC.



Step 3: With A as centre draw an arc of 2.3cm and with C as centre draw an arc of 2.6cm. Now name point D to the intersect point of both arcs. Join AD and CD. Thus, ABCD is the required quadrilateral.



Exercise 7.1

1. Construct a quadrilateral ABCD in which AB= 4.0cm, BC= 6.0cm, CD= DA=5.2 cm and AC=8.0cm.

2. Construct a quadrilateral JUMP in which JU= 3.5cm, UM= 4.0cm, MP=5.0, PJ=4.5 and PU=6.5cm.

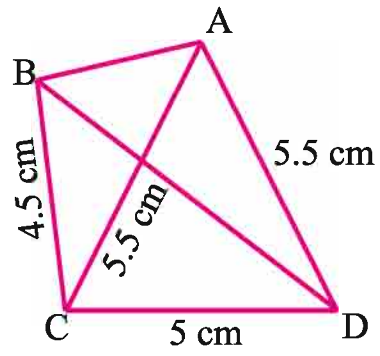
3. Construct a parallelogram MORE in which MO = 3.6cm, OR= 4.2cm, MR = 6.5cm. (Measure the remaining sides and write them in your note-book.)

4. Construct a rhombus BEST in which BE = 4.5cm, and ET = 6.0cm. Then measure the diagonal BS.

5. Construct a quadrilateral PQRS in which PQ= 4.4cm, QR= 4.0cm, RS=6.4cm, SP= 2.8cm and QS=6.6cm. Measure the diagonal PR.

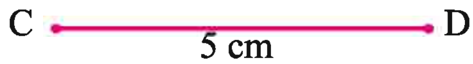
7.1.2 To Construct a Quadrilateral When three Sides and two Diagonals are Given:

Ex 2: Construct a quadrilateral ABCD in which BC= 4.5cm, AD = 5.5cm, CD = 5.0cm and diagonals AC = 5.5cm and BD = 7.0cm.

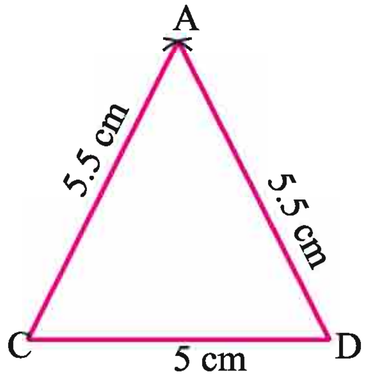


Sol: First we draw a rough sketch of quadrilateral ABCD and write down its dimension in rough figure. We may divide it into two triangles, namely ΔABC and ΔABD and by joining the point A and B we get quadrilateral ABCD.

Step-1: First of all, draw a line segment of length CD= 5.0cm.

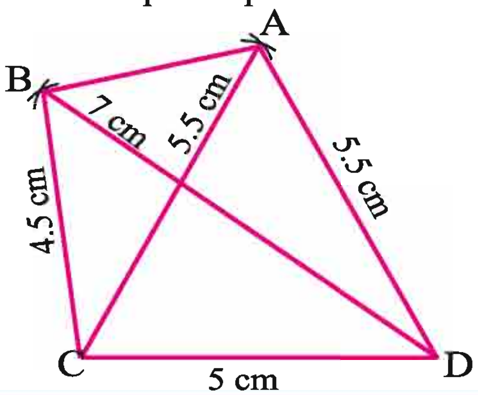


Step-2: From point C draw an arc AC = 5.5cm and from point D draw an arc of AD= 5.5cm. Now join the intersection point and give it name A. Join AC and AD.



Step-3: From point C draw an arc BC = 4.5cm and from point D draw an arc of BD= 7.0cm. Now Join the intersection point and give it name B. Join BC and BD.

Step-4: Join AB. Thus, ABCD is a required quadrilateral.



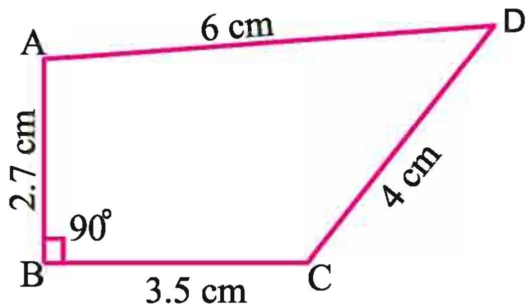
Exercise 7.2

1. Construct a quadrilateral LIFT in which LI= 4.0cm, IF= 3.0cm, TL=2.5 cm, LF = 4.5 cm and IT= 4.0cm.
2. Construct a quadrilateral ABCD in which AB = 3.8cm, BC= 3.0cm, AD= 2.3cm, AC=4.5cm and BD=3.8cm. Measure the side CD.
3. Construct a quadrilateral PQRS in which PS = 6.0cm, SR = 5.0cm, RQ = 7.5 cm, PR= 6.0cm and SQ = 10.0cm.
4. Construct a quadrilateral ABCD in which AB = BC= CD= 5.0cm and diagonal AC= 6.7cm, and BD=5.9cm.
5. Construct a quadrilateral GOLD in which GO=3.0cm, OL=2.5cm, GD=5.0cm, GL= 4cm and OD= 7cm.

7.1.3. To Construct a Quadrilateral When 4 Sides and 1 Angle are Given:

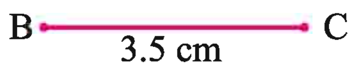
Ex 3: Construct a quadrilateral ABCD in which AB= 2.7cm, BC= 3.5cm,CD = 4.0cm, AD = 6.0cm and $\angle B = 90^\circ$.

Sol: Draw a rough sketch and write the dimensions.

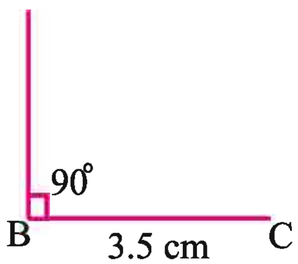


Steps of Construction:

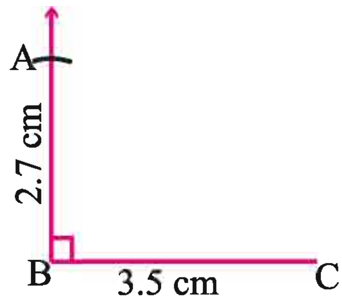
Step-1 Draw a line segment BC= 3.5cm.



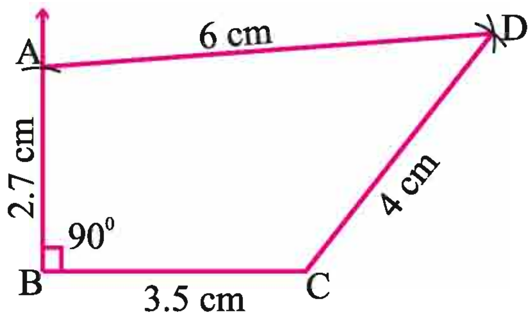
Step-2 Draw 90° at point B with the help of protractor.



Step-3 With B as centre, draw an arc AB = 2.7cm and get point A.



Step-4 With A draw an arc of 6cm and with C draw an arc of 4cm. Where two arcs meet name point D. Join AD and CD. Thus, ABCD is required quadrilateral.



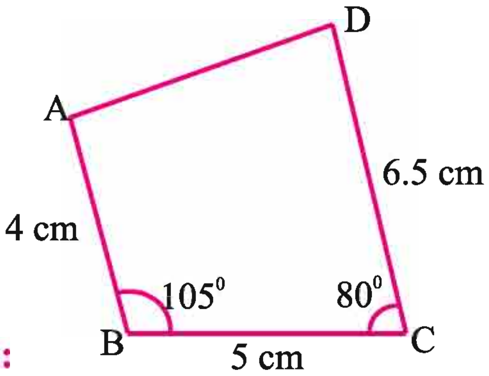
Exercise 7.3

1. Construct a quadrilateral ABCD in which AB = BC= 3.0cm, AD= CD=5.0 cm and $\angle ABC = 120^0$.
2. Construct a quadrilateral PQRS in which PQ = 2.8cm, QR =3.1cm, RS= 2.6cm, SP = 3.3cm and $\angle P = 60^0$.
3. Construct a rectange whose sides are 4.2 cm and 2.5cm. Measure its diagonal lengths.
4. Construct a rhombus in which one angle is 75^0 and one side is 5.2cm.
5. Draw a square with one side of 5.0cm.

7.1.4. To Construct a Quadrilateral when 3 Sides and 2 Angle are Given:

Ex. 4: Construct a quadrilateral ABCD in which sides AB = 4.0cm, BC = 5.0cm, CD= 6.5cm and $\angle B = 105^0$ and $\angle C = 80^0$.

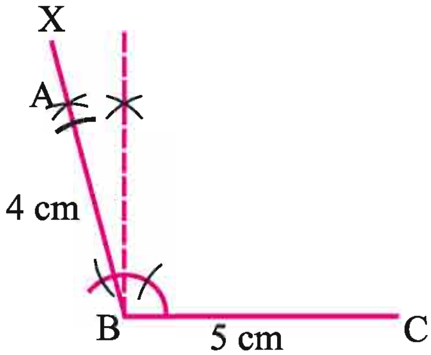
Sol: Draw a rough sketch ABCD and write the dimensions.



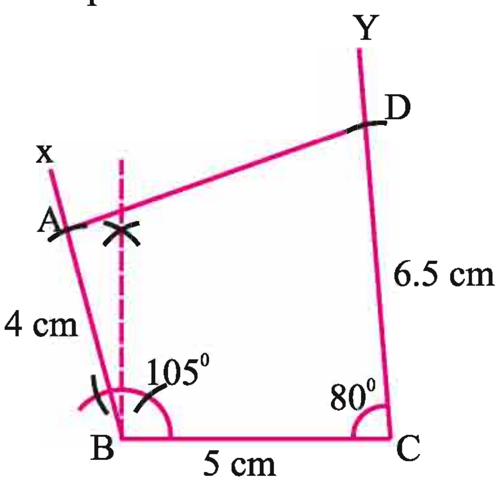
Steps of Construction:

Step-1: First draw a line segment of length BC = 5cm.

Step-2: Draw 105^0 at point B. With B as centre draw an arc of 4 cm. Mark point A at the intersection point of the arc with line BX.



Step-3: Draw 80^0 at point C. With C as centre draw an arc of 6.5cm.Mark point D at the intersection point of the arc with line CY.



Step-4: Join point A and D. Thus, ABCD is a required quadrilateral.

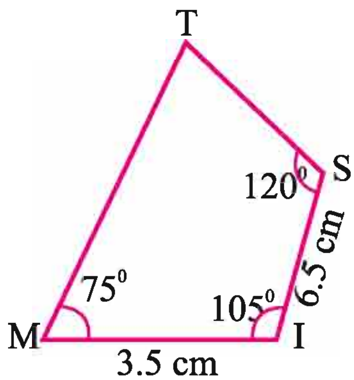
Exercise 7.4

1. Construct a quadrilateral KLMN in which KL= 4.8cm, LM=5.0cm, MN = 3.8cm and $\angle L = 72^0$, $\angle M = 105^0$.
2. Construct a quadrilateral ABCD in which AB = BC= 3.0cm, AD=5.0cm and $\angle A = 90^0$, $\angle B = 105^0$
3. Construct a quadrilateral PQRS in which QR =3.6cm, RS = 4.5cm, PS = 5.0cm, $\angle R = 75^0$ and $\angle S = 120^0$.
4. Construct a quadrilateral DEAR in which DE= 4.7cm,EA = 5.0cm, AR = 4.5cm and $\angle E = 60^0$ and $\angle A = 90^0$.
5. Construct a quadrilateral ABCD in which $\angle B = 135^0$, $\angle C = 90^0$, BC= 5.0cm, AB=9.0cm and CD = 7.0cm.

7.1.5. To Construct a Quadrilateral when two Adjacent Sides and three Angle are Given:

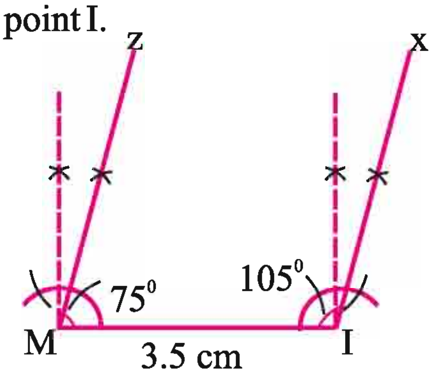
Ex. 5: Construct a quadrilateral MIST where MI = 3.5cm, IS = 6.5cm, $\angle M = 75^0$, $\angle I = 105^0$ and $\angle S = 120^0$.

Sol: Draw a rough diagram MIST and write the dimensions.

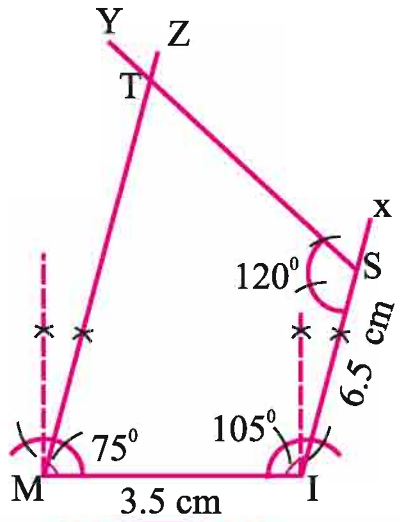


Step-1: First draw MI = 3.5cm.

Step-2: Draw an angle $\angle M = 75^0$ at point M and $\angle XIM = 105^0$ at point I.



Step-3: With I draw an arc of 6.5cm on IX, the point of intersection of arc and IX is S. Draw an angle $\angle YSI = 120^0$ at point S. T is the intersect point of MZ and SY. Thus, MIST is a required quadrilateral.



Exercise 7.5

1. Construct a quadrilateral MORE in which MO=6cm, OR=4.5cm, $\angle M= 60^0$, $\angle O = 105^0, \angle R = 105^0$.
2. Construct a quadrilateral ABCD in which $\angle B=70^0$, $\angle A=54^0$, $\angle D=105^0$ AB= 6.2 and AD=5.7cm.
3. Construct a quadrilateral PQRS in which $\angle P=75^0$, $\angle Q=85^0$, $\angle R=110^0$ PQ = 4.1 cm and QR = 3.9cm.
4. Construct a quadrilateral DNSI in which DN =2.5cm,NS = 3.7cm and $\angle I = 60^0$, $\angle N = 120^0, \angle S = 90^0$.

Some Special Cases:

Till now we learned to construct quadrilateral of such type in which five measurement are required. Is it possible to construct a quadrilateral where less number of measurement are given then this? We construct some special cases by the following examples:

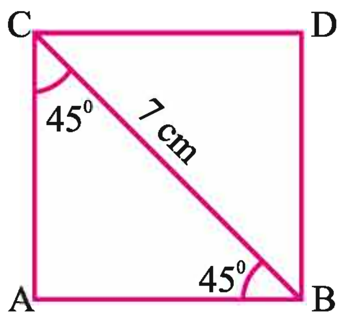
Square: There may be following conditions to construct a square:
(i) One side is given (ii) one diagonal is given

Rectangle: There may be following conditions to construct a square:
(i) Two adjacent sides are given
(ii) One side and one diagonal are given.

Rhombus: There may be following conditions to construct a square:
(i) One side and one diagonal are given
(ii) Two diagonals are given.

Ex. 6: Construct a square of diagonal 7cm.

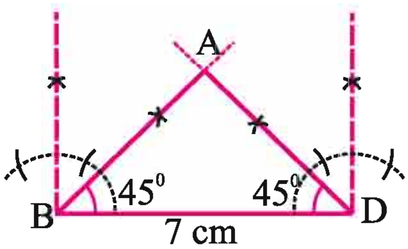
Sol: Draw a rough diagram and write the dimensions.



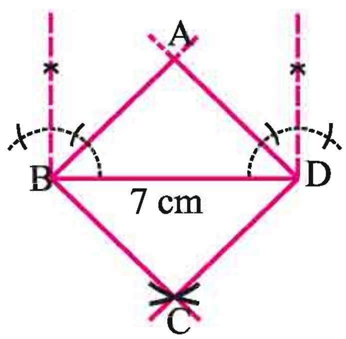
Steps of Constructions:

Step-1. First draw a diagonal BD= 7cm.

Step-2 Draw 45^0-45^0 angle at point B and D along with BD. Name the point of intersection of both angles as A and join DA and BA.

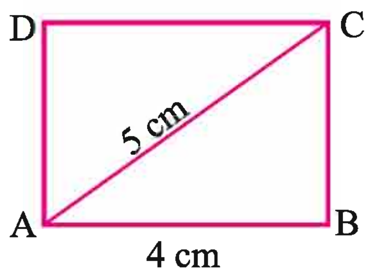


Step-3. Now in opposite direction of BD, draw an arc equal to BA with B and DA with D. Name the point of intersection as C. Join BC and CD. Thus, ABCD is the required square.



Ex. 7: Construct a rectangle ABCD of side AB = 4cm and diagonal AC= 5cm.

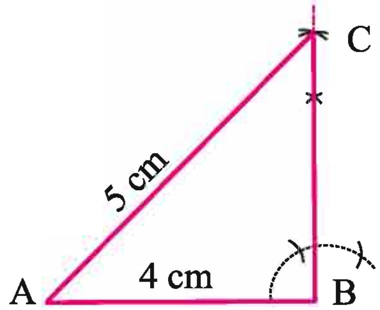
Sol: Draw a rough diagram ABCD and write the dimensions.



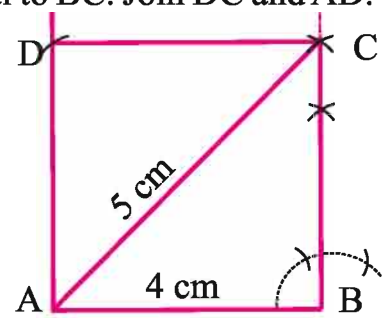
Steps of Constructons:

Step-1: First draw a line segment AB= 4cm.

Step-2: Draw a line segment BX with point B of angle 90^0. Draw an arc of 5 cm with point A which intersect BX at point C. Now join AC.

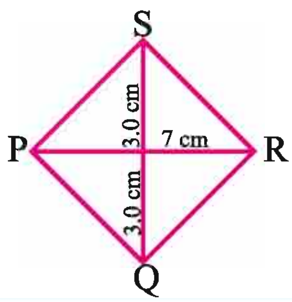


Step-3: With point C cut an arc of 4cm equal to AB, which intersect the arc cut from point A equal to BC. Join DC and AD. Thus, ABCD is a required rectangle.



Ex-8 : Construct a rhombus PQRS in which PR=7cm and QS = 6cm.

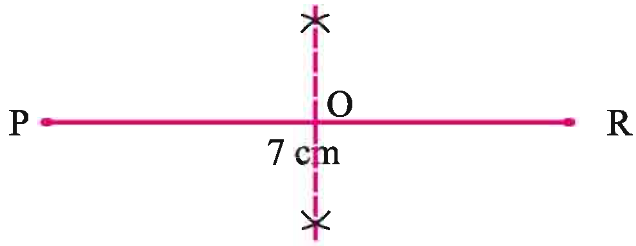
Sol: Draw a rough diagram of rhombus PQRS and write the dimensions.

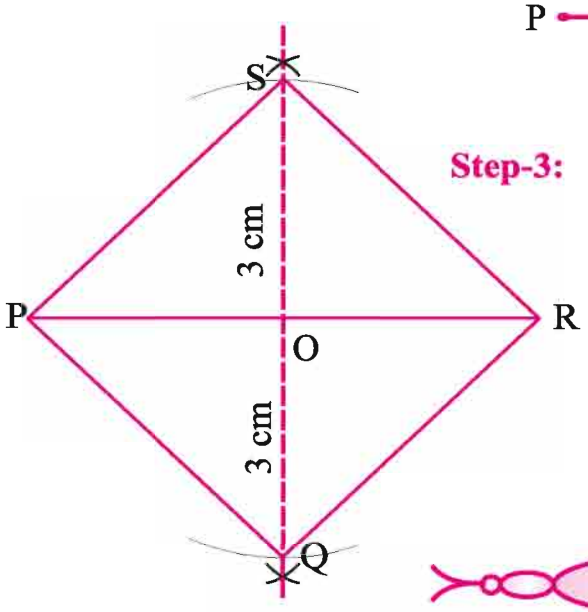


Steps of Constructions:

Step-1: First draw PR =7.

Step-2: Draw the right bisector of diagonal PR which meets at point O.





Step-3: With O as centre, draw an arc of OQ = OS = 3cm at right bisector. Join PQ,QR,RS and PS. Thus, we get exact rhombus PQRS.

Exercise 7.6

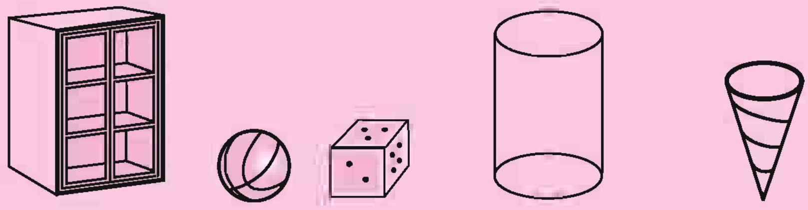
1. Construct a square in which:
   (i) Perimeter is 20cm. (ii)Sum of two adjacent sides is 9cm.
2. Construct a square PQRS in which diagonals meets at point O and PO= 2.8 cm.
3. Construct a rectangle while adjacent sides are 8cm and 6cm.
4. Construct a rectangle PQRS while PQ = 5.5cm and diagonal QS= 6.2cm.
5. Construct a rectangle EFGH while EF = 4.0 cm and diagonal EG = 5.0cm.
6. Construct a rhombus DEFG while FG = 4.8 cm and diagonal EG = 3.4cm.
7. Construct a rhombus ABCD while BC = 4.0 cm and $\angle B = 75^0$.

We Learnt

1. To construct a quadrilateral when four sides and one diagonal are given.
2. To construct a quadrilateral when three sides and two diagonals are given.
3. To construct a quadrilateral when four sides and one angle are given.
4. To construct a quadrilateral when three sides and two angles between these are given.
5. To construct a quadrilateral when two adjacent sides and three angles are given.
6. To construct rectangle, rhombus and square.

Chapter
8 Visualization of Solids

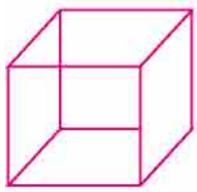
8.1 In previous class, we have studied about the various solid figures like cube, cuboids, cone, cylinder, sphere etc. Some figures are given below. Look these pictures carefully and give the answers of the given questions.

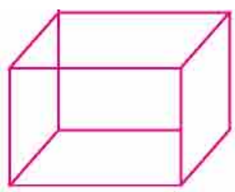


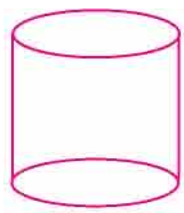
	Almirah	Ball	Dice	Cylindrical Drum	Ice cream Cone
1. Name of figure	....	....	....	....	....
2. Number of faces	....	....	....	....	....
3. All faces are plane	yes/ no	yes/no	yes/no	yes/no	yes/no
4. Figure of plane face	....	....	....	....	....

In the above table, which figures have all the faces plane? You will find that cube and cuboid have all the faces plane. A figure which has all the faces flat is called the Polyhedron. All the faces of ployhedron are polygon.

1. All faces of cube are of which shape? ________________
2. All faces of cuboid are of which shape? ________________
3. What is the shape of flat face of cylinder? ________________


Cube


Cuboid


Cylinder

You see that all the faces of cube are square and all the faces of cuboid are rectangular.

Now discuss on the following shapes:

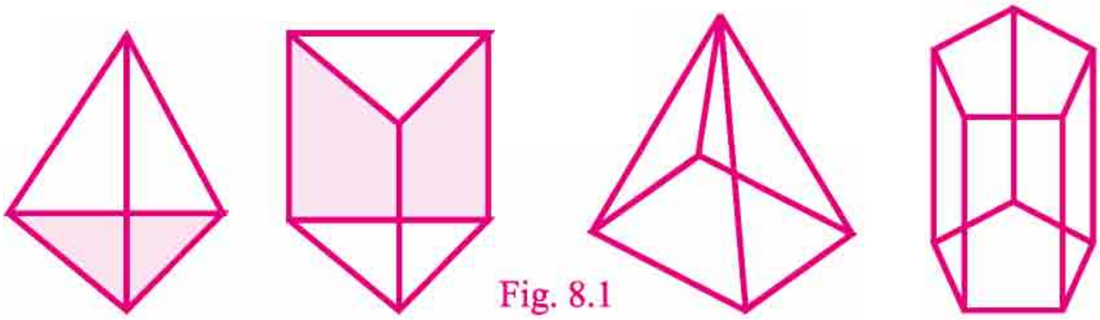


Fig. 8.1

In the above shapes, four or more than four faces are plane and these plane faces are polygon. These shapes are prism and pyramid. Let us study about these shapes.

8.2 Prism:

A polyhedron, whose base and upper part are congruent polygons and the side (Lateral faces) faces are parallelograms (rectangle or square).

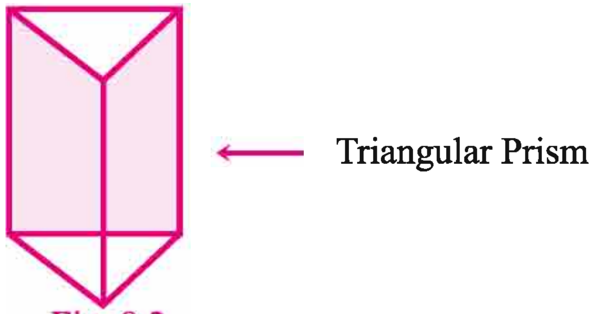


Fig. 8.2

8.2.1 Draw the Triangular Prism:

Take a triangular piece of cardboard and draw two triangles with some distance according the following figure. By joining the adjacent vertices, we get a triangular prism. You also try to make it on your note book.

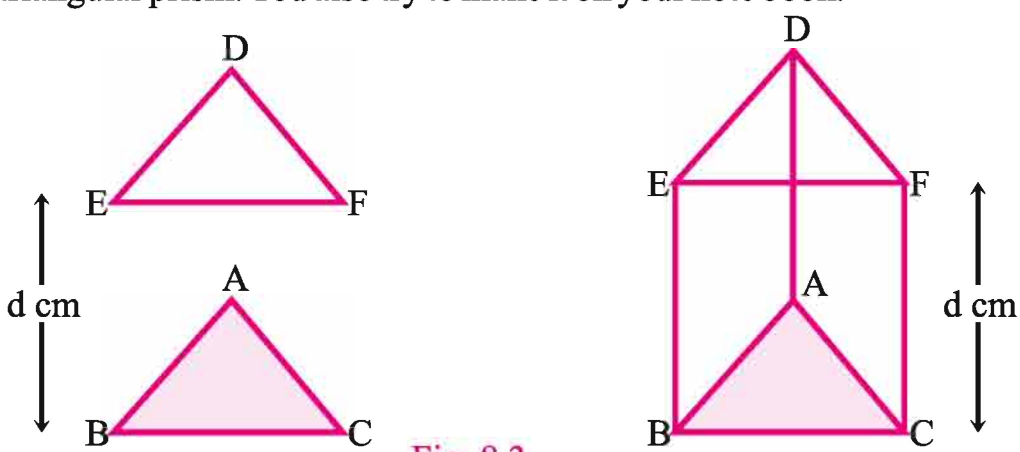


Fig. 8.3

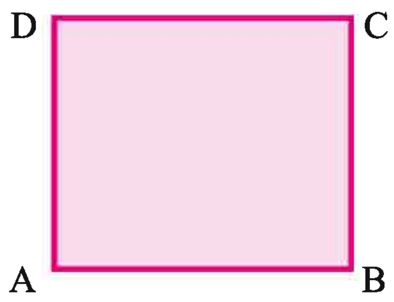
In the given triangular prism, there are three rectangular faces ABED, ADFC and BEFC and two triangular faces ABC and DEF respectively. Therefore, triangular prism has total five faces. In the triangular prism, there are the 6 vertices A,B,C,D,E and F and 9 edges AB, BC, CA,DE, EF,FD,BE,AD and CF respectively.

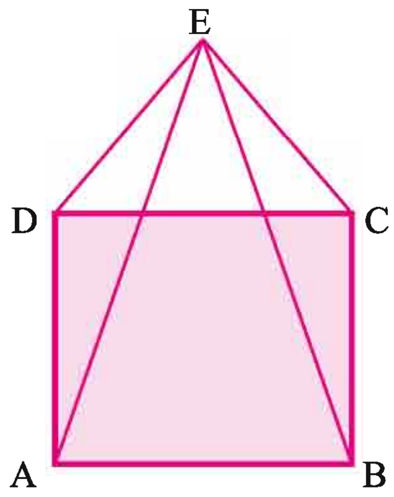
8.3 Pyramid

A polyhedron whose base is polygon and all the side faces are triangular having a common vertex, called the vertex of pyramid.

8.3.1 Draw a Square Pyramid:

Let us draw a square ABCD and shaded it. Now take a point E drove the middle of the square at some distance and join each vertex of the square to the point E. Obtained figure will be the shape of a square pyramid.





In this pyramid, there is a square face ABCD and four triangular faces EAB,EBC, ECD and EDA respectively. This pyramid has 8 edges AB, BC,CD,DA, EA, EB,EC and ED and 5 vertices A,B,C,D and E respectively.

8.4 Euler Formula for Polyhedrons

According to this formula, for every polyhedron there is a relationship between number of vertices V, edges E and faces F.

Let us see the table

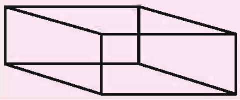
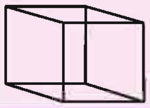
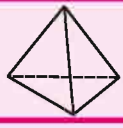
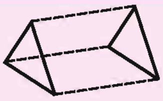
Sr. No.	Shape and its name	no. of vertices	no. of faces	no. of edges	V+F	E+2
1	 cuboid	8	6	12	8 + 6	12 + 2
2	 cube	- -	- -	12	- - - -	- - - -
3	 square pyramid	4	- -	- -	- - - -	- - - -
4	 pyramid	4	- -	- -	- - - -	- - - -
5	 prism	- -	- -	- -	- - - -	- - - -

Table 8.1

We see that values in column 6 and 7 are same. Therefore, V+F = E + 2

This relation was proposed by Euler that is why the name given to this formula on his name.

Exercise 8.1

1. Fill in the blanks:

(i) The base of pyramid is a polygon and other faces have the.............shape. (triangle/parallelogram).

(ii) Base face and top face of prism are ....of each other. (congruent/similar)

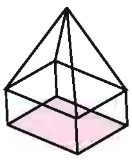
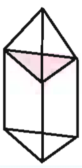
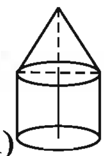
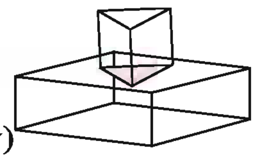
(iii) A polyhedron whose vertices are 10 and faces are 7 then its edges are.........(15/19).

(iv) Cube and cuboid are also a kind of..........(prism/pyramid).

2. Write any four solid polyhedrons surrounding you with number of its vertices, faces and edges.

3. Draw a triangular prism taking a 4cm base of equilateral triangal.

4. Recognize polyhedron shapes in the following organized shapes also tell that which shapes are used to organized these shapes.

(i)  (ii)  (iii)  (iv) 

8.5 Two-Dimensional Representation of Three-Dimensional Shapes (in net form)

In earlier classes, we studied about two dimensional representation of various solids i.e. three-dimensional shapes. Expanding the all 3d shapes, its faces are obtained in 2d shapes. Let us expand some solid shapes and see their whole faces in form of net.

8.5.1 Cuboid:

A cuboid having vertices A,B,C,D,E,F,G,H. Its visual faces are ABCD, ABGH and BGFC and shaded faces EFGH, EFCD, and AHED are side faces. By opening this cuboid we get the shape like the following figure.

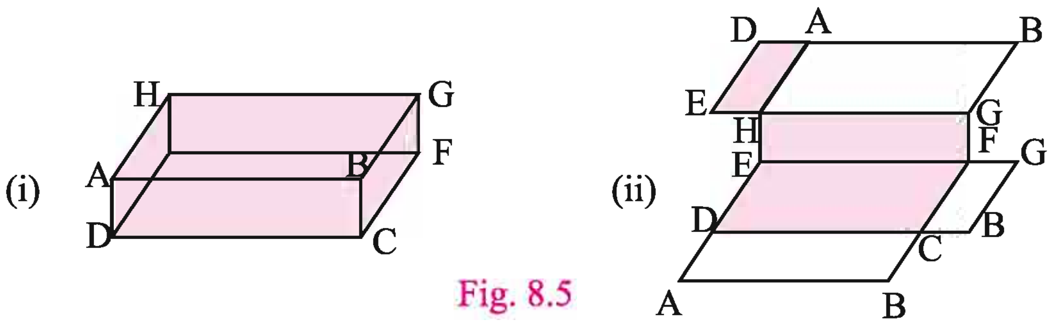


Fig. 8.5

Can the expanding figure(net) of this cuboid be of other shape. Think on this.

8.5.2 Cube

A cube having visual faces ABCD, DCGH and BFGC and shaded faces EFGH, ABFE, and AEHD are side faces. Opening this cube we get the net pattern like the following figure whose all 6 faces are equal and square.

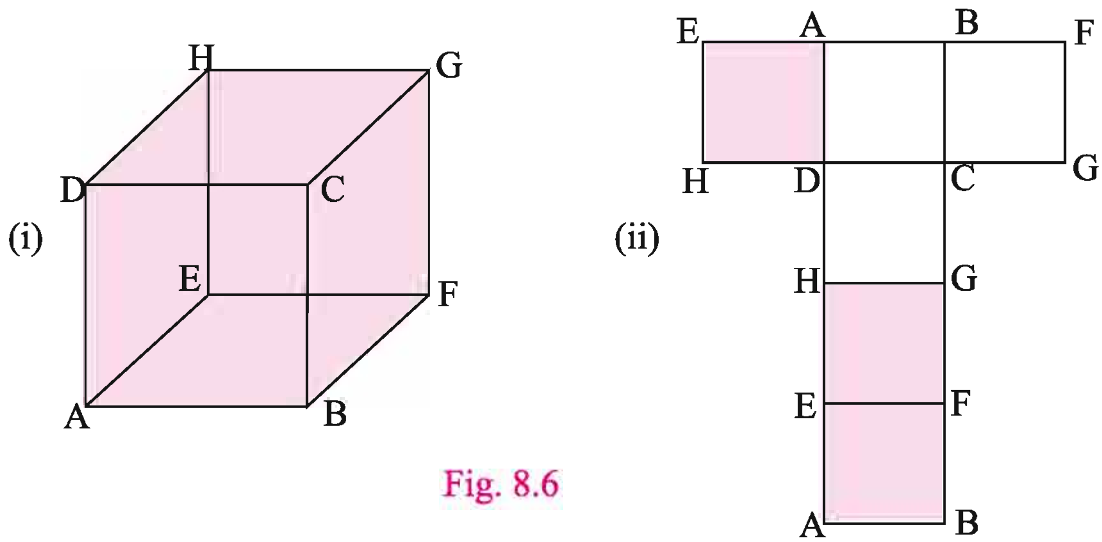


Fig. 8.6

8.5.3 Cylinder

When the solid right circular cylinder is opened with supporting a point then the curve face is obtain in form of the rectangular or square. Both parts of cylinder i.e.,top and bottom are obtain in form of circle.

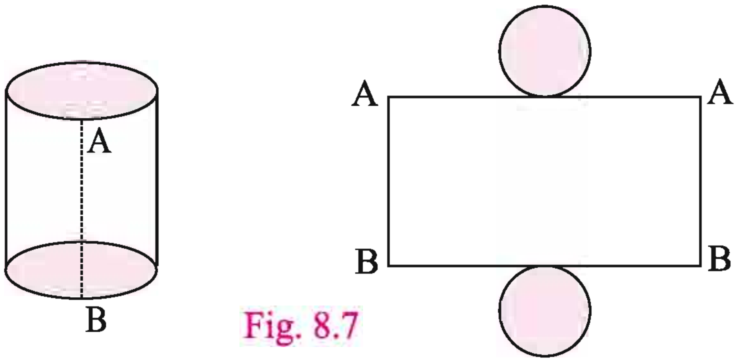


Fig. 8.7

8.5.4 Cone

There are two faces in given cone. In which one surface is curved and second surface is circular. To cut the cone supporting the AB we get the following 2d figures. One of which is equivalent to radial part and second is circular.

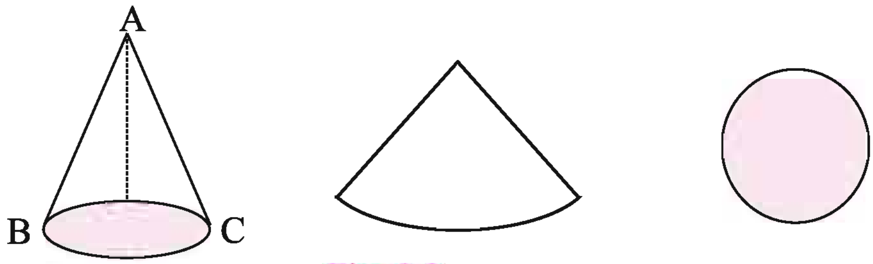


Fig. 8.8

8.5.5 Prism According to figure, a prism having the vertices A,B,C,D,E,F by expanding, three rectangular faces ABED, CADF, and BCFE are obtained respectively. Its both end are obtained in form of triangle ABC and DEF.

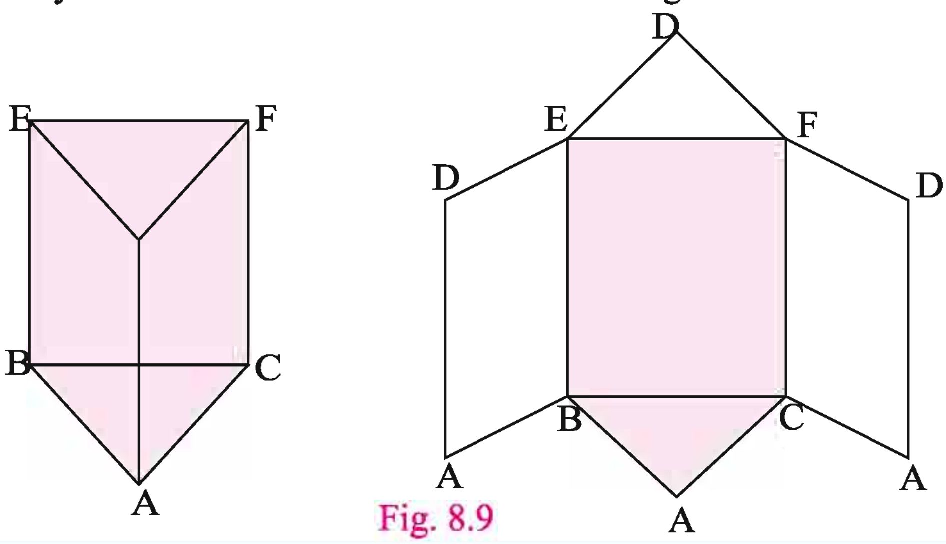


Fig. 8.9

8.5.6 Pyramid

The given pyramid has square base whose visual faces are ABE and BCE. Its side faces are ADE and CDE and its base face is ABCD.

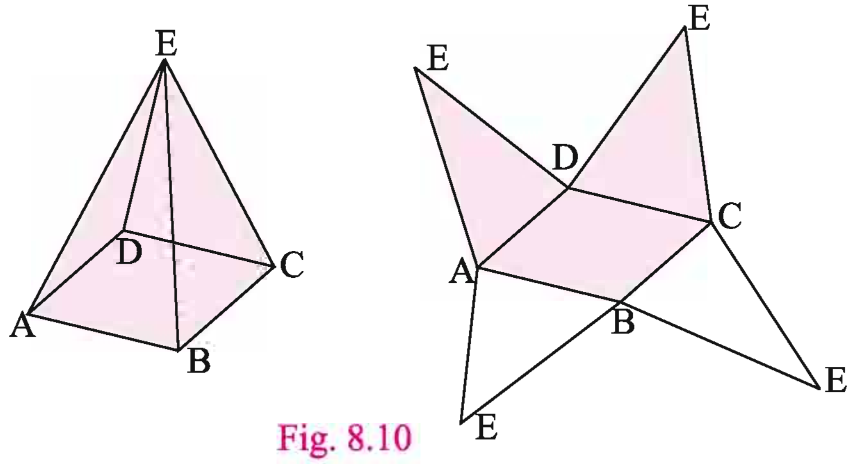
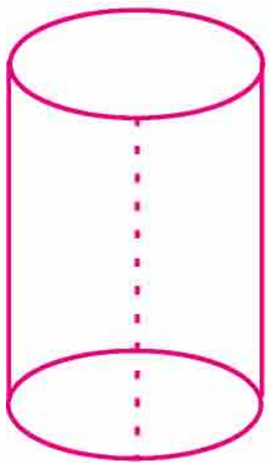


Fig. 8.10

By expanding the above pyramid we get its 2d- pattern according to the figure. In this net pattern, we get one face ABCD of square shape and four faces i.e., ABE, BCE,CDE and DAE of triangle shape.

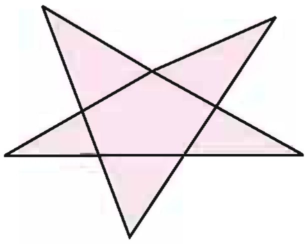
Exercise 8.2

1. Expanding the solid shape of cube and cuboid draw the different two net pattern of figure other than book.
2. After cutting the hollow right circular cylinder according to the dotted line, show the obtained 2d-shape.

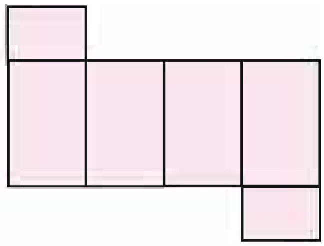


3. By folding the expanded figure (joining the faces), draw the solid figure.

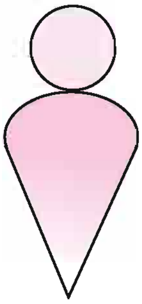
(i)



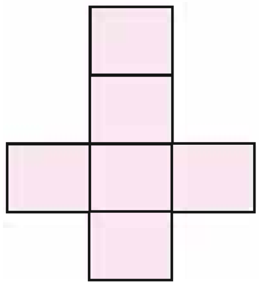
(ii)



(iii)



(iv)



We learnt

1. Polyhedron- A solid whose all the faces are plane.
2. Prism- A polyhedron, whose base and upper part are congruent polygons and the side faces are parallelogram (rectangle or square).
3. Pyramid- A polyhedron whose base is polygon and all the side faces are triangular having a common vertex.
4. Euler formula for polyhedrons-

$$\boxed{V + F = E + 2}$$

5. To represent 3d- shapes like cube, cuboid, cone, cylinder, prism, pyramid in 2d net pattern.

CHAPTER

9 Algebraic Expressions

9.1 Lets consider the given expressions

(i) $2n + 1$ (ii) 0 (iii)$5xy$ (iv) $15 + 7 + 0$ (v) $3\frac{p}{q}$ (vi) $\frac{4}{5}a^2b$

Which one of them are numeric expressions. You will find that'0', 15+7+0, are numeric expressions while $2n+1$, $5xy$, $3\frac{p}{q}$ $\frac{4}{5}a^2b$ are algebraic expressions because they are made up by the combinations of constants and variables.

As you know algebraic expressions are formed with variables and constants. The expression $3x+5$ is formed with variable x and constant 3 and 5

In the earlier class we have studied that algebraic expression can be named as monomials, binomials and trinomials according the numbers of terms contained in it in general .... Polynomial.

Monomial

x, $\frac{p}{4}$, m^2

$307q$, a^3

Binomials

$2x + y$

$5p + 4$

$3a^2 + 5b$

Trinomials

$m^2 + 2m + 5$

$a^2 + b + 3c$

$a^3 + b^3 + c^3$

$m + 2 + p + 1$

$a_1 + a_2 + a_3 + ... a_m$

Polynomial

Do and learn:-

Give five different examples of numeric and algebraic Expression. Then categorize them into monomials, binomials, and trinomials.

9.2 Power of an expression

In an expression the term with the highest power is called the power of that expression like in $7x^3y$ the power of this monomial is $3 + 1 = 4$. While in $5p^2q - 3pq + 7qr$ the term $5p^2q$ hold the highest power $2 + 1 = 3$.

9.3 Like and Unlike terms

Look at the following expression given in the table.

LIKE TERMS	UNLIKE TERMS
$4x$, $-2x$, x , $-x$	$5x, 5x^2, x^3, xy, y^2x$
$pq, 5pq, \frac{3}{5}pq$	$\frac{3}{5}pq, \frac{3}{5}p, q, p^2q, pq^2$

Do and learn:-

From the following tick the essential condition for like terms
(1) Same signs (2) Same coefficient (3) Same exponents (4) Same number of variable.

We note that like terms are terms that have the same variable and exponents but they may have different numeric coefficient. In unlike terms variables are different or the exponents of the variable are different or both are different. Even if the numeric coefficient are equal but the exponents and variables are not equal the term is not called like .

Do and learn:-

(1) Find the LIKE terms from the following

$a\,x^2y$, $2n$, $5y^2$-$7x^2$, $-3n$, $7xy$, $25y^2$

(2) Write three LIKE terms for the expression $7xy^2$

9.4 Addition and Subtraction of Algebraic Expressions

In class 7, we learnt how to add and subtract algebraic expressions for example, $7x+4x=11x$ not $11x^2$ that is $x+x \neq x^2$ $2x^2y+3xy=?$

So in addition of like terms exponent remains the same.
For example, $7x^2y-3x^2y = 4x^2y$

Do and learn:-

Fill in the blank by adding the following like terms	
$4n + (-3n) =$ …….. $5pq + 12pq =$ ……..	$-5x^2y + (-3\,x^2y) =$ ……. $2ab^2 + 11ab^2 =$ ……

In case of unlike terms neither their exponent nor their coefficient gets added. They are represented as it is with the sign + or –

Such as in, $(7a^2b) + (3a^2b^2) = 7a^2b + 3a^2b^2$

$(-3pq) - (+p^3) = -3pq - p^3$

Try to understand this through these examples

Example 1: Add: $3x^2 + 4xy + 2y^2$ and $5y^2 - xy + 7x^2$

Solution: Writing the three expressions in separate rows, with like terms one below the other, we have

$$\begin{array}{r} 3x^2+4xy+2y^2 \\ +\ 7x^2-xy+5y^2 \\ \hline 10\,x^2+3xy+7y^2 \\ \hline \end{array}$$

or

$$\begin{array}{ccc} x^2 & xy & y^2 \\ 3 & 4 & 2 \\ 7 & -1 & 5 \\ \hline 10 & 3 & 7 \\ \hline \end{array}$$

That is $10x^2 + 3xy + 7y^2$

Do and learn:-

(i) Sheela says that the sum of $2pq$ and $4pq$ is $8p^2q^2$. Is she is right ?

(ii) Raees adds $4p$ and $7q$ and gets $11pq$ as its answer. Do you agree with his answer ?

Example 2: Subtract $3xy + 9y^2$ from $15xy + 7x^2 - 3y^2$.

Solution: Writing the three expressions in separate rows, with like terms one below the other, change the sign of the subtracted expression and solve

$$\begin{array}{r} 15xy + 7x^2 - 3y^2 \\ \underset{-}{}3xy \underset{-}{+}9y^2\ \underset{-}{7} \\ \hline 18xy + 7x^2 + 6y^2 \\ \hline \end{array}$$

or

$$\begin{array}{ccc} x^2 & xy & y^2 \\ & 15 & -3 \\ 0 & -3 & -9 \\ \hline -7 & -12 & -12 \\ \hline \end{array}$$

That is $7x^2 + 12xy - 12y^2$

Note that subtraction of a number is the same as addition of its additive inverse. Thus subtracting –5 is the same as adding +5. Similarly, subtracting -3xy is the same as adding +3xy:

SECOND METHOD

=> $15xy + 7x^2 - 3y^2 - (3xy+9y^2)$

=> $15xy + 7x^2 - 3y^2 - 3xy - 9y^2$

=> $15xy - 3xy + 7x^2 - 3y^2 - 9y^2$

=> $12xy + 7x^2 - 12y^2$

Do and learn:-

1. If $A = 2y^2 + 3x - x^2$ and $B = 3x^2 - x^2$ then find A+B and A-B

9.5 Multiplication of Algebraic Expressions:

Rakesh and Leela are playing a game of arranging the stars.

```
*  *  *  *  *
*  *  *  *  *
*  *  *  *  *
```

The stars are arranged in such a way that each row contains 5 stars and there are 3 such rows. Then total number of stars = $5 \times 3 = 15$

```
*  *  *  *  *  *  *  *
*  *  *  *  *  *  *  *
*  *  *  *  *  *  *  *
-----------------
-----------------
*  *  *  *  *  *  *  *
```

If there are 8 stars in each row and there are n such rows then total number of stars = $8 \times n$

```
*  *  * --------- *
*  *  * --------- *
------- -------- -
------- -------- -
*  *  * --------- *
```

If there are p stars in each row and there are q such rows

Then total number of stars = $p \times q$

```
*  *  * -- --- *  *  *
*  *  * ------ *  *  *
------------------
-----------------
*  *  * ------- *  *  *
*  *  * ------- *  *  *
*  *  * --- ---- *  *  *
```

If there are q+3 stars in each row and there are p+2 such rows then Total number of stars = (p+2) × (q+3)

(i) Now you think of other similar situations in which two algebraic expressions have to be multiplied?

(ii) Kareena gets up and says , when we find the area of a square of side $(x+5)$ then area is given by $(x+5) \times (x+5)$

(iii) Raju says that in the same way we can find the area of the triangle. If base and height of the triangle is $(m+3)$ and 4x then

$$\text{Its area} = \frac{1}{2} \times 4x \times (m+3)$$

(iv) Sarita points out that when we buy things, we have to carry out multiplication. For example, if price of bananas per dozen = Rs x, then cost of z dozen bananas =Rs (xz).

Suppose, the price per dozen was increased by 3 rupees and the bananas needed were less by 2 dozen then, price of bananas per dozen = $(x+3)$ and bananas needed = $(z-2)$ dozens,

Therefore, we would have to pay = Rs $(x+3) \times (z-2)$

In the entire above example we have to multiply two or more than two quantities. If the quantities are given in the form of expressions then we need to find their product. Let's learn the multiplication of algebraic expression in a systematic way.

9.5.1 Multiplying a Monomial by a Monomial

We know that

$$5 \times x = x + x + x + x + x = 5x$$

And $3 \times (5x) = 5x + 5x + 5x = 15x$

Now look at the following product

(I) $x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$

(ii) $5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

(iii) $5x \times (-3y) = 5 \times x \times (-3y) = (5) \times (-3) \times x \times y = -15xy$

(iv) $5x \times 4x^2 = 5 \times x \times 4 \times x^2 = 5 \times 4 \times x \times x^2 = 20 \times x^3 = 20x^3$

(v) $5x \times (-4xyz) = (5 \times -4) \times (x \times xyz) = -20x \times xyz = -20x^2yz$

Multiplying three or more monomials

(i) $3x \times 5y \times 4z$

$= (3x \times 5y) \times 4z$

$= 15xy \times 4z$

$= 60xyz$

(ii) $2x^2y \times (-4y^2z) \times (-7z^2x) \times (2x^2yz)$

$= [2x^2y \times (-4y^2z)] \times [(-7z^2x) \times (2x^2yz)]$

$= (-8x^2y^3z) \times (-14x^3yz^3)$

$= 112x^5y^4z^4$

EXERCISE 9.1

1. Find the product of the following pairs of monomials.

(i) $3 \times 5x$ (ii) $-5p, -2q$ (iii) $7t^2, -3n^2$
(iv) $6m, 3n$ (v) $-5x^2, -2x$

2. Complete the table.

$\times$	7	x	y	$2z$	z	$-5b$	c
7	49						
x							
$2y$							
$-3a$			$-3ay$				
b							
y							
$2x^3$							
a^4					a^4z		
z^2							

3. Multiply the following monomial?

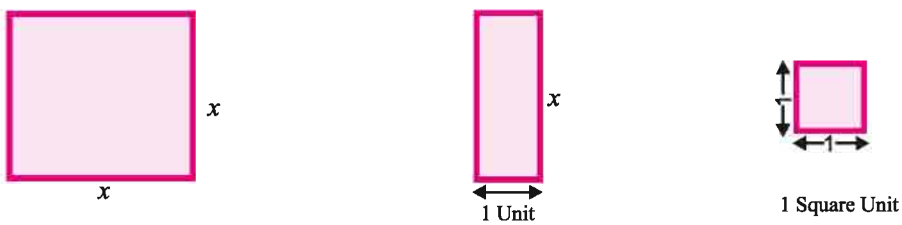
(i) xy, x^2y, xy, x (ii) m, n, mn, m^3n, mn^3
(iii) kl, lm, km, klm

4. Find the simple interest $= \dfrac{PTR}{100}$, If principal (P) $= 4x^2$, time (T) $= 5x$ and rate of interest (R) $= 5y$

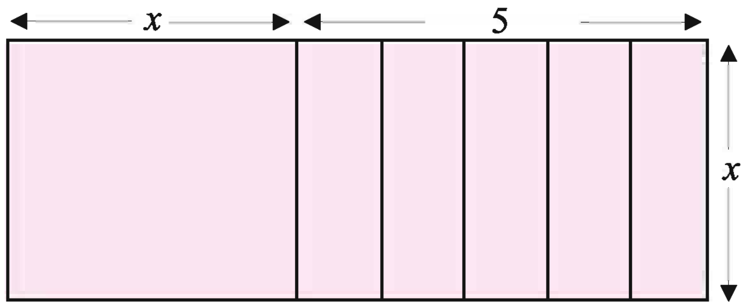
9.5.2 Multiplying a Binomial or a Trinomial by a Monomial

ACTIVITY 1

Take a square shape cardboard. let its length be x. cut rectangular shape strip of breadth 1 unit (i.e. 1cm or 1 inch) and length of x unit from it and also cut five pieces of 1 unit of square shapes.



Take a cardboard of length x and breadth $(x+5)$, i.e having area equal to $x(x+5)$. Cut it into five strips each of length x and breadth of unit length as shown in the figure.



Length of new rectangle = $x + 5$ unit
Breadth of new rectangle = x unit
Area of new rectangle = $x(x + 5)$ square unit

So , $x(x+5)$ = area of square shaped cardboard of length x + area of five strips added

$= x^2 + x + x + x + x + x$

$= x^2 + 5x$

Once again

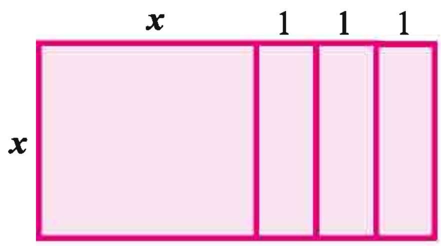
$x(x + 5) = x \times x + x \times 5$

$= x^2 + 5x$

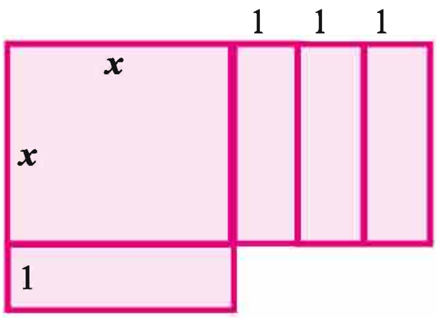
9.5.3 Multiplying a binomial by a binomial

$(x + 1)(x + 3)$

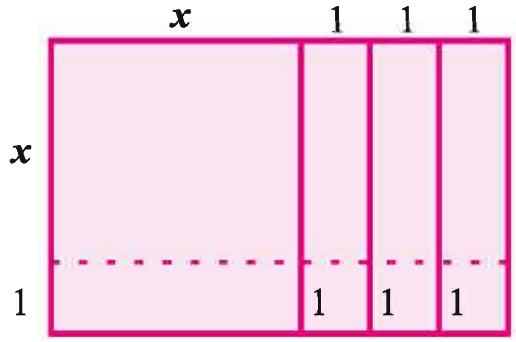
Take a cardboard of length x
Add three strips of length x and breadth 1 unit to it.



Now add one strip of same dimension below it



Now add three square of length 1 unit to the above figure so that it becomes a rectangle



Length of new rectangle = $(x+3)$ unit
Breadth of new rectangle = $(x+1)$ unit
Area of new rectangle = $(x+3)(x+1)$ square unit

So, $(x+3)(x+1)$ = area of square shaped cardboard of side x + area of four stripe + area of three square

$= x^2 + 4x + 3$

To multiply two binomials, multiply each term of the first binomial with the second binomial and then apply distributive property to multiply monomial with binomial. Then add the like terms to get the solution.

Once again $(x+3)(x+1)$

$= x(x+1) + 3(x+1)$
$= x^2 + x + 3x + 3$
$= x^2 + 4x + 3$

9.5.4 Multiplying a binomial by a trinomial

Algebraic expression $(2x + 3y)$ is binomial and $(3x + 4y - 5z)$ is trinomial. Now we multiply a binomial $(2x+3y)$ by a trinomial $(3x + 4y - 5z)$

$(2x+3y)(3x+4y-5z) = 2x(3x + 4y - 5z) + 3y(3x + 4y - 5z)$

(multiply each term of the binomial to the trinomial)

$= 2x \times 3x + 2x \times 4y - 2x \times 5z + 3y \times 3x + 3y \times 4y - 3y \times 5z$
$= 6x^2 + 8xy - 10xz + 9xy + 12y^2 - 15yz$

Now by adding or subtracting like terms we get,

$= 6x^2 + (8xy + 9xy) - 10xz + 12y^2 - 15yz$
$= 6x^2 + 17xy - 10xz + 12y^2 - 15yz$

EXERCISE 9.2

1. Multiply the given binomials.
   (i) $(2x + 5)$ and $(3x - 7)$
   (ii) $(x - 8)$ and $(3y + 5)$
   (iii) $(1.5p - 0.5q)$ and $(1.5p + 0.5q)$
   (iv) $(a + 3b)$ and $(x + 5)$
   (v) $(2lm + 3l^2)$ and $(3lm - 5l^2)$
   (vi) $(\frac{3}{4}a^2 + 3b^2)$ and $(4a^2 - \frac{5}{3}b^2)$

2. Find the product.

(i) $(3x + 8)(5 - 2x)$ (ii) $(x + 3y)(3x - y)$

(iii) $(a^2 + b)(a + b^2)$ (iv) $(p^2 - q^2)(2p + q)$

3. Simplify:

(i) $(x + 5)(x - 7)+35$ (ii) $(a^2 - 3)(b^2 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$ (iv) $(a+b)(c-d) + (a-b)(c+d) + 2(ac+bd)$

(v) $(a + b)(a^2 - ab + b^2)$ (vi) $(a + b + c)(a + b - c)$

(vii) $(a + b)(a - b) - a^2 + b^2$

9.6 What is an Identity?

Consider the equation, $x(x+5) = x^2+5x$

We shall evaluate both sides of this equation for some value of x-

	LHS	RHS
$x = 1$	$1(1+5) = 1 \times 6 = 6$	$(1)^2 + 5\times1 = 1+5 = 6$
$x = 2$	$2(2+5) = 2\times7 = 14$	$(2)^2 + 5\times2 = 4+10 = 14$

Similarly you can verify the given expression for some other values also. We have found that for any value of x, LHS = RHS.

Such an equality, true for every value of the variable is called an identity

ACTIVITY $(x + a)(x + b)$

(i) Area of square ABCD = x^2

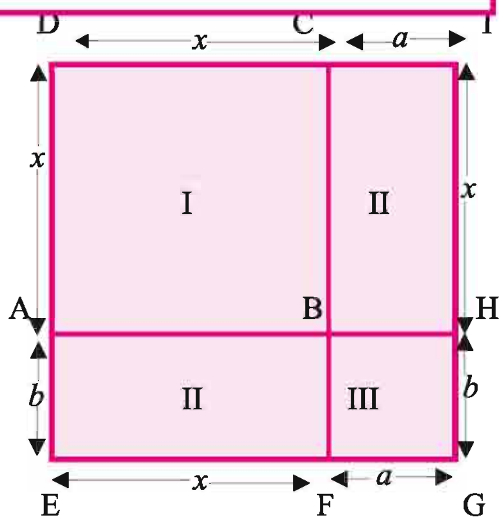
(ii) Area of rectangle AEFB = $x \times b = bx$

(iii) Area of rectangle FGHB = $a \times b = ab$

(iv) Area of rectangle BHIC = $a \times x = ax$

Area of rectangle DEGI = I +II+III+IV

$$(x+a)(x+b) = x^2 + ax + bx + ab$$
$$= x^2 + x(a+b) + ab$$



9.6.1 Standard Identities

Let us first consider the product $(a + b)(a + b)$ or $(a + b)^2$

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b)\\
&= a(a+b) + b(a+b)\\
&= a^2 + ab + ab + b^2\\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Because $ab = ba$

Hence $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$ - I

Clearly, this is an identity, since the expression on the RHS is obtained from the LHS by actual multiplication. One may verify that for any value of a and any value of b, the values of the two sides are equal.

Next we consider, $(a - b)(a - b)$ or $(a - b)^2$

$$
\begin{aligned}
(a-b)^2 &= (a-b)(a-b)\\
&= a(a-b) - b(a-b)\\
&= a^2 - ab - ba + b^2\\
&= a^2 - 2ab + b^2
\end{aligned}
$$

$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$ - II

Finally, consider (a + b) (a – b).

$$
\begin{aligned}
(a+b)(a-b) &= a(a-b) + b(a-b)\\
&= a^2 - ab + ab - b^2\\
&= a^2 - b^2
\end{aligned}
$$

Hence $\boxed{(a+b)(a-b) = a^2 - b^2}$ - III

Do and learn:-

1. Put $-b$ in place of b in Identity (I). Do you get Identity (II) ?

9.6.2 Application of Identities

We shall now see how, for many problems on multiplication of binomial expressions and also of numbers, use of the identities gives a simple alternative method of solving them.

Example 3: Solve $(2x + 3y)^2$ and $(103)^2$ with the help of identities

(I) $(2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$

$= 4x^2 + 12xy + 9y^2$

(i) $(103)^2 = (100+3)^2$
$= (100)^2 + 2(100)(3) + (3)^2$
$= 10000 + 600+9$
$= 10609$

We can directly multiply 103 by 103 to get the answer. Did you notice that we can solve $(103)^2$ by identity (1) ?

Example 4. Find the value of $(7.9)^2$

$=(7.9)^2 = (8.0 - 0.1)^2$
$= (8.0)^2 - 2 \times 8.0 \times 0.1 + (0.1)^2$
$= 64 - 1.6 + 0.01$
$= 62.41$

EXERCISE 9.3

1. Find the products of the following using suitable identity.

(i) $(x + 5)(x + 5)$ (ii) $(3x + 2)(3x + 2)$
(iii) $(5a - 7)(5a - 7)$ (iv) $(3p - \frac{1}{2})(3p - \frac{1}{2})$
(v) $(1.2m - 0.3)(1.2m + 0.3)$ (vi) $(x^2 + y^2)(x^2 - y^2)$
(vii) $(6y+7)(-6y+7)$ (viii) $(7a - 9b)(7a - 9b)$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products:

(i) $(x + 1)(x + 2)$ (ii) $(3x + 5)(3x + 1)$
(iii) $(4x - 5)(4x - 1)$ (iv) $(3a + 5)(3a - 8)$
(v) $(xyz - 1)(xyz - 2)$

3. Find the following squares by using the identities.

(i) $(b - 7)^2$ (ii) $(xy + 3z)^2$ (iii) $(6m^2 - 5n)^2$ (iv) $(\frac{3}{2}x + \frac{2}{3}y)^2$

4. Simplify

(i) $(a^2 - b^2)^2$ (ii) $(2n + 5)^2 - (2n - 5)^2$
(iii) $(7m - 8n)^2 - (7m + 8n)^2$ (iv) $(m^2 - n^2m)^2 + 2m^3n^2$

5. Show that.
   (i) $(2a + 3b)^2 - (2a - 3b)^2 = 24ab$
   (ii) $(4x + 5)^2 - 80x = (4x - 5)^2$
   (iii) $(3x - 2y) + 24xy = (3x + 2y)^2$
   (iv) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

6. Using identities, evaluate the following.
(i) 99^2 (ii) 103^2 (iii) 297×303 (iv) 78×82

7. Using $a^2 - b^2 = (a + b)(a - b)$, find
(i) $10^2 - 99^2$ (ii) $(10.3)^2 - (9.7)^2$ (iii) $153^2 - 147^2$

8. Using $a^2 - b^2 = (a + b)(a - b)$, find
(i) 103×102 (ii) 7.1×7.3 (iii) 102×99 (iv) 9.8×9.6

We Learnt

1. Expressions are formed with the help of **variables** and **constants**.
2. Expressions that contain exactly one, two and three terms are called **monomials**, **binomials** and **trinomials** respectively. In general, any expression containing one or more terms with none zero coefficients (and with variable having non –negative exponent) is called a **polynomial.**
3. In an expression the term with the highest power is called the power of that expression.
4. **Like** terms are formed with the same variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.
5. While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.
6. A monomial multiplied by a monomial always gives a monomial.
7. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
8. In carrying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial). Note that in such multiplication, we may get terms in the product which are like and have to be combined.
9. An **identity** is an equality, which is true for all values of the variables in the equality
10. The following are the standard identities

   (i) $(a + b)^2 = a^2 + 2ab + b^2$ (I)
   (ii) $(a - b)^2 = a^2 - 2ab + b^2$ (II)
   (iii) $(a + b)(a - b) = a^2 - b^2$ (III)

CHAPTER 10

FACTORIZATION

10.1 Factors

In the previous classes we have learnt the prime factorization of natural numbers

Like -

$$24 = \underline{2} \times 2 \times 2 \times \underline{3}$$
$$30 = \underline{2} \times \underline{3} \times 5$$

We see that 2 and 3 are the common prime factors in the factorization of 24 and 30.these type of prime factors are known as common factors.

Example-1 Find the common factors of 32, 56 and 72

Solution: The factors of 32, 56 and 72 is

$$32 = \underline{2 \times 2 \times 2} \times 2 \times 2$$
$$56 = \underline{2 \times 2 \times 2} \times 7$$
$$72 = \underline{2 \times 2 \times 2} \times 3 \times 3$$

Common factors $= 2 \times 2 \times 2 = 8$

Example-2 Find the common factors of 25 and 27.

Solution: The factors of 25 and 27 is

$$25 = 5 \times 5$$
$$27 = 3 \times 3 \times 3$$

They do not have any common factors therefore the common factors among them is 1

In the same way we can find the common factors of the Algebraic expression by expressing them in their factors.

10.2 Factors of Algebraic Expressions

We have seen in Class VII that the terms of algebraic expressions can be shown in tree graph as products of factors.

For example, in the algebraic expression $3xy + 5x$

The term $3xy$ has been formed by the factors 3, x and y i.e.

$$3xy = 3 \times x \times y$$
$$5x = 5 \times x$$

Observe that the factors 3, x and y of $3xy$ cannot be further factorized.
We may say that 3, x and y are 'prime' factors of $3xy$. In algebraic expressions, we use the word 'indivisible' in place of 'prime'. We say that $3 \times x \times y$ is the indivisible form of $3xy$

In the same way

$$5x\,(x+3) = 5 \times x \times (x+3)$$
$$10xy\,(x+2)(x+9) = 10 \times x \times y \times (x+2) \times (x+9)$$

> Prime factorized form of $3xy$ is $3 \times x \times y$

10.2.1 Method of Factorization

When we factorize an algebraic expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions. Expressions like $5xy$, $2x^2$, $2x^2y$, $2x(y+2)$, $6(y+1)$ are already in factor form. Their factors can be just read off from them, as we already know. On the other hand consider expressions like $6x + 3$, $2a + 4b$, $y + 5y$, $x^2 + 7x + 12$, It is not obvious what their factors are. In order to find out the factors of this expression we have the following two methods.

1) Method of common factors
2) Factorization by regrouping terms

10.2.1.1 Method of Common Factors

We begin with a simple example: Factories $6x + 9$
We shall write each term as a product of irreducible factors

$$6x = 2 \times 3 \times x$$
$$9 = 3 \times 3$$
$$6x + 9 = (2 \times 3 \times x) + (3 \times 3)$$

Notice that factor 3 is common to both the terms. Hence
$= 3[2 \times x + 3] = 3(2x + 3)$ these are the irreducible factor of $6x + 9$

<u>For Binomial</u> $ka + kb = k\,(a + b)$

<u>For Trinomial</u> $ka + kb + kc = k\,(a + b + c)$

10.2.1.2 Factorization by regrouping terms

There is no common factor in the term $3ab + 3b + 2a + 2$ .so we find the common factors from the given expression by making the group of different terms. For example the expression $3ab + 3b + 2a + 2$ have two group, the first one is $3ab + 3b$ and second is $2a + 2$.

$= \underline{3ab + 3b} + \underline{2a + 2}$
$= 3b(a+1) + 2(a+1)$
$= (a+1)(3b+2)$

Here a+1 is the common factor. Let's repeat the process.

Now if the given expression is written in the form

$3ab + 2 + 3b + 2a$

We don't get a common factor after making groups of 2 terms

Then it will not be easy to see the factorization. Rearranging the expression as

$3ab + 2 + 3b + 2a = 3ab + 3b + 2a + 2$
$= 3b(a+1) + 2(a+1)$
$= (a+1)(3b+2)$

When there is no common factor, We take 1 as the common factor.

$(3b+2)$ is the common factor in both expression. So let's repeat the process.

Please note that $(a+1)(3b+2) = (3b+2)(a+1)$

i.e. multiplication follow commutative property.

We can write it in the form as

$Ka + Kb + Pa + Pb$
$= K(a+b) + P(a+b)$
$= (K+P)(a+b)$

Example 3: Factorise $6xy - 4y + 6 - 9x$.

Solution:

Step 1 Check if there is a common factor among all terms. There is none.

Step 2 Two group forms, the first is $6xy - 4y$ and second is $6 - 9x$.

$6xy - 4y$	$6 - 9x$
$= 2y(3x-2)$	$= 3(2-3x)$

Step 3 Both the group is not same so rearranging second group we get

$6 - 9x = -(9x-6)$
$= -3(2x-3)$

Putting them together, we get

$6xy - 4y + 6 - 9x = \underline{6xy - 4y} - \underline{9x + 6}$ (by rearranging)
$= 2y(3x-2) - 3(3x-2)$
$= (3x-2)(2y-3)$

The factors of $(6xy - 4y + 6 - 9x)$ are $(3x - 2)$ and $(2y - 3)$.

EXERCISE 10.1

1. Find the common factors of the given terms.

(i) $12x$, 36 (ii) $14\,pq$, $28p^2q^2$ (iii) $6abc$, $24ab^2$, $12a^2b$
(iv) $16x^3$, $-4x^2$, $32x$ (v) $10pq$, $20qr$, $30rp$ (vi) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

2. Factorize the following expressions.(by common factors method)

(i) $6p - 12q$ (ii) $7a^2 + 14a$ (iii) $10a^2 - 15b^2 + 20c^2$
(iv) $ax^2y + bxy^2 + cxyz$ (v) $x^2yz + xy^2z + xyz^2$ (vi) $-16z + 20z^3$

3. Factorize(by regrouping method)

(i) $2xy + 3 + 2y + 3x$ (ii) $z - 7 - 7xy + xyz$
(iii) $6xy - 4y + 6 - 9x$ (iv) $15pq + 15 + 9q + 25p$

10.3 Factorization by using identities

We know that

(i) $(a+b)^2 = a^2 + 2ab + b^2$ (ii) $(a-b)^2 = a^2 - 2ab + b^2$ (iii) $(a+b)(a-b) = a^2 - b^2$

We use these identities to find the factor of the given expressions. With the help of these identities we can find the factors of the trinomial terms.

Example 4: Factorize the following

(1) $x^2 + 6x + 9$

Make the given expression in the form,

$(a+b)^2 = a^2 + 2ab + b^2$
$= x^2 + 2\times 3\times x + (3)^2$
$= (a)^2 + 2\times(a)(b) + (b)^2$
$= (x+3)^2$

NOTE THAT

(i) The given expression has three terms

(ii) Expression is of the form $a^2 + 2ab + b^2 = (x+3)^2$ where a = x, and b = 3.

Grouping Method

$x^2 + 6x + 9$
$= x^2 + 3x + 3x + 9$
$= x(x+3) + 3(x+3)$
$= (x+3)(x+3)$
$= (x+3)^2$

(2) $9a^2 - 30ab + 25b^2$
$= (3a)^2 - 2\times 3a\times 5b + (5b)^2$
$= (a)^2 - 2\times a\times b + (b)^2$
$= (3a-5b)^2$

NOTE THAT

(i) the given expression has three terms

(ii) expression is of the form $a^2 - 2ab + b^2$ where a $= 3a$ and $b = 5b$

Grouping method :

$9a^2 - 30ab + 25b^2$
$= 9a^2 - 15ab - 15ab + 25b^2$
$= 3a(30-5b) - 5b(3a-5b)$
$= (3a-5b) - 5b(3a-5b)$
$= (3a-5b)(3a-5b)$
$= (3a-5b)^2$

(3) $4x^2 - 9a^2$

$= (2x)^2 - (3a)^2$

$= (2x + 3a)(2x - 3a)$

(i) This expression contains two terms therefore we use the identity

$a^2 - b^2 = (a + b)(a - b)$

Here $a^2 = 4x^2$ and $b^2 = 9a^2$

$a = 2x$ and $b = 3a$

(4) $2x^2 + 16x + 32$

$= 2[x^2 + 18x + 16]$

$= 2[(x^2) + 2x \times 4 + (4)^2]$

$= 2(x + 4)^2$

(i) In this expression first and last term is not prefect square

(ii) 2 is common in the expression

(iii) this expression is of $(a^2 + 2ab + b^2)$

10.3.1 Factors of the form $(x + a)(x + b)$

Let us now discuss how we can factorize expressions in one variable, like $x^2 + 8x + 12$, $y^2 - 5y + 6$, $z^2 - 4z - 12$, $x^2 + 2x - 15$ etc. Observe that these expressions are not of the type $(a + b)^2$ or $(a - b)^2$, i.e., they are not perfect squares. These expressions obviously also do not fit the type $(a^2 - b^2)$ either.

They, however, seem to be of the type $x^2 + (a + b)x + ab$.

Therefore in order to factorize these type of expression we use the identity,

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

For that we have to look at the coefficients of x and the constant term. Let us see how it is done in the following example.

Do and learn

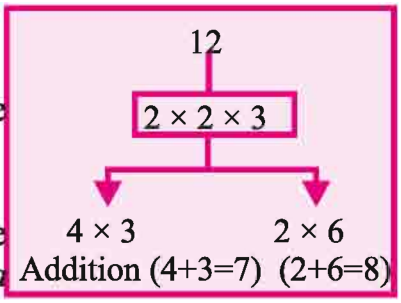
Find two integers a and b such that

$a+b$	ab	a	b
8	15	5	3
13	12		
-1	-20	-5	4
-5	4		
10	21		
-1	-12		
-11	10		
-7	10		

Table - 2

Example 5: Factories the following

Solution: (i) $x^2 + 8x + 12$



by comparing the given expression with the identity $x^2+(a+b)x+ab$ we get

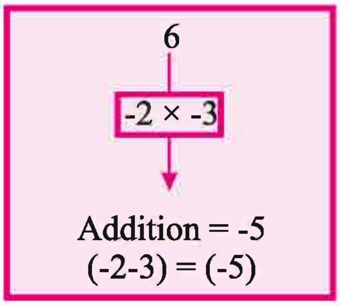
$a+b=8$ and $ab=12$

for finding the value of a and b we must select the value of a and b in such a way that the product of a and b is equal to 12 and their sum must be 8

Therefore we put $a=2$ and $b=6$ so that $ab=12$ and $a+b=8$

Therefore $x^2 + 8x + 12 = (x+2)(x+6)$

(ii) $y^2 - 5y + 6$



by comparing we get

$a+b=-5$ $\qquad ab=6$

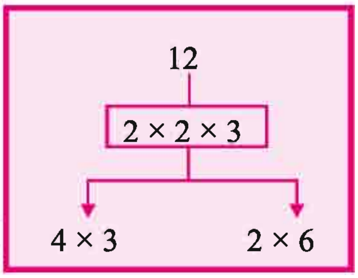
$y^2 - (2+3)y + 6$

$= \underline{y^2 - 2y} - \underline{3y + 6}$ (After grouping)

$= y(y-2) - 3(y-2)$

$= (y-2)(y-3)$

(iii) $z^2 - 4z - 12$



$a+b=-4$ $\qquad ab=-12$

$ab=-12$ that means that one among digits a or b is a negative integer. Further, $a+b=-4$, this means the one with larger numerical value is negative

Hence by taking $a=-6$ and $b=2$

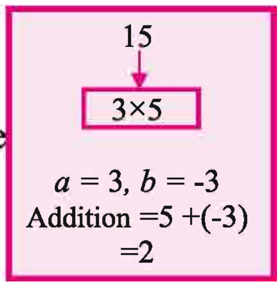
$z^2 - 4z - 12 = z^2 - 6z + 2z - 12$

$= z(z-6) + 2(z-6)$

$= (z-6)(z+2)$

Example 6: Find the factors of $x^2 + 2x - 15$

Solution: $x^2 + 2x - 15$, by comparing we get



$a+b=2$ and ab = -15

$ab=-15$, means that one of the a and b is negative

$a+b=2$, means that greater number should be of positive

$x^2 + 2x - 15 = \underline{x^2 + 5x} - \underline{3x - 15}$

$= x(x+5) - 3(x+5)$

$= (x+5)(x-3)$

EXERCISE 10.2

(1) Factorize the following expressions.

(i) $a^2 - 4$ (ii) $a^2 - 49\,b^2$ (iii) $p^3 - 121p$ (iv) $(a-b)^2 - c^2$
(v) $a^4 - b^4$ (vi) $5x^3 - 125x$ (vii) $63a^2 - 112b^2$ (viii) $9x^2y^2 - 16$
(ix) $(l+m)^2 - (l-m)^2$

(2) Factorize the following expressions.

(i) $lx^2 + mx$ (ii) $2x^3 + 2xy^2 + 2xz^2$
(iii) $a(a+b) + 4(a+b)$ (iv) $(xy+y) + x + 1$
(v) $5a^2 - 15a - 6c + 2ac$ (vi) $am^2 + bm^2 + bn^2 + an^2$

(3) Factorize the following expressions.

(i) $x^2 + 5x + 6$ (ii) $q^2 + 11q + 24$ (iii) $m^2 - 10m + 21$
(iv) $x^2 + 6x - 16$ (v) $x^2 - 7x - 18$ (vi) $k^2 - 11k - 102$
(vii) $y^2 + 2y - 48$ (viii) $d^2 - 4d - 45$ (ix) $m^2 + 16m + 63$
(x) $n^2 - 19n - 92$ (xi) $p^2 - 10p + 16$ (xii) $x^2 + 4x - 45$

10.4 Division of Algebraic Expressions

We have learnt how to add and subtract algebraic expressions. We also know how to multiply two expressions. We have not however, looked at division of one algebraic expression by another..

We recall that division is the inverse operation of multiplication. Thus,

$$5 \times 8 = 40 \text{ gives}$$
$$40 \div 8 = 5 \text{ or } 40 \div 5 = 8.$$

We may similarly follow the division of algebraic expressions. For example

(i) $3x \times 5x^2 = 15x^3$
Therefore $15x^3 \div 3x = 5x^2$
And also $15x^3 \div 5x^2 = 3x$

(ii) Similarly $5x\,(x+3) = 5x^2 + 15x$
$(5x^2 + 15x) \div 5x = x+3$
$(5x^2 + 15x) \div (x+3) = 5x$

10.4.1 Division of a Monomial by Another Monomial

$$8x^3 \div 2x = \frac{2\times2\times2\times x\times x\times x}{2\times x} = 2\times2\times x\times x = 4x^2$$

Example:7 Do the following divisions.

(i) $-20x^5 \div 5x^2$ (ii) $7a^2b^2c^2 \div 21abc$ (iii) $63p^2q^3r \div -3p^4q$

Solution (i) $-20x^5 = -2 \times 2 \times 5 \times x \times x \times x \times x \times x$

$5x^2 = 5 \times x \times x$

Therefore $-20x^5 \div 5x^2 = \dfrac{-2 \times 2 \times 5 \times x \times x \times x \times x \times x}{5 \times x \times x} = -2 \times 2 \times x \times x \times x$

$= -4x^3$

(ii) $7a^2b^2c^2 \div 21abc$

$7a^2b^2c^2 = 7\times a\times a\times b\times b\times c\times c$

$21abc = 21\times a\times b\times c$

Therefore $7a^2b^2c^2 \div 21abc = \dfrac{7\times a\times a\times b\times b\times c\times c}{21\times a\times b\times c} = \dfrac{a\times b\times c}{3} = \dfrac{abc}{3}$

(iii) $63p^2q^3r \div -3p^4q$

$63p^2q^3r = 3\times 3\times 7\times p\times p\times q\times q\times q\times r$

$-3p^4q = -3\times p\times p\times p\times p\times q$

Therefore

$63p^2q^3r \div -3p^4q = \dfrac{3\times 3\times 7\times p\times p\times q\times q\times q\times r}{-3\times p\times p\times p\times p\times q} = \dfrac{-21q^2r}{p^2}$

10.4.2 Division of a Polynomial by a Monomial

Let us consider the division of the trinomial $8y^3 + 6y^2 + 12y$ by the monomial 2y.

$$\frac{8y^3 + 6y^2 + 12y}{2y} = \frac{2y(4y^2 + 3y + 6)}{2y} = 4y^2 + 3y + 6$$

This can also be written as

$$\frac{8y^3 + 6y^2 + 12y}{2y} = \frac{8y^3}{2y} + \frac{6y^2}{2y} + \frac{12y}{2y} = 4y^2 + 3y + 6$$

Each term of the numerator is divided by the monomial of the denominator

Example:8 Divide $(18x+12x^3-6x^2)$ by $(-3x)$

Solution: $\dfrac{18x + 12x^3 - 6x^2}{-3x} = \dfrac{6x(3 + 2x^2 - x)}{-3x} = -2(3 + 2x^2 - x)$

$= 6 - 4x^2 + 2x$

$= -4x^2 + 2x - 6$

10.4.3 Division of Algebraic Expressions Continued (Polynomial ÷ Polynomial)

Consider $(7x^2 + 14x) \div (x + 2)$

$$\frac{(7x^2 + 14x)}{(x + 2)} = \frac{7x(x + 2)}{(x + 2)} = 7x$$

Example:9 Divide $(x^2 + 7x + 10)$ by $(x+2)$

Solution: $\dfrac{x^2 + 7x + 10}{x + 2} = \dfrac{x^2 + 5x + 2x + 10}{x + 2}$

$= \dfrac{x(x+5) + 2(x+5)}{x + 2} = \dfrac{(x+5)(x+2)}{x + 2} = x + 5$

10
5 2
Adding 5+2 = 7

Example: 10 Divide $p(5p^2-80)$ by $5p(p-4)$

Solution: $\dfrac{p(5p^2 - 80)}{5p(p - 4)} = \dfrac{5p(p^2 - 16)}{5p(p - 4)}$

$\dfrac{5p[(p)^2 - (4)^2]}{5p(p - 4)} = \dfrac{5p(p - 4)(p + 4)}{5p(p - 4)} = p + 4$

Do and Learn

Find the error

(I) $3x + x + 4x = 56$

$7x = 56$

$x = \dfrac{56}{7}$

We generally do not show 1 as a coefficient of any term but in addition of like term we add it with the respective terms

Find the error

(2) Find the value of $5x$ at $x = -2$

$= 5 - 2 = 3$

Find the error and also find the correct value

(3) Solution of the expression is given in the column A and B. Find which of the solution is correct

Expression	A	B
$3(x-4)$	$3x-4$	$3x-12$
$(2x)^2$	$2x^2$	$4x^2$
$(x+4)^2$	x^2+16	$x^2+8x+16$
$(x-3)^2$	x^2-9	x^2-6x+9
$\dfrac{y+1}{5}$	$y+1$	$\dfrac{y}{5}+1$

EXERCISE 10.3

1. Carry out the following divisions.

(i) $28x^4$ by $56x$ (ii) $-36y^3$ by $9y^2$

(iii) $34x^3y^3z^3$ by $51xy^2z^3$ (iv) $12a^8b^8$ by $(-6a^6b^4)$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x)$ by $3x$ (ii) $(x^3 + 2x^2 + 3x)$ by $2x$

(iii) $(p^3q^6 - p^6q^3)$ by p^3q^3 (iv) $(3x^8 - 4x^6 + 5x^4)$ by x^4

3. Work out the following divisions.

(i) $10y(6y+21) \div 5(2y+7)$ (ii) $9x^2y^2(3z-24) \div 27xy(z-8)$

(iii) $(10y+14) \div 2$ (iv) $(6x-5) \div (2x-5)$

4. Factorize the expressions and divide them as directed

(i) $(y^2+7y+10) \div (y+5)$ (ii) $(5x^2-25x+20) \div (x-1)$

(iii) $12xy(9x^2 -16y^2) \div 4xy(3x+4y)$ (iv) $4yz(z^2+6z-16) \div 2y(z+8)$

WE LEARNT

1. When we factorize an expression, we write it as a product of factors.

2. A prime factor is a factor which cannot be expressed further as a product of factors.

3. Number of expressions to be factorized are of the form or can be put into the form :

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b$

(iii) $(a+b)(a-b) = a^2 - b^2$

(iv) $x^2 + (a+b)x + ab = (x+a)(x+b)$

(v) If we have the expression of type $x^2 + (a+b)x + ab$. Then this will have the factor of $(x+a)(x+b)$ form.

4. We know that in the case of numbers, division is the inverse of multiplication. This idea is applicable also to the division of algebraic expressions.

5. In the case of divisions of algebraic expressions that we studied in this chapter, we have

Dividend = Divisor × Quotient.

In general, however, the relation is

Dividend = Divisor × Quotient + Remainder

Chapter 11

Linear Equations with One Variable

11.1 In previous classes, you have come across many algebraic expression and equations. Some expressions and equations are given below:

(i) $2x+5$ (ii) $x+y=0$ (iii) $3xy=5z$ (iv) $10x^2y$

In these, which are the equations?

You would remember that equality "= "sign is always used in equations.

In this chapter, we will study about the single variable linear equations i.e., equations having single variable and that variable should have maximum power.

$$x+2=7, \qquad 2x-3=0$$

In previous class, we learnt to solve such kind of equations which have one sided variables, with this, we also learnt to solve the equations having rational coefficient, such as:

$$5x-4=26$$

And

$$\frac{2x}{3}-\frac{x}{2}=30$$

Let us practice to solve an equation:

Equation $3x+9=15$ solution $x=2$

Sr. No.	Both Sides	New Equation	Solution
1.	Adding 2	$3x+11=17$	$x=2$
2.	Subtracting 3	$3x+6=12$	$x=$ ....
3.	Multiplying by 2	$6x+18=$......	$x=$ ....
4.	Dividing by 3	...............$=5$	$x=$ ....

In this way, by adding, subtracting, multiplying or dividing any number in both sides of the equation the formation of equation is changed but the solution remains the same. Thus, we take help of these operations accordingly to solve the equation.

Now, we would learn to solve such type of equations which may have variables on both sides.

To solve the equation $8x-13=5x-7$ means to make its formation $x=$............

So, we have to keep a variable in LHS and a constant in RHS, for this, applying same operation or transposition on both sides.

Ex 1: Solve the equation $8x - 13 = 5x - 7$

Sol: Balancing method:

$8x - 13 + 13 = 5x - 7 + 13$ (adding 13 on both sides)

The balance is not disturbed

Or $8x = 5x + 6$

Or $8x - 5x = 5x + 6 - 5x$ (subtracting $5x$ from both sides)

Or $3x = 6$

Or $\frac{3x}{3} = \frac{6}{3}$ (by dividing both sides by 3)

Or $x = 2$

Sol 2: (Transposition method)

$8x - 13 = 5x - 7$

Or $8x - 5x = -7 + 13$ (by transposition of $5x$ and -13)

Or $3x = 6$

Or $x = 6/3$ (by transposition of coefficient 3)

Or $x = 2$

Since, transposition method is the shortest and simplest way of comparison method. Thus we will use only transposition method.

Example 2: Solve the equation $\frac{x}{5} = \frac{7 - 6x}{3}$

Sol: $\frac{x}{5} = \frac{7 - 6x}{3}$

Or $3x = 35 - 30x$

(by taking LCM 15 and multiplying both side or by cross multiplication)

or $3x + 30x = 35$

or $33x = 35$

$x = \frac{35}{33}$

Example 3: Solve $\frac{6x + 1}{3} + 1 = \frac{x - 3}{6}$

Sol: $\frac{6x + 1}{3} + 1 = \frac{x - 3}{6}$

Or $\frac{6x + 4}{3} = \frac{x - 3}{6}$

Or $\frac{6x+1+3}{3} = \frac{x-3}{6}$

Or $\frac{6x+4}{3} = \frac{x-3}{6}$

Or $\frac{6x+4}{3} \times 6 = \frac{x-3}{6} \times 6$ [Multiplying on both side by LCM 6]

Or $12x + 8 = x - 3$

Or $12x - x = - 3 - 8$

Or $11x = - 11$

Or $x = \frac{-11}{11}$

Or $x = - 1$

Example 4: Solve the equation $\frac{x+1}{2} + \frac{x+2}{3} = \frac{x+3}{4} + \frac{x+4}{5}$

Sol. $\frac{x+1}{2} + \frac{x+2}{3} = \frac{x+3}{4} + \frac{x+4}{5}$

Multiplying both sides by 60 LCM of 2,3,4 and 5

$$60 \times \frac{x+1}{2} + 60 \times \frac{x+2}{3} = 60 \times \frac{x+3}{4} + 60 \times \frac{x+4}{5}$$

Or $30(x + 1) + 20(x + 2) = 15(x + 3) + 12(x + 4)$

Or $30x + 30 + 20x + 40 = 15x + 45 + 12x + 48$

Or $30x + 20x - 15x - 12x = 45 + 48 - 30 - 40$

Or $50x - 27x = 93 - 70$

Or $23x = 23$

Or $x = \frac{23}{23}$

Or $x = 1$

If coefficient or numbers are in decimal or fraction form then it is solved by changing them in simple fraction and multiply by least common multiple

Example 5: Solve $0.5m + 1.5 = 0.25m - 1.5$

Sol: $0.5m + 1.5 = 0.25m - 1.5$

$\frac{5}{10}m + \frac{15}{10} = \frac{25}{100}m - \frac{15}{10}$ (Changing decimal fraction into simple fraction)

$50m + 150 = 25m - 150$ [by multiplying 100 since LCM of 10 and 100 is 100]

$50m - 25m = -150 - 150$

$25m = -300$

$m = \frac{-300}{25}$

$m = -12$

If there is a variable in numerator and denominator in fraction then apply cross multiplication and then transpose and solve.

Example 6: Solve the equation $\frac{(2x + 5)}{(3x + 1)} = \frac{3}{11}$

Sol: By cross multiplication

$11(2x + 5) = 3(3x + 1)$

$22x + 55 = 9x + 3$

$22x - 9x = 3 - 55$

$13x = -52$

$x = \frac{-52}{13}$

$x = -4$

Do and Learn:

Solve the following equations-

1. $\frac{2x}{x + 6} = 1$ 2. $10 = x + 3$ 3. $16 = 7x - 9$ 4. $\frac{x + 5}{x} = 2\frac{2}{3}$

Exercise 11.1

1. $6x + 3 = 4x + 11$
2. $3(x + 5) = 4x + 9$
3. $3x + 2(x + 3) = 21$
4. $\frac{x + 1}{2} + \frac{x + 2}{3} = \frac{2x - 5}{7} + 9$
5. $\frac{3x - 2}{5} = 4 - \left(\frac{x + 2}{3}\right)$
6. $\frac{x + 2}{2} + \frac{x + 4}{3} = \frac{x + 6}{4} + \frac{x + 8}{5}$

7. $0.6x + 0.25x = 0.45x + 1.2$

8. $2.5x - 7 = 0.5x + 13$

9. $\frac{7x+4}{x+2} = \frac{-4}{3}$

10. $\frac{4x+8}{5x+8} = \frac{5}{6}$

11.2 Solution of word problem

Lets revise to make mathematical sentence, if there is any number x then fill in the blanks:

5 More than the number	= $x + 5$
3 Less than the number	= ....
Half of the number	= ....
7 Less half of the number	= ....
4 More than one third of the number	= ....
6 More than triple of the number	= ....
3 Less than 5 times of the number	= ....

The solution of many problems, puzzles etc of daily life can be obtained by the equations. For this purpose, following steps are used:

1. Read the given problem carefully. Find the known and unknown quantities .
2. Now write the unknown quantity in form of variable quantity x.
3. Write the all statements in mathematical form (algebraic terms and expression).
4. According to the condition of the question, equal quantities are written in form of equation.
5. Find the value of variable by solving equations.
6. Verify the answer according to the terms of the questions.

Example 7: Sum of two numbers is 60. Three times of the smaller number is equal to the double of the large number. Find the numbers.

Sol: Say, the smaller number = x

According to the question, the larger number = $60 - x$

Three times of the smaller number = $3x$

Double of the larger number = $2(60 - x)$

According to the condition of the question,

$$3x = 2(60-x)$$

$$3x = 120 - 2x$$

$3x + 2x = 120$

$5x = 120$

$x = \frac{120}{5}$

$x = 24$

Another number $= 60 - x = 60 - 24 = 36$

Ans. 24, 36

Use the following formula to make two-digit numbers-
Number = 10 × tens digit + units digit

Example 8: In a number, units digit is 3 less than the tens digit. Number is greater than 3 of the 7 times of sum of the digits. Find the numbers.

Sol: Say, tens place digit $= x$

Then, units place digit $= x$-3

Number = 10 × tens digit + units digit

$= 10 \times x + x - 3$

$= 11x - 3$

Sum of the digits $= (x + x - 3)$

According to the condition given in the question,

$11x - 3 = 7(x + x - 3) + 3$

$11x - 3 = 7x + 7x - 21 + 3$

$11x - 7x - 7x = -21 + 3 + 3$

$-3x = -15$

$x = \frac{-15}{-3}$

$x = 5$

Thus, the desired number,

$= 11x - 3$

$= 11 \times 5 - 3$

$= 55 - 3$

$= 52$

Example 9: Age of Ramesh's father exceeds Ramesh's age by 27 years. After 5 years, ratio of Ramesh's age and his father's age will be 2:3. Find their present age.

Sol: Let the age of the Ramesh $= x$ year

Then, the age of father $= (x+27)$ year

After 5 years, age of Ramesh's = $(x + 5)$ years
After 5 years, age of Father = $x + 27 + 5 = x + 32$
According to the question,

$$\frac{\text{After 5 years, age of Ramesh}}{\text{After 5 years, age of father}} = \frac{2}{3}$$

$$\frac{x+5}{x+32} = \frac{2}{3} \quad \text{(By cross multiplication)}$$

$$3(x + 5) = 2(x + 32)$$
$$3x + 15 = 2x + 64$$
$$3x - 2x = 64 - 15$$
$$x = 49$$

Age of Ramesh $x = 49$ year
Age of Father $x + 27 = 49 + 27 = 76$ years.

Example 10: Denominator of a rational number exceeds its numerator by 8. If 17 is added to numerator and 1 is subtracted from denominator then we get 3/2. Find the rational number.

Sol: Let the numerator of rational number = x
According to the question, value of denominator = $x + 8$

Thus, the rational number = $\frac{x}{x+8}$

Now adding 17 to numerator and subtracting 1 from denominator

$$\frac{x+17}{x+8-1} = \frac{3}{2}$$

$$\frac{x+17}{x+7} = \frac{3}{2}$$

$$2(x + 17) = 3(x + 7)$$
$$2x + 34 = 3x + 21$$
$$3x - 2x = 34 - 21$$
$$x = 13$$

Thus, numerator = 13 and denominator = 13 + 8 = 21

The required rational number = $\frac{13}{21}$

Exercise 11.2

1. The numerator of a rational number is less than its denominator by 3. If 5 is added to both i.e., its numerator and its denominator then it becomes $\frac{3}{4}$ Find the numbers.
2. What should be added in numerator and denominator of fraction $\frac{5}{13}$ so that the fraction become $\frac{3}{5}$.
3. What should be subtracted from numerator and denominator of fraction $\frac{15}{19}$ so that the fraction becomes $\frac{5}{7}$.
4. Ramesh distributed his capital, half of the capital to his wife, one third to his son and remaining 50,000/- to his daughter. Find the total amount of his capital.
5. 5 times of any number is 48 more than its double. Find the numbers.
6. Distribute 45 in this way that one part is 7 less than three times of another part.
7. Age of Ranu is three times of Sujal's age. After 4 years, sum of their age will be 40 years. Find their present age.
8. Length of a rectangle exceeds its breath by 6 meter. If its perimeter is 64 metre then find its length and breath.
9. Sum of the digits of a two-digit number is 12. New number formed by reversing the digit is greater than the original number by 54. Find the original number.
10. In two-digit number, first digit is four times of second digit. Adding this to the new number formed by reversing the digits, 110 is obtained. Find the numbers.

We learnt

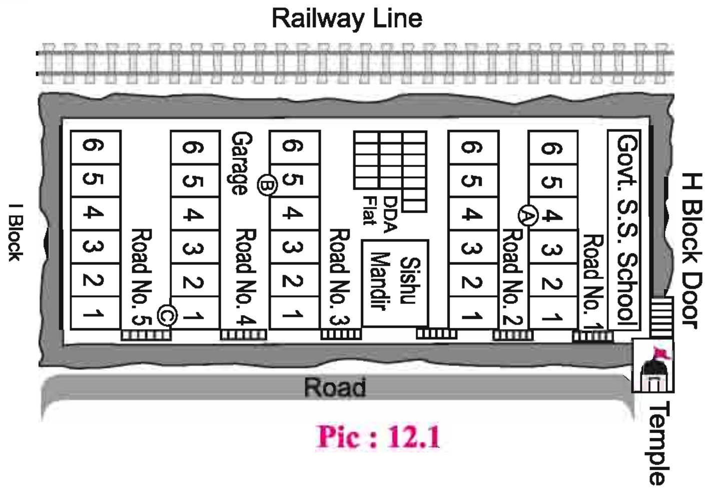
1. Equation having only linear polynomials is called a linear equation.
2. Linear equation having single variable is called single variable linear equation.
3. Substituting the value in place of variable in an equation if LHS = RHS then that value is called solution or root of an equation.
4. Like the numbers, variables in an equation can also be transposed from one side to another side.

Chapter 12

Linear Graph

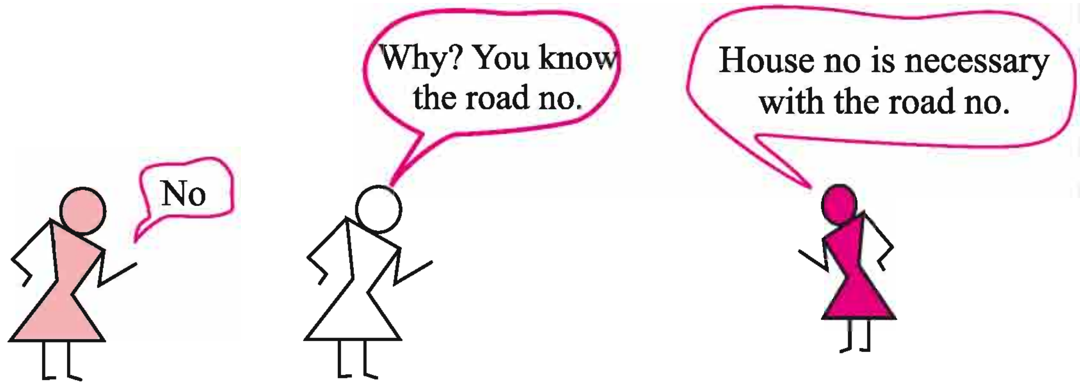
12.1 You have studied in previous classes about the number lines. On number line one can determine the location of any point and describe the position of that point ,but in our daily life, there are lot of situations where location of a point has to be interpreted with references to more than one lines.

Activity 1: Consider the following conditions.



Pic : 12.1

In figure 12.1, a map of a colony is given where 6-6 houses are located on each road. Your relatives live in this colony. You were informed that they live on road no.- 2. Can you tell their residence address easily?



In figure, A indicates that house which is situated on road no.-1 and number of that house is 4. Similarly, B indicates that house which is situated on road no. - 3 and number of that house is 5. Thus, to find out the correct location of any place, two independent information are needed.

Activity 2:

Reeta and Shyam are standing together. Reeta took a paper and mark a point A on that paper and asked Shyam to tell the location of that point A.

Pic : 12.2

Shyaam: It is situated on the left side of the paper.

Reeta: From this, location of a point A is not clear. For this, it would be necessary to tell that how far it is located from the left and how above it located from the bottom.

Shyaam: (Measuring by the scale). Well, it is located 2 cm from the left and 8 cm above from the bottom.

Reeta: Yes, now the location of a point will be found out.

Teacher: For this, two constant lines that is left corner and bottom line of the paper is fixed to determine the position of the point.

Activity 3:

30 selected students are standing at a fixed place for exercise in a play ground as shown in figure below. Actual position of these students can be determined with the help of the following three information:

i) Position of physical Teacher T.
ii) The vertical column in which he/she is standing.
iii) The horizontal row in which he/she is standing.

There are 5 columns and 6 rows in front of physical Teacher and he has to tell the position of students A,B,M,K then he will express it in this way-

A ⟶ 3, 4
B ⟶ 2, 3
M ⟶ 3, 2
K ⟶ 4, 2

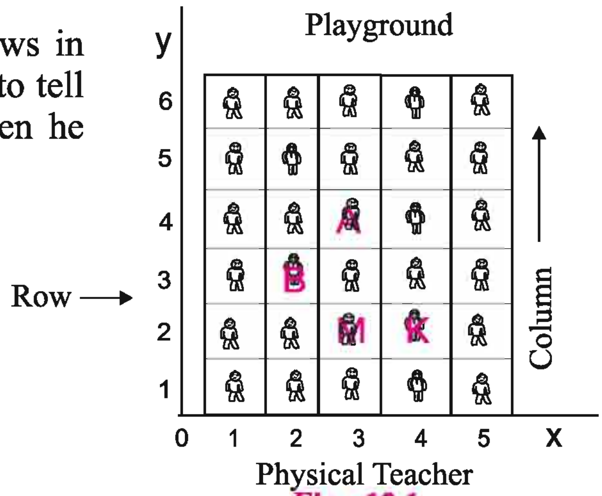


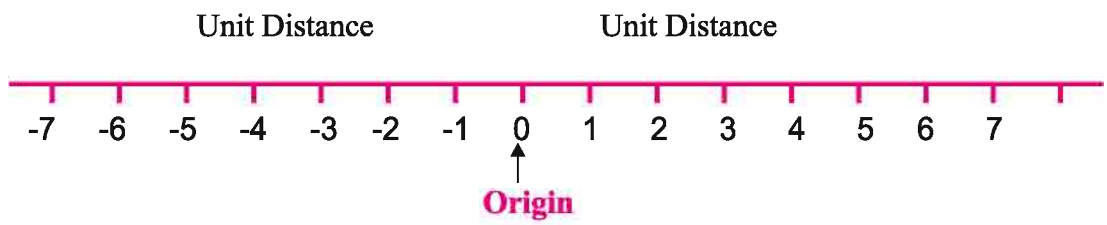
Fig : 12.1

Here it is important to note that first number represent the column and second number represent row. Note the position of student A, he is standing at third column and fourth row similarly, note the position of student B, M and K.

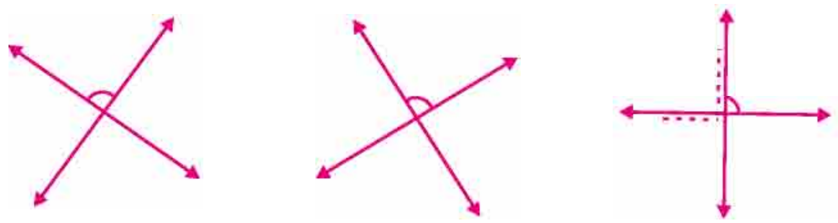
By the above example you see that the position of an object, placed on the floor, can be represented by the two perpendicular lines. If object is in a point form, then it is necessary to determine the distance of a point from the bottom line and from the left corner of the paper. Into the connection of standing for the exercise it is required to know the number of column and rows.

12.2 Cartesian System

On a number line distances from a fixed point in one direction is marked positive and in another direction it is marked negative. Point, from where distances are marked, is called origin. To mark the points on a line at equal distance, we use number line to represent numbers. If '1' unit distance represent number '1' then '3' unit distance will represent number '3' where '0' is origin.



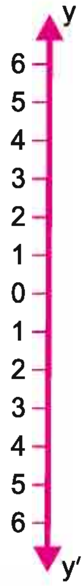
Dakarte considered representing a model in which he draws two lines perpendicular to each other in a plane and determining the position of the points in a plane with respect to these lines. Perpendicular lines may be in any directions as shown in figure 12.2 (i), (ii) and (iii). But in this chapter, when we will consider two lines in a plane to determine the location of a point, then one line will be horizontal and another will be vertical, as shown in figure (iii).



(i) (ii) (iii)

Fig : 12.2

In fact, we can obtain these lines in this way. Have two number lines and give them name X'X and Y'Y. Keep X'X horizontally. See figure (i) and numbers are written on this in a way as written on number line. Do these activities with Y'Y(see figure(ii). Difference is only that Y'Y is vertical not horizontal.

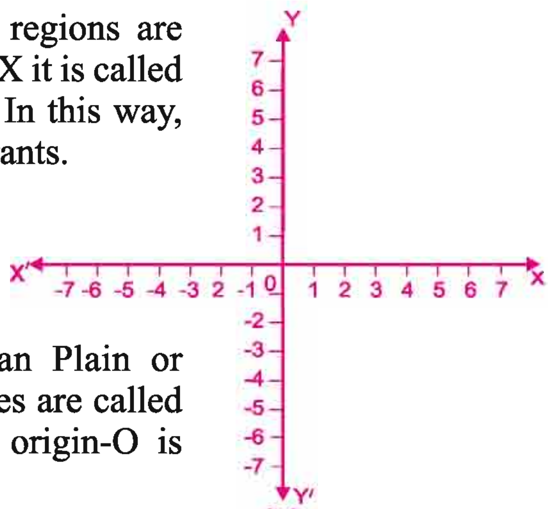


(i)

(ii)

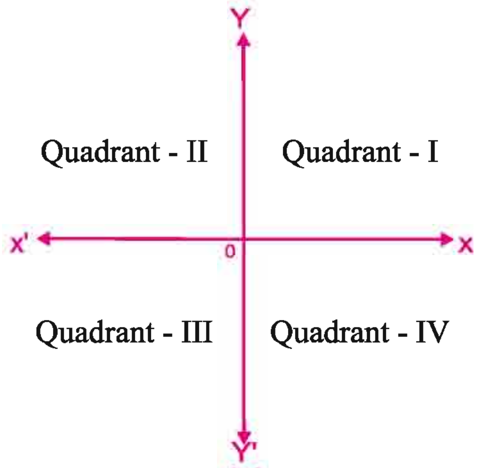
Combine the two straight lines in such a way that both of lines intersect each other at origin see figure (ii). Horizontal line X'X is called X axis and the vertical line Y'Y is called Y axis. Point, where X'X and Y'Y intersect each other is called origin. It is represented by O because positive numbers are mentioned in OX and OY directions. Therefore, OX and OY is called the positive axes of X-axis and Y-axis respectively. Similarly, OX' and OY' is called the negative axes of X-axis and Y-axis respectively.

In figure (iii), we see that both these axes divide the plane into four regions. These four regions are called Quadrants. Anti-clock wise from OX it is called I,II,III and IV quadrant (see figure (iv)). In this way, this plane contains two axes and four quadrants.



(iii)

This plane is called the Cartesian Plain or Coordinate Plain or XY-plane and the axes are called Coordinate axes. X- axis, Y- axis and origin-O is jointly called Frame of Reference.



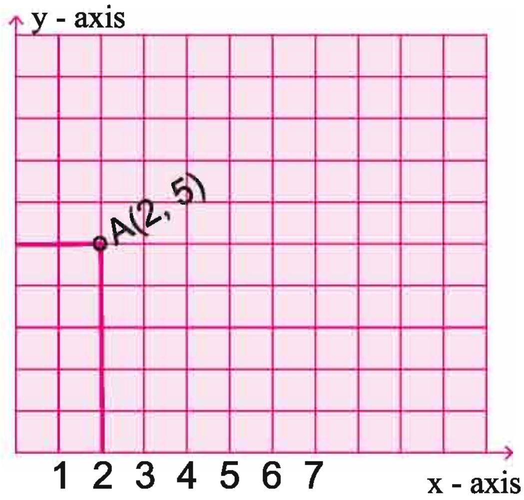
(iv)

12.3 Coordinates:

Imagine that you go to the stadium to watch the cricket match and want to arrive at your reserve seat. For this, you required two numbers. First, row number and second column number. Location of point A(2,5) can be determined by 2 unit from left corner and 5 unit from lower corner.

On squared paper, number 2 is called the x-coordinate and number 5 is y-coordinate. So we can say that point (2,5) are the coordinates. Vertical distance of a point from the x-axis is called Ordinate. Similarly, the vertical distance of a point from the y-axis is called abscissa of that point. In this way, abscissa is 2 and ordinate is 5 of point A.

While writing coordinates, it is represented by small bracket (), first abscissa then comma after that ordinate. In this way, coordinates are (x,y).

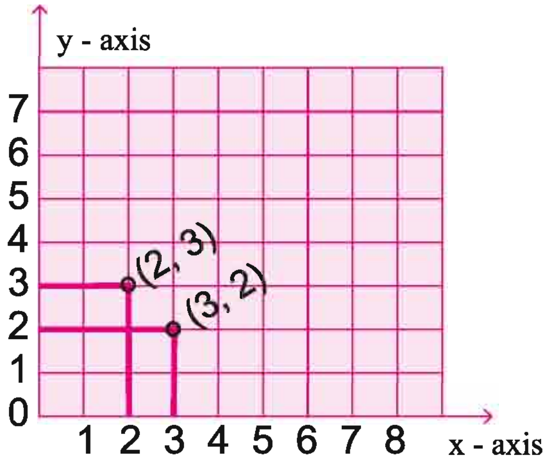


Graph 12.1

For origin, abscissa and ordinate is zero (0). Coordinates of origin are (0,0). Coordinates (x,y) and (y,x) are not equal as they represent the different points on Cartesian plane.

Do this also:

Plot the point(2,3) on a graph paper. Is it that point which represents (3,2)?



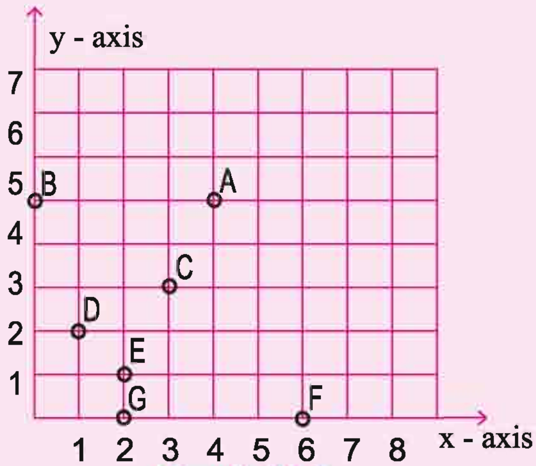
Graph 12.2

Sameer – location of both points are different.

Do and Learn:

1. Looking at the graph 12.3, for the location of the following points, select the proper alphabets.

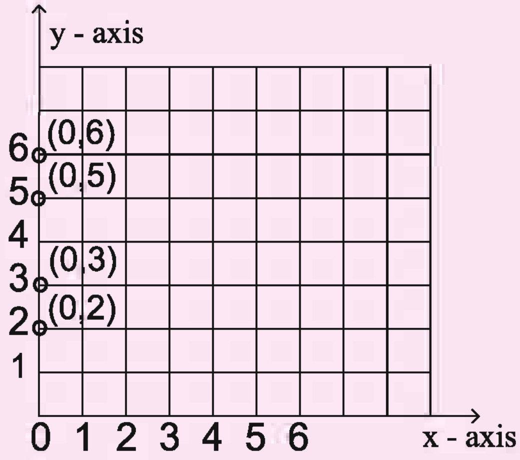
(i) (2,1)
(ii) (0,5)
(iii) (2,0)
(iv) Coordinates of point A
(v) Coordinates of point B



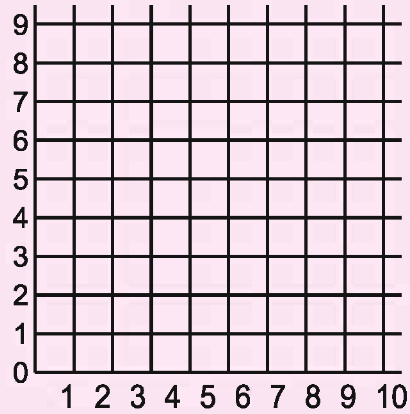
Graph 12.3

2 Plot the following coordinates on squared paper and check whether all are in a straight line. If yes, then give the name to these lines.

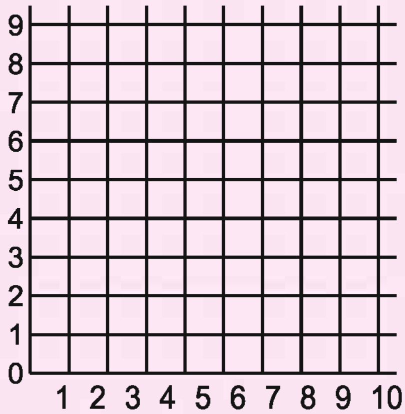
Graph 12.4 (i) (ii) (iii) (iv)



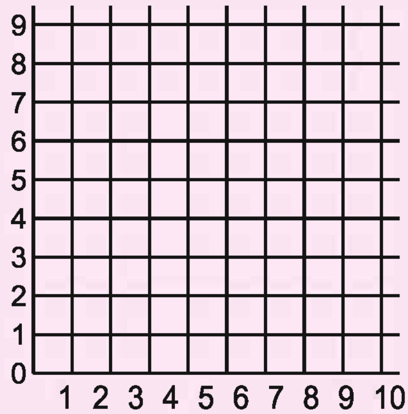
(i) (0,2); Q(0,5); R(0,6); S(0,3)



(ii) A(1,1); B(1,2); C(1,3); D(1,4)



(iii) K(1,3); L(2,3); M(3,3); N(4,3)



(iv) W(2,6);X(3,5); Y(5,3); Z(6,2)

Graph 12.4

Exercise 12.1

1.Fill in the blanks by seeing the graph 12.5 given below:

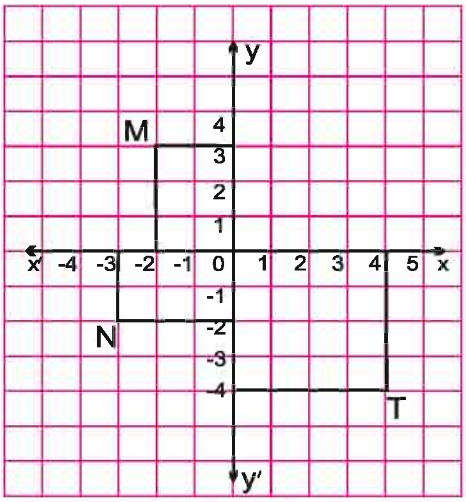
(i)Distance from x-axis of a point M is ....unit.

(ii)Distance from y-axis of a point M is ....unit.

(iii)Distance from y-axis of a point N is ....unit.

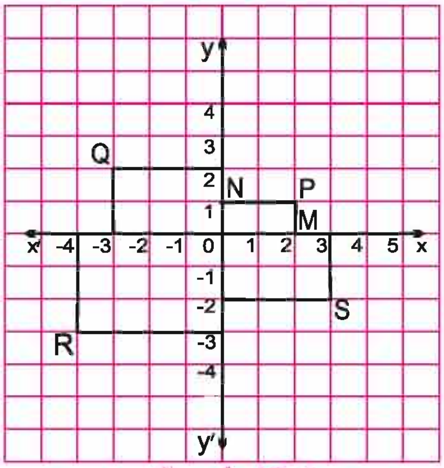
(iv)Point T is plotted on quadrant........................

(v)Distance from x-axis of a point T is ....unit.



Graph 12.5

2. Fill in the blanks by seeing the graph 12.6 given below:



Graph 12.6

(i)Abscissa ....and ordinate................are of point P. Therefore, the coordinates of point P are..............

(ii)Abscissa ....and ordinate................are of point Q. Therefore, the coordinates of point Q are..............

(iii)x-coordinate........and y-coordinate ....are of point R. Therefore, the coordinates of point R are..............

(iv)x-coordinate ....and y-coordinate ....are of point S. Therefore, the coordinates of point S are ....

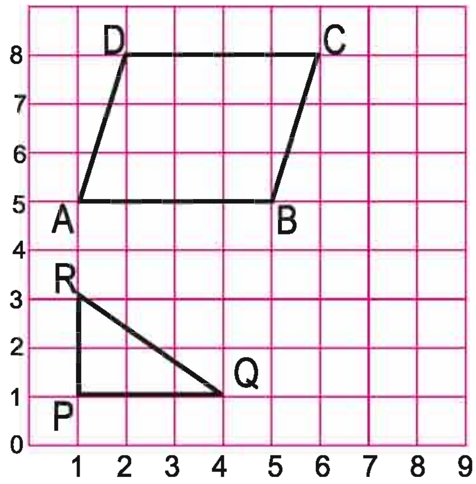
3. Plot the following points on squared paper and check whether they all are situated on straight line.

(i) A (1,1); B(1,2); C(1,3); D(1,4)

(ii) K (1,3); L(5,3); M(5,5); N(1,5)

(iii) P (2,6); Q(5,5); Y(5,3); Z(6,3)

4. Draw the following graph-12.7 on graph paper, write the answer of the questions given below.



Graph 12.7

(i) Write the coordinates of vertices of parallelogram ABCD. Find the length of side AB and DC.

(ii) Find the coordinates of vertices of triangle PQR also find the length of base PQ.

5. Write the True or False in front of each statement.

(i) Location of a point on graph paper is represented by the number pair.

(ii) Linear graph shows the change in data with respect to time-interval.

(iii) Point having x-coordinate zero and y-coordinate non-zero, is located on y-axis.

(iv) Point having y-coordinate zero and x-coordinate 5, will be located on y-axis.

(v) Coordinates of origin are (1,1).

Some Applications:

In daily life, you observed that facilities as much you consume, as you have to pay for that. If you consume much of electricity then you have to pay the electricity bill in that amount. If you consume less electricity then you would have to pay less.

Here one quantity affects the other. We say that quantity of electricity is an independent variable while the electricity bill is a dependent variable. The relationship between these variables can be represented by the following graph.

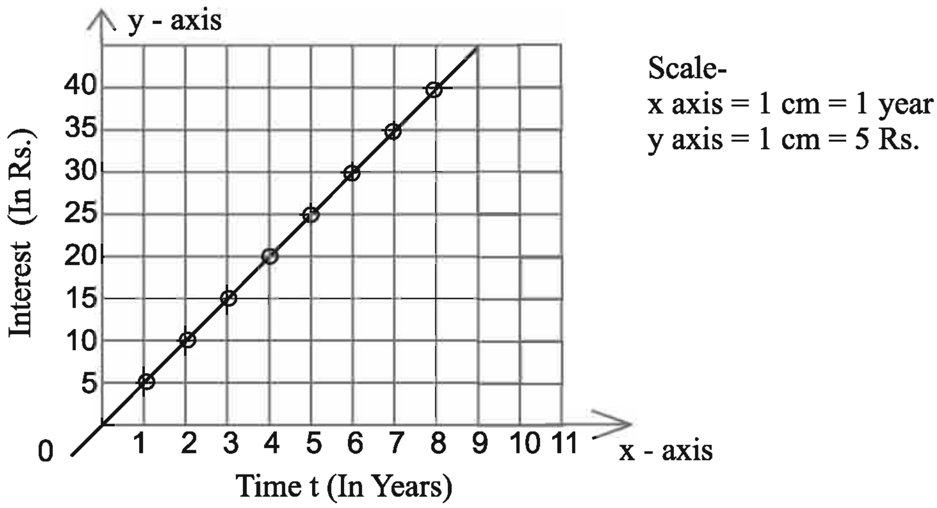
12.4 Representation of some real relationship by the Graph

Example -1: Represent the relationship by the graph between the time and simple interest at given principal amount and by the given rate of interest.

Solution: Let simple interest is 5% per year rate of interest at Rs. 100/-on given principle amount. This relationship is represented by Interest = 5 × Time. With respect to different values of time (t), interest (I) = 5 × time(t) is calculated and write below in tabular form.

Time t (In Years)	1	2	3	4	5	6	7	8
Interest I = 5t (In Rs.)	5	10	15	20	25	30	35	40

By plotting the obtaining points (1, 5); (2, 10);(3, 15);(4, 20);(5, 25);(6, 30);(7, 35);(8, 40) on graph paper, relationship between years and interest are obtain. This graph shows a straight line.

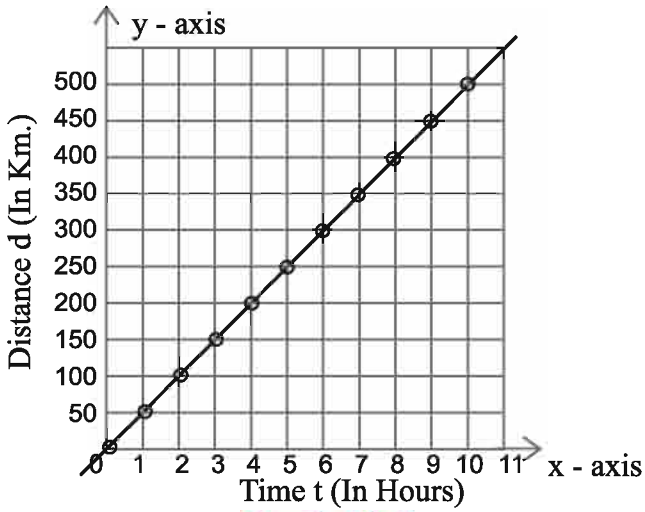


Graph 12.8

Example-2: A car travel 50 km distance in 1 hr. Relationship between distance, covered by the car, and time can be obtained by the d = 50 × t. Where time (t) is in hours and distance (d) is in km. Plot this relationship on graph paper. For different values of time (t), distance (d) can be calculated as shown in table below:

Time (In Hours)	1	2	3	4	5	6	7	8	9	10
Distance d (In Km.)	50	100	150	200	250	300	350	400	450	500

Sol: By plotting the coordinates (1,50), (2,100), (3,150), (4,200), (5,250), (6,300), (7,350), (8,400), (9,450), (10,500) on graph paper, a relationship between time and distance can be obtained which is shown by the straight line.



Scale-
on x axis = 1 cm = 1 hr
on y axis = 1 cm = 50 Km.

Graph 12.9

12.5 Read the Graph:

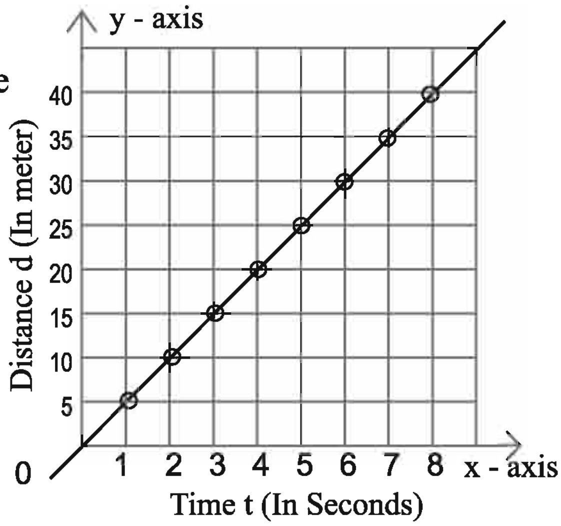
Now we draw the graph between time, interest, distance. Similarly, we can draw the graph between multiple of any number (multiple of 3 = 3,6,9, 12............), side of the square and perimeter etc. Now we see that how to read the given graph. See the following example:

Example 3: See the graph carefully and answer the following questions:

1. What is the distance travelled in 2 sec.
2. What is the distance travelled in 6 sec.
3. How much time to go to cover 20m.
4. What is the speed per second of the vehicle?

Sol: It is clear from the graph 12.10:
1. Distance covered in 2 sec. = 10 metre
2. Distance covered in 6 sec. = 30 metre
3. When distance = 20 metre
then time = 4 sec.

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{20}{4} = 5 \text{ meter/sec.}$$



Graph 12.10

Exercise 12.2

1. Length of a side of an equilateral triangle and square is x cm. Find their perimeter, draw the graph between length and perimeter.

(Hint: Perimeter of $\triangle = x + x + x$) cm.

2. Length of a rectangle is double of its breadth. Draw the graph between area and breadth of rectangle.

(Hint: area of rectangle $A = 2x \times x = 2x^2$)

3. According to the following table, draw the graph between time and simple interest.

Time	1 yr	2 yr	3yr	4 yr
Simple Interest	Rs. 60	Rs. 120	Rs. 180	Rs. 240

4. Draw a graph representing the relation between time and distance.

Time	2	4	6	8
Distance	10	20	30	40

5. According to the following table, draw the graph and explain this graph pass through the origin.

Fixed Deposit (in Rs.)	1000	2000	3000	4000	5000
Simple Interest (in Rs.)	80	160	240	320	400

We Learnt

1. Line graph, which is completely unfractional line, is called linear Graph.
2. For determining the location of any point on a squared paper, we required x-axis and y-axis.
3. Vertical distance from the x-axis is called abscissa and vertical distance from the y-axis is called ordinate of any point.
4. Relationship between dependent variable and independent variable is shown by the graph.
5. Coordinates of origin is (0,0).

CHAPTER 13

Comparison of Quantities

13.1 Percentage

We have read in previous class that ratio means comparison between two or more similar quantities.

If Luv has Rs. 5 and Kush has Rs. 10 then ratio of their Rs. will be

$$5 : 10$$

means 1:2 in simple form.

This can be written as $\frac{1}{2}$ in form of fraction.

This can be expressed in form of percentage as we know percentage means how much out of 100?

If 1 is out of 2 then

50 out of 100 will be-

Means $1 : 2 = \frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100} = 50\%$

50% is read as 50 percent

Let us understand it by taking some more example-

Harmeet and Neelam picked some tindi and kachri in basket from the field. When they came back home and counted there were 14 tindi and 6 kachri. Can you compare number of tindis and kachris? There are two types of vegetables in basket. Their comparison is 14 : 6 or 7 : 3.

Number of tidis are $\frac{7}{3}$ of number of kachris. In this way number of kachris is $\frac{3}{7}$ th of number of tindis.

See this comparison in form of percentage:

Method of Harmeet -
Total items of vegetables- 20. There are 6 kachris in 20 vegetable items % of kachris

$$\frac{6}{20} \times \frac{5}{5} = \frac{30}{100} = 30\%$$

(Denominator is made as 100)

Method of Neelam (by unitary method)-
Out of 20 items of vegetables number of kachris are 6

If the total items of vegetables are 1 then Number of kachris is $= \frac{6}{20}$

if total number of vegetables are 100 then number of kachris would be $= \frac{6}{20} \times 100 = 30\%$

There are tindis and kachris in basket

So % of tindis + % of kachris = 100

% of tindis + 30 = 100

or % of tindis = 100 - 30 = 70

So there are 70 % tindis and 30 % kachris.

Example 1 In a school of Jhalawar in Rajasthan 25% Neem, 15% Peepal plants were sown during environment pakwara (fortnight).

If there are 160 plants in total then-

(i) How many Neem plants are there?

(ii) How many Peepal plants are there?

(iii) Find the ratio of Neem and berries plants.

(iv) Determine number and percentage of Peepal plants.

(v) Number of Neem plants is less than Peepal plants by how much percentage?

Solution Total number of plants = 160

(i) Number of Neem plants is 25% of 160

$$= 160 \times \frac{25}{100}$$

= 40 plants

(ii) Number of Peepal plants is 15% of 160

$$= 160 \times \frac{15}{100}$$

= 24 plants

(iii) Neem plants: Peepal plants

40 : 24

5 : 3

(iv) Number of Peepal plants =

total number of plants shown - (Neem plants + Peepal plants)

= 160 - (40 + 24)

$= 160 - 64$

$= 94$ plants

Percentage of Peepal plants

$\because$ Peepal plants out of total 160 plants = 96 plants

$\therefore$ Peepal plants out of 100 plants $= \frac{96}{160} \times 100$

$= 60\%$

(v) Percentage difference between Neem plant and Peepal plant is

$= 60\% - 25\%$

$= 35\%$

Example 2 A students has got 10 marks in mathematics. If it is 40% then find total maximum marks of mathematics examination.

Solution Marks obtained in mathematics = 10

As 40 % of total marks are 10

Let total marks = x marks

So, 40 % of $x = 10$

Or $x \times \frac{40}{100} = 10$

Or $40x = 1000$

Or $x = \frac{1000}{40}$

Or $x = 25$ marks

Do and learn

Rashmi and Rehana picked flowers in basket from garden. In which there were 30% rose, 10% jasmine and rest ware marigold. If there were 120 flowers in basket then –

(i) What was the number of marigold flowers?

(ii) What was the number of jasmine flowers?

(iii) By what percentage were rose flowers less than jasmine flowers?

Exercise 13.1

1. Convert following ratios into percentages.
   (i) 1 : 4 (ii) 3 : 4
2. Himi travelled 240 km by bus and 360 km by train then determine,
   (i) Ratio of travelling by train and travelling by bus.
   (ii) Ratio of travelling by bus and by train.
   (iii) Ratio of travelling by train and total travelling.
   (iv) Ratio of travelling by bus and total travelling
3. In class VIII 68% students obtained grade A out of 75 students. How many students obtainable grade A?
4. A Kabaddi team of school has won 15 matches out of total matches played this year. If winning percentage was 75 then how many matches were played by the team?
5. There are 1275 trees in total in the field of Mohan. There are 36% trees that have fruits. Determine number of trees having fruits in the field.
6. Kamli spent 75% of the amount deposited in her account under Pradhan Mantri Jan Dhan Yojana. Now the amount left in her account is Rs. 600. Determine what was the amount deposited in here account?
7. According to a survey of 50,000 students under mid day meal in five districts of Rajasthan, 60% students like Pulses– Chapati, 25% like vegetable and chapatti and rest like khichdi. Determine percentage of students who like khichdi?

13.2 Profit – loss(Mark up price, Commission, VAT, discount, miscellaneous expenses, subsidy, service tax etc.)

Neelam started a lorry of readymade clothes, in which she is selling dresses of kids. She is selling a dress at Rs. 100 by purchasing it at Rs. 50 then her profit is 100 %.

After some days she started selling embroidered dresses after packing. Due to which extra expenses on each dress is Rs. 10. In this way her profit per dress is $100 - (50 + 10) =$ Rs. 40 then profit percentage is $\frac{40}{60} \times 100 = 66\frac{2}{3}\%$.

Some extra money is spent while purchasing or selling things. This is added in purchase price which is called miscellaneous price. Like carriage, transportation, agent fees, go down rent etc. are miscellaneous expenses. After some experience Neelam thought of selling clothes after getting stitched from tailor instead of purchasing and then selling. If after purchasing cloth costing Rs. 2000 if discount received is 10% then purchase price will be Rs. $2000 - 200 =$ Rs. 1800.

If 25 dresses are made by spending Rs. 950 on its stitching, weaving etc. then what will be the cost price of 25 dresses?

Rs. (1800 + 950) = Rs. 2750

Cost of one dress is Rs. $\frac{2750}{25}$ = Rs. 110. If each dress is sold at profit of 20% then what will be the selling price?

Selling price at 20% profit = (100 + 20) % of cost price

$= \frac{110 \times 120}{100}$

$= \frac{13200}{100}$ = Rs. 132

After observing increased sales in business she opened a shop. If she put on 15 % VAT (Value Added Tax) on goods sold. Then at what price customer will purchase a dress?

> VAT is tax assessed on buyer on goods / objects sold

VAT of 15% on each dress = 15 % of selling price

$= 132 \times 15\,\% = \frac{132 \times 15}{100}$

= Rs. 19.80

Selling price of a dress including VAT = Rs. 132 + 19.80

= Rs. 151.80

This type of problem can be solved in the following manner

Neelam bought a needle work (kashida) machine for Rs. 17280 including VAT of 8% then what was the price of machine (Original Price) before adding VAT? Think and tell.

Let price of machine excluding VAT	Price of machine including VAT
100	108
x	17280

First method (by ratio)

$100 : x :: 108 : 17280$

Product of extremes = product of middle terms

$100 \times 17280 = 108x$

$\frac{100 \times 17280}{108} = x$

$100 \times 160 = x$

Or x = Rs. 16000

Value of machine excluding VAT = Rs. 16000

Another method (By Unitary Method)
If S.P. of machine including VAT is Rs. 108 then Price excluding VAT will be Rs 100
If S.P. of machine including VAT is Rs. 1 then Price excluding VAT will be Rs $\frac{100}{108}$

Then for S.P. of machine including VAT as Rs. 17280 Price excluding VAT will be

$$= \frac{100}{108} \times 17280$$

$$= 100 \times 160 = \text{Rs. } 16000$$

Do and Learn

1. Reena obtained a bill of goods purchased from Khadi Bhandar. Looking at the bill answer the following questions.

Azad Khadi Bhandar

Bill number 1501 Date 5-10-2015

Mr.--------------

S.No.	Goods/ things	Quantity	Rate	Amount
1. 2. 3.	Bed sheet Kes Carpet	4 4 2	80 120 200	320 = 00 480 = 00 400 = 00 1200 = 00
	Subsidy 15%			-180 = 00 1020 = 00
	VAT 10%			102 = 00
	One thousand one hundred and twenty two only			1122 = 00
E & O.E.	Signature			

- What is the mark up price of goods purchased?
- Subsidy is calculated on which price?
- VAT is calculated on which price?

2. As compared to Rs. 5000, Rs. 4000 are less by what percentage? Is that percentage similar to percentage by which Rs. 5000 are more than Rs. 4000?

Example 3 An object marked Rs. 960 is sold for Rs. 672. What is discount percentage?

Solution Mark up price = Rs. 960

Selling price of object= Rs. 672

Discount = Mark up price – selling price

= 960 – 672

Discount = Rs. 288

Discount obtained on mark up price Rs. 960 = Rs. 288

Discount on mark up price Rs. 1 = Rs. $\frac{288}{960}$

Discount on mark up price Rs. 100 = Rs. $\frac{288}{960} \times 100$

Discount = 30%

Example 4 After giving discount of 20% on mark up price, a trouser was sold for Rs. 560. Determine mark up price of trouser.

Solution Selling Price = Rs. 560

20% discount means there is discount of Rs. 20 on mark up price Rs. 100

So selling price = Rs. 100 - 20

= Rs. 80

If selling price is Rs. 80 then mark up price of trouser is Rs. 100

If selling price is Rs. 1 then mark up price = $\frac{100}{80}$

If selling price is Rs. 560 then mark up price $= \frac{100}{80} \times 560 = 700$

Mark up price of trouser is Rs 700

Example 5 Sukhveer Singh bought a spray instrument in Rs. 4400 including 10% tax. What was the price of spray pump before including tax?

Solution Let Price excluding tax is Rs. 100

Then price including tax is Rs. 100 + 10 = Rs. 110

$\because$ If price including tax is Rs 110 then original price is Rs 100

$\therefore$ If price including tax is Rs 1 then original price is $\frac{100}{110}$

$\therefore$ If price including tax is Rs 4400 then original price is $\frac{100}{110} \times 4400$

= Rs. 4000

So price of spray pump including tax is Rs. 4000

Example 6 Ramu purchase two ceiling fans @ Rs. 1800 each. Out of which one fan was sold at 5% loss and other is sold at 12% profit. Find selling price of each fan. Tell total profit or loss.

Solution Purchase price of each fan = Rs. 1800

One fan is sold at 5% loss then

Purchase price is Rs. 100 then selling price of fan = Rs. 95

Purchase price is Rs. 1800 then selling price of fan = Rs. $\frac{95}{100} \times 1800$

= Rs. 1710

Another fan is sold at 12 % so

$\because$ If purchase price is Rs. 100 then selling price of fan is Rs 112

$\therefore$ If purchase price is Rs. 1 then selling price is Rs. $\frac{112}{100}$

$\therefore$ If purchase price is Rs. 1800 then selling price is Rs. $\frac{112}{100} \times 1800$

= Rs. 2016

Total purchase price = Rs. 1800 + Rs. 1800

= Rs. 3600

Total selling price = Rs. 1710 + Rs. 2016 = Rs. 3726

Total purchase price < total selling price so there will be profit

Profit = Rs. 3726 - Rs. 3600

= profit of Rs. 126

13.3 Simple Interest

Umesh took Rs. 2400 as loan @ 9%. If he wishes to repay loan in 3 years and 6 months then what interest he is supposed to pay?
He went to teacher. He asked how I would calculate interest for 3 years and 6 months.
Teacher said convert time duration in months to years.
One year has 12 months.

Amount taken as loan = Rs. 2400

Rate of interest = 9 % annually

Time = 3 years 6 months

Time = 3 years + $\frac{6}{12}$ years

= $(3 + \frac{1}{2})$ years

= $\frac{7}{2}$ years

$$\text{Simple interest} = \frac{\text{principal} \times \text{rate} \times \text{time}}{100}$$

$$= 2400 \times \frac{7}{2} \times \frac{9}{100}$$

= Rs. 756

Amount = Principal + Interest

= Rs. 2400 + 756

= Rs. 3156

Example 7 what amount should be lent by Eeshwar so that amount obtained as interest will be Rs. 1831.50 after 2 years and 9 months?

Solution Amount of Interest = Rs. 1831.50

Rate = 12%

Time = 2 years + $\frac{9}{12}$ months

= 2 years + $\frac{3}{4}$ = $\frac{11}{4}$

$$\text{Simple interest} = \text{Principal} \times \text{rate} \times \frac{\text{time}}{100}$$

$$1831.50 = \text{Principal} \times \frac{11}{4} \times \frac{12}{100}$$

$$1831.50 \times 100 \times 4 = \text{Principal} \times 11 \times 12$$

$$\text{Principal} \times 11 \times 12 = 1831.50 \times 100 \times 4$$

$$\text{Principal} = \frac{1831.50 \times 4 \times 100}{11 \times 12}$$

$$\text{Principal} = \frac{183150 \times 4 \times 100}{11 \times 12 \times 100}$$

$$= \text{Rs. } 5550$$

So amount given on credit by Eeshwar is Rs. 5550

Exercise 13.2

1. Mohan purchases some mattresses for Rs. 7250. After some time he sold them for Rs. 6090. Find out loss percentage.

2. Due to increase in salary of Ajit Singh by 12 % new salary becomes Rs. 25760. Find his previous salary.

3. Manjeet mark up price on a pump by increasing 40%. If he wishes to sell it after providing subsidy of 40% then find out profit or loss percentage.

4. Cost of a moped is Rs. 54000. Now price increased by 14% then tell the price to be paid for moped.

5. A businessman purchased goods for Rs. 14000. He paid Rs. 350 as auto rent and Rs. 150 as wages. For earning 5% profit, at what price he should sell goods.

6. A furniture seller sold two dressing tables at the rate of Rs. 7200. Out of them 20% profit obtained on one table and 20% loss on another. How much profit or gain percentage is obtained in whole transaction?

7. Manoj paid Rs. 6500 as interest on loan of Rs. 52000 after two years. Find percentage interest paid by Manoj.

8. In what time will principal of Rs. 3200 at rate of 8% become Rs3840?

9. Bhupendra took loan of Rs 6300 at rate of 7% for 2 years and 8 months then tell what amount will be paid by him.

13.4 Compound Interest

Bank Pass Book

Date Description Deposit Withdrawal Balance

1.4.13	Cash	2000	-	2000
1.4.14	Interest	140	-	2140
1.4.15	Interest	149.80	-	229.80

From entry of pass book tell-

- Interest is added after what period?
- How much interest is added for the first period?
- How much interest is added for the second period?
- Whether amount of interest is same for every year?

Suman after looking at bank pass book asked her mother-

Suman - Mom, Papa deposited Rs 2000 two years back in saving account. But why interest given by bank on this account increases every year?

Mother - Yes, interest received or paid normally is not simple interest. In definite period three months, six months or one year; interest is added to principal. After this definite period a new principal is obtained on adding this principal and interest. That is why every time amount of interest seems to be increased.

Suman - Then this is not called as simple interest. Then how it is calculated?

Mother - Yes this is called as compound interest. Come let us learn how to calculate compound interest.

Suman - Interest for every year is calculated separately.

Mother - See, your father has deposited Rs 2000 in bank. Compound interest rate is 7 %.

Simple interest for first year is $SI = \frac{P \times R \times T}{100}$

$$SI = \frac{2000 \times 7 \times 1}{100} = Rs\ 140$$

Amount at the end of one year is $P_1 + SI$

$= Rs(2000 + 140)$

$= Rs\ 2140 = P_2$

Simple interest for second year

$$SI_2 = \frac{P_2 \times T \times R}{100} = \frac{2140 \times 7 \times 1}{100} = Rs\ 149.80$$

Amount received / paid at the end of second year

Means amount at the end of second year $= P_2 + SI_2$

$= 2140 + 149.80$

$= \text{Rs. } 2289.80$

Suman - Mom, total Interest in two years

$= (149.80 + 140)$

$= \text{Rs } 289.80$

This is more than simple interest.

Mother - Simple interest for two years $= \dfrac{P \times T \times R}{100}$

$= \dfrac{2000 \times 2 \times 7}{100}$

$= \text{Rs. } 280$

Yes, Mom due to compound interest we received Rs. 289.80 - 280 = 9.80 more

Suman to teacher-Teacher, which is simple method to calculate simple interest?

Teacher- You have learnt how to calculate simple interest. Let us learn how to calculate compound interest.

Let principal amount for first year = P1 and Interest rate is R% then interest is calculated as follows.

Interest at the end of one year $SI_1 = \dfrac{P_1 \times T \times R}{100}$

$= \dfrac{P_1 \times 1 \times R}{100}$

$= \dfrac{P_1 R}{100}$

$\therefore$ Amount $A_1 = P_1 + \dfrac{P_1 R}{100}$

$= P_1 \left[1 + \dfrac{R}{100}\right] = P_2$ (Principal of second year)

$Si_2 = \dfrac{P_2 \times T \times R}{100} = \dfrac{P_2 R}{100}$

$SI_2 = \dfrac{P_2 R}{100}$

$= P_1 \left(1 + \dfrac{R}{100}\right) \dfrac{R}{100}$

$SI_2 = \dfrac{P_1 R}{100} \left(1 + \dfrac{R}{100}\right)$

Amount at the end of second year

$A_2 = P_2 + SI_2$

$= P_1 \left(1 + \dfrac{R}{100}\right) + \dfrac{P_1 R}{100} \left(1 + \dfrac{R}{100}\right)$

$= P_1 \left[1 + \left(\dfrac{R}{100}\right)\right]\left[1 + \left(\dfrac{R}{100}\right)\right]$

$= P_1 \left(1 + \dfrac{R}{100}\right)^2 = P_3$

(P_3 Principal of third year)

In this way for third year

$$SI_3 = \frac{P_3 \times T \times R}{100}$$
$$= \frac{P_3 \times 1 \times R}{100}$$
$$= \frac{P_3R}{100}$$

$$A_3 = P_3 + SI_3$$
$$= P_3 + \frac{P_3R}{100}$$
$$= P_3 \left(1 + \frac{R}{100}\right)$$
$$= P_1 \left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right)$$
$$= P_1 \left(1 + \frac{R}{100}\right)^3$$

After n years with R % annual compound interest rate

Amount $= P_1 \left(1 + \frac{R}{100}\right)^n$

Compound Interest $\boxed{CI = A - P}$

Means compound interest with R & annual compound interest rate for n years

$$\boxed{CI = P_1 \left(1 + \frac{R}{100}\right)^n - P_1}$$

Do and learn

Sandeep borrowed Rs 3000 for 2 years at 8% annual compound interest rate from bank for establishing a biogas plant. if interest is accumulated annually then determine

(i) What amount has to be repaid with 8% annual compound interest rate?

(ii) How much Compound interest is there?

(iii) If this money was borrowed with simple interest then less or more interest was to be paid and how much?

Suman - Sir, it is written in question that if rate of interest is accumulated annually. What does it mean?

Teacher - Yes, this mean, you have looked into different pass books (bank, saving bank pass book of post office) and experienced that rate is accumulated as quarterly, half yearly and annually. This is called transformation method.

Suman - Sir, What is transformation period?

Teacher - The time period after which interest is added to get new principal is called transformation period.

Suman - Sir, when interest is added half yearly then there will be two transformation period and when interest is added quarterly then there will be four transformation period.

Teacher - Yes, in this condition there will be change in rate also.

Suman - Sir, Why?

Teacher - If interest on Rs. 100 for one year is Rs. 8 then what will be interest half yearly, quarterly? Think.

Suman - I got it. In six months it will be half means Rs. 4 and in three months it will be one fourth means Rs. 2.

Do and learn

Complete the following blanks given in the table.

Condition of accumulation of interest	Time in years	Annual Rate	Transform-ation period	Transformed Rate
Annual	2	10%	2	10%
Half yearly	$1\frac{1}{2}$	6%	- - -	3%
Quarterly	$1\frac{1}{4}$	8%	5	- - -
Half yearly	2	14%	4	- - -
Annual	1	7%	- - -	- - -
Quarterly	Six months	16%	- - -	- - -

Suman - This means formula for compound interest will be

$$CI = P\left(1+\frac{R}{100}\right)^{n}-p$$

CI = Compound Interest
P = principal
R= Transformation rate
n= transformation period

Teacher - Very true, but if interest accumulation rate is annual then there will be no change in rate and time.

Example 8 If interest accumulation rate is half yearly and Rs 10,000 are borrowed for one year then with annual rate as 14 %. What amount is to be repaid?

Solution Interest accumulation is half yearly so time is doubled and rate is halved. Transformation period n=1×2 =2 and transformation rate = 14/2% =7% half yearly.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^{n}$$
$$= 10,000\left(1+\frac{7}{100}\right)^{2} = 10,000\left(\frac{107}{100}\right)\left(\frac{107}{100}\right)$$
$$= \text{Rs. } 11449$$

So after one year Rs 11449 will be repaid.

Example 9 What amount will be received after investing Rs 20,000 for one year and 6 months at the rate of 8% when interest is accumulated annually.

Solution Converting time period into year = 1 year + 6 months

$$= 1 \text{ year} + \frac{6}{12} \text{ years} = 1\frac{1}{2} \text{ years}$$

Condition for Interest accumulation is annual so there will be no change in rate and time. Yaman put values in formula like this

$$A = 20000\left(1+\frac{8}{100}\right)^{1\frac{1}{2}}$$

Degree is in form of fraction, how will I solve it?

Teacher - For first year rate is 8% treat amount obtained as principal and for next half year rate will be halved means 4 %.

Yaman - Sir, like this.

$$A = 20000 \times \left(\frac{108}{100}\right) \times \left(\frac{104}{100}\right)$$
$$= 2 \times 108 \times 104 = \text{Rs. } 22464$$

13.5 Practical problems based on increasing rate

While solving questions, Suman started thinking whether she can solve problems related to percentage increase rate through formula of compound interest. She asked this thing to her teacher. **Suman** - Can I solve problems related to percentage increase through formula of compound interest?

Teacher - Yes, you can apply this formula in following conditions-

(i) Increase (or decrease) in population

(ii) If rate of growth of bacteria is known, then to determine rate of increase.

(iii) To determine value of an object if in intermediary years there is increase or decrease in its value.

Example 10 There is a target of increase in number of students at Yoga centre by 20%. If in year 2014 this number is 300 then in year 2016 on completion of target how many students will be there in yoga centre.

Solution Initial number = 300

Rate of increase = 20%

Time period n = 2 years

Number of students after 2 years = initial value $\left(1+\frac{R}{100}\right)^n$

$$=300\times\left(1+\frac{20}{100}\right)^2$$

$$=300\times\frac{120}{100}\times\frac{120}{100}=432$$

Example 11 In a district number of road accidents in year 2014 is 3000. Due to awareness through road safety attempts there was 15 % decrease in road accidents then how many road accidents occurred in 2016.

Solution Initial value 8000 , rate = 15% (negative due to decrease in rate) time period = 2 years Value after two years?

Number of road accidents in year 2016 = Initial value $\left(1+\frac{-R}{100}\right)^n$

$$=8000\left(1+\frac{-15}{100}\right)^2$$

$$=8000\left(1-\frac{15}{100}\right)^2$$

$$=8000\times\frac{85}{100}\times\frac{85}{100}=\text{Rs. }5780$$

Example 12 Malaria is spreading in a city. For its prevention fogging, spray of kerosene, removing water logging and applying cow dung, DDT spray used due to which number of malaria patients get reduced by 5%. If number of patients in this week is 6859 then what was the number of malaria patients three weeks back?

Solution Rate = -5% per week, assume initial value = x

Period n = 3 weeks, number of patients in last week = 6859

Final value = initial value $\left(1+\frac{R}{100}\right)^n$

$$6859 = x\left(1-\frac{5}{100}\right)$$

$$= x \times \frac{95}{100} \times \frac{95}{100} \times \frac{95}{100}$$

$$\frac{6859 \times 100 \times 100 \times 100}{95 \times 95 \times 95} = x$$

$$\frac{6859 \times 20 \times 20 \times 20}{19 \times 19 \times 19} = x$$

$$x = 8000$$

So, number of patients suffering from malaria three weeks back was 8000.

Exercise 13.3

1. Number of visitors on the first day of book fair in the city was 3000 which increased to 3600 on next day. Determine increase in fare visitors.
2. Price of a television is Rs. 30,000. Value of object decreases (devaluates) by 20 % then determine its value after 2 years.
3. Kapil took a loan of Rs. 52800 at annual rate of 12% from a bank for purchasing a scooter when accumulated rate is annual. After one year and 6 months what amount is to be paid for repaying loan?
4. In year 2013 number of road accidents was 10,000.By traffic police awareness programs were run for avoiding road accidents due to which it decreases by 20% then what was the number of road accidents in 2015?
5. Determine compound interest on Rs. 10,000 for 2 years at 8% annual rate if interest is calculated at annual rate.
6. Payal took loan of Rs. 12,000 for parlour from a nationalized bank. How much amount she will repay after 2 years 6 months at annual rate of 8%. When interest is accumulated annually.
7. Calculate compound interest on Rs. 18,000 for $1\frac{1}{2}$ year @ 10% when interest is calculated half yearly.
8. Vishnu invested Rs. 80,000 at annual rate of 14% if interest accumulates half yearly then determines what amount he will receive? If time is (i) 6 months (ii) 1 year

9. Khushwant borrowed Rs. 12,500 for 3 years @ 5 % annually on simple interest. If same amount is borrowed @ 5% annually on compound interest then what extra amount Khushwant has to pay?

13.6 Understanding of direct and Inverse relationship and problems based on it

In a hostel varnishing was going on. In 2 days varnish of 3 rooms was done by two . In the hostel all 18 rooms are of equal measure. Students are discussing on varnishing work.

Mahaveer - Three rooms are painted in 2 days then calculate time required to paint all 18 rooms.

Gurumeet - Number of days Work done number of painted rooms

Number of days	Work done number of painted rooms
2 ↓	3 ↓
x	18

There is direct relation between number of days and amount of work done means if number of workers is constant then on increasing number of days work done also increases.

$$2 : x :: 3 : 18$$

Product of extremes = product of middle terms

$$2 \times 18 = 3x$$

$$\frac{2 \times 18}{3} = x$$

$$12 = x$$

Mahaveer - Yes this is true they will paint the whole hostel (means all 18 rooms) in 12 days.

Gurumeet - Then is there any relationship between number of workers and work done.

Mahaveer - I think if there are large numbers of workers work will be done earlier. Let us ask teacher.

Gurumeet - Sir, there is direct relationship between number of workers and number of days.

Teacher - Yes, there is direct relation.

Direct relation between two variables is like this when one variable increases then the other also increases and when one decreases then other also decreases. But sometimes one variable decreases when another variable increases and when second variable is decreased then first increases. This relation is inverse relation.

Do and learn

1.Read following statements and find it is direct/ inverse relation.

1.	A ladder is slipping by the side of wall height of top of ladder from bottom and distance from bottom to top of wall	Inverse relation
2.	Between distance and time of car moving with uniform velocity.	……….....
3.	Between number of persons and sufficiency of food material in days (when quantity of food material is constant)	…………
4.	Between water and tank and population of village(when amount of water and its distribution is same	…………

2.Think about such examples about direct and inverse relationship and discuss them with friends.

Example 13 In a field 4 workers complete work of ploughing in 8 days. If this work has to be done in 2 days then how many workers will be required.

Solution Here there is inverse relation between number of workers and time required for completion of work

Number of workers	time required for completion of work (in days)
4 ↓	8 ↑
x	2

Assume x workers are required for the task.

$4 : x :: 2 : 8$ (due to inverse relation)

Product of extremes = product of middle terms

$$4 \times 8 = 2x$$

$$\frac{4 \times 8}{2} = x$$

$$16 = x$$

So to complete work in two days 16 workers will be required.

Example 14 Dharmesh wishes to move from Dungarpur to Jalore by car

(i) If car requires 5 litre petrol for moving a distance of 90 km then what distance it will travel in 20 litres.

(ii) While reaching Jalore from Dungarpur after travelling 6 hours with an average speed of 60 km/hr. What will be average speed while returning if it takes $4\frac{1}{2}$ hours?

Solution There is direct relation between distance (km) travelled by vehicle and amount of fuel (litre).

Distance travelled (km)	Amount of fuel (in litres)
90 ↓	5 ↓
x	20

Let it will travel a distance of x km.

$$90 : x :: 5 : 20$$

Product of extremes = product of middle values

$$90 \times 20 = 5x$$

$$\frac{90 \times 20}{5} = x$$

$$360 \text{ km} = x$$

So, with 20 litres of petrol that car will travel a distance of 360 km.

(iii) There is inverse relation between average speed and time

Average speed of vehicle (Km per hour)	Time required to travel a definite distance (hour)
↓ 60	6 ↑
x	$4\frac{1}{2}$

Assume while returning average speed of car is x km/hr.

$$60 : x :: 4\frac{1}{2} : 6$$

$$60 : x :: \frac{9}{2} : 6$$

Product of extremes = product of middle values

$$60 \times 6 = \frac{9}{2} x$$

$$\frac{\overset{20}{\cancel{60}} \times \overset{2}{\cancel{6}} \times 2}{\underset{3}{\cancel{9}}} = x \qquad x = 80$$

While returning average speed of car is 80 km per hour.

Exercise 13.4

1. Vimla travelled 200 km distance by bus and she gave fare of Rs. 180. What rent she has to pay for travelling a distance of 500 km.
2. Shadow of a 10 metre long tree is 18m in morning. What will be height of shadow of 120 m high tower at same time?
3. If weight of 5 books is 2.5 kg then 30 kg will be weight of how many books?
4. A bus is moving with a uniform speed of 45 km per hour then what time bus will take to travel a distance of 225km.
5. Mamta can fill 30 parindahs with 15 litres of water then tell how many liters of water will be required for filling 120 such parindahs.
6. 100 liters of water can be saved by washing 5 cars with jug and buckets instead of tap. In this way how many liters of water can be saved by washing 20 such cars?
7. 9 workers took 16 days to complete pucca boundary walls of school. If number of workers is 12 then wall can be prepared in how many days?
8. A camp has food for 40 soldiers for 20 days. After 5 days 10 more soldiers joined then rest of the food will be sufficient for how many days?
9. Under Swachh Bharat Mission 15 volunteers clean their village in 4 days. I village needs to be cleaned in 3 days then how many workers will be required?
10. In a school under shramdan 2 students clean for 5 hours. If same part needs to be cleaned in 3 days then how many workers will be required?
11. Madhu prepares food for 12 days from biogas plant by putting on 80 kilograms of cowdung. Then to prepare food for 60 days how much cow dung will be required?

We learnt

1. Subsidy on mark up price is called discount.
   Discount = Mark up price – selling price

2. Percentage discount = $\dfrac{\text{Discount}}{\text{Mark up price}} \times 100$

3. Extra expenses on any object after purchasing are added to purchase is called miscellaneous expense.
   Actual purchase price = buying price + miscellaneous price
4. On selling an object VAT (Value - added Tax) is charged by government. This is included in amount of bill.
5. Simple Interest = Principal Rate × $\frac{\text{Time}}{100}$
   Amount = Principal + Interest
6. Interest calculated on total Amount of last year (A = P + I) is called compound interest.
7. (i) When interest is accumulated annually then

   Total amount (A) = $P\left(1+\frac{R}{100}\right)^n$ where P = principal, R = Rate, n= time period

   (ii) When interest is accumulated half yearly then

   Total Amount = $P\left(1+\frac{R}{200}\right)^{2n}$

   Where $\frac{R}{2}$ is half year rate of interest and 2n = number of half years.

8. When two quantities are related such that by increase or decrease of one quantity another quantity increase or decrease in same ratio then they are called directly proportional.
9. When two quantities are related such that by increase or decrease of one quantity another quantity decrease or increase in same ratio then they are called inversely proportional.

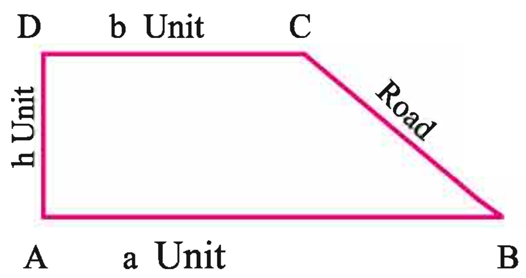
Chapter 14

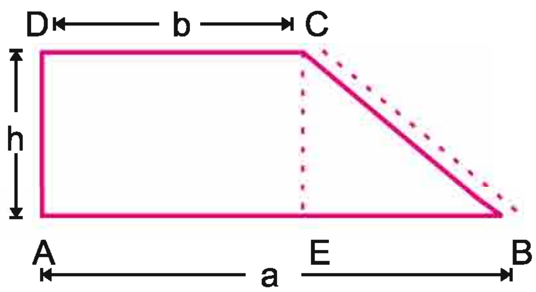
AREA

14.1 In the previous classes we have some method about how to find the area of quadrilateral. Now in this chapter we are going to discuss few more methods to find the area of quadrilateral (like trapezium, parallelogram and simple quadrilateral).

14.2 Trapezium

Let us discuss about the shape of the given plots. What are the techniques you will apply to calculate the area of the given figure. By looking at one pair of parallel side of the figure Aditya consider it as a trapezium. Quadrilateral which has one pair of parallel opposite side is known as trapezium.





He divides the given plot into two parts (figures),one of them is triangle and other is rectangle because he already knows how to find out the area of a rectangle and a triangle.

Area of the plot $=$ area of rectangle AECD + area of triangle ECB

$= (AE \times AD) + \frac{1}{2}(EB \times EC)$

$\because$ [$EC = AD$ and $AE = DC$]

Area of the figure ABCD $= (AE \times AD) + \frac{1}{2}(EB \times AD)$

$= AD \times [AE + \frac{1}{2} \times EB]$

$$= AD \times \left(\frac{2AE + EB}{2}\right)$$

$$= AD \times \left(\frac{AE + AE + EB}{2}\right)$$

$$= AD \times \left[\frac{(AE + EB) + AE}{2}\right]$$

$$= AD \times \left(\frac{AB + CD}{2}\right)$$

$$= h \times \frac{(a+b)}{2}$$

(where a and b are the parallel sides)

Therefore, Area of Trapezium = $\frac{1}{2}$ × height × (sum of the parallel sides)

Activity: lets find the area of the trapezium through an activity which has following steps

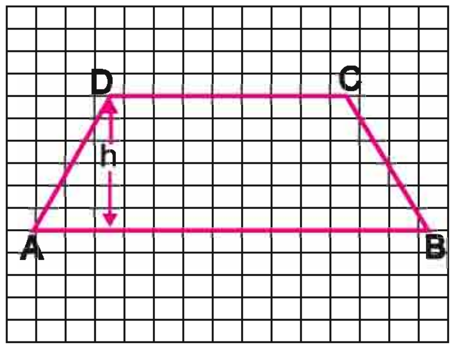
(i) Draw a trapezium of any size on the graph paper and cut it out.

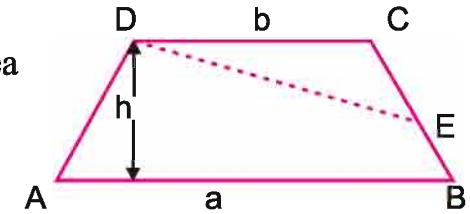
(ii) Find the mid point E of the side BC and cut the trapezium into two piece, along DE

(iii) Place the triangle DEC in such a way that point C coincides with point D.

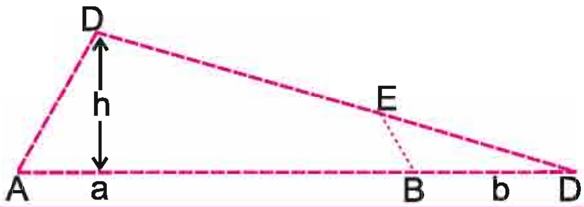
(iv) What is the length of the base of the larger triangle?

If height of the triangle is h then its area is given by, $= h \times \frac{(a+b)}{2}$





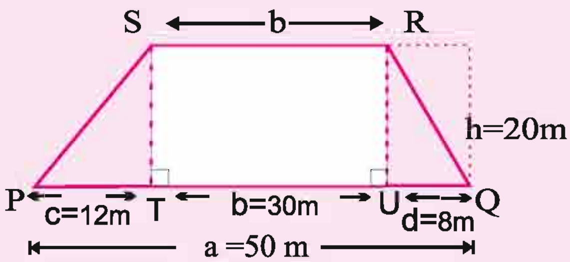
Is the area of this triangle and the area of the trapezium ABCD are same? Get the expression for the area of trapezium by using the expression for the area of triangle.



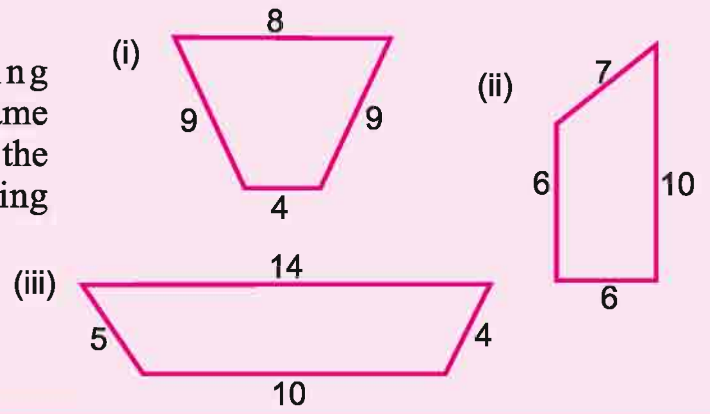
Do and learn

Azhar has a trapezium shaped farm. He divide it into three parts as shown in the figure.

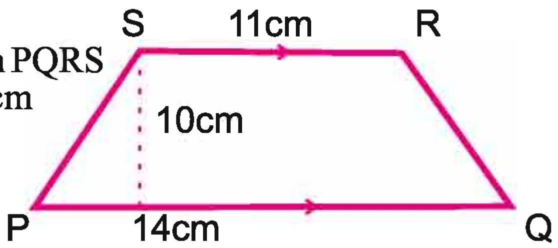
Show that the area of the trapezium PQRS = Area of Δ SPT + area of rectangle STUR +area of ΔRUQ. And compare it with the area of the trapezium $h \times \frac{(a+b)}{2}$



2 . Is trapezium having different parameter has same area? Prove with the help of the data given in the following figures



Example 1 Find the area of the trapezium.



Solution ∵ Parallel side of the trapezium PQRS are PQ = 14 cm and SR = 11 cm
Height, h = 10 cm

Area of the trapezium PQRS

$$= \frac{1}{2} \times \text{height} \times (\text{sum of parallel side})$$

$$= \frac{1}{2} \times (14 + 11) \times 10$$

$$= \frac{1}{2} \times 25 \times 10 = 125 \text{ cm}$$

Example 2 Two parallel sides of the trapezium is 12cm and 8cm.If area is equal to 60cm^2,then what is the height of the trapezium.

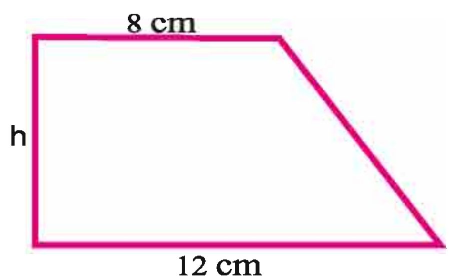
Solution Let the height of the trapezium = h

Area of the trapezium $= \frac{1}{2} \times \text{height} \times (\text{sum of parallel side})$

$$60 = \frac{1}{2} \times (12 + 8) \times h$$

$$h = \frac{60 \times 2}{20} = 6 \text{ cm}$$

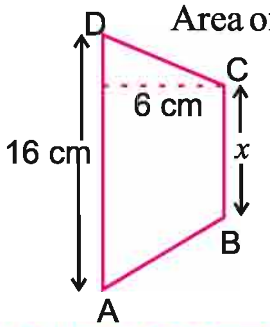
height = 6 cm



Example 3 The sides of the trapezium is given in the figure ABCD. Find the value of x if the area is equal to 78 cm^2.

Solution Since AD and BC are parallel sides

Area of trapezium = $\frac{1}{2}$ × height × sum of parallel side



$$78 = \frac{1}{2} \times 6 \times (16 + x)$$

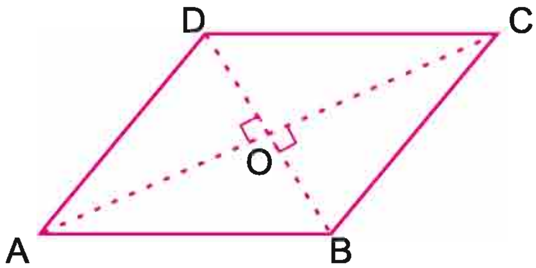
$$(16 + x) = (78 \times 2)$$

$$16 + x = 26$$

$$x = 26 - 10 = 10 \text{ cm}$$

14.3 Area of Rhombus

We have learnt that a quadrilateral that has equal side is known as Rhombus. the diagonals of the rhombus bisect orthogonally to each other. Let's explain it with the help of an example. ABCD is a rhombus having AC and BD as its diagonals. Let AC = d_1 and BD = d_2,the diagonals bisect orthogonally each other at point O .i.e the diagonals bisect and make right angle at point O.



Area of the rhombus

= Area of ΔABC + area of ΔACD

$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$

(OB and OD are the height of ΔABC and ΔACD)

$= \frac{1}{2} \times AC\,[OB + OD]$

$= \frac{1}{2} \times AC \times BD \quad (\because OB + OD = BD)$

$= \frac{1}{2} \times d_1 \times d_2$

Hence, area of the rhombus = $\frac{1}{2}$ × Product of the diagonals

In other word the area of the rhombus is equal to the half of the product of its diagonals.

Since rhombus is also a quadrilateral therefore if we know the side and height of the rhombus then

Area of the rhombus = base × height

Example: 4 The diagonal of the rhombus are respectively 10 cm and 12 cm. What is the area of the rhombus

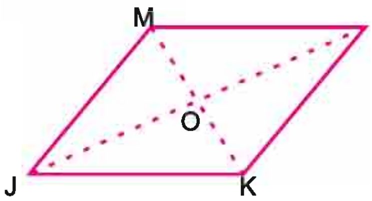
Solution: Area of the rhombus $= \frac{1}{2} \times$ product of the diagonals

$$= \frac{1}{2} \times 10 \times 12$$

$$= 60 \text{ cm}^2$$

Example: 5 According to given figure JKLM is a rhombus and has area equal to 140cm^2. If the diagonal KM= 14cm then find the value of OL.

Solution: We know that



area of the rhombus $= \frac{1}{2} \times$ product of the diagonals

Given that area of the rhombus=140 cm^2

$$140 = \frac{1}{2} \times 14 \times JL$$

$$JL = \frac{140 \times 2}{14} = 20 \text{ cm}$$

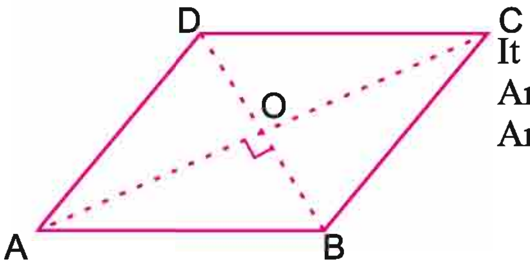
Since diagonals of the rhombus bisect each other therefore,

$$OL = \frac{1}{2} \times JL$$

$$= \frac{1}{2} \times 20 = 10 \text{ cm}$$

Example:6 The area of the rhombus shaped play ground is 21600cm^2. When Rohan walks through the larger diagonal then he has to walk 240m.calculate the total distance covered by Rohan if he walks through the boundary of the given field.

Solution: lets assume that ABCD is a rhombus shaped playing ground.



It is given that area of the ground = 21600m^2

And diagonal AC = 240m

Area of the Rhombus $= \frac{1}{2} \times \left(\text{product of the diagonals}\right)$

$$21600 = \frac{1}{2} \times 240 \times BD$$

$$BD = \frac{21600 \times 2}{240} = 180\text{m}$$

Since diagonal of the rhombus perpendicular bisect each other

Therefore, $AO = \frac{1}{2} \times AC = 120$ m and $BO = \frac{1}{2} \times BD = 90$ m

Since ΔAOB, is a triangle .so with the help of pythogorous theorem we have,

$$AB = \sqrt{(AO)^2 + (BO)^2} = \sqrt{(120)^2 + (90)^2} = 150\text{m}$$

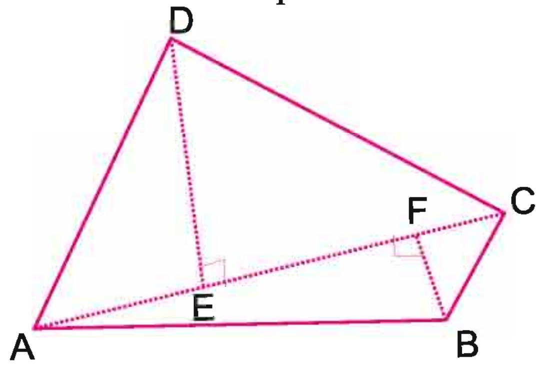
Hence distance covered along the boundary of the ground

= perimeter of the ground = $4 \times 150 = 600$m.

14.4 Area of a Quadrilateral

A general quadrilateral can be split into two triangles by drawing one of its diagonals. After this make perpendicular by each remaining vertex as giving in the figure.

Area of quadrilateral ABCD = (area of Δ ABC) + (area of Δ ADC)



$= \frac{1}{2} \times (AC \times BF) + \frac{1}{2} \times (AC \times DE)$

$= \frac{1}{2} AC \times (BF + DE)$

$= \frac{1}{2} d \times (h_1 + h_2)$.

Where , d = length of diagonal AC

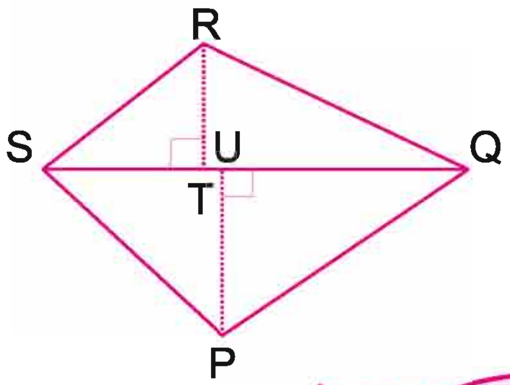
h_1 = length of perpendicular BF

h_2 = length of perpendicular DE

Area of quadrilateral $= \frac{1}{2} \times$ diagonal $\times$ (sum of length of perpendicular on the diagonal)

Example 7: The length of the diagonal SQ of the quadrilateral PQRS is 8cmand length of the perpendiculars on the diagonal RT and UP is 4 cm and 5.5cm respectively. Find the area of the quadrilateral.

Solution: Area of the quadrilateral PQRS



$= \frac{1}{2} \times$ diagonal $\times$(sum of length of perpendicular on the diagonal)

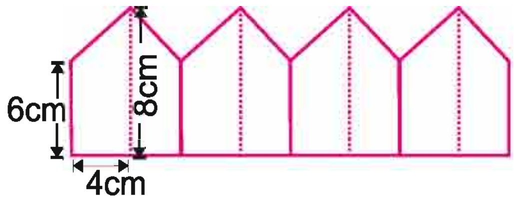
$= \frac{1}{2} \times 8 \times (4 + 5.5)$

$= \frac{1}{2} \times 8 \times 9.5$

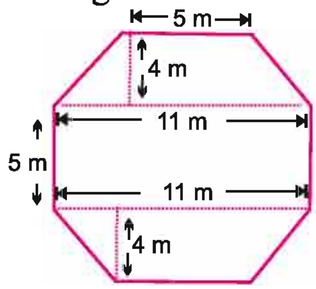
$= 38 \text{ cm}^2$

Exercise 14.1

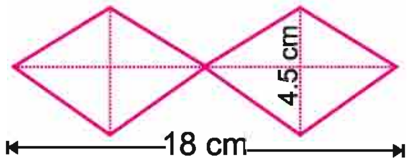
1. The length of two perpendicular sides of a trapezium are 10 cm and 16 cm perpendicular distance between them is 8 cm. find the area of the trapezium.
2. The roof of the building is in particular shape as shown in the below figure if all the dimension is of equal length then find the area of the whole design.



3. The area and height of the trapezium are 34 cm^2 and 4 cm . One of its parallel side is 10 cm. Find the length of other parallel side.
4. Top surface of a platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.



5. Length between the opposite vertex of the rhombus shaped plot are 12.5 m and 10.4 m .Find the total cost of making this plot as a flat surface if the cost of making a flat surface per square meter is Rs. 180.
6. Find the area of the combined rhombus shaped tiles as given in the figure.



7. The field of the kalyan is in the form of quadrilateral. The diagonal of this field is 220 m and the perpendiculars dropped on it from the remaining opposite vertices are 80 m and 130 m respectively. find the area of the field.
8. Fill in the blanks
   (i) Area of the rhombus is ...... product of the diagonals
   (ii) Both the diagonals of the scalene quadrilateral is always ....
   (iii) Area of the.......... quadrilateral can be found through the formula height × (sum of parallel sides) / 2
   (iv) The quadrilateral whose unequal diagonal bisect each other is called................

14.5 Area of a Polygon (Field Book)

So far we have learnt how to find out the area of the plane figure like triangles, quadrilaterals, and circles. All the above mentioned figures have some particular formulas of their areas but if we want to find out the area of the pentagon, hexagon, heptagon,....etc then what will be the processor lets discuss it briefly.

Look at the figures. Can we get some triangle or quadrilateral shaped figure by connecting any two vertices of the given figure. Is sum of the areas of these triangular and quadrilateral shaped figure formed is equal to the area of the given polygon figure?

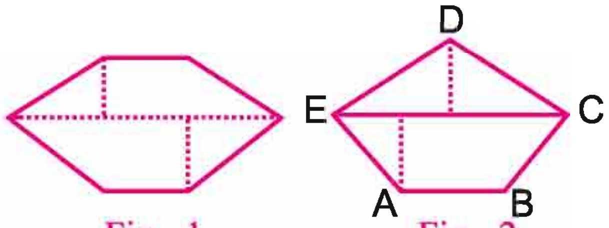
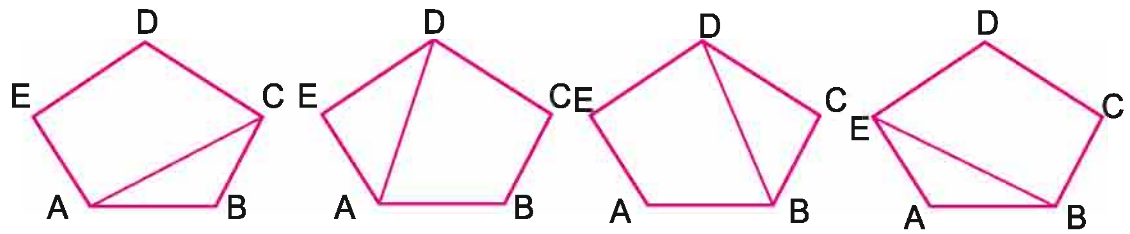


Fig - 1 Fig - 2

(i) Look at the pentagon ABCDE given in the figure 2. A triangle ECD and a quadrilateral ABCDE is formed if we join the vertices E and C. we can find the area of both the figure i.e. the triangle and the quadrilateral

Area of the polygon ABCDE = area Δ ECD + area of quadrilateral ABCE

(ii) Join different vertices of the pentagon. Calculate the area of the different figure obtained in each case. Is the area calculated in each case equal?



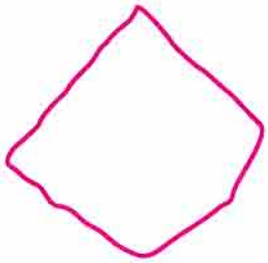
Repeat the same processor with the hexagon.

(making two quadrilaterals) (making two triangles and one quadrilateral)

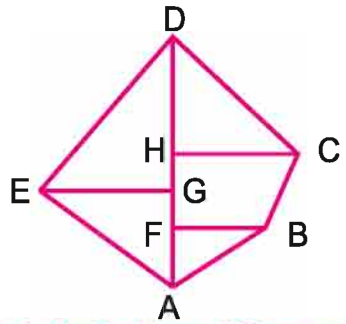
14.5.1 Area of Irregular Polygon

You have often seen around you some farm, ground and other land region which are irregular in shape .they do not resemble the shape of any polygon till we have studied.

Generally when patwari and engineer do any survey on the land which are irregular in shape then they divide that land into some small parts. These parts are in the shape of some plane geometric figures and then they can calculate the area all these small parts of the land that they have divided which is equivalent to the area of the whole land. This method of finding the area of irregular shape field is known as field book. Let's discuss in detail



irregular shape



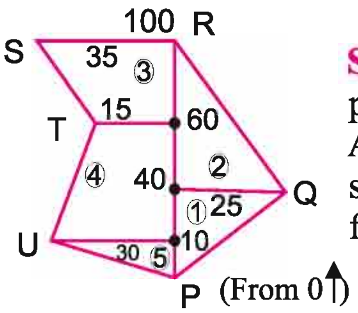
(Field book divided into different polygons)

Look at the irregular shaped land given in the figure. It is a tough task to find the area of this land.

For this we draw the straight lines along the length of the figure. Now we drop the perpendicular from the vertices on the straight line drawn. In this way we get different plane figures. From a point A on the straight line drawn we find the distance of points F , G,H,D , In the same way we can find the perpendicular distance of FB, HC, EG . So now we can calculate the area of the given figure.

Area of the whole figure = area of Δ ABC + area of quadrilateral FBCH + area of Δ HCD + area of ΔEGD + area of Δ AHE

Example 8: Find the area of the figure PQRSTU



Solution: By dividing the complete region into parts we get five figures.

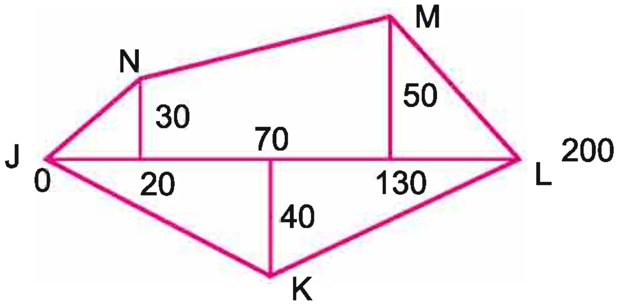
Area of PQRSTU = area of first region +area of second region + area of third region +area of fourth region +area of fifth region

$$= [\frac{1}{2}\times 40\times25+ [\frac{1}{2}\times60\times25] + [\frac{1}{2}\times(15+35)\}40] +[\frac{1}{2}\times(30+15)\times50+[\frac{1}{2}\times10\times20]$$

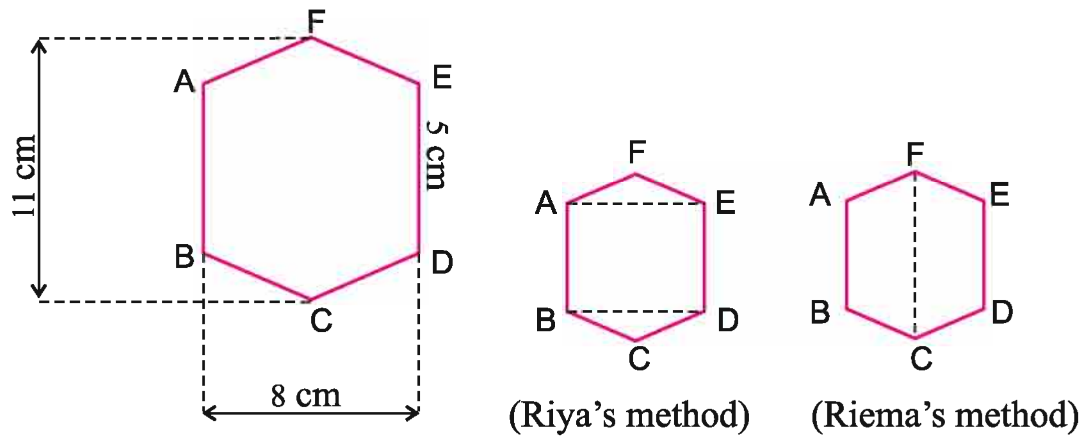
$$= 500+750+1000+1125+150 = 3525 \text{ square unit}$$

Exercise 14.2

1. Find the area of the polygon according to the measurement given in the figure.

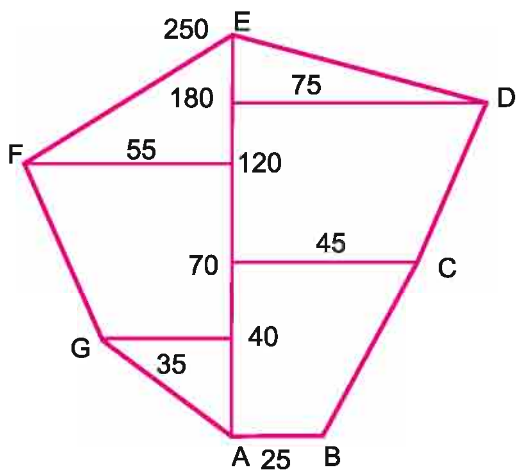


2. Each side of the hexagon ABCDEF has side of length 5cm as given in the figure. Riya and Riema find the area of the region by dividing it into two parts in two different manner. Compare the area in both the case.



(Riya's method) (Riema's method)

3. The measurement of each part of the farm of Ramlal is given in the figure. find the total cost of ploughing at the rate of Rs. 4 per meter .



We Learnt

1. Area of Trapezium = height × $\left(\frac{\text{sum of parallel sides}}{2}\right)$

2. The diagonals of the trapezium bisect each other

3. Area of Trapezium = $\frac{(\text{sum of diagonals})}{2}$

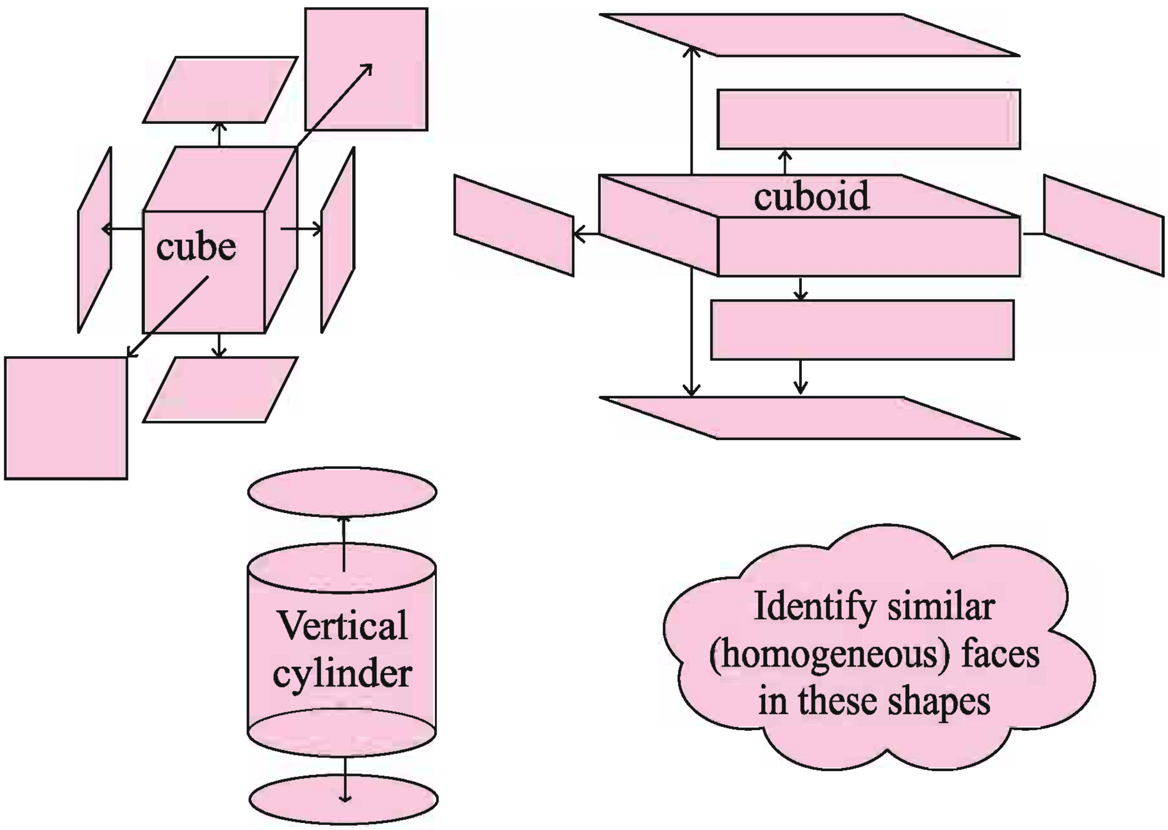
4. Area of general quadrilateral = $\frac{1}{2}$ × diagonal × (sum of length of perpendicular on the diagonal)

5. To find the area of pentagon, hexagon etc. we divide them into triangle or quadrilateral. The total area of the given polygon is equal to sum of all the triangles and quadrilateral that the polygon have been divided into.

CHAPTER 15
Surface Area and Volume

15.1 You have learnt to identify three dimensional shapes in form of two dimensional. You have also learnt to form cube, cuboid, cylinders etc. through two dimensional lattice.

You have seen in some shapes there are two or more faces or surfaces as same (Congruent).



Note: This is compulsory that in vertical cylinder line segment joining centre of circular faces is perpendicular on base.

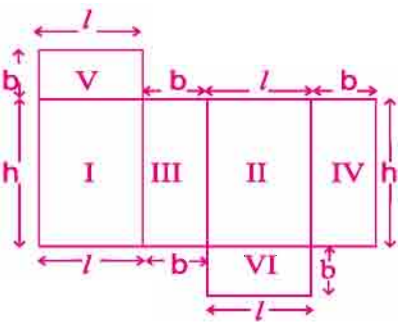
15.2 Surface Area of Cube and Cuboids –

Gopal and Ramesh want to make colourful cube and cuboids for decoration in house. Gopal prepared cube of side 4 cm. Ramesh prepared a cuboid which is 5 cm long, 4 cm broad and 3 cm high.

Now to make them attractive they want to stick colourful papers on them. Both are in this dilemma that how much paper should be bought from the market? Gopal said to Ramesh "We can add area of each face." Listening to their conversation Gopal's elder brother said this is called surface area means area covered by all faces of a solid is called its surface area.

15.2.1Cuboids

After cutting and opening a cuboidal box its lattice appears as shown in the figure. Write dimensions on each of the face. You will see three pairs of two-two congruent faces of a cuboid.



Surface area of the whole cuboid

= Area I + Area II + Area III + Area IV + Area V + Area VI

$= h \times l + h \times l + b \times h + b \times h + l \times b + l \times b$
$= 2hl + 2bh + 2lb$
$= 2(lb + bh + hl)$

Where l, b and h are length, breadth and height.

In this way total surface area of cuboid of Ramesh is $2(5\times4 + 4\times3 + 3\times5)$

$= 2(20 + 12 + 15)$

$= 2(47)$

$= 94$ square cm.

For painting all four walls of a cuboid ceiling and bottom (floor) of cuboid are to be left. Area of only four walls is to be calculated. This is called lateral surface area of a cuboid.

Means lateral surface area of a cuboid = total surface area – area of ceiling and floor

$= 2(lb + bh + hl) - 2lb$
$= 2lb + 2bh + 2hl - 2lb$
$= 2bh + 2hl$
$= 2\times(l + b)\times h$ Square unit

= Perimeter of base (floor) × height

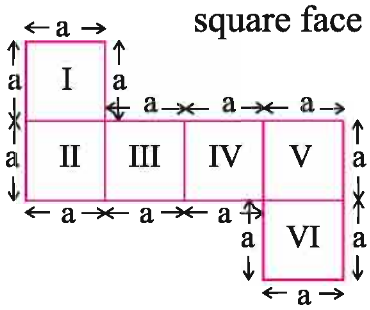
Area of ceiling and floor is independent of height. It depends only on length and breadth.

15.2.2 Total surface area of a cube

You know a cube is such cuboid whose all dimensions length, breadth and height are same. Area of one face of a cube is a^2. All six faces are squares. Each side is represented by a.

So total Surface area of a cube = 6 × area of square face $= 6 \times a^2$

$= 6a^2$



In this way total surface area of cube of Gopal $= 6 \times 4^2$

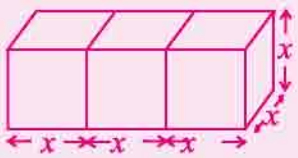
$= 6 \times 16$

= 96 square cm.

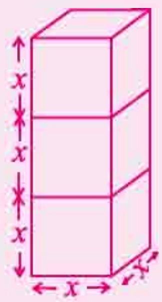
Do and learn

If a cuboid is formed by sticking three cubes of side x then what will be the dimensions of cuboid.

Condition I



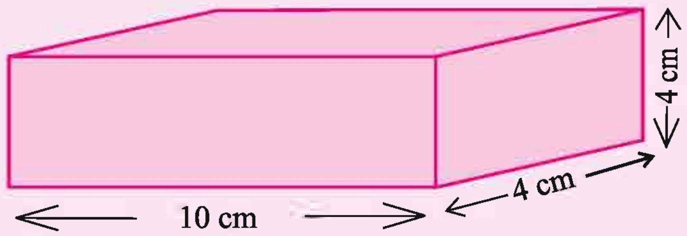
Condition II



Condition I		Condition II	
Length	= ---------	Length	= ---------
Breadth	= ---------	Breadth	= ---------
Height	= ---------	Height	= ---------

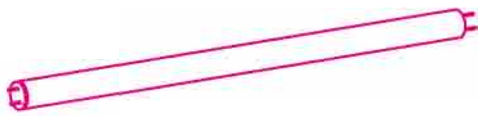
Do and learn

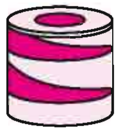
1. Find out surface area of a cube having side 3 cm. What will be the surface area of 5 such cubes?
2. What will be the total surface area of 5 cubes having side 3 cm? if they are sticked one after one then how much surface area will be decreased.
3. Find out lateral surface area of the given cuboid



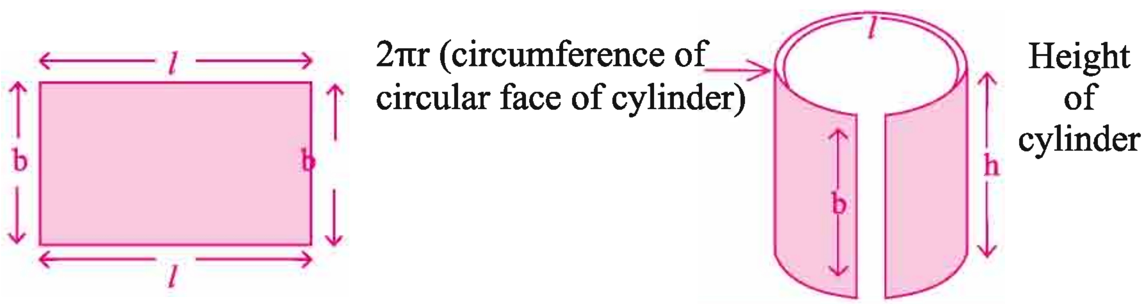
15.3 Surface area of cylinder

A tin of cold drink, pipe, tube of tube light, round pillar etc. are example of vertical cylinder. Area left after excluding area of both circular faces is called curved surface area. To determine its area we will do an activity.

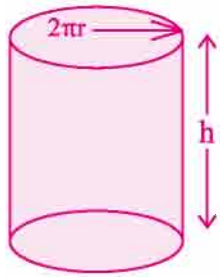




Activity – Take a rectangular paper and mark length and breadth on its corner. According to the figure fold the given paper in direction of its length and stick its end with help of a tape without overlapping. In this way a curved surface of cylinder will be formed.



Length of rectangular paper (l) is transformed into circumference ($2\pi r$) of circular base of cylinder and breadth (b) has taken form of height (h) of cylinder.



In this way curved surface area of cylinder= area of rectangular paper $= L \times b$

$= 2\pi r \times h \quad = 2\pi rh$ square units

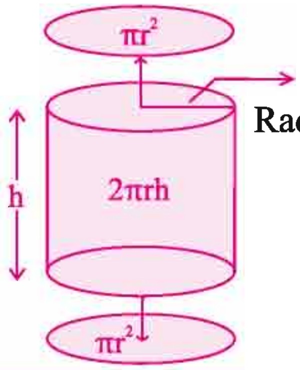
Now find curved surface area of cylinder by folding rectangular paper along its breadth.

If two congruent circular faces of cylinder are included with curved faces then it is called total surface area of cylinder. How its area will be calculated?

Total surface area of cylinder = area of curved surface + area of both circular faces

$= 2\pi rh + 2 \times \pi r^2$

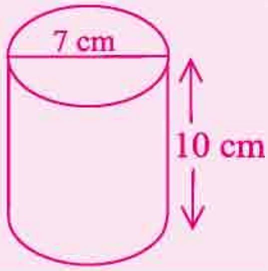
$= 2\pi r\,(h + r)$ Square unit



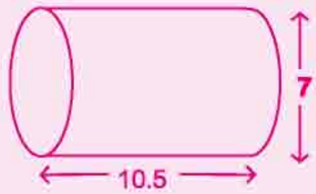
Do and learn

Determine total surface areas of following cylinders

(i)



(ii)



Example 1 Internal dimensions of cubical room of Vijay are 12m, 8m and 4m. He wishes to paint its all four walls. Determine cost of painting with Rs 5 per meter square. Tell him if he also wish to get the ceiling painted then cost will increase by what amount?

Solution Length of room (l) = 12m

Breadth (b) = 8m

Height (h) = 4m

lateral surface area (four walls) = perimeter of base × height

$= 2(l + b) \times h$

$= 2(12 + 8) \times 4$

$= 2 \times 20 \times 4$

$= 160\ m^2$

Expenditure of painting per meter square = Rs 5

Total expenditure of painting four walls of room = 160 × 5 = Rs 800

Area of ceiling = l × b = 12 × 8 = Rs 96

Extra expenses for painting ceiling = 96 × 5 = Rs 480

Means if Vijay will get the ceiling painted the expense will increase by Rs 480 and becomes Rs 1280.

Example 2 Manisha has a cuboidal tin having no lid whose side is 45cm. She wants to stick a colour paper on its external surface, find area of paper required?

Solution Total surface area of cube $= 6a^2$

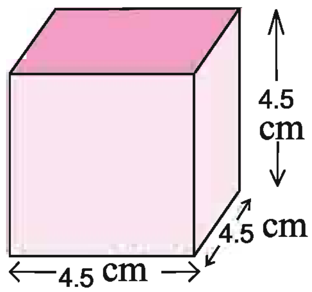
Area excluding top (lid) $= 6a^2 - a^2$

$= 5a^2$

$= 5 \times (45)^2$

$= 5 \times 2025$

$= 10125$ Square cm



Example 3 Find the height of such cylinder whose radius is 7 cm and total surface area is 968 square cm.

Solution Let the height of cylinder = h cm

Radius = 7cm

Total surface area of cylinder $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (h + 7)$ Square cm

But according to the question total surface area is 968 cm^2

So

$$2 \times \frac{22}{7} \times 7 \times (h + 7) = 968$$

$$(h + 7) = \frac{968}{2 \times 22}$$

$$h + 7 = 22$$

$$h = 15 \text{ cm}$$

So height of cylinder = 15 cm

Example 4 A company labeled its product after packing it in a cylindrical tin of 20 cm height and 14 cm diameter. While labeling 2 cm distance left on both sides then determine area of label.

Solution Height of cylindrical tin = 20 cm

Height of applied label $= 20 - (2 \times 2)$

$= 16$ cm

Diameter of tin (2r) $= 14$cm

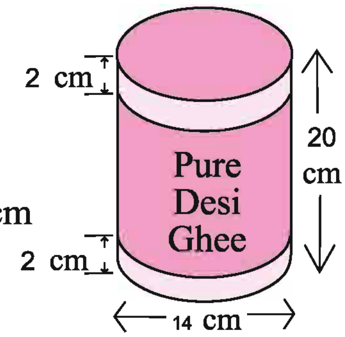
Area of label $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times 16 \text{ cm}$

$= 44 \times 16 \text{ cm}^2$

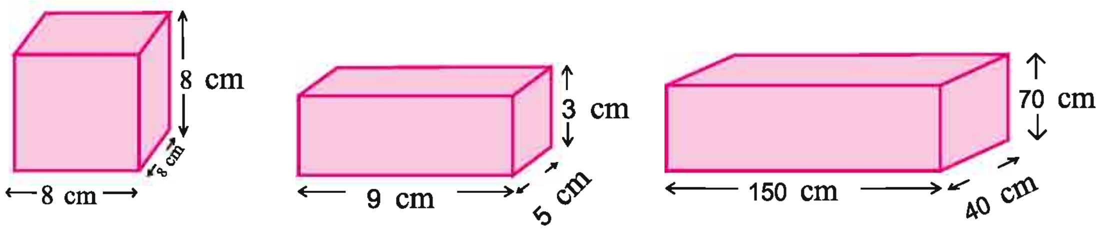
$= 704 \text{ cm}^2$

So area of label $= 704 \text{ cm}^2$



Exercise 15.1

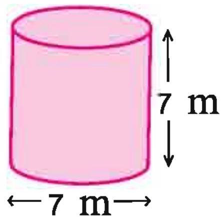
1. On the basis of given measure– determine surface area of cuboidal wooden log, cuboidal brick and box.

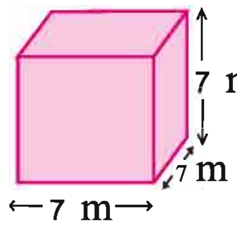


Wooden log brick box

2. Determine side of a cube whose total surface area is 600 square cm.

3. In the given figure whose surface area is more? (π =22/7)





4. Find the area of curved surface if area of base of cylindrical tank is 176 Cm^2 and height is 30 cm.
5. Form a sheet of 8 square meter, a closed cylindrical tank is formed which has one meter height and 140 cm diameter. How much sheet will be left after making tank?
6. How many paint tins having spread capacity of 100 cm^2 will be required to paint external surface of box having dimensions 80 cm × 50 cm × 25 cm.
7. There are 25 cylindrical pillars in a building. Each pillar has radius of 28 cm and height of 4 m. find expenditure of painting curved surface area of all pillars at the rate of Rs. 8 per meter square.
8. Curved surface area of a hollow cylinder is 4224 cm^2. A rectangular sheet having width 33 cm is formed cutting it along its height. Find perimeter of sheet.
9. To make a road plain a roller has to complete 750 rounds. if the diameter of roller is 84 cm and length 1meter then find the area of road.
10. A cube is made by arranging 64 cubes having side of 1 cm, find total surface area of cube so formed.

15.4 Volume and Standard Unit

Place covered by a three dimensional object is called volume. Example volume of an almirah placed in a room is more than volume of flour tin.

You know we use squares of unit side to determine area of any shape. In this way to identify volume of a solid we have to determine number of unit cubes included in solid Volume is measured in unit given below.

1cm × 1cm × 1cm = 1 cm^3 or 1 cubic cm

1m × 1m × 1m = 1 m^3 or 1 cubic meter

1 cubic meter = 100 cm × 100 cm × 100cm = 1000000 cm^3

15.4.1 Volume of cube and cuboid

By arranging cubes of unit length find volume of cuboid so obtained-

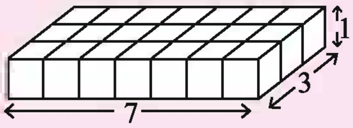
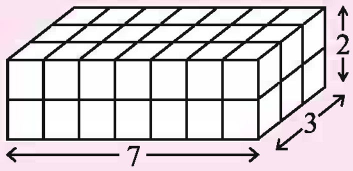
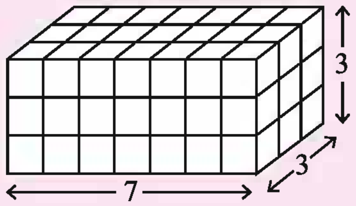
	Cuboid Volume		Length l	Breadth b	Height h	l×b×h
(i)		21 cubic unit	7 unit	3 unit	1 unit	7×3×1 =21 cubic unit
(ii)		42 cubic unit	7 unit	3 unit	2 unit	....
(iii)		63 cubic unit	7 unit	3 unit	3 unit	....

Table 15.1

After completing table we reached to following conclusion

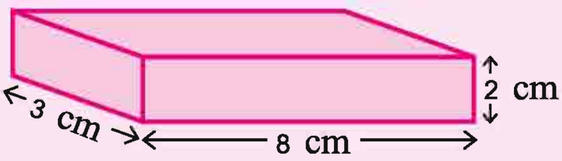
Volume of cuboid = length × breadth × height

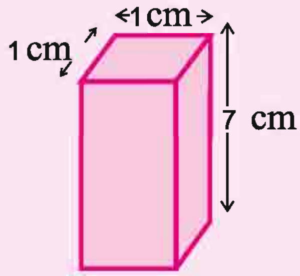
In this way volume of 7 cm long, 5 cm broad and 2 cm high cuboid

$$= \text{length} \times \text{breadth} \times \text{height}$$
$$= 7\text{ cm} \times 5\text{ cm} \times 2\text{ cm}$$
$$= 70\text{ cm}^3$$

Do and learn

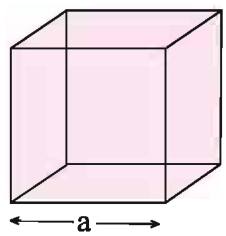
Find volume of cuboid on basis of given measures





Volume of a cube

You know that cube is such cuboid whose length, breadth and height are same if each side is represented by a.



Volume of cube = side × side × side

Volume of cube = $a \times a \times a = a^3$ cubic unit

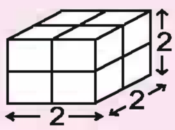
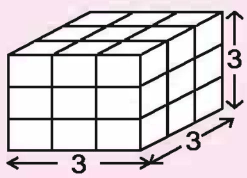
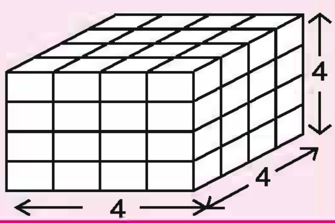
S.No	Cube	Number of unit cubes	Side	Side × side × side	$(\text{side})^3$
(i)	N 	8 cubic unit	2 unit	2 unit × 2 unit × 2 unit	8 cubic Unit
(ii)		27 cubic unit	---------	---------	---------
(iii)		64 cubic unit	---------	---------	---------

Table 15.2

Do and learn

Determine volume of the cube having side

(i) 1.5 cm (ii) 4 m

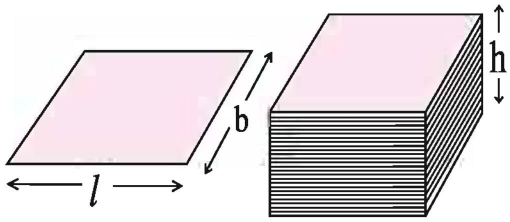
Take a rectangular paper sheet and measure its area. Now taking paper as base, place sheets of same measure on one another and make cuboid of desired height (to make it understandable paper rim can be used) multiply area of base and height and note down the result.

Determine volume of this cuboid by previously established formula. What conclusions can be drawn from both results?

Do you agree with this-

Volume of cuboid = area of base × height

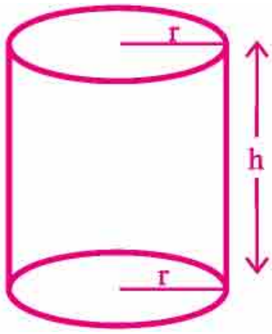
In this way volume of any shape can be determined



Note- here it is to be kept in mind that base and top of solid are congruent and line segment joining both centers are perpendicular to base and top.

15.4.2 Volume of a cylinder

Like cuboid in cylinder also base and top are congruent. And curved surface is perpendicular on base



Volume of cuboid = area of base × height

Volume of cylinder = area of base × height

$= \pi r^2 \times h$

$= \pi r^2 h$ cubic unit

Capacity

Place covered by a three dimensional object is called volume and quantity of liquid that can be filled in it is called capacity. In a cube of one cm, amount of liquid that can be filled is 1 ml. In this way its standard units are:

1 ml $= 1 \text{ cm}^3$

1 litre $= 1000 \text{ cm}^3$ or 1000 milliliter

1 kilo litre = 1000 litre or $1 \text{ m}^3 = 1000$ litre

Example 5 determine height of a cuboid whose volume is 275 cm^3 and area of base is 25 cm^2

Solution volume of cuboid = area of base × height

$275\ cm^3 = 25\ cm^2 \times$ height

Height = $\dfrac{275 cm^3}{25 cm^2} = 11$ cm

Hint:
$m \times m \times m = m^3$

Example 6 Inside a cuboidal store house of dimension 60m × 40m × 30m how many cuboidal tin can be placed, if volume of tin is 0.8 m^3

Solution volume of a cuboidal tin $= 0.8\ m^3$

Volume of a store house $= 60m \times 40m \times 30m$

$= 72000\ m^3$

Number of tins that can be placed in store house are

$= \dfrac{\text{volume of store house}}{\text{volume of a tin}}$

$= \dfrac{60m \times 40m \times 30m}{0.8m^3}$

$= \dfrac{72000\ m^3}{0.8\ m^3} = 90000$

Example 7 if weight of one meter ice cube is 900 kilogram then what will be the weight of ice cube having side 50cm.

Solution volume of cube having side as 50 cm $= (50)^3\ cm^3$

$= 125000\ cm^3$

$= \dfrac{125000}{(100 \times 100 \times 100)}$ cubic meter

$= 0.125$ cubic meter

Weight of one cubic meter ice $= 900$ kg

Weight of 0.125 cubic meter ice $= 900 \times 0.125\ m^3 = 112.5$ kg

Example 8 A 4 cm high cylinder is formed after folding (without overlapping) an 11 cm × 4 cm rectangular paper. Find its volume.

Solution height of cylinder = 4cm

Circumference of base of cylinder = 11cm.

Means $2\pi r = 11$

$$2 \times \frac{22}{7} \times r = 11$$

$$r = \frac{7}{4} \text{ cm}$$

Volume of cylinder (v) = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4$$

$$= \frac{22 \times 7}{4} \text{ cm}^3 \qquad = 38.5 \text{ cm}^3$$

Example 9 curved surface areas of a cylinder is 440 m^2 whose height is 4 m. Find volume

Solution curved surface area of cylinder = 440 m^2

$$2\pi rh = 440 \text{ m}^2$$

$$2 \times \frac{22}{7} \times r \times 4 \text{ m} = 440 \text{ m}^2$$

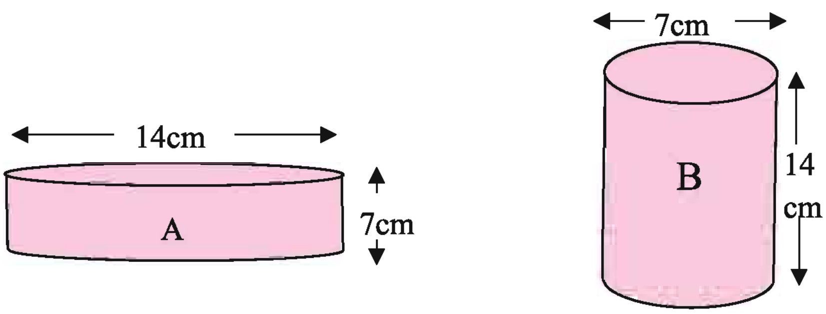
$$r = \frac{440 \times 7}{2 \times 22 \times 4} \text{ m} \quad = \frac{35}{2} \text{ m}$$

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 4 \quad = 3850 \text{ m}^3$$

Exercise 15.2

1. Dimensions of a cuboid are 60 cm × 54 cm × 30 cm. how many cubes of side 6cm can be placed in the cuboid?
2. How many wooden logs of side 6 cm can be cut from a 3m long, 50 cm broad and 25 cm high wooden pile.
3. Cylinder A has diameter of 14 cm and height of 7 cm. and cylinder B has diameter 7 cm and height 14cm. Without calculation tell volume of which cylinder is more? Verify the answer by calculation.



4. From a cylindrical milk tanker of radius 1.5 m and length 7m how many polythenes of one litre can be packed? (1 m^3= 1000 litre)
5. In what time a tap giving 60 litres of water per minute can fill a cylindrical tank of radius 3.5m and depth 3m.
6. Dimensions of a cuboidal ice is 50cm × 30cm × 20cm. Find its weight in kilogram. If weight of 1000cm^3 ice is 900 gram.
7. If side of a cube is doubled then
   (i) How many times its surface area will increase?
   (ii) How many times its volume will increase?
8. Volume of a cylindrical tank having 7 meter diameter is 770 cubic meters then find height of the tank.

We learnt

1. Area of a solid is sum total of areas of its faces.
2. Surface area of a cuboid = 2 (lb + bh + hl) where l, b, h are dimensions of cuboid
3. Surface area of cube = $6a^2$ where a = side of cube
4. Surface area of cylinder = $2\pi r(h+r)$
   R= radius of base of cylinder
   h = height of cylinder

If else not mentioned then surface area means total surface area.

5. Place covered by a three dimensional figure is called its volume.
6. Volume of a cuboid = length $\times$ breadth $\times$ height = $l \times b \times h$ cubic unit (side of a cube = a unit)
7. Volume of cube = side $\times$ side $\times$ side = a^3 cubic unit
8. Volume of cylinder = $\pi r^3 h$ cubic unit
9. If base and top of a solid are congruent and line segment joining centers of top and base is perpendicular on base (cube, cuboid and cylinder) then
   Volume = area of base $\times$ height
10. (i) 1ml = 1cm^3
    (ii) 1 litre = 1000 ml
    (iii) 1kilolitre = 1000 litre or 1m^3= 1000 litre

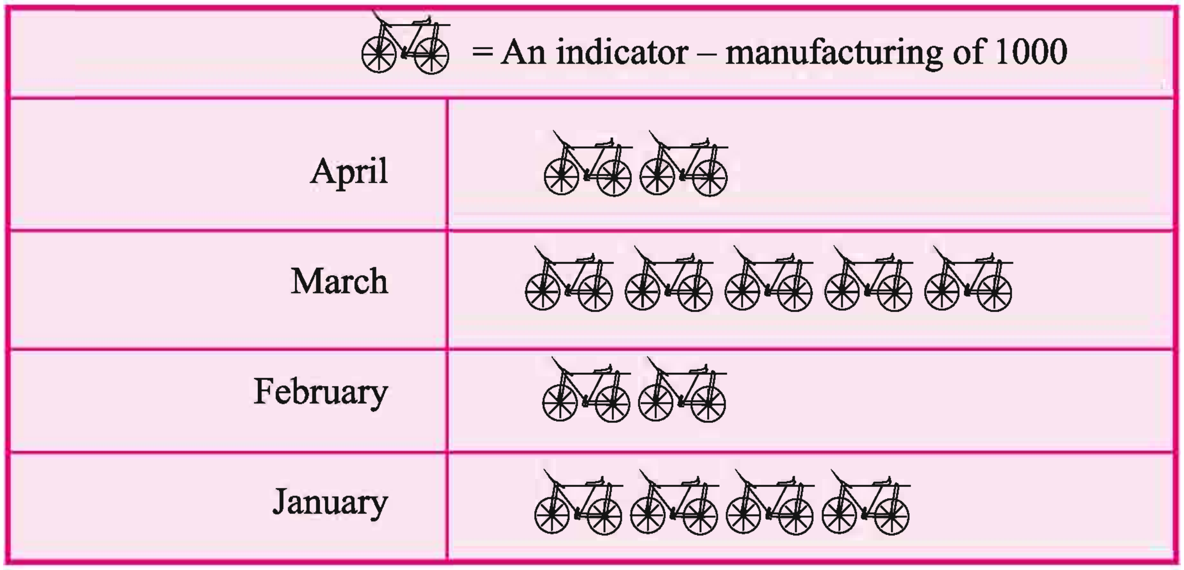
CHAPTER 16

Data Handling

16.1 We generally see around us many types of data, tables and graphs in newspaper, magazines and in doordarshan. These things give us some information. You can also get information by collecting data from your surroundings. Before collecting data we should know what we want to study. Let us assume you want to know, what is the average height of your classmate?

To know this you have to collect data related to height of your classmates. For making sense what does data tell, we represent it by graph. You have studied different types of graphs in previous classes, let us see them again.

(i) **Pictograph** – after looking at figure given below, answer the following question-



Graph 16.1

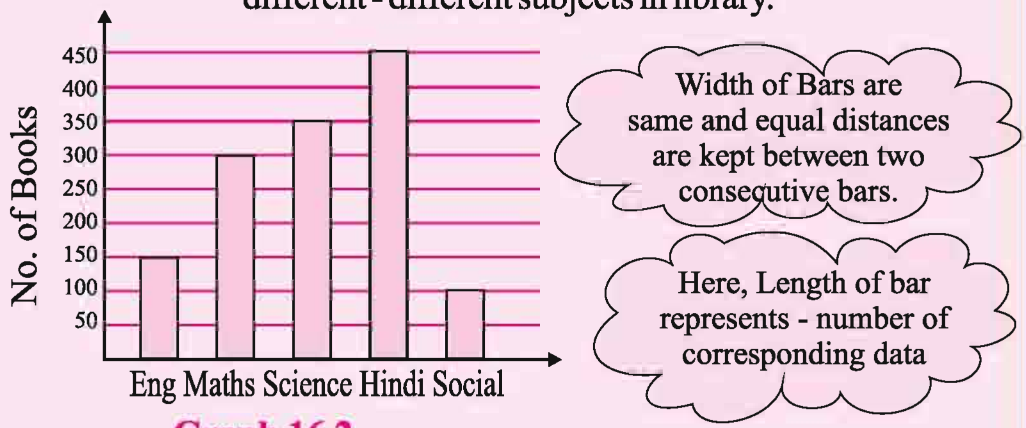
(i) How many cycles were produced in month of March?

(ii) Production of which two months is same?

(ii) Bar graph- In bar graph width of each bar is same and they are at equal distance. Height of bar is in accordance to data on scale this we can say as proportional to corresponding data.

Do and learn

◆ From given bar graph answer the following questions by looking at number of books on different - different subjects in library.



Graph 16.2

(i) Which subject has maximum number of books and how many?

(ii) Which subject has least number of books and how many?

(iii) How many books are there in the library?

(iv) What information has been given by this bar graph?

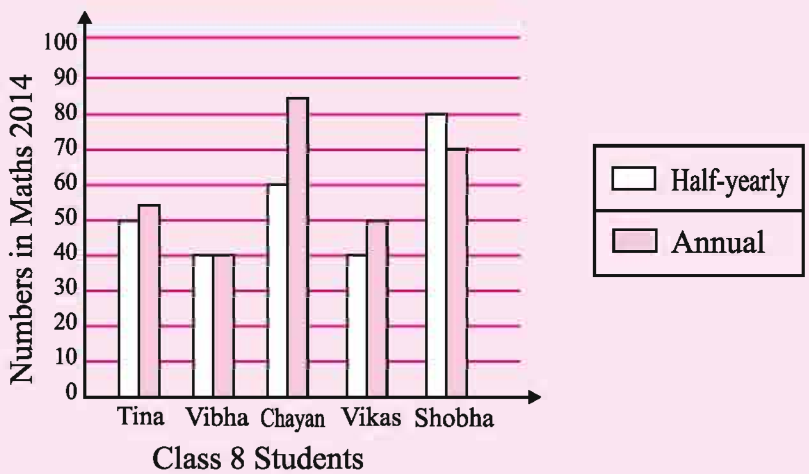
(v) Difference in the number of books of which two subjects is minimum?

(iii) Double Bar Graphs

Double bar graphs are plotted when we have to compare data of two groups.

Do and learn ◆

A double bar graph is given on marks obtained by 5 students in yearly and half yearly exam in mathematics subject



Graph 16.3

Answer the following questions on basis of graph

(i) Whose performance was similar in annual and half yearly examination?

(ii) Whose performance was better in annual as compared to half yearly?

(iii) How many students scored more than 50 in annual examination?

(iv) Which information is given in this double bar graph?

(v) What is the average of marks of half yearly? Is that less than average of annual examination?

Do and learn

Draw different graphs to show given information

Year	2010	2011	2012	2013	2014
Number of books bought for library	170	150	190	180	210

Name of village	Makadi	Akola	Ravaliya Khurd	Siwadiya
Number of men	1800	1700	1800	1500
Number of women	1600	1700	1900	1600

In mathematics examination for class VIII marks obtained by students out of 50 marks are

28, 25, 27, 8, 28, 37, 28, 28, 14, 1

15, 28, 18, 20, 36, 37, 10, 27, 15, 8

In this example each number is an observation. In this way group of observation is called raw data. For extracting meaningful conclusions, these data needed to be arranged in order (ascending or descending).

Like – data of marks obtained by students of class 8 out of 50

1, 8, 8, 10, 14, 15, 15, 18, 20, 25

27, 27, 28, 28, 28, 28, 28, 36, 37, 37

Here difference between maximum and minimum marks obtained is 37-1 = 36. This difference 36 is called range of the above data.

16.2.1 Frequency distribution

Which marks are obtained by maximum number of students and which marks are obtained by minimum number of students. For this by using tally marks, following table is formed.

Marks obtained	tally	frequency
1	|	1
8	||	2
10	|	1
14	|	1
15	||	2
18	|	1
20	|	1
25	|	1
27	||	2
28	卌	5
36	|	1
37	||	2
	Total	20

Table 16.1

From tally marks in front of marks obtained we came to know about number of students who obtained those marks. This number shows frequency of the marks. Frequency of an entry is the number, of times that entry comes in data.

Frequency of marks 28 is 5 and frequency of each mark 8, 15, 27 and 37 is 2. Table formed as above is called frequency distribution table. From this table we determine how many times an entry has appeared

16.2.2 Classification of data (In the form of class interval)

Sometimes we have such data which are diversified like – think about marks obtained by 35 students of class

10, 13, 11, 7, 8, 5, 17, 20, 3, 14, 11, 10
8, 1, 13, 8, 10, 18, 14, 5, 3, 4, 13, 14
4, 9, 12, 11, 12, 16, 20, 13, 18, 12, 14

If we make frquency distribution table for each observation (marks obtained) then it is very long and sometimes useless. So for convenience we make some class intervals like 0-5, 5-10, 10-15 etc. In this way for the above mentioned data frequency distribution table can be as follows.

Class interval	Tally marks	Frequency
0 - 5	𝍸	5
5 - 10	𝍸 ||	7
10 - 15	𝍸 𝍸 𝍸 ||	17
15 - 20	||||	4
20 - 25	||	2
	Total	35

Table 16.2

In this table all observations are included by dividing marks obtained by all 35 students into class of interval five (0-5, 5-10 etc.). In this each group is called class interval and in short, class.

When data are written in this form then they are called grouped data and obtained distribution is called grouped frequency distribution. This helps in extracting meaningful conclusions. Like,

(i) 12 students have obtained marks between 0 and 10.

(ii) Most of the students have obtained marks between 10 and 15.

(iii) 4 students have obtained 15 out of 20 marks in examination of marks 20.

(iv) Mode for this data is class interval 10-15.

See carefully observation 5 is included in both intervals 0-5 and 5-10. In this way observation 10 and 20 are included in two groups. But one observation can't be included in both groups simultaneously. To escape from this problem we include the common observation in highest class. That Means observation 5 is included in interval 5-10 not in interval 0-5. Similarly10 is included in interval 10-15 (not in 5-10).

Here to define each group, there are two numbers like in interval 5-10 there are two limits 5 and 10 out of which 5 is the lowest limit and 10 is the highest limit of the group. Can you tell lowest and highest limit in interval 15-20.

Difference of two limits of a class interval is called class size or class width. Here class interval of 5-10 is 5. What is the class size of the class intervals 20-25 and 15-20?

Do and learn

1. 1. Study the following frequency distribution table given below and answer the following questions

(i) What is measure of class interval?

(ii) Which is upper limit of class interval 300-350?

(iii) Frequency of which group is maximum?

(iv) Frequency of which two groups is same?

Class interval	Frequency
100 -150	45
150 - 200	25
200 - 250	55
250 - 300	125
300 - 350	140
350 - 400	65
400 - 450	45
Total	500

Table 16.3

2. Marks obtained by 40 students in science out of 50 marks is as follows

38, 35, 44, 30, 30, 33, 38, 40, 35, 45, 48,40, 35, 45, 38,
35, 44, 33, 40, 42, 45, 38, 35, 33, 34, 37, 47, 49, 37, 47,
40, 31, 38, 43, 31, 37, 41, 38, 45, 40

(i) Make a frequency distribution table of given data taking interval as 30-35, 35-40 etc……..

(ii) What are the class limits of class interval 35-40?

(iii) What is class measure of class interval?

16.3 Histogram – graphical representation of data

Consider the given mathematics test frequency distribution table of 40 student, about their marks obtained out of 36.

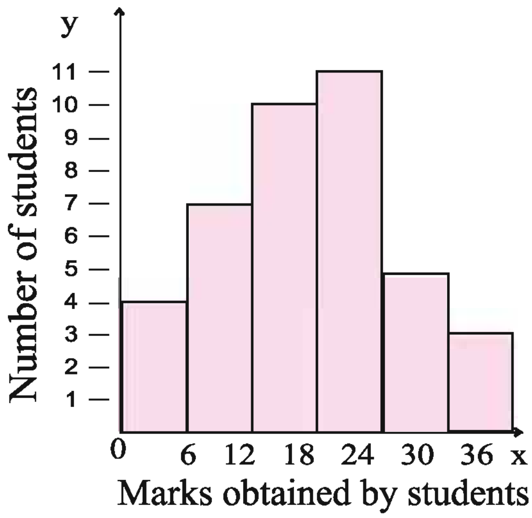
Class interval	Frequency
0 - 6	4
6 - 12	7
12 - 18	10
18 - 24	11
24 - 30	5
30 - 36	3
Total	40

Table 16.4

In graph 16.4, above frequency distribution is shown.

Is this bar graph different from those graphs which you plotted in previous class? Clearly there is no gap between bars because there is no gap between class intervals Class intervals are represented on horizontal axis and length of bar represents frequency of class interval.

Such graphical representation of data is called histogram.



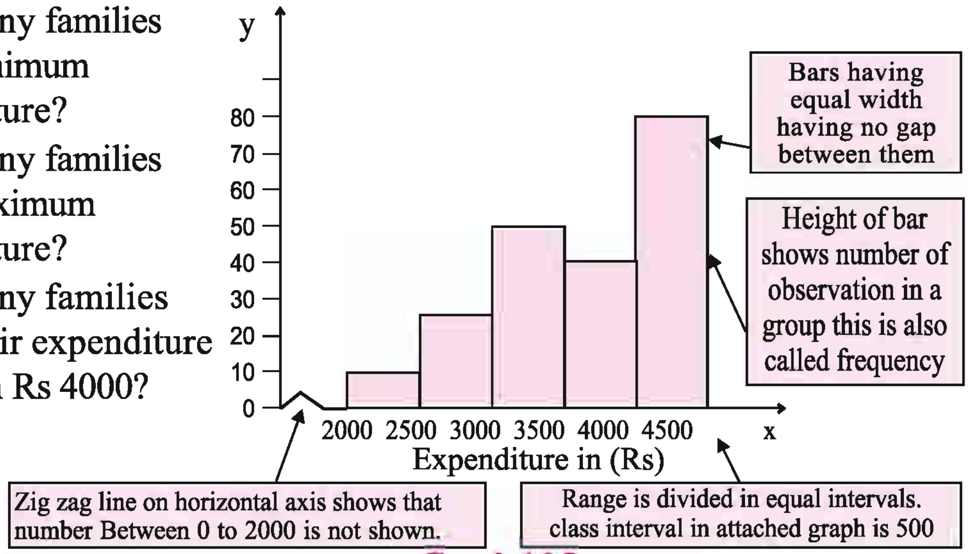
Graph -16.4

Given below is a histogram between number of families and their expenditure (graph 16.5). From this histogram 16.5 we can tell-

(i) How many families have minimum expenditure?

(ii) How many families have maximum expenditure?

(iii) How many families have their expenditure less than Rs 4000?



Graph 16.5

Exercise 16.1

1. To show which type of data you will need a histogram?
   (i) Quantity of different types of cereals available at home.
   (ii) Height of all students of your school.
   (iii) Number of cars manufactured by 5 companies.
   (iv) Number of vehicles passing a busy cross road between 8:00 A.M. to 2:00 P.M.

(v) Distance between home to school of all students of your class.(in meters)

2. Rakesh distinguishes his clothes on basis of colours and indicate like this- White (W), Red (R), Black (B), Yellow (Y), and Others (X). list formed is as follows-

R R X W R B Y R B W W X X R B Y Y X W R
B Y Y B R R X W W R W X X R Y W B Y X X

Using tally marks form a frequency distribution table. To represent it use a bar graph.

3. Payment of the Wage for a week in rupees to 30 NREGA workers of Khatwara village

830, 835, 890, 810, 815, 816, 814, 845, 898, 890,
819, 860, 832, 817, 855, 845, 804, 808, 812, 816,
885, 835, 815, 812, 878, 840, 868, 890, 806, 828

Prepare a frequency distribution table having class interval as 800-810, 810-820 using tally marks. Draw a histogram using table.

4. Number of mobiles sold in 30 days by Ms. Raja Electronics

222, 228, 238, 215, 225, 219, 217, 230, 218, 237,
214, 210, 235, 222, 220, 214, 212, 220, 237, 212,
238, 218, 210, 216, 217, 234, 233, 237, 219, 239

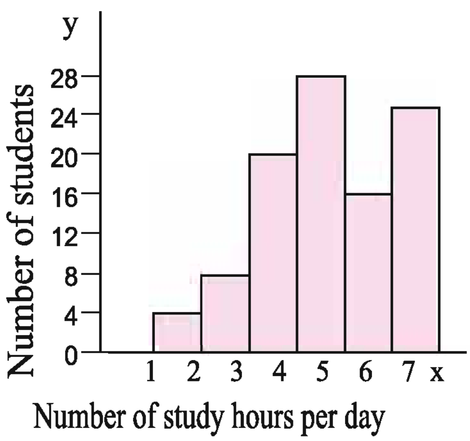
Prepare a frequency distribution table having class interval as 210-215, 215-220 using tally marks. Draw a histogram using table and answer the following questions.

(i) Number of mobiles sold in which class interval are maximum?

(ii) How many days 230 or more mobiles were sold?

(iii) On how many days less than 230 or mobiles were sold?

5. In holidays number of study hours every day of students of class 8 is shown in the graph-



Graph 16.6

Answer the following questions-

(i) How many hours did most of students studied?

(ii) How many students studied for 5 or less than 5 hours?

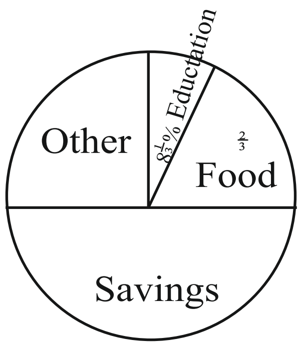
(iii) How many students studied in vacations?

(iv) Frequency of which class interval is maximum?

16.4 Circular Graph or Pie Chart

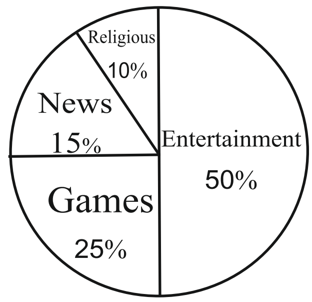
Some data are given in circular form as follows, look at them. To compare various parts of a total, a pie chart is used. Circle shows total.

(i)Monthly budget of a family



Graph 16.7 (i)

(ii) Number of viewers of different programs on television



Graph 16.7(ii)

How did you search for answers of above questions?

You know that angles formed at centre of a circle is360^0 In graph (i) education field is making minimum angle at the centre whereas in graph (ii) field of entertainment is making maximum angle. Whole circle is segregated in sectors. Size of each sector is proportional to information represented by it. Circle represented in this form is called circlular graph or pie chart.

16.4.1 Formation of pie chart

In Graph 16.7 (i) following data are represented in circular form: monthly [illegible]

[illegible]	Expenditure Expense
[illegible]	1500
[illegible]udy	750
Others	2250
saving	4500
total	9000

Table 16.5

Come, let us represent the data in the form of pie chart and understand the steps

Step (i) first of all add all the observations

$$1500 + 750 + 2250 + 4500 = 9000$$

Step (ii) each part of circle (part of total) representing each observation is determined.

Like part in total for expenditure on food

$$= \frac{\text{expense on food}}{\text{monthly income}} = \frac{1500}{9000} = \frac{1}{6}$$

So expense on food is drawn in $\frac{1}{6}$ th part of whole circle.

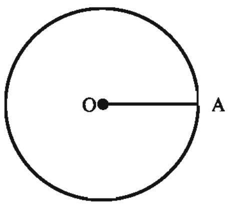
> It is easy to make $\frac{1}{2}, \frac{1}{4}, \ldots$ etc. part of a circle.
> If we have to make
> $\frac{1}{6}$ or $\frac{1}{10}$ parts then which method should be used?

Step (iii) angular measurement for each expense of total central angle (360^0) is determined.

As shown in table 16.6

Budget	expense	part of whole	part of 3600
Food	1500	$\frac{1500}{9000} = \frac{1}{6}$	$360 \times \frac{1}{6} = 60^0$
Study	750	$\frac{750}{9000} = \frac{1}{12}$	$360 \times \frac{1}{12} = 30^0$
Others	2250	$\frac{2250}{9000} = \frac{1}{4}$	$360 \times \frac{1}{4} = 90^0$
saving	4500	$\frac{4500}{9000} = \frac{1}{2}$	$360 \times \frac{1}{2} = 180^0$

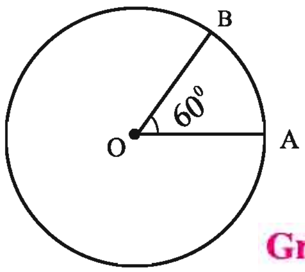
Table 16.6



Step (iv) draw a circle of any radius.
Mark its centre as O and radius as OA.

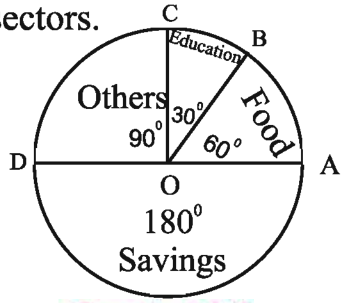
Step (v) angle of sector for expense on food is 60^0

With help of a protractor draw $AOB = \angle 60^0$



Graph 16.7

Step (vi) mark angles of rest of the sectors with the help of protractor. Whole circle will be distributed in sectors.



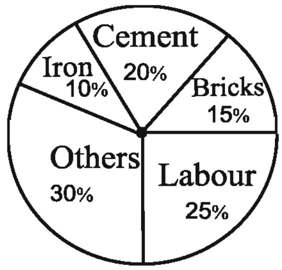
Graph 16.8

Example 1 attached pie chart (graph 16.9) represents expenses on different items in construction of house.

(i) Which item has maximum expense?

(ii) Expense of which two items is half of total expense?

(iii) If expenses on bricks is Rs 30,000 then what is the expense on iron?



Graph 16.9

Solution (i) other items have maximum expense

(ii) Cement and other expense contribute to half expense.

(iii) $\because$ 15% represents = Rs 30,000

$\therefore$ 1% will represent $= \text{Rs}\ \frac{30,000}{15}$

So 10% will represent $= \text{Rs}\ \frac{30,000}{15} \times 10 = \text{Rs}\ 20,000$

Example 2 Attendance of students in a school on a special day is as follows

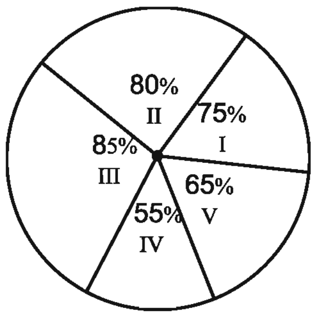
Group	I	II	III	IV	V
Number of Students	15	16	17	11	13

Draw a pie chart for the following data

Solution we find out central angle of each sector. Here total 72 students are there. From this we obtain following table.

Class	No. of Students	Part of total	Part of 360°
I	15	$\frac{15}{72}$	$360 \times \frac{15}{72} = 75^{\circ}$
II	16	$\frac{16}{72}$	$360 \times \frac{16}{72} = 80^{\circ}$
III	17	$\frac{17}{72}$	$360 \times \frac{17}{72} = 85^{\circ}$
IV	11	$\frac{11}{72}$	$360 \times \frac{11}{72} = 55^{\circ}$
V	13	$\frac{13}{72}$	$360 \times \frac{13}{72} = 65^{\circ}$

Table 16.7



Graph 16.10

Do and learn

(1) Sweets liked by students of a school is given below. Draw a pie chart on the basis of data.

Sweets	jalebi	laddu	peda	gulab jamun	others
Number of students	40%	20%	25%	10%	5%

(2) write down number of members in families of your 5 friends and show it by pie chart.

Exercise 16.2

1 Number of text books of any school is given below. Represent this data through pie chart.

Subject	Science	Maths	English	Hindi	Social Science	Total
Books	200	120	190	170	40	720

2. Description of daily activities of a child is as follows. Represent this data in the form of pie chart.

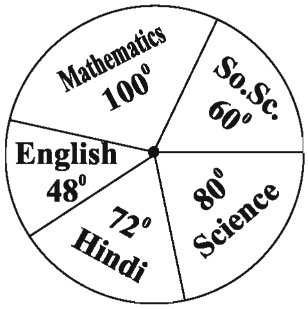
Time Spent	School	Sleep	Play	Others
Hours	8 Hours	6 Hours	2 Hours	8 Hours

3. Pie chart 16.11 shows marks obtained by student in various subjects Hindi, English, mathematics, social science and science. If total marks obtained by these students are900 then answer the following questions-

(i) In which subject did student have 250 marks?

(ii) That student obtained how many marks more in mathematics as compared to Hindi?

(iii) Check whether total marks obtained in social science and mathematics is more than total of marks obtained in English and Hindi?



Graph 16.11

4. Draw a pie chart showing following information. This table shows games liked by student of a class

Game	Kho-kho	Kabbadi	Football	Cricket
Number of Students	5	6	7	18

5. In accordance to Jan Dhan Yojana of Indian government number of accounts opened by following banks in one month in Kota city is as follows:

Name of bank	SBBJ	SBI	IDBI	BOI
Number of accounts opened	21000	18000	6000	9000

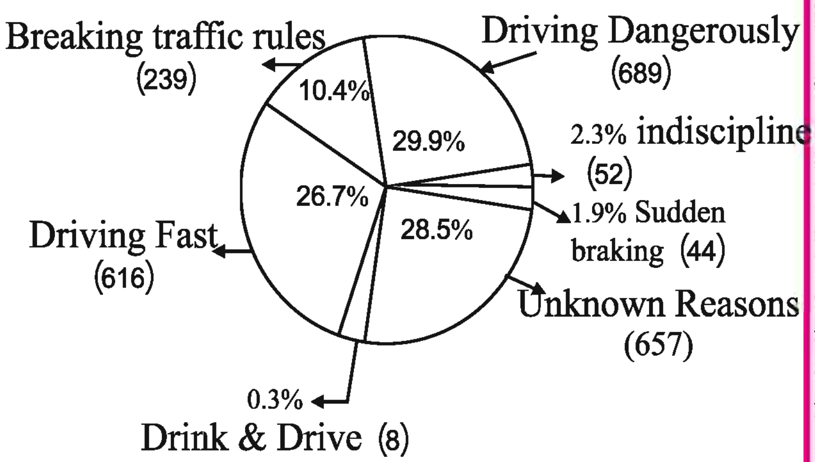
6. Various mistakes of automobile drivers are shown in pie chart. With the help of pie chart answer the following questions-

(i) Percentage of accidents by driving vehicles with fast speed?

(ii) Due to which mistake of automobile driver maximum accidents take place?

(iii) How many accidents occur due to driving of automobile in drunken state?

(iv) How many people underwent accident due to disobedience of traffic rules?



Graph 16.12

Let us discuss

Many types of accidents occur in busy life. Out of which maximum are road accidents? They are as follows-

(1) Mistake of automobile driver

(2) Mistake of sufferer

(3) Technical problem with vehicle

(4) Bad conditions of road

(5) Road condition or bad road shape

(6) Unawareness / others

Let us discuss with teacher and make a project.

16.5 Chance and probability

16.5.1 Meaning of Chance

Vishal said to Ravi – I used to keep an umbrella while going to school from home daily but it doesn't rain. Today due to mistake I didn't bring umbrella. Now it is raining.

Ravi said- Oh friend it happens to me also, for going to school when I reach bus stand on time then bus arrives late, the day on which I reach late bus departs on time.

Rakesh said-It usually happens to me when I didn't complete my homework by chance teacher didn't check homework.

Many times such situations come when by taking support of chance you are to perform work but it doesn't take place in manner you want. Can you give some more example like this?

When a person purchases lottery ticket then his chance of success or failure are not equal.Means chance of victory is less and chance of losing is more. But here we will talk about some experiments whose chances of winning and losing is similar.

16.5.2Calculation of possibilities on throwing a dice or coin

Lavina and Neema were playing; Lavina said to Neema in a dice 6 comes least time when tossed . What do you think? Does that happen?

To know whether 6 comes less times in comparison to other numbers 1,2,3,4,5 or not. Lavina and Neema threw dice for 30-30 times. A frequency table of numbers so obtained is prepared.

Table for dice thrown 30 times by Lavina

1	2	3	4	5	6
卌 III	卌 I	III	III	卌 I	IIII

Take a dice and see after throwing it for 30 times how does table appears for the numbers obtained. Is it not necessary that a number appears less times or more times. Possibility of appearance of any number on the dice is same.

This type of experiment is called random experiment.

There are 6 results 1, 2, 3, 4, 5, 6 for the experiment.

When a coin is thrown, what possible results do you get? No doubt, head and tail. Assume that you are the captain of a team and your friend is the captain of other team. You throw a coin and ask

your friend to speak head or tail.

Can you control the result of the throw?

Can you have head? Or can you have tail? No, it is not possible. This type of experiment is called random experiment. Head and tail are two outcomes of the experiment.

16.5.3 Equally likely events

Divide students your class in 3-4 groups and give one coin to each group. Tell them to throw coins several times and note down whether head appears or tail.

Each group can note down these data in two columns as per the table 1. Come let us see results of table where we are increasing number of throws.

Number of throws	tally marks	number of heads	tally marks	number of tails
40	卌 卌 卌 卌 II	22	卌 卌 卌 III	18
50	卌 卌 卌 卌 III	23	卌 卌 卌 卌 卌 II	27
60	卌 卌 卌 卌 卌 IIII	29	– – –	31
70	– – –	33	– – –	37
80	– – –	38	– – –	42
90	– – –	44	– – –	46

Table 16.8

See carefully when you increase number of throws then number of heads and number of tail appears to be same.

This is same for a dice which is thrown several times. All six results tend to have same numbers.

In this situation we can say that different results of an experiment are equally probable. Chance of occurrence of appearance of each result is same.

16.5.4 Calculation of favorable probability-

When we throw a coin then possibility of getting head is 1 out of 2 possible outcomes (head and tail). In other word, we can say probability of appearance of head = $\frac{1}{2}$ .what is the probability of getting a tail? Both of these results are equally likely.

Now, if you throw a dice then what would be the results? clearly , any one out of 1 ,2, 3, 4, 5,6. Here 6 is equally likely. What is the probability of getting 3.

The probability is $\frac{1}{6}$, where $1 \rightarrow$ = number of results giving 3
$6 \rightarrow$ = number of equally likely events

Collection of results of experiment is called event. means in throwing of a coin, to get head is an event and to get tail is also an event. To get 1, 2, 3, 4, 5, 6 while throwing a dice is an event.

Like, probability of getting an even number = $\frac{3}{6}$

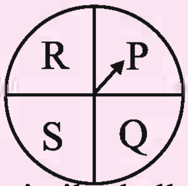
3= number of results forming the event (2, 4, 6)

6=total number of results.

Do and learn

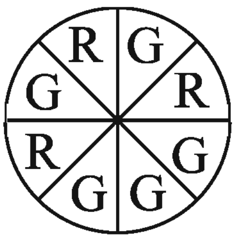
1. When a dice is thrown, what are the possible 6 events?
2. What will be the possible results, when you rotate a wheel? Form a list.

   (here result means sector at which pointer stops after rotating.)



3. You have a bag and there are similar balls having different colours. you can draw a ball without looking at this. Write the results obtained.

Example 3 If you have rotating wheel having 5 green sectors and 3 red sectors then (i) what is the probability of getting red sector? (ii) what is the probability of getting such sector which is not red?



Solution here total results of the event = (5 + 3)=8. To get red sector there are3 results.

So probability to get red sector is $\frac{3}{8}$

Number of such sectors which are not red=5

So probability of getting such sectors which are not red is $\frac{5}{8}$

Exercise 16.3

1. What is the probability of getting head on throwing a coin?
2. There are 6 white, 11 red and 7 yellow balls in a bag. What is the probability of getting a white ball from the bag?
3. What is the probability of getting a queen from a pack of well shuffled cards?
4. Find the probability of each below mentioned event when a dice is thrown-
   (i) A prime number (ii) No prime number
   (iii) Number more than 3 (iv) Number not more than 5
   (v) An odd number
5. Number from 1 to 15 is written on 15 different slips (one number on each slip). They are mixed keeping in box. A slip is taken out without looking into box. What is the probability of following?
   (i) Obtaining number 5
   (ii) Obtaining a number having two digits
   (iii) Obtaining a number having one digit

We learnt

1. The unorganized data available with us is called raw data
2. Data needs to be arranged in an order for getting meaningful conclusions out of it.
3. Frequency is that number which shows number of times a special entry appears in data
4. Data received can be arranged in groups. They can be represented as classified frequency distribution in ordered form.
5. Classified data can be represented with the help of a histogram. Histogram is a type of bar graph in which class interval is represented on horizontal axis and bar length shows frequency of class intervals. There is no gap between bars as there is no gap between the class intervals.
6. Data can be represented using pie chart or circle charts. A circular graph shows relation between a whole and its part.
7. There are some experiments in which there is equal chance of result.
8. Random experiment is that experiment in which results can't be correctly forecasted.
9. Results of an experiment are called equally likely if chance of their is appearance is equal.
10. Probability of an event = $\dfrac{\text{number of results forming event}}{\text{number of results of an experiment}}$
11. One or more results of an experiment is called event.

Answer sheet

Exercise 1

1. (i) $\frac{7}{4}$ (ii) $\frac{-19}{30}$ (iii) $\frac{-2}{3}$ (iv) $\frac{34}{15}$ (v) $\frac{-107}{35}$ (vi) $\frac{-28}{57}$

2. (i) $\frac{23}{12}$ (ii) $\frac{44}{9}$ (iii) $\frac{-1}{2}$ (iv) $\frac{-22}{63}$ (v) $\frac{-27}{91}$ (vi) $\frac{-104}{15}$

3. (i) $\frac{13}{3}$ (ii) 1 (iii) $\frac{6}{35}$ (iv) $\frac{35}{12}$ (v) $\frac{9}{10}$ (vi) $\frac{-133}{18}$

4. (i) -10 (ii) 1 (iii) $\frac{-3}{2}$ (iv) $\frac{91}{36}$ (v) $\frac{-19}{55}$ (vi) $\frac{-3}{4}$

5. (i) $\frac{59}{30}$ (ii) $\frac{-11}{9}$ (iii) $\frac{7}{9}$ (iv) $\frac{-3}{7}$

6. (i) $\frac{-181}{315}$ (ii) 0

7. (i) $\frac{-7}{19}$ (ii) $\frac{9}{5}$ (iii) $\frac{-3}{7}$ (iv) $\frac{5}{9}$ (v) $\frac{-13}{17}$ (vi) $\frac{21}{31}$

8. (i) $\frac{-1}{17}$ (ii) $\frac{-17}{11}$ (iii) $\frac{5}{3}$ (iv) $\frac{-19}{13}$

9. $\frac{-75}{49}$

10. (i) rational (ii) positive (iii) not defined (iv) zero (v) one
(vi) reciprocal (vii) left (viii) right (ix) zero (x) one

11. (i) $\frac{-21}{8}, \frac{-9}{4}, \frac{-3}{2}, \frac{-3}{4}, \frac{-3}{8}$ etc. (ii) $\frac{5}{12}, \frac{5}{24}, \frac{5}{8}, \frac{25}{48}$ etc.

(iii) $\frac{-17}{48}, \frac{1}{24}, \frac{21}{48}$ etc.

Exercise 2.1

1. 243, 100, 2700

2. (i) 2 (ii) 2 (iii) 5 (iv) 7

3. (i) 3 (ii) 2 (iii) 3 (iv) 5

4. 6

Exercise 2.2

1. (i) Correct (ii) Correct (iii) wrong (iv) Correct (v)wrong (vi) Correct

2. (i) 4 (ii) 7 (iii) 18 (iv) 42 (v) 15 (vi) 22 (vii) 36 (viii) 45

Exercise 3.1

1. (i) $\frac{1}{343}$ (ii) $\frac{16}{25}$ (iii) -5 (iv) $\frac{16}{9}$

2. (i) -125 (ii) $\frac{1}{8}$ (iii) $\frac{16}{81}$

3. (i) $\frac{1}{2^6}$ (ii) $\frac{2^4}{5^3}$ (iii) $\left(\frac{-2}{3}\right)^3$ (iv) $\left(\frac{-1}{2}\right)^3$ (v) $-\left(\frac{5}{7}\right)^2$

4. (i) 243 (ii) $\frac{1}{32}$ (iii) $\frac{243}{32}$ (iv) $\frac{-1}{128}$ (v) $\frac{-32}{3125}$

5. (i) 4^3 (ii) $(-5)^3$ (iii) $\frac{2}{3}$ (iv) $\left(\frac{-1}{5}\right)^5$

6. (i) 729 (ii) 64 (iii) 625 (iv) 256 (v) $\frac{1}{256}$ (vi) $\frac{1}{729}$

7. (i)1 (ii) 1 (iii) 1 (iv) 1 (v) 1 (vi) 2 (vii) 2

8. (i) $\frac{1}{2^3}$ (ii) $\frac{1}{3^5}$ (iii) $\frac{1}{a^4}$ (iv) $\frac{1}{(-2)^5}$ (v) $\frac{1}{(-x)^3}$ (vi) 5^3 (vii) y^3 (viii) $\left(\frac{2}{3}\right)^3$

9. (i)11664 (ii) $\frac{375}{576}$ (iii) $\frac{243}{1372}$ (iv) $\frac{1}{256}$ (v) $\frac{625}{81}$ (vi) $\frac{1}{64}$

10. 29

Exercise 3.2

1. (i) $\frac{3}{5}$ (ii) $\frac{25}{36}$ (iii) $\frac{125}{512}$ (iv) 15

2. (i) 125 (ii) $625t^4$ (iii) $\frac{614656}{3}$ (iv) $\frac{1}{20}$ (v) $\frac{1458}{625}$

3. (i) 3 (ii) 2 (iii) 2

4. (i) $\frac{80}{243}$ (ii) $\frac{8}{3}$

Exercise 3.3

1. (i)1.28×10^8 (ii) 1.68×10^9 (iii) 5×10^{-4} (iv) 1.7×10^{-7}
   (v) 3.97×10^{-10} (vi) 4.358×10^{-8}

2. (i)4000000000 (ii) 2450000000 (iii) 56172900 (iv) 0.0000085
   (v) 0.00000302 (vi) 0.0007

3. (i)2×10^{-4} (ii) 1.6×10^{-19} (iii) 1×10^{-6} (iv) 1.6×10^{-3}

Exercise 4.1

1. 3
2. 1
3. 0,9
4. 0, 3, 6, 9
5. (i) divisible by 3. (ii) divisible by 3.9,11 (iii) divisible by 3.9
6. Not completely divisible by 11.
7. 0, 3, 6, 9

Exercise 4.2

1. (i) A= 8 B= 9 (ii) A= 4 B= 3 C= 1 (iii) A= 2 B= 5
   (iv) A= 2 B= 7 (v) A= 8 B= 1 (vi) A= 6
   (vii) A= 2 B= 5 C= 1 (viii) A= 7 B= 4
2. (i) 4 (ii) 3 (iii) 1 (iv) 9 (v) 6 (vi) 4 (vii) 7 (viii) 3

Exercise 5

1. (i) 10605 (ii) 165 (iii) 1458 (iv) 15609
2. (i) 2448 (ii)783 (iii) 1224 (iv) 378 (v) 10626
   (vi) 23436 (vii) 884640 (viii) 5832 (ix) 970299
3. (i) Quotient = 142
   Remainder = 11
   (ii) Quotient = 74
   Remainder = 14
   (iii) Quotient = 90
   Remainder = 16

Exercise 6.1

1. (1) (iii), (iv), (v), (vi).
   (2) (i),(ii).
   (3) (i) concave (ii) concave (iii) convex (iv) convex (v) convex (vi) convex

2. (i) a, b, c, d, e (ii) p, q, r, s, t

3. Closed figure with 3 or more similar sides.
   (i) Pentagon (ii) Hexagon (iii) Octagon

4. (i) 60° (ii) 140° (iii) 108° (iv) x =90° y =120° z =150°
   (v) x = 60°, y =100°, z =120°, w = 80°

5. 8 6. 24 7. 15 8. 144

9. No 10. 111° 11. 135° 12. 900°

Exercise 6.2

1. (i) supplementary (ii) equal (iii) 80° (iv) rhombus (v) similar

2. x = 70°, y = 70°, z = 110°

3. (i) x = 90°, y = 45°, z = 45° (ii) x = 28° y = 112° z = 28°

4. 30°, 150°, 30°, 150°

5. (i) x = 6, y = 9 (ii) x = 3, y = 13

6. x = 3 7. x = 5cm , y = 12cm , z = 13cm 8. x = 90°, y = 50°

Exercise 7.1 to 7.6

Methodology based

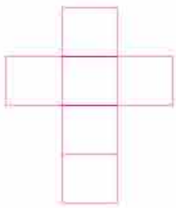
Exercise 8.1

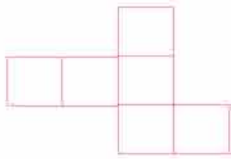
1 (i) Triangle (ii) congruent (iii) 15 (iv) prism

4. (i) and (ii) are polyhedron
   (i) cuboid and pyramid
   (ii) prism and pyramid

Exercise 8.2

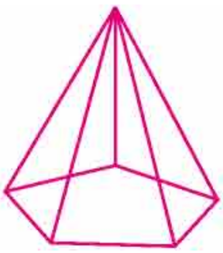
1.



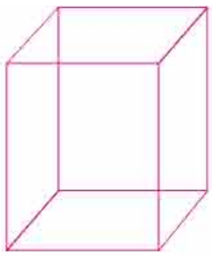


2.

3. (i)

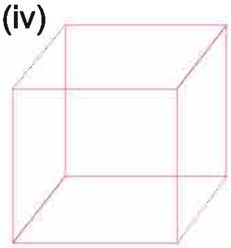


(ii)



(iii)

(iv)



Exercise 9.1

1. (i) $15x$ (ii) 10pq (iii) $-21l^2n^2$ (iv) 18mn (v) $10x^3$

2.

X	7	x	y	2z	a	-5b	c
7	49	$7x$	7y	14z	7a	-35b	7c
x	$7x$	x^2	xy	$2zx$	ax	$-5bx$	cx
2y	14y	$2xy$	$2y^2$	4yz	2ay	-10by	2cy
-3a	-21a	$-3ax$	-3ay	-6az	$-3a^2$	15ab	3ac
b	7b	bx	by	2bz	ab	$-5b^2$	bc
y	7y	xy	y^2	2yz	ay	-5by	cy
$2x^3$	$14x^3$	$2x^4$	$2x^3y$	$4x^3z$	$2x^3a$	$-10x^3b$	$2x^3c$
a^4	$7a^4$	a^4x	a^4y	$2a^4z$	a^5	$-5a^4b$	a^4c
z^2	$7z^2$	xz^2	yz^2	$2z^3$	az^2	$-5bz^2$	cz^2

3. (i) x^5y^3 (ii) m^6n^6 (iii) $k^3l^3m^3$

4. x^3y

Exercise 9.2

1. (i) $6x^2+x-35$ (ii) $3xy+5x-24y-40$ (iii) $2.25p^2-0.25q^2$ (iv) $ax+5a+3bx+15b$
   (v) $6l^2m^2-lm^2-15m^4$ (vi) $3a^4+\frac{43}{4}a^2b^2-5b^4$

2. (i) $-6x^2-x+40$ (ii) $3x^2+8xy-3y^2$ (iii) $a^3+a^2b^2+ab+b^3$ (iv) $2p^3+p^2q-2pq^2-q^3$

3. (i) x^2-2x (ii) $3a^2+a^2b^2-3b^2-4$ (iii) $t^3-s^3+s^2t^2-ts$ (iv) $4ab+2bc+2bd$
   (v) a^3+b^3 (vi) $a^2+2ab+b^2-c^2$ (vii) 0

Exercise 9.3

1. (i) $x^2+10x+25$ (ii) $9x^2+12x+4$ (iii) $25a^2-70a+49$ (iv) $9p^2-3p+\frac{1}{4}$
   (v) $1.44m^2-0.72m+0.09$ (vi) x^4-y^4 (vii) $49-36y^2$ (viii) $49a^2-81b^2$

2. (i) x^2+3x+2 (ii) $9x^2+18x+5$ (iii) $16x^2-24x+5$ (iv) $9a^2-9a-40$ (v) $x^2y^2z^2-3xyz+2$

3. (i) $b^2-14b+49$ (ii) $x^2y^2+6xyz+9z^2$ (iii) $36m^4-60m^2n+25n^2$ (iv) $\frac{9}{4}x^2+2xy+\frac{4}{9}y^2$

4. (i) $a^4-2a^2b^2+b^4$ (ii) 40n (iii) $98m^2+128n^2$ (iv) $m^4+n^4m^2$

6. (i) 9801 (ii) 10609 (iii) 89991 (iv) 6396

7. (i) 400 (ii) 12 (iii) 1800

8. (i) 10506 (ii) 51.83 (iii) 10098 (iv) 94.08

Exercise 10.1

1. (i) 12 (ii) 14pq (iii) 6ab (iv) $4x$ (v) 10 (vi) x^2y^2

2. (i) 6(p-2q) (ii) 7a(a+2) (iii) $5(2a^2-3b^2+4c^2)$ (iv) $xy(ax+by+cz)$
   (v) $xyz(x+y+z)$ (vi) $4z(-8+5z^2)$

3. (i) $(x+1)(2y+3)$ (ii) $(xy+1)(z-7)$ (iii) $(2y-3)(3x-2)$ (iv) (5p+3)(3q+5)

Exercise 10.2

1. (i) (a+2)(a-2) (ii) (a+7b)(a-7b) (iii) p(p+11)(p-11) (iv) (a-b+c)(a-b-c)
   (v) $(a^2+b^2)(a+b)(a-b)$ (vi) $5x(x+5)(x-5)$ (vii) 7(3a+4b)(3a-4b)
   (viii) $(3xy+4)(3xy-4)$ (ix) 4lm

2. (i) $x(lx+m)$ (ii) $2x(x^2+y^2+z^2)$ (iii) (a+b)(a+4) (iv) $(x+1)(y+1)$
   (v) (a-3)(5a+2c) (vi) $(a+b)(m^2+n^2)$

3. (i) $(x+2)(x+3)$ (ii) (q+8)(q+3) (iii) (m-7)(m-3) (iv) $(x+8)(x-2)$
   (v) $(x-9)(x+2)$ (vi) $(x-17)(x+6)$ (vii) (y+8)(y-6) (viii) (d-9)(d+5)
   (ix) (m+9)(m+7) (x) (n-23)(n+4) (xi) (p-8)(p-2) (xii) $(x+9)(x-5)$

Exercise 10.3

1. (i) $\frac{1}{2}x^3$ (ii) -4y (iii) $\frac{2}{3}x^2y$ (iv) $-2a^2b^4$

2. (i) $(\frac{5}{3})x-2$ (ii) $(\frac{x^2}{2})+x+\frac{3}{2}$ (iii) q^3-p^3 (iv) $3x^4-4x^2+5$

3. (i) 6y (ii) xy (iii) 5y+7 (iv) 3

4. (i) (y+2) (ii) $5(x-4)$ (iii) $3(3x-4y)$ (iv) 2z(z−2)

Exercise 11.1

(1) 4 (2) 6 (3) 3 (4) 13 (5) 4

(6) 2 (7) 3 (8) 10 (9) $(\frac{-4}{5})$ (10) 8

Exercise 11.2

(1) $\frac{4}{7}$ (2) 7 (3) 5 (4) Rs 300000 (5) 16

(6) 32, 13 (7) 8, 24 years (8) 19 m, 13 m (9) 39 (10) 28 or 82

Exercise 12.1

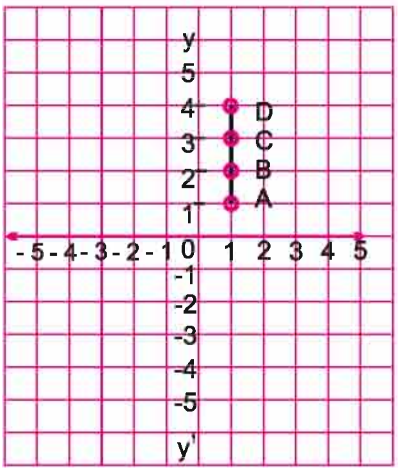
(1) (i) 3 (ii) -2 (iii) -3 (iv) four (v) - 4

(2) (i) x-coordinate = 2, y-coordinate = 1, coordinates (2, 1)
(ii) x-coordinate = -3, y-coordinate = 2, coordinates (-3, 2)

(iii) -4,-3, (-4,-3)

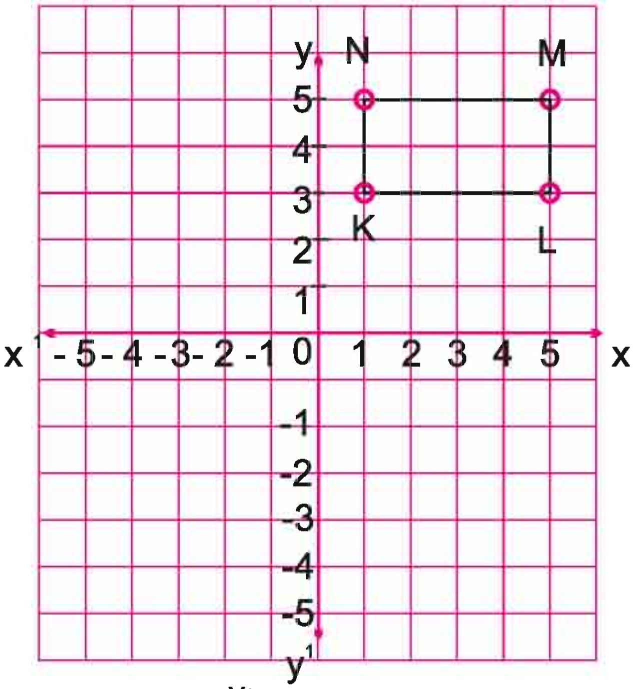
(iv) 3,-2, (3,-2)

(3)
(i)



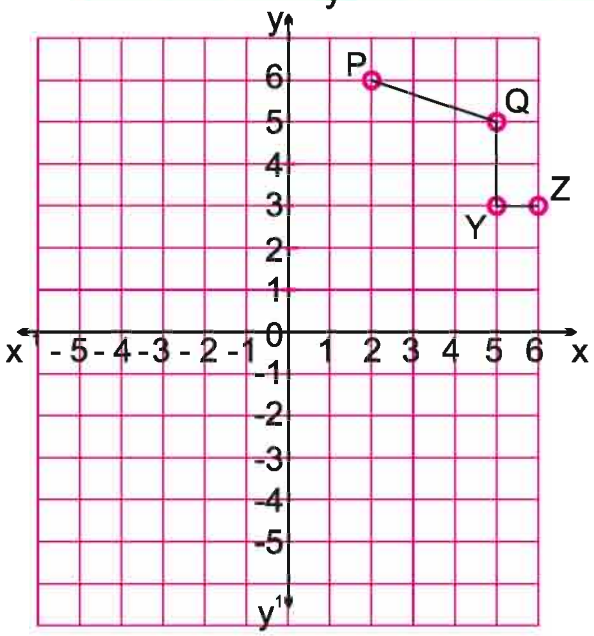
Yes

(ii)



No

(iii)



No

(4) (i) A (1, 5), B (5, 5), C (6, 8), D (2, 8)

Length of AB = 4 units, Length of DC = 4 units

(ii) P (1, 1), Q (4, 1), R (1, 3) ; Length of PQ = 3 units

(5) (i) true (ii) true (iii) true (iv) false (v) false

Exercise 12.2

(1) Equilateral triangle

Length	1	2	3	4	5	6
Perimeter	3	6	9	12	15	18

Square

Length	1	2	3	4	5	6
Perimeter	4	8	12	16	20	24

(2)

Breadth of Rectangle	1	2	3	4	5	
Area	2	8	18	32	50	

(3), (4), (5) Draw the graph as per the given table.

Exercise 13.1

(1) (i) 25% (ii) 75%

(2) (i) 3:2 (ii) 2:3 (iii) 3:5 (iv) 2:5

(3) 51 students

(4) 20 matches (5) 459 trees

(6) Rs. 2400 (7) 15%

Exercise 13.2

(1) 16% loss (2) Rs 23000 (3) 16% loss (4) Rs 61560 (5) Rs. 15225

(6) 4% loss (7) 6.25% (8) 2 years 6 months (9) 7476

Exercise 13.3

(1) 20% (2) Rs 19200 (3) Rs 62,684.16 (4) 6400 (5) Rs 1,664

(6) Rs 16,686.86 (7) 2837.25 (8) (i) Rs 85,600 (ii) Rs 91,592

(9) Rs 95.31

Exercise 13.4

(1) Rs 450 (2) 216 metre (3) 15 books (4) 5 hours

(5) 60 litres (6) 400 litres (7) 12 days (8) 12 days

(9) 20 (10) 20 students (11) 400 kilograms

Exercise 14.1

1. 104 sq. cm 2. 224 sq. cm

3. 7 cm 4. 119 sq. m

5. Rs. 11,700 6. 40.5 sq. cm

7. 23100 sq. m

8. (i) half (ii) different (iii) Trapezium (iv) Rhombus

Exercise 14.2

1. 10450 sq. m
2. 64 sq. cm
3. 19550 sq. m

Exercise 15.1

1. 384 sq. cm, 174 sq. cm, 38600 sq. cm
2. 10 cm
3. Cylinder: 231 sq. m. Cube: 294 sq. m. Surface area of cube is more than that of cylinder.
4. 5280 sq. cm
5. 0.52 sq. m
6. 145 paint boxes.
7. Rs. 1408
8. 322 meter
9. 1980 sq. m
10. 96 sq. cm

Exercise 15.2

1. 450 cubes
2. 24 wood pieces
3. Cylinder A.
   (i) 1078 cubic cm (ii) 539 cubic cm
4. 49500
5. 1925 minutes
6. 27 kg
7. (i) 4 times (ii) 8 times
8. 20 m

Exercise 16.1

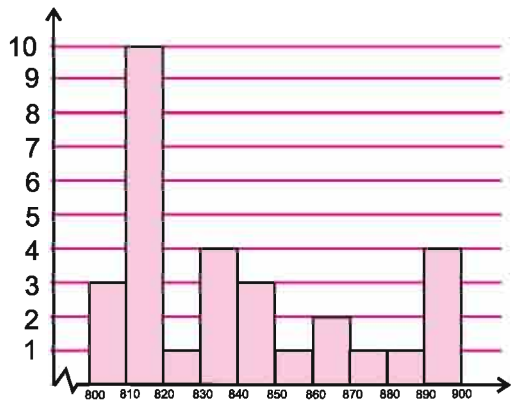
1. The data of situations of (ii), (iv) and (v) can be divided in class intervals.

2.

Colour	Tally Marks	No. of clothes
W	卌 III	8
R	卌 卌	10
B	卌 I	6
Y	卌 II	7
X	卌 IIII	9
	Total	40

3.

Weekly wages	Tally Marks	No. of workers
800-810	III	3
810-820	卌 卌	10
820-830	I	1
830-840	IIII	4
840-850	III	3
850-860	I	1
860-870	II	2
870-880	I	1
880-890	I	1
890-900	IIII	4
	Total	30



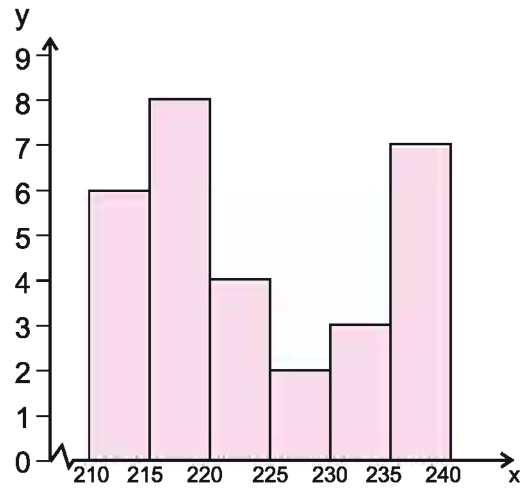
4.

No. of mobiles	Tally Marks	No. of days
210-215	卌 I	6
215-220	卌 III	8
220-225	IIII	4
225-230	II	2
230-235	III	3
235-240	卌 II	7
	Total	30

(i) 215-220

(ii) 10 days

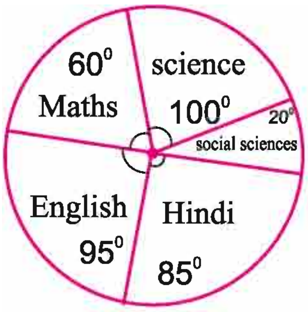
(iii) 20 days



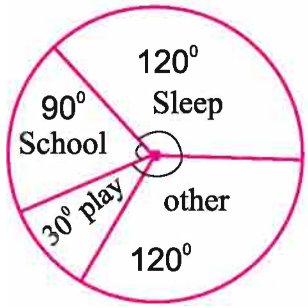
5. (i) 4 – 5 hours (ii) 60 students (iii) 100 students (iv) 4 – 5

Exercise 16.2

(1)

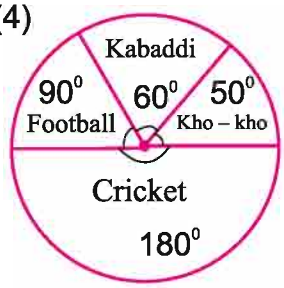


(2)

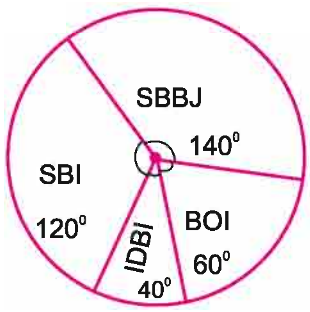


(3) **(i)** Maths
(ii) 250-180 = 70 number
(iii) Yes

(4)



(5)



6. (i) 26.7% (ii) 29.9% (iii) 0.3% (iv) 10.4%

Exercise 16.3

(1) $\frac{1}{2}$ (2) $\frac{6}{24} = \frac{1}{4}$ (3) $\frac{1}{13}$

(4) (i) (a) $\frac{1}{2}$ (ii) $\frac{3}{6} = \frac{1}{2}$

(iii) (a) $\frac{3}{6} = \frac{1}{2}$ (iv) $\frac{5}{6}$

(v) $\frac{1}{2}$

(5) (i) $\frac{1}{15}$ (ii) $\frac{6}{15} = \frac{2}{5}$ (iii) $\frac{9}{15} = \frac{3}{5}$

Brahmagupta

Brahmagupta was an Indian mathematician, born in 598 AD in Bhinmal in Jalore district. He became the head of the astronomical observatory at Ujjain. At the age of 30, he composed the theoretical treatise 'Brāhmasphuṭasiddhānta' on astronomy. The book consists of 24 chapters.

The 12th chapter of the book is '*Ganitadhyaya*'. It is named "Mathematics," because the arithmetic operations and proportions, and the "practical mathematics," such as mixture and series, treated there occupied the major part of the mathematics of Brahmagupta's milieu.

The 18th chapter of the book is '*Kuttakaadhyaya*'. In this chapter, Brahmagupta gave the solution of the general linear equation. He also introduced new methods for solving quadratic equations and gave equations to solve systems of simultaneous indeterminate equations.

The 2nd chapter of the book deals with trigonometry.

His other well known work is mathematical treatise '*Khanda-khādyaka*', a more practical text. Of particular interest to mathematics in this second work by Brahmagupta is the interpolation formula he uses to compute values of sines. It this book, rules of sines (*Jya*) and cosines (*Cojya*) are available for plane trigonometry and spherical trigonometry.

These manuscripts are available at 'The Bhandarkar Oriental Research Institute', Pune, Maharashtra. These books had been translated into Arabic and Persian and the knowledge was transitioned to Arabic countries. From there, it was conveyed to western Europe through Arabs.

Some of the important contributions made by Brahmagupta in mathematics are-

(1) Easy methods to find square roots and cube-roots.

(2) Defining the rules and properties of *Zero*.

(3) Providing two equivalent solutions to the general quadratic equation.

(4) To find the area of triangle and cyclic quadrilaterals.

स्थूलफलं त्रिचतुर्भुजबाहु प्रतिबाहु योग दसघातः।
भुजयोगार्धचतुष्टय भुजोनघातात् पदं सूक्ष्मतम्।।

The area is the square root from the product of the halves of the sums of the sides diminished by each side of the quadrilateral.

$$\text{Area of a cyclic quadrilateral} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

[where a, b, c, d are the sides of the cyclic quadrilateral and 2s = a + b + c + d]

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

[where a, b, c are the sides of the triangle and 2s = a + b + c]

(5) To construct a cyclic quadrilateral with integer sides, etc are considered as great achievement even in today's world of mathematics.