MATHEMATICS Part - I

STANDARD NINE

The coordination committee formed by GR No. Abhyas - 2116/(Pra.Kra.43/16) SD - 4
Dated 25.4.2016 has given approval to prescribe this textbook in its meeting held on 3.3.2017

MATHEMATICS

Part - I

STANDARD NINE

महाराष्ट्र राज्य पाठ्यपुस्तक निर्मिती व अभ्यासक्रम संशोधन मंडळ

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

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First Edition : 2017

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Smt. Taruben Popat (Invitee)

Cover and Illustrations : Dhanashri Mokashi

Computer Drawings : Sandeep Koli, Mumbai

Co-ordination : Ujjwala Godbole
I/C Special Officer for Mathematics

Translation : Smt. Prajakti Gokhale
Smt. Taruben Popat
Smt. Mrinalini Desai

Scrutiny : Dr Jayashri Atre
Shri. V. D. Godbole

Co-ordination :
Dhanavanti Hardikar
Academic Secretary for Languages
Santosh Pawar
Subject Assistant, English

Production :
Sachchitanand Aphale
Chief Production Officer
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Typesetting : D.T.P Section
Textbook Bureau, Pune.

Paper : 70 GSM Cream wove

Printer : SHIV OFFSET PRINTERS, SANGLI

Print Order No. : N/PB/2017-18/50,000

Publisher

Vivek Uttam Gosavi, Controller
Maharashtra State Textbook Bureau,
Prabhadevi, Mumbai - 400 025.

The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens :

JUSTICE, social, economic and political ;

LIBERTY of thought, expression, belief, faith and worship ;

EQUALITY of status and of opportunity ; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation ;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

Preface

Dear Students,

Welcome to the ninth standard!

You are now going to begin your studies at the secondary level after completing your primary education curriculum. You had only one Mathematics textbook up to the eighth standard, now you will use two textbooks – Mathematics Part-I and Mathematics Part-II.

In this Mathematics Part-I textbook, you will get acquainted with several topics in the areas of Numbers, Algebra, Commercial Mathematics and Data Handling. These topics are useful for all students in various fields. Algebra and Statistics will provide the foundation for higher studies.

Different activities are given in the textbook to help you understand the different concepts. Other activities have been provided for revision and additional practice. You are expected to do all these. You can also explore the internet to get more information regarding concepts in the textbook and to obtain more practice examples. We expect you to do the activities, solve the examples and draw inferences after discussing them with your friends. You will get through the course joyfully if you follow the three point plan of – a deep study of the textbook, activity-based learning and ample practice.

So come on! Let us study Mathematics in the company of our teachers, parents, friends and the internet. Best wishes to you for your studies!

(Dr Sunil Magar)
Director
Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

Pune
Date : 28 April, 2017
Akshaya Tritiya
Indian Solar Year :
8 Vaishakh 1939

It is expected that students will develop the following competencies after studying the syllabus of Mathematics Part I in Standard IX

Area	Topic	Competency statement
1. **Knowledge of Numbers**	1.1 Sets 1.2 Real numbers and quadratic surds	Students will be able to – • know the different sets of numbers in the number system. • determine the subset relation between pairs of sets. • identify finite and infinite sets. • use Venn diagrams to show relation between different sets. • construct examples on sets. • understand that every point on a number line is associated with a real number. • identify the surds of order two and perform mathematical operations on them.
2. **Algebra**	2.1 Polynomial 2.2 Linear equations in two variables	• identify polynomials and do operations on them. • solve word problems using two variables.
3. **Commercial Mathematics**	3.1 Financial planning 3.2 Ratio and proportion	• know about the tax system and compute taxes. • compute income tax for salaried persons. • use theorem on equal ratios. • solve word problems based on direct and inverse variation.
4. **Data handling (Statistics)**	4.1 Frequency distribution 4.2 Measures of central tendency	• prepare grouped and ungrouped frequency distribution tables. • prepare cumulative frequency tables. • find and make use of measures of central tendencies for the given data.

Instructions for teachers

The textbook ‘Mathematics Part I’ contains many fundamental concepts. Some concerts are developed according to the principle ‘from concrete to abstract’. The book also contains concepts in Economics related to Mathematics and some extension of Mathematics of the area of statistics. Teachers are expected to study them in detail. It is also expected that a teacher should make use of activities, discussions, question-answers, group projects etc. while teaching the subject. The teacher should read the textbook thoroughly, note the activities given in the book and encourage the students to do them. The teacher should also try to invent new activities.

It is more important to understand basic concepts in Mathematics rather than the calculations. Many examples are included in the book which will develop student’s logical thinking. The teachers are advised to construct such examples with the help of the students. In the textbook, some examples are star marked, which indicates that they require a little higher order of thinking. If students solve examples logically but with a different method, please do encourage them.

In the process of evaluation, it is advised to think of open ended questions and activity sheets. Teachers should endeavour to develop such methods of evaluation.

The list of practicals given in the text-book should be taken as a specimen. Teachers can frame different practicals of their own. Different activities in the textbook are included in the practicals. Encourage the students to do those activities also. We hope that the evaluation method based on them will be helpful to develop different competencies in further studies.

List of some practicals (specimen)

(1) Consider your class as a universal set and draw Venn diagrams for the set of students who play Kho-kho, Kabbadi or any other games.

(2) Represent $2 + \sqrt{3}$, $5 - \sqrt{2}$ etc. on the number line.

(3) Divide a polynomial of degree three or four by a linear polynomial using different methods of division and check whether the answers are unique.

(4) By using the given tables compute the income tax of a salaried person whose statements of income and investments are given.

(5) Prepare a group frequency distribution table for the given numerical data.

(6) Find the percentage of various components using an easily available strip of medical tablets.

(7) To solve some challenging problems based on Linear equations using two variables.

Index

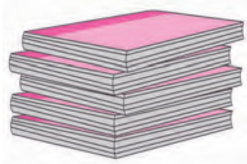
1 Sets

Let's study.

- Sets - Introduction
- Types of sets
- Venn diagrams
- Equal sets, subset
- Universal set
- Intersection and Union of sets
- Number of elements in a set

Let's recall.

Some pictures are given below. It contains the group of things you know.

				1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ...
Flower bouquet	Bunch of keys	Flock of birds	Pile of note-books	Collection of numbers

We use special word for each of the collection given above. In all the above examples we can clearly list the objects of that collection. We call the collection of such objects as **'Set'.**

Now, observe the collection. 'Happy children in the village', 'Brilliant students of the class'. In both the examples the words 'Happy' and 'Brilliant' are relative terms, because the exact meaning of these words 'to be happy' and 'to be brilliant' differ from person to person. Therefore, these collections are not sets.

See the examples given below and decide whether it is a set or not.

(1) Days of a week (2) Months in a year

(3) Brave children in the class (4) First 10 counting numbers

(5) Strong forts of Maharashtra (6) Planets in our solar system.

Let's learn.

Sets

If we can definitely and clearly decide the objects of a given collection then that collection is called a set.

Generally the name of the set is given using capital letters A, B, C,.....,Z

The members or elements of the set are shown by using small letters a, b, c, ...

If a is an element of set A, then we write it as '$a \in A$' and if a is not an element of set A then we write '$a \notin A$' .

Now let us observe the set of numbers.

N = { 1, 2, 3, . . .} is a set of natural numbers.

W = {0, 1, 2, 3, . . .} is a set of whole numbers.

I = {..., -3, -2, -1, 0, 1, 2, ...} is a set of integers.

Q is a set of rational numbers.

R is a set of real numbers.

Methods of writing sets

There are two methods of writing set.

(1) Listing method or roster method

In this method, we write all the elements of a set in curly bracket. Each of the elements is written only once and separated by commas. The order of an element is not important but it is necessary to write all the elements of the set.

e.g. the set of odd numbers between 1 and 10, can be written as

as, A = {3, 5, 7, 9} or A = {7, 3, 5, 9}

If an element comes more than once then it is customary to write that element only once. e.g. in the word 'remember' the letters 'r, m, e' are repeated more than once. So the set of letters of this word is written as A = {r, e, m, b}

(2) Rule method or set builder form

In this method, we do not write the list of elements but write the general element using variable followed by a vertical line or colon and write the property of the variable.

e.g. $A = \{x \mid x \in N, 1 < x < 10\}$ and read as 'set A is the set of all 'x' such that 'x' is a natural number between 1 and 10'.

e.g. B = { x | x is a prime number between 1 and 10}

set B contains all the prime numbers between 1 and 10. So by using listing method set B can be written as B = {2, 3, 5, 7}

Q is the set of rational numbers which can be written in set builder form as

$$Q = \{\frac{p}{q} \mid p, q \in I, q \neq 0\}$$

and read as 'Q' is set of all numbers in the form $\frac{p}{q}$ such that p and q are integers where q is a non-zero number.'

Illustrations : In the following examples each set is written in both the methods.

Rule method or Set builder form	**Listing method or Roster method**
A = { x | x is a letter of the word 'DIVISION'.}	A = {D, I, V, S, O, N}
B = { y | y is a number such that $y^2 = 9$}	B = { -3, 3}
C = {z | z is a multiple of 5 and is less than 30}	C = { 5, 10, 15, 20, 25}

Ex. : Fill in the blanks given in the following table.

Listing or Roster Method	Rule Method
A = { 2, 4, 6, 8, 10, 12, 14}	A = {x | x is an even natural number less than 15}
....	B = { x | x is a perfect square number between 1 to 20}
C = { a, e, i, o, u}	....
....	D = { y | y is a colour in the rainbow}
....	P = {x | x is an integer and , $-3 < x < 3$}
M = {1, 8, 27, 64, 125.......}	....

Practice set 1.1

(1) Write the following sets in roster form.

(i) Set of even numbers (ii) Set of even prime numbers from 1 to 50

(iii) Set of negative integers (iv) Seven basic sounds of a sargam (sur)

(2) Write the following symbolic statements in words.

(i) $\frac{4}{3} \in Q$ (ii) $-2 \notin N$ (iii) P = {p | p is an odd number}

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(3) Write any two sets by listing method and by rule method.

(4) Write the following sets using listing method.

(i) All months in the indian solar year.

(ii) Letters in the word 'COMPLEMENT'.

(iii) Set of human sensory organs.

(iv) Set of prime numbers from 1 to 20.

(v) Names of continents of the world.

(5) Write the following sets using rule method.

(i) A = { 1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

(ii) B = { 6, 12, 18, 24, 30, 36, 42, 48}

(iii) C = {S, M, I, L, E}

(iv) D = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

(v) X = $\{a, e, t\}$

Let's learn.

Types of sets

Name of set	Definition	Example
Singleton Set	A set consisting of a single element is called a singleton set.	A = {2} A is the set of even prime numbers.
Empty Set or Null Set	If there is not a single element in the set which satisfies the given condition then it is called a Null set or an empty set. Null set is represented by { } or a symbol ϕ (phi)	B = $\{x \mid x$ is natural number between 2 and 3.} $\therefore$ B = { } or ϕ
Finite Set	If a set is a null set or number of elements are limited and countable then it is called as 'Finite set'.	C = {p| p is a number from 1 to 22 divisible by 4.} C = {4, 8, 12, 16, 20}
Infinite Set	If number of elements in a set is unlimited and uncountable then the set is called 'Infinite set'.	N = {1, 2, 3, . . . }

Ex. Write the following sets using listing method and classify into finite or infinite set.

(i) A = $\{x \mid x \in$ N and x is an odd number$\}$ (ii) B = $\{x \mid x \in$ N and $3x - 1 = 0\}$

(iii) C = $\{x \mid x \in$ N, and x is divisible by 7 $\}$

(iv) D = $\{(a, b) \mid a, b \in$ W, $a + b = 9\}$ (v) E = $\{x \mid x \in$ I, $x^2 = 100\}$

(vi) F = $\{(a, b) \mid a, b \in$ Q, $a + b = 11\}$

Solution : (i) A = $\{x \mid x \in$ N and x is an odd number.$\}$

A = $\{1, 3, 5, 7, ...\}$ This is an infinite set.

(ii) B = $\{x \mid x \in$ N and $3x - 1 = 0\}$

$3x - 1 = 0$ $\therefore$ $3x = 1$ $x = \frac{1}{3}$

But $\frac{1}{3} \notin$ N $\therefore$ B = $\{ \quad \}$ $\therefore$ B is finite set.

(iii) C = $\{x \mid x \in$ N and x is divisible by 7.$\}$

C = $\{7, 14, 21, . . . \}$ This is an infinite set.

(iv) D = $\{(a , b) \mid a, b \in$ W, $a + b = 9\}$

We have to find the pairs of a and b such that, a and b are whole numbers and $a + b = 9$.

Let us first write the value of a and then the value of b. By keeping this order set D can be written as

D = $\{(0, 9), (1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1), (9, 0)\}$,

In this set, number of pairs are finite and could be counted

$\therefore$ Set D is a finite set.

(v) E = $\{x \mid x \in$ I, $x^2 = 100\}$

E = $\{-10, 10\}$. $\therefore$ E is a finite set

(vi) F = $\{(a, b) \mid a, b \in$ Q, $a + b = 11\}$

F = $\{(6, 5), (3, 8), (3.5, 7.5), (-15, 26), . . .\}$ infinitely many such pairs can be written.

$\therefore$ F is an infinite set.

Remember this !

N, W, I, Q, R all these sets are infinite sets.

Let's learn.

Equal sets

Two sets A and B are said to be equal, if every element of set A is in set B and every element of set B is in set A.

'Set A and set B are equal sets', symbolically it is written as A = B.

Ex (1) A = { x | x is a letter of the word 'listen'.} $\therefore$ A = { l, i, s, t, e, n}

B = { y | y is a letter of the word 'silent'.} $\therefore$ B = { s, i, l, e, n, t}

Though the elements of set A and B are not in the same order but all the elements are identical.

$\therefore$ A = B

Ex (2) $A = \{x \mid x = 2n, n \in N, 0 < x \leq 10\}$, $A = \{2, 4, 6, 8, 10\}$

$B = \{ y \mid y$ is an even number, $1 \leq y \leq 10\}$, $B = \{2, 4, 6, 8, 10\}$

$\therefore$ A and B are equal sets.

Now think of the following sets.

C = {1, 3, 5, 7} D = { 2, 3, 5, 7}

Are C and D equal sets ? Obviously 'No'

Because $1 \in C, 1 \notin D, 2 \in D, 2 \notin C$

$\therefore$ C and D are not equal sets. It is written as $C \neq D$

Ex (3) If A = {1, 2, 3} and B = { 1, 2, 3, 4}then $A \neq B$ verify it.

Ex (4) $A = \{x \mid x$ is prime number and $10 < x < 20\}$ and B = {11, 13, 17, 19}

Here A = B. Verify,

Practice set 1.2

(1) Decide which of the following are equal sets and which are not ? Justify your answer.

$A = \{ x \mid 3x - 1 = 2\}$

B = { x | x is a natural number but x is neither prime nor composite}

$C = \{x \mid x \in N, x < 2\}$

(2) Decide whether set A and B are equal sets. Give reason for your answer.

A = Even prime numbers $B = \{x \mid 7x - 1 = 13\}$

(3) Which of the following are empty sets ? why ?

(i) A = { a | a is a natural number smaller than zero.}

(ii) $B = \{x \mid x^2 = 0\}$ (iii) $C = \{ x \mid 5x - 2 = 0, x \in N\}$

(4) Write with reasons, which of the following sets are finite or infinite.

(i) A = { $x \mid x < 10$, x is a natural number}

(ii) B = {$y \mid y < -1$, y is an integer}

(iii) C = Set of students of class 9 from your school.

(iv) Set of people from your village.

(v) Set of apparatus in laboratory

(vi) Set of whole numbers

(vii) Set of rational number

Let's learn.

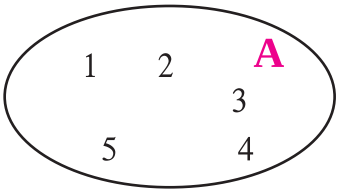
Venn diagrams

British logician John Venn was the first to use closed figures to represent sets. Such representations are called 'Venn diagrams'. Venn diagrams are very useful, in order to understand and illustrate the relationship among sets and to solve the examples based on the sets.

Let us understand the use of Venn diagrams from the following example.

e.g. A = { 1, 2, 3, 4, 5}

Set A is shown by Venn diagram.

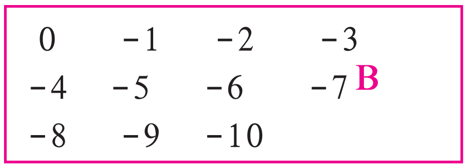


1834–1883

John Venn is the first Mathematician who gave the Mathematical form to 'logic' and 'probability'. His famous book is 'Logic of chance'.

B = {$x \mid -10 \leq x \leq 0$, x is an integer}

Venn diagram given alongside represents the set B.



Subset

If A and B are two given sets and every element of set B is also an element of set A then B is a subset of A which is symbolically written as $B \subseteq A$. It is read as 'B is a subset of A' or 'B subset A'.

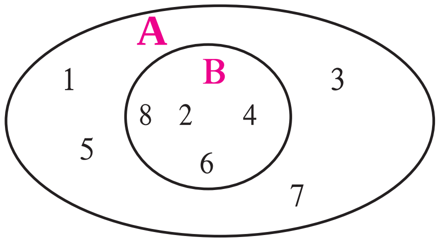
Ex (1) A = { 1, 2, 3, 4, 5, 6, 7, 8}

B = {2, 4, 6, 8}

Every element of set B is also an element of set A.

$\therefore$ $B \subseteq A$.

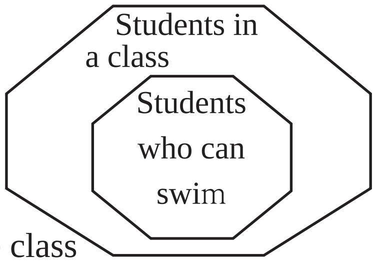
This can be represented by Venn diagram as shown above.



Activity : Set of students in a class and set of students in the same class who can swim, are shown by the Venn diagram.

Observe the diagram and draw

Venn diagrams for the following subsets.



(1) (i) set of students in a class

(ii) set of students who can ride bicycles in the same class

(2) A set of fruits is given as follows.

{guava, orange, mango, jack fruit, chickoo, jamun, custard apple, papaya, plum}

Show these subsets. (i) fruit with one seed (ii) fruit with more than one seed.

Let's see some more subsets.

Ex (2) N = set of natural numbers. I = set of integers.

Here $N \subseteq I$. because all natural numbers are integers..

Ex (3) $P = \{ x \mid x$ is square root of $25\}$ $S = \{ y \mid y \in I, -5 \leq y \leq 5\}$

Let's write set P as $P = \{-5, 5\}$

Let's write set S as $S = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

Here every element of set P is also an element of set S.

$\therefore P \subseteq S$

Remember this !

(i) Every set is a subset of itself. i.e. $A \subseteq A$

(ii) Empty set is a subset of every set i.e. $\phi \subseteq A$

(iii) If A = B then $A \subseteq B$ and $B \subseteq A$

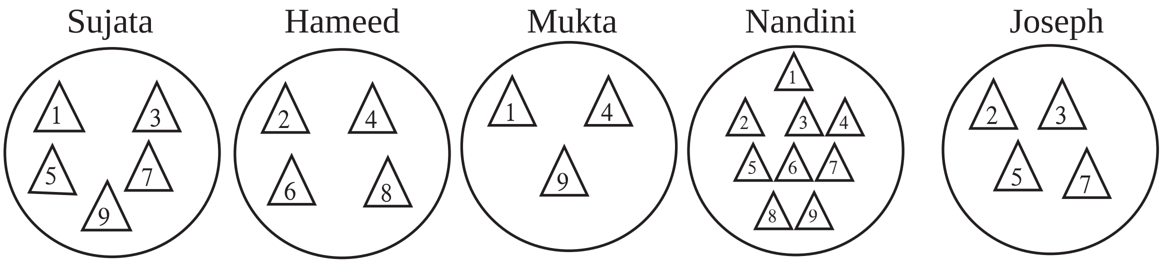
(iv) If $A \subseteq B$ and $B \subseteq A$ then A = B

Ex. If A = { 1, 3, 4, 7, 8} then write all possible subsets of A.

i.e. P = { 1, 3}, T = {4, 7, 8}, V = {1, 4, 8}, S = {1, 4, 7, 8}

In this way many subsets can be written. Write five more subsets of set A.

Activity : Every student should take 9 triangular sheets of paper and one plate. Numbers from 1 to 9 should be written on each triangle. Everyone should keep some numbered triangles in the plate. Now the triangles in each plate form a subset of the set of numbers from 1 to 9.



Look at the plates of Sujata, Hameed, Mukta, Nandini, Joseph with the numbered triangles. Guess the thinking behind selecting these numbers. Hence write the subsets in set builder form.

Let's discuss.

Ex.. Some sets are given below.

$A = \{ ..., -4, -2, 0, 2, 4, 6, ...\}$ $\quad$ $B = \{1, 2, 3, ...\}$

$C = \{..., -12, -6, 0, 6, 12, 18.....\}$ $\quad$ $D = \{..., -8, -4, 0, 4, 8, ...\}$

$I = \{ ..., -3, -2, -1, 0, 1, 2, 3, 4, ...\}$

Discuss and decide which of the following statements are true.

(i) A is a subset of sets B, C and D.

(ii) B is a subset of all the sets which are given above.

Let's learn.

Universal set

Think of a bigger set which will accommodate all the given sets under consideration which in general is known as Universal set. So that the sets under consideration are the subsets of this Universal set.

Ex (1) Suppose we want to study the students in class 9 who frequently remained absent. Then we have to think of all the students of class 9 who are in the school. So all the students in a school or the students of all the divisions of class 9 in the school is the Universal set.

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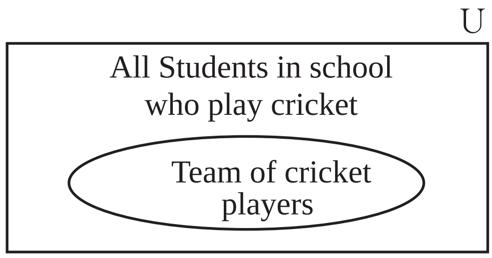
Let us see the another example.

Ex (2) A cricket team of 15 students is to be selected from a school. Here all the students from school who play cricket is the Universal set. A team of 15 cricket players is a subset of that Universal set.

Generally, the universal set is denoted by 'U' and in Venn diagram it is represented by a rectangle.



Complement of a set

Suppose U is an universal set. If $B \subseteq U$, then the set of all elements in U, which are not in set B is called the complement of B. It is denoted by B' or B^C.

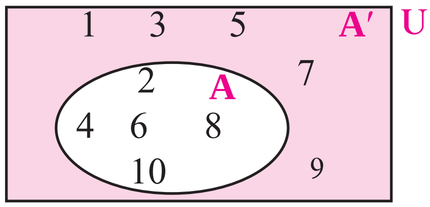
B' is defined as follows.

$B' = \{x \mid x \in U, \text{ and } x \notin B\}$

Ex (1) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 6, 8, 10\}$

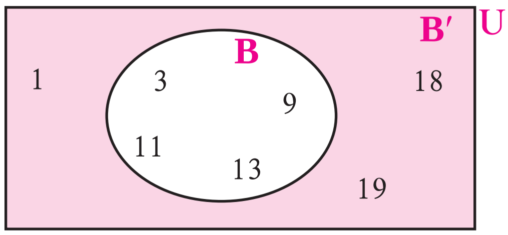
$\therefore\ A' = \{1, 3, 5, 7, 9\}$



Ex (2) Suppose $U = \{1, 3, 9, 11, 13, 18, 19\}$

$B = \{3, 9, 11, 13\}$

$\therefore\ B' = \{1, 18, 19\}$



Find $(B')'$ and draw the inference.

$(B')'$ is the set of elements which are not in B' but in U.

is $(B')' = B$?

Understand this concept with the help of Venn diagram.

Complement of a complement is the given set itself.

Remember this !

Properties of complement of a set.

(i) No elements are common in A and A'.

(ii) $A \subseteq U$ and $A' \subseteq U$

(iii) Complement of set U is empty set. $U' = \phi$

(iv) Complement of empty set is U. $\phi' = U$

Practice set 1.3

(1) If A = {a, b, c, d, e}, B = { c, d, e, f }, C = {b, d}, D = {a, e} then which of the following statements are true and which are false ?

(i) $C \subseteq B$ (ii) $A \subseteq D$ (iii) $D \subseteq B$ (iv) $D \subseteq A$ (v) $B \subseteq A$ (vi) $C \subseteq A$

(2) Take the set of natural numbers from 1 to 20 as universal set and show set X and Y using Venn diagram.

(i) X = { x | $x \in$ N, and $7 < x < 15$}

(ii) Y = { y | $y \in$ N, y is prime number from 1 to 20}

(3) U = {1, 2, 3, 7, 8, 9, 10, 11, 12}

P = {1, 3, 7, 10}

then (i) show the sets U, P and P$'$ by Venn diagram. (ii) Verify $(P')' = P$

(4) A = {1, 3, 2, 7} then write any three subsets of A.

(5) (i) Write the subset relation between the sets.

P is the set of all residents in Pune.

M is the set of all residents in Madhya Pradesh.

I is the set of all residents in Indore.

B is the set of all residents in India.

H is the set of all residents in Maharashtra.

(ii) Which set can be the universal set for above sets ?

(6*) Which set of numbers could be the universal set for the sets given below?

(i) A = set of multiples of 5, B = set of multiples of 7.

C = set of multiples of 12

(ii) P = set of integers which are multiples of 4.

T = set of all even square numbers.

(7) Let all the students of a class is an Universal set. Let set A be the students who secure 50% or more marks in Maths. Then write the complement of set A.

Let's learn.

Operations on sets

Intersection of two sets

Suppose A and B are two sets. The set of all common elements of A and B is called the intersection of set A and B. It is denoted as $A \cap B$ and read as A intersection B.

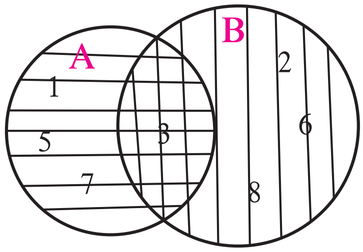
$\therefore$ $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Ex (1) A = { 1, 3, 5, 7}   B = { 2, 3, 6, 8}

Let us draw Venn diagram.

The element 3 is common in set A and B.

$\therefore$ $A \cap B = \{3\}$



Ex (2) A = {1, 3, 9, 11, 13}   B = {1, 9, 11}

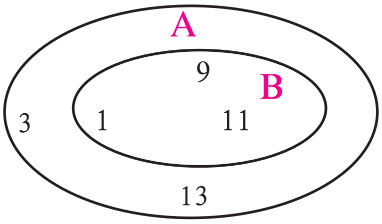
The elements 1, 9, 11 are common in set A and B.

$\therefore$ $A \cap B = \{1, 9, 11\}$ But $B = \{1, 9, 11\}$

$\therefore$ $A \cap B = B$

Here set B is the subset of A.

$\therefore$ If $B \subseteq A$ then $A \cap B = B$, similarly, if $B \cap A = B$, then $B \subseteq A$



Remember this !

Properties of Intersection of sets

(1) $A \cap B = B \cap A$

(2) If $A \subseteq B$ then $A \cap B = A$

(3) If $A \cap B = B$ then $B \subseteq A$

(4) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

(5) $A \cap A' = \phi$

(6) $A \cap A = A$

(7) $A \cap \phi = \phi$

Activity : Take different examples of sets and verify the above mentioned properties.

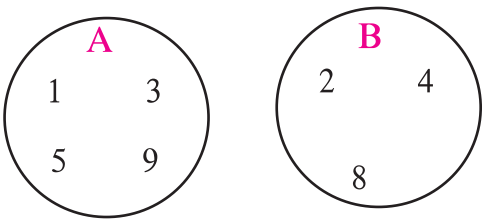
Let's learn.

Disjoint sets

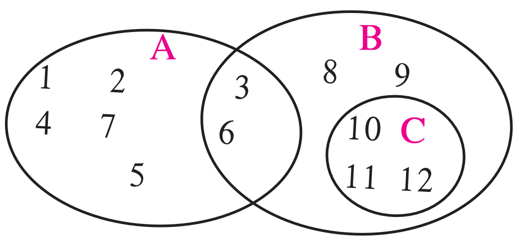
Let, A = { 1, 3, 5, 9}

and B = {2, 4, 8} are given.

Confirm that not a single element is common in set A and B. These sets are completely different from each other. So the set A and B are disjoint sets. Observe its Venn diagram.



Activity I : Observe the set A, B, C given by Venn diagrams and write which of these are disjoint sets.

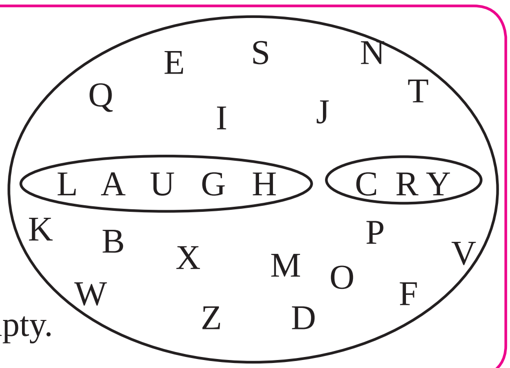


Activity II : Let the set of English alphabets be the Universal set.

The letters of the word 'LAUGH' is one set and the letter of the word 'CRY' is another set.

We can say that these are two disjoint sets.

Observe that intersection of these two sets is empty.



Union of two sets

Let A and B be two given sets. Then the set of all elements of set A and B is called the Union of two sets. It is written as $A \cup B$ and read as 'A union B'.

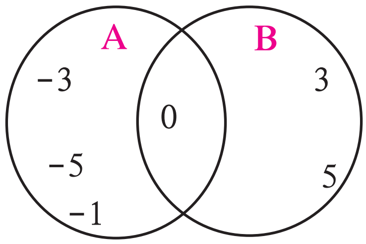
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Ex (1) $A = \{-1, -3, -5, 0\}$

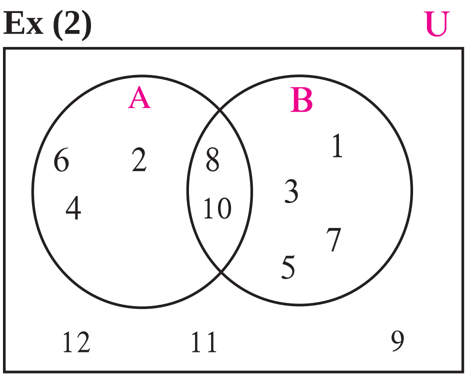
$B = \{0, 3, 5\}$

$A \cup B = \{-3, -5, 0, -1, 3, 5\}$

Note that, $A \cup B = B \cup A$



Ex (2)



Observe the Venn diagram and write the following sets using listing method.

(i) U (ii) A (iii) B (iv) $A \cup B$ (v) $A \cap B$

(vi) A' (vii) B' (viii) $(A \cup B)'$ (ix) $(A \cap B)'$

Solution : $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

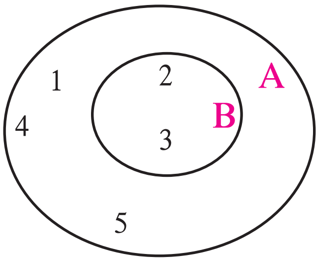
$A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 8, 10\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$ $A \cap B = \{8, 10\}$

$A' = \{1, 3, 5, 7, 9, 11, 12\}$ $B' = \{2, 4, 6, 9, 11, 12\}$

$(A \cup B)' = \{9, 11, 12\}$ $(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7, 9, 11, 12\}$

Ex (3)



$A = \{1, 2, 3, 4, 5\}$ $B = \{2, 3\}$

Let us draw its Venn diagram.

$A \cup B = \{1, 2, 3, 4, 5\}$

Observe that set A and $A \cup B$ have the same elements.

Hence, if $B \subseteq A$ then $A \cup B = A$

Remember this !

Properties of Union of sets

(1) $A \cup B = B \cup A$ (2) If $A \subseteq B$ then $A \cup B = B$

(3) $A \subseteq A \cup B$, $B \subseteq A \cup B$ (4) $A \cup A' = U$

(5) $A \cup A = A$ (6) $A \cup \phi = A$

Let's learn.

Number of elements in a set

Let A = {3, 6, 9, 12,15} is a given set with 5 elements.

Number of elements in set A is denoted as n (A). $\therefore$ n (A) = 5

Let B = { 6, 12, 18, 24, 30, 36} $\therefore$ n (B) = 6

Number of elements in Union and Intersection of sets.

Let us consider the set A and set B as given above,

$n(A) + n(B) = 5 + 6 = 11$ ----(I)

$A \cup B = \{3, 6, 9, 12, 15, 18, 24, 30, 36\}$ $\therefore$ $n(A \cup B) = 9$--------(II)

To find $A \cap B$ means to find common elements of set A and set B.

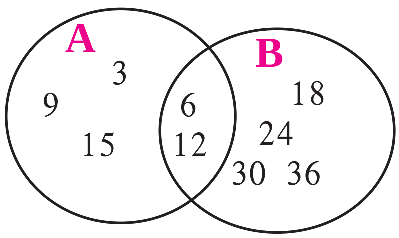
$A \cap B = \{6, 12\}$ $\therefore$ $n(A \cap B) = 2$--------(III)

In n (A) and n (B) elements in $A \cap B$ are counted twice.

$\therefore$ $n(A) + n(B) - n(A \cap B) = 5 + 6 - 2 = 9$ and $n(A \cup B) = 9$

From equations (I), (II) and (III), we can write it as follows

$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Verify the above rule for the given Venn diagram.

$n(A) = \boxed{}$, $n(B) = \boxed{}$

$n(A \cup B) = \boxed{}$, $n(A \cap B) = \boxed{}$

$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Remember this !

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

means $n(A) + n(B) = n(A \cup B) + n(A \cap B)$

Ex. Let A = {1, 2, 3, 5, 7, 9, 11, 13} B = {1, 2, 4, 6, 8, 12, 13}

Verify the above rule for the given set A and set B.

Let's learn.

Word problems based on sets

Ex. In a class of 70 students, 45 students like to play Cricket. 52 students like to play Kho-kho. All the students like to play atleast one of the two games. How many students like to play Cricket or Kho-kho ?

Solution : We will solve this example in wo ways.

Method I : Total number of students = 70

Let A be the set of students who likes to play Cricket.

Let B be the set of students who likes to play Kho-kho.

Hence the number of students who likes to play Cricket or Kho-kho is n (A $\cup$ B)

$\therefore$ n (A $\cup$ B) = 70

Number of students who likes to play both Cricket and Kho-kho = n (A $\cap$ B)

n (A) = 45, n (B) = 52

We know, n (A $\cup$ B) = n (A) + n (B) – n (A $\cap$ B) .

$\therefore$ n (A $\cap$ B) = n (A) + n (B) – n (A $\cup$ B)

= 45 + 52 – 70 = 27

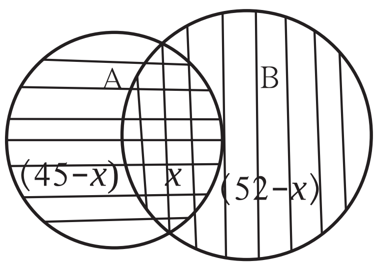
$\therefore$ Number of students who likes to play both the games are 27,

Number of students who likes to play Kho-kho are 45.

$\therefore$ Number of students who likes to play only Cricket = 45 –27 = 18

$\therefore$ A $\cap$ B is the set of students who play both the games. $\therefore$ n (A $\cap$ B)= 27

Method II : Given information can be shown by Venn diagrams as follows.



Let n (A $\cap$ B) = x, n (A) = 45, n (B) = 52,

We know that, n (A $\cup$ B) = 70

$\therefore$ n (A $\cap$ B) = x = n (A) + n (B) – n (A $\cap$ B)

= 52 + 45 – 70 = 27

Students who like to play only cricket = 45 – 27

=18

Practice set 1.4

(1) If n (A) = 15, n (A $\cup$ B) = 29, n (A $\cap$ B) = 7 then n (B) = ?

(2) In a hostel there are 125 students, out of which 80 drink tea, 60 drink coffee and 20 drink tea and coffee both. Find the number of students who do not drink tea or coffee.

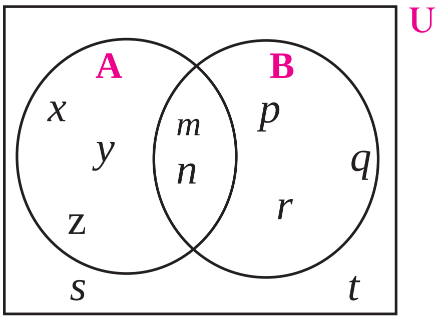
(3) In a competitive exam 50 students passed in English. 60 students passed in Mathematics. 40 students passed in both the subjects. None of them fail in both the subjects. Find the number of students who passed at least in one of the subjects ?

(4*) A survey was conducted to know the hobby of 220 students of class IX. Out of which 130 students informed about their hobby as rock climbing and 180 students informed about their hobby as sky watching. There are 110 students who follow both the hobbies. Then how many students do not have any of the two hobbies ? How many of them follow the hobby of rock climbing only ? How many students follow the hobby of sky watching only ?

(5) Observe the given Venn diagram and write the following sets.

(i) A (ii) B (iii) A $\cup$ B (iv) U

(v) A$'$ (vi) B$'$ (vii) (A $\cup$ B)$'$



Problem set 1

(1) Choose the correct alternative answer for each of the following questions.

(i) If M = {1, 3, 5}, N = {2, 4, 6}, then M $\cap$ N = ?

(A) {1, 2, 3, 4, 5, 6} (B) {1, 3, 5} (C) ϕ (D) {2, 4, 6}

(ii) P = {x | x is an odd natural number, 1 < $x \leq$ 5}
How to write this set in roster form?

(A) {1, 3, 5} (B) {1, 2, 3, 4, 5} (C) {1, 3} (D) {3, 5}

(iii) P = {1, 2, ..., 10}, What type of set P is ?

(A) Null set (B) Infinite set (C) Finite set (D) None of these

(iv) M $\cup$ N = {1, 2, 3, 4, 5, 6} and M = {1, 2, 4} then which of the following represent set N ?

(A) {1, 2, 3} (B) {3, 4, 5, 6} (C) {2, 5, 6} (D) {4, 5, 6}

(v) If $P \subseteq M$, then Which of the following set represent $P \cap (P \cup M)$?

(A) P (B) M (C) $P \cup M$ (D) $P' \cap M$

(vi) Which of the following sets are empty sets ?

(A) set of intersecting points of parallel lines (B) set of even prime numbers.

(C) Month of an english calendar having less than 30 days.

(D) $P = \{x \mid x \in I, -1 < x < 1\}$

(2) Find the correct option for the given question.

(i) Which of the following collections is a set ?

(A) Colours of the rainbow (B) Tall trees in the school campus.

(C) Rich people in the village (D) Easy examples in the book

(ii) Which of the following set represent $N \cap W$?

(A) {1, 2, 3, ...} (B) {0, 1, 2, 3, ...} (C) {0} (D) { }

(iii) $P = \{x \mid x$ is a letter of the word ' indian'} then which one of the following is set P in listing form ?

(A) {i, n, d} (B) {i, n, d, a} (C) {i, n, d, i, a} (D) {n, d, a}

(iv) If T = {1, 2, 3, 4, 5} and M = {3, 4, 7, 8} then $T \cup M$ = ?

(A) {1, 2, 3, 4, 5, 7} (B) {1, 2, 3, 7, 8}

(C) {1, 2, 3, 4, 5, 7, 8} (D) {3, 4}

(3) Out of 100 persons in a group, 72 persons speak English and 43 persons speak French. Each one out of 100 persons speak at least one language. Then how many speak only English ? How many speak only French ? How many of them speak English and French both ?

(4) 70 trees were planted by Parth and 90 trees were planted by Pradnya on the occasion of Tree Plantation Week. Out of these; 25 trees were planted by both of them together. How many trees were planted by Parth or Pradnya ?

(5) If $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$ then $n(A \cap B)$ = ?

(6) In a class, 8 students out of 28 have a dog as their pet animal at home, 6 students have a cat as their pet animal. 10 students have dog and cat both, then how many students do not have a dog or cat as their pet animal at home ?

(7) Represent the union of two sets by Venn diagram for each of the following.

(i) A ={3, 4, 5, 7} B ={1, 4, 8}

(ii) $P = \{a, b, c, e, f\}$ $Q = \{l, m, n, e, b\}$

(iii) X = $\{x \mid x$ is a prime number between 80 and 100$\}$

Y = $\{y \mid y$ is an odd number between 90 and 100 $\}$

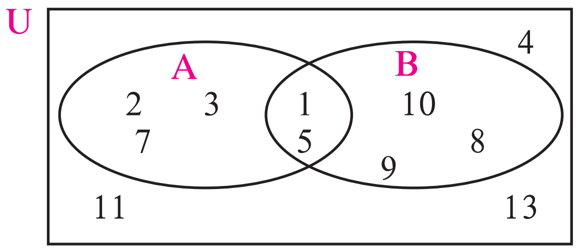
(8) Write the subset relations between the following sets..

X = set of all quadrilaterals. Y = set of all rhombuses.

S = set of all squares. T = set of all parallelograms.

V = set of all rectangles.

(9) If M is any set, then write $M \cup \phi$ and $M \cap \phi$.

(10*) 

Observe the Venn diagram and write the given sets U, A, B, $A \cup B$ and $A \cap B$.

(11) If $n(A) = 7$, $n(B) = 13$, $n(A \cap B) = 4$, then $n(A \cup B) = ?$

Activity I : Fill in the blanks with elements of that set.

U = {1, 3, 5, 8, 9, 10, 11, 12, 13, 15}

A = {1, 11, 13} B = {8, 5, 10, 11, 15} A' = {........} B' = {........}

$A \cap B$ = {............} $A' \cap B'$ = {...............}

$A \cup B$ = {............} $A' \cup B'$ = {............}

$(A \cap B)'$ = {............} $(A \cup B)'$ = {............}

Verify : $(A \cap B)' = A' \cup B'$, $(A \cup B)' = A' \cap B'$

Activity II : Collect the following information from 20 families nearby your house

(i) Number of families subscribing for Marathi Newspaper.

(ii) Number of families subscribing for English Newspaper.

(iii) Number of families subscribing for both English as well as Marathi Newspaper.

Show the collected information using Venn diagram.

2 Real Numbers

Let's study.

- Properties of rational numbers
- Properties of irrational numbers
- Surds
- Comparison of quadratic surds
- Operations on quadratic surds
- Rationalization of quadratic surds.

Let's recall.

In previous classes we have learnt about Natural numbers, Integers and Real numbers.

N = Set of Natural numbers = {1, 2, 3, 4, ...}

W = Set of Whole numbers = {0, 1, 2, 3, 4,...}

I = Set of Integers = {..., –3, –2, –1, 0, 1, 2, 3,...}

Q = Set of Rational numbers = $\{\frac{p}{q} \mid p, q \in I, q \neq 0\}$

R = Set of Real numbers.

$N \subseteq W \subseteq I \subseteq Q \subseteq R$.

Order relation on rational numbers :

$\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers where $q > 0$, $s > 0$

(i) If $p \times s = q \times r$ then $\frac{p}{q} = \frac{r}{s}$ (ii) If $p \times s > q \times r$ then $\frac{p}{q} > \frac{r}{s}$

(iii) If $p \times s < q \times r$ then $\frac{p}{q} < \frac{r}{s}$

Let's learn.

Properties of rational numbers

If a, b, c are rational numbers then

Property	Addition	Multiplication
1. Commutative	$a + b = b + a$	$a \times b = b \times a$
2. Associative	$(a + b) + c = a + (b + c)$	$a \times (b \times c) = (a \times b) \times c$
3. Identity	$a + 0 = 0 + a = a$	$a \times 1 = 1 \times a = a$
4. Inverse	$a + (-a) = 0$	$a \times \frac{1}{a} = 1 \quad (a \neq 0)$

Let's recall.

Decimal form of any rational number is either terminating or non-terminating recurring type.

Terminating type	Non terminating recurring type
(1) $\frac{2}{5} = 0.4$	(1) $\frac{17}{36} = 0.472222... = 0.47\dot{2}$
(2) $-\frac{7}{64} = -0.109375$	(2) $\frac{33}{26} = 1.2692307692307... = 1.2\overline{692307}$
(3) $\frac{101}{8} = 12.625$	(3) $\frac{56}{37} = 1.513513513... = 1.\overline{513}$

Let's learn.

To express the recurring decimal in $\frac{p}{q}$ form.

Ex. (1) Express the recurring decimal 0.777.... in $\frac{p}{q}$ form.

Solution : Let $x = 0.777... = 0.\dot{7}$

$\therefore\ 10x = 7.777... = 7.\dot{7}$

$\therefore\ 10x - x = 7.\dot{7} - 0.\dot{7}$

$\therefore\ 9x = 7$

$\therefore\ x = \frac{7}{9}$

$\therefore\ 0.777... = \frac{7}{9}$

Ex. (2) Express the recurring decimal 7.529529529... in $\frac{p}{q}$ form..

Solution : Let $x = 7.529529... = 7.\overline{529}$

$\therefore\ 1000x = 7529.529529... = 7529.\overline{529}$

$\therefore\ 1000x - x = 7529.\overline{529} - 7.\overline{529}$

$\therefore\ 999x = 7522.0$ $\qquad \therefore\ x = \frac{7522}{999}$

$\therefore\ 7.\overline{529} = \frac{7522}{999}$

Use your brain power!

How to convert $2.4\dot{3}$ in $\frac{p}{q}$ form ?

Remember this !

(1) Note the number of recurring digits after decimal point in the given rational number. Accordingly multiply it by 10, 100, 1000

e.g. In $2.\dot{3}$, digit 3 is the only recurring digit after decimal point, hence to convert $2.\dot{3}$ in $\frac{p}{q}$ form multiply $2.\dot{3}$ by 10.

In $1.\overline{24}$ digits 2 and 4 both are recurring therefore multiply $1.\overline{24}$ by 100.

In $1.\overline{513}$ digits 5, 1 and 3 are recurring so multiply $1.\overline{513}$ by1000.

(2) Notice the prime factors of the denominator of a rational number. If the prime factors are 2 or 5 only then its decimal expansion is terminating. If the prime factors are other than 2 or 5 also then its decimal expansion is non terminating and recurring.

Practice set 2.1

1. Classify the decimal form of the given rational numbers into terminating and non-terminating recurring type.

   (i) $\frac{13}{5}$ (ii) $\frac{2}{11}$ (iii) $\frac{29}{16}$ (iv) $\frac{17}{125}$ (v) $\frac{11}{6}$

2. Write the following rational numbers in decimal form.

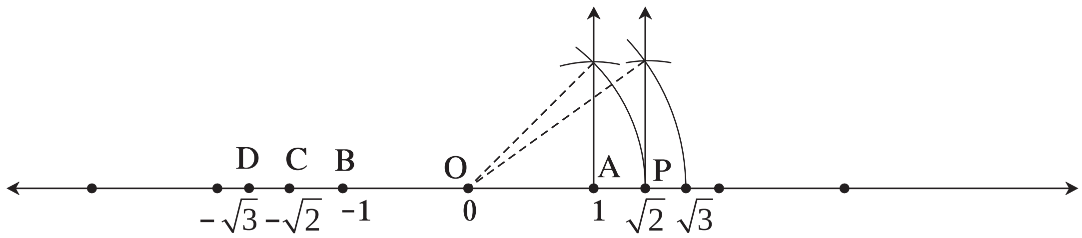
   (i) $\frac{127}{200}$ (ii) $\frac{25}{99}$ (iii) $\frac{23}{7}$ (iv) $\frac{4}{5}$ (v) $\frac{17}{8}$

3. Write the following rational numbers in $\frac{p}{q}$ form.

   (i) $0.\dot{6}$ (ii) $0.\overline{37}$ (iii) $3.\overline{17}$ (iv) $15.\overline{89}$ (v) $2.\overline{514}$

Let's recall.

The numbers $\sqrt{2}$ and $\sqrt{3}$ shown on a number line are not rational numbers means they are irrational numbers.



On a number line OA = 1 unit. Point B which is left to the point O is at a distance of 1 unit. Co-ordinate of point B is –1. Co-ordinate of point P is $\sqrt{2}$ and its opposite number $-\sqrt{2}$ is shown by point C. The co-ordinate of point C is $-\sqrt{2}$. Similarly, opposite of $\sqrt{3}$ is $-\sqrt{3}$ which is the co-ordinate of point D.

Let's learn.

Irrational and real numbers

$\sqrt{2}$ is irrational number. This can be proved using indirect proof.

Let us assume that $\sqrt{2}$ is rational. So $\sqrt{2}$ can be expressed in $\frac{p}{q}$ form.

$\frac{p}{q}$ is the reduced form of rational number hence p and q have no common factor other than 1.

$\sqrt{2} = \frac{p}{q}$ $\qquad\qquad$ $\therefore \quad 2 = \frac{P^2}{q^2}$ (Squaring both the sides)

$\therefore \quad 2q^2 = p^2$

$\therefore \quad p^2$ is even.

$\therefore \quad p$ is also even means 2 is a factor of p. ....(I)

$\therefore \quad p = 2t \qquad \therefore\ p^2 = 4t^2 \qquad t \in I$

$\therefore \quad 2q^2 = 4t^2 \ (\because\ p^2 = 2q^2) \qquad \therefore\ q^2 = 2t^2 \qquad \therefore\ q^2$ is even. $\therefore\ q$ is even.

$\therefore \quad$ 2 is a factor of q. .... (II)

From the statement (I) and (II), 2 is a common factor of p and q both.

This is contradictory because in $\frac{p}{q}$; we have assumed that p and q have no common factor except 1.

$\therefore \quad$ Our assumption that $\sqrt{2}$ is rational is wrong.

$\therefore \quad \sqrt{2}$ is irrational number.

Similarly, one can prove that $\sqrt{3}$, $\sqrt{5}$ are irrational numbers.

Numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ can be shown on a number line.

The numbers which are represented by points on a number line are real numbers.

In a nutshell, **Every point on a number line is associated with a unique a 'Real number' and every real number is associated with a unique point on the number line.**

We know that every rational number is a real number. But $\sqrt{2}$, $\sqrt{3}$, $-\sqrt{2}$, π, $3 + \sqrt{2}$ etc. are not rational numbers. It means that **Every real number may not be a rational number**.

Decimal form of irrational numbers

Find the square root of 2 and 3 using devision method.

Square root of 2

	1.41421...
1	$2.\overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$....
+1	−1
24	100
+4	−96
281	400
+ 1	−281
2824	11900
+ 4	−11296
28282	60400
+ 2	− 56564
28284 1	0383600

$\therefore \sqrt{2} = 1.41421...$

Square root of 3

	1.732....
1	$3.\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$....
+1	−1
27	200
+7	−189
343	1100
+ 3	−1029
3462	007100
+ 2	−6924
3464	0176

$\therefore \sqrt{3} = 1.732...$

Note that in the above division, numbers after decimal point are unending, means it is non-terminating. Even no group of numbers or a single number is repeating in its quotient. So decimal expansion of such numbers is non terminating, non-recurring.

$\sqrt{2}, \sqrt{3}$ are irrational numbers hence 1.4142... and 1.732... are irrational numbers too. Moreover, **a number whose decimal expansion is non-terminating, non-recurring is irrational.**

Number π

Activity I

Draw three or four circles of different radii on a card board. Cut these circles. Take a thread and measure the length of circumference and diameter of each of the circles. Note down the readings in the given table.

No.	radius (r)	diameter (d)	circumference (c)	Ratio = $\frac{c}{d}$
1	7 cm			
2	8 cm			
3	5.5 cm			

From table the ratio $\frac{c}{d}$ is nearly 3.1 which is constant. This ratio is denoted by π (pi).

Activity II

To find the approximate value of π, take the wire of length 11 cm, 22 cm and 33 cm each. Make a circle from the wire. Measure the diameter and complete the following table..

Circle No.	Circumference (c)	Diameter (d)	Ratio of (c) to (d)
1	11 cm		
2	22 cm		
3	33 cm		

Verify ratio of circumference to the diameter of a circle is approximately $\frac{22}{7}$.

Ratio of the circumference of a circle to its diameter is constant number which is irrational. This constant number is represented by the symbol π. So the approximate value of π is $\frac{22}{7}$ or 3.14.

The great Indian mathematician Aryabhat in 499 CE has calculated the value of π as $\frac{62832}{20000} = 3.1416$.

We know that, $\sqrt{3}$ is an irrational number because its decimal expansion is non-terminating, non-recurring. Now let us see whether $2 + \sqrt{3}$ is irrational or not ?

Let us assume that, $2 + \sqrt{3}$ is not an irrational number.

If $2 + \sqrt{3}$ is rational then let $2 + \sqrt{3} = \frac{p}{q}$. $\therefore$ We get $\sqrt{3} = \frac{p}{q} - 2$.

In this equation left side is an irrational number and right side rational number, which is contradictory, so $2 + \sqrt{3}$ is not a rational but it is an irrational number.

Similarly it can be proved that $2\sqrt{3}$ is irrational. The sum of two irrational numbers can be rational or irrational. Let us verify it for different numbers.

i.e., $2 + \sqrt{3} + (-\sqrt{3}) = 2$, $4\sqrt{5} \div \sqrt{5} = 4$, $(3 + \sqrt{5}) - (\sqrt{5}) = 3$,

$2\sqrt{3} \times \sqrt{3} = 6$ $\sqrt{2} \times \sqrt{5} = \sqrt{10}$, $2\sqrt{5} - \sqrt{5} = \sqrt{5}$

Remember this !

Properties of irrational numbers

(1) Addition or subtraction of a rational number with irrational number is an irrational number.

(2) Multiplication or division of non zero rational number with irrational number is also an irrational number.

(3) Addition, subtraction, multiplication and division of two irrational numbers can be either a rational or irrational number.

Let's learn.

Properties of order relation on Real numbers

1. If a and b are two real numbers then only one of the relations holds good. i.e. $a = b$ or $a < b$ or $a > b$
2. If $a < b$ and $b < c$ then $a < c$
3. If $a < b$ then $a + c < b + c$
4. If $a < b$ and $c > 0$ then $ac < bc$ and If $c < 0$ then $ac > bc$

Verify the above properties using rational and irrational numbers.

Square root of a Negative number

We know that, if $\sqrt{a} = b$ then $b^2 = a$.

Hence if $\sqrt{5} = x$ then $x^2 = 5$.

Similarly we know that square of any real number is always non-negative. It means that square of any real number is never negative. But $(\sqrt{-5})^2 = -5$ $\therefore$ $\sqrt{-5}$ is not a real number.

Hence square root of a negative real number is not a real number.

Practice set 2.2

(1) Show that $4\sqrt{2}$ is an irrational number.

(2) Prove that $3 + \sqrt{5}$ is an irrational number.

(3) Represent the numbers $\sqrt{5}$ and $\sqrt{10}$ on a number line.

(4) Write any three rational numbers between the two numbers given below.

(i) 0.3 and –0.5 (ii) –2.3 and –2.33

(iii) 5.2 and 5.3 (iv) –4.5 and –4.6

Let's learn.

Root of positive rational number

We know that, if $x^2 = 2$ then $x = \sqrt{2}$ or $x = -\sqrt{2}$, where. $\sqrt{2}$ and $-\sqrt{2}$ are irrational numbers. This we know, $\sqrt[3]{7}$, $\sqrt[4]{8}$, these numbers are irrational numbers too.

If n is a positive integer and $x^n = a$, then x is the n[th] root of a . $x = \sqrt[5]{2}$. This root may be rational or irrational.

For example, $2^5 = 32$ $\therefore$ 2 is the 5[th] root of 32, but if $x^5 = 2$ then $x = \sqrt[5]{2}$, which is an irrational number.

Surds

We know that 5 is a rational number but $\sqrt{5}$ is not rational. Square root or cube root of any real number is either rational or irrational number. Similarly n^{th} root of any number is either rational or irrational.

If n is an integer greater than 1 and if a is a positive real number and n^{th} root of a is x then it is written as $x^n = a$ or $\sqrt[n]{a} = x$

If a is a positive rational number and n^{th} root of a is x and if x is an irrational number then x is called a surd. (surd is an irrational root)

In a surd $\sqrt[n]{a}$ the symbol $\sqrt{\ }$ is called **radical sign,** n is the **Order of the surd** and a is called **radicand**.

(1) Let $a = 7$, $n = 3$, then $\sqrt[3]{7}$ is a surd because $\sqrt[3]{7}$ is an irrational number.

(2) Let $a = 27$, $n = 3$ then $\sqrt[3]{27}$ is not a surd because $\sqrt[3]{27} = 3$ is not an irrational number. .

(3) $\sqrt[3]{8}$ is a surd or not ?

Let $\sqrt[3]{8} = p$ $\quad p^3 = 8$. $\quad$ Cube of which number is 8 ?

We know 2 is cube-root of 8 or cube of 2 is 8.

$\therefore$ $\sqrt[3]{8}$ is not a surd.

(4) Whether $\sqrt[4]{8}$ is surd or not ?

Here $a = 8$, Order of surd $n = 4$; but 4^{th} root of 8 is not a rational number.

$\therefore$ $\sqrt[4]{8}$ is an irrational number. $\therefore$ $\sqrt[4]{8}$ is a surd.

This year we are going to study surds of order 2 only, means $\sqrt{3}$, $\sqrt{7}$, $\sqrt{42}$ etc.

The surds of order 2 is called **Quadratic surd.**

Simplest form of a surd

(i) $\sqrt{48} = \sqrt{16\times3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\qquad$ (ii) $\sqrt{98} = \sqrt{49\times2} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... these type of surds are in the simplest form which cannot be simplified further.

Similar or like surds

$\sqrt{2}, -3\sqrt{2}, \frac{4}{5}\sqrt{2}$ are some like surds.

If p and q are rational numbers then $p\sqrt{a}$, $q\sqrt{a}$ are called like surds. Two surds are said to be like surds if their order is equal and radicands are equal.

$\sqrt{45}$ and $\sqrt{80}$ are the surds of order 2, so their order is equal but radicands are not same, Are these like surds? Let us simplify it as follows :

$\sqrt{45} = \sqrt{9\times5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$ and $\sqrt{80} = \sqrt{16\times5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

$\therefore$ $3\sqrt{5}$ and $4\sqrt{5}$ are now similar or like surds

means $\sqrt{45}$ and $\sqrt{80}$ are similar surds.

Remember this !

In the simplest form of the surds if order of the surds and redicand are equal then the surds are similar or like surds.

Let's learn.

Comparison of surds

Let a and b are two positive real numbers and
If $a < b$ then $a \times a < b \times a$
If $a^2 < ab$ ...(I) Similarly $ab < b^2$ ...(II)
$\therefore$ $a^2 < b^2$ ...[from (I) and (II)]
But if $a > b$ then $a^2 > b^2$ and if $a = b$ then $a^2 = b^2$
hence if $a < b$ then $a^2 < b^2$

Here a and b both are real numbers so they may be rational numbers or surds. By using above properties, let us compare the surds.

(i) $6\sqrt{2}, 5\sqrt{5}$

$\sqrt{36} \times \sqrt{2}$ $\boxed{?}$ $\sqrt{25} \times \sqrt{5}$

$\sqrt{72}$ $\boxed{?}$ $\sqrt{125}$

But 72 $\boxed{<}$ 125

$\therefore 6\sqrt{2}$ $\boxed{<}$ $5\sqrt{5}$

Or

$(6\sqrt{2})^2 \boxed{} (5\sqrt{5})^2$,

$72 < 125$

$\therefore 6\sqrt{2}$ $\boxed{<}$ $5\sqrt{5}$

(ii) $8\sqrt{3}, \sqrt{192}$

$\sqrt{64} \times \sqrt{3}$ $\boxed{?}$ $\sqrt{192}$

$\sqrt{192}$ $\boxed{?}$ $\sqrt{192}$

But 192 $\boxed{=}$ 192

$\therefore$ $\sqrt{192}$ $\boxed{=}$ $\sqrt{192}$

$\therefore$ $8\sqrt{3}$ $\boxed{=}$ $\sqrt{192}$

(iii) $7\sqrt{2}, 5\sqrt{3}$

$\sqrt{49} \times \sqrt{2}$ $\boxed{?}$ $\sqrt{25} \times \sqrt{3}$

$\sqrt{98}$ $\boxed{?}$ $\sqrt{75}$

But 98 $\boxed{>}$ 75

$\therefore 7\sqrt{2}$ $\boxed{>}$ $5\sqrt{3}$

Or

$(7\sqrt{2})^2 \boxed{} (5\sqrt{3})^2$,

$98 > 75$

$\therefore 7\sqrt{2}$ $\boxed{>}$ $5\sqrt{3}$

Operations on like surds

Mathematical operations like addition, subtraction, multiplication and division can be done on like surds.

Use your brain power !

$\sqrt{9+16} \overset{?}{=} \sqrt{9}+\sqrt{16}$

$\sqrt{100+36} \overset{?}{=} \sqrt{100}+\sqrt{36}$

Ex (1) Simplify : $7\sqrt{3} + 29\sqrt{3}$

Solution : $7\sqrt{3} + 29\sqrt{3} = (7 + 29)\sqrt{3} = 36\sqrt{3}$

Ex (2) Simplify : $7\sqrt{3} - 29\sqrt{3}$

Solution : $7\sqrt{3} - 29\sqrt{3} = (7 - 29)\sqrt{3} = -22\sqrt{3}$

Ex (3) Simplify : $13\sqrt{8} + \frac{1}{2}\sqrt{8} - 5\sqrt{8}$

Solution : $13\sqrt{8} + \frac{1}{2}\sqrt{8} - 5\sqrt{8} = (13 + \frac{1}{2} - 5)\sqrt{8} = (\frac{26+1-10}{2})\sqrt{8}$

$$= \frac{17}{2}\sqrt{8} = \frac{17}{2}\sqrt{4\times 2}$$

$$= \frac{17}{2} \times 2\sqrt{2} = 17\sqrt{2}$$

Ex (4) Simplify : $8\sqrt{5} + \sqrt{20} - \sqrt{125}$

Solution : $8\sqrt{5} + \sqrt{20} - \sqrt{125} = 8\sqrt{5} + \sqrt{4\times 5} - \sqrt{25\times 5}$

$$= 8\sqrt{5} + 2\sqrt{5} - 5\sqrt{5}$$

$$= (8 + 2 - 5)\sqrt{5}$$

$$= 5\sqrt{5}$$

Ex. (5) Multiply the surds $\sqrt{7} \times \sqrt{42}$

Solution : $\sqrt{7} \times \sqrt{42} = \sqrt{7\times 42} = \sqrt{7\times 7\times 6} = 7\sqrt{6}$ ($7\sqrt{6}$ is an irrational number.)

Ex. (6) Divide the surds : $\sqrt{125} \div \sqrt{5}$

Solution : $\frac{\sqrt{125}}{\sqrt{5}} = \sqrt{\frac{125}{5}} = \sqrt{25} = 5$ (5 is a rational number.)

Ex. (7) $\sqrt{50} \times \sqrt{18} = \sqrt{25\times 2} \times \sqrt{9\times 2} = 5\sqrt{2} \times 3\sqrt{2} = 15 \times 2 = 30$

Note that product and quotient of two surds may be rational numbers which can be observed in the above Ex. 6 and Ex. 7.

Downloaded from https:// www.studiestoday.com

Rationalization of surd

If the product of two surds is a rational number, each surd is called a rationalizing factor of the other surd.

Ex. (1) If surd $\sqrt{2}$ is multiplied by $\sqrt{2}$ we get $\sqrt{2\times 2} = \sqrt{4}$. $\sqrt{4} = 2$ is a rational number.

$\therefore$ $\sqrt{2}$ is rationalizing factor of $\sqrt{2}$.

Ex. (2) Multiply $\sqrt{2} \times \sqrt{8}$

$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$ is a rational number.

$\therefore$ $\sqrt{2}$ is the rationalizing factor of $\sqrt{8}$.

Similarly $8\sqrt{2}$ is also a rationalizing factor of $\sqrt{2}$.

because $\sqrt{2} \times 8\sqrt{2} = 8\sqrt{2} \times \sqrt{2} = 8 \times 2 = 16$.

$\sqrt{6}$, $\sqrt{16}$ $\sqrt{50}$ are the rationalizing factors of $\sqrt{2}$.

Remember this !

Rationalizing factor of a given surd is not unique. If a surd is a rationalizing factor of a given surd then a surd obtained by multiplying it with any non zero rational number is also a rationalizing factor of the given surd.

Ex. (3) Find the rationalizing factor of $\sqrt{27}$

Solution : $\sqrt{27} = \sqrt{9\times 3} = 3\sqrt{3}$ $\therefore$ $3\sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$ is a rational number.

$\therefore$ $\sqrt{3}$ is the rationalizing factor of $\sqrt{27}$.

Note that, $\sqrt{27} = 3\sqrt{3}$ means $3\sqrt{3} \times 3\sqrt{3} = 9 \times 3 = 27$.

Hence $3\sqrt{3}$ is also a rationalizing factor of $\sqrt{27}$. In the same way $4\sqrt{3}$, $7\sqrt{3}$, ... are also the rationalizing factors of $\sqrt{27}$. Out of all these $\sqrt{3}$ is the simplest rationalizing factor.

Ex. (4) Rationalize the denominator of $\frac{1}{\sqrt{5}}$

Solution : $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ ....(multiply numerator and denominator by $\sqrt{5}$.)

Ex. (5) Rationalize the denominator of $\frac{3}{2\sqrt{7}}$.

Solution : $\frac{3}{2\sqrt{7}} = \frac{3}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{2\times 7} = \frac{3\sqrt{7}}{14}$

...(multiply $2\sqrt{7}$ by $\sqrt{7}$ is sufficient to rationalize.)

Remember this !

We can make use of rationalizing factor to rationalize the denominator.
It is easy to use the numbers with rational denominator, that is why we rationalize it.

Practice set 2.3

(1) State the order of the surds given below.

(i) $\sqrt[3]{7}$ (ii) $5\sqrt{12}$ (iii) $\sqrt[4]{10}$ (iv) $\sqrt{39}$ (v) $\sqrt[3]{18}$

(2) State which of the following are surds. Justify.

(i) $\sqrt[3]{51}$ (ii) $\sqrt[4]{16}$ (iii) $\sqrt[5]{81}$ (iv) $\sqrt{256}$ (v) $\sqrt[3]{64}$ (vi) $\sqrt{\dfrac{22}{7}}$

(3) Classify the given pair of surds into like surds and unlike surds.

(i) $\sqrt{52}$, $5\sqrt{13}$ (ii) $\sqrt{68}$, $5\sqrt{3}$ (iii) $4\sqrt{18}$, $7\sqrt{2}$

(iv) $19\sqrt{12}$, $6\sqrt{3}$ (v) $5\sqrt{22}$, $7\sqrt{33}$ (vi) $5\sqrt{5}$, $\sqrt{75}$

(4) Simplify the following surds.

(i) $\sqrt{27}$ (ii) $\sqrt{50}$ (iii) $\sqrt{250}$ (iv) $\sqrt{112}$ (v) $\sqrt{168}$

(5) Compare the following pair of surds.

(i) $7\sqrt{2}$, $5\sqrt{3}$ (ii) $\sqrt{247}$, $\sqrt{274}$ (iii) $2\sqrt{7}$, $\sqrt{28}$

(iv) $5\sqrt{5}$, $7\sqrt{2}$ (v) $4\sqrt{42}$, $9\sqrt{2}$ (vi) $5\sqrt{3}$, 9 (vii) 7, $2\sqrt{5}$

(6) Simplify.

(i) $5\sqrt{3} + 8\sqrt{3}$ (ii) $9\sqrt{5} - 4\sqrt{5} + \sqrt{125}$

(iii) $7\sqrt{48} - \sqrt{27} - \sqrt{3}$ (iv) $\sqrt{7} - \dfrac{3}{5}\sqrt{7} + 2\sqrt{7}$

(7) Multiply and write the answer in the simplest form.

(i) $3\sqrt{12} \times \sqrt{18}$ (ii) $3\sqrt{12} \times 7\sqrt{15}$

(iii) $3\sqrt{8} \times \sqrt{5}$ (iv) $5\sqrt{8} \times 2\sqrt{8}$

(8) Divide, and write the answer in simplest form.

(i) $\sqrt{98} \div \sqrt{2}$ (ii) $\sqrt{125} \div \sqrt{50}$ (iii) $\sqrt{54} \div \sqrt{27}$ (iv) $\sqrt{310} \div \sqrt{5}$

(9) Rationalize the denominator.

(i) $\dfrac{3}{\sqrt{5}}$ (ii) $\dfrac{1}{\sqrt{14}}$ (iii) $\dfrac{5}{\sqrt{7}}$ (iv) $\dfrac{6}{9\sqrt{3}}$ (v) $\dfrac{11}{\sqrt{3}}$

Let's recall.

We know that,

If $a > 0$, $b > 0$ then $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$(a+b)(a-b) = a^2 - b^2$; $\left(\sqrt{a}\right)^2 = a$; $\sqrt{a^2} = a$

Multiply.

Ex. (1) $\sqrt{2}\,(\sqrt{8} + \sqrt{18})$

$= \sqrt{2 \times 8} + \sqrt{2 \times 18}$

$= \sqrt{16} + \sqrt{36}$

$= 4 + 6$

$= 10$

Ex. (2) $(\sqrt{3} - \sqrt{2})(2\sqrt{3} - 3\sqrt{2})$

$= \sqrt{3}\left(2\sqrt{3} - 3\sqrt{2}\right) - \sqrt{2}\left(2\sqrt{3} - 3\sqrt{2}\right)$

$= \sqrt{3} \times 2\sqrt{3} - \sqrt{3} \times 3\sqrt{2} - \sqrt{2} \times 2\sqrt{3} + \sqrt{2} \times 3\sqrt{2}$

$= 2 \times 3 - 3\sqrt{6} - 2\sqrt{6} + 3 \times 2$

$= 6 - 5\sqrt{6} + 6$

$= 12 - 5\sqrt{6}$

Let's learn.

Binomial quadratic surd

- $\sqrt{5} + \sqrt{3}$; $\frac{3}{4} + \sqrt{5}$ are the binomial quadratic surds form. $\sqrt{5} - \sqrt{3}$; $\frac{3}{4} - \sqrt{5}$ are also binomial quadratic surds.

Study the following examples.

- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
- $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$
- $(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7}) = (\sqrt{3})^2 - (\sqrt{7})^2 = 3 - 7 = -4$
- $(\frac{3}{2} + \sqrt{5})(\frac{3}{2} - \sqrt{5}) = (\frac{3}{2})^2 - (\sqrt{5})^2 = \frac{9}{4} - 5 = \frac{9 - 20}{4} = -\frac{11}{4}$

The product of these two binomial surds $(\sqrt{5} + \sqrt{3})$ and $(\sqrt{5} - \sqrt{3})$ is a rational number, hence these are the conjugate pairs of each other.

Each binomial surds in the conjugate pair is the rationalizing factor for other.

Note that for $\sqrt{5} + \sqrt{3}$, the conjugate pair of binomial surd is $\sqrt{5} - \sqrt{3}$ or $\sqrt{3} - \sqrt{5}$.

Similarly for $7 + \sqrt{3}$, the conjugate pair is $7 - \sqrt{3}$ or $\sqrt{3} - 7$.

Remember this !

The product of conjugate pair of binomial surds is always a rational number.

Let's learn.

Rationalization of the denominator

The product of conjugate binomial surds is always a rational number - by using this property, the rationalization of the denominator in the form of binomial surd can be done.

Ex..(1) Rationalize the denominator $\frac{1}{\sqrt{5}-\sqrt{3}}$.

Solution : The conjugate pair of $\sqrt{5}-\sqrt{3}$ is $\sqrt{5}+\sqrt{3}$.

$$\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{\sqrt{5}+\sqrt{3}}{2}$$

Ex. (2) Rationalize the denominator $\frac{8}{3\sqrt{2}+\sqrt{5}}$.

Solution : The conjugate pair of $3\sqrt{2}+\sqrt{5}$ is $3\sqrt{2}-\sqrt{5}$

$$\frac{8}{3\sqrt{2}+\sqrt{5}} = \frac{8}{3\sqrt{2}+\sqrt{5}} \times \frac{3\sqrt{2}-\sqrt{5}}{3\sqrt{2}-\sqrt{5}}$$

$$= \frac{8\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{8\times3\sqrt{2} - 8\sqrt{5}}{9\times2 - 5} = \frac{24\sqrt{2} - 8\sqrt{5}}{18 - 5} = \frac{24\sqrt{2} - 8\sqrt{5}}{13}$$

Practice set 2.4

(1) Multiply

(i) $\sqrt{3}(\sqrt{7}-\sqrt{3})$ (ii) $(\sqrt{5}-\sqrt{7})\sqrt{2}$ (iii) $(3\sqrt{2}-\sqrt{3})(4\sqrt{3}-\sqrt{2})$

(2) Rationalize the denominator.

(i) $\frac{1}{\sqrt{7}+\sqrt{2}}$ (ii) $\frac{3}{2\sqrt{5}-3\sqrt{2}}$ (iii) $\frac{4}{7+4\sqrt{3}}$ (iv) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Let's learn.

Absolute value

If x is a real number then absolute value of x is its distance from zero on the number line which is written as $|x|$, and $|x|$ is read as Absolute Value of x or modulus of x.

If $x > 0$ then $|x| = x$ — If x is positive then absolute value of x is x.

If $x = 0$ then $|x| = 0$ — If x is zero then absolute value of x is zero.

If $x < 0$ then $|x| = -x$ — If x is negative then its absolute value is opposite of x.

Ex. (1) $|3| = 3$, $|-3| = -(-3) = 3$, $|0| = 0$

The absolute value of any real number is never negative.

Ex. (2) Find the value.

(i) $|9-5| = |4| = 4$ (ii) $|8-13| = |-5| = 5$

(iii) $|8| - |-3| = 5$ (iv) $|8| \times |4| = 8 \times 4 = 32$

Ex. (3) Solve $|x - 5| = 2$.

Solution : $|x - 5| = 2$ $\therefore x - 5 = +2$ or $x - 5 = -2$

$\therefore x = 2 + 5$ or $x = -2+5$

$\therefore x = 7$ or $x = 3$

Practice set 2.5

(1) Find the value.

i) $|15 - 2|$ (ii) $|4 - 9|$ (iii) $|7| \times |-4|$

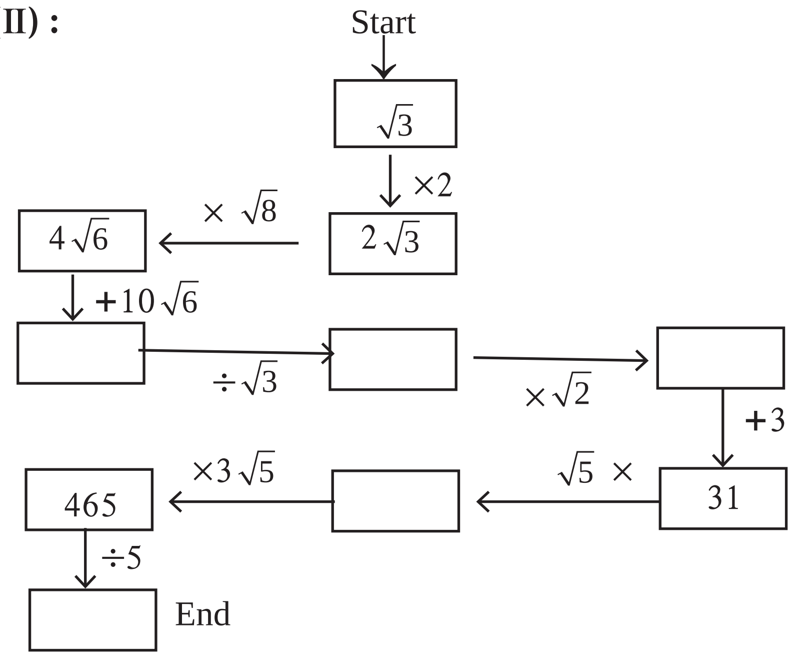
(2) Solve.

(i) $|3x-5| = 1$ (ii) $|7-2x| = 5$ (iii) $\left|\frac{8-x}{2}\right| = 5$ (iv) $\left|5+\frac{x}{4}\right| = 5$

Activity (I) : There are some real numbers written on a card sheet. Use these numbers and construct two examples each of addition, subtraction, multiplication and division. Solve these examples.

12, $2\sqrt{5}$, ..., $-\frac{3}{4}$, $-9\sqrt{-5}$, $9\sqrt{2}$, -11, $3\sqrt{11}$, $5\sqrt{7}$, $-3\sqrt{2}$

Activity (II) :



Problem set 2

(1) Choose the correct alternative answer for the questions given below.

(i) Which one of the following is an irrational number ?

(A) $\sqrt{\frac{16}{25}}$ (B) $\sqrt{5}$ (C) $\frac{3}{9}$ (D) $\sqrt{196}$

(ii) Which of the following is an irrational number?

(A) 0.17 (B) $1.\overline{513}$ (C) $0.27\overline{46}$ (D) 0.101001000.....

(iii) Decimal expansion of which of the following is non-terminating recurring ?

(A) $\frac{2}{5}$ (B) $\frac{3}{16}$ (C) $\frac{3}{11}$ (D) $\frac{137}{25}$

(iv) Every point on the number line represent, which of the following numbers?

(A) Natural numbers (B) Irrational numbers

(C) Rational numbers (D) Real numbers.

(v) The number $0.\dot{4}$ in $\frac{p}{q}$ form is .....

(A) $\frac{4}{9}$ (B) $\frac{40}{9}$ (C) $\frac{3.6}{9}$ (D) $\frac{36}{9}$

(vi) What is $\sqrt{n}$, if n is not a perfect square number ?

(A) Natural number (B) Rational number

(C) Irrational number (D) Options A, B, C all are correct.

(vii) Which of the following is not a surd ?

(A) $\sqrt{7}$ (B) $\sqrt[3]{17}$ (C) $\sqrt[3]{64}$ (D) $\sqrt{193}$

(viii) What is the order of the surd $\sqrt[3]{\sqrt{5}}$?

(A) 3 (B) 2 (C) 6 (D) 5

(ix) Which one is the conjugate pair of $2\sqrt{5}+\sqrt{3}$?

(A) $-2\sqrt{5}+\sqrt{3}$ (B) $-2\sqrt{5}-\sqrt{3}$ (C) $2\sqrt{3}-\sqrt{5}$ (D) $\sqrt{3}+2\sqrt{5}$

(x) The value of $|12-(13+7)\times 4|$ is ....

(A) –68 (B) 68 (C) –32 (D) 32.

(2) Write the following numbers in $\frac{p}{q}$ form.

(i) 0.555 (ii) $29.\overline{568}$ (iii) 9.315 315 ... (iv) 357.417417... (v) $30.\overline{219}$

(3) Write the following numbers in its decimal form..

(i) $\frac{-5}{7}$ (ii) $\frac{9}{11}$ (iii) $\sqrt{5}$ (iv) $\frac{121}{13}$ (v) $\frac{29}{8}$

(4) Show that $5+\sqrt{7}$ is an irrational number.

(5) Write the following surds in simplest form.

(i) $\frac{3}{4}\sqrt{8}$ (ii) $-\frac{5}{9}\sqrt{45}$

(6) Write the simplest form of rationalising factor for the given surds.

(i) $\sqrt{32}$ (ii) $\sqrt{50}$ (iii) $\sqrt{27}$ (iv) $\frac{3}{5}\sqrt{10}$ (v) $3\sqrt{72}$ (vi) $4\sqrt{11}$

(7) Simplify.

(i) $\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75}$ (ii) $5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$ (iii) $\sqrt{216}-5\sqrt{6}+\sqrt{294}-\frac{3}{\sqrt{6}}$

(iv) $4\sqrt{12}-\sqrt{75}-7\sqrt{48}$ (v*) $2\sqrt{48}-\sqrt{75}-\frac{1}{\sqrt{3}}$

(8) Rationalize the denominator.

(i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{2}{3\sqrt{7}}$ (iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (iv) $\frac{1}{3\sqrt{5}+2\sqrt{2}}$ (v) $\frac{12}{4\sqrt{3}-\sqrt{2}}$

3 Polynomials

Let's study.

- Introduction to Polynomials
- Degree of the polynomial
- Value of the polynomial
- Operations on polynomials
- Synthetic Division
- Remainder theorem

Let's discuss.

$p^3 - \frac{1}{2}p^2 + p$; $m^2 + 2n^3 - \sqrt{3}\,m^5$; 6 are all algebraic expressions.

Teacher : Dear Students, consider each term of the expressions $p^3 - \frac{1}{2}p^2 + p$, $m^2 + 2n^3 - \sqrt{3}\,m^5$, 6 and state the power of each variable.

Madhuri : In the expressions $p^3 - \frac{1}{2}p^2 + p$ powers of p are 3, 2, 1 respectively.

Vivek : Sir, in the expression $m^2 + 2n^3 - \sqrt{3}\,m^5$ the powers of the variable are 2, 3, 5 respectively.

Rahul : Sir, apparently there is no variable in the expression 6. But $6 = 6 \times 1 = 6 \times x^0$. Therefore, the power of the variable is 0.

Teacher : In all algebraic expressions given above the powers of the variable are positive integers or zero. i.e. whole numbers.

In an algebraic expression, if the powers of the variables are whole numbers then that algebraic expression is known as **polynomial.** 6 is also a polynomial. 6, $-$ 7, $\frac{1}{2}$, 0, $\sqrt{3}$ etc. are constant numbers can be called as **Constant polynomial.** 0 is also a constant polynomial.

Are $\sqrt{y} + 5$ and $\frac{1}{y} - 3$ polynomials?

Sara : Sir, $\sqrt{y} + 5$ is not a polynomial, because $\sqrt{y} + 5 = y^{\frac{1}{2}} + 5$, here power of y is $\frac{1}{2}$ which is not a whole number.

John : Sir, $\frac{1}{y} - 3$ is also not a polynomial because $\frac{1}{y} - 3 = y^{-1} - 3$, here power of y is $-$ 1 which is not a whole number.

Teacher : Write any five algebraic expressions which are not polynomials.

Explain why these expressions are not polynomials ? Justify your answer.

- Is every algebraic expression a polynomial ?
- Is every polynomial an algebraic expression ?

Types of polynomials (based on number of terms)

$2x$, 5	$x^4 + x$, $m^2 - 3m$	$\frac{1}{2}y^2 - 2y + 5$, $x^3 - \sqrt{3}\ x^2 + 5x$
Only one term in the polynomial	Two terms in the polynomial	Three terms in the polynomial
Monomial	Binomial	Trinomial

Polynomials are written as $p(x)$, $q(m)$, $r(y)$ according to the variable used.

For example, $p(x) = x^3 + 2x^2 + 5x - 3$, $q(m) = m^2 + \frac{1}{2}m - 7$, $r(y) = y^2 + 5$

Let's learn.

Degree of a polynomial in one variable

Teacher : In the polynomial $2x^7 - 5x + 9$ which is the highest power of the variable ?

Jija : Sir, the highest power is 7.

Teacher : In case of a polynomial in one variable, the highest power of the variable is called the **Degree of the polynomial.**

Now tell me, what is the degree of the given polynomial?

Ashok : Sir, the degree of the given polynomial $2x^7 - 5x + 9$ is 7.

Teacher : What is the degree of the polynomial 10 ?

Radha : $10 = 10 \times 1 = 10 \times x^0$ therefore the degree of the polynomial 10 is 0.

Teacher : Just like 10, **degree of any non zero constant polynomial is 0. Degree of zero polynomial is not defined.**

Degree of a polynomial in more than one variable

The highest sum of the powers of variables in each term of the polynomial is the degree of the polynomial.

Ex. $3m^3n^6 + 7m^2n^3 - mn$ is a polynomial in two variables m and n. Degree of the polynomial is 9. (as sum of the powers $3 + 6 = 9$, $2 + 3 = 5$, $1 + 1 = 2$)

Activity I : Write an example of a monomial, a binomial and a trinomial having variable x and degree 5.

Monomial ☐ Binomial ☐ Trinomial ☐

Activity II : Give example of a binomial in two variables having degree 5. ☐

Types of polynomial (based on degree)

Polynomial in one variable

$3x - 1$, $7y$	$2y^2 + y + 1$, $-3x^2$	$x^3 + x^2 + 2x + \sqrt{3}$, $m - m^3$
Degree 1	Degree 2	Degree 3
Linear polynomial	Quadratic polynomial	Cubic polynomial
Standard form $ax + b$ here a and b are coefficients and $a \neq 0$	Standard form $ax^2 + bx + c$ here a, b, c are coefficients and $a \neq 0$	Standard form $ax^3 + bx^2 + cx + d$ here a, b, c, d are coefficients and $a \neq 0$

Polynomial : $a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$ is a polynomial in x with degree n

a_n, a_{n-1}, ..., a_2, a_1, a_0 are the coefficients and $a_n \neq 0$

Standard form, coefficient form and index form of a polynomial

$p(x) = x - 3x^2 + 5 + x^4$ is a polynomial in x, which can be written in descending powers of its variable as $x^4 - 3x^2 + x + 5$. This is called the standard form of the polynomial. But in this polynomial there is no term having power 3 of the variable we can write it as $0x^3$. It can be added to the polynomial and it can be rewritten as $x^4 + 0x^3 - 3x^2 + x + 5$. This form of the polynomial is called **Index form** of the polynomial.

One can write the coefficients of the variables by considering all the missing terms in the standard form of the polynomial. For example : $x^3 - 3x^2 + 0x - 8$ can be written as $(1, -3, 0, -8)$. This form of the polynomial is called **Coefficient form.**

Polynomial $(4, 0, -5, 0, 1)$ can be written by using variable y as

$4y^4 + 0y^3 - 5y^2 + 0y + 1$. This form is called **Index form** of the polynomial.

Ex. $p(m) = 3m^5 - 7m + 5m^3 + 2$

Write the polynomial in standard form	$3m^5 + 5m^3 - 7m + 2$
Write it in the index form by considering all the missing terms with coefficient zero.	$3m^5 + 0m^4 + 5m^3 + 0m^2 - 7m + 2$
Write it in a coefficient form	$(3, 0, 5, 0, -7, 2)$
Degree of the polynomial	5

Ex (1) Write the polynomial $x^3 + 3x - 5$ in coefficient form.

Solution: $x^3 + 3x - 5 = x^3 + 0x^2 + 3x - 5$

$\therefore$ given polynomial in coefficient form is $(1, 0, 3, -5)$

Ex (2) $(2, -1, 0, 5, 6)$ is the coefficient form of the polynomial. Represent it in index form.

Solution: Coefficient form of the polynomial is $(2, -1, 0, 5, 6)$

$\therefore$ index form of the polynomial is $2x^4 - x^3 + 0x^2 + 5x + 6$ i.e. $2x^4 - x^3 + 5x + 6$

Practice set 3.1

1. State whether the given algebraic expressions are polynomials ? Justify.

   (i) $y + \frac{1}{y}$ (ii) $2 - 5\sqrt{x}$ (iii) $x^2 + 7x + 9$

   (iv) $2m^{-2} + 7m - 5$ (v) 10

2. Write the coefficient of m^3 in each of the given polynomial.

   (i) m^3 (ii) $\frac{-3}{2} + m - \sqrt{3}\,m^3$ (iii) $\frac{-2}{3}m^3 - 5m^2 + 7m - 1$

3. Write the polynomial in x using the given information.

   (i) Monomial with degree 7 (ii) Binomial with degree 35

   (iii) Trinomial with degree 8

4. Write the degree of the given polynomials.

(i) $\sqrt{5}$ (ii) x° (iii) x^2 (iv) $\sqrt{2}\,m^{10} - 7$ (v) $2p - \sqrt{7}$

(vi) $7y - y^3 + y^5$ (vii) $xyz + xy - z$ (viii) $m^3n^7 - 3m^5n + mn$

5. Classify the following polynomials as linear, quadratic and cubic polynomial.

(i) $2x^2 + 3x + 1$ (ii) $5p$ (iii) $\sqrt{2}\ y - \frac{1}{2}$

(iv) $m^3 + 7m^2 + \frac{5}{2}m - \sqrt{7}$ (v) a^2 (vi) $3r^3$

6. Write the following polynomials in standard form.

(i) $m^3 + 3 + 5m$ (ii) $-7y + y^5 + 3y^3 - \frac{1}{2} + 2y^4 - y^2$

7. Write the following polynomials in coefficient form.

(i) $x^3 - 2$ (ii) $5y$ (iii) $2m^4 - 3m^2 + 7$ (iv) $-\frac{2}{3}$

8. Write the polynomials in index form.

(i) (1, 2, 3) (ii) (5, 0, 0, 0, − 1) (iii) (− 2, 2, − 2, 2)

9. Write the appropriate polynomials in the boxes.

$x + 7$, x^2, $x^3 + x^2 + x + 5$, $2x^2 + 5x + 10$, $x^3 + 9$, $3x^2 + 5x$

Quadratic polynomial	Binomial
....	....
Cubic polynomial	Trinomial
....	....
Linear polynomial	Monomial
....	....

Let's recall.

(1) Coefficients are added or subtracted while adding or subtracting like algebraic terms, e.g. $5m^3 - 7m^3 = (5 - 7)m^3 = -2m^3$

(2) While multiplying or dividing two algebraic terms, we multiply or divide their coefficients. We also use laws of indices.

$-4y^3 \times 2y^2z = -8y^5z$; $12a^2b \div 3ab^2 = \frac{4a}{b}$

Let's learn.

Operations on polynomials

The methods of addition, subtraction, multiplication and division of polynomials is similar to the operation of algebraic expressions.

Ex (1) Subtract : $5a^2 - 2a$ from $7a^2 + 5a + 6$.

Solution : $(7a^2 + 5a + 6) - (5a^2 - 2a)$

$$= \underline{7a^2 + 5a} + 6 - \underline{5a^2 + 2a}$$

$$= 7a^2 - 5a^2 + 5a + 2a + 6$$

$$= 2a^2 + 7a + 6$$

Ex (2) Multiply : $-2a \times 5a^2$

Solution : $-2a \times 5a^2 = -10a^3$

Ex (3) Multiply :$(m^2 - 5) \times (m^3 + 2m - 2)$

Solution : $(m^2 - 5) \times (m^3 + 2m - 2)$

$$= m^2 (m^3 + 2m - 2) - 5 (m^3 + 2m - 2)$$

(Each term of second polynomials is multiplied by each term of first polynomial)

$$= m^5 + 2m^3 - 2m^2 - 5m^3 - 10m + 10$$

$$= m^5 + 2m^3 - 5m^3 - 2m^2 - 10m + 10$$ (Like terms taken together.)

$$= m^5 - 3m^3 - 2m^2 - 10m + 10$$

Here the degree of the product is 5.

Ex (4) Add : $3m^2n + 5mn^2 - 7mn$ and $2m^2n - mn^2 + mn$.

Solution : $(3m^2n + 5mn^2 - 7mn) + (2m^2n - mn^2 + mn)$

$$= 3m^2n + 5mn^2 - 7mn + 2m^2n - mn^2 + mn$$

$$= \underline{3m^2n + 2m^2n} + \underline{5mn^2 - mn^2} - \underline{7mn + mn}$$ (Like terms are arranged.)

$$= 5m^2n + 4mn^2 - 6mn$$ (Like terms are added.)

Let's recall.

Degree of one polynomial is 3 and the degree of other polynomials is 5. Then what is the degree of their product ?

What is the relation between degree of multiplicand and degree of a multiplier with degree of their product ?

Ex (5) Divide $(2 + 2x^2) \div (x + 2)$ and write the answer in the given form

Dividend = Divisor × Quotient + Remainder

Solution : Let us write the polynomial in standard form. $p(x) = 2 + 2x^2$

$2 + 2x^2 = 2x^2 + 0x + 2$

Method I :

$$
\begin{array}{r|l}
 & \quad 2x - 4 \\
\hline
x + 2 & 2x^2 + 0x + 2 \\
 - & 2x^2 + 4x \\
 & \; - \quad - \\
\hline
 & \qquad -4x + 2 \\
 - & \qquad -4x - 8 \\
 & \qquad + \quad + \\
\hline
 & \qquad\qquad 10
\end{array}
$$

Dividend = divisor × quotient + remainder

$2 + 2x^2 = (x + 2) \times (2x - 4) + 10$

$q(x)$, divisor $= (x + 2)$

$s(x)$, quotient $= 2x - 4$ and

$r(x)$, remainder $= 10$

$\therefore \; p(x) = q(x) \times s(x) + r(x).$

Method II : Linear method of division :

Divide $(2x^2 + 2) \div (x + 2)$

To get the term $2x^2$ multiply $(x + 2)$ by $2x$ and subtract $4x$.

$2x(x+2) - 4x = 2x^2$

$\therefore$ Dividend $= 2x^2 + 2 = 2x(x+2) - 4x + 2$ ...(I)

To get the term $-4x$ multiply $(x+2)$ by -4 and add 8.

$-4(x+2) + 8 = -4x$

$\therefore \; (2x^2 + 2) = 2x(x+2) - 4(x+2) + 8 + 2$ ...from (I)

$\therefore \; (2x^2 + 2) = (x + 2)(2x - 4) + 10$

Dividend = divisor × quotient + remainder.

Remember this !

Euclid's division lemma

If s(x) and $p(x)$ are two polynomials such that degree of s(x) is greater than or equal to the degree of $p(x)$ and after dividing s(x) by $p(x)$ the quotient is $q(x)$ then s(x) = $p(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) < 0$.

Practice set 3.2

(1) Use the given letters to write the answer.

(i) There are 'a' trees in the village Lat. If the number of trees increases every year by 'b', then how many trees will there be after 'x' years?

(ii) For the parade there are y students in each row and x such row are formed. Then, how many students are there for the parade in all ?

(iii) The tens and units place of a two digit number is m and n respectively. Write the polynomial which represents the two digit number.

(2) Add the given polynomials.

(i) $x^3 - 2x^2 - 9$; $5x^3 + 2x + 9$

(ii) $-7m^4 + 5m^3 + \sqrt{2}$; $5m^4 - 3m^3 + 2m^2 + 3m - 6$

(iii) $2y^2 + 7y + 5$; $3y + 9$; $3y^2 - 4y - 3$

(3) Subtract the second polynomial from the first.

(i) $x^2 - 9x + \sqrt{3}$; $-19x + \sqrt{3} + 7x^2$

(ii) $2ab^2 + 3a^2b - 4ab$; $3ab - 8ab^2 + 2a^2b$

(4) Multiply the given polynomials.

(i) $2x$; $x^2 - 2x - 1$ (ii) $x^5 - 1$; $x^3 + 2x^2 + 2$ (iii) $2y + 1$; $y^2 - 2y^3 + 3y$

(5) Divide first polynomial by second polynomial and write the answer in the form 'Dividend = Divisor × Quotient + Remainder'.

(i) $x^3 - 64$; $x - 4$ (ii) $5x^5 + 4x^4 - 3x^3 + 2x^2 + 2$; $x^2 - x$

(6*) Write down the information in the form of algebraic expression and simplify.

There is a rectangular farm with length $(2a^2 + 3b^2)$ metre and breadth $(a^2 + b^2)$ metre. The farmer used a square shaped plot of the farm to build a house. The side of the plot was $(a^2 - b^2)$ metre. What is the area of the remaining part of the farm ?

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Activity : Read the following passage, write the appropriate amount in the boxes and discuss.

Govind, who is a dry land farmer from Shiralas has a 5 acre field. His family includes his wife, two children and his old mother. He borrowed one lakh twenty five thousand rupees from the bank for one year as agricultural loan at 10 p.c.p.a. He cultivated soyabean in x acres and cotton and tur in y acres. The expenditure he incurred was as follows :

He spent Rs. 10,000 on seeds. The expenses for fertilizers and pesticides for the soyabean crop was 2000 x rupees and 4000 x^2 rupees were spent on wages and cultivation of land. He spent 8000 y rupees on fertilizers and pesticides and rupees 9000 y^2 for wages and cultivation of land for the cotton and tur crops.

Let us write the total expenditure on all the crops by using variables x and y.

$\boxed{} + \boxed{2000\,x} + \boxed{4000\,x^2} + \boxed{8000\,y} + \boxed{}$ rupees

He harvested 5 x^2 quintals soyabean and sold it at Rs. 2800 per quintal. The cotton crop yield was $\frac{5}{3}y^2$ quintals which fetched Rs. 5000 per quintal. The tur crop yield was $4y$ quintals and was sold at Rs. 4000 per quintal. Let us write the total income in rupees that was obtained by selling the entire farm produce, with the help of an expression using variables x and y.

$\boxed{} + \boxed{} + \boxed{}$ rupees

Let's learn.

Synthetic division

We know, how to divide one polynomial by other polynomial. Now we will learn an easy method for division of polynomials when divisor is of the form $x + a$ or $x - a$.

Ex (1) Divide the polynomial $(3x^3 + 2x^2 - 1)$ by $(x + 2)$.

Solution : Let us write the dividend polynomial in the coefficient form.

Index form of the dividend polynomial is $3x^3 + 2x^2 - 1 = 3x^3 + 2x^2 + 0\,x - 1$

$\therefore$ coefficient form of the given polynomial = (3, 2, 0, – 1)

Divisor polynomial = $x + 2$

Let us use the following steps for synthetic division.

(1) Draw one horizontal and one vertical line as shown alongside.

$$\begin{array}{c|ll} & \cdots\cdots\cdots\cdots & \text{First row} \\ & \cdots\cdots\cdots\cdots & \text{Second row} \\ \hline & \cdots\cdots\cdots\cdots & \text{Third row} \end{array}$$

(2) Divisor is $x + 2$. Hence take opposite number of 2 which is -2
Write -2 to the left of the vertical line as shown. Write the coefficient form of the dividend polynomial in the first row.

$$\begin{array}{r|rrrrl} -2 & 3 & 2 & 0 & -1 & \text{First row} \\ & \downarrow & & & & \\ \hline & 3 & & & & \text{Third row} \end{array}$$

(3) Write the first coefficient as it is in the third row.

(4) The product of 3 in the third row with divisor -2 is -6. Write this -6 in the second row below the coefficient 2. Addition of 2 and -6 which is -4, is to be written in the third row.

$$\begin{array}{r|rrrrl} -2 & 3 & 2 & 0 & -1 & \\ & \downarrow & -6 & 8 & \boxed{-16} & \\ \hline & 3 & -4 & 8 & -17 & \text{Remainder} \end{array}$$

Similarly by multiplying and adding, last addition is the remainder, which is (-17) and coefficient form of the Quotient is $(3, -4, 8)$.

$\therefore$ Quotient $= 3x^2 - 4x + 8$ and Remainder $= -17$

$\therefore$ $3x^3 + 2x^2 - 1 = (x + 2)(3x^2 - 4x + 8) - 17$

This method is called the **method of synthetic division**. The same division can be done by linear method of division as shown below.

$$\begin{aligned} 3x^3 + 2x^2 - 1 &= 3x^2(x + 2) - 6x^2 + 2x^2 - 1 \\ &= 3x^2(x + 2) - 4x^2 - 1 \\ &= 3x^2(x + 2) - 4x^2 - 8x + 8x - 1 \\ &= 3x^2(x + 2) - 4x(x + 2) + 8x - 1 \\ &= 3x^2(x + 2) - 4x(x + 2) + 8x + 16 - 16 - 1 \\ &= 3x^2(x + 2) - 4x(x + 2) + 8(x + 2) - 17 \end{aligned}$$

$\therefore$ $3x^3 + 2x^2 - 1 = (x + 2)(3x^2 - 4x + 8) - 17$

Ex (2) Divide $(2y^4 - 3y^3 + 5y - 4) \div (y - 1)$

Solution : Synthetic division : Dividend $= 2y^4 - 3y^3 + 5y - 4 = 2y^4 - 3y^3 + 0y^2 + 5y - 4$

Divisor $= y - 1$ Opposite of -1 is 1.

$$\begin{array}{c|ccccc}
1 & 2 & -3 & 0 & 5 & -4 \\
 & & 2 & -1 & -1 & 4 \\
\hline
 & 2 & -1 & -1 & 4 & \boxed{0}
\end{array} \text{ Remainder}$$

Coefficient form of the quotient is $(2, -1, -1, 4)$.

$\therefore$ Quotient $= 2y^3 - y^2 - y + 4$ and Remainder $= 0$

Linear method : $2y^4 - 3y^3 + 5y - 4 = 2y^3(y - 1) + 2y^3 - 3y^3 + 5y - 4$

$$= 2y^3(y - 1) - y^2(y - 1) - y^2 + 5y - 4$$

$$= 2y^3(y - 1) - y^2(y - 1) - y(y - 1) + 4y - 4$$

$$= (2y^3 - y^2 - y + 4)(y - 1)$$

Remember this !

In the division by synthetic method the divisor polynomial is in the form $x + a$ or $x - a$ whose degree is 1.

Practice set 3.3

1. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

(i) $(2m^2 - 3m + 10) \div (m - 5)$ (ii) $(x^4 + 2x^3 + 3x^2 + 4x + 5) \div (x + 2)$

(iii) $(y^3 - 216) \div (y - 6)$ (iv) $(2x^4 + 3x^3 + 4x - 2x^2) \div (x + 3)$

(v) $(x^4 - 3x^2 - 8) \div (x + 4)$ (vi) $(y^3 - 3y^2 + 5y - 1) \div (y - 1)$

Let's learn.

Value of a polynomial

In a polynomial if variable is replaced by a number then we get the value of that polynomial. For example if we replace x by 2 in the polynomial $x + 7$ we get $2 + 7 = 9$ which is the value of that polynomial.

If $p(x)$ is a polynomial in x then the value of the polynomial for $x = a$ is written as $p(a)$.

Ex (1) Find the value of the polynomial $p(x) = 2x^2 - 3x + 5$ for $x = 2$.

Solution : Polynomial $p(x) = 2x^2 - 3x + 5$

Put $x = 2$ in the given polynomial,

$$\therefore\ p(2) = 2 \times 2^2 - 3 \times 2 + 5$$
$$= 2 \times 4 - 6 + 5$$
$$= 8 - 6 + 5$$
$$\therefore\ p(2) = 7$$

Ex (2) Find the value of $p(y) = 2y^3 - 2y + \sqrt{7}$ for $y = -2$

Solution : $p(y) = 2y^3 - 2y + \sqrt{7}$

$$\therefore\ p(-2) = 2 \times (-2)^3 - 2 \times (-2) + \sqrt{7}$$
$$= 2 \times (-8) - 2 \times (-2) + \sqrt{7}$$
$$= -16 + 4 + \sqrt{7}$$
$$= -12 + \sqrt{7}$$

$\therefore$ For $y = -2$ the value of polynomial is $-12 + \sqrt{7}$.

Ex (3) If $p(x) = 2x^2 - x^3 + x + 2$ then find $p(0)$.

Solution : $p(x) = 2x^2 - x^3 + x + 2$

$$\therefore\ p(0) = 2 \times 0^2 - 0^3 + 0 + 2$$
$$= 2 \times 0 - 0 + 0 + 2$$
$$= 2$$

Ex (4) If the value of the polynomial $m^2 - am + 7$ for $m = -1$ is 10, then find the value of a.

Solution : $p(m) = m^2 - am + 7$

$$\therefore\ p(-1) = (-1)^2 - a \times (-1) + 7$$
$$= 1 + a + 7$$
$$= 8 + a$$

But $p(-1) = 10$ (given.)

$$\therefore\ 8 + a = 10$$
$$\therefore\ a = 10 - 8$$
$$\therefore\ a = 2$$

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Practice set 3.4

(1) For $x = 0$ find the value of the polynomial $x^2 - 5x + 5$.

(2) If $p(y) = y^2 - 3\sqrt{2}y + 1$ then find $p(3\sqrt{2})$.

(3) If $p(m) = m^3 + 2m^2 - m + 10$ then $p(a) + p(-a) = ?$

(4) If $p(y) = 2y^3 - 6y^2 - 5y + 7$ then find $p(2)$.

Remember this !

To find the value of a polynomial for a given value of the variable put the value in place of the variable in each term of the polynomial.

Let's learn.

Remainder Theorem

There is a relation between the value of $p(x)$ for $x = -(a \times 1)$ that is $p(-a)$, and the remainder when $p(x)$ is divided by $(x + a)$.

To understand this relation let's learn the following example .

Ex. Divide $p(x) = (4x^2 - x + 2)$ by $(x + 1)$

[Note that here $(x + a)$ is $(x + 1)$]

Solution :

Dividend polynomial = $4x^2 - x + 2$

Divisor polynomial = $x + 1$

$$
\begin{array}{rl}
 & 4x - 5 \quad \text{Division} \\
\text{Divisor } x + 1 \big) & 4x^2 - x + 2 \quad \text{Dividend} \\
- & \underline{\overset{-}{4x^2} \overset{-}{+ 4x}} \\
 & -5x + 2 \\
- & \underline{\overset{+}{-5x} \overset{+}{-} 5} \\
 & 7 \quad \text{Remainder}
\end{array}
$$

Quotient = $4x - 5$ Remainder = 7 .... (I)

Let's divide by synthetic method.

Coefficient form of $p(x)$ is $(4, -1, 2)$

Divisor polynomial = $x + 1$

Opposite of 1 is -1

$$
\begin{array}{r|rrr}
-1 & 4 & -1 & 2 \\
 & & -4 & 5 \\
\hline
 & 4 & -5 & \boxed{7}
\end{array}
\quad \text{Remainder}
$$

Quotient = $4x - 5$ Remainder = 7

Now we will find the relation between remainder and value of the polynomial as follows:

In the dividend polynomial $4x^2 - x + 2$ put $x = -1$.

$p(x) = 4x^2 - x + 2$

$\therefore\ p(-1) = 4 \times (-1)^2 - (-1) + 2$

$= 4 \times 1 + 1 + 2$

$= 4 + 1 + 2$

$= 7$

$\therefore$ value of the polynomial $p(x)$ for $x = -1$ is 7. ...... (II)

From the statement (I) and (II), the remainder when $p(x) = 4x^2 - x + 2$ is divided by $(x + a)$ that is $x + 1$ and the value of the polynomial $p(x)$ for $x = -1$, that is $p(-1)$, both are equal.

Hence we get the following property.

If the polynomial $p(x)$ is divided by $(x + a)$ then the **remainder** is $p(-a)$ means it is same as the **value of the polynomial** $p(x)$ for $x = -a$

This is known as the **Remainder theorem.**

Let's prove the theorem using Euclid's division lemma.

If $p(x)$ is divided by $(x + a)$

$p(x) = q(x) \times (x + a) + r(x)$ [$q(x)$ = Quotient, $r(x)$ = Remainder]

If, $r(x) \neq 0$, then by rule the degree of the polynomial $r(x)$ is less than 1 means 0. Therefore $r(x)$ is a real number.

$\therefore$ $r(-a)$ is also a real number.

Now, $p(x) = q(x) \times (x + a) + r(x)$ ....(I)

By putting $x = -a$ in (I) we get,

$$p(-a) = q(-a) \times (a - a) + r(-a)$$

$$= q(-a) \times 0 + r(-a) \text{.........(II)}$$

$$\therefore\ p(-a) = r(-a) \text{ ....from (I) and (II)}$$

Activity : Verify the following examples.

(1) Divide $p(x) = 3x^2 + x + 7$ by $x + 2$. Find the Remainder.

(2) Find the value of $p(x) = 3x^2 + x + 7$ when $x = -2$.

(3) See whether remainder obtained by division is same as the value of $p(-2)$. Take one more example and verify.

Ex (1) Divide $x^4 - 5x^2 - 4x$ by $x + 3$ and find the remainder.

Solution : **By Remainder Theorem**

Dividend polynomial $p(x) = x^4 - 5x^2 - 4x$

Divisor $= x + 3$

take $x = -3$.

$\therefore \quad p(x) = x^4 - 5x^2 - 4x$

$p(-3) = (-3)^4 - 5(-3)^2 - 4(-3)$

$= 81 - 45 + 12$

$p(-3) = 48$

By Synthetic Divison

Index form $x^4 + 0x^3 - 5x^2 - 4x + 0$

Coefficient form $= (1, 0, -5, -4, 0)$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -5 & -4 & 0 \\ & & -3 & 9 & -12 & 48 \\ \hline & 1 & -3 & 4 & -16 & \boxed{48} \end{array}$$

Remainder $= 48$

Ex (2) By using remainder theorem divide the polynomial $x^3 - 2x^2 - 4x - 1$ by $x - 1$ and find the remainder.

Solution : $p(x) = x^3 - 2x^2 - 4x - 1$

Divisor $= x - 1 \quad \therefore \quad$ take $x = 1$

$\therefore$ Remainder $= p(1) = 1^3 - 2 \times 1^2 - 4 \times 1 - 1$ ...(by remainder theorem)

$= 1 - 2 \times 1 - 4 - 1$

$p(1) = 1 - 2 - 4 - 1 = -6$

$\therefore$ Remainder $= -6$

Ex (3) If the polynomial $t^3 - 3t^2 + kt + 50$ is divided by $(t-3)$, the remainder is 62. Find the value of k.

Solution : When given polynomial is divided by $(t-3)$ the remainder is 62. It means the value of the polynomial when $t = 3$ is 62.

$p(t) = t^3 - 3t^2 + kt + 50$

By remainder theorem,

Remainder $= p(3) = 3^3 - 3 \times 3^2 + k \times 3 + 50$

$= 27 - 3 \times 9 + 3k + 50$

$= 27 - 27 + 3k + 50$

$= 3k + 50$

But remainder is 62.

$\therefore\ 3k + 50 = 62$

$\therefore\ 3k = 62 - 50$

$\therefore\ 3k = 12$

$\therefore\ k = \frac{12}{3}$

$\therefore\ k = 4$

Remember this !

If a polynomial $p(x)$ is divided by $(x + a)$ then the remainder is $p(-a)$ where 'a' is a real number.

$p(x) = s(x) \times (x - a) + r(x)$ where degree of $r(x) < 1$ or $r(x) = 0$
In this equation by putting $x = a$ we get, $p(a) = 0 + r(a) = r(a)$.

Hence if $r(a) = 0$ means $(x - a)$ is a factor of $p(x)$.

Let's learn.

Factor Theorem

If 21 is divided by 7 then remainder is 0, therefore we say that 7 is a factor of 21. In the same way when a given polynomial is divided by the divisor polynomial and if the remainder is 0 then we say that divisor polynomial is the factor of the dividend polynomial.

Ex (1) If $p(x) = (x^3 + 4x - 5)$ is divided by $(x - 1)$ then find the remainder and hence determine whether $(x - 1)$ is a factor of $p(x)$ or not ?

Solution : $p(x) = x^3 + 4x - 5$

$p(1) = (1)^3 + 4(1) - 5$

$= 1 + 4 - 5$

$= 0$

As per the remainder theorem,

Remainder $= 0$

$\therefore\ (x - 1)$ is a factor of $p(x)$.

Ex (2) If $p(x) = x^3 + 4x - 5$ is divided by $x + 2$ then find the remainder and hence determine whether $(x + 2)$ is a factor of $p(x)$ or not.

Solution : $p(x) = x^3 + 4x - 5$

$p(-2) = (-2)^3 + 4(-2) - 5$

$p(-2) = -8 - 8 - 5$

$= -21$

As per the remainder theorem,

Remainder $= -21$ $\therefore$ Remainder $\neq 0$

$\therefore\ (x + 2)$ is not a factor of $p(x)$.

Activity : Verify that $(x - 1)$ is a factor of the polynomial $x^3 + 4x - 5$.

Remember this !

$p(x)$ is a polynomial and a is any real number, and if $p(a) = 0$ then $(x - a)$ is the factor of $p(x)$.

Conversely if $(x - a)$ is the factor of the polynomial $p(x)$ then $p(a) = 0$

Ex (1) Check whether, $x - 2$ is a factor of the polynomial $x^3 - x^2 - 4$ by using factor theorem.

Solution : $p(x) = x^3 - x^2 - 4$ Divisor $= x - 2$

$\therefore\ p(2) = 2^3 - 2^2 - 4 = 8 - 4 - 4 = 0$

$\therefore$ By factor theorem $(x - 2)$ is a factor of the polynomial $(x^3 - x^2 - 4)$.

Ex (2) If $(x - 1)$ is the factor of the polynomial $(x^3 - 2x^2 + mx - 4)$ then find the value of m.

Solution : $(x - 1)$ is factor of $p(x)$. $\therefore\ p(1) = 0$

$p(x) = x^3 - 2x^2 + mx - 4$

$p(1) = 1^3 - 2 \times 1^2 + m \times 1 - 4 = 0$

$\therefore\ 1 - 2 \times 1 + m - 4 = 0$

$\therefore\ 1 - 2 + m - 4 = 0$ $\therefore\ m - 5 = 0$ $\therefore\ m = 5$

Activity : We have seen the example of expenditure and income (in terms of polynomials) of Govind who is a dry land farmer. He has borrowed rupees one lakh twenty, five thousand from the bank as an agriculture loan and repaid the said loan at 10 p.c.p.a. He had spent ₹ 10,000 on seeds. The expenses on soyabean crop was ₹ $2000x$ for fertilizers and pesticides and ₹ $4000x^2$ was spent on wages and cultivation. He spent ₹ $8000y$ on fertilizers and pesticides and ₹ $9000y^2$ on cultivation and wages for cotton and tur crop.

His total income was rupees $14000x^2 + \frac{25000}{3}y^2 + 16000y$.

By taking $x = 2$, $y = 3$ write the income-expenditure account of Govind's farming.

Solution :

Credit (Income)		**Debit (Expenses)**	
₹ 1,25,000	Bank loan	₹ 1,37,000	loan paid with interest for seeds
₹	Income from soyabean	₹	For seeds
₹	Income from cotton	₹	Fertilizers and pesticides for soyabean
₹	Income from tur	₹	Wages and cultivation charges for soyabean
₹	Total income	₹	Fertilizers and pesticides for cotton & tur
		₹	Wages and cultivation charges for cotton & tur
		₹	Total expenditure

Practice set 3.5

(1) Find the value of the polynomial $2x - 2x^3 + 7$ using given values for x.

(i) $x = 3$ (ii) $x = -1$ (iii) $x = 0$

(2) For each of the following polynomial, find $p(1)$, $p(0)$ and $p(-2)$.

(i) $p(x) = x^3$ (ii) $p(y) = y^2 - 2y + 5$ (iii) $p(x) = x^4 - 2x^2 - x$

(3) If the value of the polynomial $m^3 + 2m + a$ is 12 for $m = 2$, then find the value of a.

(4) For the polynomial $mx^2 - 2x + 3$ if $p(-1) = 7$ then find m.

(5) Divide the first polynomial by the second polynomial and find the remainder using factor theorem.

(i) $(x^2 - 7x + 9)$; $(x + 1)$ (ii) $(2x^3 - 2x^2 + ax - a)$; $(x - a)$

(iii) $(54m^3 + 18m^2 - 27m + 5)$; $(m - 3)$

(6) If the polynomial $y^3 - 5y^2 + 7y + m$ is divided by $y + 2$ and the remainder is 50 then find the value of m.

(7) Use factor theorem to determine whether $x + 3$ is factor of $x^2 + 2x - 3$ or not.

(8) If $(x - 2)$ is a factor of $x^3 - mx^2 + 10x - 20$ then find the value of m.

(9) By using factor theorem in the following examples, determine whether $q(x)$ is a factor $p(x)$ or not.

(i) $p(x) = x^3 - x^2 - x - 1$, $q(x) = x - 1$

(ii) $p(x) = 2x^3 - x^2 - 45$, $q(x) = x - 3$

(10) If $(x^{31} + 31)$ is divided by $(x + 1)$ then find the remainder.

(11) Show that $m - 1$ is a factor of $m^{21} - 1$ and $m^{22} - 1$.

(12*) If $x - 2$ and $x - \frac{1}{2}$ both are the factors of the polynomial $nx^2 - 5x + m$, then show that $m = n = 2$

(13) (i) If $p(x) = 2 + 5x$ then $p(2) + p(-2) - p(1)$.

(ii) If $p(x) = 2x^2 - 5\sqrt{3}x + 5$ then $p(5\sqrt{3})$.

Let's recall.

In previous classes we have learnt how to find the factors of the polynomials. Let's revise it with some examples.

Factorize.

Ex (1) $4x^2 - 25$

$= (2x)^2 - (5)^2$

$= (2x + 5)(2x - 5)$

Ex (2) $3x^2 + 7x + 2$

$= \underline{3x^2 + 6x} + \underline{x + 2}$

$= 3x(x + 2) + 1(x + 2)$

$= (x + 2)(3x + 1)$

Ex (3) $63x^2 + 5x - 2$

$= 63x^2 + 14x - 9x - 2$

$= 7x(9x + 2) - 1(9x + 2)$

$= (9x + 2)(7x - 1)$

Ex (4) $6x^2 - 5x - 6$

$= 6x^2 - 9x + 4x - 6$

$= 3x(2x - 3) + 2(2x - 3)$

$= (2x - 3)(3x + 2)$

Let's learn.

Factors of polynomials

Sometimes polynomial can be written in the form $ax^2 + bx + c$ and hence it is easy to find its factors.

Ex (1) Factorise : $(y^2-3y)^2 - 5(y^2-3y) - 50$.

Solution : Let $(y^2-3y) = x$

$\therefore\ (y^2-3y)^2 - 5(y^2-3y) - 50 = x^2 - 5x - 50$

$= x^2 - 10x + 5x - 50$

$= x(x - 10) + 5(x - 10)$

$= (x - 10)(x + 5)$

$= (y^2-3y - 10)(y^2-3y + 5)$

$= [y^2-5y + 2y - 10](y^2-3y + 5)$

$= [y(y - 5) + 2(y - 5)](y^2-3y + 5)$

$= (y - 5)(y + 2)(y^2-3y + 5)$

Ex (2) Factorise.

$(x + 2)(x - 3)(x - 7)(x - 2) + 64$

Solution : $(x + 2)(x - 3)(x - 7)(x - 2) + 64$

$= (x + 2)(x - 7)(x - 3)(x - 2) + 64$

$= (x^2 - 5x - 14)(x^2 - 5x + 6) + 64$

$= (m - 14)(m + 6) + 64$ . . . . (putting $x^2 - 5x = m$)

$= m^2 - 14m + 6m - 84 + 64$

$= m^2 - 8m - 20$

$= (m - 10)(m + 2)$

$= (x^2 - 5x - 10)(x^2 - 5x + 2)$ .... (replace m with $x^2 - 5x$)

Practice set 3.6

(1) Find the factors of the polynomials given below.

(i) $2x^2 + x - 1$ (ii) $2m^2 + 5m - 3$ (iii) $12x^2 + 61x + 77$

(iv) $3y^2 - 2y - 1$ (v) $\sqrt{3}x^2 + 4x + \sqrt{3}$ (vi) $\frac{1}{2}x^2 - 3x + 4$

(2) Factorize the following polynomials.

(i) $(x^2 - x)^2 - 8(x^2 - x) + 12$

(ii) $(x - 5)^2 - (5x - 25) - 24$

(iii) $(x^2 - 6x)^2 - 8(x^2 - 6x + 8) - 64$

(iv) $(x^2 - 2x + 3)(x^2 - 2x + 5) - 35$

(v) $(y + 2)(y - 3)(y + 8)(y + 3) + 56$

(vi) $(y^2 + 5y)(y^2 + 5y - 2) - 24$

(vii) $(x - 3)(x - 4)^2 (x - 5) - 6$

Problem set 3

(1) Write the correct alternative answer for each of the following questions.

(i) Which of the following is a polynomial ?

(A) $\frac{x}{y}$ (B) $x^{\sqrt{2}} - 3x$ (C) $x^{-2} + 7$ (D) $\sqrt{2}x^2 + \frac{1}{2}$

(ii) What is the degree of the polynomial $\sqrt{7}$?

(A) $\frac{1}{2}$ (B) 5 (C) 2 (D) 0

(iii) What is the degree of the 0 polynomial ?

(A) 0 (B) 1 (C) undefined (D) any real number

(iv) What is the degree of the polynomial $2x^2 + 5x^3 + 7$?

(A) 3 (B) 2 (C) 5 (D) 7

(v) What is the coefficient form of $x^3 - 1$?

(A) (1, – 1) (B) (3, – 1) (C) (1, 0, 0, – 1) (D) (1, 3, – 1)

(vi) $p(x) = x^2 - 7\sqrt{7}x + 3$ then $p(7\sqrt{7}) = $?

(A) 3 (B) $7\sqrt{7}$ (C) $42\sqrt{7} + 3$ (D) $49\sqrt{7}$

(vii) When $x = -1$, what is the value of the polynomial $2x^3 + 2x$?

(A) 4 (B) 2 (C) – 2 (D) – 4

(viii) If $x-1$, what is a factor of the polynomial $3x^2 + mx$ then find the value of m.

(A) 2 (B) – 2 (C) – 3 (D) 3

(ix) Multiply $(x^2 - 3)(2x - 7x^3 + 4)$ and write the degree of the product.

(A) 5 (B) 3 (C) 2 (D) 0

(x) Which of the following is a linear polynomial ?

(A) $x + 5$ (B) $x^2 + 5$ (C) $x^3 + 5$ (D) $x^4 + 5$

(2) Write the degree of the polynomial for each of the following.

(i) $5 + 3x^4$ (ii) 7 (iii) $ax^7 + bx^9$ (a, b are constants.)

(3) Write the following polynomials in standard form.

(i) $4x^2 + 7x^4 - x^3 - x + 9$ (ii) $p + 2p^3 + 10p^2 + 5p^4 - 8$

(4) Write the following polynomial in coefficient form.

(i) $x^4 + 16$ (ii) $m^5 + 2m^2 + 3m + 15$

(5) Write the index form of the polynomial using variable x from its coefficient form.

(i) (3, −2, 0, 7, 18) (ii) (6, 1, 0, 7) (iii) (4, 5, −3, 0)

(6) Add the following polynomials.

(i) $7x^4 - 2x^3 + x + 10$; $3x^4 + 15x^3 + 9x^2 - 8x + 2$

(ii) $3p^3q + 2p^2q + 7$; $2p^2q + 4pq - 2p^3q$

(7) Subtract the second polynomial from the first.

(i) $5x^2 - 2y + 9$; $3x^2 + 5y - 7$ (ii) $2x^2 + 3x + 5$; $x^2 - 2x + 3$

(8) Multiply the following polynomials.

(i) $(m^3 - 2m + 3)(m^4 - 2m^2 + 3m + 2)$ (ii) $(5m^3 - 2)(m^2 - m + 3)$

(9) Divide polynomial $3x^3 - 8x^2 + x + 7$ by $x - 3$ using synthetic method and write the quotient and remainder.

(10) For which the value of m, $x + 3$ is the factor of the polynomial $x^3 - 2mx + 21$?

(11) At the end of the year 2016, the population of villages Kovad, Varud, Chikhali is $5x^2 - 3y^2$, $7y^2 + 2xy$ and $9x^2 + 4xy$ respectively. At the beginning of the year 2017, $x^2 + xy - y^2$, $5xy$ and $3x^2 + xy$ persons from each of the three villages respectively went to another village for education then what is the remaining total population of these three villages ?

(12) Polynomials $bx^2 + x + 5$ and $bx^3 - 2x + 5$ are divided by polynomial $x - 3$ and the remainders are m and n respectively. If $m - n = 0$ then find the value of b.

(13) Simplify. $(8m^2 + 3m - 6) - (9m - 7) + (3m^2 - 2m + 4)$

(14) Which polynomial is to be subtracted from $x^2 + 13x + 7$ to get the polynomial $3x^2 + 5x - 4$?

(15) Which polynomial is to be added to $4m + 2n + 3$ to get the polynomial $6m + 3n + 10$?

4 Ratio and Proportion

Let's study.

- **Ratio**
- **Operations on equal ratios**
- **Continued proportion**
- **Properties of ratios**
- **Theorem of equal ratios**
- ***k* method**

Let's recall.

In earlier standards, we have learnt about ratio and proportion. We have also solved examples based on it. Let us discuss following example.

Ex. The rawa ladoo prepared by Vimal are tasty, for which she takes 1 bowl of ghee, 3 bowls of rawa and 2 bowls of sugar.

Here proportion of rawa and sugar is 3 : 2 or $\frac{3}{2}$.

If 12 units of rawa is used, how many units of sugar are required ?

Let the number of bowls of sugar required be x.

$\therefore$ from above information, $\frac{3}{2} = \frac{12}{x}$ $\therefore$ $3x = 24$ $\therefore$ $x = 8$

That is for preparation of ladoo, with 12 units of rawa requires 8 units of sugar. Alternatively we can solve the above example in the following way.

$3k$ bowls of rawa, $2k$ bowls of sugar is required because $\frac{3k}{2k} = \frac{3}{2}$

If $3k = 12$ then $k = 4$ $\therefore$ $2k = 2 \times 4 = 8$ bowls of sugar is required.

Let's learn.

Ratio and proportion

The concept of ratio of two numbers can be extended to three or more numbers.

Let us see the above example of ladoos. The proportion of ghee, rawa and sugar is 1 : 3 : 2.

Here proportion of ghee and rawa is 1 : 3 and that of rawa and sugar is 3 : 2, This means the proportion of ghee, rawa and sugar is 1 : 3 : 2.

Let us take k bowls of ghee, $3k$ bowls of rawa and $2k$ bowls of sugar.

Hence for 12 bowls of rawa, how much quantity of ghee and sugar is required can be found as follows.

Now $3k = 12$ $\therefore$ $k = 4$ and $2k = 8$.

$\therefore$ 4 bowls of ghee and 8 bowls of sugar is required.

The same concept can be extended for proportion of 4 or more entities.

If a, b, c, d are in the ratio 2 : 3 : 7 : 4 then let us assume that the numbers are $2m$, $3m$, $7m$, $4m$. From the given information, value of m can be determined. For example if the sum of these four numbers is 48, we find these numbers.

$2m + 3m + 7m + 4m = 16\,m = 48$

$\therefore \quad m = 3$

$\therefore \quad 2m = 6, \;\; 3m = 9, \;\; 7m = 21, \;\; 4m = 12$

$\therefore$ required numbers are 6, 9, 21, 12

Ex (1) The proportion of compounds of nitrogen, phosphorous and potassium in certain fertilizer is 18 : 18 : 10. Here compound of nitrogen is 18%, compound of phosphorous is 18% and that of potassium is 10%. Remaining part is of other substances. Find the weight of each of the above compounds in 20 kg of fertilizer.

Solution : Let the weight of nitrogen compound in 20 kg of fertilizer be x kg.

$\therefore \quad \frac{18}{100} = \frac{x}{20} \qquad\qquad \therefore \quad x = \frac{18 \times 20}{100} = 3.6$

$\therefore$ weight of nitrogen compound is 3.6 kg

The percentage of phosphorous compound is also 18%.

$\therefore$ Weight of compound of phosphorous is 3.6 kg

If we assume the weight of potassium compound y kg then

$\frac{10}{100} = \frac{y}{20} \quad \therefore y = 2 \qquad\qquad \therefore$ weight of potassium compound is 2 kg.

Direct proportion

A car covers a distance of 10 km consuming 1 litre of petrol.

It will cover a distance of $20 \times 10 = 200$ km consuming 20 litre of petrol .

Consuming 40 litre of petrol, it will cover a distance of $40 \times 10 = 400$ km.

Let us write this information in tabular form

Petrol : x litre	1	20	40	
Distance : y km	10	200	400	
$\frac{x}{y}$	$\frac{1}{10}$	$\frac{20}{200} = \frac{1}{10}$	$\frac{40}{400} = \frac{1}{10}$	$\frac{x}{y} = \text{k}$

The ratio of consumption of petrol (in litre) and distance covered by the car (in kilometres), is constant. In such case, it is said that the two quantities are in direct proportion or in direct variation.

Inverse proportion

A car takes two hours to cover a distance of 100 km at the speed of 50 km/hr. A bullock-cart travels 5 km in 1 hour. To cover a distance of 100 km at the speed of 5 km/hr, the bullock-cart takes 20 hours.

We know that, **Speed $\times$ time = distance**

By using the relation let us put the above information in a tabular form.

Vehicle	Speed/hr (x)	Time (y)	$x \times y$	$x \times y = \text{k}$
Car	50	2	100	
Bullock-cart	5	20	100	

Hence, we see that, the product of speed of the vehicle and time is constant. In such a case it is said that the quantities are in inverse proportion or in inverse variation.

Let's recall.

Properties of ratio

(1) Ratio of numbers a and b is written as $a : b$ or $\frac{a}{b}$. a is called the predecessor (first term) and b is called successor (Second term).

(2) In the ratio of two numbers, if the second term is 100 then it is known as a percentage.

(3) The ratio remains unchanged, if its terms are multiplied or divided by non-zero number.
e.g.. 3 : 4 = 6:8 = 9:12, Similarly 2:3:5 = 8:12:20. If k is a non-zero number,
then $a : b = ak : bk$ $\qquad$ $a : b : c = ak : bk : ck$

(4) The quantities taken in the ratio must be expressed in the same unit.

(5) The ratio of two quantities is unitless.
For example The ratio of 2 kg and 300 g is not 2 : 300 ,
but it is 2000 : 300 as (2 kg = 2000 gm) i.e. 20 : 3

Ex (1) The ratio of ages of Seema and Rajashree is 3 : 1. The ratio of ages of Rajashree and Atul is 2 : 3. Then find the ratio of ages of Seema, Rajashree and Atul.

Solution : Seema's age : Rajashree's age = 3 : 1 Rajashree's age : Atul's age = 2 : 3

Second term of first ratio should be the same as the first term of second ratio.

Hence to get the continuous ratio, multiplying each term of the first ratio by 2. We get

3:1 = 6:2 .

$$\frac{\text{Seema's age}}{\text{Rajashree's age}} = \frac{6}{2}, \quad \frac{\text{Rajashree's age}}{\text{Atul's age}} = \frac{2}{3}$$

$\therefore$ Seema's age : Rajashree's age : Atul's age 6 : 2 : 3.

Ex (2) The length of a rectangular field is 1.2 km and its breadth is 400 metre. Find the ratio of length to breadth.

Solution : Here the length is in kilometer and breadth is in meter. In order to find the ratio of length to breadth, they must be expressed in same unit. Hence we convert kilometre to meter.

1.2 km= 1.2 ×1000 = 1200 m

$\therefore$ ratio of 1200 m , to 400 m is $\frac{1200}{400} = \frac{3}{1}$, that is 3 : 1

Ex (3) The ratio of expenditure and income of Mahesh is 3 : 5. Find the percentage of expenses to his income.

Solution : The ratio of expenditure to income is 3 : 5. To convert it into percentage, convert second term into 100.

$\frac{3}{5} = \frac{3\times20}{5\times20} = \frac{60}{100}$ $\therefore$ $\frac{\text{Expenditure}}{\text{Income}} = \frac{60}{100} = 60\%$ $\therefore$ Mahesh spends 60% of his income.

Ex (4) The ratio of number of mango trees to chikoo trees in an orchard is 2 : 3. If 5 more trees of each type are planted the ratio of trees would be 5 : 7. Then find the number of mango and chickoo trees in the orchard.

Solution : The ratio of trees is 2 : 3.

Let the number of mango trees = $2x$ and chikoo trees = $3x$

From given condition, $\frac{2x+5}{3x+5} = \frac{5}{7}$

$14x + 35 = 15x + 25$

$\therefore$ $x = 10$

$\therefore$ number of mango trees in the orchard = $2x$ = 2 ×10= 20

and number of chikoo trees = $3x$ = 3 ×10= 30

Ex (5) The ratio of two numbers is 5 : 7. If 40 is added in each number, then the ratio becomes 25 : 31, Find the numbers.

Solution : Let the first number be $5x$ and and second number be $7x$.

From the given condition, $\frac{5x+40}{7x+40}=\frac{25}{31}$

$$31(5x+40)=25(7x+40)$$

$$155x+1240=175x+1000$$

$$1240-1000 = 175x-155x$$

$$240=20x$$

$$x=12$$

$\therefore$ first number = 5 × 12 = 60 and second number = 7 × 12 = 84

$\therefore$ given numbers are 60 and 84.

Practice set 4.1

(1) From the following pairs of numbers, find the reduced form of ratio of first number to second number.

(i) 72, 60 (ii) 38,57 (iii) 52,78

(2) Find the reduced form of the ratio of the first quantity to second quantity.

(i) 700 ₹, 308 ₹ (ii) 14 ₹, 12 ₹. 40 paise.

(iii) 5 litre, 2500 ml (iv) 3 years 4 months, 5 years 8 months

(v) 3.8 kg, 1900 gm (vi) 7 minutes 20 seconds, 5 minutes 6 seconds.

(3) Express the following percentages as ratios in the reduced form.

(i) 75 : 100 (ii) 44 : 100 (iii) 6.25% (iv) 52 : 100 (v) 0.64%

(4) Three persons can build a small house in 8 days. To build the same house in 6 days, how many persons are required?

(5) Convert the following ratios into percentage.

(i) 15 : 25 (ii) 47 : 50 (iii) $\frac{7}{10}$ (iv) $\frac{546}{600}$ (v) $\frac{7}{16}$

(6) The ratio of ages of Abha and her mother is 2 : 5. At the time of Abha's birth her mothers age was 27 year. Find the present ages of Abha and her mother.

(7) Present ages of Vatsala and Sara are 14 years and 10 years respectively. After how many years the ratio of their ages will become 5 : 4?

(8) The ratio of present ages of Rehana and her mother is 2 : 7. After 2 years, the ratio of their ages will be 1 : 3. What is Rehana's present age ?

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Let's learn.

Comparison of ratios

The numbers a, b, c, d being positive, comparison of ratios $\frac{a}{b}$, $\frac{c}{d}$ can be done using following rules :

(i) If $ad > bc$ then $\frac{a}{b} > \frac{c}{d}$ (ii) If $ad < bc$ then $\frac{a}{b} < \frac{c}{d}$

(iii) If $ad = bc$ then $\frac{a}{b} = \frac{c}{d}$

Compare the following pairs of ratios

Ex (1) $\frac{4}{9}, \frac{7}{8}$

Solution : 4×8 $\boxed{?}$ 7×9

$32 < 63$

$\therefore \quad \frac{4}{9} < \frac{7}{8}$

Ex (2) $\frac{\sqrt{13}}{\sqrt{8}}, \frac{\sqrt{7}}{\sqrt{5}}$

$\sqrt{13} \times \sqrt{5},$ $\boxed{?}$ $\sqrt{8} \times \sqrt{7}$

$\sqrt{65}$ $\boxed{?}$ $\sqrt{56}$

$\sqrt{65} > \sqrt{56}$

$\therefore \quad \frac{\sqrt{13}}{\sqrt{8}} > \frac{\sqrt{7}}{\sqrt{5}}$

Ex (3) If a and b are integers and $a < b$, $b \neq \pm 1$ then compare $\frac{a-1}{b-1}, \frac{a+1}{b+1}$.

Solution : $a < b \quad \therefore a - 1 < b - 1$

Now consider the subtraction $\frac{a-1}{b-1} - \frac{a+1}{b+1}$

$$\frac{a-1}{b-1} - \frac{a+1}{b+1} = \frac{(a-1)(b+1)-(a+1)(b-1)}{(b-1)(b+1)}$$

$$= \frac{(ab-b+a-1)-(ab+b-a-1)}{b^2-1}$$

$$= \frac{ab-b+a-1-ab-b+a+1}{b^2-1}$$

$$= \frac{2a-2b}{b^2-1}$$

$$= \frac{2(a-b)}{b^2-1} \quad \ldots\ldots\ldots. (1)$$

Now $a < b \quad \therefore a - b < 0$

also $b^2 - 1 > 0$ because $b \neq \pm 1$

$\frac{2(a-b)}{b^2-1} < 0 \ldots\ldots\ldots. (2)$

$\frac{a-1}{b-1} - \frac{a+1}{b+1} < 0$.....from (1) & (2)

$\frac{a-1}{b-1} < \frac{a+1}{b+1}$

Ex (4) If $a : b = 2 : 1$ and $b : c = 4 : 1$ then find the value of $\left(\dfrac{a^4}{32b^2c^2}\right)^3$.

Solution : $\dfrac{a}{b} = \dfrac{2}{1}$ $\quad \therefore\ a = 2\,b$ $\qquad \dfrac{b}{c} = \dfrac{4}{1}$ $\quad \therefore\ b = 4c$

$a = 2\,b = 2 \times 4c = 8c$ $\quad \therefore\ a = 8\,c$

Now substituting the values $a = 8\,c,\ \ b = 4c$

$$\left(\frac{a^4}{32b^2c^2}\right)^3 = \left(\frac{(8c)^4}{32\times 4^2\times c^2\times c^2}\right)^3$$

$$= \left[\frac{8\times 8\times 8\times 8\times c^4}{32\times 16\times c^2\times c^2}\right]^3$$

$$= (8)^3$$

$$\therefore \left(\frac{a^4}{32b^2c^2}\right)^3 = 512$$

Practice set 4.2

(1) Using the property $\dfrac{a}{b} = \dfrac{ak}{bk}$, fill in the blanks substituting proper numbers in the following.

(i) $\dfrac{5}{7} = \dfrac{....}{28} = \dfrac{35}{....} = \dfrac{....}{3.5}$ $\qquad$ (ii) $\dfrac{9}{14} = \dfrac{4.5}{....} = \dfrac{....}{42} = \dfrac{....}{3.5}$

(2) Find the following ratios.

(i) The ratio of radius to circumference of the circle.

(ii) The ratio of circumference of circle with radius r to its area.

(iii) The ratio of diagonal of a square to its side, if the length of side is 7 cm.

(iv) The lengths of sides of a rectangle are 5 cm and 3.5 cm. Find the ratio of its perimeter to area.

(3) Compare the following pairs of ratios.

(i) $\dfrac{\sqrt{5}}{3}, \dfrac{3}{\sqrt{7}}$ $\qquad$ (ii) $\dfrac{3\sqrt{5}}{5\sqrt{7}}, \dfrac{\sqrt{63}}{\sqrt{125}}$ $\qquad$ (iii) $\dfrac{5}{18}, \dfrac{17}{121}$

(iv) $\dfrac{\sqrt{80}}{\sqrt{48}}, \dfrac{\sqrt{45}}{\sqrt{27}}$ $\qquad$ (v) $\dfrac{9.2}{5.1}, \dfrac{3.4}{7.1}$

(4) (i) □ABCD is a parallelogram. The ratio of $\angle$A and $\angle$B of this parallelogram is 5 : 4. Find the measure of $\angle$ B.

(ii) The ratio of present ages of Albert and Salim is 5 : 9. Five years hence ratio of their ages will be 3 : 5. Find their present ages.

(iii) The ratio of length and breadth of a rectangle is 3 : 1, and its perimeter is 36 cm. Find the length and breadth of the rectangle.

(iv) The ratio of two numbers is 31 : 23 and their sum is 216. Find these numbers.

(v) If the product of two numbers is 360 and their ratio is 10 : 9, then find the numbers.

(5*) If $a : b = 3 : 1$ and $b : c = 5 : 1$ then find the value of (i) $\left(\frac{a^3}{15b^2c}\right)^3$ (ii) $\frac{a^2}{7bc}$

(6*) If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$ then find the ratio $\frac{a}{b}$.

(7) $(x + 3) : (x + 11) = (x - 2) : (x + 1)$ then find the value of x.

Let's learn.

Operations on equal ratios

Using the properties of equality, we can perform some operations on ratios. Let's study them.

Let us learn some properties of the equal ratios, if a, b, c, d, are positive integers.

(I) Invertendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

$$\frac{a}{b} = \frac{c}{d}$$

$$\therefore \quad a \times d = b \times c$$

$$\therefore \quad b \times c = a \times d$$

$$\therefore \quad \frac{b \times c}{a \times c} = \frac{a \times d}{a \times c} \quad \text{...(dividing both sides by } a \times c \text{)}$$

$$\frac{b}{a} = \frac{d}{c}$$

$\therefore$ If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$. **This property is known as Invertendo**

(II) Alternando : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$

$$\frac{a}{b} = \frac{c}{d}$$

$$\therefore \quad a \times d = b \times c$$

$$\frac{a \times d}{c \times d} = \frac{b \times c}{c \times d} \quad \text{...(dividing both sides by } c \times d\text{)}$$

$$\frac{a}{c} = \frac{b}{d}$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$. **This property is known as Alternando.**

(III) Componendo : If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$

$$\frac{a}{b}=\frac{c}{d}$$

$$\frac{a}{b}+1=\frac{c}{d}+1 \quad \text{...(adding 1 to both sides)}$$

$$\frac{a+b}{b}=\frac{c+d}{d}$$

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$. **This property is known as Componendo.**

(IV) Dividendo : If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$

$$\therefore \quad \frac{a}{b}=\frac{c}{d}$$

$$\therefore \quad \frac{a}{b}-1=\frac{c}{d}-1 \quad \text{...(subtracting 1 from both sides)}$$

$$\therefore \quad \frac{a-b}{b}=\frac{c-d}{d}$$

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$. **This property is known as Dividendo**

(V) Componendo-Dividendo : If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$, $a \neq b, c \neq d$

जर $\frac{a}{b}=\frac{c}{d}$

$$\therefore \quad \frac{a+b}{b}=\frac{c+d}{d} \quad \text{...(using componendo) ....(1)}$$

$$\therefore \quad \frac{a-b}{b}=\frac{c-d}{d} \quad \text{...(using dividendo) ....(2)}$$

$$\therefore \quad \frac{a+b}{a-b}=\frac{c+d}{c-d} \quad \text{...from (1) and (2)}$$

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$. **This property is known as Componendo-dividendo.**

General form of Componendo and Dividendo

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$ ...(performing componendo once)

$$\frac{a+2b}{b}=\frac{c+2d}{d} \quad \text{...(performing componendo twice)}$$

Generally $\frac{a+mb}{b}=\frac{c+md}{d}$ ... (performing componendo m times) ...(I)

Similarly if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-mb}{b}=\frac{c-md}{d}$ ...(performing dividendo m time) ...(II)

and if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+mb}{a-mb}=\frac{c+md}{c-md}$ ...[dividing (I) by (II)]

Remember this !

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{b}{a}=\frac{d}{c}$ (Invertendo)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{c}=\frac{b}{d}$ (Alternando)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$ (Componendo)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$ (Dividendo)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Componendo-Dividendo)

Solved Examples :

Ex (1) If $\frac{a}{b}=\frac{5}{3}$ then find the ratio $\frac{a+7b}{7b}=$.

Method I

Solution : If $\frac{a}{b}=\frac{5}{3}$ then $\frac{a}{5}=\frac{b}{3}=k$,

...(using alternando)

$\therefore\ a = 5k,\ b = 3k$

$$\therefore \frac{a+7b}{7b}=\frac{5k+7\times 3k}{7\times 3k}$$

$$=\frac{5k+21k}{21k}$$

$$=\frac{26k}{21k} \quad =\frac{26}{21}$$

Method II

$$\frac{a}{b}=\frac{5}{3}$$

$$\therefore \frac{a}{7b}=\frac{5}{21}$$

$$\therefore \frac{a+7b}{7b}=\frac{5+21}{21} \quad \text{...(using componendo)}$$

$$\therefore \frac{a+7b}{7b}=\frac{26}{21}$$

Ex. (2) If $\frac{a}{b}=\frac{7}{4}$ then find the ratio $\frac{5a-b}{b}$.

Method I

Solution : $\frac{a}{b}=\frac{7}{4}$

$\therefore \frac{a}{7}=\frac{b}{4}$ ...(using alternando)

Let $\frac{a}{7}=\frac{b}{4}=m$

$\therefore\ a = 7m,\quad b = 4m$

$$\therefore \frac{5a-b}{b}=\frac{5(7m)-4m}{4m}$$

$$=\frac{35m-4m}{4m}$$

$$=\frac{31}{4}$$

Method II

$$\frac{a}{b}=\frac{7}{4}$$

$$\frac{5a}{b}=\frac{5\times 7}{4}$$

$$=\frac{35}{4}$$

$$\frac{5a-b}{b}=\frac{35-4}{4} \quad \text{...(using dividendo)}$$

$$\frac{5a-b}{b}=\frac{31}{4}$$

Ex. (3) If $\frac{a}{b} = \frac{7}{3}$ then find the value of the ratio $\frac{a+2b}{a-2b}$.

Solution : Method I :

Let $a = 7m$, $b = 3m$

$$\therefore \frac{a+2b}{a-2b} = \frac{7m+2\times 3m}{7m-2\times 3m}$$

$$= \frac{7m+6m}{7m-6m}$$

$$= \frac{13m}{m} = \frac{13}{1}$$

Method II : $\therefore \frac{a}{b} = \frac{7}{3}$

$$\therefore \frac{a}{2b} = \frac{7}{6} \quad \text{...(multiplying both sides by } \frac{1}{2}\text{)}$$

$$\therefore \frac{a+2b}{a-2b} = \frac{7+6}{7-6} \quad \text{(using componendo - dividendo)}$$

$$\therefore \frac{a+2b}{a-2b} = \frac{13}{1}$$

Ex (4) If $\frac{a}{3} = \frac{b}{2}$ then find the value of the ratio $\frac{5a+3b}{7a-2b}$.

Solution : **Method I**

$$\frac{a}{3} = \frac{b}{2}$$

$$\therefore \frac{a}{b} = \frac{3}{2} \text{........ (using Alternando)}$$

Now dividing each term of $\frac{5a+3b}{7a-2b}$ by b.

$$\frac{\frac{5a}{b}+\frac{3b}{b}}{\frac{7a}{b}-\frac{2b}{b}} = \frac{5\left(\frac{a}{b}\right)+3}{7\left(\frac{a}{b}\right)-2}$$

$$= \frac{5\left(\frac{3}{2}\right)+3}{7\left(\frac{3}{2}\right)-2}$$

$$= \frac{\frac{15}{2}+3}{\frac{21}{2}-2}$$

$$= \frac{15+6}{21-4}$$

$$= \frac{21}{17}$$

Method II

$$\frac{a}{3} = \frac{b}{2}$$

Let $\frac{a}{3} = \frac{b}{2} = t$.

$\therefore$ by substituting $a = 3t$ and $b = 2t$,

$$\frac{5a+3b}{7a-2b} = \frac{5(3t)+3(2t)}{7(3t)-2(2t)} \quad (t \neq 0)$$

$$= \frac{15t+6t}{21t-4t}$$

$$= \frac{21t}{17t}$$

$$= \frac{21}{17}$$

Ex (5) If $\frac{x}{y}=\frac{4}{5}$ then find the value of the ratio $\frac{4x-y}{4x+y}$.

Solution :

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{4x}{y} = \frac{16}{5} \qquad \text{...(multiplying both sides by 4)}$$

$$\therefore \frac{4x+y}{4x-y} = \frac{16+5}{16-5} \qquad \text{...(using componendo-dividendo)}$$

$$\therefore \frac{4x+y}{4x-y} = \frac{21}{11}$$

$$\therefore \frac{4x-y}{4x+y} = \frac{11}{21}$$

$\therefore$

Ex (6) If $5x = 4y$ then find the value of the ratio $\frac{3x^2+y^2}{3x^2-y^2}$.

Solution :

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{x^2}{y^2} = \frac{16}{25}$$

$$\therefore \frac{3x^2}{y^2} = \frac{48}{25} \qquad \text{...(multiplying both sides by 3)}$$

$$\therefore \frac{3x^2+y^2}{3x^2-y^2} = \frac{48+25}{48-25} \qquad \text{...(using componendo-dividendo)}$$

$$\therefore \frac{3x^2+y^2}{3x^2-y^2} = \frac{73}{23}$$

$\therefore$

Let's learn.

Application of properties of equal ratios

To solve some types of equations, it is convenient to use properties of equal ratios rather than using other methods.

Ex (1) Solve the equation. $\frac{3x^2+5x+7}{10x+14}=\frac{3x^2+4x+3}{8x+6}$

Solution :

$$\frac{3x^2+5x+7}{10x+14}=\frac{3x^2+4x+3}{8x+6}$$

$$\frac{\left(6x^2+10x+14\right)}{10x+14}=\frac{\left(6x^2+8x+6\right)}{8x+6} \qquad \text{....(multiplying both sides by 2)}$$

$$\frac{(6x^2+10x+14)-(10x+14)}{10x+14}=\frac{(6x^2+8x+6)-(8x+6)}{8x+6} \quad \text{...(using dividendo)}$$

$$\therefore \quad \frac{6x^2}{10x+14}=\frac{6x^2}{8x+6}$$

This equation is true for $x = 0$ $\quad\therefore\quad$ $x = 0$ is a solution of the given equation.

If $x \neq 0$ then $x^2 \neq 0$, $\therefore$ dividing by $6x^2$, $\quad \frac{1}{10x+14}=\frac{1}{8x+6}$

$$\therefore \quad 8x+6=10x+14$$

$$\therefore \quad 6-14=10x-8x$$

$$\therefore \quad -8=2x$$

$$\therefore \quad x=-4$$

$\therefore$ $x=-4$ or $x=0$ are the solutions of the given equation.

Ex (2) Solve. $\quad \frac{\sqrt{x+7}+\sqrt{x-2}}{\sqrt{x+7}-\sqrt{x-2}}=\frac{5}{1}$

Solution : $\quad \frac{(\sqrt{x+7}+\sqrt{x-2})+(\sqrt{x+7}-\sqrt{x-2})}{(\sqrt{x+7}+\sqrt{x-2})-(\sqrt{x+7}-\sqrt{x-2})}=\frac{5+1}{5-1}$ ...(using componendo-dividendo)

$$\therefore \quad \frac{2\sqrt{x+7}}{2\sqrt{x-2}}=\frac{6}{4}$$

$$\therefore \quad \frac{\sqrt{x+7}}{\sqrt{x-2}}=\frac{3}{2}$$

$$\therefore \quad \frac{x+7}{x-2}=\frac{9}{4} \quad \text{...(squaring both sides of the equation)}$$

$$\therefore \quad 4x+28=9x-18$$

$$\therefore \quad 28+18=9x-4x$$

$$\therefore \quad 46=5x$$

$$\therefore \quad \frac{46}{5}=x$$

$\therefore$ $x=\frac{46}{5}$ is the solution of the given equation

Activity :

Take 5 pieces of card paper. Write the following statements, one on each paper.

(i) $\dfrac{a+b}{b}=\dfrac{c+d}{d}$ (ii) $\dfrac{a}{c}=\dfrac{b}{d}$ (iii) $\dfrac{a}{b}=\dfrac{ac}{bd}$ (iv) $\dfrac{c}{d}=\dfrac{c-a}{d-b}$ (v) $\dfrac{a}{b}=\dfrac{rc}{rd}$

a, b, c, d are positive numbers and $\dfrac{a}{b}=\dfrac{c}{d}$ is given. Which of the above statements are true or false, write at the back of each card, if false explain why.

Practice set 4.3

(1) If $\dfrac{a}{b}=\dfrac{7}{3}$ then find the values of the following ratios.

(i) $\dfrac{5a+3b}{5a-3b}$ (ii) $\dfrac{2a^2+3b^2}{2a^2-3b^2}$ (iii) $\dfrac{a^3-b^3}{b^3}$ (iv) $\dfrac{7a+9b}{7a-9b}$

(2) If $\dfrac{15a^2+4b^2}{15a^2-4b^2}=\dfrac{47}{7}$ then find the values of the following ratios.

(i) $\dfrac{a}{b}$ (ii) $\dfrac{7a-3b}{7a+3b}$ (iii) $\dfrac{b^2-2a^2}{b^2+2a^2}$ (iv) $\dfrac{b^3-2a^3}{b^3+2a^3}$

(3) If $\dfrac{3a+7b}{3a-7b}=\dfrac{4}{3}$ then find the value of the ratio $\dfrac{3a^2-7b^2}{3a^2+7b^2}$.

(4) Solve the following equations.

(i) $\dfrac{x^2+12x-20}{3x-5}=\dfrac{x^2+8x+12}{2x+3}$ (ii) $\dfrac{10x^2+15x+63}{5x^2-25x+12}=\dfrac{2x+3}{x-5}$

(iii) $\dfrac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2}=\dfrac{17}{8}$ (iv*) $\dfrac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}}=\dfrac{4}{1}$

(v) $\dfrac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9}=\dfrac{61}{36}$ (vi) $\dfrac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3}=\dfrac{61}{189}$

Activity : In the following activity, the values of a and b can be changed. That is by changing $a : b$ we can create many examples. Teachers should give lot of practice to the students and encourage them to construct their own examples.

$\dfrac{a}{b}=\dfrac{3}{4}$

$\dfrac{5a^2+2b^2}{5a^2-2b^2}=$ ------

$\dfrac{3a}{4b}=$ ------

$\dfrac{2a-b}{2a+b}=$ ------

$\dfrac{a^2+b^2}{b^2}=$ ---

$\dfrac{a}{2b}=$ ------

Let's learn.

Theorem on equal ratios

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{b}=\frac{a+c}{b+d}=\frac{c}{d}$ This property is called the theorem of equal ratios.

Prrof : Let $\frac{a}{b}=\frac{c}{d}=k$. $\therefore$ $a = bk$ and $c = dk$

$$\therefore \quad \frac{a+c}{b+d}=\frac{bk+dk}{b+d}=\frac{k(b+d)}{b+d}=k$$

$$\therefore \quad \frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$$

We know that, $\frac{a}{b}=\frac{al}{bl}$

$\therefore$ If $\frac{a}{b}=\frac{c}{d}=k$, then $\frac{al}{bl}=\frac{cm}{dm}=\frac{al+cm}{bl+dm}=k$

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=$........ (finite terms) and if l, m, n are non-zero numbers

then each ratio = $\frac{al+cm+en+...}{bl+dm+fn+...}$ (finite terms) is the general form of the above theorem.

Use your brain power !

In a certain gymnasium, there are 35 girls and 42 boys in the kid's section, 30 girls and 36 boys in the children's section and 20 girls and 24 boys in the teens' section. What is the ratio of the number of boys to the number of girls in every section ?

For physical exercises, all three groups gathered on the ground. Now what is the ratio of number of boys to the number of girls ?

From the answers of the above questions, did you verify the theorem of equal ratios ?

Ex (1) Fill in the blanks in the following statements.

(i) $\frac{a}{3}=\frac{b}{7}=\frac{4a+9b}{..........}$ (ii) $\frac{x}{3}=\frac{y}{5}=\frac{z}{4}=\frac{5x-3y+4z}{...............}$

Solution : (i) $\frac{a}{3}=\frac{b}{7}=\frac{4a+9b}{4\times 3+9\times 7}=\frac{4a+9b}{12+63}=\frac{4a+9b}{75}$

(ii) $\frac{x}{3}=\frac{y}{5}=\frac{z}{4}=\frac{5\times x}{5\times 3}=\frac{-3\times y}{-3\times 5}=\frac{4\times z}{4\times 4}$

$$\therefore \quad =\frac{5x}{15}=\frac{-3y}{-15}=\frac{4z}{16}$$

$$=\frac{5x-3y+4z}{15-15+16} \quad \text{----(by the theorem of equal ratio)}$$

$$=\frac{5x-3y+4z}{16}$$

Ex (2) If $\dfrac{a}{(x-2y+3z)}=\dfrac{b}{(y-2z+3x)}=\dfrac{c}{(z-2x+3y)}$ and $x+y+z\neq 0$

then prove that each ratio = $\dfrac{a+b+c}{2(x+y+z)}$

Solution : Let $\dfrac{a}{(x-2y+3z)}=\dfrac{b}{(y-2z+3x)}=\dfrac{c}{(z-2x+3y)}=$ k.

$\therefore$ by theorem of equal ratios

$$k=\frac{a+b+c}{(x-2y+3z)+(y-2z+3x)+(z-2x+3y)}$$
$$=\frac{a+b+c}{2x+2y+2z}$$
$$=\frac{a+b+c}{2(x+y+z)}$$
$$\therefore \quad \frac{a}{x-2y+3z}=\frac{b}{y-2z+3x}=\frac{c}{z-2x+3y}=\frac{a+b+c}{2(x+y+z)}$$

Ex (3) If $\dfrac{y}{b+c-a}=\dfrac{z}{c+a-b}=\dfrac{x}{a+b-c}$ then prove that $\dfrac{a}{z+x}=\dfrac{b}{x+y}=\dfrac{c}{y+z}$.

Solution : By invertendo, we get

$$\frac{b+c-a}{y}=\frac{c+a-b}{z}=\frac{a+b-c}{x}$$

Now let $\dfrac{b+c-a}{y}=\dfrac{c+a-b}{z}=\dfrac{a+b-c}{x}=k.$

$\therefore$ by theorem of equal ratios

$$k=\frac{(c+a-b)+(a+b-c)}{z+x}=\frac{2a}{z+x} \quad .....(I)$$

$$k=\frac{(a+b-c)+(b+c-a)}{x+y}=\frac{2b}{x+y} \quad ....(II)$$

$$k=\frac{(b+c-a)+(c+a-b)}{y+z}=\frac{2c}{y+z} \quad ......(III)$$

$$\therefore \quad \frac{2a}{z+x}=\frac{2b}{x+y}=\frac{2c}{y+z}$$
$$\therefore \quad \frac{a}{z+x}=\frac{b}{x+y}=\frac{c}{y+z}$$

Ex (4) Solve : $\dfrac{14x^2-6x+8}{10x^2+4x+7}=\dfrac{7x-3}{5x+2}$

Solution : By observation, we see that multiplying by $2x$ the predecessor and the successor of right hand side, we get two terms of the predecessor and the successor of the left hand side.

But before multiplying, we must ensure that $x\neq 0$.

If x = 0 then $\frac{14x^2-6x+8}{10x^2+4x+7} = \frac{8}{7}$ and $\frac{7x-3}{5x+2}=\frac{-3}{2}$

$\therefore$ $\frac{8}{7}=\frac{-3}{2}$ Which is a contradiction.

$\therefore$ $x \neq 0$

$\therefore$ multiplying predecessor and successor of RHS by $2x$.

$$\frac{14x^2-6x+8}{10x^2+4x+7}=\frac{2x(7x-3)}{2x(5x+2)}=k$$

$$\therefore \frac{14x^2-6x+8}{10x^2+4k+7}=\frac{14x^2-6x}{10x^2+4x}=k$$

$$\therefore \frac{14x^2-6x+8-14x^2+6x}{10x^2+4x+7-10x^2-4x}=\frac{8}{7}=k$$

$$\therefore \quad k=\frac{8}{7}$$

$$\therefore \quad \frac{7x-3}{5x+2}=\frac{8}{7}$$

$$\therefore \quad 49x-21=40x+16$$

$$\therefore \quad 49x-40x=16+21$$

$$\therefore \quad 9x=37 \qquad \therefore \ x=\frac{37}{9}$$

Practice set 4.4

(1) Fill in the blanks of the following

(i) $\frac{x}{7}=\frac{y}{3}=\frac{3x+5y}{.......}=\frac{7x-9y}{......}$ (ii) $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=\frac{a-2b+3c}{......}=\frac{......}{6-8+14}$

(2) $5m-n=3m+4n$ then find the values of the following expressions.

(i) $\frac{m^2+n^2}{m^2-n^2}$ (ii) $\frac{3m+4n}{3m-4n}$

(3) (i) If $a(y+z) = b(z+x) = c(x+y)$ and out of a, b, c no two of them are equal then show that, $\frac{y-z}{a(b-c)}=\frac{z-x}{b(c-a)}=\frac{x-y}{c(a-b)}$.

(ii) If $\frac{x}{3x-y-z}=\frac{y}{3y-z-x}=\frac{z}{3z-x-y}$ and $x+y+z \neq 0$ then show that the value of each ratio is equal to 1.

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(iii) If $\dfrac{ax+by}{x+y}=\dfrac{bx+az}{x+z}=\dfrac{ay+bz}{y+z}$ and $x+y+z\neq 0$ then show that $\dfrac{a+b}{2}$.

(iv) If $\dfrac{y+z}{a}=\dfrac{z+x}{b}=\dfrac{x+y}{c}$ then show that $\dfrac{x}{b+c-a}=\dfrac{y}{c+a-b}=\dfrac{z}{a+b-c}$.

(v) If $\dfrac{3x-5y}{5z+3y}=\dfrac{x+5z}{y-5x}=\dfrac{y-z}{x-z}$ then show that every ratio = $\dfrac{x}{y}$.

(4) Solve. (i) $\dfrac{16x^2-20x+9}{8x^2+12x+21}=\dfrac{4x-5}{2x+3}$ (ii) $\dfrac{5y^2+40y-12}{5y+10y^2-4}=\dfrac{y+8}{1+2y}$

Let's learn.

Continued Proportion

Let us consider the ratios 4 : 12 and 12 : 36. They are equal ratios. In the two ratios, the successor (second term) of the first ratio is equal to the predecessor (first term) of the second ratio. Hence 4, 12, 36 are said to be in continued proportion.

If $\dfrac{a}{b}=\dfrac{b}{c}$ **then *a, b, c* are in continued proportion.**

If $ac = b^2$, then dividing both sides by bc we get $\dfrac{a}{b}=\dfrac{b}{c}$.

$\therefore$ if $ac = b^2$, then a, b, c are in continued proportion.

When a, b, c are in continued proportion then b is known as **Geometric mean** of a and c or **Mean proportional** of a and c.

Hence all the following statements convey the same meaning.

$\therefore$ (1) $\dfrac{a}{b}=\dfrac{b}{c}$ (2) $b^2= a\,c$ (3) a, b, c are in continued proportion.

(4) b is the geometric mean of a and c.

(5) b is the mean proportional of a and c.

We can generalise the concept of continued proportion

If $\dfrac{a}{b}=\dfrac{b}{c}=\dfrac{c}{d}=\dfrac{d}{e}=\dfrac{e}{f}$ then a, b, c, d, e and f are said to be in continued proportion.

Ex (1) If x is the geometric mean of 25 and 4, then find the value of x.

Solution : x is the geometric mean of 25 and 4

$$\therefore\ x^2 = 25\times 4$$
$$\therefore\ x^2 = 100$$
$$\therefore\ x = 10$$

Ex (2) If $4\,a^2b$, $8\,ab^2$, p are in continued proportion then find the value of p .

Solution : From given information, $4\,a^2b$, $8\,ab^2$, p are in continued proportion

$$\therefore \quad \frac{4a^2b}{8ab^2} = \frac{8ab^2}{p}$$

$$p = \frac{8ab^2 \times 8ab^2}{4a^2b} = 16b^3$$

Ex (3) Which number should be subtracted from 7, 12 and 18 such that the resultant numbers are in continued proportion?

Solution : Let x be subtracted from 7, 12 and 18 such that resultant numbers are in continued proportion.

$(7-x)$, $(12-x)$, $(18-x)$ are in continued proportion.

$\therefore \quad (12-x)^2 = (7-x)(18-x)$

$\therefore \quad 144 - 24x + x^2 = 126 - 25x + x^2$

$\therefore \quad -24x + 25x = 126 - 144$

$\therefore \quad x = -18$

Tally

$(7-x) = 7-(-18) = 25$

$(12-x) = 12-(-18) = 30$

$(18-x) = 18-(-18) = 36$

$30^2 = 900$ and $25 \times 36 = 900$

25, 30, 36 are in continued proportion

$\therefore$ If -18 is subtracted from 7, 12, 18 the resultant numbers are in continued proportion.

k - method

The k-method is used to solve examples based on equal ratios, i.e. equal proportions. In this simple method every equal ratio is assumed to be equal to k.

Ex (1) If $\frac{a}{b} = \frac{c}{d}$ then show tha t $\frac{5a-3c}{5b-3d} = \frac{7a-2c}{7b-2d}$

Solution : Let $\frac{a}{b} = \frac{c}{d} = k \quad \therefore\ a = bk,\ c = dk$

Substituting values of a and c in both sides,

$$\text{LHS} = \frac{5a-3c}{5b-3d} = \frac{5(bk)-3(dk)}{5b-3d} = \frac{k(5b-3d)}{(5b-3d)} = k$$

$$\text{RHS} = \frac{7a-2c}{7b-2d} = \frac{7(bk)-2(dk)}{7b-2d} = \frac{k(7b-2d)}{7b-2d} = k$$

$\therefore$ LHS = RHS.

$$\therefore \frac{5a-3c}{5b-3d} = \frac{7a-2c}{7b-2d}$$

Ex (2) If a, b, c are in continued proportion then show that, $\frac{(a+b)^2}{ab} = \frac{(b+c)^2}{bc}$.

Solution : a, b, c are in continued proportion. Let $\frac{a}{b} = \frac{b}{c} = k$.

$\therefore b = ck, \quad a = bk = ck \times k = ck^2$

Substituting values of a and b.

$$\text{LHS} = \frac{(a+b)^2}{ab} = \frac{(ck^2+ck)^2}{(ck^2)(ck)} = \frac{c^2k^2(k+1)^2}{c^2k^3} = \frac{(k+1)^2}{k}$$

$$\text{RHS} = \frac{(b+c)^2}{bc} = \frac{(ck+c)^2}{(ck)c} = \frac{c^2(k+1)^2}{c^2k} = \frac{(k+1)^2}{k}$$

$\therefore$ LHS = RHS. $\qquad \therefore \frac{(a+b)^2}{ab} = \frac{(b+c)^2}{bc}$

Ex (3) If a, b, c are in continued proportion then show that $\frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2}$

Solution : a, b, c are in continued proportion.

$$\therefore \frac{a}{b} = \frac{b}{c}$$

Let, $\frac{a}{b} = \frac{b}{c} = k \quad \therefore b = ck$ and $a = ck^2$

$$\text{LHS} = \frac{a}{c} = \frac{ck^2}{c} = k^2$$

$$\text{RHS} = \frac{a^2+ab+b^2}{b^2+bc+c^2}$$

$$= \frac{(k^2c)^2 + k^2c(ck) + (ck)^2}{(ck)^2 + (ck)(c) + c^2}$$

$$= \frac{k^4c^2 + k^3c^2 + c^2k^2}{c^2k^2 + c^2k + c^2}$$

$$= \frac{c^2k^2(k^2+k+1)}{c^2(k^2+k+1)}$$

$$= k^2$$

$\therefore$ LHS = RHS

$$\therefore \frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2}$$

Ex (4) Five numbers are in continued proportion. The first term is 5 and the last term is 80. Find these numbers.

Solution : Let the numbers in continued proportion be a, ak, ak^2, ak^3, ak^4.

Here $a = 5$ and $ak^4 = 80$

$\therefore 5 \times k^4 = 80$

$\therefore k^4 = 16$

$\therefore k = 2 \quad \because 2^4 = 16$

$ak = 5 \times 2 = 10 \qquad ak^2 = 5 \times 4 = 20$

$ak^3 = 5 \times 8 = 40 \qquad ak^4 = 5 \times 16 = 80$

$\therefore$ the numbers are 5, 10, 20, 40, 80.

Practice set 4.5

(1) Which number should be subtracted from 12, 16 and 21 so that resultant numbers are in continued proportion?

(2) If $(28-x)$ is the mean proportional of $(23-x)$ and $(19-x)$ then find the vaue of x.

(3) Three numbers are in continued proportion, whose mean proportional is 12 and the sum of the remaining two numbers is 26, then find these numbers.

(4) If $(a + b + c)(a - b + c) = a^2 + b^2 + c^2$ show that a, b, c are in continued proportion.

(5) If $\frac{a}{b} = \frac{b}{c}$ and $a, b, c > 0$ then show that,

(i) $(a + b + c)(b - c) = ab - c^2$

(ii) $(a^2 + b^2)(b^2 + c^2) = (ab + bc)^2$

(iii) $\frac{a^2 + b^2}{ab} = \frac{a + c}{b}$

(6) Find mean proportional of $\frac{x+y}{x-y}, \frac{x^2-y^2}{x^2y^2}$

Activity : Observe the political map of India from a Geography text book. Study the scale of this map.

From the given scale find the straight line distances between various cities like (i) New Delhi to Bengaluru (ii) Mumbai to Kolkata, (iii) Jaipur to Bhuvaneshvar.

Problem set 4

(1) Select the appropriate alternative answer for the following questions.

(i) If 6 : 5 = y : 20 then what will be the value of y ?

(A) 15 (B) 24 (C) 18 (D) 22.5

(ii) What is the ratio of 1 mm to 1 cm ?

(A) 1 : 100 (B) 10 : 1 (C) 1 : 10 (D) 100 : 1

(iii*) The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age ?

(A) 3 : 2 (B) 2 : 3 (C) 4 : 3 (D) 3 : 4

(iv) 24 Bananas were distributed between Shubham and Anil in the ratio 3 : 5, then how many bananas did Shubham get ?

(A) 8 (B) 15 (C) 12 (D) 9

(v) What is the mean proportional of 4 and 25 ?

(A) 6 (B) 8 (C) 10 (D) 12

(2) For the following numbers write the ratio of first number to second number in the reduced form.

(i) 21, 48 (ii) 36, 90 (iii) 65, 117 (iv) 138, 161 (v) 114, 133

(3) Write the following ratios in the reduced form.

(i) Radius to the diameter of a circle.

(ii) The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.

(iii) The ratio of perimeter to area of a square, having side 4 cm.

(4) Check whether the following numbers are in continued proportion.

(i) 2, 4, 8 (ii) 1, 2, 3 (iii) 9, 12, 16 (iv) 3, 5, 8

(5) a, b, c are in continued proportion. If $a = 3$ and $c = 27$ then find b.

(6) Convert the following ratios into percentages..

(i) 37 : 500 (ii) $\frac{5}{8}$ (iii) $\frac{22}{30}$ (iv) $\frac{5}{16}$ (v) $\frac{144}{1200}$

(7) Write the ratio of first quantity to second quantity in the reduced form.

(i) 1024 MB, 1.2 GB [(1024 MB = 1 GB)]

(ii) 17 Rupees, 25 Rupees 60 paise (iii) 5 dozen, 120 units

(iv) 4 sq.m, 800 sq.cm (v) 1.5 kg, 2500 gm

(8) If $\frac{a}{b} = \frac{2}{3}$ then find the values of the following expressions.

(i) $\frac{4a+3b}{3b}$ (ii) $\frac{5a^2+2b^2}{5a^2-2b^2}$

(iii) $\frac{a^3+b^3}{b^3}$ (iv) $\frac{7b-4a}{7b+4a}$

(9) If a, b, c, d are in proportion, then prove that

(i) $\frac{11a^2+9ac}{11b^2+9bd} = \frac{a^2+3ac}{b^2+3bd}$

(ii*) $\sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$

(iii) $\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{c^2+cd+d^2}{c^2-cd+d^2}$

(10) If a, b, c are in continued proportion, then prove that

(i) $\dfrac{a}{a+2b} = \dfrac{a-2b}{a-4c}$ (ii) $\dfrac{b}{b+c} = \dfrac{a-b}{a-c}$

(11) Solve : $\dfrac{12x^2+18x+42}{18x^2+12x+58} = \dfrac{2x+3}{3x+2}$

(12) If $\dfrac{2x-3y}{3z+y} = \dfrac{z-y}{z-x} = \dfrac{x+3z}{2y-3x}$ then prove that every ratio = $\dfrac{x}{y}$.

(13*) If $\dfrac{by+cz}{b^2+c^2} = \dfrac{cz+ax}{c^2+a^2} = \dfrac{ax+by}{a^2+b^2}$ then prove that $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$.

5 Linear Equations in Two Variables

Let's study.

- Introduction
  Linear equation in two variables
- Simultaneous equations
- Solving simultaneous equations
- Word problems based on simultaneous equations

Let's recall.

Ex. Solve the following equations.

(1) $m + 3 = 5$ (2) $3y + 8 = 22$ (3) $\frac{x}{3} = 2$ (4) $2p = p + \frac{4}{9}$

$m = \square$ $y = \square$ $x = \square$ $p = \square$

(5) Which number should be added to 5 to obtain 14 ?

$\square + 5 = 14$

$x + 5 = 14$

$x = \square$

(6) Which number should be subtracted from 8 to obtain 2 ?

$8 - \square = 2$

$8 - y = 2$

$y = \square$

In all above equations, degree of the variable is 1. These are called as Linear equations.

Let's learn.

Linear equations in two variables

Find two numbers whose sum is 14.

Using variables x and y for the two numbers, we can form the equation $x + y = 14$.

This is an equation in two variables.

We can find many values of x and y satisfying the condition.

e.g. $9 + 5 = 14$ $7 + 7 = 14$ $8 + 6 = 14$ $4 + 10 = 14$

$(-1) + 15 = 14$ $15 + (-1) = 14$ $2.6 + 11.4 = 14$ $0 + 14 = 14$

$100 + (-86) = 14$ $(-100) + (114) = 14$ $\square + \square = 14$ $\square + \square = 14$

Hence above equation has many solutions like ($x = 9, y = 5$); ($x = 7, y = 7$); ($x = 8, y = 6$) etc..

Convensionally, the solution $x = 9, y = 5$ is written as an ordered pair (9, 5) where 9 is the value of x and 5 is the value of y. To satisfy the equation $x + y = 14$, we can get infinite ordered pairs like (9,5), (7,7), (8,6), (4,10), (10,4), (–1,15), (2.6, 11.4), … etc. All of these are the solutions of $x + y = 14$.

Consider second example.

Find two numbers such that their difference is 2.

Let the greater number be x and the smaller number be y.

Then we get the equation $x - y = 2$

For the values of x and y, we can get following equations.

$10 - 8 = 2$	$9 - 7 = 2$	$8 - 6 = 2$	$(-3) - (-5) = 2$	$5.3 - 3.3 = 2$
$15 - 13 = 2$	$100 - 98 = 2$	$\square - \square = 2$	$\square - \square = 2$	

Here if we take values $x = 10$ and $y = 8$, then the ordered pair (10, 8) satisfies the above equation. Here we cannot write as (8, 10) because (8, 10) will imply $x = 8$ and $y = 10$ and it does not satisfy the equation $x - y = 2$. Therefore, note that, the order of numbers in the pair indicating solution is very important.

Now let us write the solutions of $x - y = 2$ in the form of ordered pairs.

(7, 5), (–2, –4), (0, –2), (5.2, 3.2), (8, 6) etc. There are infinite solutions.

Find the solution of $4m - 3n = 2$.

Construct 3 different equations and find their solutions.

Now, observe the first two equations.

$x + y = 14$ I

$x - y = 2$ II

Solution of equation I : (9, 5), (7, 7), (8, 6)...

Solutions of Equation II : (7, 5), (–2, –4), (0, –2), (5.2, 3.2), (8, 6)...

(8, 6) is the only common solution of both the equations. This solution satisfies both the equations. Hence it is the unique common solution of both the equations.

Remember this !

When we consider two linear equations in two variables simultaneously and we get unique common solution, then such set of equations is known as **Simultaneous equations**.

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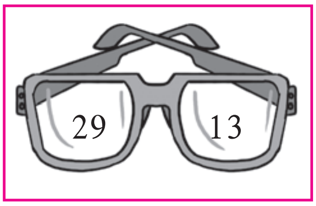
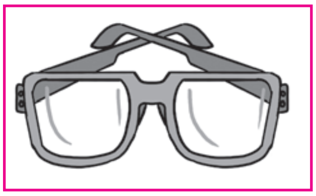
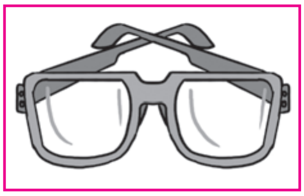
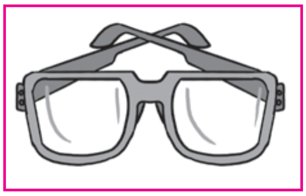
Activity : On the glasses of following spectacles, write numbers such that





(i) Their sum is 42 and difference is 16 (ii) Their sum is 37 and difference is 11





(iii) Their sum is 54 and difference is 20 (iv) Their sum is ... and difference is ...

Let's recall.

$x+y = 5$ and $2x + 2y = 10$ are two equations in two variables. Find five different solutions of $x+y = 5$, verify whether same solutions satisfy the equation $2x + 2y = 10$ also.

Observe both equations.

Find the condition where two equations in two variables have all solutions in common.

Let's learn.

Elimination method of solving simultaneous equations

By taking different values of variables we have solved the equations $x + y = 14$ and $x - y = 2$. But every time, it is not easy to solve by this method, e.g. , $2x + 3y = -4$ and $x - 5y = 11$. Try to solve these equations by taking different values of x and y. By this method observe that it is not easy to obtain the solution.

Therefore to solve simultaneous equations we use different method. In this method, we eliminate one of the variables to obtain equations in one variable. We can solve and find the value of one of the two variables and then substituting this value in one of the given equations we can find the value of the other variable.

Study the following example to understand this method.

Ex (1) Solve $x + y = 14$ and $x - y = 2$.

Solution : By adding both the equations we get an equation in one variable

$$
\begin{array}{rrcll}
 & x + y & = & 14 & \text{.........I} \\
+ & x - y & = & 2 & \text{.........II} \\
\hline
 & 2x + 0 & = & 16 & \\
 & 2x & = & 16 & \\
 & x & = & 8 &
\end{array}
$$

Substituting $x = 8$ in the equation (I)

$$x + y = 14$$

$$\therefore\ 8 + y = 14$$

$$\therefore\ y = 6$$

Here (8, 6) is the solution of first equation. Let us check, whether it satisfies the second equation also.

$$x - y = 8 - 6 = 2 \text{ is true.}$$

$\therefore$ (8,6) is the solution for both the equations.

Hence (8, 6) is the solution of simultaneous equations $x + y = 14$ and $x - y = 2$.

Ex (2) Sum of the ages of mother and son is 45 years. If son's age is subtracted from twice of mother's age then we get answer 54. Find the ages of mother and son.

It becomes easy to solve a problem if we make use of variables.

Solution : Let the mother's today's age be x years and son's today's age be y years.

From the first condition $x+y = 45$ I

From the second condition $2x-y = 54$ II

Adding equations (I) and (II) $\quad 3x+0 = 99$

$$3x = 99$$

$$x = 33$$

Substituting $x = 33$ in equation (I), $\quad 33+y = 45$

$$y = 45-33$$

$$y = 12$$

Verify that $x=33$ and $y = 12$ is the solution of second equation.

Today's age of mother = 33 and today's age of son = 12.

General form of linear equation in two variables

The general form of a linear equation in two variables is $ax + by + c = 0$ where a, b, c are real numbers and a and b are non-zero at the same time.

In this equation the index of both the variables is 1. Hence it is a linear equation.

Ex (1) Solve the following equations

$$3x + y = 5 \text{........... (I)}$$
$$2x + 3y = 1 \text{........(II)}$$

Solution : To eliminate one of the variables, we observe that in both equations, not a single coefficient is equal or opposite number. Hence we will make one of them equal.

Multiply both sides of the equation (I) by 3.

$$\therefore \quad 3x \times 3 + 3 \times y = 5 \times 3$$

$$\begin{aligned} \therefore \quad 9x + 3y &= 15 \quad \text{.......(III)} \\ 2x + 3y &= 1 \quad \text{.......(II)} \end{aligned}$$

Now subtracting eqn (II) from eqn (III)

$$\begin{array}{rcl} 9x + 3y & = & 15 \\ \underset{-}{+} 2x \underset{-}{+} 3y & \underset{-}{=} & 1 \\ \hline 7x & = & 14 \\ x & = & 2 \end{array}$$

Substituting $x = 2$ in one of the equations.

$$\begin{aligned} 2x + 3y &= 1 \\ \therefore \quad 2 \times 2 + 3y &= 1 \\ \therefore \quad 4 + 3y &= 1 \\ \therefore \quad 3y &= -3 \\ \therefore \quad y &= -1 \end{aligned}$$

Verify that $(2, -1)$ satisfies the second equation.

Ex (2) Solve the following simultaneous equations.

$$3x - 4y - 15 = 0 \text{ .... (I)}$$
$$y + x + 2 = 0 \text{ ....(II)}$$

Solution : Let us write the equations by shifting constant terms to RHS

$$3x - 4y = 15 \text{........(I)}$$
$$x + y = -2 \text{ .... (II)}$$

To eliminate y, multiply second equation by 4 and add to equation (I).

$$\begin{array}{rcl} 3x - 4y & = & 15 \\ +\quad 4x + 4y & = & -8 \\ \hline 7x & = & 7 \\ x & = & 1 \end{array}$$

Substituting $x = 1$ in the equation (II).

$$\begin{aligned} x + y &= -2 \\ \therefore 1 + y &= -2 \\ \therefore y &= -2 - 1 \\ \therefore y &= -3 \end{aligned}$$

$(1, -3)$ is the solution of the above equations. Verify that it satisfies equation (I) also.

Use your brain power!

$3x - 4y - 15 = 0$ and $y + x + 2 = 0$. Can these equations be solved by eliminating x ? Is the solution same ?

Let's learn.

Substitution method of solving simultaneous equations

There is one more method to eliminate a variable. We can express one variable in terms of other from one of the equations. Then substituting it in the other equation we can eliminate the variable. Let us discuss this method from following examples.

Ex (1) Solve $8x + 3y = 11$; $3x - y = 2$

Solution : $8x + 3y = 11$.................. (I)

$3x - y = 2$.......................(II)

In Equation (II), it is easy to express y in terms of x.

$3x - y = 2$

$3x - 2 = y$

Substituting $y = 3x - 2$ in equation (I).

$8x + 3y = 11$

$\therefore\ 8x + 3(3x-2) = 11$

$\therefore\ 8x + 9x - 6 = 11$

$\therefore\ 17x - 6 = 11$

$\therefore\ 17x = 11 + 6 = 17$

$\therefore\ x = 1$

Now, substituting this value of x in the equation $y = 3x - 2$.

$y = 3 \times 1 - 2$

$\therefore\ y = 1$

$\therefore$ (1, 1) is the solution of the given equations

Ex (2) Solve. $3x - 4y = 16$; $2x - 3y = 10$

Solution : $3x-4y=16$..........(I)

$2x - 3y = 10$.............(II)

Writing x in terms of y from equation (I).

$3x - 4y = 16$

$3x = 16 + 4y$

$x = \dfrac{16+4y}{3}$

Substituting this value of x in equation (II)

$2x - 3y = 10$

$2\left(\dfrac{16+4y}{3}\right) - 3y = 10$

$\dfrac{32+8y}{3} - 3y = 10$

$\dfrac{32+8y-9y}{3} = 10$

$32 + 8y - 9y = 30$

$32 - y = 30 \quad \therefore\ y = 2$

Now, substituting $y = 2$ in equation (I)

$3x - 4y = 16$

$\therefore\ 3x - 4\times2 = 16$

$\therefore\ 3x - 8 = 16$

$\therefore\ 3x = 16 + 8$

$\therefore\ 3x = 24$

$\therefore\ x = 8$

$\therefore\ x = 8$ and $y = 2$

$\therefore$ (8, 2) is the solution of the given equations.

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Practice set 5.1

(1) By using variables x and y form any five linear equations in two variables.

(2) Write five solutions of the equation $x + y = 7$.

(3) Solve the following sets of simultaneous equations.

(i) $x + y = 4$; $2x - 5y = 1$ (ii) $2x + y = 5$; $3x - y = 5$

(iii) $3x - 5y = 16$; $x - 3y = 8$ (iv) $2y - x = 0$; $10x + 15y = 105$

(v) $2x + 3y + 4 = 0$; $x - 5y = 11$ (vi) $2x - 7y = 7$; $3x + y = 22$

Let's learn.

Word problems based on simultaneous equations

While solving word problems, converting the given information into mathematical form is an important step in this process. In the following flow-chart, the procedure for finding solutions of word problems is given.

Steps	Example
Read the given word problem carefully and try to understand it.	Sum of two numbers is 36. If 9 is subtracted from 8 times the first number, we get second number. Find the numbers.
↓	↓
From given information, use proper variables for given quantities	Let first number = x and second number = y
↓	↓
Form the mathematical statements from above variables	Sum of the numbers 36 $\therefore x + y = 36$ 8 times first number = $8x$ 8 times first number $- 9 = 8x - 9$ $\therefore$ Second number $= y = 8x - 9$
↓	↓
Use suitable method to solve the equations	$x + y = 36$ $+\ 8x - y = 9$ $\therefore\ 9x = 45$ $\therefore\ x = 5$ $x + y = 36$ $\therefore\ 5 + y = 36$ $\therefore\ y = 36 - 5$ $\therefore\ y = 31$
↓	↓
Find the solution	$x = 5,\ y = 31$
↓	↓
Substituting the values of the variables in both the equations, verify your result.	$31 + 5 = 36$ ....(I) $31 = 8 \times 5 - 9$ ....(II)
↓	↓
Write the answer.	$\therefore$ the numbers are 5 and 31.

Word Problems

Now we will see various types of word problems.

(1) Problems regarding age.

(2) Problems regarding numbers.

(3) Problems based on fractions.

(4) Problems based on money transactions.

(5) Problems based on geometrical properties

(6) Problems based on speed, distance, time.

Ex (1) Sum of two numbers is 103. If greater number is divided by smaller number then the quotient is 2 and the remainder is 19. Then find the numbers.

Solution : Step 1 : To understand the given problem.

Step 2 : Use proper variables for given quantities. Also note the rule

dividend = divisor × quotient + remainder.

Let the greater number be x and the smaller number be y

Step 3 : Given information : Sum of the numbers = 103

$x + y = 103$ is the first equation.

By dividing greater number by smaller numbers quotient is 2
and remainder is 19.

$x = 2 \times y + 19$ ...(dividend = divisor × quotient + remainder)

$x - 2y = 19$ is the second equation.

Step 4 : Let us find the solution of the equations.

$$x + y = 103 \qquad \text{................(I)}$$
$$x - 2y = 19 \qquad \text{................(II)}$$

Subtracting eqn. (II) from eqn. (I)

$$\begin{array}{rcl} x + y & = & 103 \\ x - 2y & = & 19 \\ - \quad + & & - \\ \hline 0 + 3y & = & 84 \\ \therefore \quad y & = & 28 \end{array}$$

Step 5 : Substituting value of y in equation $x + y = 103$.

$\therefore \quad x + 28 = 103$

$\therefore \quad x = 103 - 28$

$\therefore \quad x = 75$

Step 6 : Given numbers are 75 and 28.

Ex (2) Salil's age is 23 years more than half of the Sangram's age. Five years ago, the sum of their ages was 55 years. Find their present ages.

Solution : Let Salil's present age be x and Sangram's present age be y .

Salil's age is 23 years more than half of the Sangram's age $\therefore$ $x = \frac{y}{2} + \boxed{}$

Five years ago Salil's age = $x - 5$. Five years ago Sangram's age = $y - 5$

The sum of their ages five years ago = 55

$\boxed{} + \boxed{} = 55$

Finding the solution by solving equations

$2x = y + 46$ $\qquad$ $2x - y = 46$ ....(I)

$(x - 5) + (y-5) = 55$

$x + y = 65$ $\qquad$ ....(II)

Adding equation (I) and (II)

$$\begin{array}{rl} & 2x - y = 46 \\ + & x + y = 65 \\ \hline \therefore & 3x = 111 \\ \therefore & x = 37 \end{array}$$

Substituting $x = 37$ in equation (II)

$$\begin{aligned} x+y &= 65 \\ \therefore\ 37 + y &= 65 \\ \therefore\ y &= 65 - 37 \\ \therefore\ y &= 28 \end{aligned}$$

Salil's present age is 37 years and Sangram's present age is 28 years.

Ex (3) A two digit number is 4 times the sum of its digits. If we interchange the digits, the number obtained is 9 less than 4 times the original number. Then find the number.

Solution : Let the units place digit in original number be x, and tens place be y.

	Digit in tens place	Digit in units place	Number	Sum of the digits
For original number	y	x	$10y + x$	$y + x$
Number obtained by interchanging the digits	x	y	$10x + y$	$x + y$

From first condition, $10y + x = 4(y + x)$

$\therefore 10y + x = 4y + 4x$

$\therefore x - 4x + 10y - 4y = 0$

$\therefore -3x + 6y = 0$ $\therefore$ $-3x = -6y$ $\qquad$ $\therefore x = 2y$ .....(I)

From second condition, $10x + y = 2(10y+x)-9$

$$10x+y = 20y + 2x-9$$

$$10x-2x+y-20y = -9$$

$$8x - 19y = -9 \qquad \text{.............(II)}$$

$$x = 2y \qquad \text{..............(I)}$$

Substituting $x = 2y$ in equation (II).

$$16y - 19y = -9 \qquad \text{...............(I)}$$

$$\therefore\ -3y = -9$$

$$\therefore\ y = 3$$

Substituting $y = 3$ in equation (I).. $x - 2y = 0$

$$x - 2 \text{ x } 3 = 0 \qquad \therefore\ x - 6 = 0 \qquad \therefore\ x = 6$$

Original two digit number : $10y + x = 10 \times 3 + 6$

$$= 36$$

Ex (4) The population of a certain town was 50,000. In a year, male population was increased by 5% and female population was increased by 3%. Now the population became 52020. Then what was the number of males and females in the previous year?

Solution : Let the number of males in previous year be x, number of females be y

By first condition $\square + \square = 50000$ ....(I)

Male population increased by 5% $\therefore$ number of males $= \dfrac{\square}{\square}x$

Female population increased by 3% $\therefore$ number of females $= \dfrac{\square}{\square}y$.

From second condition $\dfrac{\square}{\square}x + \dfrac{\square}{\square}y = 52020$

$$\square x + \square y = 5202000 \qquad \text{.......(II)}$$

Multiplying equation (I) by 103

$$\square x + \square y = 5150000 \qquad \text{.......(III)}$$

Subtracting equation (III) from equation (II) .

$$2x = 5202000 - 5150000$$

$$2x = 52000$$

$\therefore$ number of males $= x = \square$ $\qquad \therefore$ number of females $= y = \square$

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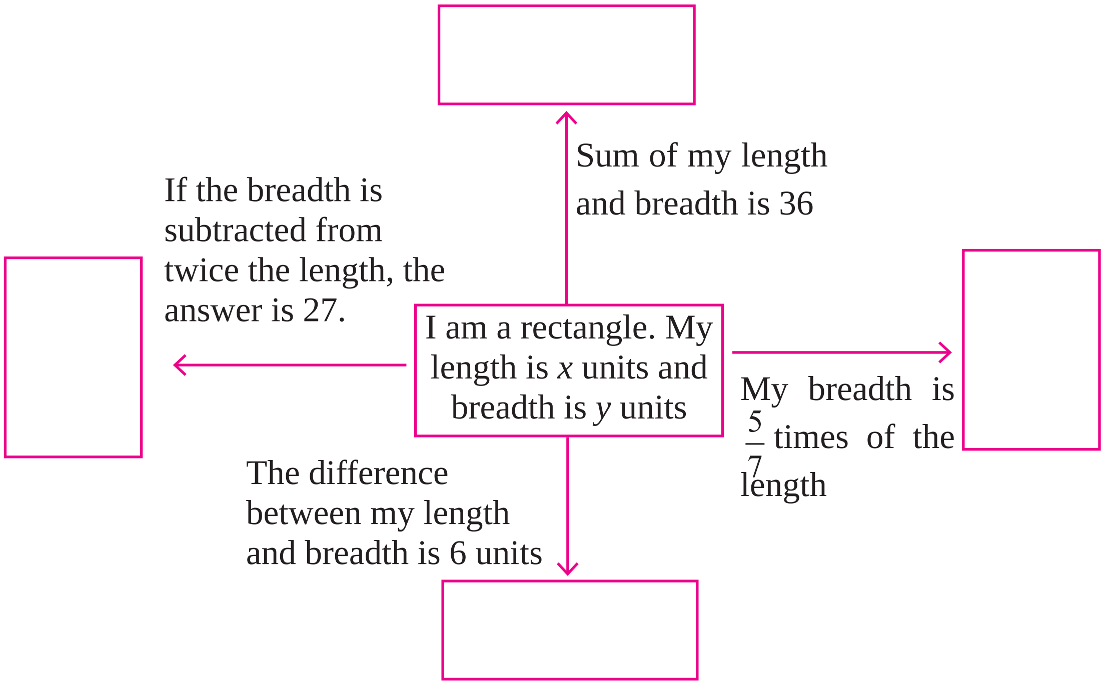
Activity I : There are instructions written near the arrows in the following diagram. From this information form suitable equations and write in the boxes indicated by arrows. Select any two equations from these boxes and find their solutions. Also verify the solutions. By taking one pair of equations at a time, how many pairs can be formed ? Discuss the solutions for these pairs.



Practice set 5.2

(1) In an envelope there are some 5 rupee notes and some 10 rupee notes. Total amount of these notes together is 350 rupees. Number of 5 rupee notes are less by 10 than number of 10 rupee notes. Then find the number of 5 rupee and 10 rupee notes.

(2) The denominator of a fraction is 1 more than twice its numerator. If 1 is added to numerator and denominator respectively, the ratio of numerator to denominator is 1 : 2. Find the fraction.

(3) The sum of ages of Priyanka and Deepika is 34 years. Priyanka is elder to Deepika by 6 years. Then find their today's ages.

(4) The total number of lions and peacocks in a certain zoo is 50. The total number of their legs is140. Then find the number of lions and peacocks in the zoo.

(5) Sanjay gets fixed monthly income. Every year there is a certain increment in his salary. After 4 years, his monthly salary was Rs. 4500 and after 10 years his monthly salary became 5400 rupees, then find his original salary and yearly increment.

(6) The price of 3 chairs and 2 tables is 4500 rupees and price of 5 chairs and 3 tables is 7000 rupees, then find the price of 2 chairs and 2 tables.

(7) The sum of the digits in a two-digits number is 9. The number obtained by interchanging the digits exceeds the original number by 27. Find the two-digit number.

(8*) In Δ ABC, the measure of angle A is equal to the sum of the measures of $\angle$ B and $\angle$ C. Also the ratio of measures of $\angle$ B and $\angle$ C is 4 : 5. Then find the measures of angles of the triangle.

(9*) Divide a rope of length 560 cm into 2 parts such that twice the length of the smaller part is equal to $\frac{1}{3}$ of the larger part. Then find the length of the larger part.

(10) In a competitive examination, there were 60 questions. The correct answer would carry 2 marks, and for incorrect answer 1 mark would be subtracted. Yashwant had attempted all the questions and he got total 90 marks. Then how many questions he got wrong ?

Problem set 5

(1) Choose the correct alternative answers for the following questions.

(i) If $3x + 5y = 9$ and $5x + 3y = 7$ then What is the value of $x + y$?

(A) 2 (B) 16 (C) 9 (D) 7

(ii) 'When 5 is subtracted from length and breadth of the rectangle, the perimeter becomes 26.' What is the mathematical form of the statement ?

(A) $x - y = 8$ (B) $x + y = 8$ (C) $x + y = 23$ (D) $2x + y = 21$

(iii) Ajay is younger than Vijay by 5 years. Sum of their ages is 25 years. What is Ajay's age ?

(A) 20 (B) 15 (C) 10 (D) 5

(2) Solve the following simultaneous equations.

(i) $2x + y = 5$; $3x - y = 5$

(ii) $x - 2y = -1$; $2x - y = 7$

(iii) $x + y = 11$; $2x - 3y = 7$

(iv) $2x + y = -2$; $3x - y = 7$

(v) $2x - y = 5$; $3x + 2y = 11$

(vi) $x - 2y = -2$; $x + 2y = 10$

(3) By equating coefficients of variables, solve the following equations.

(i) $3x - 4y = 7$; $5x + 2y = 3$

(ii) $5x + 7y = 17$; $3x - 2y = 4$

(iii) $x - 2y = -10$; $3x - 5y = -12$

(iv) $4x + y = 34$; $x + 4y = 16$

(4) Solve the following simultaneous equations.

(i) $\frac{x}{3} + \frac{y}{4} = 4$; $\frac{x}{2} - \frac{y}{4} = 1$

(ii) $\frac{x}{3} + 5y = 13$; $2x + \frac{y}{2} = 19$

(iii) $\frac{2}{x} + \frac{3}{y} = 13$; $\frac{5}{x} - \frac{4}{y} = -2$

(5*) A two digit number is 3 more than 4 times the sum of its digits. If 18 is added to this number, the sum is equal to the number obtained by interchanging the digits. Find the number.

(6) The total cost of 6 books and 7 pens is 79 rupees and the total cost of 7 books and 5 pens is 77 rupeess. Find the cost of 1 book and 2 pens.

(7*) The ratio of incomes of two persons is 9 : 7. The ratio of their expenses is 4 : 3. Every person saves rupees 200, find the income of each.

(8*) If the length of a rectangle is reduced by 5 units and its breadth is increased by 3 units, then the area of the rectangle is reduced by 8 square units. If length is reduced by 3 units and breadth is increased by 2 units, then the area of rectangle will increase by 67 square units. Then find the length and breadth of the rectangle.

(9*) The distance between two places A and B on road is 70 kilometers. A car starts from A and the other from B. If they travel in the same direction, they will meet after 7 hours. If they travel towards each other they will meet after 1 hour, then find their speeds.

(10*) The sum of a two digit number and the number obtained by interchanging its digits is 99. Find the number.

Activity : Find the fraction.

$$\frac{\boxed{\text{Numerator } x}}{\boxed{\text{Denominator } y}}$$

If numerator is multiplied by 3 and 3 is subtracted from the denominator then the fraction obtained is $\frac{18}{11}$

Equation I

$$11x - 6y + 18 = 0$$

If numerator is increased by 8 and denominator is doubled then the resulting fraction is $\frac{1}{2}$

Equation II

$$x - y + 8 = 0$$

$\therefore$ Given fraction = $\frac{\square}{\square}$

Verify the answer obtained.

6 Financial Planning

Let's study.

- **Introduction to financial planning**
- **Tax structure**
- **Savings and Investments**
- **Computation of Income -tax**

Let's discuss.

Anagha : Shall we buy computer ?
Mother : Ok, let's buy it next year.
Anagha : Mamma, why not now ?
Mother : Anagha, you don't know how expensive it is !
Anagha : You mean we will have to save up for it, right ?
Mother : Yes, thats the thing.

We often hear such conversations.

Everyone requires money to meet a variety of needs. That is why after spending on the necessities of the present, everyone tries to save money to make provisions for the future needs. That is what we call 'Saving' money. In order to protect our savings or even to make them grow, we keep them as fixed deposits or buy immovable properties such as a house, land etc. That is what we call 'investment'.

Every investor, first spends the amount required to meet primary necessities and saves the remaining amount. One also uses these savings to make a carefully considered investments. This is called **financial planning.** The main purpose of financial planning is protection and growth of the wealth.

Financial planning is useful for making provisions for the predictable and unpredictable expenses that each of us has to meet in our life.

Predictable expenses	Unpredictable expenses
(1) Education of children and other expenses for them	(1) Natural disasters
(2) Capital for a profession or business	(2) Medical expenses for a family member.
(3) Buying a vehicle	(3) Loss due to an accident
(4) Buying or building a house.	(4) Sudden death
(5) Old age requirements.	

The above considerations make it quite clear why financial planning is a must. However some important points must be kept in mind as we plan our finances.

Let's learn.

Savings

(1) It is for our own benefit to keep our savings safe and to make them grow. Our savings remain safe in a bank or in a post office. Money saved in a bank is also useful for cashminus transactions. This way, we do not have to carry large amounts of cash or worry about losing it or getting it stolen.

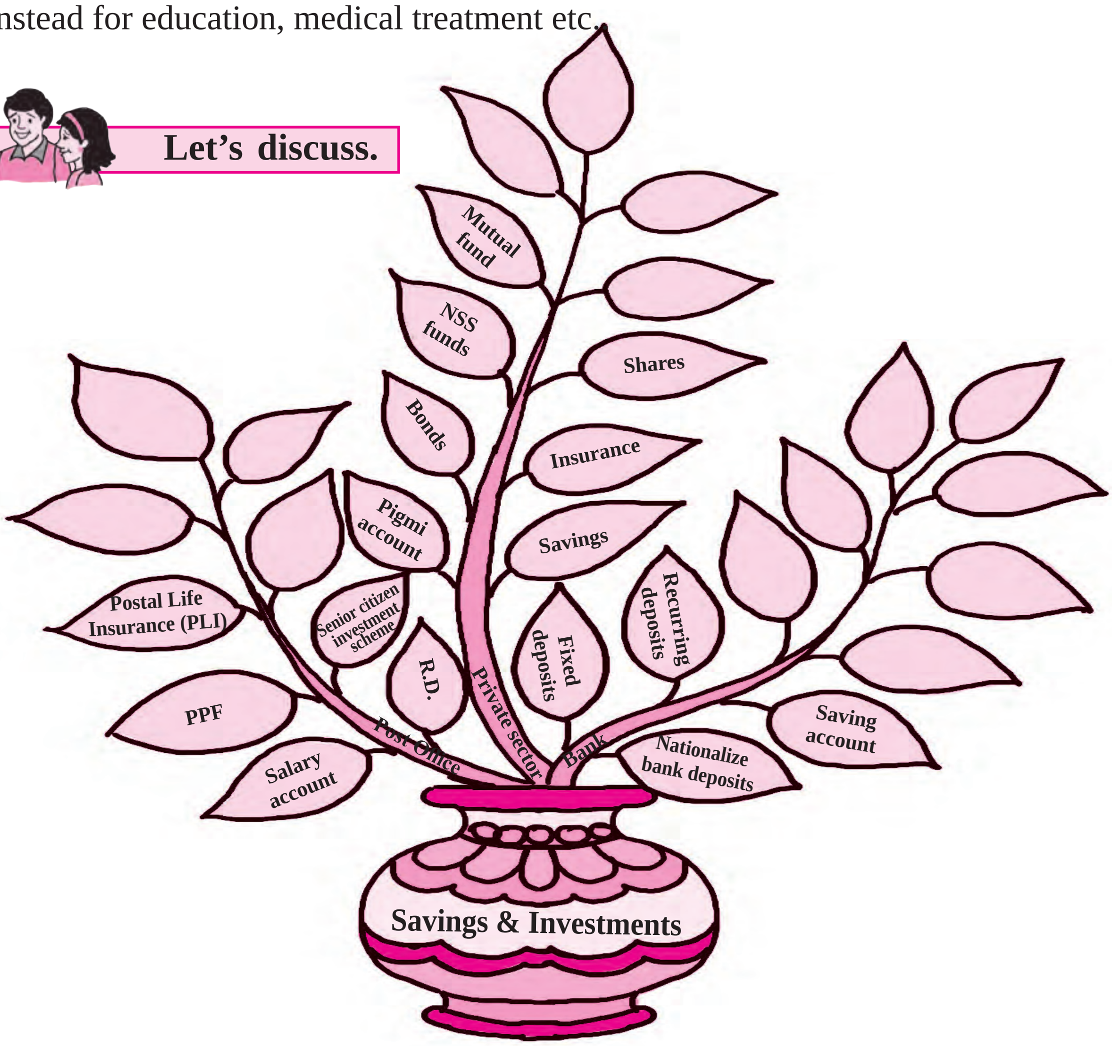
(2) If the money we get or earn is in the form of cash and we keep it as it is, without investing it, its value diminishes with time. For example, if today your can buy two pencils for ten rupees, a few years hence, you may be able to buy only one for that amount.

(3) If the amount invested is used for expanding a business, to start an industry or other such purposes, it contributes to the growth of the national production.

(4) If some part of the income is spent for a socially useful cause everyone benefits from it in the long run.

(5) After spending on necessities it is beneficial to limit spending on luxuries and to save, instead for education, medical treatment etc.

Let's discuss.



Observe the above picture, which shows some modes of investment. Discuss them. Find out other modes of investment and write them in the blank spaces in the picture.

Let's learn.

Investments

Investments are of many types. Investors often favour institutions like banks and postal departments for investing their money because it is safe there. There is a certain risk in investing money in shares, mutual funds, etc. That is because this money is invested in a business or industry and if that incurs a loss, the investor suffers the loss too. On the other hand, if it makes a profit the money is safe and there is the opportunity to get a dividend too.

An investor must take two important points into account when making an investment namely the risk and the gain. It is possible to make big gains by taking greater risk. However it must be kept in mind that the greater risk can also lead to greater loss.

Study the following examples based on income and investment.

Ex(1) Shamrao's income in 2015-16 after paying all taxes is Rs. 6,40,000 . He pays Rs. 2000 per month for insurance and 20% of his annual income into his provident fund. He puts aside Rs. 500 per month for emergencies. How much money does he have for yearly spending ?

Solution : (i) Annual income = 6,40,000 rupees

(ii) Insurance premium = $2000 \times 12 = 24{,}000$ rupees

(iii) Contribution to provident fund = $6{,}40{,}000 \times \frac{20}{100} = 1{,}28{,}000$ rupees

(iv) Amount put aside for emergency = $500 \times 12 = 6000$ rupees

$\therefore$ Total planned expenditure = $24{,}000 + 1{,}28{,}000 + 6{,}000 = 1{,}58{,}000$ rupees

$\therefore$ Amount available for yearly expenses = $6{,}40{,}000 - 1{,}58{,}000 = 4{,}82{,}000$ rupees

Ex(2) Mr. Shah invested Rs. 3,20,000 in a bank at 10% compound interest . He also invested Rs. 2,40,000 in mutual funds. At market rates he got Rs. 3,05,000 after 2 years. How much did he gain ? Which of his investments was more profitable ?

Solution : (i) We shall first calculate the compound interest on the money invested in the bank.

Compound interest = Amount - Principal

That is, $I = A - P$

$$= P\left(1+\frac{r}{100}\right)^n - P$$

$$= P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$= 3{,}20{,}000\left[\left(1+\frac{10}{100}\right)^2 - 1\right]$$

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$$= 3{,}20{,}000\left[(1.1)^2 - 1\right]$$

$$= 3{,}20{,}000\,[1.21 - 1]$$

$$= 3{,}20{,}000 \times 0.21$$

$$= 67{,}200 \text{ rupees}$$

Mr. Shah invested Rs. 3,20,000 in the bank and got Rs. 67,200 as interest.

Let us see percentage of interest obtained on the investment.

Percentage of interest = $\dfrac{100 \times 67200}{3{,}20{,}000} = 21$

$\therefore$ The investment in the bank gave a profit of 21%.

(ii) The amount Mr. Shah got at the end of 2 years from the mutual fund = 3,05,000 rupees

$\therefore$ The gain from the mutual fund = 3,05,000 – 2,40,000 = 65,000 rupees

$\therefore$ Percentage gain = $\dfrac{65000 \times 100}{2{,}40{,}000} = 27.08$

The investment in the mutual fund yielded a profit of 27.08% .

It is clear that Mr. Shah's investment in the mutual fund was more profitable.

Ex(3) Mr. Shaikh invested Rs. 4,00,000 in a glass industry. After 2 years he received Rs. 5,20,000 from the industry. Putting aside the original investment, he invested his gains in a fixed deposit and in shares in the ratio 3 : 2. How much amount did he invested originally in each of the schemes ?

Solution : Mr. Shaikh's profit at the end of 2 years = 5,20,000 – 4,00,000 = 1,20,000 rupees

Amount invested in the fixed deposit $= \dfrac{3}{5} \times 1{,}20{,}000$

$= 3 \times 24{,}000$

$= 72{,}000$ rupees

Amount invested in shares $= \dfrac{2}{5} \times 1{,}20{,}000$

$= 2 \times 24{,}000$

$= 48{,}000$ rupees

Mr. Shaikh invested 72000 rupees in the fixed deposit and 48,000 rupees in shares.

Ex(4) The ratio of Mr. Anil's monthly income to expenditure is 5 : 4,. For Mr. Aman the same figure is 3 : 2. Also, 4% of Aman's monthly income is equal to 7% of Anil's monthly income. If Anil's monthly expenditure is 96,000 rupees

(i) Find Aman's annual income. (ii) Savings made by Mr. Anil and Mr. Aman.

Solution: We know that savings = Income – Expenditure

Anil's income to expenditrue 5 : 4

Suppose Anil's income is $5x$.

Anil's expenditure is $4x$

Aman's income to expenditrue 3 : 2

Suppose Aman's income is $3y$.

Aman's expenditure is $2y$

Anil's monthly income is 9600 rupees,

$$\therefore\ 5x = 9600$$
$$x = 1920$$

Monthly expenditure = $4x = 4 \times 1920 = 7680$ rupees.

Anil's monthly expenditure is 7680 rupees. $\therefore$ Anil's saving is 1920 rupees.

4% of Aman's income = 7% of Anil's income

$$\therefore\ \frac{4}{100} \times 3y = 9600 \times \frac{7}{100}$$

$$\therefore\ 12y = 9600 \times 7$$

$$\therefore\ y = \frac{9600 \times 7}{12} = 5600$$

Aman's income $= 3y = 3 \times 5600 = 16{,}800$ rupees

Aman's expenditure $= 2y = 2 \times 5600 = 11{,}200$ rupees

$\therefore$ Aman's savings = 16,800 – 11,200 = 5,600 rupees

Aman's monthly income is Rs. 16,800 and Aman's saving is Rs. 5,600

Anil's monthly saving is 1,920 rupees.

Activity I : Amita invested some part of 35000 rupees at 4% and the rest at 5% interest for one year. Altogether her gain was Rs. 1530. Find out the amounts she had invested at the two different rates. Write your answer in words.

Invested Rs. x at 4% rate

Invested Rs. y at 5% rate

$\boxed{} + \boxed{} = 35000$ ..... (I)

Interest

$\frac{4}{100}x + \frac{5}{100}y = 1530$ ..... (II)

$x = \boxed{}$ $y = \boxed{}$

Activity: (1) With your parent's help write down the income and expenditure of your family for one week. Make 7 columns for the seven days of the week. Write all expenditure under such heads as provisions, education, medical expenses, travel, clothes and miscellaneous. On the credit side write the amount received for daily expenses, previous balance and any other new income.

(2) In the holidays, write the accounts for the whole month.

Activity II : Study the Income Expenditure of Govind on page no.52 Discuss the methods a farmer may use who does dry land farming to enhance his income. Some students have expressed their opinions.

Sohil : Farmers get money only when they sell their produces. This profit must be sufficient to sustain for the whole year. So, financial planning is very important for him.

Prakash : His income will increase if agricultural products gets a reasonable price.

Nargis : A law of economics states that if the supply of a commodity far exceeds its demand then its price falls. Naturally profits will also be reduced.

Rita : If the farm production is in excess and if there is a fear of fall in prices, it can be stocked and sent to the market only when prices recover again.

Azam : For that, we need good warehouses.

Reshma : Credit should be easily available to farmers at low rates of interest.

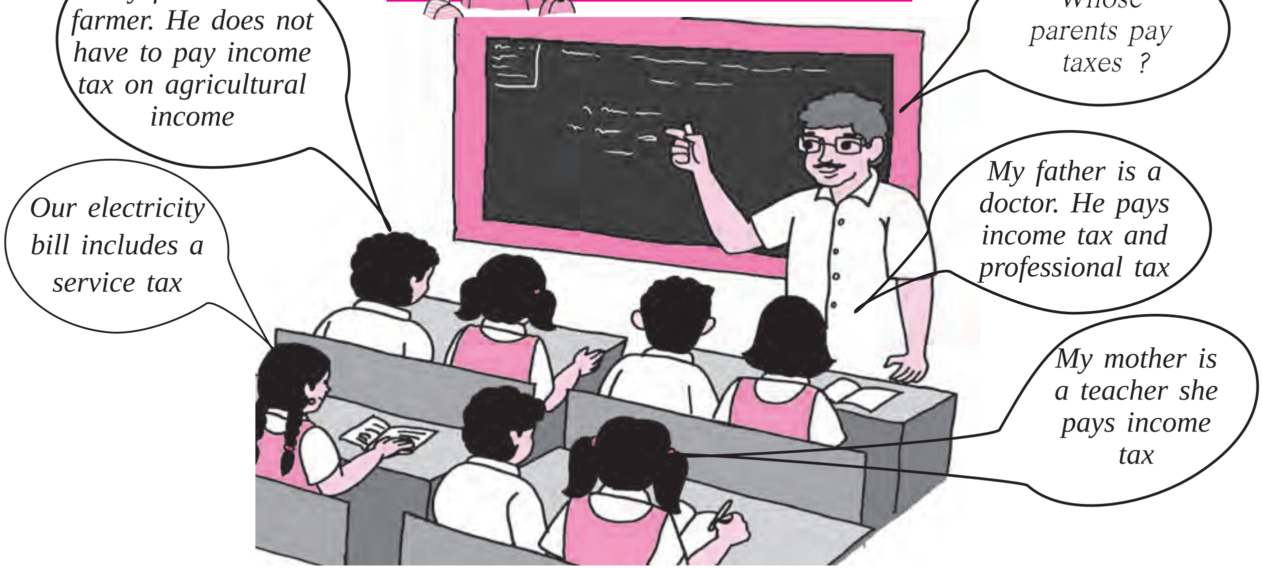
Vatsala : Other farm based businesses like dairy and poultry can provide additional income. Besides, dung and urine obtained from farm animals, can also provide good quality manure.

Kunal : If then start agro-processing units and make preserves like squashes, jams, pickles, pulps or dried vegetables they could sell these packed products all the year round. They could even start producing more of those tins which can be exported.

Practice set 6.1

1. Alka spends 90% of the money that she receives every month, and saves Rs. 120. How much money does she get monthly ?
2. Sumit borrowed a capital of Rs. 50,000 to start his food products business. In the first year he suffered a loss of 20%. He invested the remaining capital in a new sweets business and made a profit of 5%. How much was his profit or loss computed on his original capital ?
3. Nikhil spent 5% of his monthly income on his children's education, invested 14% in shares, deposited 3% in a bank and used 40% for his daily expenses. He was left with a balance of Rs. 19,000. What was his income that month ?.
4. Mr. Sayyad kept Rs. 40,000 in a bank at 8% compound interest for 2 years. Mr. Fernandes invested Rs. 1,20,000 in a mutual fund for 2 years. After 2 years, Mr. Fernandes got Rs. 1,92,000. Whose investment turned out to be more profitable ?
5. Sameera spent 90% of her income and donated 3% for socially useful causes. If she left with Rs. 1750 at the end of the month, what was her actual income ?

Let's discuss.



What is a tax ? Which are different types of taxes ? Find out more information on following websites

ICT Tools or Links — www.incometaxindia.gov.in, www.mahavat.gov.in, www.gst.gov.in

Let's learn.

Levying of taxes or Taxation

The government makes many plans for the development of the country. It requires large amounts of money for implementing these schemes. By charging different types of taxes the funds are generated for implementation of these schemes.

Utility of taxes

- Provision of infrastructure / basic amenities.
- Implementing various welfare schemes.
- Implementing schemes of development and research in various fields.
- Maintaining law and order.
- Giving aid to people affected by natural disasters.
- Defence of the country and its citizens etc.

Types of taxes

Direct taxes	Indirect taxes
Taxes which are paid directly by the taxpayer are called direct taxes. Examples : Income tax, wealth tax, profession, customs duty, etc.	Taxes which are not paid directly by the taxpayer are called indirect taxes. Examples : Central sales tax, value added tax, service tax, excise duty, etc.

The types of taxes listed above are in accordance with the existing tax structure.

Project : Obtain more information about different types of taxes from employees and professionals who pay taxes.

Let's learn.

Income tax

If the income earned in India by an individual, institute or authorised industry exceeds the limit specified under the Income Tax Act, income tax is levied on it. In this chapter, we shall consider only those taxes which are to be paid by individuals. Income tax is levied by the central Government in India, income tax is levied under following two acts.

(1) Income Tax Act 1961 which came into force on. 01.04.1962.

(2) The act passed every year by parliament which makes financial provisions.

Every year sometime in February the finance Minister presents the budget for the next financial year. It has proposals for the income tax rates. Once parliament passes the budget the proposed rates become applicable in the following year.

Income tax rates are fixed every year in the budget.

Some Income tax related terms :

- **An assessee :** Any person liable to pay income tax according to the Income Tax Rules is termed an assessee.
- **Financial year :** The period of one year during which the taxable income has been earned is called a financial year. In our country, at present, the financial year is from 1st April to 31st March.
- **Assessment year :** The financial year immediately following a particular financial year is called the assessment year. The tax payable for the previous financial year is calculated during the current year. i.e. the assessment year.

Financial year and Assessment year will be clear from the table below.

Financial Year	Assessment Year
2016-17 : 01-04-2016 to 31-03-2017	2017-18
2017-18 : 01-04-2017 to 31-03-2018	2018-19

- **Permanent Account Number** (PAN) : On applying for it, every tax-payer gets a unique ten digit alphanumeric number from the Income Tax Department. (PAN). We are required to mention this number in many important documents and financial transactions.

Use of the PAN : It is binding to write our PAN on the challan used for paying our income tax to the IT Department or our Income Tax Returns and other official correspondence. PAN card can also be used as a proof of identity.

Let's learn.

Computation of income tax

As income tax is a tax levied on income, it is necessary to know about the different sources of income.

There are five main heads of income.

(1) Income from salary.
(2) Income from house/property.
(3) Income from business or profession
(4) Income from Capital gain
(5) Income from other sources.

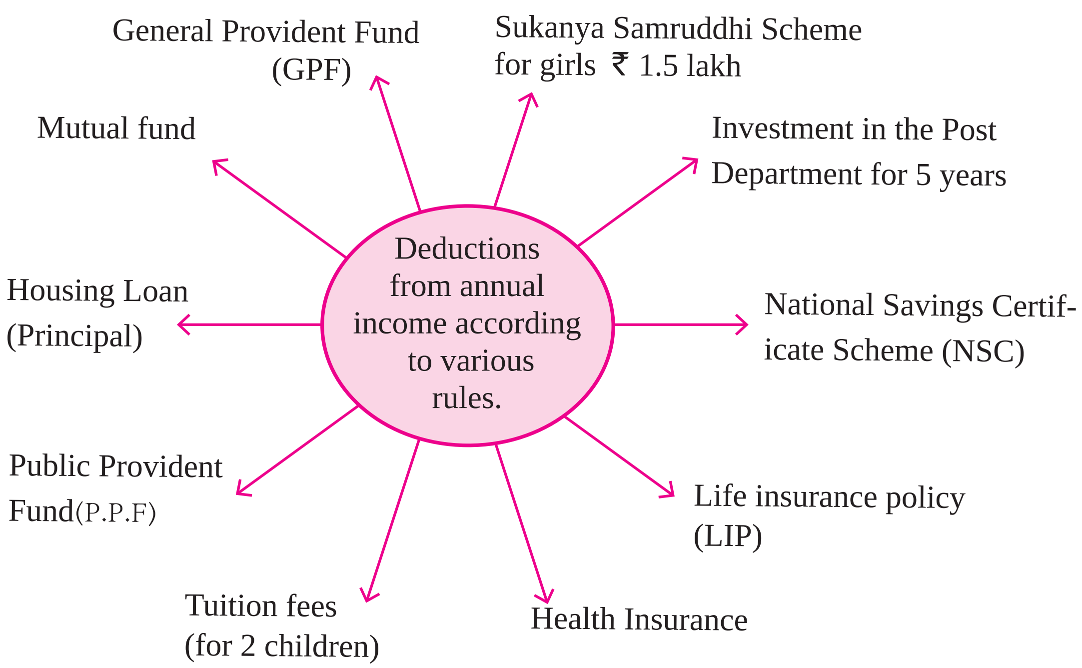
Important considerations for computing the income tax payable by a salaried employee :

The total annual income (Gross Total Income) is taken into account for calculating the tax payable. According to the sections 80C, 80D, 80G etc. of the Income Tax Act some deductions can be availed from the total annual income. The amount remaining after these deductions are made, is called taxable income. Income tax is levied on this taxable income.

Every year, the rules for computing income tax are changed. Hence, it is important to know the latest rules when actually calculating the tax payable.

No tax is levied up to a certain limit of taxable income. This is called the basic exemption limit.

- Farmer's income from agricultural produce is exempt from taxation.
- Under section 80 G of IT Act donations to the Prime Minister's relief fund, Chief Minister's Relief Fund and certain other donations recognized to institutions/ organizations are exempt from taxation.
- Under section 80 D, installments of premium for health insurance are exempt from taxation.
- Generally, the maximum premissible deduction to various kinds of savings under section 80C is Rs. 1,50,000.



Tax rates according to age of taxpayers are fixed in each year's budget. Samples of tables showing tax rates for different income slabs are given below.

Table I

Individuals (up to the age of 60 years)			
Taxable Income slabs (In Rupees)	Income Tax	Education cess	Secondary and Higher Education cess
Up to 2,50,000	Nil	Nil	Nil
2,50,001 to 5,00,000	5% (On taxable income minus two lakh fifty thousand)	2% of Income tax	1% of Income tax
5,00,001 to 10,00,000	₹ 12,500 + 20% (On taxable income minus five lakh)	2% of Income tax	1% of Income tax
More than 10,00,000	₹ 1,12,500 + 30% (On taxable income minus ten lakh)	2% of Income tax	1% of Income tax
(Surcharge equal to 10% of income tax payable by individuals having an annual income of 50 lakh to one crore rupees and 15% of income tax by individuals having an annual income greater that one crore rupees)			

Activity : Use Table I given above and write the appropriate amount/figure in the boxes for the example given below.

Ex. Mr. Mehta's annual income is Rs. 4,50,000

- If he does not have any savings by which he can claim deductions from his income, to which slab does his taxable income belong ? $\boxed{}$
- What is the amount on which he will have to pay income tax and at what percent rate ?
  on ₹ $\boxed{}$ percentage $\boxed{}$
- On what amount will the cess be levied ? $\boxed{}$

Table II

Senior citizens (Age 60 to 80 years)			
Taxable Income slabs (In Rupees)	Income Tax	Education cess	Secondary and Higher Education cess
Upto 3,00,000	Nil	Nil	Nil
3,00,001 to 5,00,000	5% (On taxable income minus three lakh)	2% of Income tax	1% of Income tax
5,00,001 to 10,00,000	₹ 10,000 + 20% (On taxable income minus five lakh)	2% of Income tax	1% of Income tax
More than 10,00,000	₹ 1,10,000 + 30% (On taxable income minus ten lakh)	2% of Income tax	1% of Income tax
(Surcharge equal to 10% of income tax payable by individuals having an annual income of 50 lakh to one crore rupees and 15% of income tax by individuals having an annual income greater that one crore rupees)			

Activity : Use table II to carry out the following activity

Ex. Mr. Pandit is 75 years of age. Last year his annual income was 13,25,000 rupees. How much is his taxable income ? How much tax does he have to pay ?

13,25,000 – 10,00,000 = 3,25,000

According to the table he must first pay Rs. 1,10,000 as income tax. In addition, on 3,25,000 rupees he has to pay 30% income tax.

$3,25,000 \times \frac{30}{100} = \boxed{}$ rupees.

Therefore, his total income tax amounts to $\boxed{} + \boxed{} = \boxed{}$

Besides this, education cess willl be 2% of income tax $\boxed{} \times \frac{2}{100} = \boxed{}$.

A secondary and higher education cess at 1% of income tax $= \boxed{} \times \frac{1}{100} = \boxed{}$

∴ Total income tax = Income tax + education cess + secondary and higher education cess.

$= \boxed{} + \boxed{} + \boxed{}$

$= \boxed{₹\ 2,13,725}$

Table III

Super senior citizens (Age above 80 years)			
Taxable Income slabs (In Rupees)	Income Tax	Education cess	Secondary and Higher Education cess
Upto 5,00,000	Nil	Nil	Nil
5,00,001 to 10,00,000	20% (On taxable income minus five lakh)	2% of Income tax	1% of Income tax
More than 10,00,000	₹ 1,00,000 + 30% (On taxable income minus ten lakh)	2% of Income tax	1% of Income tax
(Surcharge equal to 10% of income tax payable by individuals having an annual income of 50 lakh to one crore rupees and 15% of income tax by individuals having an annual income greater that one crore rupees)			

Project : Obtain information about sections 80C, 80G, 80D of the Income Tax Act.
Study a PAN card and make a note of all the information it contains..
Obtain information about all the devices and means used for carrying out cash minus transactions.

From the following solved examples we will learn how the tables given and the deductions available to individuals are used to compute income tax.

Ex(1) Mr. Mhatre is 50 years old. His gross total income is Rs. 12,00,000. He has invested in the following amounts in different schemes.

(i) Insurance premium : ₹ 90,000 (ii) Investment in provident fund : ₹ 25,000

(iii) Investment in PPF : ₹ 15,000 (iv) National Savings Certificate : ₹ 20,000

Find out the permissible deductions, taxable income, and the income tax payable.

Solution : (1) Total Yearly income = 12,00,000 rupees.

(2) Total savings under section 80C.

Savings	Amount of Savings (rupees)
(i) Insurance premium	90,000
(ii) Provident Fund	25,000
(iii) Public Provident Fund	15,000
(iv) National Savings Certificate	20,000
Total	1,50,000

According to section 80C, a maximum deduction of Rs. 1,50,000 is permissible.

(3) $\therefore$ Taxable Income = Amount in [1] – Amount in [2]

= 12,00,000 – 1,50,000 = 10,50,000

(4) We shall use Table I to calculate Mr. Mhatre's total income tax .

Mr. Mhatre's taxable income = ₹ 10,50,000 which is greater than ten lakh rupees.

$\therefore$ According to Table (I) Income tax = ₹ 1,12,500 + 30% (of total income minus 10 lakh)

$\therefore$ 10,50,000 – 10,00,000 = 50,000

$\therefore$ Income tax $= 1,12,500 + 50,000 \times \frac{30}{100}$

$= 1,12,500 + 15,000$

$= 1,27,500$

We must also include 2% education cess and 1% secondary and higher education cess.

Education cess $= 1,27,500 \times \frac{2}{100} = 2550$ rupees

Secondary and higher education cess $= 1,27,500 \times \frac{1}{100} = 1275$ rupees

$\therefore$ Total income tax = 1,27,500 + 2550 + 1275 = 1,31,325 rupees

Mr. Mhatre’s tax payable = ₹ 1,31,325

Ex(2) Mr. Ahmed, a 62 year old senior citizen is employed in a private company. His total annual income is Rs.6,20,000. He has contributed Rs. 1,00,000 to the Public Provident Fund and paid a premium of Rs. 80,000 for the year for health insurance and a donation of Rs. 10,000 to CM's Relief Fund. What is tax payable ?

Solution : (1) Total Yearly income = 6,20,000 rupees

(2) Total deduction (According to 80C)

(i) Public Provident Fund	=	1,00,000 rupees
(ii) Insurance	=	80,000 rupees
		1,80,000 rupees

(iii) Section 80C permits a maximum deduction of Rs. 1,50,000 rupees.

(3) Amt. given to CM's Relief Fund (According to 80 G) = 10000 rupees.

(4) Taxable income = (1) – [(2) + (3)]

= 6,20,000 – [1,50,000 + 10000]

= 4,60,000 rupees

From table II we see that the taxable income is in the slab 3 lakh to 5 lakh rupees.

$\therefore$ Income tax $= (\text{Taxable income} - 3,00,000)\times \frac{5}{100}$

$= (4,60,000 - 3,00,000)\times \frac{5}{100}$

$= 1,60,000 \times \frac{5}{100}$

= 8000 rupees

Education cess is levied on income tax.

Education cess $= 8,000 \times \frac{2}{100} = 160$

Secondary and higher education cess : $8,000\times \frac{1}{100} = 80$

$\therefore$ Total Income tax = 8000 + 160 + 80 = ₹ 8,240

$\therefore$ tax payable by Mr. Ahmed is ₹ 8,240.

Ex(3) Mrs. Hinduja's age is 50 years. Last year her taxable income was Rs. 16,30,000. How much income tax has she to pay ?

Solution : Mrs Hinduja's taxable income is in the bracket of Rs. 10,00,000 and above.

Let us use Table I to compute her income tax. Accordingly, for income greater than Rs. 10,00,000.

Income tax = Rs. 1,12,500 + 30% of total income minus ten lakh

Mrs. Hinduja's income minus ten lakh $= 16,30,000 - 10,00,000$

$= 6,30,000$ rupees

From table I

Income tax $= 1,12,500 + 6,30,000 \times \frac{30}{100}$

$= 1,12,500 + 30 \times 6,300$

$= 1,12,500 + 1,89,000$

$= 3,01,500$ rupees

On this we compute

1% secondary and higher education cess $= \frac{1}{100} \times 3,01,500 =$ ₹ 3015

2% education cess $= \frac{2}{100} \times 3,01,500 =$ ₹ 6030

$\therefore$ total income tax payable $= 3,01,500 + 3015 + 6030$

$= 3,10,545$

$\therefore$ total income tax payable is 3,10,545 rupees

Practice set 6.2

(1) Observe the table given below. Check and decide, whether the individuals have to pay income tax.

S. No.	Individuals	Age	Taxable Income (₹)	Will have to pay income tax or not
(i)	Miss Nikita	27	₹ 2,34,000	
(ii)	Mr. Kulkarni	36	₹ 3,27,000	
(iii)	Miss Mehta	44	₹ 5,82,000	
(iv)	Mr. Bajaj	64	₹ 8,40,000	
(v)	Mr. Desilva	81	₹ 4,50,000	

(2) Mr. Kartarsingh (age 48 years) works in a private company. His monthly income after deduction of allowances is Rs. 42,000 and every month he contributes Rs. 3000 to GPF. He has also bought Rs. 15,000 worth of NSC (National Savings Certificate) and donated Rs. 12,000 to the PM's Relief Fund. Compute his income tax.

Problem set 6

(1) Write the correct alternative answer for each of the following quesitons.

(i) For different types of investments what is the maximum permissible amount under section 80C of income tax ?

(A) 1,50,000 rupees (B) 2,50,000 rupees (C) 1,00,000 rupees (D) 2,00,000rupees

(ii) A person has earned his income during the financial year 2017-18. Then his assessment year is ....

(A) 2016-17 (B) 2018-19 (C) 2017-18 (D) 2015-16

(2) Mr. Shekhar spends 60% of his income. From the balance he donates Rs. 300 to an orphanage. He is then left with Rs. 3,200. What is his income ?

(3) Mr. Hiralal invested Rs. 2,15,000 in a Mutual Fund. He got Rs. 3,05,000 after 2 years. Mr. Ramniklal invested Rs. 1,40,000 at 8% compound interest for 2 years in a bank. Find out the percent gain of each of them. Whose investment was more profitable ?

(4) At the start of a year there were Rs. 24,000 in a savings account. After adding Rs. 56,000 to this the entire amount was invested in the bank at 7.5% compound interest. What will be the total amount after 3 years ?

(5) Mr. Manohar gave 20% part of his income to his elder son and 30% part to his younger son. He gave 10% of the balance as donation to a school. He still had Rs. 1,80,000 for himself. What was Mr. Manohar's income ?

(6^*) Kailash used to spend 85% of his income. When his income increased by 36% his expenses also increased by 40% of his earlier expenses. How much percentage of his earning he saves now ?

(7^*) Total income of Ramesh, Suresh and Preeti is 8,07,000 rupees. The percentages of their expenses are 75%, 80% and 90% respectively. If the ratio of their savings is 16 : 17 : 12, then find the annual saving of each of them.

(8) Compute the income tax payable by following individuals.

(i) Mr. Kadam who is 35 years old and has a taxable income of Rs. 13,35,000.

(ii) Mr. Khan is 65 years of age and his taxable income is Rs. 4,50,000.

(iii) Miss Varsha (Age 26 years) has a taxable income of Rs. 2,30,000.

ICT Tools or Links

Visit www.incometaxindia.gov.in which is a website of the Government of India. Click on the 'incometax calculator' menu. Fill in the form that gets downloaded using an imaginary income and imaginary deductible amounts and try to compute the income tax payable for this income.

7 Statistics

Let's study.

- Sub-divided bar-diagram
- Percentage bar-diagram
- Primary and secondary data
- Grouped and ungrouped frequency distribution
- Cumulative frequency distribution
- Mean, Median and Mode for ungrouped data

Let's recall.

In earlier standards, we have learnt how to draw a simple bar-diagram and a joint bar-diagram. Also, we have observed various graphs from newspapers, magazines, television etc. and gathered information from them.

It is very important to decide according to the nature of the data, what diagram or graph would be suitable to represent it.

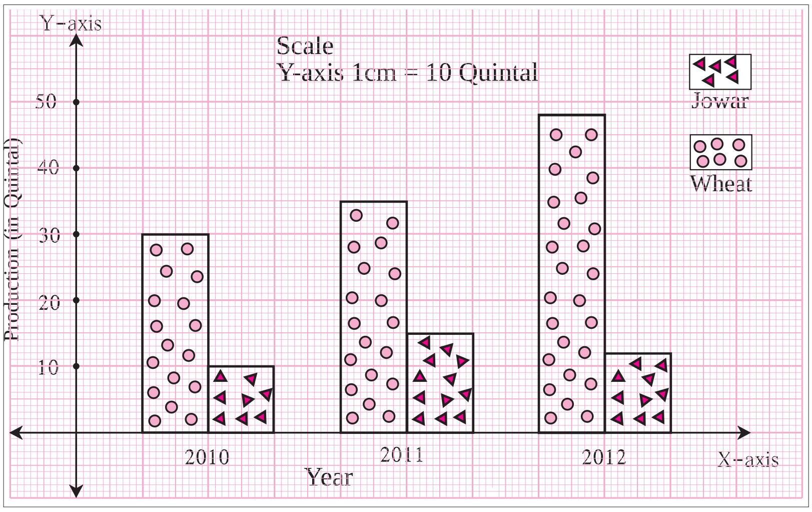
A farmer has produced Wheat and Jowar in his field. The following joint bar diagram shows the production of Wheat and Jowar. From the given diagram answer the following questions :

(i) Which crop production has increased consistently in 3 years ?

(ii) By how many quintals the production of Jowar has reduced in 2012 as compared to 2011?

(iii) What is the difference between the production of Wheat in 2010 and 2012?

(iv) Complete the following table using this diagram.



Year \ Production (in Quintal)	Wheat	Jowar	Total
2010			
2011			
2012	48	12	60

Let's learn

Sub-divided bar diagram

To compare the information in the given data, we can also draw another type of bar-diagram

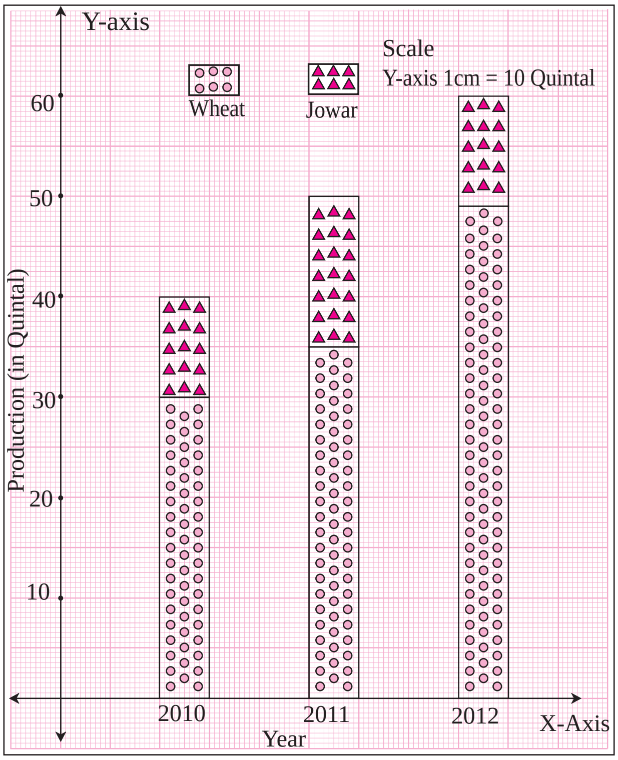
To draw it, we add the numerical values of the entities, decide a scale and show the total by a bar proportional to the scale. Then we divide the bar in parts, proportional to the entities we had added. Hence this type of diagram is called a sub-divided bar diagram.

Now let us show the information in the previous example by a sub-divided bar diagram.

(i) Show the total production of the year 2010 by a bar. The height of the bar should be to the decided scale.

(ii) Show the production of wheat by lower part of the bar, the height of which is to the scale.

(iii) Obviously, remaining part of the bar denotes the production of Jowar for the year.



(iv) Similarly draw divided bars to show productions of the years 2011 and 2012.

When two quantities are compared using percentages, it is more informative. We have studied this before. For example, if there is Rs.600 profit on Rs.2,000 and Rs.510 profit on Rs.1,500; Rs.600 looks greater amount. But if we calculate their percentages they are 30% and 34% respectively. Hence it is clear that Rs.510 profit on Rs.1,500 is a more profitable transaction.

Percentage bar diagram

To compare the given information, in a different way, it is converted into percentages and then a sub-divided bar diagram is drawn. Such diagram is known as 'Percentage bar-diagram'.

The information in the previous example is converted into percentages as shown in the adjacent table.

Year	Production of Wheat (Quintal)	Production of Jowar (Quintal)	Percentage production of Wheat as compared to total production
2010	30	10	$\frac{30}{40} \times 100 = 75\%$
2011	35	15	$\frac{35}{50} \times 100 = 70\%$
2012	48	12	$\frac{48}{60} \times 100 = 80\%$

The information is shown in the percentage bar diagram by following steps

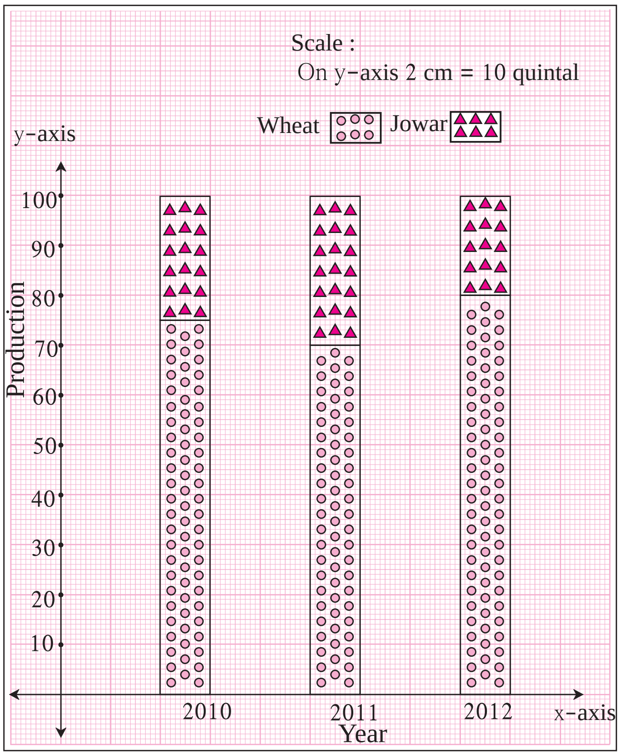
(i) Yearly productions of Wheat and Jowar are converted into percentages.

(ii) The height of each bar to scale is taken as 100.

(iii) The percentage of production of Wheat is shown by the lower part of the bar to the scale.

(iv) The remaining upper part of the bar shows percentage production of Jowar.

Information of more than two entities can be shown by a sub divided bar diagram or by a percentage bar diagram.



Solved examples :

Ex. 1. In the neighbouring figure, percentage bar-diagram is given. Percentage expenses on different items of two families are given. Answer the following questions based on it :

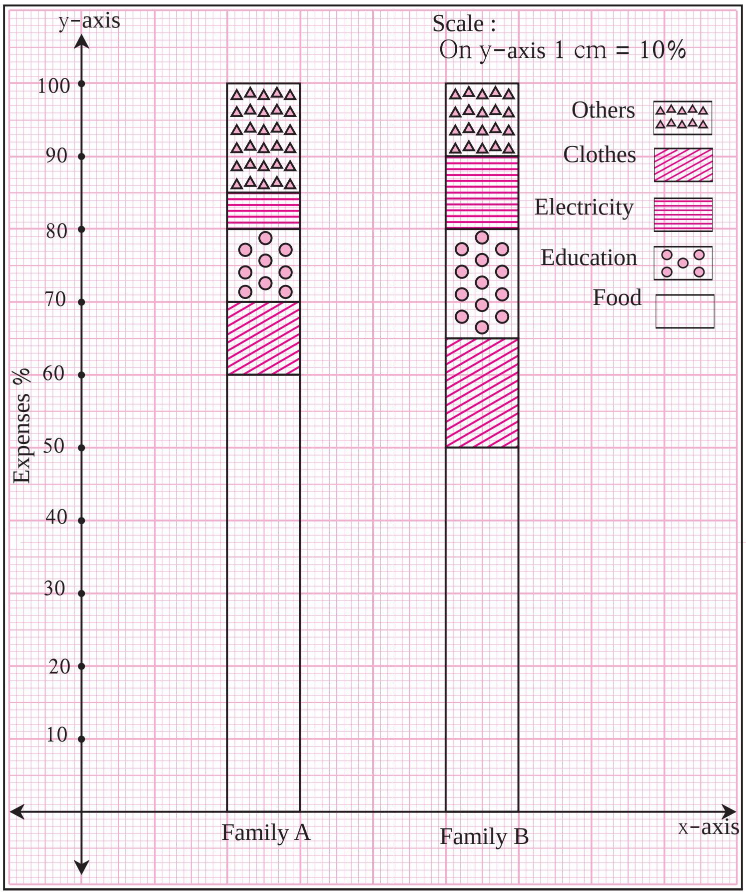
(i) Write the percentage expenses of every component for each family.

(ii) Which family spends more percent of expenses on food as compared to the other and by how much ?

(iii) What are the percentage expenses on other items ?

(iv) Which family shows more percentage expenses on electricity?

(v) Which family's percentage expense is more on education ?



Solution :

(i)

Expenses / Family	Food	Clothes	Education	Electricity	Others
A	60%	10%	10%	5%	15%
B	50%	15%	15%	10%	10%

(ii) Food expenses of family A are more by 10% as compared with family B.

(iii) Other expenses of family A are 15% and that of family B are 10%.

(iv) Percentage expenses on electricity of family B is greater.

(v) Percentage expenses on education are more of family B.

Practice set 7.1

(1) The following table shows the number of Buses and Trucks in nearest lakh units. Draw percentage bar-diagram. (Approximate the percentages to the nearest integer)

Year	No. of Trucks	No. of Buses
2005-2006	47	9
2007-2008	56	13
2008-2009	60	16
2009-2010	63	18

(2) In the table given below, the information is given about roads. Using this draw sub-divided and percentage bar-diagram (Approximate the percentages to the nearest integer)

Year	Permanent Roads (Lakh km.)	Temporary Roads (Lakh km.)
2000-2001	14	10
2001-2002	15	11
2003-2004	17	13
2007-2008	20	19

Activity : In the following table, the information of number of girls per 1000 boys is given in different States. Fill in the blanks and complete the table.

States	Boys	Girls	Total	Percentage of boys	Percentage of girls
Assan	1000	960	1960	$\frac{1000}{1960} \times \frac{100}{1} = 51\%$	$100 - 51 = 49\%$
Bihar	1000	840	1840		
Punjab	1000	900			
Keala	1000	1080			
Maha-rashtra	1000	900			

Draw percentage bar-diagram from this information and discuss the findings from the diagram.

Use your brain power! On page 111, for the given activity, the information of number of girls per 1000 boys is given for five states. The literacy percentage of these five States is given below.

Assam (73%), Bihar (64%), Punjab (77%), Kerala (94%), Maharashtra (83%).

Think of the number of girls and the literacy percentages in the respective states. Can you draw any conclusions from it ?

Let's discuss.

To show following information diagrammatically, which type of bar-diagram is suitable?

(1) Literacy percentage of four villages.

(2) The expenses of a family on various items.

(3) The numbers of girls and boys in each of five divisions.

(4) The number of people visiting a science exhibition on each of three days.

(5) The maximum and minimum temperature of your town during the months from January to June.

(6) While driving a two-wheeler, number of people wearing helmets and not wearing helmet in 100 families.

Let's learn.

Statistics

Suppose, a large group (population) is to be studied with a particular aspect. (For example, blood pressures of senior citizens in a locality) For the purpose, a sufficiently small part of the group is selected randomly. This small group represents the large group (sample). The necessary information is gathered from the representative group which, in general, is numerical in most of the cases. The analysis of the information enables us to draw conclusions. The study of this type is called 'Statistics'.

The word Statistics is originated from the Latin word ‘status’, which means situation of a state. This suggests that in ancient times statistics was used for administrative purposes. Today, it is used in many fields of knowledge.

Sir Ronald Aylmer Fisher (17 February 1890 - 29 July 1962) is known as Father of Statistics.

Data collection

Teacher : Suppose, you want to know how much agricultural land is owned by every family in the village. What will you do?

Robert : We will visit each house in the village and record the information about agricultural land owned by them.

Teacher : Correct, my dear students, when we collect information of a group it is called as ‘data’. Generally it is numerical. We must know the purpose of collecting it. If some one collects the information personally by asking questions, taking measurements, etc. it is called as the 'Primary Data'.

Afrin : So, the data collected regarding agricultural land, as Robert said, is primary data.

Teacher : Yes, well said Afrin !

Ramesh : But what to do if we want to collect the above data in a short time ?

Teacher : What Ramesh is saying is right. In this situation we have to use another method of data collection. Think what it could be ?

Ketaki : We can go to villageTalathi office and can get the information from their records.

Teacher : Correct, in some situations, because of lack of time, lack of resources, we can't collect information personally. In such cases, we have to use the information, already collected in the form of records, information published in journals, case-studies etc. The data collected from such sources is known as 'Secondary data'. So as suggested by Ketaki, the data collected from village Talathi office, regarding agricultural land is secondary data.

See the following examples :

(i) The chart made from information published in news paper is secondary data.

(ii) The feedback of customers in a restaurant regarding quality of the food is primary data.

(iii) The heights of students recorded by actual measurements is primary data.

Primary data	Secondary data
1. It requires more time. 2. It is up to date and detailed information. 3. It is correct and reliable.	1. It is readily available, so needs less time 2. It is taken from already collected data. It is not necesssarily up to date. It may lac in details also. 3. It may be less reliable.

Activity : You gather information for several reasons. Take a few examples and discuss whether the data is primary or secondary.

Practice set 7.2

(1) Classify following information as primary or secondary data.

(i) Information of attendance of every student collected by visiting every class in a school.

(ii) The information of heights of students was gathered from school records and sent to the head office, as it was to be sent urgently.

(iii) In the village Nandpur, the information collected from every house regarding students not attending school.

(iv) For science project, information of trees gathered by visiting a forest.,

Let's recall.

Classification of data

Ex.(1) The record of marks out of 20 in Mathematics in the first unit test is as follows.

20, 6, 14, 10, 13, 15, 12, 14, 17, 17, 18, 11, 19, 9, 16, 18, 14, 7, 17, 20,
8, 15, 16, 10, 15, 12, 18, 17, 12, 11, 11, 10, 16, 14, 16, 18, 10, 7, 17, 14,
20, 17, 13, 15, 18, 20, 12, 12, 15, 10

What is the above information called ? -Primary data

What is each of the numbers in the data called ? -A score

Answer the following questions, from the above information.

(i) How many students scored 15 marks ?
(ii) How many students scored more than 15 marks ?
(iii) How many students scored less than 15 marks ?
(iv) What is the lowest score of the group ?
(v) What is the highest score the group ?

Let's discuss.

(1) Was it easy to find out the answers of the above questions ? Did you refer the data frequently ?

(2) What should we do to find answers easily ?

Shamim : We had to refer the data frequently. It was tedious and boaring. If we write the data in ascending or descending order the above answers could be found easily.

According to Shamim's suggestion, lets us arrange the data in ascending order.

6, 7, 7, 8, 9, 10, 10, 10, 10, 10, 11, 11, 11, 12, 12, 12, 12, 12, 13, 13, 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17, 18, 18, 18, 18, 18, 19, 20, 20, 20, 20

Verify that the ascending order of scores helps to find the answers of the questions in Ex. (1) easily.

Let's recall.

Martin : Writing the data in a tabular form can also make the above work easy, We have studied this in previous year. This table is known as 'frequency distribution table'.

Teacher : Correct Martin ! Now let us prepare a table of the information given in example (1).

In example (1), the lowest score is 6 and the highest score is 20. Hence in the table, we write, numbers from 6 to 20 in the column of scores. In second column we record tally marks and in last column, frequency by counting the tally marks. (Complete the table)

Frequency Distribution Table

Score	Tally Marks	Frequency (No.of students)
6	|	1
7	||	2
8		
9		
10	卌	5
11		
12		
13		
14		
15		
16		
17	卌 |	6
18		
19		
20	||||	4
		Total N = 50

Make it sure that the sum of all frequencies, N is 50.

Let's discuss.

(i) Is the above table is very long ?

(ii) If the number of observations are more, is it difficult to make a table ?

Grouped frequency distribution table

Teacher : From above discussion, we conclude that, when number of observations is large, preparing a table is time consuming. What can be done to condense the data and save time ?

Rohit : In this situation, we can group the scores in the data.

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Teacher : Well done Rohit! If we group the scores, that means if we make their classes, then the data will be condensed and time can be saved. Such a table is known as grouped frequency distribution table.

These are two methods of preparing a grouped frequency distribution table.

(1) Inclusive method, (2) Exclusive method.

(1) Inclusive method (Discrete classes)

6, 7, 7, 8, 9, 10, 10, 10, 10, 10, 11, 11, 11, 12, 12, 12, 12, 12, 13, 13, 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17, 18, 18, 18, 18, 18, 19, 20, 20, 20, 20

In the above scores the smallest is $\boxed{}$ and the largest is $\boxed{}$. The difference between largest and smallest scores is 20 – 6 = 14. This difference is called as 'Range of the data'.

By noticing the range, how can we classify the data into convenient classes ?

We can take classes like this.

(i) 6 to 8, 9 to 11, 12 to 14, 15 to 17, 18 to 20 or

(ii) 6 to 10, 11 to 15, 16 to 20.

By taking classes 6 – 10, 11 – 15 & 16 – 20, let us prepare frequency distribution table.

Grouped Frequency Distribution Table (inclusive method)

Class	Tally marks	Frequency (No.of students)
6 – 10	𝍸 𝍸	10
11 – 15	.....	.....
16 – 20	.....	20
		N = 50

While preparing this table, 6, 10 and all scores between them are included in the class 6–10 hence such classes are known as 'Inclusive Classes' of discrete class.

Let's learn.

Basic terms in statistics :

(1) Class : When the observations are divided into suitable groups, each of the groups is called a 'Class'.

(2) Class-Limit : The end values of the classes are called class-limits.
For the class 6-10, the lower class limit is 6 and the upper class limit is 10.

(3) Frequency : The total number of observations in to each class is called the 'frequency' of the that class.
In the above table, there are 20 observations in the class 11 to 15. Hence frequency of the class 11 – 15 is 20

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4. Class width or Class Size or Class-interval : When continuous classes are given, the difference between upper class limit and lower class limit is known as class-width.

For example, if 5 − 10, 10 − 15, 15−20, ...are given classes,

class width of 5−10 is 10 − 5 = 5

5. Class mark : The average of the lower class limit and the upper class limit for a given class is known as class mark

$$\text{Class mark} = \frac{\text{Lower class limit + Upper class limit}}{2}$$

For example, class mark the for class 11 to 15 $= \dfrac{\square + \square}{2} = \dfrac{26}{2} = 13$

(2) Exclusive method (Continuous classes)

Ex. 6, 10, 10.5, 11, 15.5, 19, 20, 12, 13 are the given observations. By taking classes 6-10, 11-15, 16-20 prepare grouped frequency distribution table

Solution :

Classes	Tally marks	Frequency (f)
6-10	||	2
11-15	|||	3
16-20	||	2

In the above table, we could not include observations 10.3 and 15.7.

Because the numbers 10.3 and 15.7 cannot be included in any of the classes 6-10, 11-15, 16-20. Hence in order to include them, we have to change the structure of the classes. Therefore if we take class intervals as 5-10 10-15, 15-20 the problem will be solved. The scores 10.3 and15.7 can be included in the classes 10-15 and 15-20 respectively. But still a question arises. In which interval the score 10 should be included ? In 5-10 or 10-20 ? To overcome the difficulty, we follow a convension. We will include the score 10 in the class 10-15 instead of 5-10. That is the upper class limit of a class should be excluded from the class. Therefore, this is called the exclusive method of classification.

Now taking classes accordingly and as per the convention of exclusion, we can prepare the table as follows.

Grouped frequency distribution table (Exclusive method)

Class interval Marks	Tally marks	Frequency (No. of students)
5-10	|	1
10-15	$\cancel{\|\|\|\|}$	5
15-20	||	2
20-25	|	1

Remember this !

Frequency Distribution Table

Ungrouped

Ages of 9th std. students	No. of Students
14	12
15	23
16	10

Grouped

Inclusive Method
(Discrete classes)

Shoe size	No.of Students
2-4	12
5-7	29
8-10	7

Exclusive Method
(Continuous classes)

Height (cm)	No.of students
145-150	18
150-155	27
155-160	3

Practice set 7.3

(1) For class interval 20-25 write the lower class limit and the upper class limit.

(2) Find the class-mark of the class 35-40.

(3) If class mark is 10 and class width is 6 then find the class.

(4) Complete the following table.

Classes (age)	Tally marks	Frequency (No. of students)
12-13	$\cancel{||||}$	$\square$
13-14	$\cancel{||||}$ $\cancel{||||}$ $||||$	$\square$
14-15		$\square$
15-16	$||||$	$\square$
		$N=\sum f = 35$

(5) In a 'tree plantation' project of a certain school there are 45 students of 'Harit Sena.' The record of trees planted by each student is given below :

3, 5, 7, 6, 4, 3, 5, 4, 3, 5, 4, 7, 5 , 3, 6, 6, 5, 3, 4, 5, 7, 3, 5, 6, 4, 4, 3,
5, 6, 6, 4, 3, 5, 7, 3, 4, 5, 7, 6, 4, 3, 5, 4, 4, 7.

Prepare a frequency distribution table of the data.

(6) The value of π upto 50 decimal places is given below :

3.14159265358979323846264338327950288419716939937510

From this information prepare an ungrouped frequency distribution table of digits appearing after the decimal point.

(7) In the tables given below, class-mark and frequencies is given. Construct the frequency tables taking inclusive and exclusive classes.

(i)

Class width	Frequency
5	3
15	9
25	15
35	13

(ii)

Class width	Frequency
22	6
24	7
26	13
28	4

(8) In a school, 46 students of 9th standard, were told to measure the lengths of the pencils in their compass-boxes in centimeters. The data collected was as follows.

16, 15, 7, 4.5, 8.5, 5.5, 5, 6.5, 6, 10, 12,
13, 4.5, 4.9, 16, 11, 9.2, 7.3, 11.4, 12.7, 13.9, 16,
5.5, 9.9, 8.4, 11.4, 13.1, 15, 4.8, 10, 7.5, 8.5, 6.5,
7.2, 4.5, 5.7, 16, 5.7, 6.9, 8.9, 9.2, 10.2, 12.3, 13.7,
14.5, 10

By taking inclusive classes 0-5, 5-10, 10-15.... prepare a grouped frequency distribution table.

(9) In a village, the milk was collected from 50 milkmen at a collection center in litres as given below :

27, 75, 5, 99, 70, 12, 15, 20, 30, 35, 45, 80,
77, 90, 92, 72, 4, 33, 22, 15, 20, 28, 29, 14,
16, 20, 72, 81, 85, 10, 16, 9, 25, 23, 26, 46,
55, 56, 66, 67, 51, 57, 44, 43, 6, 65, 42, 36,
7, 35.

By taking suitable classes, prepare grouped frequency distribution table.

(10) 38 people donated to an organisation working for differently abled persons. The amount in rupees were as follows :

101, 500, 401, 201, 301, 160, 210, 125, 175, 190, 450, 151,
101, 351, 251, 451, 151, 260, 360, 410, 150, 125, 161, 195,
351, 170, 225, 260, 290, 310, 360, 425, 420, 100, 105, 170,
250, 100.

(i) By taking classes 100-149, 150-199, 200-249... prepare grouped frequency distribution table.

(ii) From the table, find the number of people who donated rupees 350 or more.

Let's learn.

Less than Cumulative frequency less than the upper class limit

Ex. : The following information is regarding marks in mathematics, obtain out of 40, scored by 50 students of 9th std. in the first unit test.

Class Interval	Frequency (no.of students)
0-10	02
10-20	12
20-30	20
30-40	16
	Total N = 50

(1) From the table, fill in the blanks in the following statements.

(i) For class interval 10-20 the lower class limit is $\boxed{}$ and upper class limit is $\boxed{}$

(ii) How many students obtained marks less than 10 ? $\boxed{2}$

(iii) How many students obtained marks less than 20 ? $2 + \boxed{} = 14$

(iv) How many students obtained marks less than 30 ? $\boxed{} + \boxed{} = 34$

(v) How many students obtained marks less than 40 ? $\boxed{} + \boxed{} = 50$

Remember this !

The sum of the frequency of a cenrtain class and all the frequencies of previous classes is called as cumulative frequency less than the upper class limit for that given class. In short, it is also called as 'less than type' cumulative frequency.

The Meaning of less than type cumulative frequency :

Classes marks	Frequency	Less than type cumulative frequency
0-10	2	2
10-20	12	$2 + 12 = \boxed{}$
20-30	20	$\boxed{} + 20 = 34$
30-40	16	$34 + \boxed{} = 50$
Total 50		

Class	Cumulative frequency	Meaning of less than type cumulative frequency
0-10	2	2 students got less than 10 marks
10-20	14	14 students got less than 20 marks
20-30	34	34 students got less than 30 marks
30-40	50	50 students got less than 40 marks
Total 50		

(2) Cumulative frequency more than or equal to the lower class limit

Classes	Frequency	Clumulative freq.
0-10	2	50
10-20	12	50 – 2 = 48
20-30	20	48 – 12 = 36
30-40	16	36 – 20 = 16
Total 50		

Classes	Cum. freq.	Meaning of cumulative frequency more than or equal to the lower class limit
0-10	50	50 students got 0 or more than 0 marks
10-20	48	48 students got 10 or more than 10 marks
20-30	36	36 students got 20 or more than 20 marks
30-40	16	16 students got 30 or more than 30 marks

Ex. A sports club has organised a table-tennis tournaments. The following table gives the distribution of players ages. Find the cumulative frequencies equal to or more than the lower class limit and complete the table.

Solution : Equal to lower limit or more than lower limit type of cumulative table.

Age (Year)	Tally marks	Frequency (No. of students)	Equal to lower limit or more than lower limit
10-12	卌 ||||	09	50
12 – 14	卌 卌 卌 卌 |||	☐	☐ – 9 = 41
14-16	卌 卌 |||	☐	41 – 23 = ☐
15 – 16	卌	05	☐ – 13 = ☐
		Total N = 50	

Practice set 7.4

(1) Complete the following cumulative freuqency table :

Class (Height in cm)	Frequency (No. of students)	Less than type frequency
150-153	05	05
153-156	07	05 + ☐ = ☐
156-159	15	☐ + 15 = ☐
159-162	10	☐ + ☐ = 37
162-165	05	37 + 5=42
165-168	03	☐ + ☐ = 45
	Total N = 45	

(2) Complete the following Cumulative Frequency Table :

Class (Monthly income in Rs.)	Frequency (No. of individuals)	More than or equal to type cumulative frequency
1000-5000	45	....
5000-10000	19	....
10000-15000	16	....
15000-20000	02	....
20000-25000	05	....
	Total N = 87	

(3) The data is given for 62 students in a certain class regarding their mathematics marks out of 100. Take the classes 0-10, 10-20.. and prepare frequency distribution table and cumulative frequency table more than or equal to type.

55, 60, 81, 90, 45, 65, 45, 52, 30, 85, 20, 10,
75, 95, 09, 20, 25, 39, 45, 50, 78, 70, 46, 64,
42, 58, 31, 82, 27, 11, 78, 97, 07, 22, 27, 36,
35, 40, 75, 80, 47, 69, 48, 59, 32, 83, 23, 17,
77, 45, 05, 23, 37, 38, 35, 25, 46, 57, 68, 45,
47, 49.

From the prepared table, answer the following questions :

(i) How many students obtained marks 40 or above 40 ?

(ii) How many students obtained marks 90 or above 90 ?

(iii) How many students obtained marks 60 or above 60 ?

(iv) What is the cumulative frequency of equal to or more than type of the class 0-10?

(4) Using the data in example (3) above, prepare less than type cumulative frequency table and answer the following questions.

(i) How many students obtained less than 40 marks ?

(ii) How many students obtained less than 10 marks ?

(iii) How many students obtained less than 60 marks ?

(iv) Find the cumulative frequency of the class 50-60.

Let's learn.

Measures of central tendency

Central Tendency : If the data collected in a survey of a group is sufficiently large, then it generally shows a peculiar property. The numbers in the data tend to cluster around a certain number. This property is called the **central tendency** of the group.

The number around which the numbers in the data tend to cluster is called **measure** of central tendency. It is supposed that the measure is a representative of the data.

In statistics, the measures of central tendency mainly used are as follows.

The following measures of central tendency are used :

(1) Mean : The arithmatical average of all observations in the given data is known as its 'Arithmetic mean' or simply 'mean'.

$$\text{Mean} = \frac{\text{The sum of all observations in the data}}{\text{Total number of observation}}$$

Ex. (1) Find the mean of numbers 25, 30, 27, 23 and 25.

Solution : $\frac{25+30+27+23+25}{5} = \frac{130}{5} = 26$

Ex. (2) The first unit test of 40 marks was conducted for a class of 35 students. The marks obtained by the students were as follows. Find the mean of the marks.

40,	35,	30,	25,	23,	20,	14,	15,	16,	20,	17,	37,
37,	20,	36,	16,	30,	25,	25,	36,	37,	39,	39,	40,
15,	16,	17,	30,	16,	39,	40,	35,	37,	23,	16.	

Solution : Here, we can add all observations, but it will be a tedious job. Here 3 students obtained 30 marks each. So their sum is 30 + 30 + 30 = 90, which is 30 × 3. In this way the sum of marks of all students is worked out in the following table.

In statistics, it is convenient to use the Greek letter Σ (sigma) to show the sum of numbers.

In the adjacent table, 956 is the sum of the products 14 × 1 + 15 × 6+...+40 × 3. These are the products of the frequencies and the scores; in short of f 's and x's.

The product of first frequency and first score is 14 × 1, which we write as $f_1 \times x_1$.

The product of first frequency and first score is 15 × 2, which we write as $f_2 \times x_2$.

In general, the product of i^{th} frequency and i^{th} score is written as $f_i \times x_i$.

So, using the letter Σ, the sum of the products $f_1 \times x_1 + f_2 \times x_2 + \ldots + f_i \times x_i$, is in short written as $\Sigma f_i \times x_i$

Marks (x_i)	No. of students (f_i)	$f_i \times x_i$
14	1	14 × 1 = 14
15	2	15 × 2 = ...
16	5	16 × .... = ...
17	2	17 × 2 = 34
20	3	.... × 3 = ...
23	2	23 × 2 = ...
25	3	25 × 3 = ...
30	3	.... × .... = ...
35	2	35 × 2 = 70
36	2	.... × .... = ...
37	4	..... × .... = ...
39	3	39 × 3 = 117
40	3	.... × .... = 120
	N= $\boxed{}$	$\sum f_i x_i = 956$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{956}{35}$$

$= 27.31$ marks (approximately)

$\therefore$ mean of the given data is 27.31.

(2) Median : The scores are arranged in ascending or descending order. The number appearing exactly at the middle position in this order is known as 'Median' of the observations.

If the number of observations is even then the median is the average of the middle two numbers.

Ex. (1) Find the median of 54, 63, 66, 72, 98, 87, 92.

Solution : Let us write the given observations in the ascending order.

54, 63, 66, **72,** 78, 87, 92

Here the 4th number is at the middle position, which is 72

$\therefore$ Median of the scores = 72

Ex. (2) Find the median of the data. 30, 25, 32, 23, 42, 36, 40, 33, 21, 43

Solution : Let us write the given observations in the ascending order.

21, 23, 25, 30, **32, 33,** 36, 40, 42, 43

Here number of observations = 10 which is an even number.

$\therefore$ the 5th and 6th numbers are in the middle position.

Those numbers are 32 and 33

$$\therefore \text{ median } = \frac{32+33}{2} = \frac{65}{2} = 32.5$$

Let's recall.

If the number of observations is '*n*' and

(i) if 'n' is odd, which observation is the median of the data ?

(ii) if '*n*' is even, the average of which two numbers is the median ?

(3) Mode : The score which is repeated maximum number of times in the given data is known as the 'mode' of the data.

Ex. (1) Find the mode of 90, 55, 67, 55, 75, 75, 40, 35, 55, 95

Solution : If the data is arranged in ascending order, it is easy to find the observation repeating maximum number of times.

Ascending order of given data. 35, 40, 55, 55, 55, 67, 75, 75 90, 95

The observation repeated maximum number of times = 55.

$\therefore$ mode for the given data is 55.

Ex (2) The ages of workers in a certain factory are given in the following table.

Age (Year)	19	21	25	27	30
Workers	5	15	13	15	7

Find the mode of their ages.

Solution : Here the maximum frequency is 15; but this is the frequency is of two observations.

$\therefore$ Mode = 21 and 29 $\qquad\therefore$ mode for ages is 21 years and 29 years.

Practice set 7.5

(1) Yield of soyabean per acre in quintal in Mukund's field for 7 years was 10, 7, 5, 3, 9, 6, 9. Find the mean of yield per acre.

(2) Find the median of the observations, 59, 75, 68, 70, 74, 75, 80.

(3) The marks (out of 100) obtained by 7 students in Mathematics' examination are given below. Find the mode for these marks.

99, 100, 95, 100, 100, 60, 90

(4) The monthly salaries in rupees of 30 workers in a factory are given below.

5000, 7000, 3000, 4000, 4000, 3000, 3000, 3000, 8000, 4000,
4000, 9000, 3000, 5000, 5000, 4000, 4000, 3000, 5000, 5000,
6000, 8000, 3000, 3000, 6000, 7000, 7000, 6000, 6000, 4000

From the above data find the mean of monthly salary.

(5) In a basket there are 10 tomatoes. The weight of each of these tomatoes in grams is as follows 60, 70, 90, 95, 50, 65, 70, 80, 85, 95.

Find the median of the weights of tomatoes.

(6) A hockey player has scored following number of goals in 9 matches. 5, 4, 0, 2, 2, 4, 4, 3, 3. Find the mean, median and mode of the data.

(7) The calculated mean of 50 observations was 80. It was later discovered that observation 19 was recorded by mistake as 91. What was the correct mean?

(8) Following 10 observations are arranged in ascending order as follows.
2, 3, 5, 9, $x + 1$, $x + 3$, 14, 16, 19, 20
If the median of the data is 11, find the value of x.

(9) The mean of 35 observations is 20, out of which mean of first 18 observations is 15 and mean of last 18 observation is 25. Find the 18th observation.

(10) The mean of 5 observations is 50. One of the observations was removed from the data, hence the mean became 45. Find the observation which was removed.

(11) There are 40 students in a class, out of them 15 are boys. The mean of marks obtained by boys is 33 and that for girls is 35. Find out the mean of all students in the class.

(12) The weights of 10 students (in kg) are given below :

40, 35, 42, 43, 37, 35, 37, 37, 42, 37. Find the mode of the data.

(13) In the following table, the information is given about the number of families and the siblings in the families less than 14 years of age. Find the mode of the data.

No. of siblings	1	2	3	4
Families	15	25	5	5

(14) Find the mode of the following data.

Marks	35	36	37	38	39	40
No. of students	09	07	09	04	04	02

Which is a suitable measure of Central Tendency ? The answer of this question is related to the purpose of the survey.

For example, the number of runs scored by a player in continuous 11 matches are 41, 58, 35, 80, 23, 12, 63, 48, 107, 9, 73 respectively. To find his overall performance we have to consider the runs he scored in each match. Hence, the suitable measure in this example is mean.

If a company has to decide, the number of shirts to be manufactured 'different sizes'. For this out of 34, 36, 38, 40, 42, 44, which size shirts are used by maximum customers are to be found. By observation it can be found. Here the mode is useful to decide, how many shirts to be manufactured of each size.

Problem Set 7

(1) Write the correct alternative answer for each of the following questions.

(i) Which of the following data is not primary ?

(A) By visiting a certain class, gathering information about attendence of students.

(B) By actual visit to homes, to find number of family members.

(C) To get information regarding plantation of soyabean done by each farmer from the village Talathi.

(D) Review the cleanliness status of canals by actually visitiing them.

(ii) What is the upper class limit for the class 25-35 ?

(A) 25 (B) 35 (C) 60 (D) 30

(iii) What is the class-mark of class 25-35 ?

(A) 25 (B) 35 (C) 60 (D) 30

(iv) If the classes are 0-10, 10-20, 20-30... then in which class should the observation 10 be included ?

(A) 0-10 (B) 10-20 (C) 0-10 and 10-20 in these 2 classes (D) 20-30

(v) If $\bar{x}$ is the mean of $x_1, x_2, \ldots\ldots\ldots x_n$ and $\bar{y}$ is the mean of $y_1, y_2, \ldots\ldots y_n$ and $\bar{z}$ is the mean of $x_1, x_2, \ldots\ldots\ldots x_n, y_1, y_2, \ldots\ldots\ldots y_n$ then $\bar{z} = $?

(A) $\frac{\bar{x}+\bar{y}}{2}$ (B) $\bar{x}+\bar{y}$ (C) $\frac{\bar{x}+\bar{y}}{n}$ (D) $\frac{\bar{x}+\bar{y}}{2n}$

(vi) The mean of five numbers is 80, out of which mean of 4 numbers is 46, find the 5th number :

(A) 4 (B) 20 (C) 434 (D) 66

(vii) Mean of 100 observations is 40. The 9th observation is 30. If this is replaced by 70 keeping all other observations same, find the new mean.

(A) 40.6 (B) 40.4 (C) 40.3 (D) 40.7

(viii) What is the the mode of 19, 19, 15, 20, 25, 15, 20, 15?

(A) 15, (B) 20 (C) 19 (D) 25

(ix) What is the median of 7, 10, 7, 5, 9, 10 ?

(A) 7 (B) 9 (C) 8 (D) 10

(x) From following table, what is the cumulative frequency of less than type for the class 30-40 ?

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	3	12	13	2

(A) 13 (B) 15 (C) 35 (D) 22

(2) The mean salary of 20 workers is Rs.10,250. If the salary of office superintendent is added, the mean will increase by Rs.750. Find the salary of the office superintendent.

(3) The mean of nine numbers is 77. If one more number is added to it then the mean increases by 5. Find the number added in the data.

(4) The monthly maximum temperature of a city is given in degree celcius in the following data. By taking suitable classes, prepare the grouped frequency distribution table

29.2, 29.0, 28.1, 28.5, 32.9, 29.2, 34.2, 36.8, 32.0, 31.0,
30.5, 30.0, 33, 32.5, 35.5, 34.0, 32.9, 31.5, 30.3, 31.4,
30.3, 34.7, 35.0, 32.5, 33.5, 29.0, 29.5, 29.9, 33.2, 30.2

From the table, answer the following questions.

(i) For how many days the maximum temperature was less than 34^{o}C ?

(ii) For how many days the maximum temperature was 34^{o}C or more than 34^{o}C ?

(5) If the mean of the following data is 20.2, then find the value of p.

x_i	10	15	20	25	30
f_i	6	8	p	10	6

(6) There are 68 students of 9th standard from model Highschool, Nandpur. They have scored following marks out of 80, in written exam of mathematics.

70, 50, 60, 66, 45, 46, 38, 30, 40, 47, 56, 68,
80, 79, 39, 43, 57, 61, 51, 32, 42, 43, 75, 43,
36, 37, 61, 71, 32, 40, 45, 32, 36, 42, 43, 55,
56, 62, 66, 72, 73, 78, 36, 46, 47, 52, 68, 78,
80, 49, 59, 69, 65, 35, 46, 56, 57, 60, 36, 37,
45, 42, 70, 37, 45, 66, 56, 47

By taking classes 30-40, 40-50, ... prepare the less than type cumulative frequency table

Using the table, answer the following questions :

(i) How many students, have scored marks less than 80 ?

(ii) How many students have scored marks less than 40 ?

(iii) How many students have scored marks less than 60 ?

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(7) By using data in example (6), and taking classes 30-40, 40-50... prepare equal to or more than type cumulative frequency table and answer the following questions based on it.

(i) How many students have scored marks 70 or more than 70 ?

(ii) How many students have scored marks 30 or more than 30 ?

(8) There are 10 observations arranged in ascending order as given below.
45,47,50,52,x, x+2, 60,62,63,74. The median of these observations is 53.
Find the value of x. Also find the mean and the mode of the data.

Fun with maths

Pascal's Triangle (Meru prastar)

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

....

....

....

This arrangement is knowns as Pascal's triangle. Write the remaining 3 lines of above arrangement. The numbers obtained in above arrangement in horizontal lines denote the coefficients of the expansion of $(x + y)^n$. See the following expansions :

$(x + y)^0 = 1$

$(x + y)^1 = 1x + 1y$

$(x + y)^2 = 1x^2 + 2xy + 1y^2$

$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

Observe the degrees of x and y in the above expansions.
Try to write the expansion of $(x + y)^{10}$ from above arrangement.

Answers

1. Sets

Practice set 1.1

(1) (i) {2, 4, 6, 8, ...} (ii) {2} (iii) {–1, –2, –3, ...} (iv) {sa, re, ga, ma, pa, dha, ni}

(2) (i) $\frac{4}{3}$ is an element of set Q. (ii) –2 is not an element of set N

(iii) Set P is a set of all p's such that p is an odd number.

(4) (i) A = {Chaitra, Vaishakh, Jyeshth, Ashadh, Shravan, Bhadra, Ashwin, Kartik, Agrahayan, Paush, Magh, Phalgun}

(ii) X = {C, O, M, P, L, E, N, T} (iii) Y = {Nose, Ears, Eyes, Tounge, Skin}

(iv) Z = {2, 3, 5, 7, 11, 13, 17, 19}

(v) E = {Asia, Africa, Europe, Australia, Antarctica, South America, North America}

(5) (i) $A = \{x \mid x = n^2, n \in N, n \leq 10\}$ (ii) $B = \{x \mid x = 6n, n \in N, n < 9\}$

(iii) C = {y | y is a letter in the word 'SMILE'}

(iv) D = {z | z is a day of a week} (v) X = {y | y is a letter in the word 'eat' }

Practice set 1.2

(1) A = B = C (2) A = B (3) A and C are empty sets.

(4) (i), (iii), (iv), (v) are finite sets (ii), (vi), (vii) are infinte sets

Practice set 1.3

(1) (i), (ii), (iii), (v) are false and (iv), (vi) are true statements.

(4) {1}, {3}, {2}, {7}, {1, 3}, {1, 2}, {1, 7}, {3, 2}, {3, 7}, {2, 7}, {1, 3, 2}, {1, 2, 7}, {3, 2, 7}, {1, 3, 2, 7} any three like these sets..

(5) (i) $P \subseteq H$, $P \subseteq B$, $I \subseteq M$, $I \subseteq B$, $H \subseteq B$, $M \subseteq B$ (ii) set B

(6) (i) N, W, I any of these sets. (ii) N, W, I any of these sets.

(7) Set of students getting marks less than 50% in Maths.

Practice set 1.4

(1) n (B) = 21 (2) Number of students who do not take any of the drinks = 5

(3) Total number of students = 70

(4) The number of students who do not like rock climbing and sky-watching = 20

The students who like only rock climbing = 20,

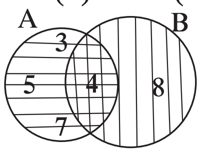
The students who like only sky watching = 70

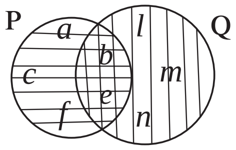
(5) (i) $A = \{x, y, z, m, n\}$ (ii) $B = \{p, q, r, m, n\}$

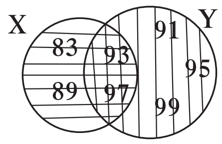
(iii) $A \cup B = \{x, y, z, m, n, p, q, r\}$ (iv) $U = \{x, y, z, m, n, p, q, r, s, t\}$

(v) $A' = \{p, q, r, s, t\}$ (vi) $B' = \{x, y, z, s, t\}$ (vii) $(A \cup B)' = \{s, t\}$

Problem set 1

(1) (i) (C) (ii) (D) (iii) (C) (iv) (B) (v) (A) (vi) (A)
(2) (i) (A) (ii) (A) (iii) (B) (iv) (C)
(3) People speaking only English 57, People speaking only french 28,
People speaking both languages 15
(4) 135 (5) 12 (6) 4
(7) (i) 

(ii) 

(iii) 

(8) $S \subseteq X$, $V \subseteq X$, $S \subseteq X$, $T \subseteq X$, $S \subseteq Y$, $S \subseteq V$, $S \subseteq T$, $V \subseteq T$, $Y \subseteq T$,
(9) $M \cup \phi = M$, $M \cap \phi = \phi$
(10) U = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13}, A = {1, 2, 3, 5, 7} B = {1, 5, 8, 9, 10}
$M \cup B$ = {1, 2, 3, 5, 7, 8, 9, 10}, $A \cap B$ = {1, 5}

(11) $n(A \cup B) = 16$

2. Real Numbers

Practice set 2.1

(1) Terminating (i), (iii), (iv) Non recurring non terminating (ii), (v)

(2) (i) 0.635 (ii) $0.\overline{25}$ (iii) $3.\overline{285714}$ (iv) 0.8 (v) 2.125

(3) (i) $\frac{2}{3}$ (ii) $\frac{37}{99}$ (iii) $\frac{314}{99}$ (iv) $\frac{1574}{99}$ (v) $\frac{2512}{999}$

Practice set 2.2

(4) (i) Infinitely many numbers like –0.4, –0.3, 0.2

(ii) Infinitely many numbers like –2.310, –2.320, –2.325

(iii) Infinitely many numbers like 5.21, 5.22, 5.23

(iv) Infinitely many numbers like –4.51, –4.55, –4.58

Practice set 2.3

(1) (i) 3 (ii) 2 (iii) 4 (iv) 2 (v) 3
(2) (i), (iii), (vi) are surds and (ii), (iv), (v) are not surds.
(3) Like surds : (i), (iii), (iv) and unlike surds : (ii), (v), (vi)
(4) (i) $3\sqrt{3}$ (ii) $5\sqrt{2}$ (iii) $5\sqrt{10}$ (iv) $4\sqrt{7}$ (v) $2\sqrt{42}$
(5) (i) $7\sqrt{2} > 5\sqrt{3}$ (ii) $\sqrt{247} < \sqrt{274}$ (iii) $2\sqrt{7} = \sqrt{28}$

(iv) $5\sqrt{5} < 7\sqrt{5}$ (v) $4\sqrt{42} > 9\sqrt{2}$ (vi) $5\sqrt{3} < 9$ (vii) $7 > 2\sqrt{5}$

(6) (i) $13\sqrt{5}$ (ii) $10\sqrt{5}$ (iii) $24\sqrt{3}$ (iv) $\frac{12}{5}\sqrt{7}$

(7) (i) 54 (ii) $126\sqrt{5}$ (iii) $6\sqrt{10}$ (iv) 80

(8) (i) 7 (ii) $\sqrt{\frac{5}{2}}$ (iii) $\sqrt{2}$ (iv) $\sqrt{62}$.

(9) (i) $\frac{3}{5}\sqrt{5}$ (ii) $\frac{\sqrt{14}}{14}$ (iii) $\frac{5\sqrt{7}}{7}$ (iv) $\frac{2}{9}\sqrt{3}$ (v) $\frac{11}{3}\sqrt{3}$

Practice set 2.4

(1) (i) $-3+\sqrt{21}$ (ii) $\sqrt{10}-\sqrt{14}$ (iii) $-18+13\sqrt{6}$

(2) (i) $\frac{\sqrt{7}-\sqrt{2}}{5}$ (ii) $\frac{3\left(2\sqrt{5}+3\sqrt{2}\right)}{2}$ (iii) $28-16\sqrt{3}$ (iv) $4-\sqrt{15}$

Practice set 2.5

(1) (i) 13 (ii) 5 (iii) 28 (2) 2 or $\frac{4}{3}$ (ii) 1 or 6 (iii) –2 or 18 (iv) 0 or –40

Problem set 2

(1) (i) B (ii) D (iii) C (iv) D (v) A

(vi) C (vii) C (viii) C (ix) C (x) B

(2) (i) $\frac{555}{1000}$ (ii) $\frac{29539}{999}$ (iii) $\frac{9306}{999}$ (iv) $\frac{357060}{999}$ (v) $\frac{30189}{999}$

(3) (i) $-0.\overline{714285}$ (ii) $0.\overline{81}$ (iii) 2.2360679... (iv) $9.\overline{307692}$ (v) 3.625

(5) (i) $\frac{3}{2}\sqrt{2}$ (ii) $-\frac{5}{3}\sqrt{5}$

(6) (i) $\sqrt{2}$ (ii) $\sqrt{2}$ (iii) $\sqrt{3}$ (iv) $\sqrt{10}$ (v) $\sqrt{2}$ (vi) $\sqrt{11}$

(7) (i) $6\sqrt{3}$ (ii) $\frac{34}{3}\sqrt{3}$ (iii) $\frac{15}{2}\sqrt{6}$ (iv) $-25\sqrt{3}$ (v) $\frac{8}{3}\sqrt{3}$

(8) (i) $\frac{\sqrt{5}}{5}$ (ii) $\frac{2\sqrt{7}}{21}$ (iii) $\sqrt{3}+\sqrt{2}$ (iv) $\frac{3\sqrt{5}-2\sqrt{2}}{37}$ (v) $\frac{6\left(4\sqrt{3}+\sqrt{2}\right)}{23}$

3. Polynomials

Practice set 3.1

(1) (i) No, because index of y in $\frac{1}{y}$ is (–1) .

(ii) No, because index of x in the term $5\sqrt{x}$ is $\left(\frac{1}{2}\right)$.

(iii) Yes. (iv) No, because index of m in the term $2m^{-2}$ is (–2) . (v) Yes.

(2) (i) 1 (ii) $-\sqrt{3}$, (iii) $-\frac{2}{3}$

(3) (i) x^7 (ii) $2x^{35}-7$ (iii) x^8-2x^5+3 other polynomials like these.

(4) (i) 0 (ii) 0 (iii) 2 (iv) 10 (v) 1 (vi) 5 (vii) 3 (viii) 10

(5) (i) Quadratic (ii) Linear (iii) Linear (iv) Cubic (v) Quadratic (vi) Cubic

(6) (i) $m^3 + 5m + 3$ (ii) $y^5 + 2y^4 + 3y^3 - y^2 - 7y - \frac{1}{2}$

(7) (i) (1, 0, 0, –2) (ii) (5, 0) (iii) (2, 0, –3, 0, 7) (iv) $\left(\frac{-2}{3}\right)$

(8) (i) $x^2 + 2x + 3$ (ii) $5x^4 - 1$ (iii) $-2x^3 + 2x^2 - 2x + 2$

(9) Quadratic polynomial : x^2; $2x^2 + 5x + 10$; $3x^2 + 5x$;

Cubic polynomial : $x^3 + x^2 + x + 5$; $x^3 + 9$ Linear polynomial : $x + 7$;

Binomial : $x + 7$, $x^3 + 9$; Trinomial : $2x^2 + 5x + 10$; Monomial : x^2

Practice set 3.2

(1) (i) $a + bx$ (ii) xy (iii) $10n + m$

(2) (i) $6x^3 - 2x^2 + 2x$ (ii) $-2m^4 + 2m^3 + 2m^2 + 3m - 6 + \sqrt{2}$ (iii) $5y^2 + 6y + 11$

(3) (i) $-6x^2 + 10x$ (ii) $10ab^2 + a^2b - 7ab$

(4) (i) $2x^3 - 4x^2 - 2x$ (ii) $x^8 + 2x^7 + 2x^5 - x^3 - 2x^2 - 2$ (iii) $-4y^4 + 7y^2 + 3y$

(5) (i) $x^3 - 64 = (x - 4)(x^2 + 4x + 16) + 0$

(ii) $5x^5 + 4x^4 - 3x^3 + 2x^2 + 2 = (x^2 - x)(5x^3 + 9x^2 + 6x + 8) + (8x + 2)$

(6) $a^4 + 7a^4b^2 + 2b^4$

Practice set 3.3

(1) (i) Quotient = $2m + 7$, Remainder = 45

(ii) Quotient = $x^3 + 3x - 2$, Remainder = 9

(iii) Quotient = $y^2 + 6y + 36$, Remainder = 0

(iv) Quotient = $2x^3 - 3x^2 + 7x - 17$, Remainder = 51

(v) Quotient = $x^3 - 4x^2 + 13x - 52$, Remainder = 200

(vi) Quotient = $y^2 - 2y + 3$, Remainder = 2

Practice set 3.4

(1) 5 (2) 1 (3) $4a^2 + 20$ (4) –11

Practice set 3.5

(1) (i) –41 (ii) 7 (iii) 7 (2) (i) 1, 0, –8 (ii) 4, 5, 13 (iii) –2, 0, 10

(3) 0 (4) 2 (5) (i) 17 (ii) $2a^3 - a^2 - a$ (iii) 1544 (6) 92 (7) Yes

(8) 2 (9) (i) No (ii) Yes (10) 30 (11) Yes

(13) (i) –3 (ii) 80

Practice set 3.6

(1) (i) $(x + 1)(2x - 1)$ (ii) $(m + 3)(2m - 1)$ (iii) $(3x + 7)(4x + 11)$

(iv) $(y - 1)(3y + 1)$ (v) $(x + \sqrt{3})(\sqrt{3}x + 1)$ (vi) $(x - 4)\left(\frac{1}{2}x - 1\right)$

(2) (i) $(x - 3)(x + 2)(x - 2)(x + 1)$ (ii) $(x - 13)(x - 2)$

(iii) $(x-8)(x+2)(x-4)(x-2)$ (iv) $(x^2-2x+10)(x^2-2x-2)$

(v) $(y^2+5y-22)(y+4)(y+1)$ (vi) $(y+6)(y-1)(y+4)(y+1)$

(vii) $(x^2-8x+18)(x^2-8x+13)$

Problem set 3

(1) (i) D (ii) D (iii) C (iv) A (v) C (vi) A (vii) D (viii) C (ix) A (x) A

(2) (i) 4 (ii) 0 (iii) 9

(3) (i) $7x^4-x^3+4x^2-x+9$ (ii) $5p^4+2p^3+10p^2+p-8$

(4) (i) (1, 0, 0, 0, 16) (ii) (1, 0, 0, 2, 3, 15)

(5) (i) $3x^4-2x^3+0x^2+7x+18$ (ii) $6x^3+x^2+0x+7$ (iii) $4x^3+5x^2-3x+0$

(6) (i) $10x^4+13x^3+9x^2-7x+12$ (ii) $p^3q+4p^2q+4pq+7$

(7) (i) $2x^2-7y+16$ (ii) x^2+5x+2

(8) (i) $m^7-4m^5+6m^4+6m^3-12m^2+5m+6$

(ii) $5m^5-5m^4+15m^3-2m^2+2m-6$

(9) Remainder = 19 (10) $m=1$ (11) Total population $=10x^2+5y^2-xy$

(12) $b=\frac{1}{2}$ (13) $11m^2-8m+5$ (14) $-2x^2+8x+11$ (15) $2m+n+7$

4. Ratio and Proportion

Practice set 4.1

(1) (i) 6 : 5 (ii) 2 : 3 (iii) 2 : 3
(2) (i) 25 : 11 (ii) 35 : 31 (iii) 2 : 1 (iv) 10 : 17 (v) 2 : 1 (vi) 220 : 153
(3) (i) 3 : 4 (ii) 11 : 25 (iii) 1 : 16 (iv) 13 : 25 (v) 4 : 625
(4) 4 people (5) (i) 60% (ii) 94% (iii) 70% (iv) 91% (v) 43.75%
(6) Abha's age 18 years, Mother's age 45 years (7) After 6 years
(8) Present age of Rehana is 8 years.

Practice set 4.2

(1) (i) 20, 49, 2.5 respectively (ii) 7, 27, 2.25 respectively

(2) (i) $1:2\pi$ (ii) $2:r$ (iii) $\sqrt{2}:1$ (iv) 34 : 35

(3) (i) $\frac{\sqrt{5}}{3}<\frac{3}{\sqrt{7}}$ (ii) $\frac{3\sqrt{5}}{5\sqrt{7}}=\frac{\sqrt{63}}{\sqrt{125}}$ (iii) $\frac{5}{18}>\frac{17}{121}$

(iv) $\frac{\sqrt{80}}{\sqrt{48}}=\frac{\sqrt{45}}{\sqrt{27}}$ (v) $\frac{9.2}{5.1}>\frac{3.4}{7.1}$

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(4) (i) 80° (ii) Present age of Albert is 25 years, Present age of Salim is 45 years

(iii) Length 13.5 cm, Breadth 4.5 cm (iv) 124, 92 (v) 20, 18

(5) (i) 729 (ii) 45 : 7 (6) 2 : 125 (7) $x = 5$

Practice set 4.3

(1) (i) 22 : 13 (ii) 125 : 71 (iii) 316 : 27 (iv) 38 : 11

(2) (i) 3 : 5 (ii) 1 : 6 (iii) 7 : 43 (iv) 71 : 179 (3) 170 : 173

(4) (i) $x = 8$ (ii) $x = 9$ (iii) $x = 2$ (iv) $x = 6$ (v) $x = \frac{9}{14}$ (vi) $x = 3$

Practice set 4.4

(1) (i) 36, 22 (ii) 16, $2a - 2b + 2c$

(2) (i) 29 : 21 (ii) 23 : 7 (4) (i) $x = 2$ (ii) $y = 1$

Practice set 4.5

(1) $x = 4$ (2) $x = \frac{347}{14}$ (3) 18, 12, 8 or 8, 12, 18 (6) $\frac{x+y}{xy}$

Problem set 4

(1) (i) B (ii) A (iii) B (iv) D (v) C

(2) (i) 7 : 16 (ii) 2 : 5 (iii) 5 : 9 (iv) 6 : 7 (v) 6 : 7

(3) (i) 1 : 2 (ii) 5 : 4 (iii) 1 : 1

(4) (i) and (iii) are in continued proportion. (ii) and (iv) are not in continued proportion.

(5) $b = 9$

(6) (i) 7.4% (ii) 62.5% (iii) 73.33% (iv) 31.25% (v) 12%

(7) (i) 5 : 6 (ii) 85 : 128 (iii) 1 : 2 (iv) 50 : 1 (v) 3 : 5

(8) (i) $\frac{17}{9}$ (ii) 19 (iii) $\frac{35}{27}$ (iv) $\frac{13}{29}$

(11) $x = 9$

5. Linear Equations in Two Variables

Practice set 5.1

(3) (i) $x = 3;\ y = 1$ (ii) $x = 2;\ y = 1$ (iii) $x = 2;\ y = -2$

(iv) $x = 6;\ y = 3$ (v) $x = 1;\ y = -2$ (vi) $x = 7;\ y = 1$

Practice set 5.2

(1) 30 notes of ₹ 5 and 20 notes of ₹ 10.

(2) $\frac{5}{9}$ (3) Priyanka's age is 20 years, Deepika's age is 14 years

(4) 20 lions, 30 peacocks

(5) Initial salary ₹ 3900, Yearly increment ₹ 150

(6) ₹ 4000 (7) 36 (8) $\angle A = 90^0$, $\angle B = 40^0$, $\angle C = 50^0$

(9) 420 cm (10) 10

Problem set 5

(1) (i) A (ii) C (iii) C

(2) (i) $x = 2;\ y = 1$ (ii) $x = 5;\ y = 3$ (iii) $x = 8;\ y = 3$

(iv) $x = 1;\ y = -4$ (v) $x = 3;\ y = 1$ (vi) $x = 4;\ y = 3$

(3) (i) $x = 1;\ y = -1$ (ii) $x = 2;\ y = 1$ (iii) $x = 26;\ y = 18$ (iv) $x = 8;\ y = 2$

(4) (i) $x = 6;\ y = 8$ (ii) $x = 9;\ y = 2$ (iii) $x = \frac{1}{2};\ y = \frac{1}{3}$ (5) 35

(6) ₹ 71 (7) ₹ 1800 and ₹ 1400 is the monthly income of each person respectively.

(8) length 347 units, breadth 207 units (9) 40 km/hr, 30 km/hr

(10) (i) 54, 45 (ii) 36, 63 etc.

6. Financial planning

Practice set 6.1

(1) ₹ 1200 (2) Capital after second years ₹ 42,000, 16% loss on initial capital.

(3) Monthly income ₹ 50,000 (4) Shri. Farnandis (5) ₹ 25,000

Practice set 6.2

(1) (i) Need not pay income tax (ii) Needs to pay (iii) Needs to pay

(iv) Needs to pay (v) Need not pay income tax

(2) ₹ 9836.50

Problem set 6

(1) (i) A (ii) B (2) Income ₹ 8750

(3) 36.73% profit of Hiralal, 16.64% profit of Ramniklal. Hiralal's profit is more.

(4) ₹ 99383.75 (5) ₹ 4,00,000 (6) 12.5%

(7) Savings of Ramesh is ₹ 48000 ; Savings of Suresh is ₹ 51000 ; Savings of Priti is ₹ 36000
(8) (i) ₹ 213000 (ii) ₹ 7500 (iii) No tax.

7. Statistics

Practice set 7.2

(1) Primary data : (i), (iii), (v)   Secondary data : (ii), (iv)

Practice set 7.3

(1) Lower limit of class = 20, Upper limit of class = 25   (2) 37.5   (3) 7–13

Practice set 7.4

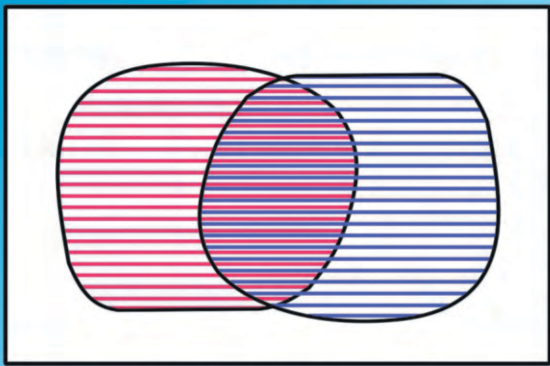
(3) (i) 38 (ii) 3 (iii) 19 (iv) 62   (4) (i) 24 (ii) 3 (iii) 43 (iv) 43

Practice set 7.5

(1) 7 quintal   (2) 74   (3) 100   (4) ₹ 4900   (5) 75 gram
(6) Mean = 3, Median = 3, Mode = 4   (7) 78.56   (8) $x = 9$ (9) 20   (10) 70
(11) 34.25   (12) 37 kg   (13) 2   (14) 35 and 37

Problem set 7

(1) (i) C   (ii) B   (iii) D   (iv) B   (v) A (vi) D
(vii) B   (viii) A   (ix) C   (x) C
(2) ₹ 26000   (3) ₹ 127
(4) (i) 24   (ii) 06
(5) $p = 20$
(6) (i) 66   (ii) 14   (iii) 45
(7) (i) 11   (ii) 68
(8) $x = 52$, Mean = 55.9, Mode = 52



$$x + y = 4$$

$$2x + 3y = 3$$

$$x = \boxed{}, \quad y = \boxed{}$$

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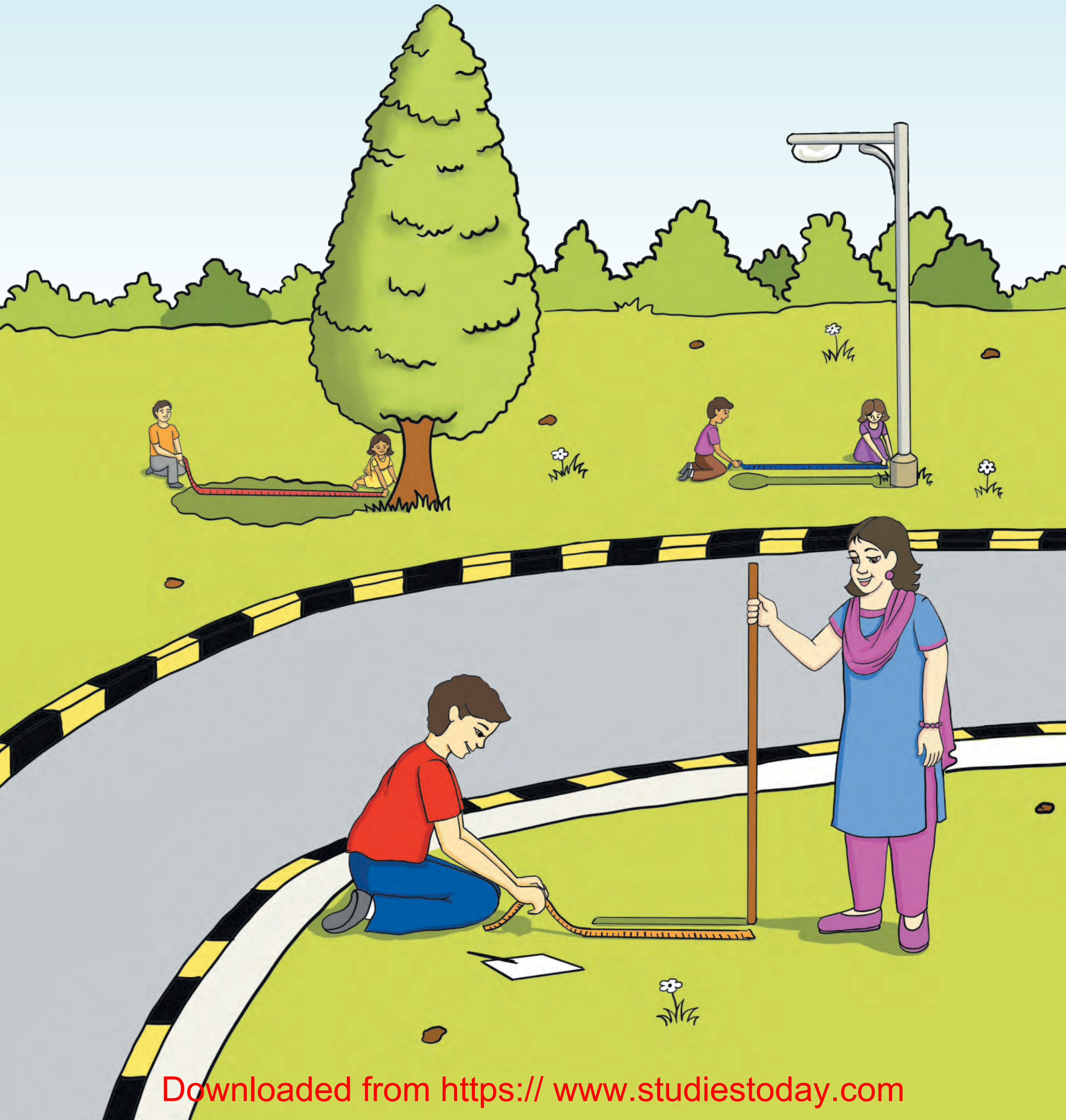
MATHEMATICS

Part - II

STANDARD NINE



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MATHEMATICS

Part-II

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Maharashtra State Bureau of Textbook Production and
Curriculum Research, Pune.

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First Edition : 2017

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Cover and Illustrations :
Dhanashri Mokashi

Computer Drawings :
Sandeep Koli, Mumbai

Co-ordination : Ujjwala Godbole
I/C Special Officer for Mathematics

Translation : Dr Jayashri Atre
Shri. V. D. Godbole
Smt. Mrinalini Desai

Scrutiny : Smt. Prajakti Gokhale
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Co-ordination :
Dhanavanti Hardikar
Academic Secretary for Languages
Santosh Pawar
Subject Assistant, English

Production :
Sachchitanand Aphale
Chief Production Officer
Sanjay Kamble, Production Officer
Prashant Harne, Asst. Production Officer

Typesetting : D.T.P Section

Textbook Bureau, Pune.

Paper : 70 GSM Cream wove

Printer : ADARASH COMBINED EXAMINATION, KOLHAPUR

Print Order No. : N/PB/2017-18/50,000

Publisher

Vivek Uttam Gosavi, Controller
Maharashtra State Textbook Bureau,
Prabhadevi, Mumbai - 400 025.

The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens :

JUSTICE, social, economic and political ;

LIBERTY of thought, expression, belief, faith and worship ;

EQUALITY of status and of opportunity ; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation ;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

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NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

Preface

Dear Students,

Welcome to the ninth standard!

You are now going to begin your studies at the secondary level after completing your primary education curriculum. You had only one Mathematics textbook up to the eighth standard, now you will use two textbooks – Mathematics Part-I and Mathematics Part-II.

Up to the eighth standard you have verified the properties of lines, triangles, quadrilaterals, circles, etc. given in the textbook. Now you are going to give logical proofs of these and some more properties. The skill of logical reasoning is of utmost importance in all fields of life. This textbook gives you an opportunity to learn the skill gradually.

Different activities are given in the textbook to help you understand different concepts. Other activities have been provided for revision and additional practice. You are expected to do all these and learn the proofs of properties. Discuss the reason behind every step of a proof and learn the property.

In this textbook, Mathematics-Part II, two new topics namely Trigonometry and Co-ordinate Geometry are introduced. These topics will provide a foundation for higher studies. The study of Surface Area and Volume will be useful in day to day life.

Use of internet will also help you to understand the subject. You will get through the course joyfully if you follow the three point plan of – a deep study of the textbook, activity-based learning and ample practice.

So come on! Let us study Mathematics in the company of our teachers, parents, friends and the internet. Best wishes to you for your studies!

(Dr Sunil Magar)
Director
Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

Pune
Date : 28 April, 2017
Akshaya Tritiya
Indian Solar Year :
8 Vaishakh 1939

It is expected that students will develop the following competencies after studying Mathematics Part II syllabus in Standard IX

Area	Topic	Competency statement
1. **Geometry**		The students will be able to –
	1.1 Euclidean Geometry	• write 'what is given' and 'what is to be proved' from the given statement. • write the proof of the given statements by using logical conclusions.
	1.2 Parallel lines and pairs of angles	• identify the pairs of angles made by a transversals of parallel lines. • understand the properties of pairs of angles and make use of them.
	1.3 Theorems on angles and sides of a triangle.	• write 'Given' 'To prove' and 'proof' of the statements.
	1.4 Similar triangles	• identify similar triangles and write the ratios of corresponding sides.
	1.5 Circle	• prove the properties of circle using tests of congruence of triangles.
	1.6 Geometric constructions	• construct triangles if different type of information is given.
	1.7 Quadrilateral	• write proofs of the properties of different types of quadrilaterals. • use ICT tools to verify the properties of triangle, quadrilateral and circle.
3. **Co-ordinate Geometry**	2.1 Basics of co-ordinate Geometry	• explain the meaning of co-ordinates of a point in a plane. • describe a point by its co-ordinates. • use ICT tools to find the co-ordinates of a point.
3. **Mensuration**	3.1 Surface area and Volume	• find the surface area and volume of a sphere and a cone.
4. **Trigonometry**	4.1 Introduction to trigonometry	• tell the different trigonometric ratios using similar triangles and Pythagoras theorem and make use of it.

Instructions for teachers

It is expected that the teachers should go through the text book Mathematics Part-II for std IX thoroughly. The book contains many activities and practicals. Try to understand the purpose behind them.

The activities are of two types, (1) to write the proofs and (2) practical verification of properties and theorems. A teacher should make use of discussion, question-answers, group activities etc. to carry out the activities and make the text book more useful. A teacher is also expected to encourage the students to do the activities in the book and help them to invent new ones.

It is more important to write the proofs pursuing logical thinking than doing them by heart. The text book contains a variety of examples to enhance students' logical thinking. Teachers should construct more such examples with the help of students. Examples, which require a little higher thinking ability, are star-marked. Teachers should encourage the students who write proofs logically correct but thinking in a different way.

In the process of evaluation, it is advised to make use of open ended questions and of activity-sheets. Teachers should endeavour to develop such methods of evaluation.

The list of practicals given in the text book should be considered as specimen. Teachers can frame different practicals as well as teaching aids of their own using available material. Different activities given in the text book are included in the practicals. We hope that the evaluation method based on all these will be helpful to develop different competencies for further studies.

List of some practicals (specimen)

(1) To find the distance between two points on a number line.
(2) To verify the properties of angles made by a transversal of parallel lines.
(3) To verify the properties of sides and angles of a triangle using Geometric instruments.
(4) To verify the property of median on hypotenuse of a right angled triangle.
(5) To do the construction of a triangle with given specific conditions.
(6) An activity is given in the book to derive the formula of the surface area of a cone. Using the same activity, derive the formula for the area of a circle which is πr^2.
(7) To draw proportionate map of a room on a graph paper by considering the measurements of the things inside the room.
(8) By drawing X and Y-axes on the school ground, ask students to tell the co-ordinates of a students' positions on the ground.
(9) To find the volume of a cylindrical vessel using formula. Then fill the vessel completely with water and find the volume of the water. Compare both the measurements.
Similar activities can be done for different three dimensional objects.

Index

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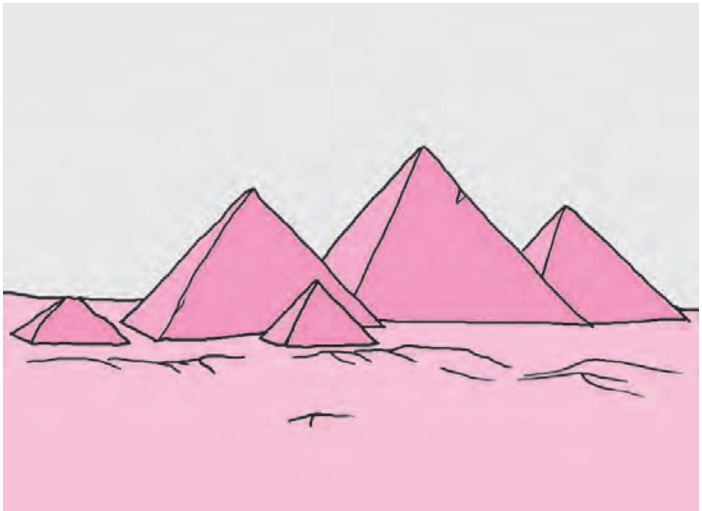
1 Basic concepts in Geometry

Let's study.

- **Point, line and plane**
- **Co-ordinates of a points and distance**
- **Betweenness**
- **Conditional statements**
- **Proof**



Did you recognise the adjacent picture ? It is a picture of pyramids in Egypt, built 3000 years before Christian Era. How the people were able to build such huge structures in so old time ? It is not possible to build such huge structures without developed knowledge of Geometry and Engineering

The word Geometry itself suggests the origin of the subject. It is generated from the Greek words Geo (Earth) and Metria (measuring). So it can be guessed that the subject must have evolved from the need of measuring the Earth, that is land .

Geometry was developed in many nations in different periods and for different constructions. The first Greek mathematician, Thales, had gone to Egypt. It is said that he determined height of a pyramid by measuring its shadow and using properties of similar triangles.

Ancient Indians also had deep knowledge of Geometry. In vedic period, people used geometrical properties to build altars. The book shulba-sutra describes how to build different shapes by taking measurements with the help of a string. In course of time, the mathematicians Aaryabhat, Varahamihir, Bramhagupta, Bhaskaracharya and many others have given valuable contribution to the subject of Geometry.

Let's learn.

Basic concepts in geometry (Point, Line and Plane)

We do not define numbers. Similarly we do not define a point, line and plane also. These are some basic concepts in Geometry. Lines and planes are sets of points. Keep in mind that the word 'line' is used in the sense 'straight line'.

Co-ordinates of points and distance

Observe the following number line.

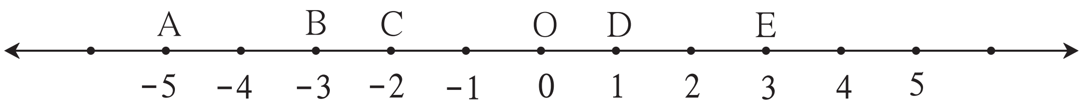


Fig. 1.1

Here, the point D on the number line denotes the number 1. So, it is said that 1 is the co-ordinate of point D. The point B denotes the number – 3 on the line. Hence the co-ordinate of point B is – 3. Similarly the co-ordinates of point A and E are – 5 and 3 respectively.

The point E is 2 unit away from point D. It means the distance between points D and E is 2. Thus, we can find the distance between two points on a number line by counting number of units. The distance between points A and B on the above number line is also 2.

Now let us see how to find distance with the help of co-ordinates of points.

To find the distance between two points, consider their co-ordinates and subtract the smaller co-ordinate from the larger.

The co-ordinates of points D and E are 1 and 3 respectively. We know that 3 > 1.

Therefore, distance between points E and D = 3 – 1 = 2

The distance between points E and D is denoted as d (E,D). This is the same as l(ED), that is, the length of the segment ED.

d (E, D) = 3 – 1 = 2

$\therefore$ l(ED) = 2

d (E, D) = l(ED) = 2

Similarly d (D, E) = 2

d (C, D) = 1 – (– 2)

= 1 + 2 = 3

$\therefore$ d (C, D) = l(CD) = 3

Similarly d (D, C) = 3

Now, let us find d(A,B). The co-ordinate of A is – 5 and that of B is – 3; – 3 > – 5

$\therefore$ d (A, B) = – 3 – (– 5) = – 3+5 = 2.

From the above examples it is clear that the distance between two distinct points is always a positive number.

Note that, if the two points are not distinct then the distance between them is zero.

Remember this !

- The distance between two points is obtained by subtracting the smaller co-ordiante from the larger co-ordinate.
- The distance between any two points is a non-negative real number.

Let's learn.

Betweenness

If P, Q, R are three distinct collinear points, there are three possibilities.

Fig. 1.2

(i) Point Q is between P and R

(ii) Point R is between P and Q

(iii) Point P is between R and Q

If d (P, Q) + d (Q, R) = d (P, R) then it is said that point Q is between P and R. The betweeness is shown as P - Q - R.

Solved examples

Ex (1) On a number line, points A, B and C are such that

d (A, B) = 5, d (B,C) = 11 and d (A, C) = 6.

Which of the points is between the other two ?

Solution : Which of the points A, B and C is between the other two, can be decided as follows.

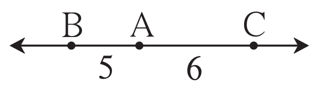


Fig. 1.3

d(B,C) = 11 . . . . (I)

d(A,B) + d(A,C)= 5+6 = 11 . . . . (II)

$\therefore$ d (B, C) = d (A, B) + d (A, C) . . . . [from (I) and (II)]

Point A is between point B and point C.

Ex (2) U, V and A are three cities on a straight road. The distance between U and A is 215 km, between V and A is 140 km and between U and A is 75 km. Which of them is between the other two ?

Solution : d (U,A) = 215; d (V,A) = 140; d (U,V) = 75

d (U,V) + d (V,A) = 75 + 140 = 215; d (U,A) = 215

$\therefore$ d (U,A) = d (U,V) + d (V,A)

$\therefore$ The city V is between the cities U and A.

Ex (3) The co-ordinate of point A on a number line is 5. Find the co-ordinates of points on the same number line which are 13 units away from A.

Solution : As shown in the figure, let us take points T and D to the left and right of A respectively, at a distance of 13 units.

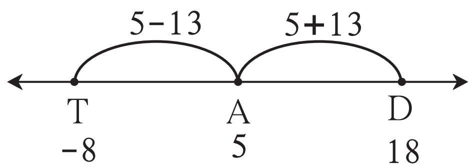


Fig. 1.4

The co-ordinate of point T, which is to the left of A ,will be $5 - 13 = -8$

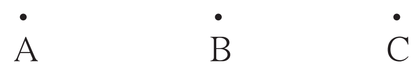
The co-ordinate of point D, which is to the right of A, will be $5 + 13 = 18$

$\therefore$ the co-ordinates of points 13 units away from A will be – 8 and 18.

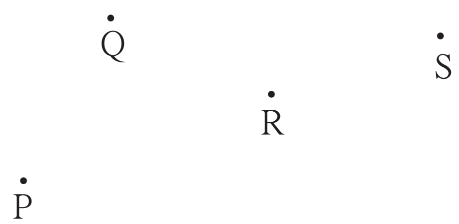
Verify your answer : $d(A,D) = d(A,T) = 13$

Activity

(1) Points A, B, C are given aside. Check, with a stretched thread, whether the three points are collinear or not. If they are collinear, write which one of them is between the other two.



(2) Given aside are four points P, Q, R, and S. Check which three of them are collinear and which three are non collinear. In the case of three collinear points, state which of them is between the other two.



(3) Students are asked to stand in a line for mass drill. How will you check whether the students standing are in a line or not ?

(4) How had you verified that light rays travel in a straight line ?

Recall an experiment in science which you have done in a previous standard.

Practice set 1.1

1. Find the distances with the help of the number line given below.

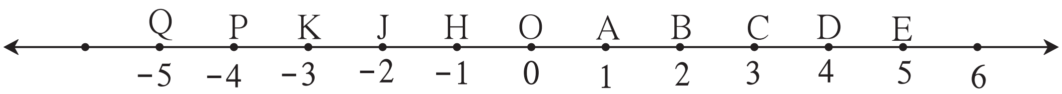


Fig. 1.5

(i) d(B,E) (ii) d(J, A) (iii) d(P, C) (iv) d(J, H)

(v) d(K, O) (vi) d(O, E) (vii) d(P, J) (viii) d(Q, B)

2. If the co-ordinate of A is x and that of B is y, find d(A, B) .

(i) $x = 1, y = 7$ (ii) $x = 6, y = -2$ (iii) $x = -3, y = 7$

(iv) $x = -4, y = -5$ (v) $x = -3, y = -6$ (vi) $x = 4, y = -8$

3. From the information given below, find which of the point is between the other two. If the points are not collinear, state so.

(i) $d(P, R) = 7$, $d(P, Q) = 10$, $d(Q, R) = 3$

(ii) $d(R, S) = 8$, $d(S, T) = 6$, $d(R, T) = 4$

(iii) $d(A, B) = 16$, $d(C, A) = 9$, $d(B, C) = 7$

(iv) $d(L, M) = 11$, $d(M, N) = 12$, $d(N, L) = 8$

(v) $d(X, Y) = 15$, $d(Y, Z) = 7$, $d(X, Z) = 8$

(vi) $d(D, E) = 5$, $d(E, F) = 8$, $d(D, F) = 6$

4. On a number line, points A, B and C are such that $d(A,C) = 10$, $d(C,B) = 8$ Find d(A, B) considering all possibilities.

5. Points X, Y, Z are collinear such that $d(X,Y) = 17$, $d(Y,Z) = 8$, find d(X,Z) .

6. Sketch proper figure and write the answers of the following questions.

(i) If A - B - C and $l(AC) = 11$, $l(BC) = 6.5$, then l(AB) =?

(ii) If R - S - T and $l(ST) = 3.7$, $l(RS) = 2.5$, then l(RT) =?

(iii) If X - Y - Z and $l(XZ) = 3\sqrt{7}$, $l(XY) = \sqrt{7}$, then l(YZ) =?

7. Which figure is formed by three non-collinear points ?

Let's learn.

In the book, Mathematics - Part I for std IX, we have learnt union and intersection of sets in the topic on sets. Now, let us describe a segment, a ray and a line as sets of points.

(1) Line segment :

The union set of point A, point B and points between A and B is called segment AB. Segment AB is written as seg AB in brief. Seg AB means seg BA. Point A and point B are called the end points of seg AB. The distance between the end points of a segment is called the length of the segment. That is $l(AB) = d\,(A,B)$

$l(AB) = 5$ is also written as AB = 5.

Fig. 1.6

(2) Ray AB :

Suppose, A and B are two distinct points. The union set of all points on seg AB and the points P such that A - B - P, is called ray AB. Here point A is called the end point of ray AB.

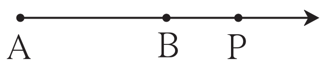


Fig. 1.7

(3) Line AB :

The union set of points on ray AB and opposite ray of ray AB is called line AB. The set of points of seg AB is a subset of points of line AB.

(4) Congruent segments :

If the length of two segments is equal then the two segments are congruent.

If $l(AB) = l(CD)$ then seg AB $\cong$ seg CD

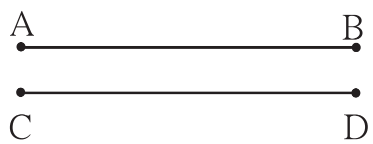


Fig. 1.8

(5) Properties of congruent segements :

(i) Reflexivity : seg AB $\cong$ seg AB

(ii) Symmetry : If seg AB $\cong$ seg CD then seg CD $\cong$ seg AB

(iii) Transitivity : If seg AB $\cong$ seg CD and seg CD $\cong$ seg EF then seg AB $\cong$ seg EF

(6) Midpoint of a segment :

If A-M-B and seg AM $\cong$ seg MB, then M is called the midpoint of seg AB.

Every segment has one and only one midpoint.

Fig. 1.9

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(7) **Comparison of segments :**

If length of segment AB is less than the length of segment CD, it is written as seg AB < seg CD or seg CD > seg AB.

The comparison of segments depends upon their lengths.

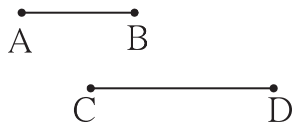

Fig. 1.10

(8) **Perpendicularity of segments or rays :**

If the lines containing two segments, two rays or a ray and a segment are perpendicular to each other then the two segments, two rays or the segment and the ray are said to be perpendicular to each other.

In the figure 1.11, seg AB $\perp$ line CD,

seg AB $\perp$ ray CD.

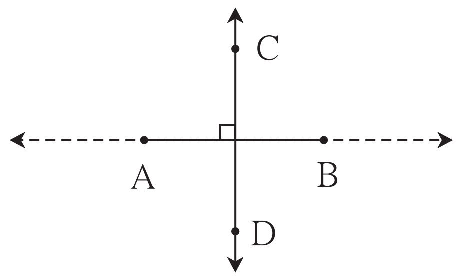

Fig. 1.11

(9) **Distance of a point from a line :**

If seg CD $\perp$ line AB and the point D lies on line AB then the length of seg CD is called the distance of point C from line AB.

The point D is called the foot of the perpendicular.

If l(CD) = a, then the point C is at a distance of 'a' from the line AB.

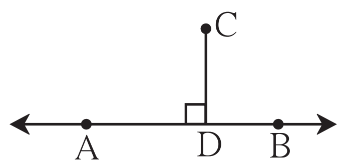

Fig. 1.12

Practice set 1.2

1. The following table shows points on a number line and their co-ordinates. Decide whether the pair of segments given below the table are congruent or not.

Point	A	B	C	D	E
Co-ordinate	–3	5	2	–7	9

(i) seg DE and seg AB (ii) seg BC and seg AD (iii) seg BE and seg AD

2. Point M is the midpoint of seg AB. If AB = 8 then find the length of AM.

3. Point P is the midpoint of seg CD. If CP = 2.5, find l(CD).

4. If AB = 5 cm, BP = 2 cm and AP = 3.4 cm, compare the segments.

5. Write the answers to the following questions with reference to figure 1.13.

(i) Write the name of the opposite ray of ray RP

(ii) Write the intersection set of ray PQ and ray RP.

(iii) Write the union set of ray PQ and ray QR.

(iv) State the rays of which seg QR is a subset.

(v) Write the pair of opposite rays with common end point R.

(vi) Write any two rays with common end point S.

(vii) Write the intersection set of ray SP and ray ST.

Fig. 1.13

6. Answer the questions with the help of figure 1.14.

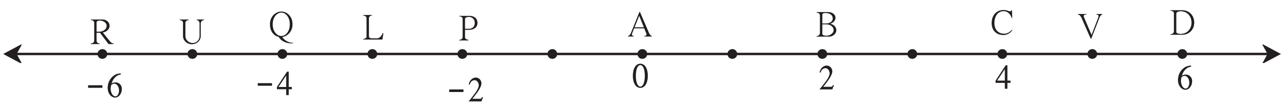

Fig. 1.14

(i) State the points which are equidistant from point B.

(ii) Write a pair of points equidistant from point Q.

(iii) Find d (U,V), d (P,C), d (V,B), d (U, L).

Let's learn.

Conditional statements and converse

The statements which can be written in the 'If-then' form are called conditional statements. The part of the statement following 'If' is called the antecedent, and the part following 'then' is called the consequent.

For example, consider the statement : The diagonals of a rhombus are perpendicular bisectors of each other.

The statement can be written in the conditional form as, 'If the given quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.'

If the antecedent and consequent in a given conditional statement are interchanged, the resulting statement is called the **converse** of the given statement.

If a conditional statement is true, its converse is not necessarily true. Study the following examples.

Conditional statement : If a quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.

Converse : If the diagonals of a quadrilateral are perpendicular bisectors of each other then it is a rhombus.

In the above example, the statement and its converse are true.

Now consider the following example,

Conditional statement : If a number is a prime number then it is even or odd.

Converse : If a number is even or odd then it is a prime number.

In this example, the statement is true, but its converse is false.

Let's learn.

Proofs

We have studied many properties of angles, triangles and quadrilaterals through activities.

In this standard we are going to look at the subject of Geometry with a different point of view, which was originated by the Greek mathematician Euclid, who lived in the third century before Christian Era. He gathered the knowledge of Geometry prevailing at his time and streamlined it. He took for granted some self evident geometrical statements which were accepted by all and called them **Postulates.** He showed that on the basis of the postulates some more properties can be proved logically.

Properties proved logically are called **Theorems.**

Some of Euclid's postulates are given below.

(1) There are infinite lines passing through a point.

(2) There is one and only one line passing through two points.

(3) A circle of given radius can be drawn taking any point as its centre.

(4) All right angles are congruent with each other.

(5) If two interior angles formed on one side of a transversal of two lines add up to less than two right angles then the lines produced in that direction intersect each other.

Euclid

We have verified some of these postulates through activities.

A property is supposed to be true if it can be proved logically. It is then called a **Theorem.** The logical argument made to prove a theorem is called its **proof.**

When we are going to prove that a conditional statement is true, its antecedent is called 'Given part' and the consequent is called 'the part to be proved'.

There are two types of proofs, **Direct** and **Indirect.**

Let us give a direct proof of the property of angles made by two intersecting lines.

Theorem : The opposite angles formed by two intersecting lines are of equal measures.

Given : Line AB and line CD intersect at point O such that A - O - B, C - O - D.

To prove : (i) $\angle AOC = \angle BOD$

(ii) $\angle BOC = \angle AOD$

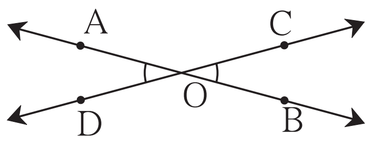

Fig. 1.15

Proof : $\angle AOC + \angle BOC = 180°$ . . . . (I) (angles in linear pair)

$\angle BOC + \angle BOD = 180°$ . . . . (II) (angles in linear pair)

$\angle AOC + \angle BOC = \angle BOC + \angle BOD$ . . . . [from (I) and (II)]

$\therefore \angle AOC = \angle BOD$. . . . . . . eliminating $\angle BOC$.

Similarly, it can be proved that $\angle BOC = \angle AOD$.

Indirect proof :

This type of proof starts with an assumption that the consequence is false. Using it and the properties accepted earlier, we start arguing step by step and reach a conclusion. The conclusion is contradictory with the antecedent or a property which is already accepted. Hence, the assumption that the consequent is false goes wrong. So it is accepted that the consequent is true.

Study the following example.

Statement : A prime number greater than 2 is odd.

Conditional statement : If p is a prime number greater than 2 then it is odd.

Given : p is a prime number greater than 2. That is, 1 and p are the only divisors of p.

To prove : p is an odd number.

Proof : Let us suppose that p is not an odd number.

So p is an even number

$\therefore$ a divisor of p is 2 ..... (I)

But it is given that p is a prime number greater than 2 ....(given)

$\therefore$ 1 and p are the only divisors of p ..... (II)

Statements (I) and (II) are contradictory.

$\therefore$ the assumption , that p is not odd is false.

This proves that a prime number greater than 2 is odd.

Practice set 1.3

1. Write the following statements in 'if-then' form.
   (i) The opposite angles of a parallelogram are congruent.
   (ii) The diagonals of a rectangle are congruent.
   (iii) In an isosceles triangle, the segment joining the vertex and the mid point of the base is perpendicular to the base.
2. Write converses of the following statements.
   (i) The alternate angles formed by two parallel lines and their transversal are congruent.
   (ii) If a pair of the interior angles made by a transversal of two lines are supplementary then the lines are parallel.
   (iii) The diagonals of a rectangle are congruent.

Problem set 1

1. Select the correct alternative from the answers of the questions given below.

   (i) How many mid points does a segment have ?
   (A) only one (B) two (C) three (D) many

   (ii) How many points are there in the intersection of two distinct lines ?
   (A) infinite (B) two (C) one (D) not a single

   (iii) How many lines are determined by three distinct points ?
   (A) two (B) three (C) one or three (D) six

   (iv) Find d(A, B), if co-ordinates of A and B are – 2 and 5 respectively.
   (A) – 2 (B) 5 (C) 7 (D) 3

   (v) If P - Q - R and d(P,Q) = 2, d(P,R) = 10, then find d(Q,R).
   (A) 12 (B) 8 (C) $\sqrt{96}$ (D) 20

2. On a number line, co-ordinates of P, Q, R are 3, – 5 and 6 respectively. State with reason whether the following statements are true or false.

   (i) d(P,Q) + d(Q,R) = d(P,R) (ii) d(P,R) + d(R,Q) = d(P,Q)
   (iii) d(R,P) + d(P,Q) = d(R,Q) (iv) d(P,Q) – d(P,R) = d(Q,R)

3. Co-ordinates of some pairs of points are given below. Hence find the distance between each pair.

   (i) 3, 6 (ii) – 9, – 1 (iii) – 4, 5 (iv) x, – 2
   (v) x + 3, x– 3 (vi) – 25, – 47 (vii) 80, – 85

4. Co-ordinate of point P on a number line is – 7. Find the co-ordinates of points on the number line which are at a distance of 8 units from point P.

5. Answer the following questions.

(i) If A - B - C and d(A,C) = 17, d(B,C) = 6.5 then d (A,B) = ?

(ii) If P - Q - R and d(P,Q) = 3.4, d(Q,R)= 5.7 then d(P,R) = ?

6. Co-ordinate of point A on a number line is 1. What are the co-ordinates of points on the number line which are at a distance of 7 units from A ?

7. Write the following statements in conditional form.

(i) Every rhombus is a square.

(ii) Angles in a linear pair are supplementary.

(iii) A triangle is a figure formed by three segments.

(iv) A number having only two divisors is called a prime number.

8. Write the converse of each of the following statements.

(i) If the sum of measures of angles in a figure is 180^0, then the figure is a triangle.

(ii) If the sum of measures of two angles is 90^0 then they are complement of each other.

(iii) If the corresponding angles formed by a transversal of two lines are congruent then the two lines are parallel.

(iv) If the sum of the digits of a number is divisible by 3 then the number is divisible by 3.

9. Write the antecedent (given part) and the consequent (part to be proved) in the following statements.

(i) If all sides of a triangle are congruent then its all angles are congruent.

(ii) The diagonals of a parallelogram bisect each other.

10*. Draw a labelled figure showing information in each of the following statements and write the antecedent and the consequent.

(i) Two equilateral triangles are similar.

(ii) If angles in a linear pair are congruent then each of them is a right angle.

(iii) If the altitudes drawn on two sides of a triangle are congruent then those two sides are congruent.

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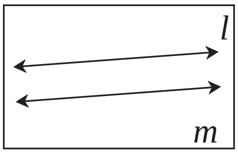
2 Parallel Lines

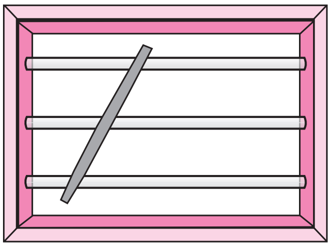
Let's study.

- Properties of angles formed by parallel lines and its transversal
- Tests of parallelness of two lines
- Use of properties of parallel lines

Let's recall.

Parallel lines : The lines which are coplanar and do not intersect each other are called parallel lines.





Hold a stick across the horizontal parallel bars of a window as shown in the figure. How many angles are formed ?

- Do you recall the pairs of angles formed by two lines and their transversal ?

  In figure 2.1, line *n* is a transversal of line *l* and line *m*.

  Here, in all 8 angles are formed. Pairs of angles formed out of these angles are as follows :

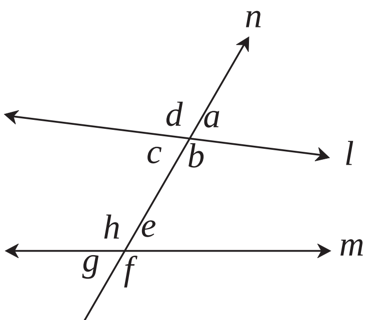


Fig. 2.1

Pairs of corresponding angles

(i) ∠d, ∠h

(ii) ∠*a*, $\boxed{}$

(iii) ∠c, $\boxed{}$

(iv) ∠b, $\boxed{}$

Pairs of alternate interior angles

(i) ∠c, ∠e (ii) ∠b, ∠h

Pairs of alternate exerior angles

(i) ∠d, ∠*f*

(ii) ∠*a*, ∠g

Pairs of interior angles on the same side of the transversal

(i) ∠c, ∠h

(ii) ∠b, ∠e

Some important properties :

(1) When two lines intersect, the pairs of opposite angles formed are congruent.

(2) The angles in a linear pair are supplementary.

(3) When one pair of corresponding angles is congruent, then all the remaining pairs of corresponding angles are congruent.

(4) When one pair of alternate angles is congruent, then all the remaining pairs of alternate angles are congruent.

(5) When one pair of interior angles on one side of the transversal is supplementary, then the other pair of interior angles is also supplementary.

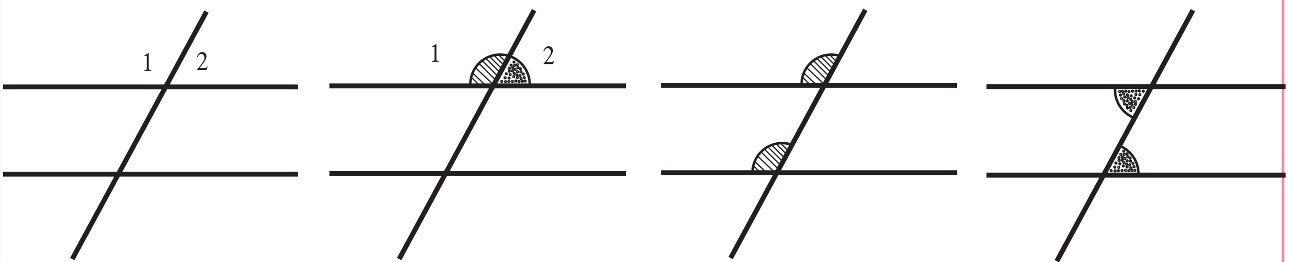
Let's learn.

Properties of parallel lines

Activity

To verify the properties of angles formed by a transversal of two parallel lines.

Take a piece of thick coloured paper. Draw a pair of parallel lines and a transversal on it. Paste straight sticks on the lines. Eight angles will be formed. Cut pieces of coloured paper, as shown in the figure, which will just fit at the corners of ∠1 and ∠2. Place the pieces near different pairs of corresponding angles, alternate angles and interior angles and verify their properties.



Let's learn.

We have verified the properties of angles formed by a transversal of two parallel lines. Let us now prove the properties using Euclid's famous fifth postulate given below.

If sum of two interior angles formed on one side of a transversal of two lines is less than two right angles then the lines produced in that direction intersect each other.

Interior angle theorem

Theorem : If two parallel lines are intersected by a transversal, the interior angles on either side of the transversal are supplementary.

Given : line $l \parallel$ line m and line n is their transversal. Hence as shown in the figure $\angle a$, $\angle \mathrm{b}$ are interior angles formed on one side and $\angle \mathrm{c}$, $\angle \mathrm{d}$ are interior angles formed on other side of the transversal.

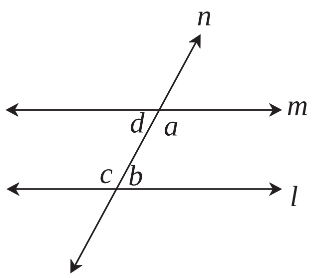


Fig. 2.2

To prove : $\angle a + \angle \mathrm{b} = 180^\circ$

$\angle \mathrm{d} + \angle \mathrm{c} = 180^\circ$

Proof : Three possibilities arise regarding the sum of measures of $\angle a$ and $\angle \mathrm{b}$.

(i) $\angle a + \angle \mathrm{b} < 180^\circ$ (ii) $\angle a + \angle \mathrm{b} > 180^\circ$ (iii) $\angle a + \angle \mathrm{b} = 180^\circ$

Let us assume that the possibility (i) $\angle a + \angle \mathrm{b} < 180^\circ$ is true.

Then according to Euclid's postulate, if the line l and line m are produced will intersect each other on the side of the transversal where $\angle a$ and $\angle \mathrm{b}$ are formed.

But line l and line m are parallel lines ....given

$\therefore \angle a + \angle \mathrm{b} < 180^\circ$ impossible ....(I)

Now let us suppose that $\angle a + \angle \mathrm{b} > 180^\circ$ is true.

$\therefore \angle a + \angle \mathrm{b} > 180^\circ$

But $\angle a + \angle \mathrm{d} = 180^\circ$

and $\angle \mathrm{c} + \angle \mathrm{b} = 180^\circ$ . . . . . angles in linear pairs

$\therefore \angle a + \angle \mathrm{d} + \angle \mathrm{b} + \angle \mathrm{c} = 180^\circ + 180^\circ = 360^\circ$

$\therefore \angle \mathrm{c} + \angle \mathrm{d} = 360^\circ - (\angle a + \angle \mathrm{b})$

If $\angle a + \angle \mathrm{b} > 180^\circ$ then $[360^\circ - (\angle a + \angle \mathrm{b})] < 180^\circ$

$\therefore \angle \mathrm{c} + \angle \mathrm{d} < 180^\circ$

$\therefore$ In that case line l and line m produced will intersect each other on the same side of the transversal where $\angle c$ and $\angle d$ are formed.

$\therefore \angle c + \angle d < 180$ is impossible.

That is $\angle a + \angle b > 180^\circ$ is impossible...... (II)

$\therefore$ the remaining possibility,

$\angle a + \angle b = 180^\circ$ is true......from (I) and (II)

$\therefore$ $\angle a + \angle b = 180^\circ$ Similarly, $\angle c + \angle d = 180^\circ$

Note that, in this proof, because of the contradictions we have denied the possibilities $\angle a + \angle b > 180^\circ$ and $\angle a + \angle b < 180^\circ$.

Therefore, this proof is an example of indirect proof.

Corresponding angles and alternate angles theorems

Theorem : The corresponding angles formed by a transversal of two parallel lines are of equal measure.

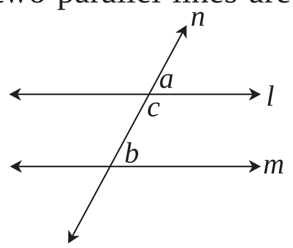


Fig. 2.3

Given : line l || line m
line n is a transversal.

To prove : $\angle a = \angle b$

Proof : $\angle a + \angle c = 180^\circ$ ....(I) angles in linear pair

$\angle b + \angle c = 180^\circ$ ....(II) property of interior angles of parallel lines

$\angle a + \angle c = \angle b + \angle c$ ....from (I) and (II)

$\therefore$ $\angle a = \angle b$

Theorem : The alternate angles formed by a transversal of two parallel lines are of equal measures.

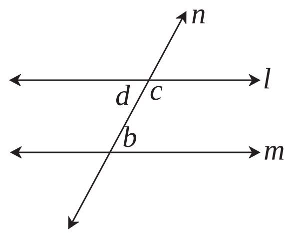


Fig. 2.4

Given : line l || line m
line n is a transversal.

To prove : $\angle d = \angle b$

Proof : $\angle d + \angle c = 180^\circ$................(I) angles in linear pair

$\angle c + \angle b = 180^\circ$ ....(II) property of interior angles of parallel line

$\angle d + \angle c = \angle c + \angle b$................from (I) and (II)

$\therefore$ $\angle d = \angle b$

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Practice set 2.1

1. In figure 2.5, line RP || line MS and line DK is their transversal. ∠DHP = 85°
Find the measures of following angles.
(i) ∠RHD (ii) ∠PHG
(iii) ∠HGS (iv) ∠MGK

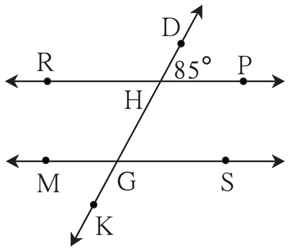


Fig. 2.5

2. In figure 2.6, line *p* || line *q* and line *l* and line *m* are transversals. Measures of some angles are shown. Hence find the measures of ∠*a*, ∠b, ∠c, ∠d.

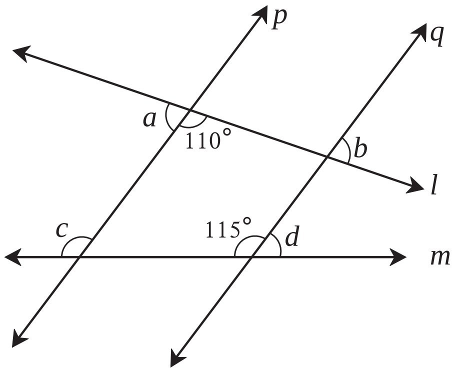


Fig. 2.6

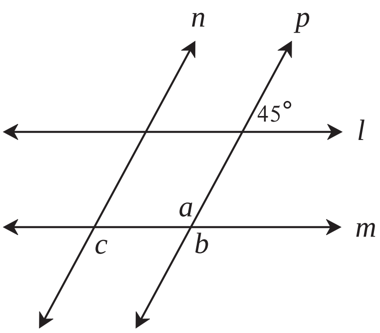


Fig. 2.7

3. In figure 2.7, line *l* || line *m* and line *n* || line *p*. Find ∠*a*, ∠b, ∠c from the given measure of an angle.

4*. In figure 2.8, sides of ∠PQR and ∠XYZ are parallel to each other. Prove that, ∠PQR ≅ ∠XYZ

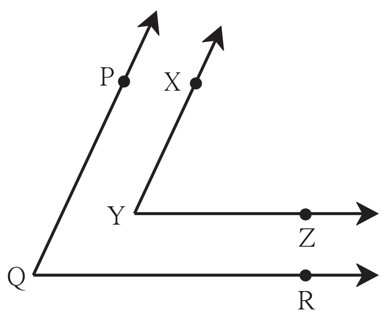


Fig. 2.8

5. In figure 2.9, line AB || line CD and line PQ is transversal. Measure of one of the angles is given. Hence find the measures of the following angles.

(i) ∠ART (ii) ∠CTQ

(iii) ∠DTQ (iv) ∠PRB

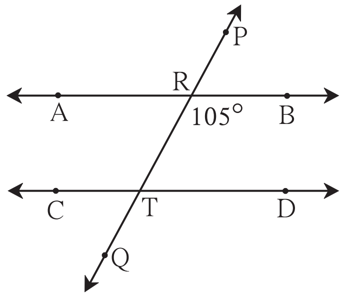


Fig. 2.9

Let's learn.

Use of properties of parallel lines

Let us prove a property of a triangle using the properties of angles made by a transversal of parallel lines.

Theorem : The sum of measures of all angles of a triangle is 180°.

Given : Δ ABC is any triangle.

To prove : $\angle ABC + \angle ACB + \angle BAC = 180^\circ$.

Construction : Draw a line parallel to seg BC and passing through A. On the line take points P and Q such that, P - A - Q.

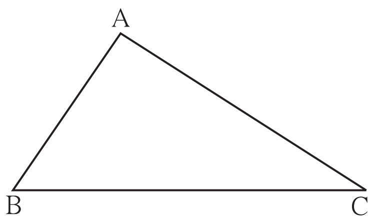


Fig. 2.10

Proof : Line PQ || line BC and seg AB is a transversal.

$\therefore$ $\angle ABC = \angle PAB$.......alternate angles.....(I)

line PQ || line BC and seg AC is a transversal.

$\therefore$ $\angle ACB = \angle QAC$.......alternate angles.....(II)

$\therefore$ From I and II ,

$\angle ABC + \angle ACB = \angle PAB + \angle QAC$ . . . (III)

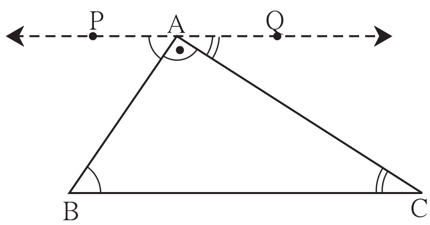


Fig. 2.11

Adding $\angle BAC$ to both sides of (III).

$\angle ABC + \angle ACB + \angle BAC = \angle PAB + \angle QAC + \angle BAC$

$= \angle PAB + \angle BAC + \angle QAC$

$= \angle PAC + \angle QAC$ ...($\because \angle PAB + \angle BAC = \angle PAC$)

$= 180^\circ$ ...Angles in linear pair

That is, sum of measures of all three angles of a triangle is 180°.

Let's discuss.

In fig. 2.12, How will you decide whether line l and line m are parallel or not ?

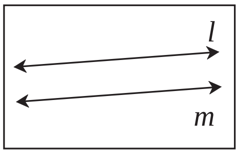


Fig. 2.12

Let's learn.

Tests for parallel lines

Whether given two lines are parallel or not can be decided by examining the angles formed by a transversal of the lines.

(1) If the interior angles on the same side of a transversal are supplementary then the lines are parallel.

(2) If one of the pairs of alternate angles is congruent then the lines are parallel.

(3) If one of the pairs of corresponding angles is congruent then the lines are parallel.

Interior angles test

Theorem : If the interior angles formed by a transversal of two distinct lines are supplementary, then the two lines are parallel.

Given : Line XY is a transversal of line AB and line CD.

$\angle BPQ + \angle PQD = 180°$

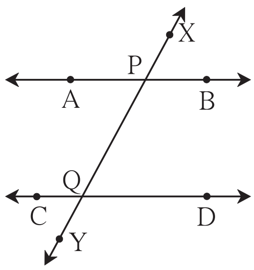


Fig. 2.13

To prove : line AB $||$ line CD

Proof : We are going to give an indirect proof. Let us suppose that the statement to be proved is wrong. That is, we assume, line AB and line CD are not parallel, means line AB and CD intersect at point T. So Δ PQT is formed.

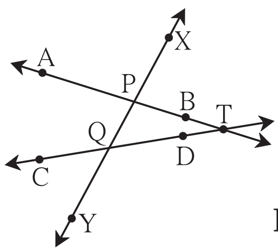


Fig. 2.14

$\therefore$ $\angle TPQ + \angle PQT + \angle PTQ = 180°$ ....sum of angles of a triangle

but $\angle TPQ + \angle PQT = 180°$ ....given

That is the sum of two angles of the triangle is $180°$.

But sum of three angles of a triangle is $180°$.

$\therefore$ $\angle PTQ = 0°$.

$\therefore$ line PT and line QT means line AB and line CD are not distinct lines.

But, we are given that line AB and line CD are distinct lines.

$\therefore$ we arrive at a contradiction.

$\therefore$ our assumption is wrong. Hence line AB and line CD are parallel.

Thus it is proved that if the interior angles formed by a transversal are supplementary, then the lines are parallel.

This property is called **interior angles test** of parallel lines.

Alternate angles test

Theorem : If a pair of alternate angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m.

$\angle a$ and $\angle b$ is a congruent pair of alternate angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180°$ . ....angles in linear pair

$\angle a = \angle b$ .... given

$\therefore\ \angle b + \angle c = 180°$

But $\angle b$ and $\angle c$ are interior angles on the same side of the transversal.

$\therefore$ line $l \parallel$ line m .... interior angles test

This property is called the **alternate angles test** of parallel lines.

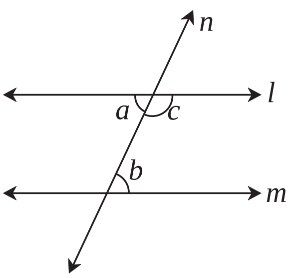


Fig. 2.15

Corresponding angles Test

Theorem : If a pair of corresponding angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m.

$\angle a$ and $\angle b$ is a congruent pair of corresponding angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180°$ ....angles in linear pair

$\angle a = \angle b$ .... given

$\therefore\ \angle b + \angle c = 180°$

That is a pair of interior angles on the same side of the transversal is congruent.

$\therefore$ line $l \parallel$ line m ....interior angles test

This property is called the **corresponding angles test** of parallel lines.

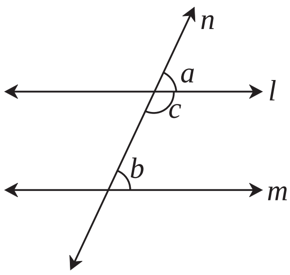


Fig. 2.16

Corollary I : If a line is perpendicular to two lines in a plane, then the two lines are parallel to each other.

Given : Line $n \perp$ line l and line $n \perp$ line m

To prove : line $l \parallel$ line m

Proof : line $n \perp$ line l and line $n \perp$ line m ...given

$\therefore$ $\angle a = \angle c = 90°$

$\angle a$ and $\angle c$ are corresponding angles formed by transversal n of line l and line m.

$\therefore$ line $l \parallel$ line m ....corresponding angles test

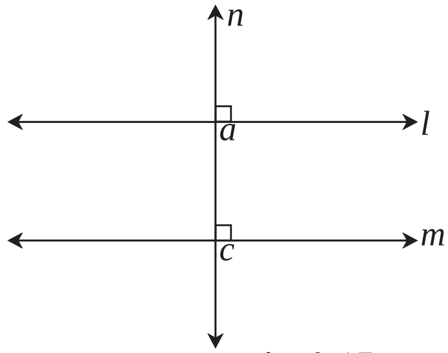


Fig. 2.17

Corollary II : If two lines in a plane are parallel to a third line in the plane then those two lines are parallel to each other. Write the proof of the corollary.

Practice set 2.2

1. In figure 2.18, $y = 108°$ and $x = 71°$
Are the lines m and n parallel ? Justify ?

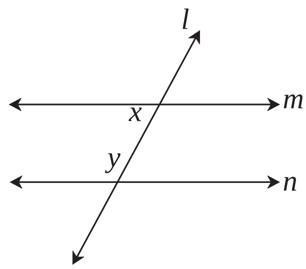


Fig. 2.18

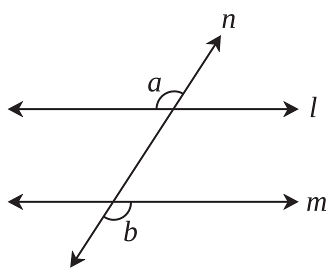


Fig. 2.19

2. In figure 2.19, if $\angle a \cong \angle b$ then prove that line $l \parallel$ line m.

3. In figure 2.20, if $\angle a \cong \angle b$ and $\angle x \cong \angle y$ then prove that line $l \parallel$ line n.

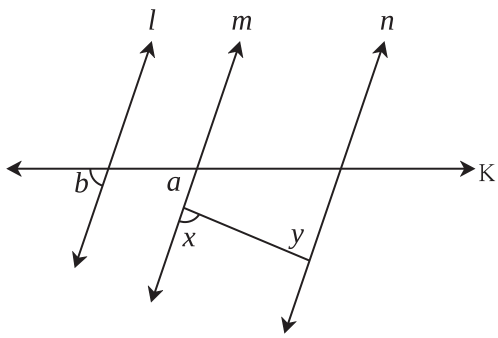


Fig. 2.20

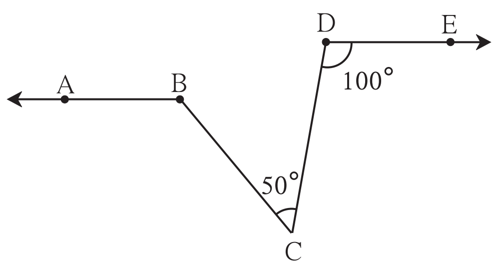


Fig. 2.21

4. In figure 2.21, if ray BA $\parallel$ ray DE, $\angle C = 50°$ and $\angle D = 100°$. Find the measure of $\angle ABC$.
(Hint **:** Draw a line passing through point C and parallel to line AB.)

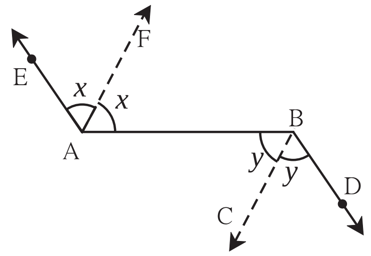
5. 

Fig. 2.22

In figure 2.22, ray AE || ray BD, ray AF is the bisector of $\angle$EAB and ray BC is the bisector of $\angle$ABD. Prove that line AF || line BC.

6. A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of $\angle$BPQ and $\angle$PQC respectively.

Prove that line AB || line CD.

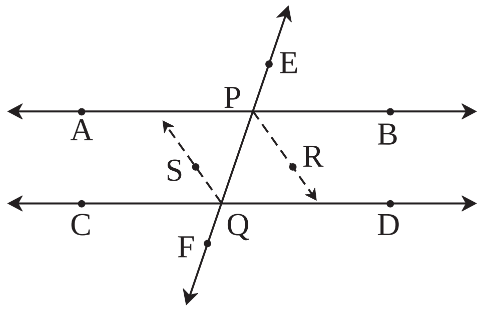


Fig. 2.23

Problem set 2

1. Select the correct alternative and fill in the blanks in the following statements.

(i) If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is ....

(A) 0° (B) 90° (C) 180° (D) 360°

(ii) The number of angles formed by a transversal of two lines is ....

(A) 2 (B) 4 (C) 8 (D) 16

(iii) A transversal intersects two parallel lines. If the measure of one of the angles is 40° then the measure of its corresponding angle is ....

(A) 40° (B) 140° (C) 50° (D) 180°

(iv) In Δ ABC, $\angle A = 76^\circ$, $\angle B = 48^\circ$, $\therefore$ $\angle C =$ ....

(A) 66° (B) 56° (C) 124° (D) 28°

(v) Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is ....

(A) 105° (B) 15° (C) 75° (D) 45°

2*. Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of $\angle$QPR respectively. Ray PB and ray PA are perpendicular to each other. Draw a figure showing all these rays and write -

(i) A pair of complementary angles (ii) A pair of supplementary angles.

(iii) A pair of congruent angles.

3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of $\angle x$ and $\angle y$ and hence show that line $l \parallel$ line m.

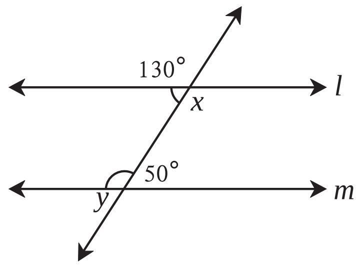


Fig. 2.24

5. Line AB $\parallel$ line CD $\parallel$ line EF and line QP is their transversal. If $y : z = 3 : 7$ then find the measure of $\angle x$. (See figure 2.25.)

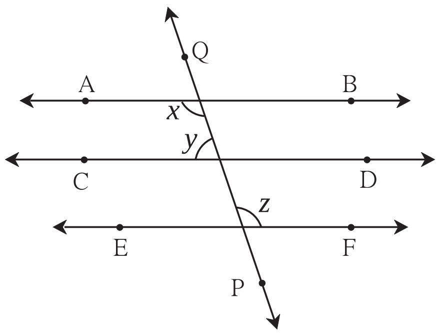


Fig. 2.25

6. In figure 2.26, if line $q \parallel$ line r, line p is their transversal and if $a = 80°$ find the values of f and g.

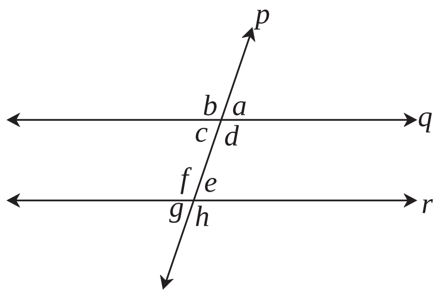


Fig. 2.26

7. In figure 2.27, if line AB $\parallel$ line CF and line BC $\parallel$ line ED then prove that $\angle ABC = \angle FDE$.

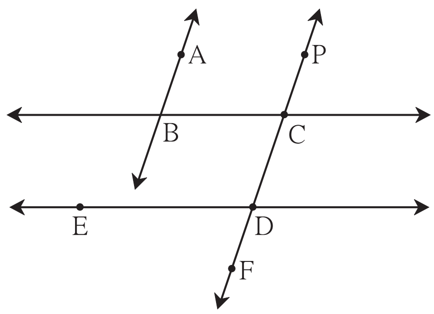


Fig. 2.27

8. In figure 2.28, line PS is a transversal of parallel line AB and line CD. If Ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that ☐ QXRY is a rectangle.

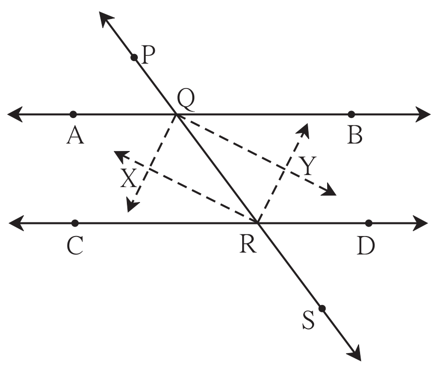


Fig. 2.28

3 Triangles

Let's study.

- Theorem of remote interior angles of a triangle
- Congruence of triangles
- Theorem of isosceles triangle
- Property of 30°- 60°- 90° angled triangle
- Median of a triangle
- Property of median on hypotenuse of a right angled triangle
- Perpendicular bisector theorem
- Angle bisector theorem
- Similar triangles

Activity :

Draw a triangle of any measure on a thick paper. Take a point T on ray QR as shown in fig. 3.1. Cut two pieces of thick paper which will exactly fit the corners of $\angle$P and $\angle$Q. See that the same two pieces fit exactly at the corner of $\angle$PRT as shown in the figure.

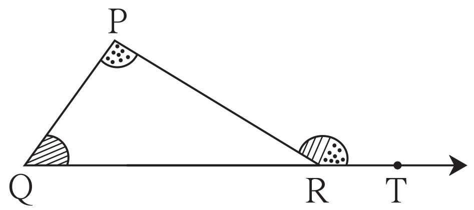


Fig. 3.1

Let's learn.

Theorem of remote interior angles of a triangle

Theorem : The measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

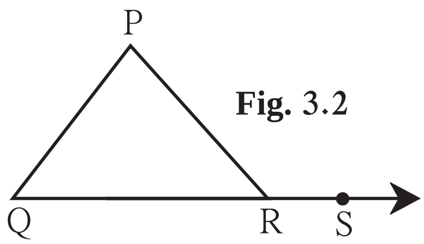


Fig. 3.2

Given : $\angle$PRS is an exterior angle of Δ PQR.

To prove : $\angle PRS = \angle PQR + \angle QPR$

Proof : The sum of all angles of a triangle is 180°.

$\therefore$ $\angle PQR + \angle QPR + \angle PRQ = 180^\circ$.......(I)

$\angle PRQ + \angle PRS = 180^\circ$.....angles in linear pair......(II)

$\therefore$ from (I) and (II)

$\angle PQR + \angle QPR + \angle PRQ = \angle PRQ + \angle PRS$

$\therefore$ $\angle PQR + \angle QPR = \angle PRS$ ....eliminating $\angle$PRQ from both sides

$\therefore$ the measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

Use your brain power! Can we give an alternative proof of the theorem drawing a line through point R and parallel to seg PQ in figure 3.2 ?

Let's learn.

Property of an exterior angle of triangle

The sum of two positive numbers a and b, that is $(a + b)$ is greater than a and greater than b also. That is, $a + b > a$, $a + b > b$

Using this inequality we get one property relaed to exterior angle of a triangle.

If $\angle PRS$ is an exterior angle of Δ PQR then

$\angle PRS > \angle P$, $\angle PRS > \angle Q$

$\therefore$ an exterior angle of a triangle is greater than its remote interior angle.

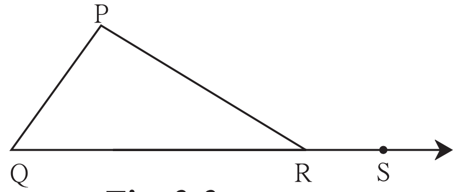


Fig. 3.3

Solved examples

Ex (1) The measures of angles of a triangle are in the ratio 5 : 6 : 7. Find the measures.

Solution : Let the measures of the angles of a triangle be $5x$, $6x$, $7x$.

$$\therefore \quad 5x + 6x + 7x = 180^\circ$$

$$18x = 180^\circ$$

$$x = 10^\circ$$

$5x = 5 \times 10 = 50^\circ$, $6x = 6 \times 10 = 60^\circ$, $7x = 7 \times 10 = 70^\circ$

$\therefore$ the measures of angles of the triangle are 50°, 60° and 70°.

Ex (2) Observe figure 3.4 and find the measures of $\angle PRS$ and $\angle RTS$.

Solution : $\angle PRS$ is an exterior angle of Δ PQR.

So from the theorem of remote interior angles,

$\angle PRS = \angle PQR + \angle QPR$

$= 40^\circ + 30^\circ$

$= 70^\circ$

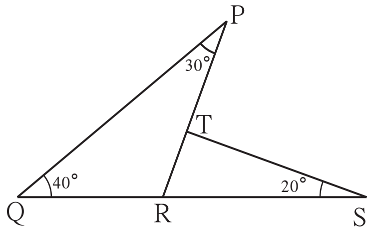


Fig. 3.4

In Δ RTS

$\angle TRS + \angle RTS + \angle TSR = \boxed{}$ .... sum of all angles of a triangle

$\therefore \boxed{} + \angle RTS + \boxed{} = 180^\circ$

$\therefore \angle RTS + 90^\circ = 180^\circ$

$\therefore \angle RTS = \boxed{}$

Ex (3) Prove that the sum of exterior angles of a triangle, obtained by extending its sides in the same direction is 360°.

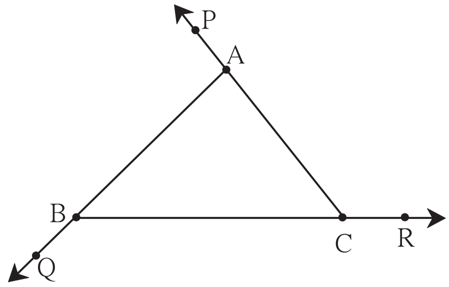


Fig. 3.5

Given : $\angle PAB$, $\angle QBC$ and $\angle ACR$ are exterior angles of Δ ABC

To prove : $\angle PAB + \angle QBC + \angle ACR = 360^\circ$

Proof : **Method I**

Considering exterior $\angle PAB$ of Δ ABC,

$\angle ABC$ and $\angle ACB$ are its remote interior angles.

$\angle PAB = \angle ABC + \angle ACB$ ----(I)

Similarly, $\angle ACR = \angle ABC + \angle BAC$ ----(II)..theorem of remote interior angles

and $\angle CBQ = \angle BAC + \angle ACB$ ---- (III)

Adding (I), (II) and (III),

$\angle PAB + \angle ACR + \angle CBQ$

$= \angle ABC + \angle ACB + \angle ABC + \angle BAC + \angle BAC + \angle ACB$

$= 2\angle ABC + 2\angle ACB + 2\angle BAC$

$= 2\,(\angle ABC + \angle ACB + \angle BAC)$

$= 2 \times 180^\circ$ . . . . . sum of interior angles of a triangle

$= 360^\circ$

Method II

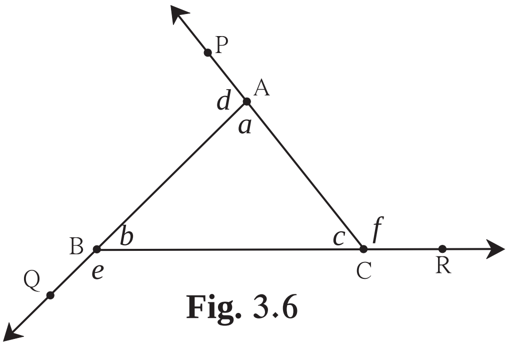


Fig. 3.6

$\angle c + \angle f = 180^\circ$ . . . . (angles in linear pair)

Also, $\angle a + \angle d = 180^\circ$

and $\angle b + \angle e = 180^\circ$

$\therefore\ \angle c + \angle f + \angle a + \angle d + \angle b + \angle e = 180^\circ \times 3 = 540^\circ$

$\angle f + \angle d + \angle e + (\angle a + \angle b + \angle c) = 540^\circ$

$\therefore\ \angle f + \angle d + \angle e + 180^\circ = 540^\circ$

$\therefore\ f + d + e = 540^\circ - 180^\circ$

$= 360^\circ$

Ex (4) In figure 3.7, bisectors of $\angle B$ and $\angle C$ of Δ ABC intersect at point P.

Prove that $\angle BPC = 90 + \frac{1}{2} \angle BAC$.

Complete the proof filling in the blanks.

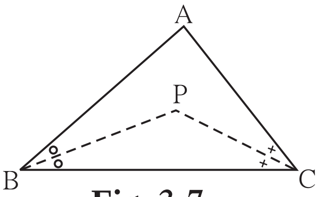


Fig. 3.7

Proof : In Δ ABC,

$\angle BAC + \angle ABC + \angle ACB = \boxed{}$ ...... sum of measures of angles of a triangle

$\therefore \frac{1}{2} \angle BAC + \frac{1}{2} \angle ABC + \frac{1}{2} \angle ACB = \frac{1}{2} \times \boxed{}$

....multiplying each term by $\frac{1}{2}$

$\therefore \frac{1}{2} \angle BAC + \angle PBC + \angle PCB = 90^\circ$

$\therefore \angle PBC + \angle PCB = 90^\circ - \frac{1}{2} \angle BAC$ ......(I)

In Δ BPC

$\angle BPC + \angle PBC + \angle PCB = 180^\circ$ ......sum of measures of angles of a triangle

$\therefore \angle BPC + \boxed{} = 180^\circ$ ......from (I)

$$\therefore \angle BPC = 180^\circ - (90^\circ - \frac{1}{2} \angle BAC)$$

$$= 180^\circ - 90^\circ + \frac{1}{2} \angle BAC$$

$$= 90^\circ + \frac{1}{2} \angle BAC$$

Practice set 3.1

1. In figure 3.8, $\angle ACD$ is an exterior angle of Δ ABC. $\angle B = 40^\circ$, $\angle A = 70^\circ$. Find the measure of $\angle ACD$.

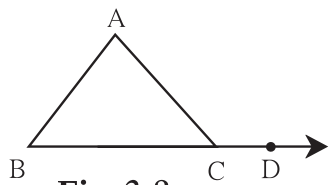


Fig. 3.8

2. In Δ PQR, $\angle P = 70^\circ$, $\angle Q = 65^\circ$ then find $\angle R$.

3. The measures of angles of a triangle are x°, $(x-20)^\circ$, $(x-40)^\circ$. Find the measure of each angle.

4. The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.

5. In figure 3.9, measures of some angles are given. Using the measures find the values of x, y, z.

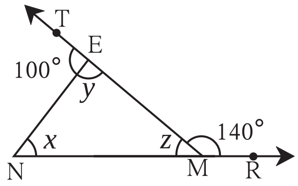


Fig. 3.9

6. In figure 3.10, line AB $\parallel$ line DE. Find the measures of $\angle$DRE and $\angle$ARE using given measures of some angles.

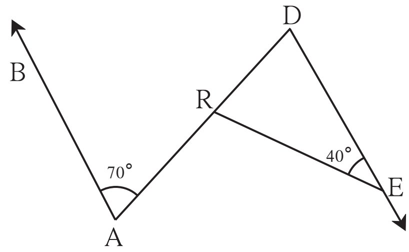


Fig. 3.10

7. In Δ ABC, bisectors of $\angle$A and $\angle$B intersect at point O. If $\angle C = 70°$. Find measure of $\angle$AOB.

8. In Figure 3.11, line AB $\parallel$ line CD and line PQ is the transversal. Ray PT and ray QT are bisectors of $\angle$BPQ and $\angle$PQD respectively.
Prove that m$\angle PTQ = 90°$.

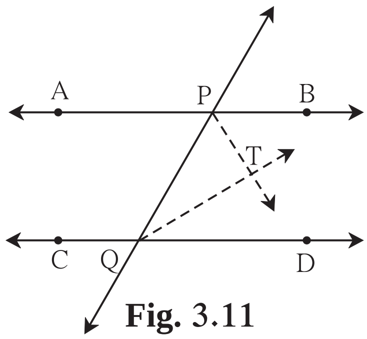


Fig. 3.11

9. Using the information in figure 3.12, find the measures of $\angle a$, $\angle b$ and $\angle c$.

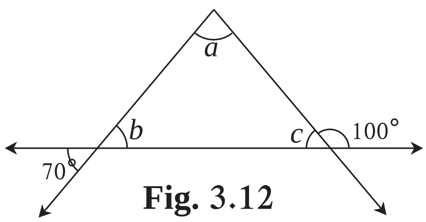


Fig. 3.12

10. In figure 3.13, line DE $\parallel$ line GF ray EG and ray FG are bisectors of $\angle$DEF and $\angle$DFM respectively.
Prove that,
(i) $\angle DEG = \frac{1}{2}$ $\angle$EDF (ii) EF = FG.

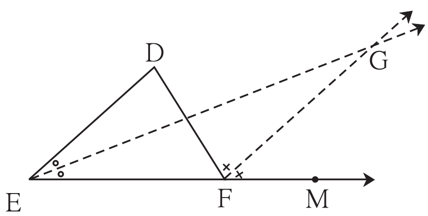


Fig. 3.13

Let's learn.

Congruence of triangles

We know that, if a segment placed upon another fits with it exactly then the two segmetns are congruent. When an angle placed upon another fits with it exactly then the two angles are congruent. Similarly, if a triangle placed upon another triangle fits exactly with it then the two triangles are said to be congruent. If Δ ABC and Δ PQR are congruent is written as $\Delta ABC \cong \Delta PQR$

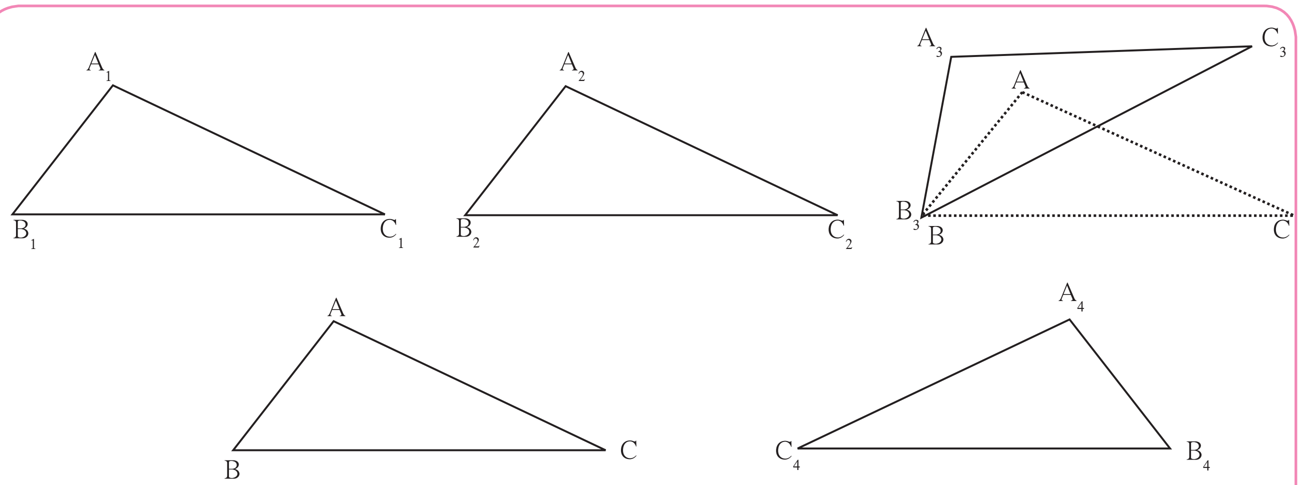


Fig. 3.14

Activity : Draw Δ ABC of any measure on a card-sheet and cut it out.

Place it on a card-sheet. Make a copy of it by drawing its border. Name it as $\Delta A_1B_1C_1$

Now slide the Δ ABC which is the cut out of a triangle to some distance and make one more copy of it. Name it $\Delta A_2B_2C_2$

Then rotate the cut out of triangle ABC a little, as shown in the figure, and make another copy of it. Name the copy as $\Delta A_3B_3C_3$. Then flip the triangle ABC, place it on another card-sheet and make a new copy of it. Name this copy as $\Delta A_4B_4C_4$.

Have you noticed that each of $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$, $\Delta A_3B_3C_3$ and $\Delta A_4B_4C_4$ is congruent with Δ ABC ? Because each of them fits exactly with Δ ABC.

Let us verify for $\Delta A_3B_3C_3$. If we place $\angle A$ upon $\angle A_3$, $\angle B$ upon $\angle B_3$ and $\angle C$ upon $\angle C_3$, then only they will fit each other and we can say that $\Delta ABC \cong \Delta A_3B_3C_3$.

We also have $AB = A_3B_3$, $BC = B_3C_3$, $CA = C_3A_3$.

Note that, while examining the congruence of two triangles, we have to write their angles and sides in a specific order, that is with a specific one-to-one correspondence.

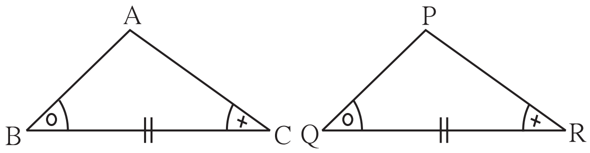
If $\Delta ABC \cong \Delta PQR$, then we get the following six equations :

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ . .. . (I) and $AB = PQ$, $BC = QR$, $CA = RP$ . . . . (II)

This means, with a one-to-one correspondence between the angles and the sides of two triangles, we get three pairs of congruent angles and three pairs of congruent sides.

Given six equations above are true for congruent triangles. For this let us see three specific equations are true then all six equations become true and hence two triangles congruent.

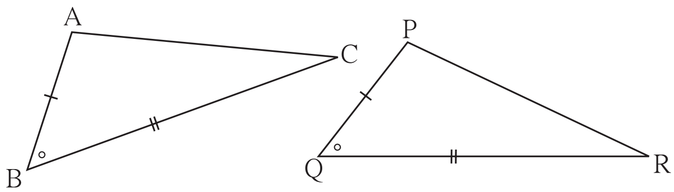
(1) In a correspondence, if two angles of ΔABC are equal to two angles of ΔPQR and the sides included by the respective pairs of angles are also equal, then the two triangles are congruent.



This property is called as angle-side-angle test, which in short we write A-S-A test.

Fig. 3.15

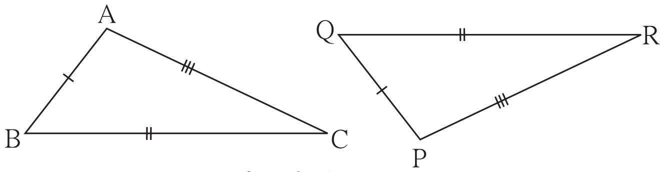
(2) In a correspondence, if two sides of Δ ABC are equal to two sides of Δ PQR and the angles included by the respective pairs of sides are also equal, then the two triangles are congruent.



This property is called as side-angle-side test, which in short we write S-A-S test.

Fig. 3.16

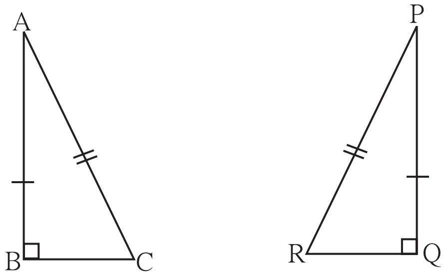
(3) In a correspondence, if three sides of Δ ABC are equal to three sides of Δ PQR , then the two triangles are congruent.



This property is called as side-side-side test, which in short we write S-S-S test.

Fig. 3.17

(4) If in Δ ABC and Δ PQR, $\angle$B and $\angle$Q are right angles, hypotenuses are equal and AB = PQ, then the two triangles are congruent.



This property is called the hypotenuse side test.

Fig. 3.18

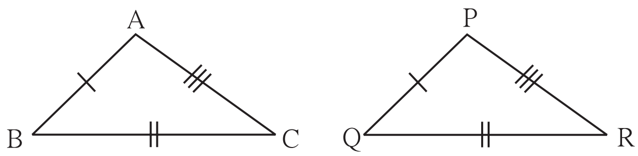
Remember this !

We have constructed triangles using the given information about parts of triangles. (For example, two angles and the included side, three sides, two sides and an included angle). We have experienced that the triangle constructed with any of these information is unique. So if by some one-to-one correspondence between two triangles, these three parts of one triangle are congruent with corresponding three parts of the other triangle then the two triangles are congruent. Then we come to know that in that correspondence their three angles and three sides are congruent. If two triangles are congruent then their respective angles and respective sides are congruent. This property is useful to solve many problems in Geometry.

Practice set 3.2

1. In each of the examples given below, a pair of triangles is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.

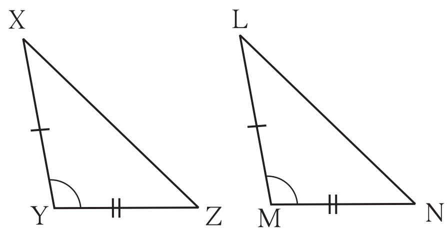
(i)



By . . . . test

Δ ABC $\cong \Delta$ PQR

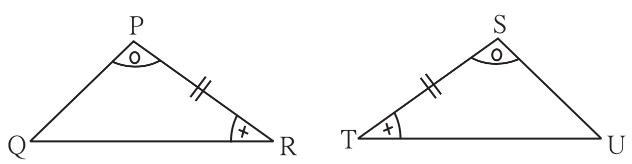
(ii)



By . . . . test

Δ XYZ $\cong \Delta$ LMN

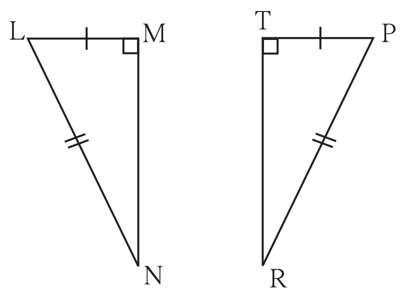
(iii)



By . . . . test

Δ PRQ $\cong \Delta$ STU

(iv)



By . . . . test

Δ LMN $\cong \Delta$ PTR

Fig. 3.19

2. Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.

(i)

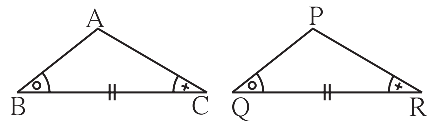


Fig. 3.20

From the information shown in the figure,

in Δ ABC and Δ PQR

$\angle ABC \cong \angle PQR$

seg BC $\cong$ seg QR

$\angle ACB \cong \angle PRQ$

$\therefore$ Δ ABC $\cong$ Δ PQR .... $\boxed{\quad\quad}$ test

$\therefore \angle BAC \cong$ $\boxed{\quad\quad}$ ....corresponding angles of congruent triangles.

seg AB $\cong$ $\boxed{\quad\quad}$
and $\boxed{\quad\quad}$ $\cong$ seg PR $\Big\}$ ... corresponding sides of congruent triangles

(ii)

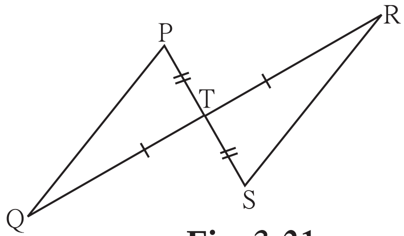


Fig. 3.21

From the information shown in the figure,,

In Δ PTQ and Δ STR

seg PT $\cong$ seg ST

$\angle PTQ \cong \angle STR$....vertically opposite angles

seg TQ $\cong$ seg TR

$\therefore$ Δ PTQ $\cong$ Δ STR .... $\boxed{\quad\quad}$ test

$\therefore \angle TPQ \cong$ $\boxed{\quad\quad}$
and $\boxed{\quad\quad}$ $\cong \angle TRS$ $\Big\}$ ..... corresponding angles of congruent triangles.

seg PQ $\cong$ $\boxed{\quad\quad}$ corresponding sides of congruent triangles.

3. From the information shown in the figure, state the test assuring the congruence of Δ ABC and Δ PQR. Write the remaining congruent parts of the triangles.

Fig. 3.22

4. As shown in the following figure, in Δ LMN and Δ PNM, LM = PN, LN = PM. Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.

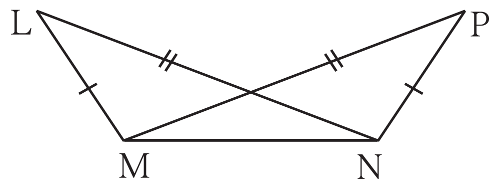


Fig. 3.23

5. In figure 3.24, seg AB $\cong$ seg CB and seg AD $\cong$ seg CD.
Prove that
Δ ABD $\cong$ Δ CBD

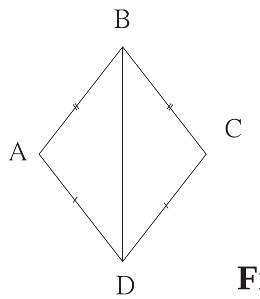


Fig. 3.24

Please note : corresponding sides of congruent triangles in short we write c.s.c.t. and corresponding angles of congruent triangles in short we write c.a.c.t.

6. In figure 3.25, $\angle P \cong \angle R$
seg PQ $\cong$ seg RQ
Prove that,
Δ PQT $\cong$ Δ RQS

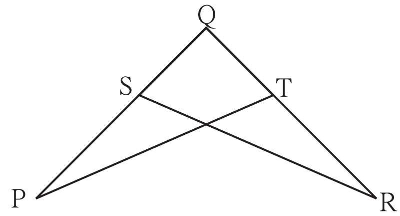


Fig. 3.25

Let's learn.

Isosceles triangle theorem

Theorem : If two sides of a triangle are congruent then the angles opposite to them are congruent.

Given : In Δ ABC, side AB $\cong$ side AC

To prove : $\angle ABC \cong \angle ACB$

Construction : Draw the bisector of $\angle BAC$ which intersects side BC at point D.

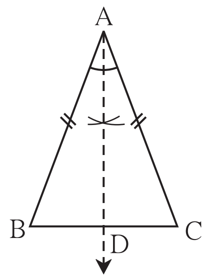


Fig. 3.26

Proof : In Δ ABD and Δ ACD

seg AB $\cong$ seg AC .... given

$\angle BAD \cong \angle CAD$........construction

seg AD $\cong$ seg AD .... common side

$\therefore$ Δ ABD $\cong$ Δ ACD ...... $\boxed{\quad}$

$\therefore \angle ABD \cong$ $\boxed{\quad}$ .... (c.a.c.t.)

$\therefore \angle ABC \cong \angle ACB$ $\because$ B - D - C

Corollary : If all sides of a triangle are congruent then its all angles are congruent. (write the proof of this corollary.)

Converse of isosceles triangle theorem

Theorem : If two angles of a triangle are congruent then the sides opposite to them are congruent.

Given : In Δ PQR, $\angle PQR \cong \angle PRQ$

To prove : Side PQ $\cong$ side PR

Construction : Draw the bisector of $\angle P$ intersecting side QR at point M

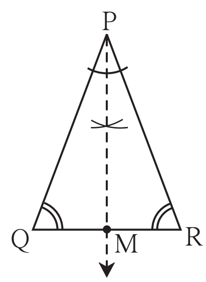


Fig. 3.27

Proof : In Δ PQM and Δ PRM

$\angle PQM \cong$ $\boxed{\quad}$........ given

$\angle QPM \cong \angle RPM$........$\boxed{\quad}$

seg PM $\cong$ $\boxed{\quad}$ .... common side

$\therefore$ Δ PQM $\cong$ Δ PRM ...... $\boxed{\quad}$ test

$\therefore$ seg PQ $\cong$ seg PR....... c.s.c.t.

Corollary : If three angles of a triangle are congruent then its three sides also are congruent. (Write the proof of this corollary yourself.)

Both the above theorems are converses of each other also.

Similarly the corollaries of the theorems are converses of each other.

Use your brain power!

(1) Can the theorem of isosceles triangle be proved doing a different construction ?

(2) Can the theorem of isosceles triangle be proved without doing any construction ?

Let's learn.

Property of 30° - 60° - 90° triangle

Activity I

Every student in the group should draw a right angled triangle, one of the angles measuring 30°. The choice of lengths of sides should be their own. Each one should measure the length of the hypotenuse and the length of the side opposite to 30° angle.

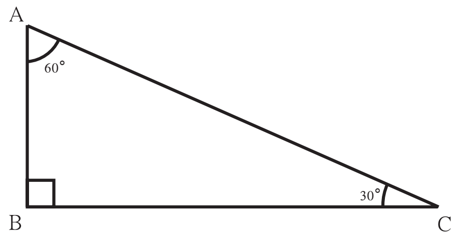


Fig. 3.28

One of the students in the group should fill in the following table.

Triangle Number	1	2	3	4
Length of the side opposite to 30° angle				
Length of the hypotenuse				

Did you notice any property of sides of right angled triangle with one of the angles measuring 30°?

Activity II

The measures of angles of a set square in your compass box are 30°,60° and 90°. Verify the property of the sides of the set square.

Let us prove an important property revealed from these activities.

Theorem : If the acute angles of a right angled triangle have measures 30° and 60°, then the length of the side opposite to 30° angle is half the length of the hypotenuse.

(Fill in the blanks and complete the proof .)

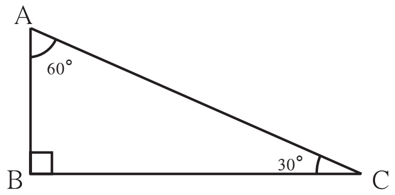


Fig. 3.29

Given : In Δ ABC

$\angle B = 90^\circ$, $\angle C = 30^\circ$, $\angle A = 60^\circ$

To prove : $AB = \frac{1}{2} AC$

Construction : Take a point D on the extended seg AB such that AB = BD. Draw seg DC.

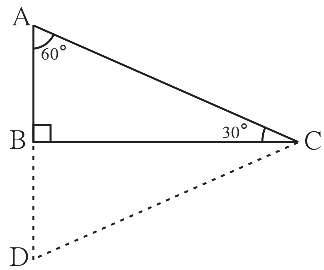


Fig. 3.30

Proof : Δ ABC and Δ DBC

seg AB $\cong$ seg DB .... $\boxed{}$

$\angle ABC \cong \angle DBC$ .... $\boxed{}$

seg BC $\cong$ seg BC .... $\boxed{}$

$\therefore$ $\Delta ABC \cong \Delta DBC$ ..... $\boxed{}$

$\therefore$ $\angle BAC \cong \angle BDC$ .... (c.a.c.t.)

In Δ ABC, $\angle BAC = 60^\circ$ $\therefore$ $\angle BDC = 60^\circ$

$\angle DAC = \angle ADC = \angle ACD = 60^\circ$ ...$\because$ sum of angles of Δ ADC is 180°

$\therefore$ Δ ADC is an equilateral triangle.

$\therefore$ AC = AD = DC .... corollary of converse of isosceles triangle theorem

But $AB = \frac{1}{2} AD$........ construction

$\therefore AB = \frac{1}{2} AC$ .... $\because$ AD = AC

Activity

With the help of the Figure 3.29 above fill in the blanks and complete the proof of the following theorem.

Theorem : If the acute angles of a right angled triangle have measures 30° and 60° then the length of the side opposite to 60° angle is $\frac{\sqrt{3}}{2} \times$ hypotenuse

Proof : In the above theorem we have proved $AB = \frac{1}{2} AC$

$AB^2 + BC^2 = \boxed{}$ .... Pythagoras theorem

$\frac{1}{4} AC^2 + BC^2 = \boxed{}$

$\therefore BC^2 = AC^2 - \frac{1}{4} AC^2$

$\therefore BC^2 = \boxed{}$

$\therefore BC = \frac{\sqrt{3}}{2} AC$

Activity : Complete the proof of the theorem.

Theorem : If measures of angles of a triangle are 45°, 45°, 90° then the length of each side containing the right angle is $\frac{1}{\sqrt{2}} \times$ hypotenuse.

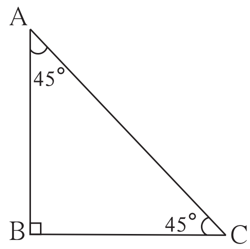


Fig. 3.31

Proof : In Δ ABC, $\angle B = 90^\circ$ and $\angle A = \angle C = 45^\circ$

$\therefore$ BC = AB

By Pythagoras theorem

$AB^2 + BC^2 = \boxed{\quad}$

$AB^2 + \boxed{\quad} = AC^2$ ... $\because$ (BC = AB)

$\therefore 2AB^2 = \boxed{\quad}$

$\therefore AB^2 = \boxed{\quad}$

$\therefore AB = \frac{1}{\sqrt{2}} AC$

This property is called 45°- 45°- 90° theorem.

Remember this !

(1) If the acute angles of a right angled triangle are 30°, 60° then the length of side opposite to 30° angle is half of hypotenuse and the length of side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ hypotenuse . This property is called $30^\circ - 60^\circ - 90^\circ$ theorem.

(2) If acute angles of a right angled triangle are 45°, 45° then the length of each side containing the right angle is $\frac{\text{hypotenuse}}{\sqrt{2}}$.

This property is called $45^\circ - 45^\circ - 90^\circ$ theorem

Let's recall.

Median of a triangle

The segment joining a vertex and the mid-point of the side opposite to it is called a **Median** of the triangle.

In Figure 3.32, point D is the mid point of side BC.

$\therefore$ seg AD is a median of Δ ABC.

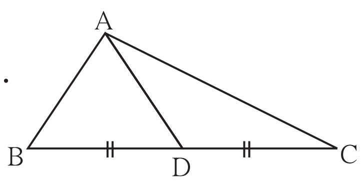


Fig. 3.32

Activity I : Draw a triangle ABC. Draw medians AD, BE and CF of the triangle. Let their point of concurrence be G, which is called the centroid of the triangle. Compare the lengths of AG and GD with a divider. Verify that the length of AG is twice the length of GD. Similarly, verify that the length of BG is twice the length of GE and the length of CG is twice the length of GF. Hence note the following property of medians of a triangle of a triangle.

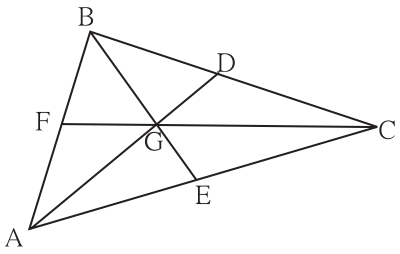


Fig. 3.33

The point of concurrence of medians of a triangle divides each median in the ratio 2 : 1.

Activity II : Draw a triangle ABC on a card board. Draw its medians and denote their point of concurrence as G. Cut out the triangle.

Now take a pencil. Try to balance the triangle on the flat tip of the pencil. The triangle is balanced only when the point G is on the flat tip of the pencil.

This activity shows an important property of the **centroid** (point of concurrence of the medians) of the triangle.

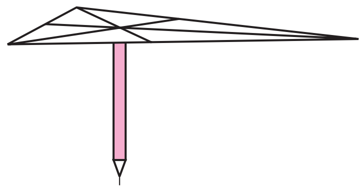


Fig. 3.34

Let's learn.

Property of median drawn on the hypotenuse of right triangle

Activity : In the figure 3.35, Δ ABC is a right angled triangle. seg BD is the median on hypotenuse.

Measure the lengths of the following segments

AD =......... DC =............ BD = ...

From the measurements verify that BD = $\frac{1}{2}$ AC .

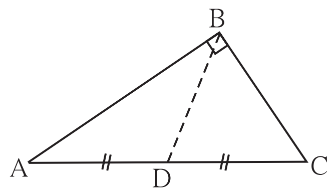


Fig. 3.35

Now let us prove the property, the length of the median is half the length of the hypotenuse.

Theorem : In a right angled triangle, the length of the median of the hypotenuse is half the length of the hypotenuse.

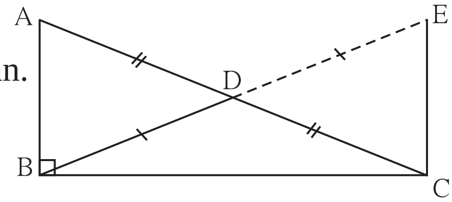


Fig. 3.36

Given : In Δ ABC, $\angle B = 90^\circ$, seg BD is the median.

To prove : BD = $\frac{1}{2}$ AC

Construction : Take point E on the ray BD such that B - D - E and l(BD) = l(DE). Draw seg EC.

Proof : (Main steps are given. Write the steps in between with reasons and complete the proof.)

Δ ADB $\cong$ Δ CDE .... by S-A-S test

line AB $||$ line EC ....by test of alternate angles

Δ ABC $\cong$ Δ ECB .... by S-A-S test

BD = $\frac{1}{2}$ AC

Remember this !

In a right angled triangle, the length of the median on its hypotenuse is half the length of the hypotenuse.

Practice set 3.3

1. Find the values of x and y using the information shown in figure 3.37. Find the measure of $\angle$ABD and m$\angle$ACD.

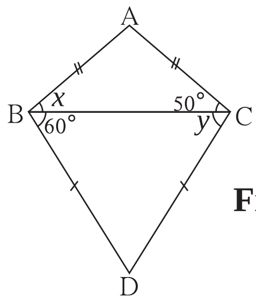


Fig. 3.37

2. The length of hypotenuse of a right angled triangle is 15. Find the length of median of its hypotenuse.

3. In Δ PQR, $\angle Q = 90^\circ$, PQ = 12, QR = 5 and QS is a median. Find l(QS).

4. In figure 3.38, point G is the point of concurrence of the medians of Δ PQR . If GT = 2.5, find the lengths of PG and PT.

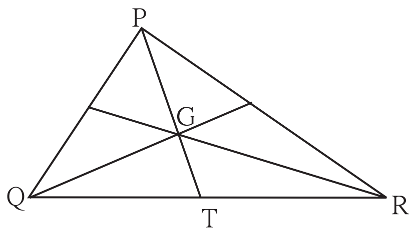


Fig. 3.38

Let's recall.

Activity : Draw a segment AB of convenient length. Lebel its mid-point as M. Draw a line l passing through the point M and perpendicular to seg AB.

Did you notice that the line l is the perpendicular bisector of seg AB ?

Now take a point P anywhere on line l. Compare the distance PA and PB with a divider. What did you find ? You should have noticed that PA = PB. This observation shows that any point on the perpendicular bisector of a segment is equidistant from its end points.

Now with the help of a compass take any two points like C and D, which are equidistant from A and B. Did all such points lie on the line l ? What did you notice from the observation ? Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

These two properties are two parts of the perpendicular bisector theorem. Let us now prove them.

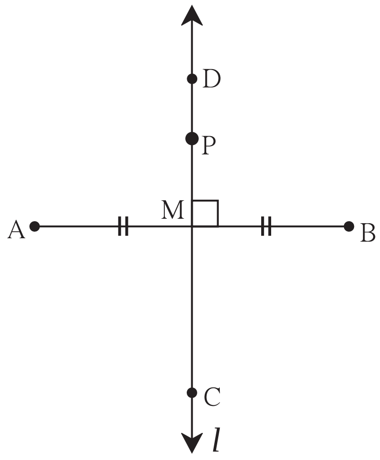


Fig. 3.39

Let's learn.

Perpendicular bisector theorem

Part I : Every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Given : line l is the perpendicular bisector of seg AB at point M.

Point P is any point on l,

To prove : PA = PB

Construction : Draw seg AP and seg BP.

Proof : In Δ PMA and Δ PMB

seg PM $\cong$ seg PM .... common side

$\angle$PMA $\cong$ $\angle$PMB ....each is a right angle

seg AM $\cong$ seg BM ....given

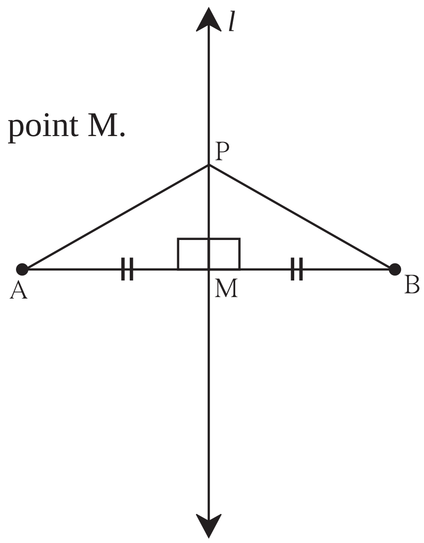


Fig. 3.40

$\therefore$ Δ PMA $\cong$ Δ PMB ...... S-A-S test
$\therefore$ seg PA $\cong$ seg PB ....c.s.c.t.
$\therefore$ l (PA) = l (PB)
Hence every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Part II : Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. That is, PA = PB.

To prove: Point P is on the perpendicular bisector of seg AB.

Construction : Take mid-point M of seg AB and draw line PM.

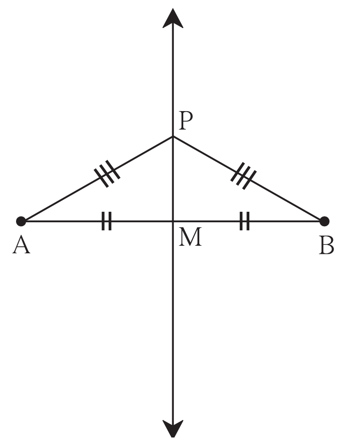


Fig. 3.41

Proof : In Δ PAM and Δ PBM

seg PA $\cong$ seg PB .... $\boxed{}$

seg AM $\cong$ seg BM .... $\boxed{}$

seg PM $\cong$ $\boxed{}$ .... common side

$\therefore$ Δ PAM $\cong$ Δ PBM ...... $\boxed{}$ test.

$\therefore$ $\angle$PMA $\cong$ $\angle$PMB.......c.a.c.t.

But $\angle$PMA + $\boxed{}$ = 180°

$\angle$PMA + $\angle$PMA = 180° .... ($\because$ $\angle$PMB = $\angle$PMA)

2 $\angle$PMA = $\boxed{}$

$\therefore$ $\angle$PMA = 90°

$\therefore$ seg PM $\perp$ seg AB ......(1)

But Point M is the midpoint of seg AB. ......construction .... (2)

$\therefore$ line PM is the perpendicular bisector of seg AB. So point P is on the perpendicular bisector of seg AB

Angle bisector theorem

Part I : Every point on the bisector of an angle is equidistant from the sides of the angle.

Given : Ray QS is the bisector of $\angle$PQR .
Point A is any point on ray QS
seg AB $\perp$ ray QP seg AC $\perp$ ray QR

To prove : seg AB $\cong$ seg AC

Proof : Write the proof using test of congruence of triangles.

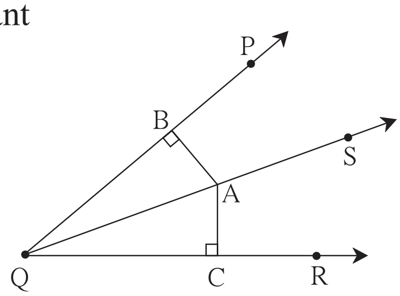


Fig. 3.42

Part II : Any point equidistant from sides of an angle is on the bisector of the angle.

Given : A is a point in the interior of $\angle PQR$.

seg $AC \perp$ ray QR   seg $AB \perp$ ray QP

and AB = AC

To prove : Ray QA is the bisector of $\angle PQR$.

That is $\angle BQA = \angle CQA$

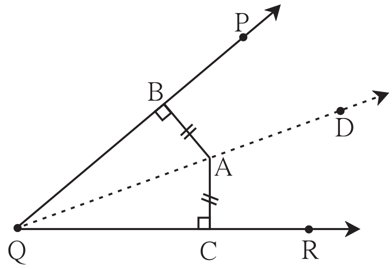


Fig. 3.43

Proof : Write the proof using proper test of congruence of triangles.

Let's recall.

Activity

As shown in the figure, draw Δ XYZ such that XZ > side XY

Find which of $\angle Z$ and $\angle Y$ is greater.

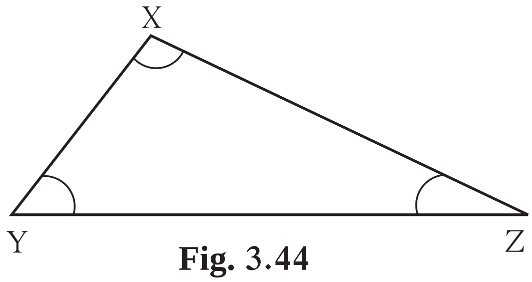


Fig. 3.44

Let's learn.

Properties of inequalities of sides and angles of a triangle

Theorem : If two sides of a triangle are unequal, then the angle opposite to the greater side is greater than angle opposite to the smaller side.

Given : In Δ XYZ, side XZ > side XY

To prove : $\angle XYZ > \angle XZY$

Construction : Take point P on side XZ such that XY = XP, Draw seg YP.

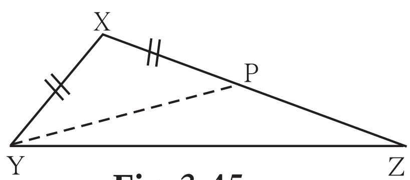


Fig. 3.45

Proof : In Δ XYP

XY = XP ....construction

$\therefore$ $\angle XYP = \angle XPY$.....isosceles triangle theorem .....(I)

$\angle XPY$ is an exterior angle of Δ YPZ.

$\therefore$ $\angle XPY > \angle PZY$ ....exterior angle theorem

$\therefore$ $\angle XYP > \angle PZY$ ....from (I)

$\therefore$ $\angle XYP + \angle PYZ > \angle PZY$ ....If a > b and c > 0 then a + c > b

$\therefore$ $\angle XYZ > \angle PZY$, that is $\angle XYZ > \angle XZY$

Theorem : If two angles of a triangle are unequal then the side opposite to the greater angle is greater than the side opposite to smaller angle.

The theorem can be proved by indirect proof. Complete the following proof by filling in the blanks.

Given : In ΔABC, $\angle B > \angle C$

To prove : $AC > AB$

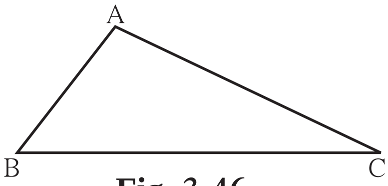


Fig. 3.46

Proof : There are only three possibilities regarding the lengths of side AB and side AC of Δ ABC

(i) $AC < AB$ (ii) $\boxed{\quad}$ (iii) $\boxed{\quad}$

(i) Let us assume that $AC < AB$.

If two sides of a triangle are unequal then the angle opposite to greater side is $\boxed{\quad}$.

$\therefore \angle C > \boxed{\quad}$

But $\angle C < \angle B$ .... (given)

This creates a contradiction.

$\therefore \boxed{\quad} < \boxed{\quad}$ is wrong.

(ii) If $AC = AB$

then $\angle B = \angle C$

But $\boxed{\quad} > \boxed{\quad}$ ...... (given)

This also creates a contradiction.

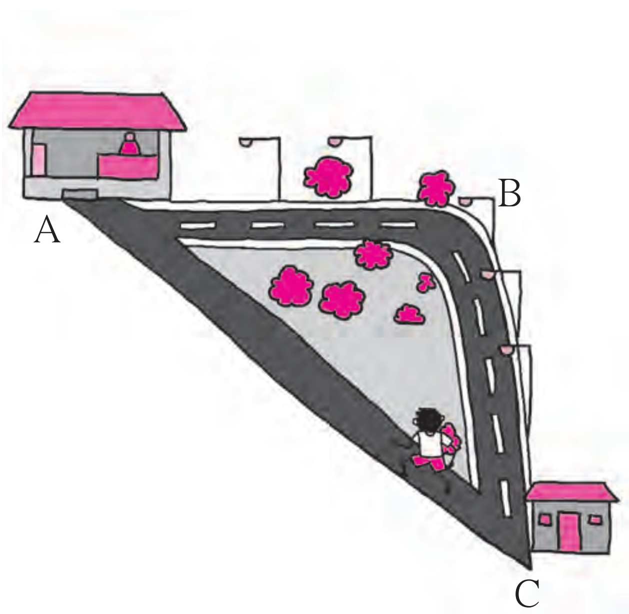
$\therefore \boxed{\quad} = \boxed{\quad}$ is wrong

$\therefore$ $AC > AB$ is the only remaining possibility.

$\therefore AC > AB$

Let's recall.

As shown in the adjacent picture, there is a shop at A. Sameer was standing at C. To reach the shop, he choose the way $C \rightarrow A$ instead of $C \rightarrow B \rightarrow A$, because he knew that the way $C \rightarrow A$ was shorter than the way $C \rightarrow B \rightarrow A$. So which property of a triangle had he realised ?



The sum of two sides of a triangle is greater than its third side.

Let us now prove the property.

Theorem : The sum of any two sides of a triangle is greater than the third side.

Given : Δ ABC is any triangle.

To prove : AB + AC > BC

AB + BC > AC

AC + BC > AB

Construction : Take a point D on ray BA such that AD = AC.

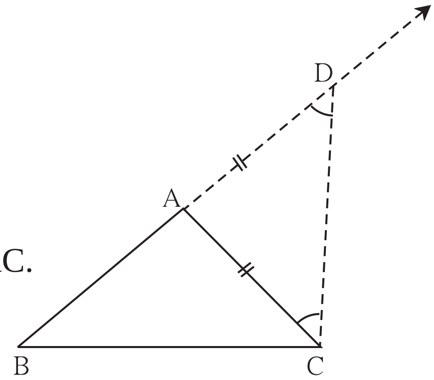


Fig. 3.47

Proof : In Δ ACD, AC = AD ..... construction

$\therefore$ $\angle$ACD = $\angle$ADC ...... c.a.c.t.

$\therefore$ $\angle$ACD + $\angle$ACB > $\angle$ADC

$\therefore$ $\angle$BCD > $\angle$ADC

$\therefore$ side BD > side BC ....the side opposite to greater angle is greater

$\therefore$ BA + AD > BC ....$\because$ BD = BA + AD

BA + AC > BC .... $\because$ AD = AC

Similarly we can prove that AB + BC > AC

and BC + AC > AB.

Practice set 3.4

1. In figure 3.48, point A is on the bisector of $\angle$XYZ.

If AX = 2 cm then find AZ.

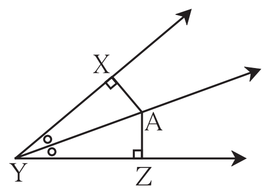


Fig. 3.48

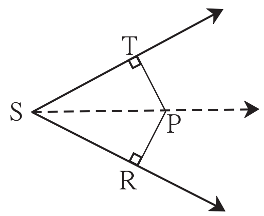
2. 

Fig. 3.49

In figure 3.49, $\angle$RST = 56°, seg PT $\perp$ ray ST, seg PR $\perp$ ray SR and seg PR $\cong$ seg PT

Find the measure of $\angle$RSP.

State the reason for your answer.

3. In Δ PQR, PQ = 10 cm, QR = 12 cm, PR = 8 cm. Find out the greatest and the smallest angle of the triangle.

4. In Δ FAN, $\angle$F = 80°, $\angle$A = 40° . Find out the greatest and the smallest side of the triangle. State the reason.

5. Prove that an equilateral triangle is equiangular.

6. Prove that, if the bisector of $\angle BAC$ of Δ ABC is perpendicular to side BC, then Δ ABC is an isosceles triangle.

7. In figure 3.50, if seg PR $\cong$ seg PQ, show that seg PS > seg PQ.

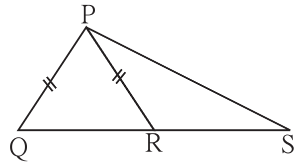


Fig. 3.50

8. In figure 3.51, in Δ ABC, seg AD and seg BE are altitudes and AE = BD.

Prove that seg AD $\cong$ seg BE

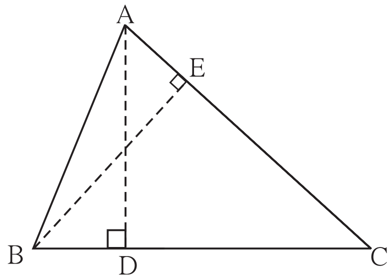
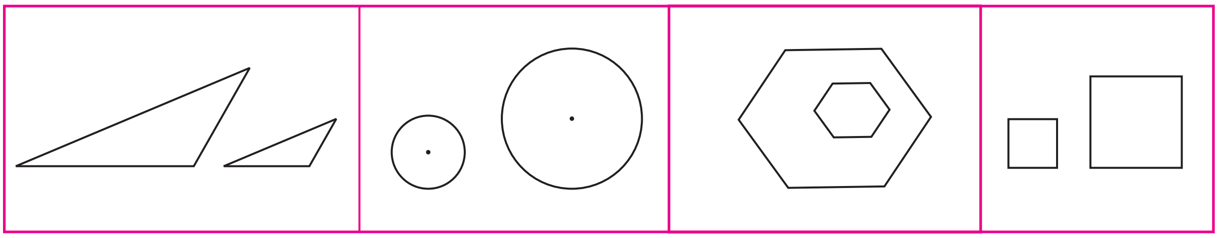


Fig. 3.51

Let's learn.

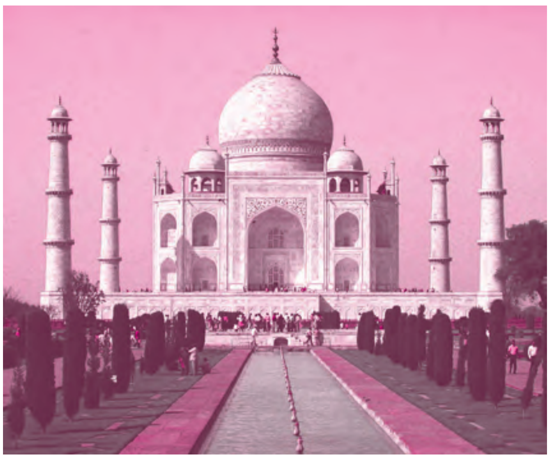
Similar triangles

Observe the following figures.



The pairs of figures shown in each part have the same shape but their sizes are different. It means that they are not congruent.

Such like looking figures are called similar figures.



We find similarity in a photo and its enlargement, also we find similarity between a road-map and the roads.

The proportionality of all sides is an important property of similarity of two figures. But the angles in the figures have to be of the same measure. If the angle between this roads is not the same in its map, then the map will be misleading.

ICT Tools or Links

Take a photograph on a mobile or a computer. Recall what you do to reduce it or to enlarge it. Also recall what you do to see a part of the photograph in detail.

Now we shall learn properties of similar triangles through an activity.

Activity : On a card-sheet, draw a triangle of sides 4 cm, 3 cm and 2 cm. Cut it out. Make 13 more copies of the triangle and cut them out from the card sheet.
Note that all these triangular pieces are congruent. Arrange them as shown in the following figure and make three triangles out of them.

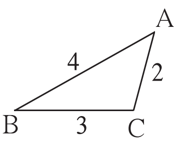


Fig. 3.52

1 triangle

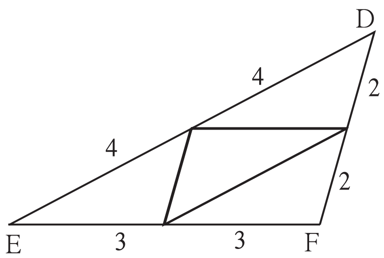


Fig. 3.53

4 triangles

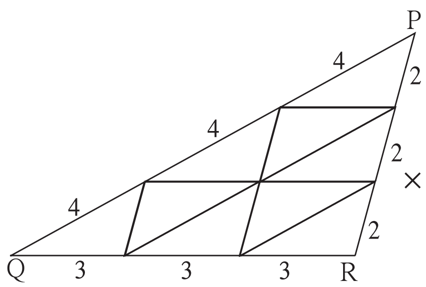


Fig. 3.54

9 triangles

Δ ABC and Δ DEF are similar in the correspondence ABC $\leftrightarrow$ DEF.

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

and $\dfrac{AB}{DE} = \dfrac{4}{8} = \dfrac{1}{2}$; $\quad \dfrac{BC}{EF} = \dfrac{3}{6} = \dfrac{1}{2}$; $\quad \dfrac{AC}{DF} = \dfrac{2}{4} = \dfrac{1}{2}$,

........the corresponding sides are in proportion.

Similarly, consider Δ DEF and Δ PQR. Are their angles congruent and sides proportional in the correspondence DEF $\leftrightarrow$ PQR ?

Let's learn.

Similarity of triangles

In Δ ABC and Δ PQR, If (i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and

(ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$; then Δ ABC and Δ PQR are called similar triangles.

'Δ ABC and Δ PQR are similar' is written as '$\Delta ABC \sim \Delta PQR$'.

Let us learn the relation between the corresponding angles and corresponding sides of similar triangles through an activity.

Activity : Draw a triangle $\Delta A_1B_1C_1$ on a card-sheet and cut it out. Measure $\angle A_1$, $\angle B_1$, $\angle C_1$.

Draw two more triangles $\Delta A_2B_2C_2$ and $\Delta A_3B_3C_3$ such that

$\angle A_1 = \angle A_2 = \angle A_3$, $\angle B_1 = \angle B_2 = \angle B_3$, $\angle C_1 = \angle C_2 = \angle C_3$

and $B_1C_1 > B_2C_2 > B_3C_3$. Now cut these two triangles also. Measure the lengths of the three triangles. Arrange the triangles in two ways as shown in the figure.

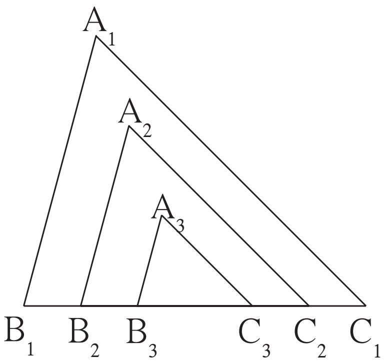


Fig. 3.55

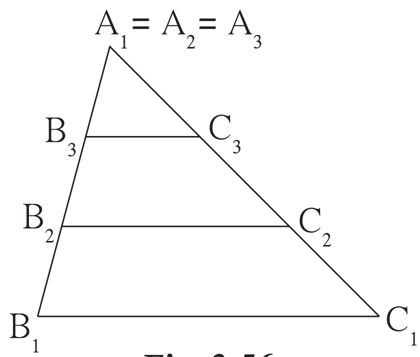


Fig. 3.56

Check the ratios $\frac{A_1B_1}{A_2B_2}$, $\frac{B_1C_1}{B_2C_2}$, $\frac{A_1C_1}{A_2C_2}$. You will notice that the ratios are equal.

Similarly, see whether the ratios $\frac{A_1C_1}{A_3C_3}$, $\frac{B_1C_1}{B_3C_3}$, $\frac{A_1B_1}{A_3B_3}$ are equal.

From this activity note that, when corresponding angles of two triangles are equal, the ratios of their corresponding sides are also equal. That is, their corresponding sides are in the same proportion.

We have seen that, in Δ ABC and Δ PQR if

(i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, then (ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

This means, if corresponding angles of two triangles are equal then the corresponding sides are in the same proportion.

This rule can be proved elaborately. We shall use it to solve problems.

Remember this !

- If corresponding angles of two triangles are equal then the two triangles are similar.
- If two triangles are similar then their corresponding sides are in proportion and corresponding angles are congruent.

Ex. Some information is shown in Δ ABC and Δ PQR in figure 3.57. Observe it. Hence find the lengths of side AC and PQ.

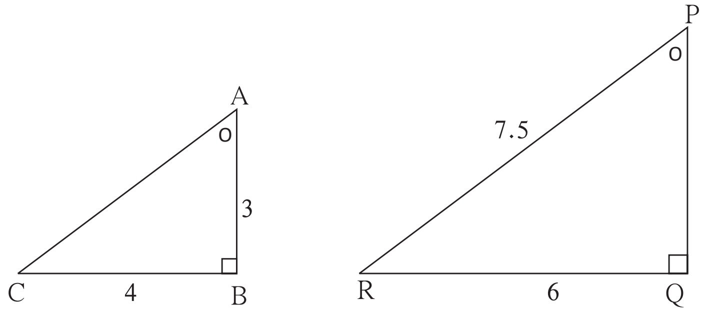


Fig. 3.57

Solution : The sum of all angles of a triangle is 180°.

It is given that,

$\angle A = \angle P$ and $\angle B = \angle Q$ $\quad \therefore \angle C = \angle R$

$\therefore$ Δ ABC and Δ PQR are equiangular triangles.

$\therefore$ there sides are propotional.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\therefore \frac{3}{PQ} = \frac{4}{6} = \frac{AC}{7.5}$$

$\therefore$ $4 \times PQ = 18$

$$\therefore PQ = \frac{18}{4} = 4.5$$

Similarly $6 \times AC = 7.5 \times 4$

$$\therefore AC = \frac{7.5 \times 4}{6} = \frac{30}{6} = 5$$

Practice set 3.5

1. If Δ XYZ ~ Δ LMN, write the corresponding angles of the two triangles and also write the ratios of corresponding sides.

2. In Δ XYZ, XY = 4 cm, YZ = 6 cm, XZ = 5 cm, If Δ XYZ ~ Δ PQR and PQ = 8 cm then find the lengths of remaining sides of Δ PQR.

3. Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.

Let's recall.

While preparing a map of a locality, you have to show the distances between different spots on roads with a proper scale. For example, 1 cm = 100 m, 1 cm = 50 m etc. Did you think of the properties of triangle ? Keep in mind that side opposite to greater angle is greater.

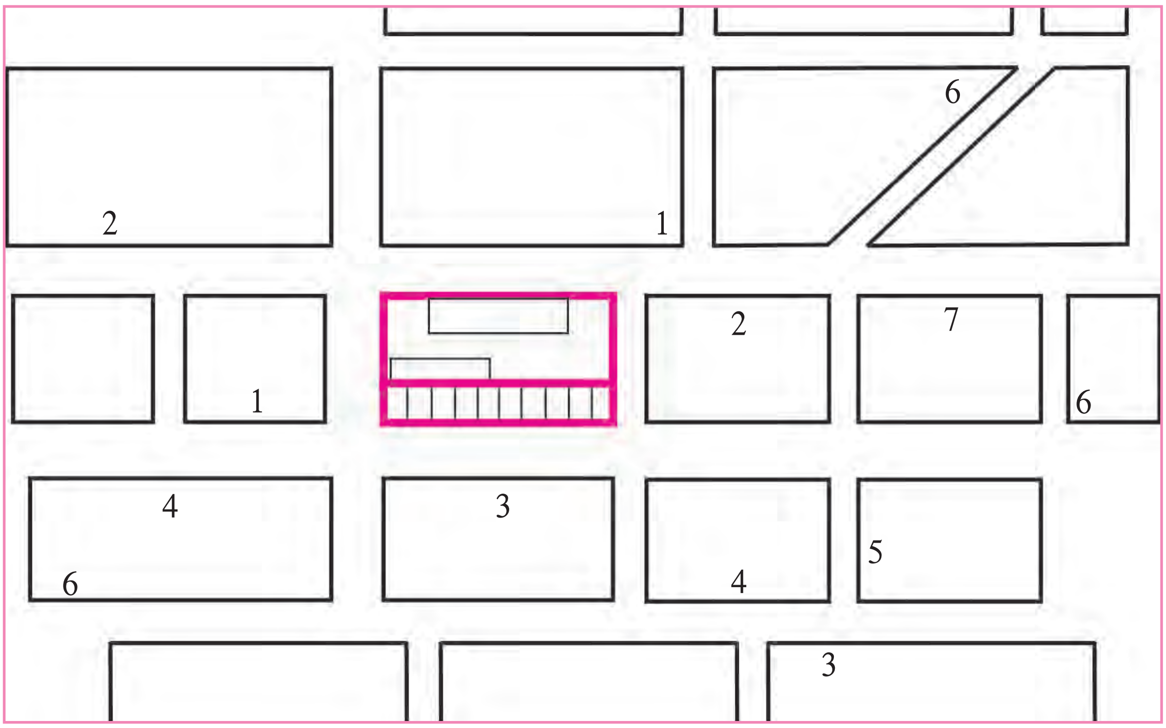
Project :

Prepare a map of road surrounding your school or home, upto a distance of about 500 metre.

How will you measure the distance between two spots on a road ?

While walking, count how many steps cover a distance of about two metre. Suppose, your three steps cover a distance of 2 metre. Considering this proportion 90 steps means 60 metre. In this way you can judge the distances between different spots on roads and also the lengths of roads. You have to judge the measures of angles also where two roads meet each other. Choosing a proper scale for lengths of roads, prepare a map. Try to show shops, buildings, bus stops, rickshaw stand etc. in the map.

A sample map with legend is given below



Legend: 1. Book store 2. Bus stop 3. Stationery shop 4. Bank 5. Medical store 6. Restaurant 7. Cycle shop

Problem set 3

1. Choose the correct alternative answer for the following questions.

(i) If two sides of a triangle are 5 cm and 1.5 cm, the lenght of its third side cannot be . . . .

(A) 3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

(ii) In Δ PQR, If $\angle R > \angle Q$ then . . . .

(A) QR > PR (B) PQ > PR (C) PQ < PR (D) QR < PR

(iii) In Δ TPQ, $\angle T = 65°$, $\angle P = 95°$ which of the following is a true statement ?

(A) PQ < TP (B) PQ < TQ (C) TQ < TP < PQ (D) PQ < TP < TQ

2. Δ ABC is isosceles in which AB = AC. Seg BD and seg CE are medians. Show that BD = CE.

3. In Δ PQR, If PQ > PR and bisectors of $\angle Q$ and $\angle R$ intersect at S. Show that SQ > SR.

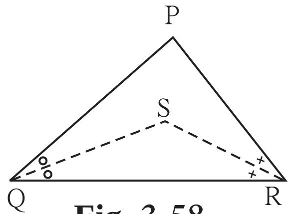


Fig. 3.58

4. In figure 3.59, point D and E are on side BC of Δ ABC, such that BD = CE and AD = AE. Show that Δ ABD $\cong$ Δ ACE.

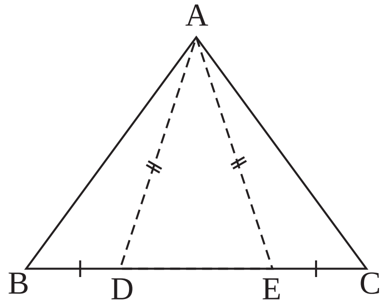


Fig. 3.59

5. In figure 3.60, point S is any point on side QR of Δ PQR

Prove that : PQ + QR + RP > 2PS

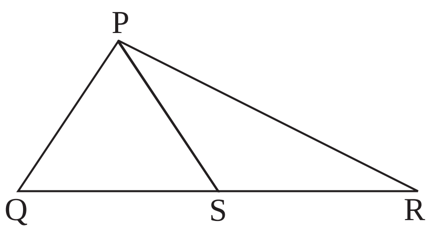


Fig. 3.60

6. In figure 3.61, bisector of $\angle BAC$ intersects side BC at point D. Prove that AB > BD

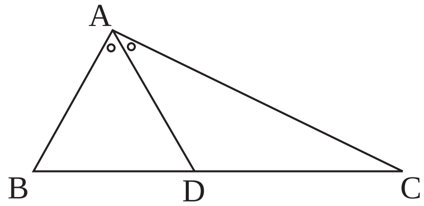


Fig. 3.61

7. In figure 3.62, seg PT is the bisector of $\angle QPR$. A line through R intersects ray QP at point S. Prove that PS = PR

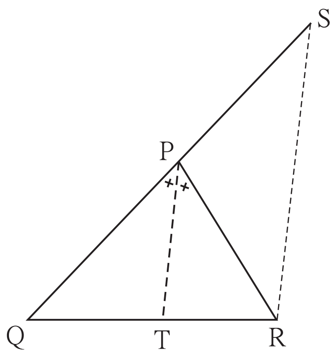


Fig. 3.62

8. In figure 3.63, seg AD $\perp$ seg BC. seg AE is the bisector of $\angle CAB$ and C - E - D. Prove that

$$\angle DAE = \frac{1}{2}(\angle C - \angle B)$$

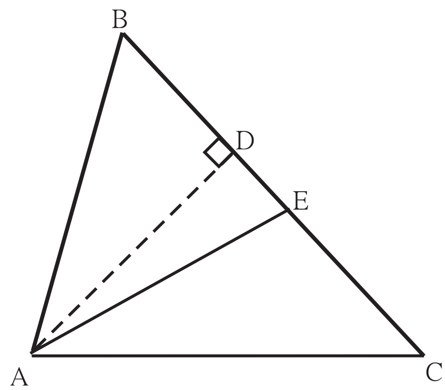


Fig. 3.63

Use your brain power!

We have learnt that if two triangles are equiangular then their sides are in proportion. What do you think if two quadrilaterals are equiangular ? Are their sides in proportion? Draw different figures and verify.

Verify the same for other polygons.

4 Constructions of Triangles

Let's study.

To construct a triangle, if following information is given.

- **Base, an angle adjacent to the base and sum of lengths of two remaining sides.**
- **Base, an angle adjacent to the base and difference of lengths of remaining two sides.**
- **Perimeter and angles adjacent to the base.**

Let's recall.

In previous standard we have learnt the following triangle constructions.

* To construct a triangle when its three sides are given.
* To construct a triangle when its base and two adjacent angles are given.
* To construct a triangle when two sides and the included angle are given.
* To construct a right angled triangle when its hypotenuse and one side is given.

Perpendicular bisector Theorem

- Every point on the perpendicular bisector of a segment is equidistant from its end points.
- Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.

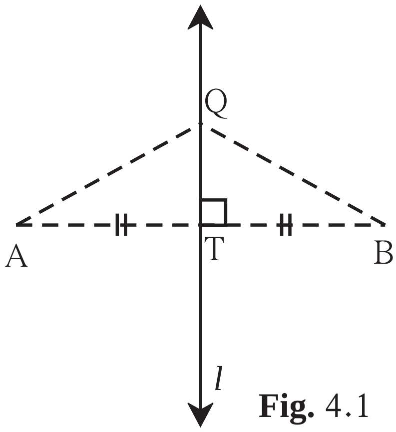


Fig. 4.1

Let's learn.

Constructions of triangles

To construct a triangle, three conditions are required. Out of three sides and three angles of a triangle two parts and some additional information about them is given, then we can construct a triangle using them.

We frequently use the following property in constructions.

If a point is on two different lines then it is the intersecrtion of the two lines.

Construction I

To construct a triangle when its base, an angle adjacent to the base and the sum of the lengths of remaining sides is given.

Ex. Construct Δ ABC in which BC = 6.3 cm, $\angle$B = 75° and AB + AC = 9 cm.
Solution : Let us first draw a rough figure of expected triangle.

Explanation : As shown in the rough figure, first we draw seg BC = 6.3 cm of length. On the ray making an angle of 75° with seg BC, mark point D such that

BD = AB + AC = 9 cm

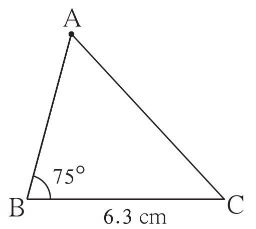
Now we have to locate point A on ray BD.

BA + AD = BA + AC = 9

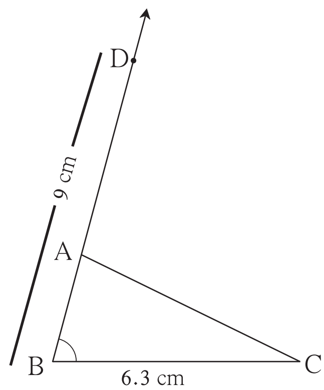
$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg CD.

$\therefore$ the point of intersection of ray BD and the perpendicular bisector of seg CD is point A.



Rough figure 4.2



Rough figure 4.3

Steps of construction

(1) Draw seg BC of length 6.3 cm.

(2) Draw ray BP such that m$\angle$ PBC = 75°.

(3) Mark point D on ray BP such that d(B,D) = 9 cm

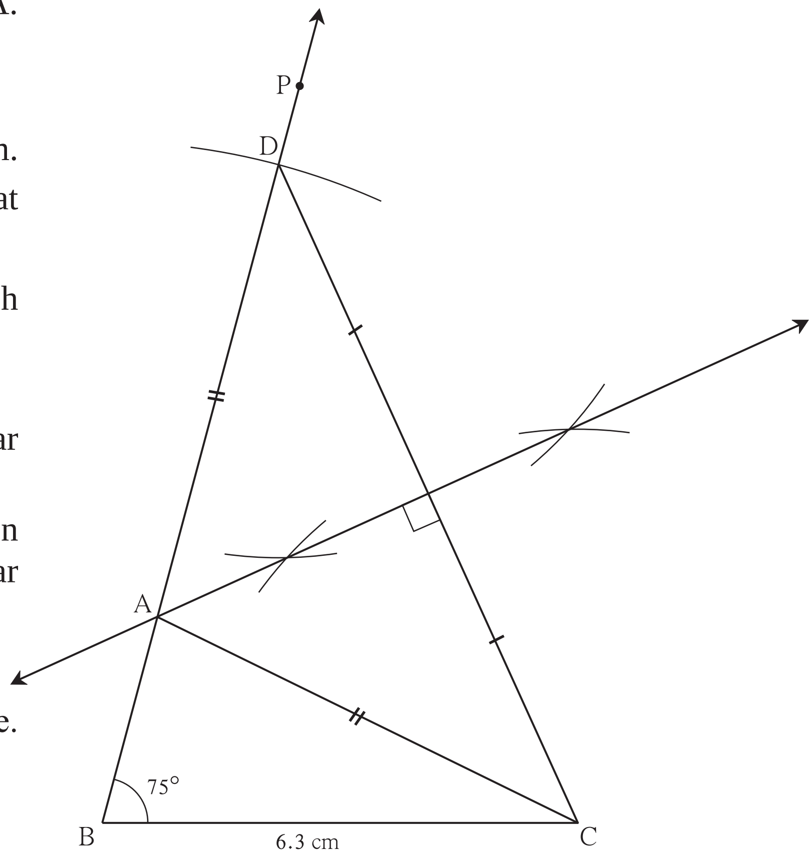
(4) Draw seg DC.

(5) Construct the perpendicular bisector of seg DC .

(6) Name the point of intersection of ray BP and the perpendicular bisector of CD as A.

(7) Draw seg AC.

Δ ABC is the required triangle.



Fair fig. 4.4

Practice set 4.2

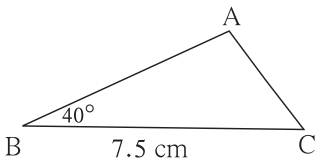
1. Construct Δ PQR, in which QR = 4.2 cm, m$\angle$Q = 40° and PQ + PR = 8.5 cm
2. Construct Δ XYZ, in which YZ = 6 cm, XY + XZ = 9 cm. $\angle$XYZ = 50°
3. Construct Δ ABC, in which BC = 6.2 cm, $\angle$ACB = 50°, AB + AC = 9.8 cm
4. Construct Δ ABC, in which BC = 5.2 cm, $\angle$ACB = 45° and perimeter of Δ ABC is 10 cm

Construction II

To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.

Ex (1) Construct Δ ABC, such that BC = 7.5 cm, $\angle$ABC = 40°, AB – AC = 3 cm.

Solution : Let us draw a rough figure.



Rough Figure 4.5

Explanation : AB – AC = 3 cm $\therefore$ AB > AC

Draw seg BC. We can draw the ray BL such that $\angle$LBC = 40°. We have to locate point A on ray BL. Take point D on ray BL such that BD = 3 cm.

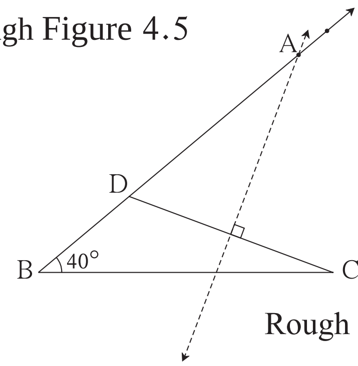
Now , B-D-A and BD = AB – AD = 3.

It is given that AB – AC = 3

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg DC.

$\therefore$ point A is the intersection of ray BL and the perpendicular bisector of seg DC.



Rough figure 4.6

Steps of construction

(1) Draw seg BC of length7.5 cm.

(2) Draw ray BL such that $\angle$LBC = 40°

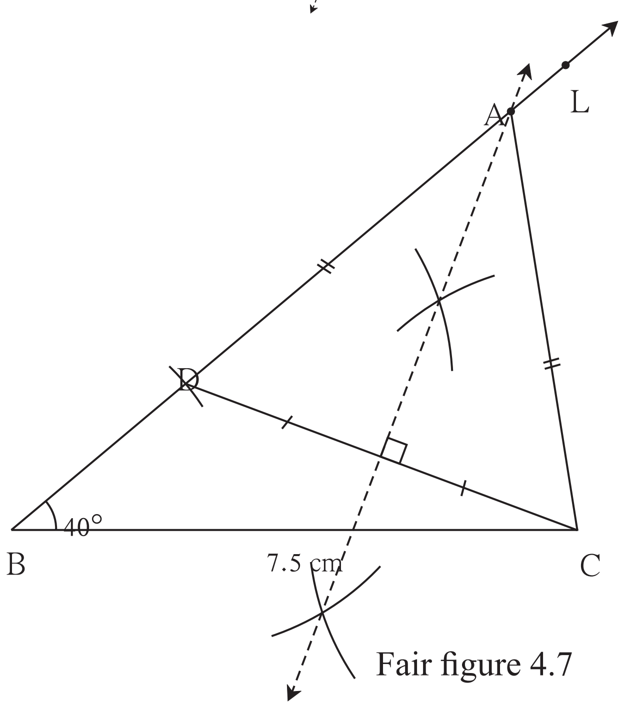
(3) Take point D on ray BL such that BD = 3 cm.

(4) Construct the perpendicular bisector of seg CD.

(5) Name the point of intersection of ray BL and the perpendicular bisector of seg CD as A.

(6) Draw seg AC.

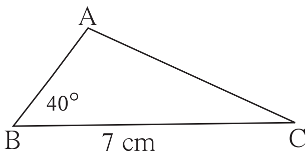
Δ ABC is required triangle.



Fair figure 4.7

Ex. 2 Construct Δ ABC, in which side BC = 7 cm, $\angle B = 40^\circ$ and AC – AB = 3 cm.

Solution : Let us draw a rought figure.



Rough figure 4.8

seg BC = 7 cm. AC > AB.

We can draw ray BT such that

$\angle$ TBC = 40°

Point A is on ray BT. Take point D on opposite ray of ray BT such that

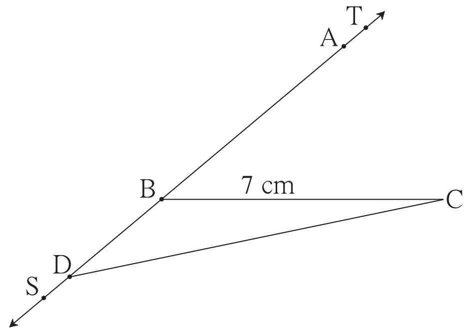
BD = 3 cm.

Now AD = AB + BD = AB + 3 = AC

($\because$ AC – AB = 3 cm.)

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg CD.



Rough figure 4.9

Steps of construction

(1) Draw BC of length 7 cm.

(2) Draw ray BT such that $\angle$ TBC = 40°

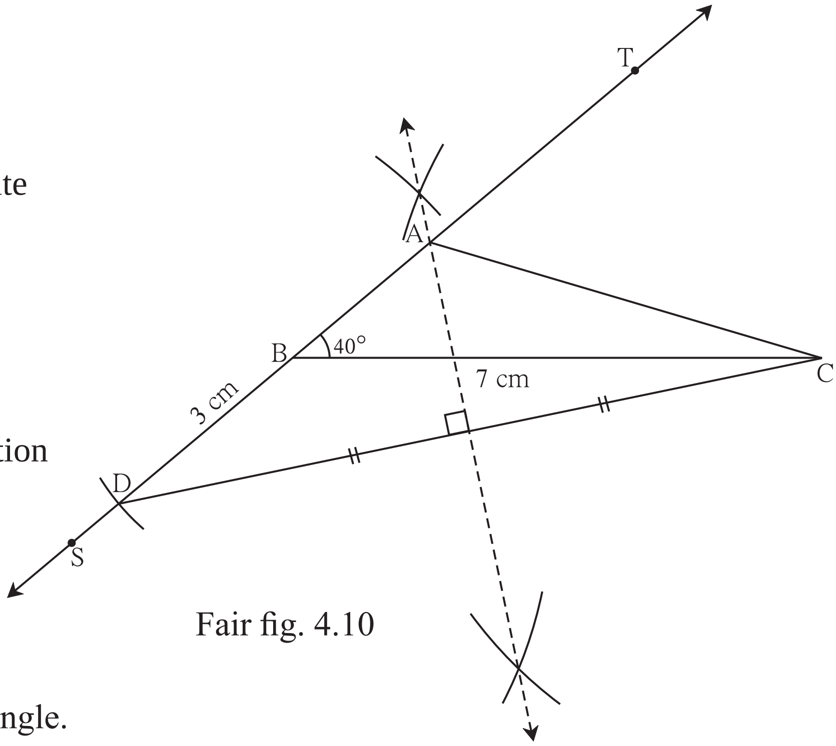
(3) Take point D on the opposite ray BS of ray BT such that BD = 3 cm.

(4) Construct perpendicular bisector of seg DC.

(5) Name the point of intersection of ray BT and the perpendicular bisector of DC as A.

(6) Draw seg AC.

Δ ABC is the required triangle.



Fair fig. 4.10

Practice set 4.2

1. Construct Δ XYZ, such that YZ = 7.4 cm, $\angle XYZ = 45^\circ$ and XY – XZ = 2.7 cm.

2. Construct Δ PQR, such that QR = 6.5 cm, $\angle PQR = 60^\circ$ and PQ – PR = 2.5 cm.

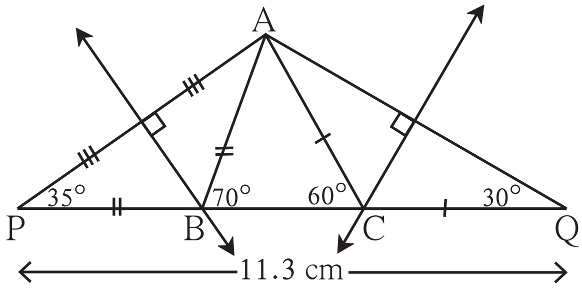
3. Construct Δ ABC, such that BC = 6 cm, $\angle ABC = 100^\circ$ and AC – AB = 2.5 cm.

Construction III

To construct a triangle, if its perimeter, base and the angles which include the base are given.

Ex. Construct Δ ABC such that AB + BC + CA = 11.3 cm, $\angle B = 70^\circ$, $\angle C = 60^\circ$.

Solution : Let us draw a rough figure.



Rough Fig. 4.11

Explanation : As shown in the figure, points P and Q are taken on line BC such that,

PB = AB, CQ = AC

$\therefore$ PQ = PB + BC + CQ = AB + BC +AC = 11.3 cm.

Now in ΔPBA, PB = BA

$\therefore$ $\angle APB = \angle PAB$ and $\angle APB + \angle PAB$ = extieror angleABC = 70°

......theorem of remote interior angles

$\therefore$ $\angle APB = \angle PAB = 35^\circ$ Similarly, $\angle CQA = \angle CAQ = 30^\circ$

Now we can draw Δ PAQ, as its two angles and the included side is known.

Since BA = BP , point B lies on the perpendicular bisector of seg AP.

Similarly, CA = CQ, therefore point C lies on the perpendicular bisector of seg AQ

$\therefore$ by constructing the perpendicular bisectors of seg AP and AQ we can get points B and C, where the perpendicular bisectors intersect line PQ.

Steps of construction

(1) Draw seg PQ of 11.3 cm length.

(2) Draw a ray making angle of 35° at point P.

(3) Draw another ray making an angle of 30° at point Q.

(4) Name the point of intersection of the two rays as A.

(5) Draw the perpendicular bisector of seg AP and seg AQ. Name the points as B and C respectively where the perpendicular bisectors intersect line PQ.

(6) Draw seg AB and seg AC.

Δ ABC is the required triangle.

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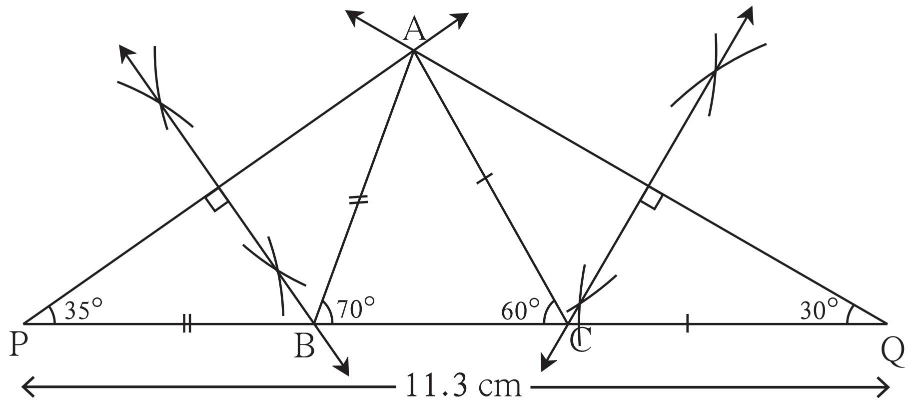

Final Fig. 4.12

Practice set 4.3

1. Construct Δ PQR, in which $\angle Q = 70°$, $\angle R = 80°$ and PQ + QR + PR = 9.5 cm.
2. Construct Δ XYZ, in which $\angle Y = 58°$, $\angle X = 46°$ and perimeter of triangle is 10.5 cm.
3. Construct Δ LMN, in which $\angle M = 60°$, $\angle N = 80°$ and LM + MN + NL = 11 cm.

Problem set 4

1. Construct Δ XYZ, such that XY + XZ = 10.3 cm, YZ = 4.9 cm, $\angle XYZ = 45°$.
2. Construct Δ ABC, in which $\angle B = 70°$, $\angle C = 60°$, AB + BC + AC = 11.2 cm.
3. The perimeter of a triangle is 14.4 cm and the ratio of lengths of its side is 2 : 3 : 4. Construct the triangle.
4. Construct Δ PQR, in which PQ – PR = 2.4 cm, QR = 6.4 cm and $\angle PQR = 55°$.

ICT Tools or Links

Do constructions of above types on the software Geogebra and enjoy the constructions. The third type of construction given above is shown on Geogebra by a different method. Study that method also.

5 Quadrilaterals

Let's study.

- Parallelogram
- Tests of parallelogram
- Rhombus
- Rectangles
- Square
- Trapezium
- Mid point theorem

Let's recall.

1. Write the following pairs considering □ABCD

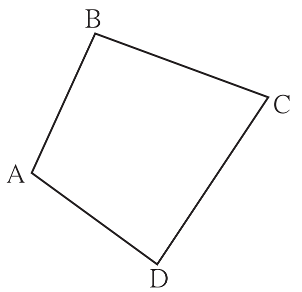

Fig. 5.1

Pairs of adjacent sides:
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of adjacent angles :
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of opposite sides (1) ..... , ... (2) ..... , ...
Pairs of opposite angles (1) ..... , ... (2) ..... , ...

Let's recall types of quadrilaterals and their properties .

I am a quadrilateral

My both pairs of opposite sides are parallel

My properties

- Opposite sides congruent
- Opposite angels ......
- Diagonals .....

My all angles are right angles

My properties

- Opposite sides .....
- Diagonals .....

My all sides are equal in length

My properties

- Opposite sides .....
- Diagonals .....

My all angles are equal and all sides are equal

My properties

- Diagonals .....

My only one pair of opposite sides is parallel

You know different types of quadrilaterals and their properties. You have learned then through different activities like measuring sides and angles, by paper folding method etc. Now we will study these properties by giving their logical proofs.

A property proved logically is called a proof.

In this chapter you will learn that how a rectangle, a rhombus and a square are parallelograms. Let us start our study from parallelogram.

Let's learn.

Parallelogram

A quadrilateral having both pairs of opposite sides parallel is called a parallelogram.

For proving the theorems or for solving the problems we need to draw figure of a parallelogram frequently. Let us see how to draw a parallelogram.

Suppose we have to draw a parallelogram □ABCD.

Method I :

- Let us draw seg AB and seg BC of any length and making an angle of any measure with each other.
- Now we want seg AD and seg BC parallel to each other. So draw a line parallel to seg BC through the point A.
- Similarly we will draw line parallel to AB through the point C. These lines will intersect in point D.

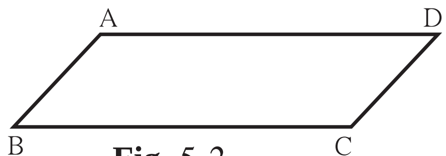

Fig. 5.2

So constructed quadrilateral ABCD will be a parallelogram.

Method II :

- Let us draw seg AB and seg BC of any length and making angle of any measure between them.
- Draw an arc with compass with centre A and radius BC.
- Similarly draw an arc with centre C and radius AB intersecting the arc previously drawn.
- Name the point of intersection of two arcs as D.

Draw seg AD and seg CD.

Quadrilateral so formed is a parallelogram ABCD

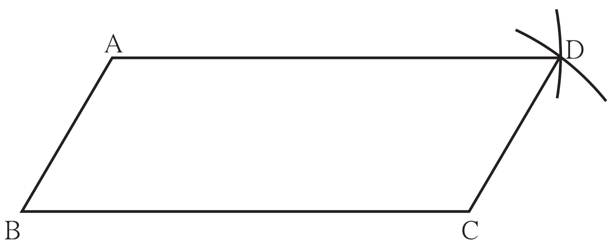

Fig. 5.3

In the second method we have actually drawn □ABCD in which opposite sides are equal. We will prove that a quadrilateral whose opposite sides are equal, is a parallelogram.

Activity I Draw five parallelograms by taking various measures of lengths and angles.

For the proving theorems on parallelogram, we use congruent triangles. To understand how they are used, let's do the following activity.

Activity II

- Draw a parallellogram ABCD on a card sheet. Draw diagonal AC. Write the names of vertices inside the triangle as shown in the figure. Then cut is out.

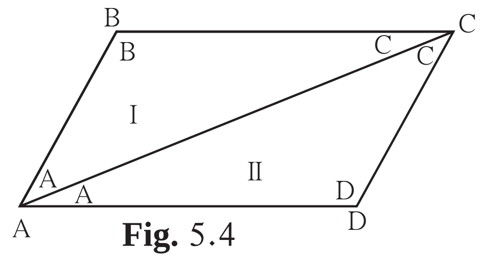

Fig. 5.4

- Fold the quadrilateral on the diagonal AC and see whether ΔADC and ΔCBA match with each other or not.

- Cut □ABCD along diagonals AC and separate ΔADC and ΔCBA. By rotating and flipping ΔCBA, check whether it matchs exactly with ΔADC.

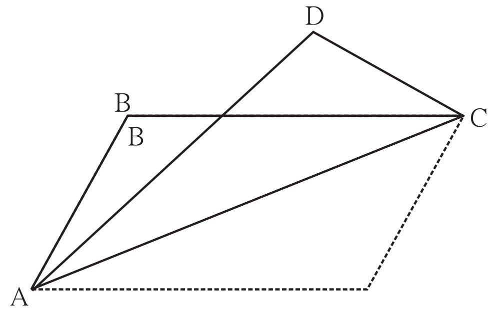

Fig. 5.5

What did you find ? Which sides of ΔCBA match with which sides of ΔADC ? Which angles of ΔCBD match with which angles of ΔADC ?

Side DC matches with side AB and side AD matches with side CB. Similarly $\angle$B matches with $\angle$D.

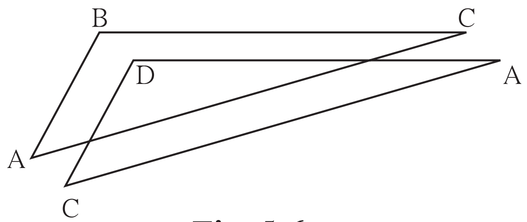

Fig. 5.6

So we can see that opposite sides and angles of a parallelogram are congruent.

We will prove these properties of a parallelogram.

Theorem 1. Opposite sides and opposite angles of a parallelogram are congruent.

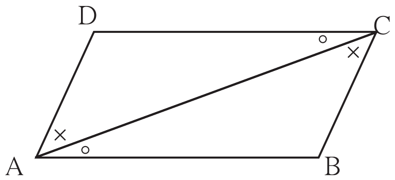

Fig. 5.7

Given : □ABCD is a parallelogram.
It means side AB $\parallel$ side DC,
side AD $\parallel$ side BC.

To prove : seg AD $\cong$ seg BC ; seg DC $\cong$ seg AB
$\angle$ADC $\cong$ $\angle$CBA, and $\angle$DAB $\cong$ $\angle$BCD.

Construction : Draw diagonal AC.

Proof : seg DC $\parallel$ seg AB and diagonal AC is a transversal.

$\therefore$ $\angle$DCA $\cong$ $\angle$BAC ....(1)
and $\angle$DAC $\cong$ $\angle$BCA ....(2) } ..... alternate angles

Now , in ΔADC and ΔCBA,

$\angle$DAC $\cong$ $\angle$BCA .... from (2)

$\angle$DCA $\cong$ $\angle$BAC .... from (1)

seg AC $\cong$ seg CA .... common side

$\therefore$ ΔADC $\cong$ ΔCBA .... ASA test

$\therefore$ side AD $\cong$ side CB .... c.s.c.t.

and side DC $\cong$ side AB .... c.s.c.t.,

Also, $\angle$ADC $\cong$ $\angle$CBA .... c.a.c.t.

Similarly we can prove $\angle$DAB $\cong$ $\angle$BCD.

Use your brain power!

In the above theorem, to prove $\angle$DAB $\cong$ $\angle$BCD, is any change in the construction needed ? If so, how will you write the proof making the change ?

To know one more property of a parallelogram let us do the following activity.

Activity : Draw a parallelogram PQRS. Draw diagonals PR and QS. Denote the intersection of diagonals by letter O. Compare the two parts of each diagonal with a divider. What do you find ?

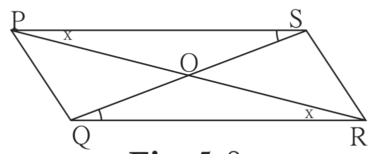

Fig. 5.8

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Theorem : Diagonals of a parallelogram bisect each other.

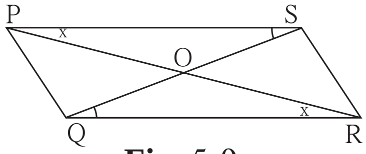

Fig. 5.9

Given : $\square$PQRS is a parallelogram. Diagonals PR and QS intersect in point O.

To prove : seg PO $\cong$ seg RO,
seg SO $\cong$ seg QO.

Proof : In ΔPOS and ΔROQ

$\angle$OPS $\cong$ $\angle$ORQ .... alternate angles

side PS $\cong$ side RQ .... opposite sides of parallelogram

$\angle$PSO $\cong$ $\angle$RQO .... alternate angles

$\therefore \Delta$POS $\cong$ ΔROQ .... ASA test

$\therefore$ seg PO $\cong$ seg RO ....
and seg SO $\cong$ seg QO .... $\Big\}$..... corresponding sides of congruent triangles

Remember this !

- Adjacent angles of a parallelogram are supplementary.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.

Solved Examples

Ex (1) $\square$PQRS is a parallelogram. PQ = 3.5, PS = 5.3 $\angle$Q = 50° then find the lengths of remaining sides and measures of remaining angles.

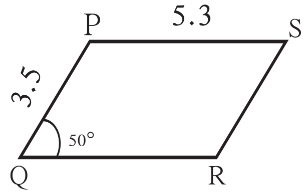

Fig. 5.10

Solution : $\square$PQRS is a parallelogram.

$\therefore \angle Q + \angle P = 180°$ .... interior angles are supplementary.

$\therefore\ 50° + \angle P = 180°$

$\therefore\ \angle P = 180° - 50° = 130°$

Now , $\angle P = \angle R$ and $\angle Q = \angle S$ ....opposite angles of a parallelogram.

$\therefore\ \angle R = 130°$ and $\angle S = 50°$

Similarly, PS = QR and PQ = SR ....opposite sides of a parallelogram.

$\therefore$ QR = 5.3 and SR = 3.5

Ex (2) □ABCD is a parallelogram. If $\angle A = (4x + 13)^\circ$ and $\angle D = (5x - 22)^\circ$ then find the measures of $\angle B$ and $\angle C$.

Solution : Adjacent angles of a parallelogram are supplementary.

$\angle A$ and $\angle D$ are adjacent angles.

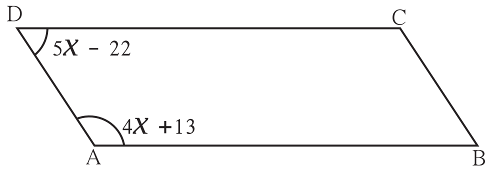

Fig. 5.11

$\therefore (4x + 13)^\circ + (5x - 22)^\circ = 180$

$\therefore 9x - 9 = 180$

$\therefore 9x = 189$

$\therefore x = 21$

$\therefore \angle A = 4x + 13 = 4 \times 21 + 13 = 84 + 13 = 97^\circ$ $\quad\quad \therefore \angle C = 97^\circ$

$\angle D = 5x - 22 = 5 \times 21 - 22 = 105 - 22 = 83^\circ$ $\quad\quad \therefore \angle B = 83^\circ$

Practice set 5.1

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135^\circ$ then what is the measure of $\angle XWZ$ and $\angle YZW$?
If l(OY)= 5 cm then l(WY)= ?

2. In a parallelogram ABCD, If $\angle A = (3x + 12)^\circ$, $\angle B = (2x - 32)^\circ$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.

3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.

4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.

5*. Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.

6. In the figure 5.12, □PQRS and □ABCR are two parallelograms.
If $\angle P = 110^\circ$ then find the measures of all angles of □ABCR.

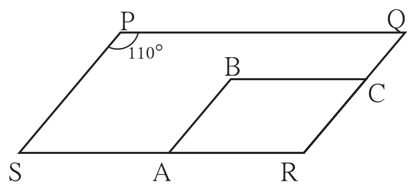

Fig. 5.12

7. In figure 5.13 □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.

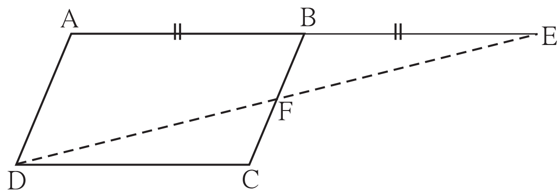

Fig. 5.13

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Let's recall.

Tests for parallel lines

1. If a transversal interesects two lines and a pair of corresponding angles is congruent then those lines are parallel.
2. If a transversal intersects two lines and a pair of alternate angles is corgruent then those two lines are parallel.
3. If a transversal intersects two lines and a pair of interior angles is supplementary then those two lines are parallel.

Let's learn.

Tests for parallelogram

Suppose, in □PQRS, PS = QR and PQ = SR and we have to prove that □PQRS is a parallelogram. To prove it, which pairs of sides of □PQRS should be shown parallel ?

Which test can we use to show the sides parallel ? Which line will be convenient as a transversal to obtain the angles necessary to apply the test ?

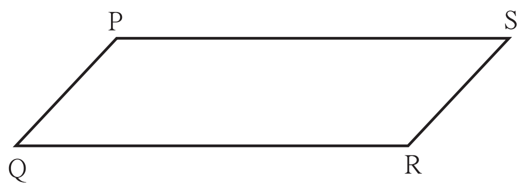


Fig. 5.14

Theorem : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.

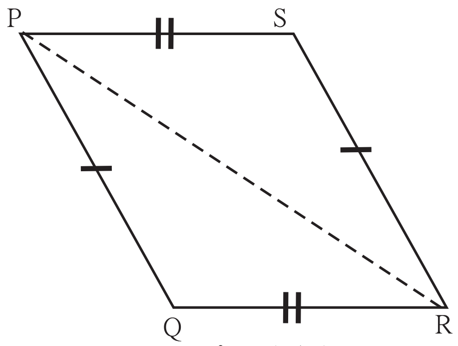


Fig. 5.15

Given : In □PQRS
side PS ≅ side QR
side PQ ≅ side SR

To prove : □PQRS is a parallelogram.

Construction : Draw diagonal PR

Proof : In Δ SPR and Δ QRP
side PS ≅ side QR ....given
side SR ≅ side QP .... given
side PR ≅ side RP .... common side
$\therefore \Delta$ SPR ≅ Δ QRP ...... sss test
$\therefore \angle$SPR ≅ $\angle$QRP .... c.a.c.t.
Similarly, $\angle$PRS ≅ $\angle$RPQ ..... c.a.c.t.
$\angle$SPR and $\angle$QRP are alternate angles formed by the transversal PR of seg PS and seg QR.

$\therefore$ side PS $\parallel$ side QR ......(I) alternate angles test for parallel lines.

Similarly $\angle PRS$ and $\angle RPQ$ are the alternate angles formed by transversal PR of seg PQ and seg SR.

$\therefore$ side PQ $\parallel$ side SR ......(II) .....alternate angle test

$\therefore$ from (I) and (II) $\square$PQRS is a parallelogram.

On page 56, two methods to draw a parallelogram are given. In the second method actually we have drawn a quadrilateral of which opposite sides are equal. Did you now understand why such a quadrilateral is a parallelogram ?

Theorem : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.

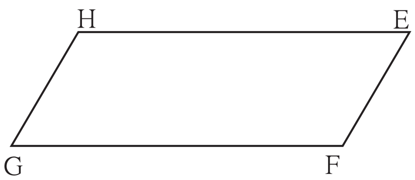


Fig. 5.16

Given : In $\square$EFGH $\angle E \cong \angle G$

and $\angle$.......... $\cong$ $\angle$..........

To prove : $\square$EFGH is a ....

Proof : Let $\angle E = \angle G = x$ and $\angle H = \angle F = y$

Sum of all angles of a quadrilateral is ....

$\therefore$ $\angle E + \angle G + \angle H + \angle F =$ ....

$\therefore$ $x + y +$ .... $+$ .... $=$ ....

$\therefore$ $\square x + \square y =$ .....

$\therefore$ $x + y = 180^\circ$

$\therefore$ $\angle G + \angle H =$ ....

$\angle G$ and $\angle H$ are interior angles formed by transversal HG of seg HE and seg GF.

$\therefore$ side HE $\parallel$ side GF .... (I) interior angle test for parallel lines.

Similarly, $\angle G + \angle F =$ ....

$\therefore$ side .... $\parallel$ side .... (II) interior angle test for parallel lines.

$\therefore$ From (I) and (II), $\square$EFGH is a ....

Theorem : If the diagonals of a quadrilateral bisect each other then it is a parallelogram.

Given : Diagonals of □ABCD bisect each other in the point E.
It means seg AE ≅ seg CE
and seg BE ≅ seg DE

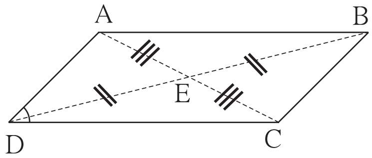


Fig. 5.17

To prove : □ABCD is a parallelogram.

Proof : Find the answers for the following questions and write the proof of your own.

1. Which pair of alternate angles should be shown congruent for proving seg AB || seg DC ? Which transversal will form a pair of alternate angles ?
2. Which triangles will contain the alternate angles formed by the transversal?
3. Which test will enable us to say that the two triangles congruent ?
4. Similarly, can you prove that seg AD || seg BC?

The three theorems above are useful to prove that a given quadrilateral is a parallelogram. Hence they are called as tests of a parallelogram.

One more theorem which is useful as a test for parallelogram is given below.

Theorem : A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

Given : In □ABCD
seg CB ≅ seg DA and seg CB || seg DA

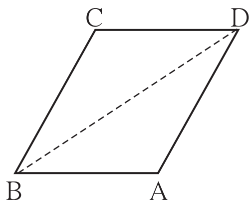


Fig. 5.18

To prove : □ABCD is a parallelogram.

Construction : Draw diagonal BD.

Write the complete proof which is given in short.

Δ CBD ≅ Δ ADB ....SAS test

∴ ∠CDB ≅ ∠ABD ..... c.a.c.t.

∴ seg CD || seg BA ..... alternate angle test for parallel lines

Remember this !

- A quadrilateral is a parallelogram if its pairs of opposite angles are congruent.
- A quadrilateral is a parallelogram if its pairs of opposite sides are congruent.
- A quadrilateral is a parallelogram if its diagonals bisect each other.
- A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

These theorems are called tests for parallelogram

Let's recall.

Lines in a note book are parallel. Using these lines how can we draw a parallelogram ?

Solved examples -

Ex (1) □PQRS is parallelogram. M is the midpoint of side PQ and N is the mid point of side RS. Prove that □PMNS and □MQRN are parallelograms.

Given : □ PQRS is a parallelogram. M and N are the midpoints of side PQ and side RS respectively.

To prove : □PMNS is a parallelogram.
□MQRN is a parallelogram.

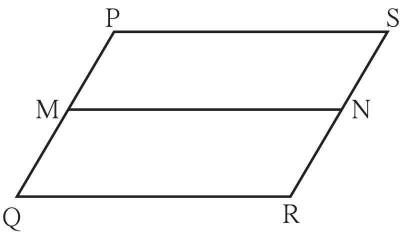


Fig. 5.19

Proof : side PQ || side SR

∴ side PM || side SN ...... (∵ P-M-Q; S-N-R) ......(I)
side PQ ≅ side SR.

∴ $\frac{1}{2}$ side PQ = $\frac{1}{2}$ side SR

∴ side PM ≅ side SN ..... (∵ M and N are midpoints.)......(II)

∴ From (I) and (II), □PMNQ is a parallelogram,

Similarly, we can prove that □MQRN is parallelogram.

Ex (2) Points D and E are the midpoints of side AB and side AC of Δ ABC respectively. Point F is on ray ED such that ED = DF. Prove that □AFBE is a parallelogram. For this example write 'given' and 'to prove' and complete the proof given below.

Given : -------------------

To prove : ----------------------

Proof : seg AB and seg EF are $\boxed{}$ of □AFBE.

seg AD ≅ seg DB.......$\boxed{}$

seg $\boxed{}$ ≅ seg $\boxed{}$.......construction.

∴ Diagonals of □AFBE $\boxed{}$ each other

∴ □AFBE is a parallelogram ...by $\boxed{}$ test.

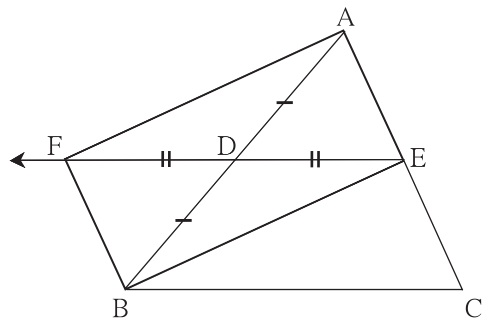


Fig. 5.20

Ex (3) Prove that every rhombus is a parallelogram.

Given : □ABCD is a rhombus

To prove : □ABCD is parallelogram.

Proof : seg AB ≅ seg BC ≅ seg CD ≅ seg DA (given)

∴ side AB ≅ side CD and side BC ≅ side AD

∴ □ABCD is a parallelogram..... opposite side test for parallelogram

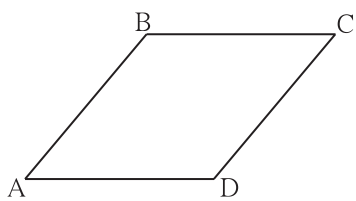


Fig. 5.21

Practice set 5.2

1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram.

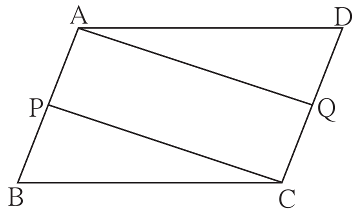


Fig. 5.22

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

3. In figure 5.23, G is the point of concurrence of medians of Δ DEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □GEHF is a parallelogram.

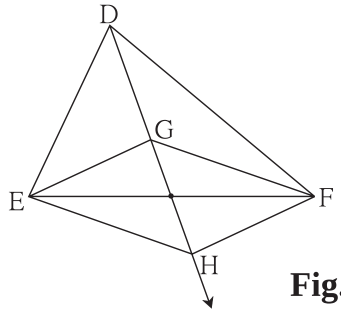


Fig. 5.23

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

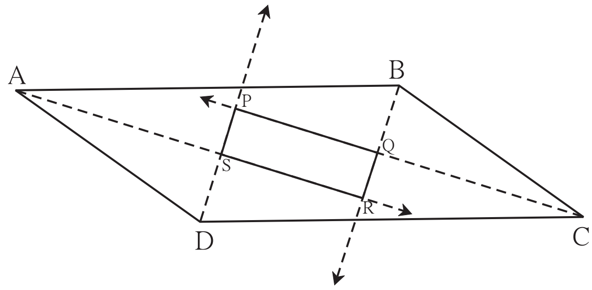


Fig. 5.24

5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.

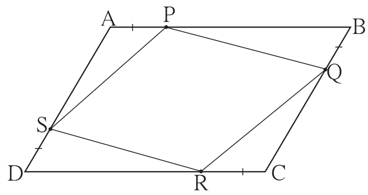


Fig. 5.25

Let's learn.

Properties of rectangle, rhombus and square

Rectangle, rhombus and square are also parallelograms. So the properties that opposite sides are equal, opposite angles are equal and diagonals bisect each other hold good in these types of quadrilaterals also. But there are some more properties of these quadrilaterals.

Proofs of these properties are given in brief. Considering the steps in the given proofs, write the proofs in detail.

Theorem : Diagonals of a rectangle are congruent.

Given : □ABCD is a rectangle

To prove : Diagonal AC ≅ diagonal BD

Proof : Complete the proof by giving suitable reasons.

Δ ADC ≅ Δ DAB ...... SAS test

$\therefore$ diagonal AC ≅ diagonal BD..... c.s.c.t.

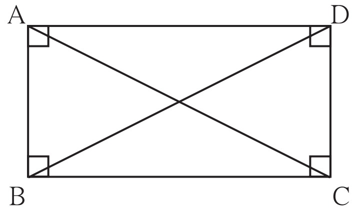


Fig. 5.26

Theorem : Diagonals of a square are congruent.

Write 'Given', 'To prove' and 'proof' of the theorem.

Theorem : Diagonals of a rhombus are perpendicular bisectors of each other.

Given : □EFGH is a rhombus

To prove : (i) Diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Diagonal HF is the perpendicular bisector of diagonal EG.

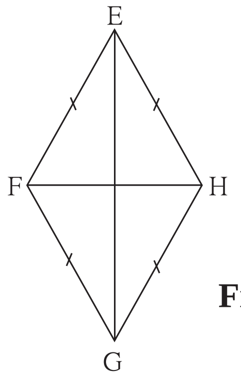


Fig. 5.27

Proof : (i) $\left.\begin{array}{l}\text{seg EF} \cong \text{seg EH} \\ \text{seg GF} \cong \text{seg GH}\end{array}\right\}$ given

Every point which is equidistant from end points of a segment is on the perpendicular bisector of the segment.

$\therefore$ point E and point G are on the perpendicular bisector of seg HF.

One and only one line passes through two distinct points.

$\therefore$ line EG is the perpendicular bisector of diagonal HF.

$\therefore$ diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Similarly, we can prove that diagonal HF is the perpendicular bisector of EG.

Write the proofs of the following statements.

- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect its opposite angles.
- Diagonals of a square bisect its opposite angles.

Remember this !

- Diagonals of a rectangle are congruent.
- Diagonals of a square are congruent.
- Diagonals of a rhombus are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect the pairs of opposite angles.
- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a square bisect opposite angles.

Practice set 5.3

1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find BO and if $\angle CAD = 35^\circ$ then find $\angle ACB$.

2. In a rhombus PQRS if PQ = 7.5 then find QR.
If $\angle QPS = 75^\circ$ then find the measure of $\angle PQR$ and $\angle SRQ$.

3. Diagonals of a square IJKL intersects at point M, Find the measures of $\angle IMJ$, $\angle JIK$ and $\angle LJK$.

4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.

5. State with reasons whether the following statements are 'true' or 'false'.
(i) Every parallelogram is a rhombus.
(ii) Every rhombus is a rectangle.
(iii) Every rectangle is a parallelogram.
(iv) Every squre is a rectangle.
(v) Every square is a rhombus.
(vi) Every parallelogram is a rectangle.

Let's learn.

Trapezium

When only one pair of opposite sides of a quadrilateral is parallel then the quadrilateral is called a trapezium.

In the adjacent figure only side AB and side DC of □ABCD are parallel to each other. So this is a trapezium. $\angle A$ and $\angle D$ is a pair of adjacent angles and so is the pair of $\angle B$ and $\angle C$. Therefore by property of parallel lines both the pairs are supplementary.

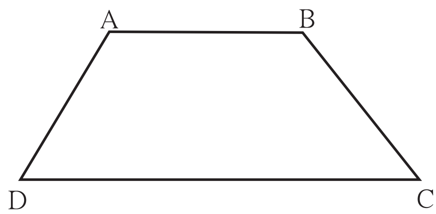


Fig. 5.28

If non-parallel sides of a trapezium are congruent then that quadrilateral is called as an **Isoceles trapezium.**

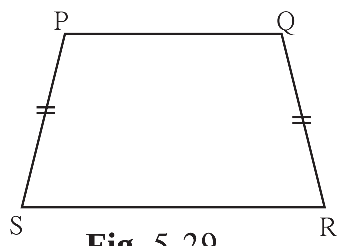


Fig. 5.29

The segment joining the midpoints of non parallel sides of a trapezium is called the median of the trapezium

Solved examples

Ex (1) Measures of angles of □ABCD are in the ratio 4 : 5 : 7 : 8. Show that □ABCD is a trapezium.

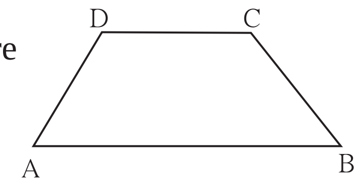


Fig. 5.30

Solution : Let measures of ∠A, ∠B, ∠C and ∠D are $(4x)^\circ$, $(5x)^\circ$, $(7x)^\circ$, and $(8x)^\circ$ respectively.

Sum of all angles of a quadrialteral is 360°.

$\therefore$ $4x + 5x + 7x + 8x = 360$

$\therefore$ $24x = 360$ $\quad\therefore$ $x = 15$

$\angle A = 4 \times 15 = 60^\circ$, $\angle B = 5 \times 15 = 75^\circ$, $\angle C = 7 \times 15 = 105^\circ$,

and $\angle D = 8 \times 15 = 120^\circ$

Now , $\angle B + \angle C = 75^\circ + 105^\circ = 180^\circ$

$\therefore$ side CD || side BA...... (I)

But $\angle B + \angle A = 75^\circ + 60^\circ = 135^\circ \neq 180^\circ$

$\therefore$ side BC and side AD are not parallel ....(II)

$\therefore$ □ABCD is a trapezium. ....[from (I) and (II)]

Ex (2) In □PQRS, side PS || side QR and side PQ ≅ side SR, side QR > side PS then prove that ∠PQR ≅ ∠SRQ

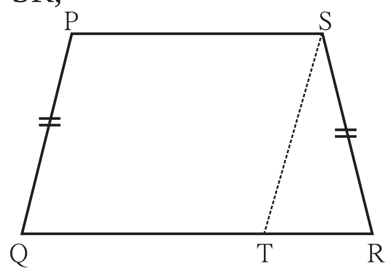


Fig. 5.31

Given : In □PQRS, side PS || side QR, side PQ ≅ side SR and side QR > side PS.

To prove : ∠PQR ≅ ∠SRQ

Construction : Draw the segment parallel to side PQ through the point S which intersects side QR in T.

Proof : In □PQRS,

seg PS || seg QT ....given

seg PQ || seg ST ....construction

$\therefore$ □PQTS is a parallelogram

$\therefore$ ∠PQT ≅ ∠STR ..... corresponding angles (I)

and seg PQ ≅ seg ST ......opposite sides of parallelogram

But seg PQ ≅ seg SR ......given

$\therefore$ seg ST ≅ seg SR

$\therefore$ ∠STR ≅ ∠SRT.....isosceles triangle theorem (II)

$\therefore$ ∠PQT ≅ ∠SRT ....[from (I) and (II)]

$\therefore$ ∠PQR ≅ ∠SRQ

Hence, it is proved that base angles of an isosceles trapezium are congruent.

Practice set 5.4

1. In □IJKL, side IJ || side KL $\angle I = 108^\circ$ $\angle K = 53^\circ$ then find the measures of $\angle J$ and $\angle L$.

2. In □ABCD, side BC || side AD, side AB ≅ side DC If $\angle A = 72^\circ$ then find the measures of $\angle B$, and $\angle D$.

3. In □ABCD, side BC < side AD (Figure 5.32) side BC || side AD and if side BA ≅ side CD then prove that $\angle ABC \cong \angle DCB$.

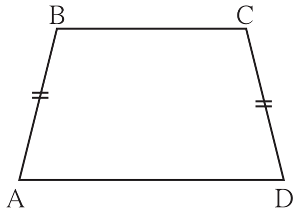


Fig. 5.32

Let's learn.

Theorem of midpoints of two sides of a triangle

Statement : The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

Given : In Δ ABC, point P is the midpoint of seg AB and point Q is the midpoint of seg AC

To prove : seg PQ || seg BC

and $PQ = \frac{1}{2}$ BC

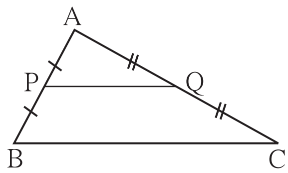


Fig. 5.33

Construction : Produce seg PQ upto R such that PQ = QR

Draw seg RC.

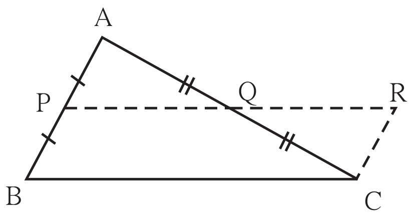


Fig. 5.34

Proof : In Δ AQP and Δ CQR

seg PQ ≅ seg QR ...... construction

seg AQ ≅ seg QC ...... given

$\angle AQP \cong \angle CQR$ ..... vertically opposite angles.

$\therefore$ Δ AQP ≅ Δ CQR .... SAS test

$\angle PAQ \cong \angle RCQ$ ..... (1) c.a.c.t.

$\therefore$ seg AP ≅ seg CR ......(2) c.s.c.t.

From (1) line AB || line CR.........alternate angle test

from (2) seg AP ≅ seg CR

Now, seg AP ≅ seg PB ≅ seg CR and seg PB || seg CR

$\therefore$ □PBCR is a parallelogram.

$\therefore$ seg PQ || seg BC and PR = BC ..... opposite sides are congruent

$PQ = \frac{1}{2}$ PR ...... (construction)

$\therefore$ $PQ = \frac{1}{2}$ BC $\because$ PR = BC

Converse of midpoint theroem

Theorem : If a line drawn through the midpoint of one side of a triangle and parallel to the other side then it bisects the third side.

For this theorem 'Given', To prove', 'construction' is given below. Try to write the proof.

Given : Point D is the midpoint of side AB of Δ ABC. Line l passing through the point D and parallel to side BC intersects side AC in point E.

To prove : AE = EC

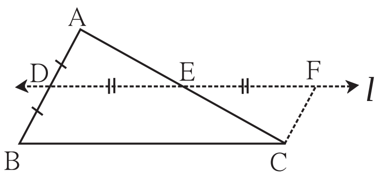


Fig. 5.35

Construction : Take point F on line l such that D-E-F and DE = EF.

Draw seg CF.

Proof : Use the construction and line l || seg BC which is given. Prove Δ ADE $\cong$ Δ CFE and complete the proof.

Ex (1) Points E and F are mid points of seg AB and seg AC of Δ ABC respectively. If EF = 5.6 then find the length of BC.

Solution : In Δ ABC, point E and F are midpoints of side AB and side AC respectively.

$EF = \frac{1}{2}$ BC ....midpoint theorem

$5.6 = \frac{1}{2}$ BC $\therefore$ BC = 5.6 × 2 = 11.2

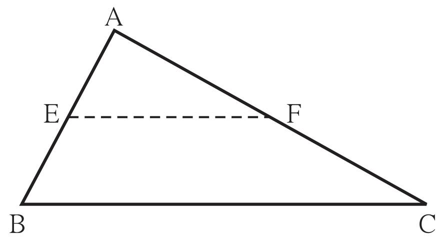


Fig. 5.36

Ex (2) Prove that the quadrilateral formed by joining the midpoints of sides of a quadrilateral in order is a parallelogram.

Given : □ABCD is a quadrilateral. P, Q, R, S are midpoints of the sides AB, BC, CD and AD respectively.

To prove : □PQRS is a parallelogram.

Construction : Draw diagonal BD

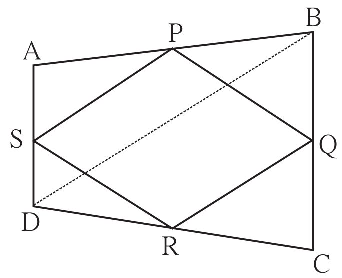


Fig. 5.37

Proof : In Δ ABD, the midpoint of side AD is S and the midpoint of side AB is P.

$\therefore$ by midpoint theorem, PS $\parallel$ DB and PS = $\frac{1}{2}$ BD .... (1)

In Δ DBC point Q and R are midpoints of side BC and side DC respectively.

$\therefore$ QR $\parallel$ BD and QR = $\frac{1}{2}$ BD ....by midpoint theorem (2)

$\therefore$ PS $\parallel$ QR and PS = QR .... from (1) and (2)

$\therefore$ $\square$PQRS is a parallelogram.

Practice set 5.5

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of Δ ABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.

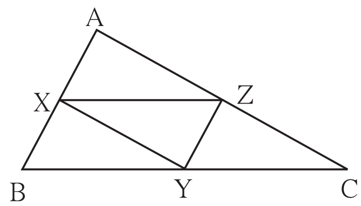


Fig. 5.38

2. In figure 5.39, $\square$PQRS and $\square$MNRL are rectangles. If point M is the midpoint of side PR then prove that, (i) SL = LR, (ii) LN = $\frac{1}{2}$SQ.

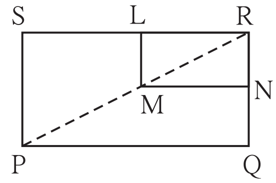


Fig. 5.39

3. In figure 5.40, ΔABC is an equilateral traingle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that Δ FED is an equilateral traingle.

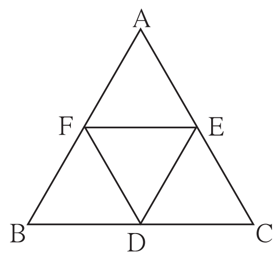


Fig. 5.40

4. In figure 5.41, seg PD is a median of Δ PQR. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.

[Hint : draw DN $\parallel$ QM.]

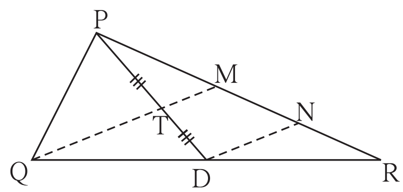


Fig. 5.41

Problem set 5

1. Choose the correct alternative answer and fill in the blanks.

(i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called ....

(A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

(ii) If the diagonal of a square is $12\sqrt{2}$ cm then the perimeter of square is ......

(A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$ cm

(iii) If opposite angles of a rhombus are $(2x)^\circ$ and $(3x - 40)^\circ$ then value of x is ...

(A) 100° (B) 80° (C) 160° (D) 40°

2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.

3. If diagonal of a square is 13 cm then find its side.

4. Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.

5. Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.

6. Diagonals of a rectangle PQRS are intersecting in point M. If $\angle$QMR = 50° then find the measure of $\angle$MPS.

7. In the adjacent Figure 5.42, if seg AB $\parallel$ seg PQ , seg AB $\cong$ seg PQ, seg AC $\parallel$ seg PR, seg AC $\cong$ seg PR then prove that, seg BC $\parallel$ seg QR and seg BC $\cong$ seg QR.

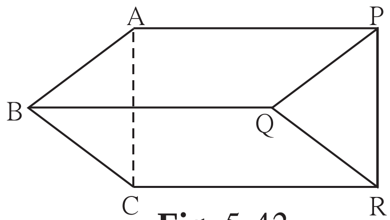


Fig. 5.42

8*. In the Figure 5.43, □ABCD is a trapezium. AB $\parallel$ DC. Points P and Q are midpoints of seg AD and seg BC respectively. Then prove that, PQ $\parallel$ AB and

$$PQ = \frac{1}{2}(AB + DC).$$

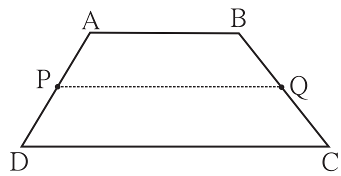


Fig. 5.43

9. In the adjacent figure 5.44, □ABCD is a trapezium. AB $\parallel$ DC. Points M and N are midpoints of diagonal AC and DB respectively then prove that MN $\parallel$ AB.

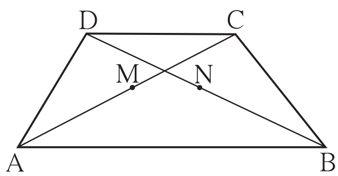


Fig. 5.44

Activity

To verify the different properties of quadrilaterals

Material : A piece of plywood measuring about 15 cm$\times$ 10 cm, 15 thin screws, twine, scissor.

Note : On the plywood sheet, fix five screws in a horizontal row keeping a distance of 2cm between any two adjacent screws. Similarly make two more rows of screws exactly below the first one. Take care that the vertical distance between any two adjacent screws is also 2cm.

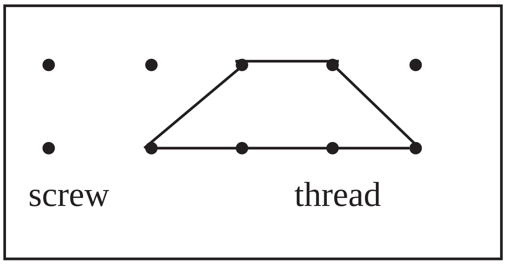


Fig. 5.45

With the help of the screws, make different types of quadrilaterals of twine. Verify the properties of sides and angles of the quadrilaterals.

Additional information

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio 2 : 1. Proof of this property is given below.

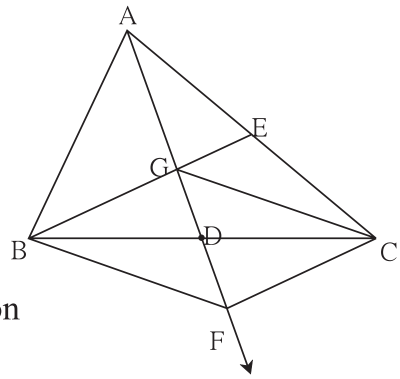


Fig. 5.46

Given : seg AD and seg BE are the medians of Δ ABC which intersect at point G.

To prove: AG : GD = 2 : 1

Construction : Take point F on ray AD such that G-D-F and GD = DF

Proof : Diagonals of □BGCF bisect each other .... given and construction

$\therefore$ □BGCF is a parallelogram.

$\therefore$ seg BE || set FC

Now point E is the midpoint of side AC of Δ AFC . ... given

seg EB || seg FC

Line passing through midpoint of one side and parallel to the other side bisects the third side.

$\therefore$ point G is the midpoint of side AF.

$\therefore$ AG = GF

But GF = 2GD .... construction

$\therefore$ AG = 2 GD

$\therefore \dfrac{AG}{GD} = \dfrac{2}{1}$ i.e. AG : GD = 2 : 1

6 Circle

Let's study.

- Circle
- Property of chord of the circle
- Incircle
- Circumcircle

Let's recall.

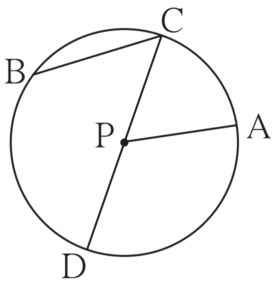


Fig. 6.1

In adjoining figure, observe the circle with center P. With reference to this figure, complete the following table.

---	seg PA	---	---	---	---	$\angle$CPA
chord	---	diameter	radius	centre	central angle	---

Let's learn.

Circle

Let us describe this circle in terms of a set of points.

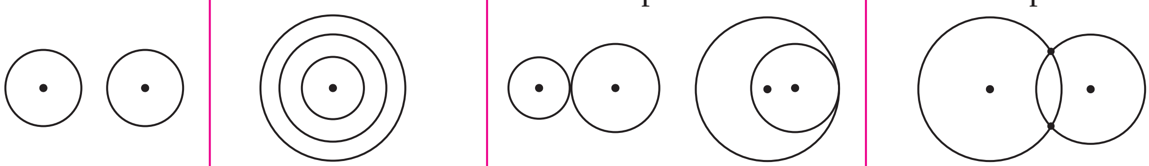
- The set of points in a plane which are equidistant from a fixed point in the plane is called a circle.

Some terms related with a circle.

- The fixed point is called the centre of the circle.
- The segment joining the centre of the circle and a point on the circle is called a radius of the circle.
- The distance of a point on the circle from the centre of the circle is also called the radius of the circle.
- The segment joining any two points of the circle is called a chord of the circle.
- A chord passing through the centre of a circle is called a diameter of the circle. A diameter is a largest chord of the circle.

Circles in a plane

Congruent circles | Concentric circles | Circles intersecting in a point | Circles intersecting in two points



- the same radii
- the same centre, different radii
- different centres, different radii, only one common point
- different centres, different radii, two common points

Fig. 6.2

Let's learn.

Properties of chord

Activity I : Every student in the group will do this activity. Draw a circle in your notebook. Draw any chord of that circle. Draw perpendicular to the chord through the centre of the circle. Measure the lengths of the two parts of the chord. Group leader will prepare a table and other students will write their observations in it.

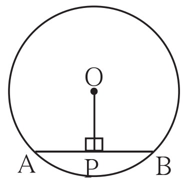


Fig. 6.3

Length \ Student	1	2	3	4	5	6
l (AP)	...... cm					
l (PB)	...... cm					

Write theproperty which you have observed.
Let us write the proof of this property.

Theorem : A perpendicular drawn from the centre of a circle on its chord bisects the chord.

Given : seg AB is a chord of a circle with centre O.
seg OP $\perp$ chord AB

To prove : seg AP $\cong$ seg BP

Proof : Draw seg OA and seg OB

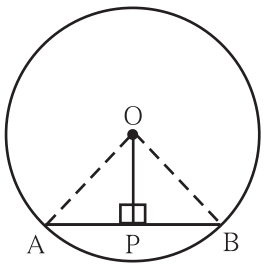


Fig. 6.4

In Δ OPA and Δ OPB

$\angle$OPA $\cong$ $\angle$OPB . . . . seg OP $\perp$ chord AB

seg OP $\cong$ seg OP . . . . common side

hypotenuse OA $\cong$ hypotenuse OB . . . . radii of the same circle

$\therefore$ Δ OPA $\cong$ Δ OPB . . . . hypotenuse side theorem

seg PA $\cong$ seg PB . . . . c.s.c.t.

Activity II : Every student from the group will do this activity. Draw a circle in your notebook. Draw a chord of the circle. Join the midpoint of the chord and centre of the circle. Measure the angles made by the segment with the chord. Discuss about the measures of the angles with your friends.
Which property do the observations suggest ?

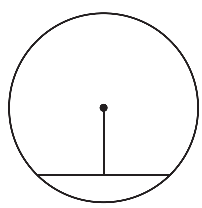


Fig. 6.5

Theorem : The segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord.

Given : seg AB is a chord of a circle with centre O and P is the midpoint of chord AB of the circle. That means seg AP $\cong$ seg PB.

To prove : seg OP $\perp$ chord AB

Proof : Draw seg OA and seg OB.

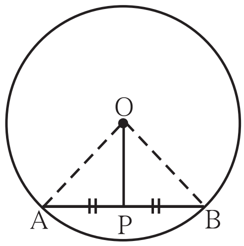


Fig. 6.6

In Δ AOP and Δ BOP

seg OA $\cong$ seg OB . . . . radii of the same circle

seg OP $\cong$ seg OP. . . . . common sides

seg AP $\cong$ seg BP . . . . given

$\therefore$ Δ AOP $\cong$ Δ BOP . . . . SSS test

$\therefore$ $\angle$OPA $\cong$ $\angle$OPB . . . . c.a.c.t. . . .(I)

Now $\angle OPA + \angle OPB = 180^\circ$ . . . angles in linear pair

$\therefore$ $\angle OPB + \angle OPB = 180^\circ$ . . . . . . from (I)

$\therefore$ $2\angle OPB = 180^\circ$

$\therefore$ $\angle OPB = 90^\circ$

$\therefore$ seg OP $\perp$ chord AB

Solved examples

Ex (1) Radius of a circle is 5 cm. The length of a chord of the circle is 8 cm. Find the distance of the chord from the centre.

Solution :

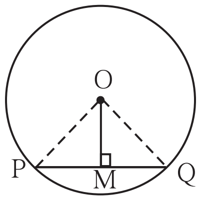


Fig. 6.7

Let us draw a figure from the given information.
O is the centre of the circle.
Length of the chord is 8 cm.
seg OM $\perp$ chord PQ.

We know that a perpendicular drawn from the centre of a circle on its chord bisects the chord.

$\therefore$ PM = MQ = 4 cm

Radius of the circle is 5 cm, means OQ = 5 cm .... given

In the right angled Δ OMQ using Pythagoras' theorem,

$OM^2 + MQ^2 = OQ^2$

$\therefore$ $OM^2 + 4^2 = 5^2$

$\therefore$ $OM^2 = 5^2 - 4^2 = 25 - 16 = 9 = 3^2$

$\therefore$ OM = 3

Hence distance of the chord from the centre of the circle is 3 cm.

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Ex (2) Radius of a circle is 20 cm. Distance of a chord from the centre of the circle is 12 cm. Find the length of the chord.

Solution : Let the centre of the circle be O. Radius = OD = 20 cm.

Distance of the chord CD from O is12 cm. seg OP $\perp$ seg CD

$\therefore$ OP = 12 cm

Now CP = PD ...... perpendicular drawn from the centre bisects the chord

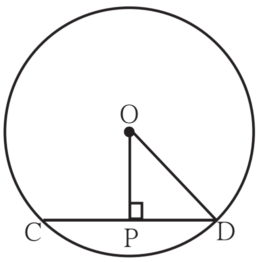


Fig. 6.8

In the right angled Δ OPD, using Pythagoras' theorem

$OP^2 + PD^2 = OD^2$

$(12)^2 + PD^2 = 20^2$

$PD^2 = 20^2 - 12^2$

$PD^2 = (20+12)(20-12)$

$= 32 \times 8 = 256$

$\therefore$ PD = 16 $\qquad \therefore$ CP = 16

CD = CP + PD = 16 + 16 = 32

$\therefore$ the length of the chord is 32 cm.

Practice set 6.1

1. Distance of chord AB from the centre of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.
2. Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the centre.
3. Radius of a circle is 34 cm and the distance of the chord from the centre is 30 cm, find the length of the chord.
4. Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the circle.
5. In figure 6.9, centre of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q. Show that AP = BQ
6. Prove that, if a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.

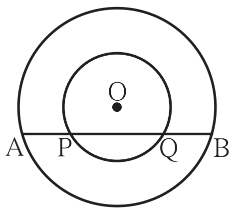


Fig. 6.9

Activity I

(1) Draw circles of convenient radii.
(2) Draw two equal chords in each circle.
(3) Draw perpendicular to each chord from the centre.
(4) Measure the distance of each chord from the centre.

Let's learn.

Relation between congruent chords of a circle and their distances from the centre

Activity II : Measure the lengths of the perpendiculars on chords in the following figures.

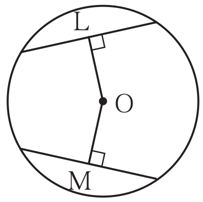


Figure (i)

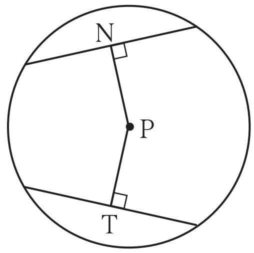


Figure (ii)

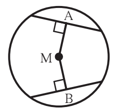


Figure (iii)

Did you find OL = OM in fig (i), PN = PT in fig (ii) and MA = MB in fig (iii) ?

Write the property which you have noticed from this activity.

Let's learn.

Properties of congruent chords

Theorem : Congruent chords of a circle are equidistant from the centre of the circle.

Given : In a circle with centre O

chord AB ≅ chord CD

OP ⊥ AB, OQ ⊥ CD

To prove : OP = OQ

Construction : Join seg OA and seg OD.

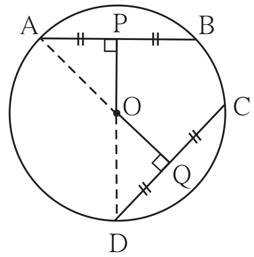


Fig. 6.10

Proof : AP = $\frac{1}{2}$ AB, DQ = $\frac{1}{2}$ CD . . . perpendicular drawn from the centre of a circle to its chord bisects the chord.

AB = CD . . . . given

∴ AP = DQ

∴ seg AP ≅ seg DQ . . . . (I) . . . segments of equal lengths

In right angled Δ APO and right angled Δ DQO

seg AP ≅ seg DQ . . . . from (I)

hypotenuse OA ≅ hypotenuse OD . . . . radii of the same circle

∴ Δ APO ≅ Δ DQO . . . . hypotenuse side theorem

seg OP ≅ seg OQ . . . . c.s.c.t.

∴ OP = OQ . . . . Length of congruent segments.

Congruent chords in a circle are equidistant from the centre of the circle.

Theorem : The chords of a circle equidistant from the centre of a circle are congruent.

Given : In a circle with centre O

seg OP $\perp$ chord AB

seg OQ $\perp$ chord CD

and OP = OQ

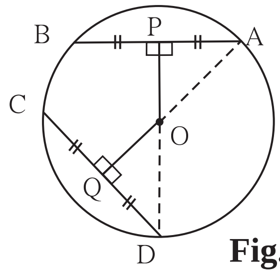


Fig. 6.11

To prove : chord AB $\cong$ chord CD

Construction : Draw seg OA and seg OD.

Proof : (Complete the proof by filling in the gaps.)

In right angled Δ OPA and right Δ OQD

hypotenuse OA $\cong$ hypotenuse OD . . . . $\boxed{\quad}$

seg OP $\cong$ seg OQ . . . . given

$\therefore$ Δ OPA $\cong$ Δ OQD . . . . $\boxed{\quad}$

$\therefore$ seg AP $\cong$ seg QD . . . . c.s.c.t.

$\therefore$ AP = QD . . . . (I)

But AP = $\frac{1}{2}$ AB, and DQ = $\frac{1}{2}$ CD . . . . . $\boxed{\quad}$

and AP = QD . . . . from (I)

$\therefore$ AB = CD

$\therefore$ seg AB $\cong$ seg CD

Note that both the theorems are converses of each other

Remember this !

Congruent chords of a circle are equidistant from the centre of the circle.
The chords equidistant from the centre of a circle are congruent.

Activity : The above two theorems can be proved for two congruent circles also.

1. Congruent chords in congruent circles are equidistant from their respective centres.
2. Chords of congruent circles which are equidistant from their respective centres are congruent.
   Write 'Given', 'To prove' and the proofs of these theorems .

Solved example

Ex. In the figure 6.12, O is the centre of the circle and AB = CD. If OP = 4 cm, find the length of OQ.

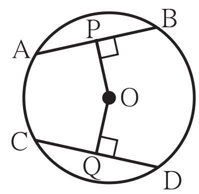


Fig. 6.12

Solution : O is the centre of the circle,

chord AB $\cong$ chord CD .....given

OP $\perp$ AB, OQ $\perp$ CD

OP = 4 cm, means distance of AB from the centre O is 4 cm.

The congruent chords of a circle are equidistant from the centre of the circle.

$\therefore$ OQ = 4 cm

Practice set 6.2

1. Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the centre of the circle ?
2. In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the centre. Find the lengths of the chords.
3. Seg PM and seg PN are congruent chords of a circle with centre C. Show that the ray PC is the bisector of $\angle$NPM.

Let's recall.

In previous standard we have verified the property that the angle bisectors of a triangle are concurrent. We denote the point of concurrence by letter I.

Let's learn.

Incircle of a triangle

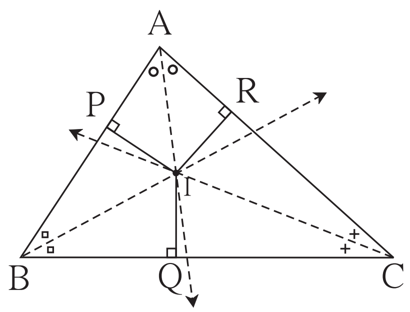


Fig. 6.13

In fig. 6.13, bisectors of all angles of a ΔABC meet in the point I. Perpendiculars on three sides are drawn from the point of concurrence.

$$IP \perp AB, \quad IQ \perp BC, \quad IR \perp AC$$

We know that, every point on the angle bisector is equidistant from the sides of the angle.

Point I is on the bisector of $\angle$B. $\therefore$ IP = IQ.

Point I is on the bisector of $\angle$C $\therefore$ IQ = IR

$$\therefore IP = IQ = IR$$

That is point I is equidistant from all the sides of ΔABC.

$\therefore$ if we draw a circle with centre I and radius IP, it will touch the sides AB, AC, BC of ΔABC internally.

This circle is called the Incircle of the triangle ABC.

Let's learn.

To construct the incircle of a triangle

Ex. Construct Δ PQR such that PQ = 6 cm, $\angle$Q = 35°, QR = 5.5 cm. Draw incircle of Δ PQR.

Draw a rough figure and show all measures in it.

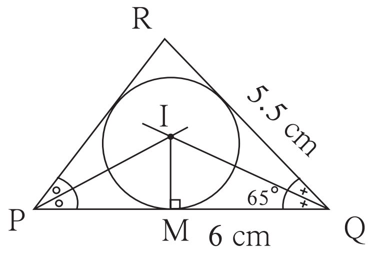
(1) Construct Δ PQR of given measures.

(2) Draw bisectors of any two angles of the triangle.

(3) Denote the point of intersection of angle bisectors as I.

(4) Draw perpendicular IM from the point I to the side PQ.

(5) Draw a circle with centre I and radius IM.



Rough fig. 6.14

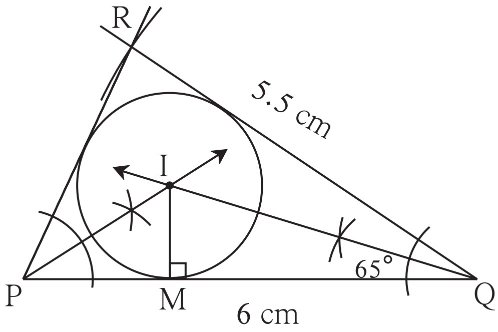


Fig. 6.15

Remember this !

The circle which touches all the sides of a triangle is called incircle of the triangle and the centre of the circle is called the incentre of the triangle.

Let's recall.

In previous standards we have verified the property that perpendicular bisectors of sides of a triangle are concurrent. That point of concurrence is denoted by the letter C.

Let's learn.

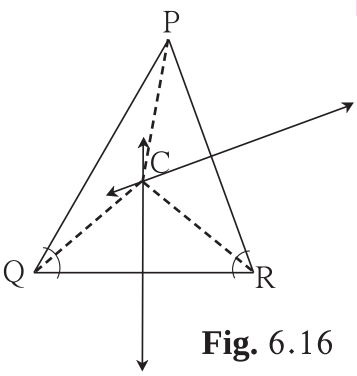


Fig. 6.16

In fig. 6.16, the perpendicular bisectors of sides of Δ PQR are intersecting at point C. So C is the point of concurrence of perpendicular bisectors.

Circumcircle

Point C is on the perpendicular bisectors of the sides of triangle PQR. Join PC, QC and RC. We know that, every point on the perpendicular bisector is equidistant from the end points of the segment.

Point C is on the perpendicular bisector of seg PQ. $\therefore$ PC = QC I

Point C is on the perpendicular bisector of seg QR. $\therefore$ QC = RC II

$\therefore$ PC = QC = RC . . . . . From I and II

$\therefore$ the circle with centre C and radius PC will pass through all the vertices of Δ PQR. This circle is called as the circumcircle.

Remember this !

Circle passing through all the vertices of a triangle is called circumcircle of the triangle and the centre of the circle is called the circumcentre of the triangle.

Let's learn.

To draw the circumcircle of a triangle

Ex. Construct Δ DEF such that DE = 4.2 cm, $\angle D = 60^\circ$, $\angle E = 70^\circ$ and draw circumcircle of it. Draw rough figure. Write the given measures.

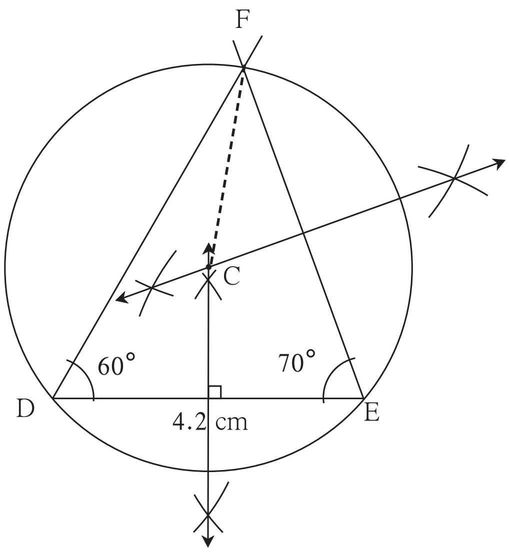


Fig. 6.18

Rough figure

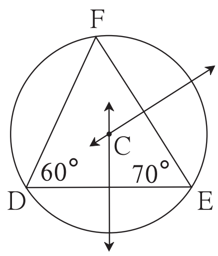


Fig. 6.17

Steps of construction **:**

(1) Draw Δ DEF of given measures.

(2) Draw perpendicular bisectors of any two sides of the triangle.

(3) Name the point of intersection of perpendicular bisectors as C.

(4) Join seg CF.

(5) Draw circle with centre C and radius CF.

Activity :
Draw different triangles of different measures and draw incircles and circumcircles of them. Complete the table of observations and discuss.

Type of triangle	Equilateral triangle	Isosceles triangles	Scalene triangle
Position of incenter	Inside the triangle	Inside the triangle	Inside the triangle
Position of circumcentre	Inside the triangle	Inside, outside on the triangle	

Type of triangle	Acute angled triangle	Right angled triangle	Obtuse angled triangle
Position of incentre			
Position of circumcircle		Midpoint of hypotenuse	

Remember this !

- Incircle of a triangle touches all sides of the triangle from inside.
- For construction of incircle of a triangle we have to draw bisectors of any two angles of the triangle.
- Circumcircle of a triangle passes through all the vertices of a triangle.
- For construciton of a circumcircle of a triangle we have to draw perpendicular bisectors of any two sides of the triangle.
- Circumcentre of an acute angled triangle lies inside the triangle.
- Circumcentre of a right angled triangle is the midpoint of its hypotenuse.
- Circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
- Incentre of any triangle lies in the interior of the triangle.

Activity : Draw any equilateral triangle. Draw incircle and circumcircle of it. What did you observe while doing this activity ?

(1) While drawing incircle and circumcircle, do the angle bisectors and perpendicular bisectors coincide with each other ?

(2) Do the incentre and circumcenter coincide with each other ? If so, what can be the reason of it ?

(3) Measure the radii of incircle and circumcircle and write their ratio.

Remember this !

- The perpendicular bisectors and angle bisectors of an equilateral triangle are coincedent.
- The incentre and the circumcentre of an equilateral triangle are coincedent.
- Ratio of radius of circumcircle to the radius of incircle of an equilateral triangle is 2 : 1

Practice set 6.3

1. Construct Δ ABC such that $\angle B = 100^\circ$, BC = 6.4 cm, $\angle C = 50^\circ$ and construct its incircle.

2. Construct Δ PQR such that $\angle P = 70^\circ$, $\angle R = 50^\circ$, QR = 7.3 cm. and construct its circumcircle.

3. Construct Δ XYZ such that XY = 6.7 cm, YZ = 5.8 cm, XZ = 6.9 cm. Construct its incircle.

4. In Δ LMN, LM = 7.2 cm, $\angle M = 105^\circ$, MN = 6.4 cm, then draw Δ LMN and construct its circumcircle.

5. Construct Δ DEF such that DE = EF = 6 cm, $\angle F = 45^\circ$ and construct its circumcircle.

Problem set 6

1. Choose correct alternative answer and fill in the blanks.

(i) Radius of a circle is 10 cm and distance of a chord from the centre is 6 cm. Hence the length of the chord is ....
(A) 16 cm (B) 8 cm (C) 12 cm (D) 32 cm

(ii) The point of concurrence of all angle bisectors of a triangle is called the ......
(A) centroid (B) circumcentre (C) incentre (D) orthocentre

(iii) The circle which passes through all the vertices of a triangle is called .....
(A) circumcircle (B) incircle (C) congruent circle (D) concentric circle

(iv) Length of a chord of a circle is 24 cm. If distance of the chord from the centre is 5 cm, then the radius of that circle is ....
(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

(v) The length of the longest chord of the circle with radius 2.9 cm is .....
(A) 3.5 cm (B) 7 cm (C) 10 cm (D) 5.8 cm

(vi) Radius of a circle with centre O is 4 cm. If l(OP) = 4.2 cm, say where point P will lie.
(A) on the centre (B) Inside the circle (C) outside the circle(D) on the circle

(vii) The lengths of parallel chords which are on opposite sides of the centre of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is .....
(A) 2 cm (B) 1 cm (C) 8 cm (D) 7 cm

2. Construct incircle and circumcircle of an equilateral Δ DSP with side 7.5 cm. Measure the radii of both the circles and find the ratio of radius of circumcircle to the radius of incircle.

3. Construct Δ NTS where NT = 5.7 cm, TS = 7.5 cm and $\angle$NTS = 110° and draw incircle and circumcircle of it.

4. In the figure 6.19, C is the centre of the circle.
seg QT is a diameter
CT = 13, CP = 5, find the length of chord RS.

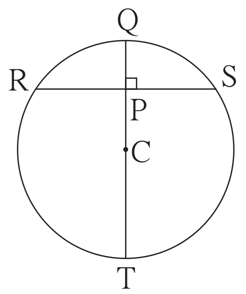


Fig. 6.19

5. In the figure 6.20, P is the centre of the circle.
chord AB and chord CD intersect
on the diameter at the point E
If $\angle$AEP $\cong$ $\angle$DEP
then prove that AB = CD.

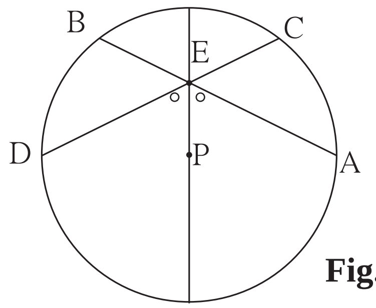


Fig. 6.20

6. In the figure 6.21, CD is a diameter of the circle with centre O. Diameter CD is perpendicular to chord AB at point E. Show that Δ ABC is an isosceles triangle.

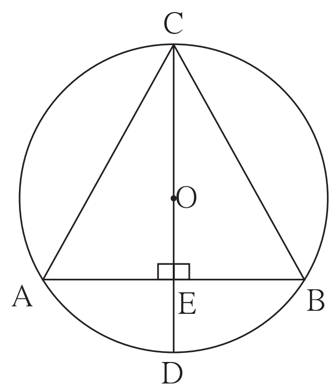
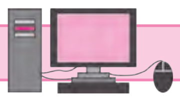


Fig. 6.21



ICT Tools or Links

Draw different circles with Geogebra software. Verify and experience the properties of chords. Draw circumcircle and incircle of different triangles. Using 'Move Option' experience how the incentre and circumcentre changes if the size of a triangle is changed.

7 Co-ordinate Geometry

Lets study

- **Axis, Origin, Quadrant**
- **Co-ordinates of a point in a plane.**
- **To plot a point.**
- **Line parallel to X-axis.**
- **Line parallel to Y-axis.**
- **Equation of a line.**

Chintu and his friends were playing cricket on the ground in front of a big building, when a visitor arrived.

Visitor : Hey Chintu, Dattabhau lives here, doesnt he ?

Chintu : Yes, on the second floor. See that window ? Thats his flat.

Visitor : But there are five windows on the second floor. It could be any of them !

Chintu : His window is the third one from the left, on the second floor..

Chintu's description of the location of Dattabhau's flat is in fact, based on the most basic concept in Co-ordinate Geometry.

It did not suffice to give only the floor number to locate the house. Its serial number from the left or from the right also needed to be given. That is two numbers had to be given in a specific sequence. Two **ordinal numbers** namely, **second** from the ground and **third** from the left had to be used.

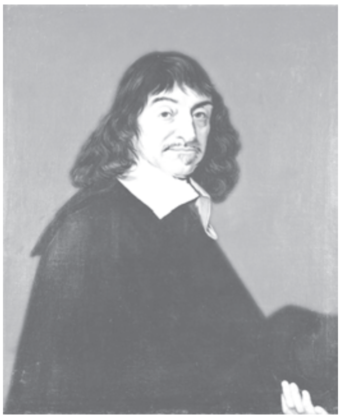
Let' Learn

Axes, origin, quadrants

We could give the location of Dattabhau's house using two ordinal numbers. Similarly, the location of a point can be fully described using its distances from two mutually perpendicular lines.

To locate a point in a plane, a horizontal number line is drawn in the plane. This number line is called the X-axis.

Rene Descartes (1596-1650)



Rene Descartes, a French mathematician of the 17th Century, proposed the co-ordinate system to describe the position of a point in a plane accurately. It is called the Cartesian co-ordinate system. Obviously the word Cartesian is derived from his name. He brought about a revolution in the field of mathematics by establishing the relationship between Algebra and Geometry.

The Cartesian co-ordinate system is the foundation of Analytical Geometry. La Geometric was Descartes' first book on mathematics. In it, he used algebra for the study of geometry and proposed that a point in a plane can be represented by an ordered pair of real numbers. This ordered pair is the 'Cartesian Co-ordinates' of a point.

Co-ordinate geometry has used in a variety of fields such as Physics, Engineering, Nautical Science, Siesmology and Art. It plays an important role in the development of technology in Geogebra. We see the inter-relationship between Algebra and Geometry quite clearly in the software Geogebra;the very name being a combination of the words 'Geometry' and 'Algebra'.

Another number line intersecting the X-axis at point marked O and perpendicular to the X-axis, is the Y-axis. Generally, the number O is represented by the same point on both the number lines. This point is called the origin and is shown by the letter O.

On the X-axis, positive numbers are shown on the right of O and negative numbers on the left.

On the Y-axis, positive numbers are shown above O and negative numbers below it.

The X and Y axes divide the plane into four parts, each of which is called a **Quadrant.** As shown in the figure, the quadrants are numbered in the anti-clocksise direction.

The points on the axes are not included in the quadrants.

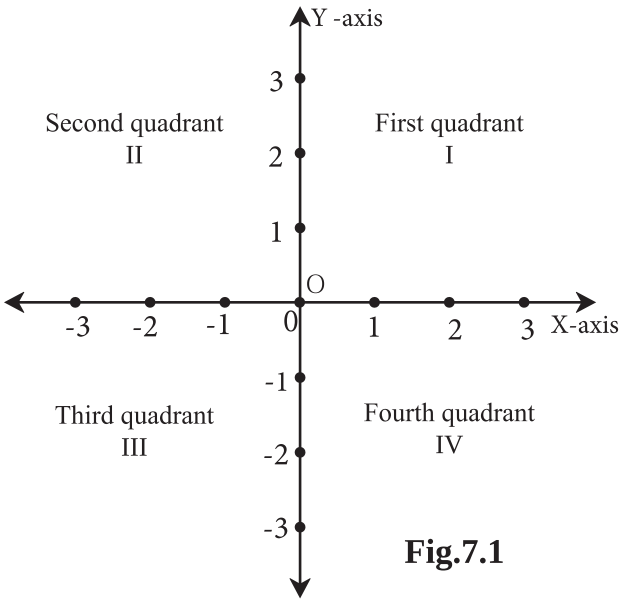


Fig.7.1

The Co-ordinates of a point in a plane

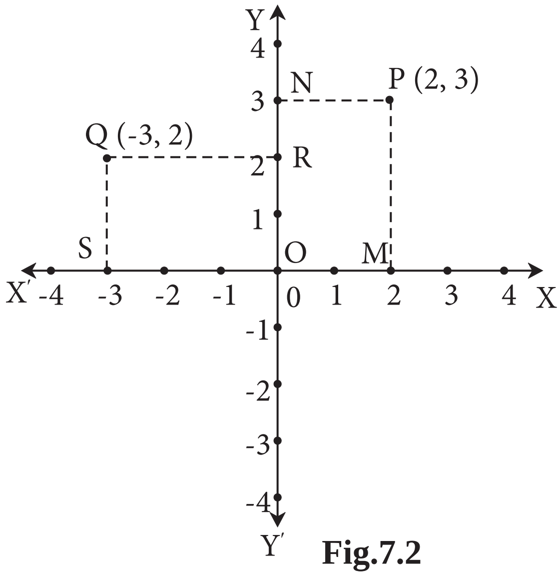


Fig.7.2

The point P is shown in the plane determined by the X-axis and the Y-axis. Its position can be determined by its distance from the two axes. To find these distances, we draw seg PM ⊥ X-axis and seg PN ⊥ Y-axis.

Co-ordinate of point M on X-axis is 2 and co-ordinate of point N on Y-axis is 3.

Therefore x co-ordinate of point P is 2 and y co-ordinate of point P is 3..

The convention for describing the position of a point is to mention x co-ordinate first. According to this convention the order of co-ordinates of point P is decided as 2, 3. The position of the point P in brief, is described by the pair (2, 3)

The order of the numbers in the pair (2, 3) is important. Such a pair of numbers is called an ordered pair.

To describe the position of point Q, we draw seg QS ⊥ X-axis and seg QR ⊥ Y-axis. The co-ordinate of point Q on the X-axis is –3 and the co-ordinate on the Y-axis is 2. Hence the co-ordinates of point Q are (–3, 2).

Ex. Write the co-ordinates of points E, F, G, T in the figure alongside.

Solution :

- The co-ordinates of point E are (2, 1)
- The co-ordinates of point F are (–3, 3)
- The co-ordinates of point G are (–4, –2).
- The co-ordinates of point T are (3, –1)

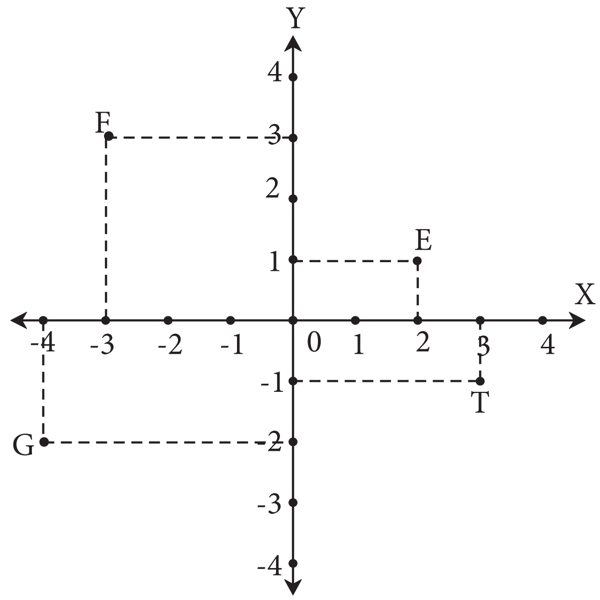


Fig.7.3

Downloaded from https:// www.studiestoday.com

Let's learn.

Co-ordinates of points on the axes

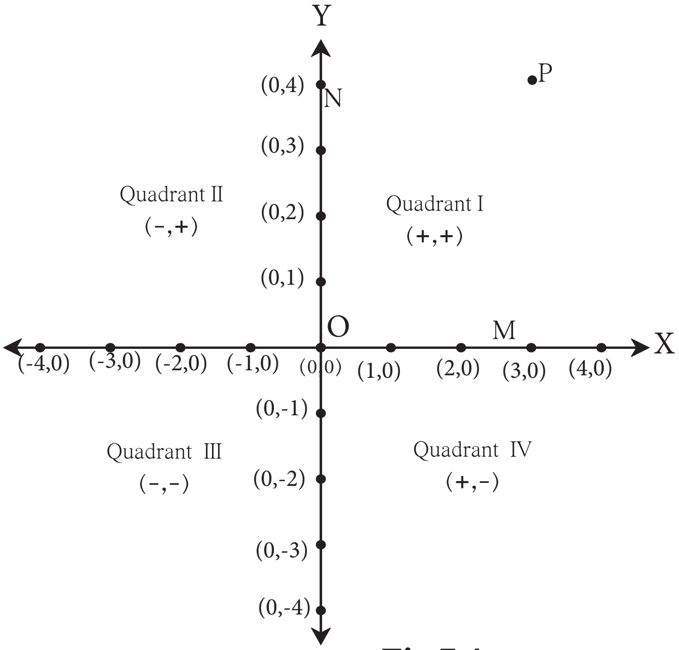


Fig.7.4

The x co-ordinate of point M is its distance from the Y-axis. The distance of point M from the X-axis is zero. Hence, the y co-ordinate of M is 0.

Thus, the co-ordinates of point M on the X-axis are (3,0).

The y co-ordinate of point N on the Y-axis is 4 units from the X-axis because N is at a distance of 4. Its x co-ordinate is 0 because its distance from the Y-axis is zero.

Hence, the co-ordinates of point N on the Y-axis are (0, 4).

Now the origin 'O' is on X-axis as well as on Y-axis. Hence, its distance from X-axis and Y-axis is zero. Therefore, the co-ordinates of O are (0, 0).

One and only one pair of co-ordinates (ordered pair) is associated with every point in a plane.

Let's Remember

- The y co-ordinate of every point on the X-axis is zero.
- The x co-ordinate of every point on the Y-axis is zero.
- The coordinates of the origin are (0, 0).

Ex. In which quadrant or on which axis are the points given below ?
A(5,7), B(–6,4), C(4,–7), D(–8,–9), P(–3,0), Q(0,8)

Solution : The x co-ordinate of A (5, 7) is positive and its y co-ordinate is positive..
$\therefore$ point A is in the first quadrant.
The x co-ordinate of B (–6, 4) is negative and y co-ordinate is positive.
$\therefore$ point B is in the second quadrant.
The x co-ordinate of C (4, –7) is positive and y co-ordinate is negative.
$\therefore$ point C is in the fourth quadrant.
The x co-ordinate of D (–8, –7) is negative and y co-ordinate is negative.
$\therefore$ point D is in the third quadrant.

The y co-ordinate of P(-3,0) is zero $\therefore$ point P is on the X-axis.

The x co-ordinate of Q (0,8) is zero $\therefore$ point Q is on the Y-axis.

Activity As shown in fig. 7.5, ask girls to sit in lines so as to form the X-axis and Y-axis.

- Ask some boys to sit at the positions marked by the coloured dots in the four quadrants.
- Now, call the students turn by turn using the initial letter of each student's name. As his or her initial is called, the student stands and gives his or her own co-ordinates. For example Rajendra (2, 2) and Kirti (-1, 0)
- Even as they have fun during this field activity, the students will learn how to state the position of a point in a plane.

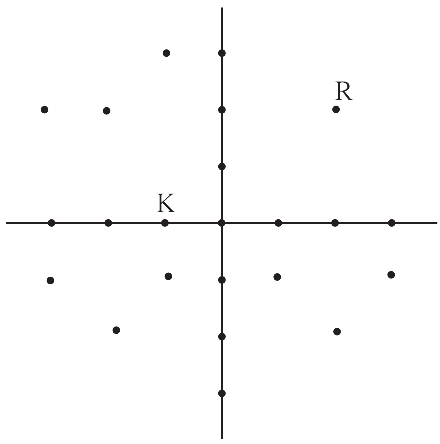


Fig. 7.5

Let's learn.

To plot the points of given co-ordinates

Suppose we have to plot the points P (4,3) and Q (-2,2)

Steps for plotting the points

(i) Draw X-axis and Y-axis on the plane. Show the origin.

(ii) To find the point P (4,3), draw a line parallel to the Y-axis through the point on X axis which represents the number 4.

Through the point on Y-axis which represents the number 3 draw a line parallel to the X-axis .

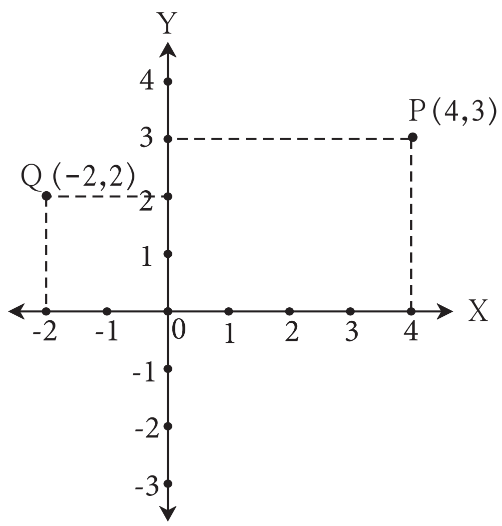


Fig. 7.6

(iii) The point of intersection of these two lines parallel to the Y and X-axis respectively, is the point P (4,3). In which quadrant does this point lie ?

(iv) In the same way, plot the point Q (–2, 2) . Is this point in the second quadrant ? Using the same method, plot the points R(–3, –4), S(3, –1)

Ex. In which quadrants or on which axis are the points given below ?

(i) (5, 3) (ii) (–2, 4) (iii) (2, –5) (iv) (0, 4)
(v) (–3, 0) (vi) (–2, 2.5) (vii) (5, 3.5) (viii) (–3.5, 1.5)
(ix) (0, –4) (x) (2, –4)

Solution :

	co-ordinates	Quadrant / axis
(i)	(5,3)	Quadrant I
(ii)	(–2,4)	Quadrant II
(iii)	(2,–5)	Quadrant IV
(iv)	(0,4)	Y–axis
(v)	(–3,0)	X–axis

	co-ordinates	Quadrant / axis
(vi)	(–2, –2.5)	Quadrant III
(vii)	(5,3.5)	Quadrant I
(viii)	(–3.5,1.5)	Quadrant II
(ix)	(0, –4)	Y–axis
(x)	(2,–4)	Quadrant IV

Practice set 7.1

1. State in which quadrant or on which axis do the following points lie.

- A(–3, 2), • B(–5, –2), • K(3.5, 1.5), • D(2, 10),
- E(37, 35), • F(15, –18), • G(3, –7), • H(0, –5),
- M(12, 0), • N(0, 9), • P(0, 2.5), • Q(–7, –3)

2. In which quadrant are the following points ?

(i) whose both co-ordinates are positive.

(ii) whose both co-ordinates are negative.

(iii) whose x co-ordinate is positive, and the y co-ordinate is negative.

(iv) whose x co-ordinate is negative and y co-ordinate is positive.

3. Draw the co-ordinate system on a plane and plot the following points.

L(–2, 4), M(5, 6), N(–3, –4), P(2, –3), Q(6, –5), S(7, 0), T(0, –5)

Let's learn.

Lines parallel to the X-axis

- On a graph paper, plot the following points
  A (5, 4), B (2, 4), C (–2, 4), D (–4, 4), E (0, 4), F (3, 4)
- Observe the co-ordinates of the given points.
- Did you notice that the y co-ordinates of all the points are equal ?
- All the points are collinear.
- To which axis is this line parallel ?
- The y co-ordinate of every point on the line DA is 4. It is constant. Therefore the line DA is described by the equation $y = 4$. If the y co-ordinate of any point is 4, will be on the line DA.

  The equation of the line parallel to the X axis at a distance of 4 units from the X-axis is $y = 4$.

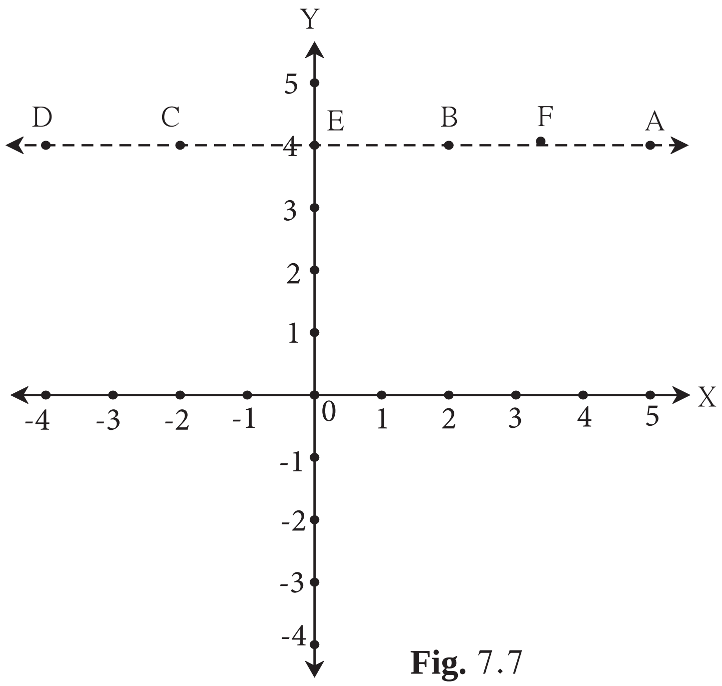


Fig. 7.7

Let's discuss.

- Can we draw a line parallel to the X-axis at a distance of 6 units from it and below the X-axis ?
- Will all of the points (–3, –6), (10, –6), $(\frac{1}{2}, -6)$ be on that line ?
- What would be the equation of this line ?

Remember this !

If b > 0, and we draw the line y = b through the point (0, b), it will be above the X-axis and parallel, to it. If b < 0, then the line $y = b$ will be below the X-axis and parallel to it.

The equation of a line parallel to the X-axis is in the form $y = b$.

Let's learn.

Lines parallel to the Y-axis

- On a graph paper, plot the following points
  P(–4, 3), Q(–4, 0), R(–4, 1), S(–4, –2), T(–4, 2), U(–4, –3)

- Observe the co–ordinates of the points.
- Did you notice that the x co–ordinate of all the points are the same ?
- Are all the points collinear ?
- To which axis is this line parallel ?
- The x co–ordinate of every point on the line PS is -4. It is constant. Therefore, the line PS can be described by the equation $x = -4$. Every point whose x co–ordinate is -4 lies on the line PS. The equation of the line parallel to the Y–axis at a distance of 4 units and to the left of Y–axis is $y = -4$.

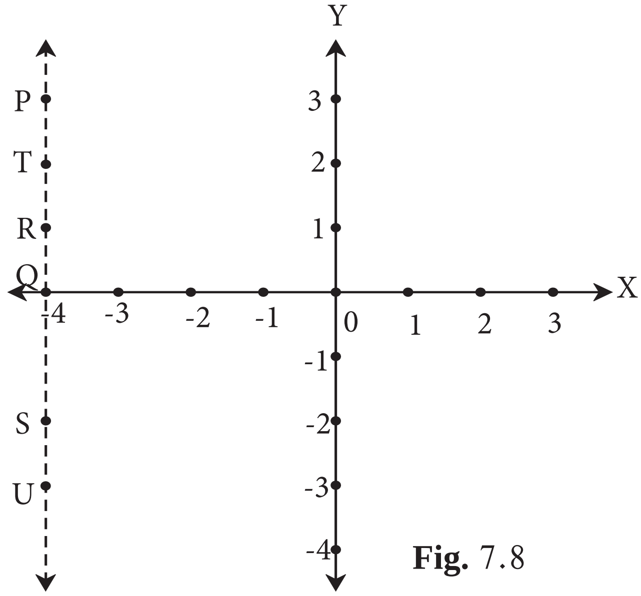


Fig. 7.8

Let's discuss

- Can we draw a line parallel to the Y-axis at a distance of 2 units from it and to its right ?
- Will all of the points (2,10), (2,8), $(2, -\frac{1}{2})$ be on that line ?
- What would be the equation of this line ?

Remember this !

If we draw the line $x = a$ parallel to the Y-axis passing through the point $(a, 0)$ and if $a > 0$ then the line will be to the right of the Y-axis. If $a < 0$, then the line will be to the left of the Y-axis.

The equation of a line parallel to the Y-axis is in the form $x = a$.

Remember this !

(1) The y co-ordinate of every point on the X-axis is zero. Conversely, every point whose y co-ordinate is zero is on the X-axis. Therefore, the equation of the X axis is $y = 0$.

(2) The x co-ordinate of every point on the Y-axis is zero. Conversely, every point whose x co-ordinate is zero is on the Y-axis. Therefore, the equation of the Y-axis is $x = 0$.

Let's learn.

Graph of a linear equations

Ex. Draw the graphs of the equations $x = 2$ and $y = -3$.

Solution : (i) On a graph paper draw the X-axis and the Y-axis.

(ii) Since it is given that $x = 2$, draw a line on the right of the Y-axis at a distance of 2 units from it and parallel to it.

(iii) Since it is given that $y = -3$, draw a line below the X-axis at a distance of 3 units from it and parallel to it.

(iv) These lines, parallel to the two axes, are the graphs of the given equations.

(v) Write the co-ordinates of the point P, the point of intersection of these two lines.

(vi) Verify that the co-ordinates of the point P are $(2, -3)$

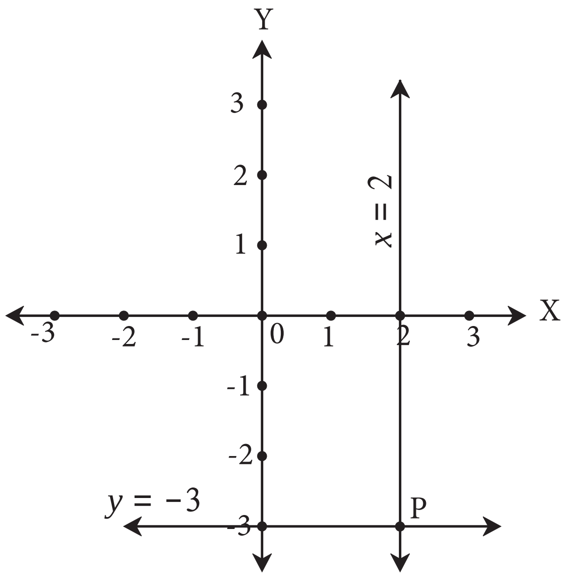


Fig. 7.9

The graph of a linear equation in the general form.

Activity : On a graph paper, plot the points (0,1) (1,3) (2,5). Are they collinear ? If so, draw the line that passes through them.

- Through which quadrants does this line pass ?
- Write the co-ordinates of the point at which it intersects the Y-axis.
- Show any point in the third quadrant which lies on this line. Write the co-ordinates of the point.

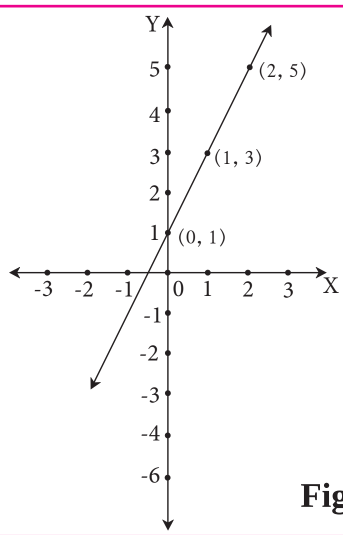


Fig. 7.10

Ex. $2x - y + 1 = 0$ is a linear equation in two variables in general form. Let us draw the graph of this equation.

Solution : $2x - y + 1 = 0$ means $y = 2x + 1$

Let us assume some values of x and find the corresponding values of y.

For example, If $x = 0$, then substituting this value of x in the equation we get $y = 1$.

Similarly, let us find the values of y when 0, 1, 2, $\frac{1}{2}$, -2 are some values of x and write these values in the table below in the form of ordered pairs.

x	0	1	2	$\frac{1}{2}$	-2
y	1	3	5	2	-3
(x, y)	(0,1)	(1,3)	(2,5)	$(\frac{1}{2}, 2)$	$(-2, -3)$

Now, let us plot these points. Let us verify that these points are collinear. Let us draw that line. The line is the graph of the equaiton $2x - y + 1 = 0$.

ICT Tools or Links

Use the Software Geogebra to draw the X and Y-axis. Plot several points. Find and study the co-ordinates of the points in 'Algebraic view'. Read the equations of lines that are parallel to the axes. Use the 'move' option to vary the positions of the lines. What are the equations of the X-axis and the Y-axis ?

Practice set 7.2

1. On a graph paper plot the points A (3,0), B(3,3), C(0,3). Join A, B and B, C. What is the figure formed ?
2. Write the equation of the line parallel to the Y-axis at a distance of 7 units from it to its left.
3. Write the equation of the line parallel to the X-axis at a distance of 5 units from it and below the X-axis.
4. The point Q(-3, -2) lies on a line parallel to the Y-axis. Write the equation of the line and draw its graph.

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5. X-axis and line $x = -4$ are parallel lines. What is the distance between them?

6. Which of the equations given below have graphs parallel to the X-axis, and which ones have graphs parallel to the Y-axis ?

(i) $x = 3$ (ii) $y - 2 = 0$ (iii) $x + 6 = 0$ (iv) $y = -5$

7. On a graph paper, plot the points A(2, 3), B(6, –1) and C(0, 5). If those points are collinear then draw the line which includes them. Write the co-ordinates of the points at which the line intersects the X-axis and the Y-axis.

8. Draw the graphs of the following equations on the same system of co-ordinates. Write the co-ordinates of their points of intersection.

$x + 4 = 0$, $y - 1 = 0$, $2x + 3 = 0$, $3y - 15 = 0$

9. Draw the graphs of the equations given below

(i) $x + y = 2$ (ii) $3x - y = 0$ (iii) $2x + y = 1$

Problem set 7

1. Choose the correct alternative answer for the following quesitons.

(i) What is the form of co-ordinates of a point on the X-axis ?
(A) (b, b) (B) $(0, b)$ (C) $(a, 0)$ (D) (a, a)

(ii) Any point on the line $y = x$ is of the form .....
(A) (a, a) (B) $(0, a)$ (C) $(a, 0)$ (D) $(a, -a)$

(iii) What is the equation of the X-axis ?
(A) $x = 0$ (B) $y = 0$ (C) $x + y = 0$ (D) $x = y$

(iv) In which quadrant does the point (–4, –3) lie ?
(A) First (B) Second (C) Third (D) Fourth

(v) What is the nature of the line which includes the points (–5,5), (6,5), (–3,5), (0,5) ?
(A) Passes through the origin,, (B) Parallel to Y–axis.
(C) Parallel to X–axis (D) None of these

(vi) Which of the points P (–1,1), Q (3,–4), R(1,–1), S (–2,–3), T (–4,4) lie in the fourth quadrant ?
(A) P and T (B) Q and R (C) only S (D) P and R

2. Some points are shown in the figure 7.11

With the help of it answer the following questions :

(i) Write the co-ordinates of the points Q and R.

(ii) Write the co-ordinates of the points T and M.

(iii) Which point lies in the third quadrant ?

(iv) Which are the points whose x and y co-ordinates are equal ?

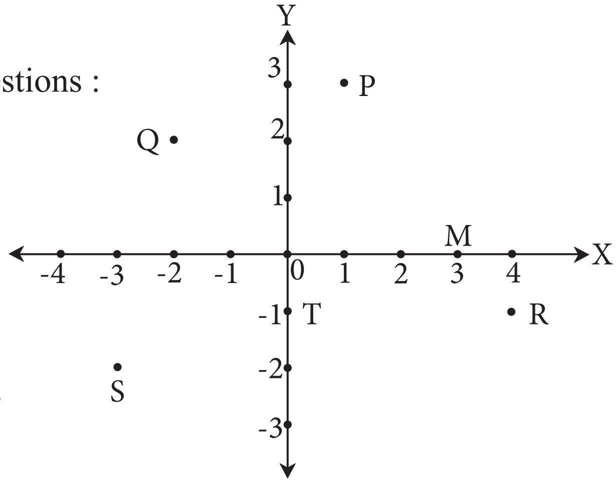


Fig.7.11

3. Without plotting the points on a graph, state in which quadrant or on which axis do the following point lie.

(i) (5, –3) (ii) (–7, –12) (iii) (–23, 4)

(iv) (–9, 5) (v) (0, –3) (vi) (–6, 0)

4. Plot the following points on the one and the same co-ordinate system.

A(1, 3), B(–3, –1), C(1, –4),
D(–2, 3), E(0, –8), F(1, 0)

5. In the graph alongside, line LM is parallel to the Y-axis. (Fig. 7.12)

(i) What is the distance of line LM from the Y-axis ?

(ii) Write the co-ordinates of the points P, Q and R.

(iii) What is the difference between the x co-ordinates of the points L and M?

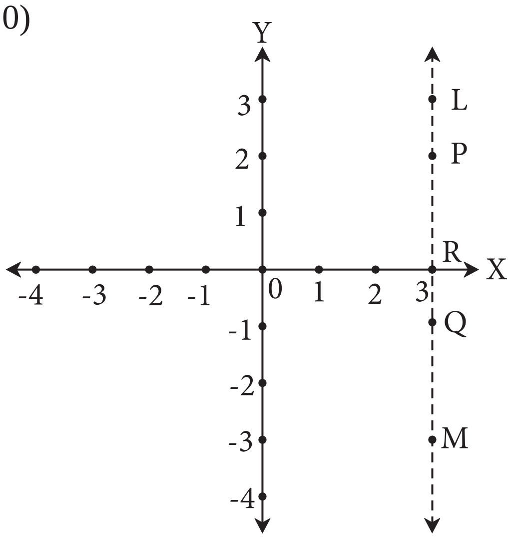


Fig.7.12

6. How many lines are there which are parallel to X-axis and having a distance 5 units?

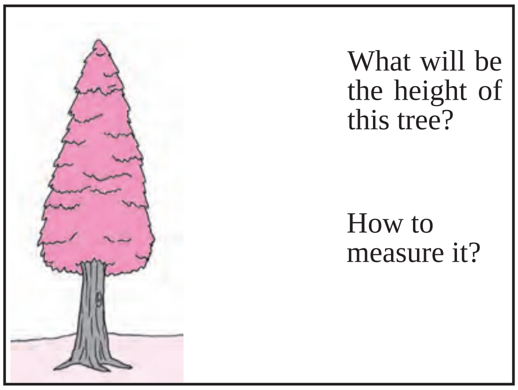
7*. If 'a' is a real number, what is the distance between the Y-axis and the line $x = a$?

8 Trigonometry

Let's study.

- **Introduction of Trigonometry**
- **Trigonometic Ratios**
- **Relations among Trigonometric Ratios**
- **Trigonometric Ratios of Particular Angles**

Introduction to Trigonometry



We can measure distances by using a rope or by walking on ground, but how to measure the distance between a ship and a light house? How to measure the height of a tall tree?

Observe the above pictures. Questions in the pictures are related to mathematics. Trigonometry, a branch of mathematics, is useful to find answers to such questions. Trigonometry is used in different branches of Engineering, Astronomy, Navigation etc.

The word Trigonometry is derived from three Greek words 'Tri' means three, 'gona' means sides and 'metron' means measurements.

Let's recall.

We have studied triangle. The subject trigonometry starts with right angled triangle, theorem of Pythagoras and similar triangles, so we will recall these topics.

- In Δ ABC, $\angle$B is a right angle and side AC opposite to $\angle$B, is hypotenuse. Side opposite to $\angle$A is BC and side opposite to $\angle$C is AB. Using Pythagoras' theorem, we can write the following statement for this triangle.

  $(AB)^2 + (BC)^2 = (AC)^2$

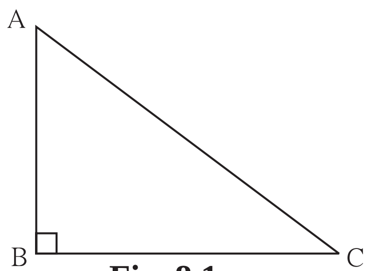


Fig. 8.1

- If $\Delta ABC \sim \Delta PQR$ then their corresponding sides are in the same proportions.

  So $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

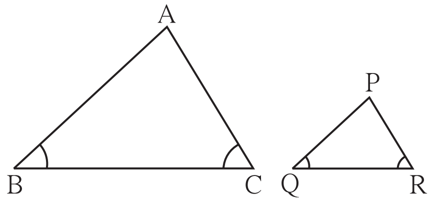


Fig. 8.2

Let us see how to find the height of a tall tree using properties of similar triangles.

Activity : This experiment can be conducted on a clear sunny day.

Look at the figure given alongside.

Height of the tree is QR, height of the stick is BC.

Thrust a stick in the ground as shown in the figure. Measure its height and length of its shadow. Also measure the length of the shadow of the tree. Rays of sunlight are parallel. So Δ PQR and Δ ABC are equiangular, means similar triangles. Sides of similar triangles are proportional.

So we get $\frac{QR}{PR} = \frac{BC}{AC}$.

Therefore, we get an equation,

height of the tree = QR = $\frac{BC}{AC} \times PR$

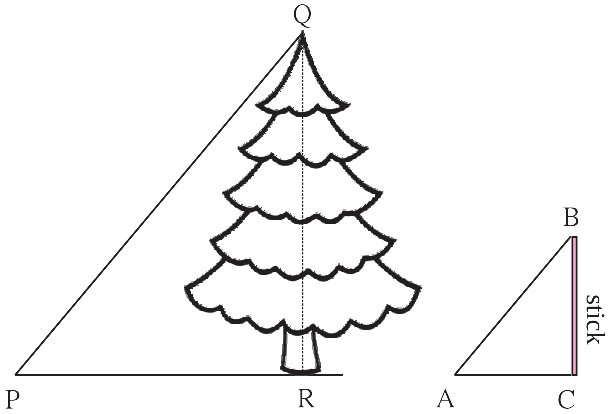


Fig.8.3

We know the values of PR, BC and AC. Substituting these values in this equation, we get length of QR, means height of the tree.

Use your brain power !

It is convenient to do this experiment between 11:30 am and 1:30 pm instead of doing it in the morning at 8'O clock. Can you tell why ?

Activity : You can conduct this activity and find the height of a tall tree in your surrounding. If there is no tree in the premises then find the height of a pole.

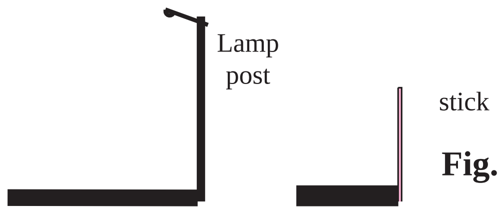


Fig. 8.4

Let's learn.

Terms related to right angled triangle

In right angled Δ ABC, $\angle$B = 90°, $\angle$ A and $\angle$ C are acute angles.

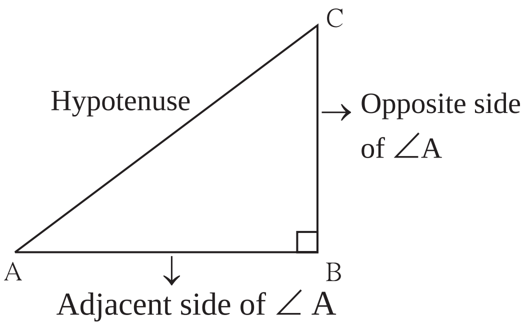


Fig. 8.5

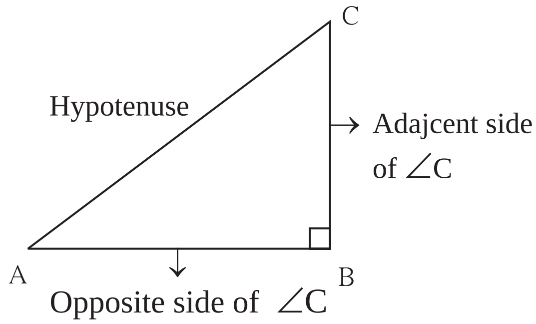


Fig. 8.6

Ex. In the figure 8.7, Δ PQR is a right angled triangle. Write-

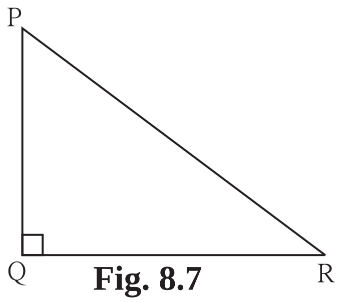


Fig. 8.7

side opposite to $\angle$ P =.....

side opposite to $\angle$ R =.....

side adjacent to $\angle$ P = ...

side adjacent to $\angle$ R = ...

Trigonometic ratios

In the adjacent Fig.8.8 some right angled triangles are shown. $\angle$B is their common angle. So all right angled triangles are similar.

Δ PQB ~ Δ ACB

$$\therefore \frac{PB}{AB} = \frac{PQ}{AC} = \frac{BQ}{BC}$$

$$\therefore \frac{PQ}{AC} = \frac{PB}{AB} \quad \therefore \frac{PQ}{PB} = \frac{AC}{AB} \quad \text{... alternando}$$

$$\frac{QB}{BC} = \frac{PB}{AB} \quad \therefore \frac{QB}{PB} = \frac{BC}{AB} \quad \text{... alternando}$$

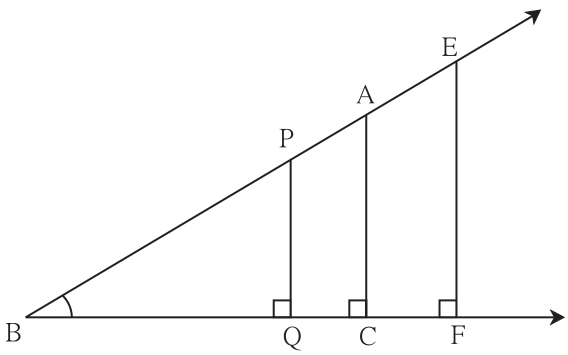


Fig. 8.8

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The figures of triangles in 8.9 and.8.10 are of the triangles separated from the figure 8.8

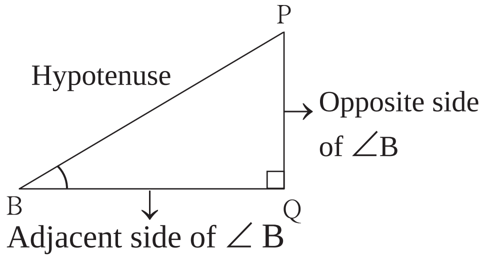

Fig.8.9

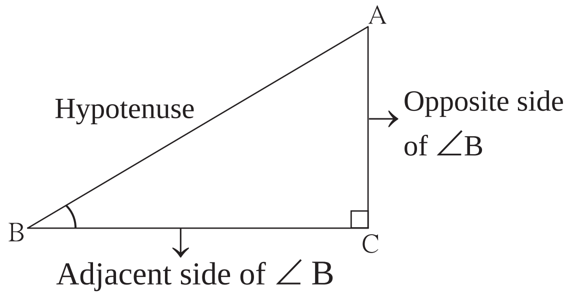

Fig.8.10

(i) In Δ PQB,

$$\frac{PQ}{PB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

In Δ ACB,

$$\frac{AC}{AB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

The ratios $\frac{PQ}{PB}$ and $\frac{AC}{AB}$ are equal.

$$\therefore \frac{PQ}{PB} = \frac{AC}{AB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

This ratio is called the 'sine' ratio of $\angle$ B, and is written in brief as **'sin B'**.

(ii) In Δ PQB and Δ ACB,

$$\frac{BQ}{PB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}} \text{ and } \frac{BC}{AB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}}$$

$$\therefore \frac{BQ}{PB} = \frac{BC}{AB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}}$$

This ratio is called as the 'cosine' ratio of $\angle$ B , and written in brief as **'cos B'**

(iii) $\frac{PQ}{BQ} = \frac{AC}{BC} = \frac{\text{Opposite side of } \angle B}{\text{Adjacent side of } \angle B}$

This ratio is called as the tangent ratio of $\angle$ B, and written in brief as **tan B.**

Ex. :

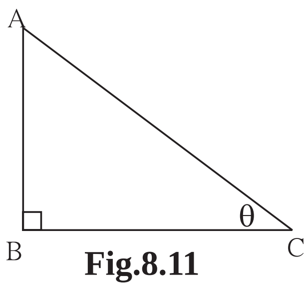

Fig.8.11

Sometimes we write measures of acute angles of a right angled triangle by using Greek letters θ (Theta), α (Alpha), β (Beta) etc.

In the adjacent figure of Δ ABC, measure of acute angle C is denoted by the letter θ. So we can write the ratios sin C, cos C, tan C as sin θ, cos θ, tan θ respectively.

$\sin C = \sin\theta = \frac{AB}{AC}$, $\cos C = \cos\theta = \frac{BC}{AC}$, $\tan C = \tan\theta = \frac{AB}{BC}$

Remember this !

- sin ratio $= \frac{\text{opposite side}}{\text{hypotenuse}}$
- cos ratio $= \frac{\text{adjacent side}}{\text{hypotenuse}}$
- tan ratio $= \frac{\text{opposite side}}{\text{adjcent side}}$
- $\sin\theta = \frac{\text{opposite side of } \angle\theta}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent side of } \angle\theta}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite side of } \angle\theta}{\text{opposite side of } \angle\theta}$

Practice set 8.1

1.

Fig. 8.12

In the Fig.8.12, $\angle R$ is the right angle of Δ PQR. Write the following ratios.
(i) sin P (ii) cos Q (iii) tan P (iv) tan Q

2.

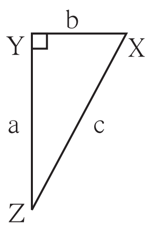

Fig. 8.13

In the right angled Δ XYZ, $\angle XYZ = 90^\circ$ and a,b,c are the lengths of the sides as shown in the figure. Write the following ratios,
(i) sin X (ii) tan Z (iii) cos X (iv) tan X.

3.

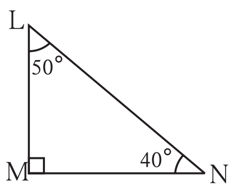

Fig. 8.14

In right angled Δ LMN, $\angle LMN = 90^\circ$
$\angle L = 50^\circ$ and $\angle N = 40^\circ$,
write the following ratios.
(i) $\sin 50^\circ$ (ii) $\cos 50^\circ$
(iii) $\tan 40^\circ$ (iv) $\cos 40^\circ$

4.

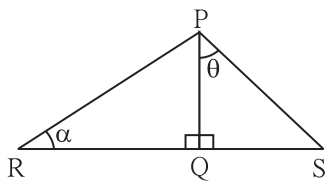

Fig. 8.15

In the figure 8.15,.$\angle PQR = 90^\circ$,
$\angle PQS = 90^\circ$, $\angle PRQ = \alpha$ and $\angle QPS = \theta$
Write the following trigonometric ratios.
(i) $\sin\alpha$, $\cos\alpha$, $\tan\alpha$
(ii) $\sin\theta$, $\cos\theta$, $\tan\theta$

Lets Learn

Relation among trigonometric ratios

In the Fig.8.16

Δ PMN is a right angled triangle.

$\angle$ M = 90°, $\angle$P and $\angle$ N are complimentary angles.

$\therefore$ If $\angle$ N = θ then $\angle$ P = 90 – θ

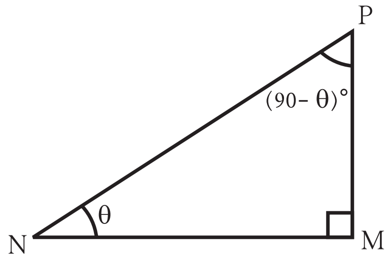

Fig.8.16

$\sin\theta = \dfrac{PM}{PN}$ ....(1)

$\cos\theta = \dfrac{NM}{PN}$ ....(2)

$\tan\theta = \dfrac{PM}{NM}$ ....(3)

$\sin(90-\theta) = \dfrac{NM}{PN}$ ....(4)

$\cos(90-\theta) = \dfrac{PM}{PN}$ ....(5)

$\tan(90-\theta) = \dfrac{NM}{PM}$ ....(6)

$\therefore$ $\sin\theta = \cos(90-\theta)$ .... from (1) and (5)

$\cos\theta = \sin(90-\theta)$ .... from (2) and (4)

Also note that $\tan\theta \times \tan(90-\theta) = \dfrac{PM}{NM} \times \dfrac{NM}{PM}$ .... from (3) and (6)

$\therefore$ $\tan\theta \times \tan(90-\theta) = 1$

Similarly, $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{PM}{PN}}{\frac{NM}{PN}} = \dfrac{PM}{PN} \times \dfrac{PN}{NM} = \dfrac{PM}{NM} = \tan\theta$

Remember this !

$\cos(90-\theta) = \sin\theta$, $\quad$ $\sin(90-\theta) = \cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$, $\quad$ $\tan\theta \times \tan(90-\theta) = 1$

*** For more information**

$$\frac{1}{\sin\theta} = \text{cosec}\,\theta, \quad \frac{1}{\cos\theta} = \sec\theta, \quad \frac{1}{\tan\theta} = \cot\theta$$

It means cosec θ, sec θ and cot θ are inverse ratios of sin θ, cos θ and tan θ respectively.

- $\sec\theta = \text{cosec}\,(90-\theta)$
- $\text{cosec}\,\theta = \sec(90-\theta)$
- $\tan\theta = \cot(90-\theta)$
- $\cot\theta = \tan(90-\theta)$

Lets Recall

Theorem of 30°- 60°-90° triangle :

We know that if the measures of angles of a triangle are 30°,60°, 90° then side opposite to 30° angle is half of the hypotenuse and side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ of hypotenuse.

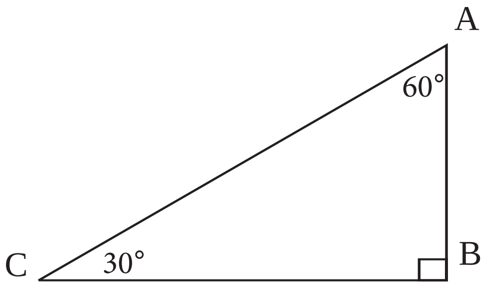

Fig. 8.17

In the Fig. 8.17, Δ ABC is a right angled triangle. ∠C = 30°, ∠A = 60°, ∠B = 90° .

$$\therefore \; AB = \frac{1}{2}AC \text{ and } BC = \frac{\sqrt{3}}{2}AC$$

Let's learn.

Trignometric ratios of 30° and 60° angles

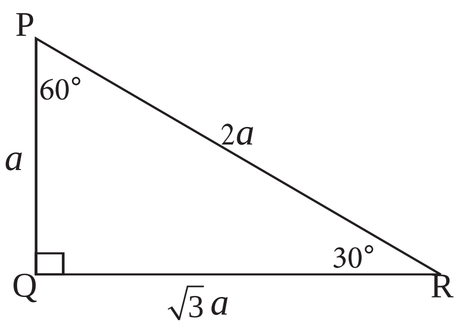

Fig. 8.18

In right angled Δ PQR if ∠R = 30°, ∠P = 60°, ∠Q = 90° and PQ = a

then $PQ = \frac{1}{2}PR$

$a = \frac{1}{2}PR$

$\therefore \; PR = 2a$

$QR = \frac{\sqrt{3}}{2}PR$

$QR = \frac{\sqrt{3}}{2} \times 2a$

$QR = \sqrt{3}\,a$

$\therefore$ If PQ = a, then PR = 2a and QR = $\sqrt{3}\,a$

(I) Trigonometric ratios of the 30° angle

$$\sin 30^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{QR}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

(II) Trigonometric ratios of 60° angle

$$\sin 60^\circ = \frac{QR}{PR} = \frac{\sqrt{3}\,a}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{QR}{PQ} = \frac{\sqrt{3}\,a}{a} = \sqrt{3}$$

In right angled Δ PQR, $\angle Q = 90^\circ$. Therefore $\angle P$ and $\angle R$ are complimentary angles of each other. Verify the relation between sine and cosine ratios of complimentary angles here also.

$\sin \theta = \cos (90 - \theta)$ $\sin 30^\circ = \cos (90^\circ - 30^\circ) = \cos 60^\circ$ $\sin 30^\circ = \cos 60^\circ$	$\cos \theta = \sin (90 - \theta)$ $\cos 30^\circ = \sin (90^\circ - 30^\circ) = \sin 60^\circ$ $\cos 30^\circ = \sin 60^\circ$

Remember this !

$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

(III) Trigonometric ratios of the 45° angle

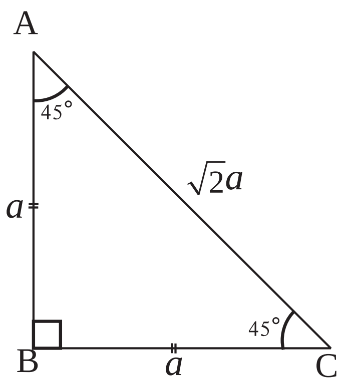

Fig.8.19

In right angled Δ ABC, $\angle B = 90^\circ$, $\angle A = 45^\circ$, $\angle C = 45^\circ$ $\therefore$ This is an isosceles triangle.

Suppose AB = a then BC = a.

Using Pythagoras' theorem, let us find the length of AC.

$$AC^2 = AB^2 + BC^2$$
$$= a^2 + a^2$$
$$AC^2 = 2a^2$$
$$\therefore\ AC = \sqrt{2}\,a$$

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In the Fig. 8.19, $\angle C = 45°$

$$\sin 45° = \frac{AB}{AC} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{BC}{AC} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{AB}{BC} = \frac{a}{a} = 1$$

Remember this !

$$\sin 45° = \frac{1}{\sqrt{2}}, \qquad \cos 45° = \frac{1}{\sqrt{2}}, \qquad \tan 45° = 1$$

(IV) Trigonometric ratios of the angle 0° and 90°

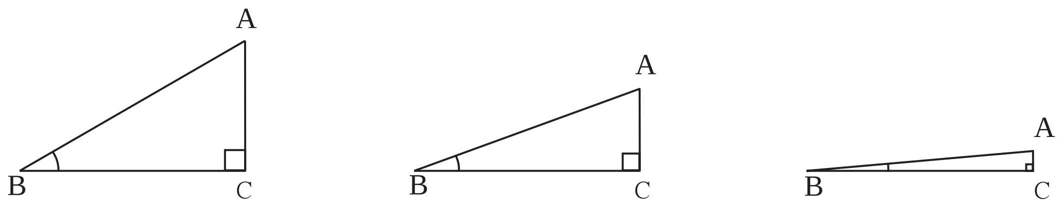


Fig.8.20

In the right angled Δ ACB, $\angle C = 90°$ and $\angle B = 30°$. We know that $\sin 30° = \frac{AC}{AB}$. Keeping the length of side AB constant, if the measure of $\angle B$ goes on decreasing the length of AC, which is opposite to $\angle B$ also goes on decreasing. So as the measure of $\angle B$ decreases, then value of $\sin\theta$ also decreases.

$\therefore$ when measure of $\angle B$ becomes 0°, then length of AC becomes 0.

$\therefore \sin 0° = \frac{AC}{AB} = \frac{0}{AB} = 0 \qquad \therefore \sin 0° = 0$

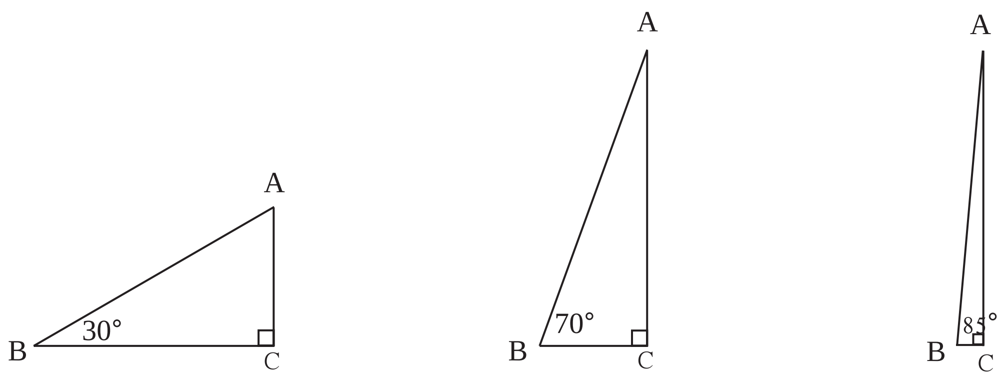


Fig.8.21

Now look at the Fig. 8.21. In this right angled triangle, as the measure of $\angle B$ increases the length of AC also increases. When measure of $\angle B$ becomes 90°, the length of AC become equal to AB

$$\therefore \sin 90^\circ = \frac{AC}{AB} \qquad \therefore \sin 90^\circ = 1$$

We know the relations between trigonometric ratios of complimentary angles.

$$\sin\theta = \cos(90-\theta) \quad \text{and} \quad \cos\theta = \sin(90-\theta)$$

$$\therefore \cos 0^\circ = \sin(90-0)^\circ = \sin 90^\circ = 1$$

$$\text{and } \cos 90^\circ = \sin(90-90)^\circ = \sin 0^\circ = 0$$

Remember this !

$\sin 0^\circ = 0,$ $\sin 90^\circ = 1,$ $\cos 0^\circ = 1,$ $\cos 90^\circ = 0$

We know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \therefore \tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

$$\text{But } \tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0}$$

But we can not do the division of 1 by 0. Note that θ is an acute angle. As it increases and reaches the value of 90°, $\tan\theta$ also increases indefinitely. Hence we can not find the definite value of tan 90.

Remember this !

Trigonometric ratios of particular ratios.

Ratios \ Measures of angles	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Solved Examples :

Ex. (1) Find the value of 2tan 45° + cos 30° – sin 60°

Solution : 2tan 45° + cos 30° – sin 60°

$$= 2 \times 1 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$= 2 + 0$

$= 2$

Ex. (2) Find the value of $\frac{\cos 56^\circ}{\sin 34^\circ}$

Solution : $56^\circ + 34^\circ = 90^\circ$ means 56 and 34 are the measures of complimentary angles.

$\sin \theta = \cos (90- \theta)$

$\therefore \sin 34^\circ = \cos (90- 34)^\circ = \cos 56^\circ$

$$\therefore \frac{\cos 56^\circ}{\sin 34^\circ} = \frac{\cos 56^\circ}{\cos 56^\circ} = 1$$

Ex. 3 In right angled Δ ACB, If $\angle C = 90^\circ$, AC = 3, BC = 4.

Find the ratios sin A, sin B, cos A, tan B

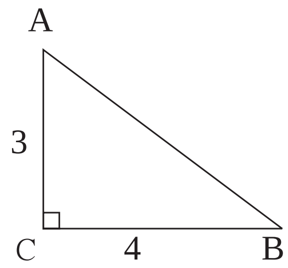


Fig. 8.22

Solution : In right angled Δ ACB, using Pythagoras' theorem,

$AB^2 = AC^2 + BC^2$

$= 3^2 + 4^2 = 5^2$

$\therefore$ AB = 5

$$\sin A = \frac{BC}{AB} = \frac{4}{5} \qquad \cos A = \frac{AC}{AB} = \frac{3}{5}$$

$$\text{and } \sin B = \frac{AC}{AB} = \frac{3}{5} \qquad \tan B = \frac{AC}{BC} = \frac{3}{4}$$

Ex. 4 In right angled triangle Δ PQR, $\angle Q = 90^\circ$, $\angle R = \theta$ and if $\sin \theta = \frac{5}{13}$ then find $\cos \theta$ and $\tan \theta$.

Solution :

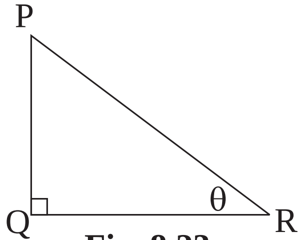


Fig. 8.23

In right angled Δ PQR, $\angle R = \theta$

$$\sin \theta = \frac{5}{13}$$

$$\therefore \frac{PQ}{PR} = \frac{5}{13}$$

$\therefore$ Let PQ = $5k$ and PR = $13k$

Let us find QR by using Pythagoras' theorem,

$$PQ^2 + QR^2 = PR^2$$
$$(5k)^2 + QR^2 = (13k)^2$$
$$25k^2 + QR^2 = 169\,k^2$$
$$QR^2 = 169\,k^2 - 25k^2$$
$$QR^2 = 144\,k^2$$
$$QR = 12k$$

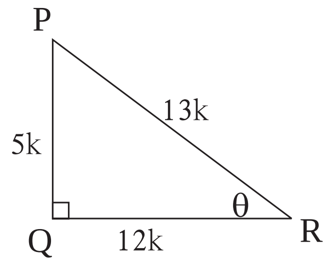


Fig. 8.24

Now, in right angled Δ PQR, PQ = $5k$, PR = $13k$ and QR = $12k$

$$\therefore \cos\theta = \frac{QR}{PR} = \frac{12k}{13k} = \frac{12}{13}, \ \tan\theta = \frac{PQ}{QR} = \frac{5k}{12k} = \frac{5}{12}$$

Use your brain power!

(1) While solving above example, why the lengths of PQ and PR are taken $5k$ and $13k$?

(2) Can we take the lengths of PQ and PR as 5 and 13 ? If so then what changes are needed in the writing of the solution.

Important Equation in Trigonometry

Δ PQR is a right angled triangle.

$\angle PQR = 90^\circ$, $\angle R = \theta$

$$\sin\theta = \frac{PQ}{PR} \quad \text{..........(I)}$$

and $$\cos\theta = \frac{QR}{PR} \quad \text{..........(II)}$$

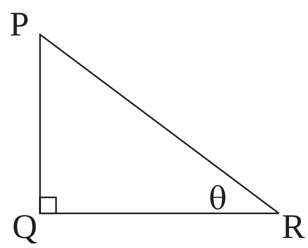


Fig.8.25

Using Pythagoras' theorem,

$PQ^2 + QR^2 = PR^2$

$$\therefore \frac{PQ^2}{PR^2} + \frac{QR^2}{PR^2} = \frac{PR^2}{PR^2}$$ .... dividing each term by PR^2

$$\therefore \left(\frac{PQ}{PR}\right)^2 + \left(\frac{QR}{PR}\right)^2 = 1$$

$\therefore (\sin\theta)^2 + (\cos\theta)^2 = 1$ ..from (I) & (II)

Remember this !

'Square of' sin θ means $(\sin\theta)^2$. It is written as $\sin^2\theta$.

We have proved the equation $\sin^2\theta + \cos^2\theta = 1$ using Pythagoras' theorem, where θ is an acute angle of a right angled triangle.

Verify that the equation is true even when $\theta = 0°$ or $\theta = 90°$

Since the equation $\sin^2\theta + \cos^2\theta = 1$ is true for any value of θ. So it is a basic trigonometrical identity.

(i) $0 \le \sin\theta \le 1$, $0 \le \sin^2\theta \le 1$ (ii) $0 \le \cos\theta \le 1$, $0 \le \cos^2\theta \le 1$

Practice set 8.2

1. In the following table, a ratio is given in each column. Find the remaining two ratios in the column and complete the table.

sin θ		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
cos θ	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
tan θ			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

2. Find the values of -

(i) $5\sin 30° + 3\tan 45°$

(ii) $\frac{4}{5}\tan^2 60° + 3\sin^2 60°$

(iii) $2\sin 30° + \cos 0° + 3\sin 90°$

(iv) $\frac{\tan 60}{\sin 60 + \cos 60}$

(v) $\cos^2 45° + \sin^2 30°$

(vi) $\cos 60° \times \cos 30° + \sin 60° \times \sin 30°$

3. If $\sin\theta = \frac{4}{5}$ then find $\cos\theta$

4. If $\cos\theta = \frac{15}{17}$ then find $\sin\theta$

Problem Set 8

1. Choose the correct alternative answer for following multiple choice questions.

(i) Which of the following statements is true ?

(A) $\sin\theta = \cos(90-\theta)$ (B) $\cos\theta = \tan(90-\theta)$

(C) $\sin\theta = \tan(90-\theta)$ (D) $\tan\theta = \tan(90-\theta)$

(ii) Which of the following is the value of $\sin 90^\circ$?

(A) $\frac{\sqrt{3}}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1

(iii) $2\tan 45^\circ + \cos 45^\circ - \sin 45^\circ =$?

(A) 0 (B) 1 (C) 2 (D) 3

(iv) $\frac{\cos 28^\circ}{\sin 62^\circ} =$?

(A) 2 (B) -1 (C) 0 (D) 1

2. In right angled Δ TSU, TS = 5, $\angle S = 90^\circ$, SU = 12 then find sin T, cos T, tan T. Similarly find sin U, cos U, tan U.

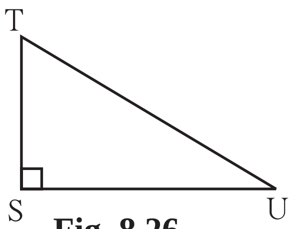


Fig. 8.26

3. In right angled Δ YXZ, $\angle X = 90^\circ$, XZ = 8 cm, YZ = 17 cm, find sin Y, cos Y, tan Y, sin Z, cos Z, tan Z.

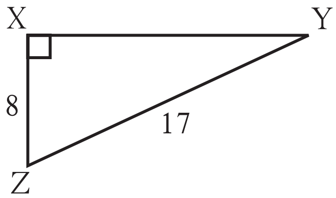


Fig. 8.27

4. In right angled Δ LMN, if $\angle N = \theta$, $\angle M = 90^\circ$, $\cos\theta = \frac{24}{25}$, find $\sin\theta$ and $\tan\theta$

Similarly, find $(\sin^2\theta)$ and $(\cos^2\theta)$.

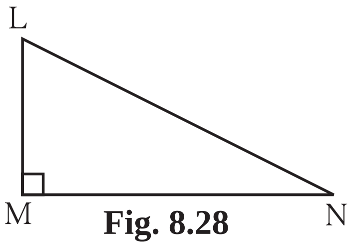


Fig. 8.28

5. Fill in the blanks.

(i) $\sin 20^\circ = \cos \boxed{}^\circ$

(ii) $\tan 30^\circ \times \tan \boxed{}^\circ = 1$

(iii) $\cos 40^\circ = \sin \boxed{}^\circ$

9 Surface Area and Volume

Let's Study

- Surface area of a cone
- Volume of a cone
- Surface area of a sphere
- Volume of a sphere

Let's recall.

We have learnt how to find the surface area and volume of a cuboid, a cube and a cylinder, in earlier standard.

Cuboid

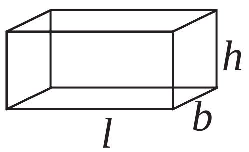


Fig.9.1

- Length, breadth and height of a cuboid are l , b, h respectively.

  (i) Area of vertical surfaces of a cuboid = $2(l + b) \times h$

  Here we have considered only 4 surfaces into consideration.

  (ii) Total surface area of a cuboid = $2(lb + bh + lh)$

  Here we have taken all 6 surfaces into consideration.

  (iii) Volume of a cuboid $= l \times b \times h$

Cube

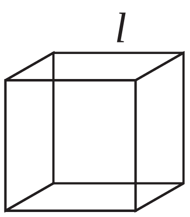


Fig.9.2

- If l is the edge of a cube,

  (i) Total surface area of a cube = $6l^2$

  (ii) Area of vertical surfaces of a cube = $4l^2$

  (iii) Volume of a cube = l^3

Cylinder

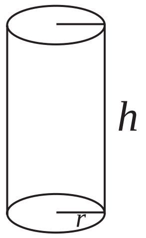


Fig.9.3

- Radius of cylinder is r and height is h.

  (i) Curved surface area of a cylinder = $2\pi rh$

  (ii) Total surface area of a cylinder = $2\pi r(r + h)$

  (iii) Volume of a cylinder = $\pi r^2 h$

Practice Set 9.1

1. Length, breadth and height of a cuboid shape box of medicine is 20cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box.
2. Total surface area of a box of cuboid shape is 500 sq. unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box ?
3. Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube.
4. Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube.
5. Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5m and 1.15m respectively. Find its length.
6. What will be the volume of a cube having length of edge 7.5 cm ?
7. Radius of base of a cylinder is 20cm and its height is 13cm, find its curved surface area and total surface area. ($\pi = 3.14$)
8. Curved surface area of a cylinder is 1980 cm^2 and radius of its base is 15cm. Find the height of the cylinder. ($\pi = \frac{22}{7}$).

Let's learn.

Terms related to a cone and their relation

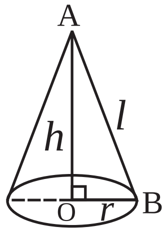


Fig.9.4

A cone is shown in the adjacent Fig.9.4. Centre of the circle, which is the base of the cone, is O and A is the vertex (apex) of the cone. Seg OB is a a radius and seg OA is perpendicular to the radius at O, means AO is perpendicular height of the cone. Slant height of the cone is the length of AB, which is shown by (l).

Δ AOB is a right angled triangle.

$\therefore$ by the Pythagoras' theorem

$$AB^2 = AO^2 + OB^2$$

$$\therefore l^2 = h^2 + r^2$$

That is, (slant height)^2 = (Perpendicular height)^2 + (Base radius)^2

Surface area of a cone

A cone has two surfaces : (i) circular base and (ii) curved surface.

Out of these two we can find the area of base of a cone because we know the formula for the area of a circle.

How to find the curved surface area of a cone ? How to derive a formula for it ?

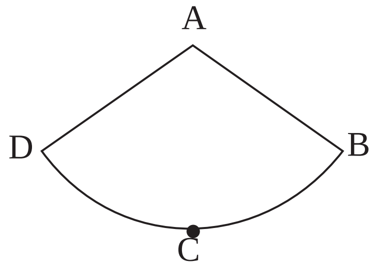


Fig.9.5

To find a formula for the curved surface area of a cone, let us see the net of the curved surface, which is a sector of a circle.

If a cone is cut along edge AB,we get its net as shown in fig.9.5.

Compare the figures 9.4 and 9.5

Have you noticed the following things ?

(i) Radius AB of the sector is the same as the slant height of the cones.

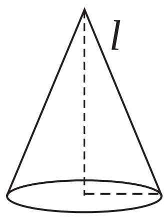
(ii) Arc BCD of the sector is the same as circumference of the base of the cone.

(iii) Curved surface area of cone = Area of sector A-BCD.

It means to find the curved surface area of a cone we have to find the area of its net that is the area of the sector.

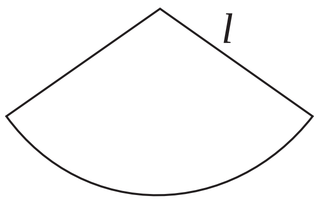
Try to understand, how it is done from the following activity.

Activity : Look at the following figures.



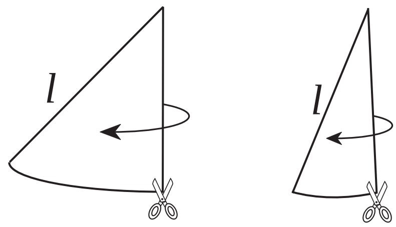
Cone

Fig. 9.6



Net of curved surface

Fig. 9.7



Pieces of the net

Fig. 9.8

Circumference of base of the circle $= 2\pi r$

As shown in the Fig.9.8, make pieces of the net as small as possible. Join them as shown in the Fig.9.9.

By Joining the small pieces of net of the cone, we get a rectangle ABCD approximately.

Total length of AB and CD is $2\pi r$.

$\therefore$ length of side AB of rectangle ABCD is πr and length of side CD is also πr.

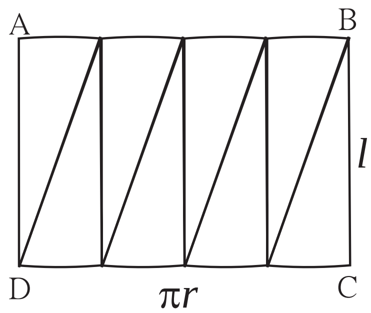


Fig. 9.9

Length of side BC of rectangle = slant height of cone $= l$.

Curved surface area of cone is equal to the area of the rectangle.

$\therefore$ curved surface area of cone = Area of rectangle $= AB \times BC = \pi r \times l = \pi rl$

Now, we can derive the formula for total surface area of a cone.

Total surface area of cone = Curved surface area + Area of base

$= \pi rl + \pi r^2$

$= \pi r(l + r)$

Did you notice a thing ? If a cone is not closed (Just like a cap of jocker or a cap in a birthday party) it will have only one surface, which is the curved surface. Then we get the surface area of the cone by the formula πrl.

Activity : Prepare a cylinder of a card sheet, keeping one of its faces open. Prepare an open cone of card sheet which will have the same base-radius and the same height as that of the cylinder.

Pour fine sand in the cone till it just fills up the cone. Empty the cone in the cylinder. Repeat the procedure till the cylinder is just filled up with sand. Note how many coneful of sand is required to fill up the cylinder.

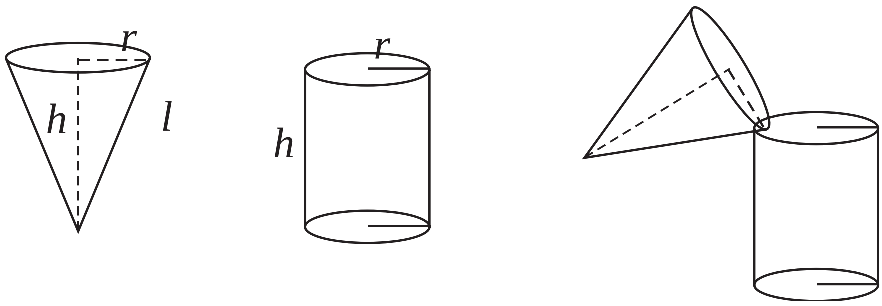


Fig. 9.10

To fill up the cylinder, three coneful of sand is required.

Let's learn.

Volume of a cone

If the base-radii and heights of a cone and a cylinder are equal then

$3 \times$ volume of cone = volume of cylinder

$\therefore$ $3 \times$ volume of cone $= \pi r^2 h$

$\therefore$ volume of cone $= \frac{1}{3} \times \pi r^2 h$

Remember this !

(i) Area of base of a cone $= \pi r^2$

(ii) Curved surface area of a cone $= \pi rl$

(iii) Total surface area of a cone $= \pi r (l + r)$

(iv) Volume of a cone $= \frac{1}{3} \times \pi r^2 h$

Solved Examples :

Ex. (1) Radius of base (r) and perpendicular height (h) of cone is given. Find its slant height (l)

(i) $r = 6$ cm, $h = 8$ cm, (ii) $r = 9$ cm, $h = 12$ cm

Solution :

(i) $r = 6$ cm, $h = 8$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (6)^2 + (8)^2$

$\therefore l^2 = 36 + 64$

$\therefore l^2 = 100$

$\therefore l = 10$ cm

(ii) $r = 9$ cm, $h = 12$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (9)^2 + (12)^2$

$\therefore l^2 = 81 + 144$

$\therefore l^2 = 225$

$\therefore l = 15$ cm

Ex. (2) Find (i) the slant height, (ii) the curved surface area and (iii) total surface area of a cone, if its base radius is 12 cm and height is 16 cm. ($\pi = 3.14$)

Solution :

(i) $r = 12$ cm, $h = 16$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (12)^2 + (16)^2$

$\therefore l^2 = 144 + 256$

$\therefore l^2 = 400$

$\therefore l = 20$ cm

(ii) Curved surface area $= \pi rl$

$= 3.14 \times 12 \times 20$

$= 753.6 \text{ cm}^2$

(iii) Total surface area of cone

$= \pi r (l + r)$

$= 3.14 \times 12(20+12)$

$= 3.14 \times 12 \times 32$

$= 1205.76 \text{ cm}^2$

Ex. (3) The total surface area of a cone is 704 sq.cm and radius of its base is 7 cm, find the slant height of the cone. ($\pi = \frac{22}{7}$)

Solution : Total surface area of cone $= \pi r (l + r)$

$$\therefore \quad 704 = \frac{22}{7} \times 7 (l + 7)$$

$$\therefore \quad \frac{704}{22} = l + 7$$

$$\therefore \quad 32 = l + 7$$

$$\therefore \quad 32 - 7 = l$$

$$\therefore \quad l = 25 \text{ cm}$$

Ex. (4) Area of the base of a cone is 1386 sq.cm and its height is 28 cm.

Find its surface area. ($\pi = \frac{22}{7}$)

Solution :

Area of base of cone $= \pi r^2$

$\therefore \quad 1386 = \frac{22}{7} \times r^2$

$\therefore \quad \frac{1386 \times 7}{22} = r^2$

$\therefore \quad 63 \times 7 = r^2$

$\therefore \quad 441 = r^2$

$\therefore \quad r = 21$ cm

$\therefore l^2 = (21)^2 + (28)^2$

$\therefore l^2 = 441 + 784$

$\therefore l^2 = 1225$

$\therefore l = 35$ cm

Surface area of cone $= \pi r l$

$= \frac{22}{7} \times 21 \times 35$

$= 22 \times 21 \times 5$

$= 2310$ sq. cm.

Practice set 9.2

1. Perpendicular height of a cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.
2. Find the volume of a cone, if its total surface area is 7128 sq.cm and radius of base is 28 cm. ($\pi = \frac{22}{7}$)
3. Curved surface area of a cone is 251.2 cm^2 and radius of its base is 8cm. Find its slant height and perpendicular height. ($\pi = 3.14$)
4. What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is Rs.10 per sq.m ?
5. Volume of a cone is 6280 cubic cm and base radius of the cone is 30 cm. Find its perpendicular height. ($\pi = 3.14$)
6. Surface area of a cone is 188.4 sq.cm and its slant height is 10cm. Find its perpendicular height ($\pi = 3.14$)
7. Volume of a cone is 1212 cm^3 and its height is 24cm. Find the surface area of the cone. ($\pi = \frac{22}{7}$)
8. The curved surface area of a cone is 2200 sq.cm and its slant height is 50 cm. Find the total surface area of cone. ($\pi = \frac{22}{7}$)
9. There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18m, find the volume of the tent.

10. In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1m. and diameter of base is 7.2 m. Find the volume of the fodder. if it is to be covered by polythin in rainy season then how much minimum polythin sheet is needed ?

($\pi = \frac{22}{7}$ and $\sqrt{17.37} = 4.17$.)

Let's learn.

Surface area of a sphere

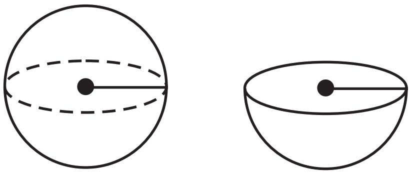


Fig. 9.11

Surface area of a sphere = $4\pi r^2$

$\therefore$ Surface area of a hollow hemisphere = $2\pi r^2$

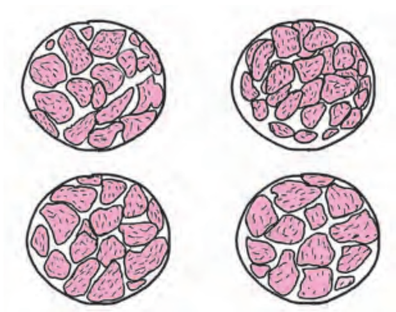
Total surface area of a solid hemisphere

= Surface area of hemisphere + Area of circle

= $2\pi r^2 + \pi r^2 = 3\pi r^2$

Take a sweet lime (Mosambe), Cut it into two equal parts.

Take one of the parts. Place its circular face on a paper. Draw its circular border. Copy three more such circles. Again, cut each half of the sweet lime into two equal parts.



Now you get 4 quarters of sweet lime. Separate the peel of a quarter part. Cut it into pieces as small as possible. Try to cover one of the circles drawn, by the small pieces.

Observe that the circle gets nearly covered.

The activity suggests that,

Curved surface area of a sphere = $4\pi r^2$

Solved Examples :

(1) Find the surface area of a sphere having radius 7 cm. ($\pi = \frac{22}{7}$)

Solution : Surface Area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (7)^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7$$

$$= 88 \times 7$$

$$= 616$$

Surface Area of sphere = 616 sq.cm.

(2) Find the radius of a sphere having surface area 1256sq.cm.($\pi = 3.14$)

Solution : Surface Area of Sphere $= 4\pi r^2$

$\therefore\ 1256 = 4 \times 3.14 \times r^2$

$\therefore\ r^2 = \frac{1256}{4 \times 3.14}$

$= \frac{31400}{314}$

$\therefore\ 100 = r^2$

$\therefore\ 10 = r$

$\therefore$ radius of the sphere is 10 cm.

Activity : Make a cone and a hemisphere of cardsheet such that radii of cone and hemisphere are equal and height of cone is equal to radius of the hemisphere. Fill the cone with fine sand. Pour the sand in the hemisphere. How many cones are required to fill the hemisphere completely ?

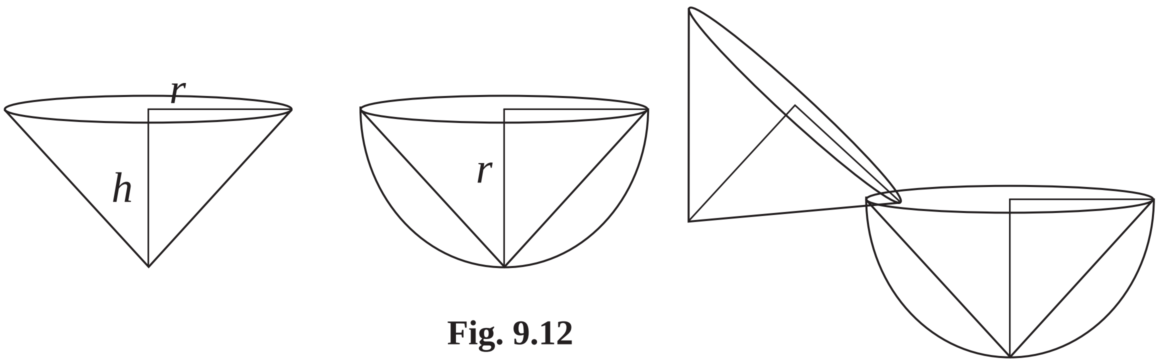


Fig. 9.12

Two conefull of sand is required to fill the hemisphere.

$\therefore\ 2 \times$ volume of cone = volume of hemisphere.

$\therefore$ volume of hemisphere $= 2 \times$ volume of cone

$$= 2 \times \frac{1}{3} \times \pi r^2 h$$

$$= 2 \times \frac{1}{3} \times \pi r^2 \times r$$

$$= \frac{2}{3}\ \pi r^3$$

$\therefore$ volume of sphere

$= 2 \times$ volume of hemisphere.

$$= \frac{4}{3}\pi r^3$$

$\therefore$ volume of sphere $= \frac{4}{3}\pi r^3$

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Remember this !

- Volume of hemisphere = $\frac{2}{3}\pi r^3$
- Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Solved Examples :

Ex. (1) Find the volume of a sphere having radius 21 cm. ($\pi = \frac{22}{7}$)

Solution : Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (21)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 88 \times 441$$

$\therefore$ volume of sphere = 38808 cubic cm.

Ex. (2) Find the radius of a sphere whose volume is 113040 cubic cm. ($\pi = 3.14$)

Solution : Volume of sphere = $\frac{4}{3}\pi r^3$

$$113040 = \frac{4}{3} \times 3.14 \times r^3$$

$$\frac{113040 \times 3}{4 \times 3.14} = r^3$$

$$\frac{28260 \times 3}{3.14} = r^3$$

$$\therefore 9000 \times 3 = r^3$$

$$\therefore r^3 = 27000$$

$$\therefore r = 30 \text{ cm}$$

$\therefore$ radius of sphere is 30 cm.

Ex. (3) Find the volume of a sphere whose surface area is 314.sq.cm. (Take $\pi = 3.14$)

Solution : Surface area of sphere = $4\pi r^2$

$$314 = 4 \times 3.14 \times r^2$$

$$\frac{314}{4 \times 3.14} = r^2$$

$$\frac{31400}{4 \times 314} = r^2$$

$$\therefore \frac{100}{4} = r^2$$

$$\therefore 25 = r^2$$

$$\therefore r = 5 \text{ cm}$$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times 5^3$$

$$= \frac{4}{3} \times 3.14 \times 125$$

$$= 523.33 \text{ cubic cm.}$$

Practice Set 9.3

1. Find the surface areas and volumes of spheres of the following radii.
   (i) 4 cm (ii) 9 cm (iii) 3.5 cm. ($\pi = 3.14$)
2. If the radius of a solid hemisphere is 5cm, then find its curved surface area and total surface area. ($\pi = 3.14$)
3. If the surface area of a sphere is 2826 cm^2 then find its volume. ($\pi = 3.14$)
4. Find the surface area of a sphere, if its volume is 38808 cubic cm. ($\pi = \frac{22}{7}$)
5. Volume of a hemisphere is 18000 π cubic cm. Find its diameter.

Problem set 9

1. If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 rotations ?
2. To make an open fish tank, a glass sheet of 2 mm gauge is used. The outer length, breadth and height of the tank are 60.4 cm, 40.4 cm and 40.2 cm respectively. How much maximum volume of water will be contained in it ?
3. If the ratio of radius of base and height of a cone is 5:12 and its volume is 314 cubic metre. Find its perpendicular height and slant height ($\pi = 3.14$)
4. Find the radius of a sphere if its volume is 904.32 cubic cm. ($\pi = 3.14$)
5. Total surface area of a cube is 864 sq.cm. Find its volume.
6. Find the volume of a sphere, if its surface area is 154 sq.cm.
7. Total surface area of a cone is 616 sq.cm. If the slant height of the cone is three times the radius of its base, find its slant height.
8. The inner diameter of a well is 4.20 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering it from inside at the rate Rs.52 per sq.m.
9. The length of a road roller is 2.1m and its diameter is 1.4m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of Rs. 7 per sq. m.

Answers

1. Basic Concepts in Geometry

Practice set 1.1

1. (i) 3 (ii) 3 (iii) 7 (iv) 1
(v) 3 (vi) 5 (vii) 2 (viii) 7

2. (i) 6 (ii) 8 (iii) 10 (iv) 1 (v) 3 (vi) 12

3. (i) P-R-Q (ii) Non collinear (iii) A-C-B (iv) Non collinear
(v) X-Y-Z (vi) Non collinear

4. 18 and 2 **5.** 25 and 9 **6.** (i) 4.5 (ii) 6.2 (iii) $2\sqrt{7}$ **7.** Triangle

Practice set 1.2

1. (i) No (ii) No (iii) Yes **2.** 4 **3.** 5 **4.** BP < AP < AB

5. (i) Ray RS or Ray RT (ii) Ray PQ (iii) Ray QR (iv) Ray QR and Ray RQ etc.
(v) Ray RQ and Ray RT etc.. (vi) Ray SR , Ray ST etc.. (vii) Point S

6. (i) Point A & Point C, Point D & Point P (ii) Point L & Point U, Point P & Point R
(iii) d (U,V) = 10, d (P,C) = 6 , d (V,B) = 3 , d (U,L) = 2

Practice set 1.3

1. (i) If a quadrilateral is a parallelogram then opposite angles of that quadrilateral are congruent.
(ii) If quadrilateral is a rectangle then diagonals are congruent.
(iii) If a triangle is an isosceles then segment joining vertex of a triangle and mid point of the base is perpendicular to the base

2. (i) If alternate angles made by two lines and its transversal are congruent then the lines are parallel.
(ii) If two parallel lines are intersected by a transversal the interior angles so formal are supplementary.
(iii) If the diagonals of a quaddrilateral are congruent then that quadrilateral is rectangle.

Problem set 1

1. (i) A (ii) C (iii) C (iv) C (v) B
2. (i) False (ii) False (iii) True (iv) False
3. (i) 3 (ii) 8 (iii) 9 (iv) 2 (v) 6 (vi) 22 (vii) 165
4. –15 and 1 **5.** (i) 10.5 (ii) 9.1 **6.** –6 and 8

2. Parallel Lines

Practice set 2.1

1. (i) 95° (ii) 95° (iii) 85° (iv) 85°

2. $\angle a = 70°$, $\angle b = 70°$, $\angle c = 115°$, $\angle d = 65°$

3. $\angle a = 135°$, $\angle b = 135°$, $\angle c = 135°$

5. (i) 75° (ii) 75° (iii) 105° (iv) 75°

Practice set 2.2

1. No. **4.** ∠ABC = 130°

Problem set 2

1. (i) C (ii) C (iii) A (iv) B (v) C **4.** $x = 130°$ $y = 50°$

5. x = 126^{0}° **6.** f = 100° $g = 80°$

3. Triangles

Practice set 3.1

1. 110° **2.** 45° **3.** 80°, 60°, 40° **4.** 30°, 60°, 90°

5. 60°, 80°, 40° **6.** ∠DRE = 70°, ∠ARE = 110°

7. ∠AOB = 125° **9.** 30°, 70°, 80°

Practice set 3.2

1. (i) SSC Test (ii) SAS Test (iii) ASA Test (iv) Hypotenuse Side Test.

2. (i) ASA Test, ∠BAC ≅ ∠QPR ,side AB ≅ side PQ, side AC ≅ side PR
(ii) SAS Test, ∠TPQ ≅ ∠TSR, ∠TQP ≅ ∠TRS, side PQ ≅ side SR

3. Hypotenuse Side Test, ∠ACB ≅ ∠QRP, ∠ABC ≅ ∠QPR, side AC ≅ side QR

4. SSS Test, ∠MLN ≅ ∠MPN, ∠LMN ≅ ∠MNP,∠LNM ≅ ∠PMN

Practice set 3.3

1. $x = 50°$, $y = 60°$, $m\angle ABD = 110°$, $m\angle ACD = 110°$.

2. 7.5 Units **3.** 6.5 Units **4.** l(PG) = 5 cm , l(PT) = 7.5 cm

Practice set 3.4

1. 2 cm **2.** 28° **3.** ∠QPR, ∠PQR **4.** greatest side NA, smallest side FN

Practice set 3.5

1. $\frac{XY}{LM} = \frac{YZ}{MN} = \frac{XZ}{LN}$, ∠X ≅ ∠L, ∠Y ≅ ∠M, ∠Z ≅ ∠N

2. l(QR) = 12 cm, l(PR) = 10 cm

Problem set 3

1. (i) D (ii) B (iii) B

5. Quadrilaterals

Practice set 5.1

1. m$\angle$XWZ = 135°, m$\angle$YZW = 45°, l (WY) = 10 cm
2. x = 40°, $\angle$C = 132°, $\angle$D = 48°
3. 25 cm, 50 cm, 25 cm, 50 cm
4. 60°, 120°, 60°, 120°
6. $\angle$A = 70°, $\angle$B = 110°, $\angle$C = 70°, $\angle$R = 110°

Practice set 5.3

1. BO = 4 cm, $\angle$ACB = 35°
2. QR = 7.5 cm, $\angle$PQR = 105°, $\angle$SRQ = 75°
3. $\angle$IMJ = 90°, $\angle$JIK = 45°, $\angle$LJK = 45°
4. side = 14.5 cm, Perimetere = 58 cm
5. (i) False (ii) False (iii) True (iv) True (v) True (vi) False

Practice set 5.4

1. $\angle$J = 127°, $\angle$L = 72° **2.** $\angle$B = 108°, $\angle$D = 72°

Practice set 5.5

1. XY = 4.5 cm,, YZ = 2.5 cm,, XZ = 5.5 cm

Problem set 5

1. (i) D (ii) C (iii) D **2.** 25 cm, **3.** $6.5\sqrt{2}$ cm
4. 24 cm, 32 cm, 24 cm, 32 cm **5.** PQ = 26 cm **6.** $\angle$MPS = 65°

6. Circle

Practice set 6.1

1. 20 cm **2.** 5 cm **3.** 32 unit **4.** 9 unit

Practice set 6.2

1. 12 cm **2.** 24 cm

Problem set 6

1. (i) A (ii) C (iii) A (iv) B (v) D (vi) C (vii) D **2.** 2:1 **4.** 24 units

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7. Co - ordinate Geometry

Practice set 7.1

1. point A : Quadrant II, point B : Quadrant III, point K : Quadrant I, point D : Quadrant I
point E : Quadrant I, point F : Quadrant IV, point G : Quadrant IV, point H : Y-Axis.
point M : X-Axis, point N : Y-Axis, point P : Y-Axis, point Q : Quadrant III

2. (i) Quadrant I (ii) Quadrant III (iii) Quadrant IV (iv) Quadrant II

Practice set 7.2

1. Square **2.** $x = -7$ **3.** $y = -5$ **4.** $x = -3$ **5.** 4

6. (i) Y-Axis, (ii) X-axis, (iii) Y-axis, (iv) X-axis,

7. To X-axis (5,0) , To Y-axis (0,5)

8. (-4,1), (-1.5, 1), (-1.5,5), (-4,5)

Problem set 7

1. (i) C (ii) A (iii) B (iv) C (v) C (vi) B

2. (i) Q (2,2), R(4,–1) (ii) T(0,–1), M(3,0) (iii) point S (iv) point O

3. (i) Quadrant IV (ii) Quadrant III
(iii) Quadrant II (iv) Quadrant II (v) Y-axis (vi) X-axis

5. (i) 3 (ii) P(3,2), Q(3,-1), R (3,0) (iii) 0 **6.** . $y = 5, y = -5$ **7.** $|a|$

8. Trigonometry

Practice set 8.1

1. (i) $\frac{QR}{PQ}$ (ii) $\frac{QR}{PQ}$ (iii) $\frac{QR}{PR}$ (iv) $\frac{PR}{QR}$

2. (i) $\frac{a}{c}$ (ii) $\frac{b}{a}$ (iii) $\frac{b}{c}$ (iv) $\frac{a}{b}$

3. (i) $\frac{MN}{LN}$ (ii) $\frac{LM}{LN}$ (iii) $\frac{LM}{MN}$ (iv) $\frac{MN}{LN}$

4. (i) $\frac{PQ}{PR}, \frac{RQ}{PR}, \frac{PQ}{RQ}$ (ii) $\frac{QS}{PS}, \frac{PQ}{PS}, \frac{QS}{PQ}$

Practice set 8.2

1. $\sin\theta : \frac{12}{37}, \frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{21}{29}, \frac{8}{17}, \frac{1}{3}$; $\cos\theta : \frac{60}{61}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, \frac{20}{29}, \frac{15}{17}, \frac{4}{5}, \frac{2\sqrt{2}}{3}$

$\tan\theta : \frac{12}{35}, \frac{11}{60}, \frac{1}{\sqrt{3}}, \sqrt{2}, \frac{3}{4}$

2. (i) $\frac{11}{2}$ (ii) $\frac{93}{20}$ (iii) 5 (iv) $\frac{2\sqrt{3}}{\sqrt{3}+1}$ (v) $\frac{3}{4}$ (vi) $\frac{\sqrt{3}}{2}$ **3.** $\frac{3}{5}$ **4.** $\frac{8}{17}$

Problem set 8

1. (i) A (ii) D (iii) C (iv) D

2. $\sin T = \frac{12}{13}$, $\cos T = \frac{5}{13}$, $\tan T = \frac{12}{5}$, $\sin U = \frac{5}{13}$, $\cos U = \frac{12}{13}$, $\tan U = \frac{5}{12}$

3. $\sin Y = \frac{8}{17}$, $\cos Y = \frac{15}{17}$, $\tan Y = \frac{8}{15}$, $\sin Z = \frac{15}{17}$, $\cos Z = \frac{8}{17}$, $\tan Z = \frac{15}{8}$

4. $\sin \theta = \frac{7}{25}$, $\tan \theta = \frac{7}{24}$, $\sin^2 \theta = \frac{49}{625}$, $\cos^2 \theta = \frac{576}{625}$

5. (i) 70 (ii) 60 (iii) 50

9. Surface Area and Volume

Practice set 9.1

1. 640 sq.cm, 1120 sq.cm. **2.** 20 Unit **3.** 81 sq.cm, 121.50 sq.cm.

4. 3600 sq.cm. **5.** 20 m **6.** 421.88 cubic cm

7. 1632.80 sq.cm, 4144.80 sq.cm. **8.** 21 cm

Practice set 9.2

1. 5 cm **2.** 36960 cubic cm. **3.** 10 cm, 6 cm **4.** ₹ 2640

5. 15 cm **6.** 8 cm **7.** 550 sq.cm **8.** 2816 sq.cm, 9856 cubic cm

9. 600 cubic metre **10.** 28.51 cubic metre, 47.18 sq.m.

Practice Set 9.3

1. (i) 200.96 sq.cm, 267.95 cubic cm. (ii) 1017.36 sq.cm, 3052.08 cubic cm.
(iii) 153.86 sq.m, 179.50 cubic cm.

2. 157 sq.cm, 235.5 sq.cm. **3.** 14130 cubic cm. **4.** 5544 sq.cm. **5.** 60 cm

Problem set 9

1. 1980 sq.m. **2.** 96801.6 cubic cm. **3.** 12 m, 13 m

4. 6 cm **5.** 1728 cubic cm. **6.** 179.67 cubic cm.

7. 21 cm **8.** 132 sq.m., ₹ 6864 **9.** 4620 sq.m, ₹ 32340

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

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