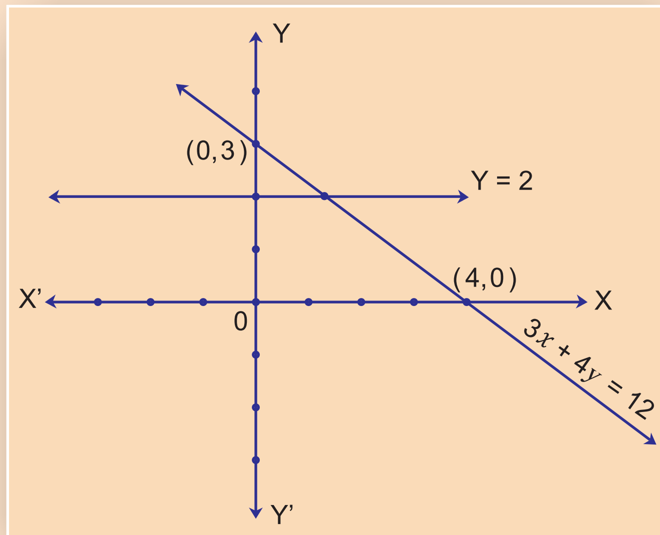
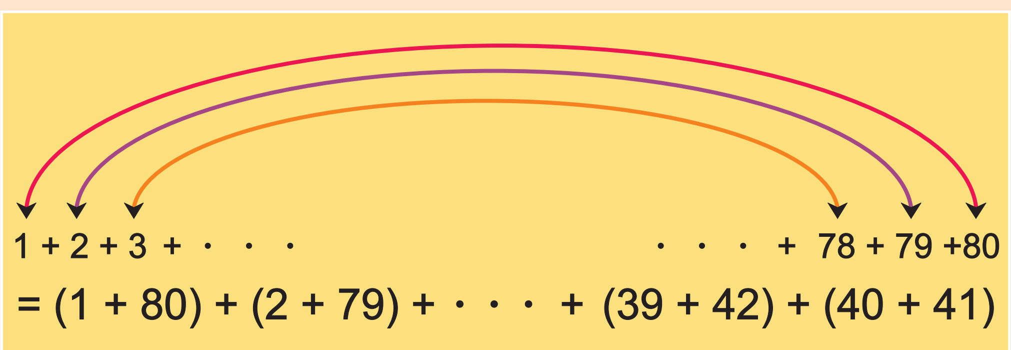
Mathematics

Part - I

STANDARD TEN





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Mathematics

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The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens :

JUSTICE, social, economic and political ;

LIBERTY of thought, expression, belief, faith and worship ;

EQUALITY of status and of opportunity ; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation ;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

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Preface

Dear Students,

Welcome to the tenth standard!

This year you will study two text books - Mathematics Part-I and Mathematics Part-II

The main areas in the book Mathematics Part-I are Algebra, Graph, Financial planning and Statistics. All of these topics were introduced in the ninth standard. This year you will study some more details of the same. The new tax system, GST is introduced in Financial planning. Wherever a new unit, formula or application is introduced, its lucid explanation is given. Each chapter contains illustrative solved examples and sets of questions for practice. In addition, some questions in practice sets are star-marked, indicating that they are challenging for talented students. After tenth standard, some students do not opt for Mathematics. This book gives them basic concepts and mathematics needed to work in other fields. The matter under the head 'For more information' is useful for those students who wish to study mathematics after tenth standard and achieve proficiency in it. So they are earnestly advised to study it. Read the book thoroughly at least once and understand the concepts.

Additional useful audio-visual material regarding each lesson will be available to you by using Q.R. code through 'App'. It will definitely be useful to you for your studies.

Much importance is given to the tenth standard examination. You are advised not to take the stress and study to the best of your ability to achieve expected success. Best wishes for it !

(Dr. Sunil Magar)
Director
Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

Pune
Date : 18 March 2018, Gudhipadva
Indian Solar Year : 27 Falgun 1939

It is expected that students will develop the following competencies after studying Mathematics– Part I syllabus in standard X

Area	Topic	Competency Statements
1. Knowledge of numbers	1.1 Arithmetic Progression	The students will be able to- • solve examples using Arithmetic Progression. • plan steps to achieve a goal in future.
2. Algebra	2.1 Quadratic Equations 2.2 Linear equations in two variables	• solve day to day problems which can be expressed in the form of quadratic equations. • decide the number of variables required to find solutions of word problems. • convert a word problem into an equation in two variables and find its solution.
3. Commercial Mathematics	3.1 Financial planning	• understand the concepts of savings and investments. • get familiar with financial transactions in business, profession etc.
4. Statistics and Probability	4.1 Probability 4.2 Graph and measures of central tendencies	• use the concept of probaility in games, voting etc. • present the collected data in the form of graphs or pictures deciding the suitable form of presentation. • find the mean, median and mode of a provided classified data.

Instructions for Teachers

Read the book in detail and understand the content thoroughly.Take the help of activities to explain different topics, to verify the formulae etc.

Practicals is also a means of evaluation. Activities given can also be used for this purpose. Encourage the students to think independently.Compliment a student if he solves an example by a different and logically correct method.

List of some practicals (Specimen)

1. On a graph paper, draw a line parallel to the X- axis or Y- axis. Write coordinates of any four points on the line. Write how the equation of the line can be obtained from the coordinates.

   [Instead of parallel lines, lines passing through the origin or intersecting the X or Y- axis can also be considered]

2. Bear a two- digit number in mind. Without disclosing it, cunstruct a puzzle. Create two algebraic relations between the two digits of the number and solve the puzzle.

   [The above practical can be extended to a three-digit number also.]

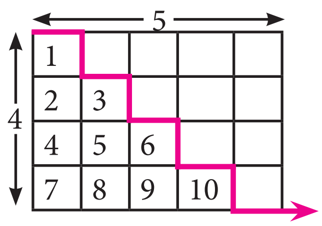
3. Read the information about contents on a food packet. Show the information by a pie diagram. For example, see the chart of contents like carbohydrates, proteins, vitamines etc. per given weight on a buiscuit packet. Show the proportion of the contents by a pie diagram. The contents can be divided into four classes as carbohydrates, proteins, fats and others.

4. Prepare a frequncy distribution table given by the teacher in Excel sheet on a computer. From the table draw a frequency polygon and a histogram in Excel.

5. Roll a die ten times and record the outcomes in the form of a table.

6. Observe the tax invoice given by your teacher. Record all of its contents. Re-calculate the taxes and verify their correctness.

7. Calculate the sum of first n natural numbers given by your teacher through the following activity. For example to find the sum of first four natural numbers, take a square-grid piece of paper of 4 × 5 squares. Then cut it as shown in the figure. Hence verify the formula $S_n = \frac{n(n+1)}{2}$ (Here $n = 4$)



$S_n = \frac{n(n+1)}{2}$ $\therefore S_4 = \frac{4\ (4+1)}{2} = \frac{4\times5}{2} = \frac{20}{2} = 10$

[Note: Here $a = 1$ and $d = 1$. The activity can be done taking different values of a and d. Similarly, you can find the sum of even or odd numbers, cubes of natural numbers etc.]

8. Write $\alpha = 6$ on one side of a card sheet and $\alpha = -6$ on its backside. Similarly, write $\beta = -3$ on one side of another card sheet and $\beta = 7$ on its backside. From these values, form different values of $(\alpha + \beta)$ and $(\alpha\beta)$; using these values form quadratic equations.

INDEX

1 Linear Equations in Two Variables

Let's study.

- Methods of solving linear equations in two variables – graphical method, Cramer's method
- Equations that can be transformed in linear equation in two variables
- Application of simultaneous equations

Let's recall.

Linear equation in two variables

An equation which contains two variables and the degree of each term containing variable is one, is called a linear equation in two variables.

$ax + by + c = 0$ is the general form of a linear equation in two variables; a, b, c are real numbers and a, b are not equal to zero at the same time.

Ex. $3x - 4y + 12 = 0$ is the general form of equation $3x = 4y - 12$

Activity : Complete the following table

No.	Equation	Is the equation a linear equation in 2 variables ?
1	$4m + 3n = 12$	Yes
2	$3x^2 - 7y = 13$	
3	$\sqrt{2}x - \sqrt{5}y = 16$	
4	$0x + 6y - 3 = 0$	
5	$0.3x + 0y - 36 = 0$	
6	$\frac{4}{x} + \frac{5}{y} = 4$	
7	$4xy - 5y - 8 = 0$	

Simultaneous linear equations

When we think about two linear equations in two variables at the same time, they are called simultaneous equations.

Last year we learnt to solve simultaneous equations by eliminating one variable. Let us revise it.

Ex. (1) Solve the following simultaneous equations.

(1) $5x - 3y = 8;\ 3x + y = 2$

Solution :

Method I : $5x - 3y = 8 \ldots$ (I)

$3x + y = 2 \ldots$ (II)

Multiplying both sides of equation (II) by 3.

$9x + 3y = 6 \ldots$ (III)

$5x - 3y = 8 \ldots$ (I)

Now let us add equations (I) and (III)

$$\begin{array}{r} 5x - 3y = 8 \\ + \ 9x + 3y = 6 \\ \hline 14x = 14 \end{array}$$

$\therefore x = 1$

substituting $x = 1$ in equation (II)

$3x + y = 2$

$\therefore 3 \times 1 + y = 2$

$\therefore 3 + y = 2$

$\therefore \quad y = -1$

solution is $x = 1, y = -1$; it is also written as $(x, y) = (1, -1)$

Method **(II)**

$5x - 3y = 8 \ldots$ (I)

$3x + y = 2 \ldots$ (II)

Let us write value of y in terms of x from equation (II) as

$y = 2 - 3x \quad \ldots$ (III)

Substituting this value of y in equation (I).

$5x - 3y = 8$

$\therefore 5x - 3(2 - 3x) = 8$

$\therefore 5x - 6 + 9x = 8$

$\therefore 14x - 6 = 8$

$\therefore 14x = 8 + 6$

$\therefore 14x = 14$

$\therefore \quad x = 1$

Substituting $x = 1$ in equation (III).

$y = 2 - 3x$

$\therefore y = 2 - 3 \times 1$

$\therefore y = 2 - 3$

$\therefore y = -1$

$x = 1, y = -1$ is the solution.

Ex. (2) Solve : $3x + 2y = 29;\ 5x - y = 18$

Solution : $3x + 2y = 29 \ldots$ (I) and $5x - y = 18 \ldots$ (II)

Let's solve the equations by eliminating 'y'. Fill suitably the boxes below.

Multiplying equation (II) by 2.

$\therefore\ 5x \times \square - y \times \square = 18 \times \square$

$\therefore\ 10x - 2y = \square \ldots$ (III)

Let's add equations (I) and (III)

$$\begin{array}{r} 3x + 2y = 29 \\ + \quad \square - \square = \square \\ \hline \square = \square \end{array} \qquad \therefore\ x = \square$$

Substituting $x = 5$ in equation (I)

$3x + 2y = 29$

$\therefore\ 3 \times \square + 2y = 29$

$\therefore\ \square + 2y = 29$

$\therefore\ 2y = 29 - \square$

$\therefore\ 2y = \square \qquad \therefore\ y = \square$

$(x, y) = (\square, \square)$ is the solution.

Ex. (3) Solve : $15x + 17y = 21;\ 17x + 15y = 11$

Solution : $15x + 17y = 21 \ldots$ (I)

$17x + 15y = 11 \ldots$ (II)

In the two equations above, the coefficients of x and y are interchanged. While solving such equations we get two simple equations by adding and subtracting the given equations. After solving these equations, we can easily find the solution.

Let's add the two given equations.

$$\begin{array}{r} 15x + 17y = 21 \\ + \quad 17x + 15y = 11 \\ \hline 32x + 32y = 32 \end{array}$$

Dividing both sides of the equation by 32.

$x + y = 1$ . . . (III)

Now, let's subtract equation (II) from (I)

$$\begin{array}{r} 15x + 17y = 21 \\ - \quad \underset{-}{17x} \underset{-}{+} 15y = \underset{-}{11} \\ \hline -2x + 2y = 10 \end{array}$$

dividing the equation by 2.

$-x + y = 5$ . . . (IV)

Now let's add equations (III) and (V).

$$\begin{array}{r} x + y = 1 \\ + \quad -x + y = 5 \\ \hline \therefore \; 2y = 6 \end{array} \qquad \therefore \; y = 3$$

Place this value in equation (III).

$x + y = 1$

$\therefore \; x + 3 = 1$

$\therefore \; x = 1 - 3 \qquad \therefore \; x = -2$

$(x, y) = (-2, 3)$ is the solution.

Practice Set 1.1

(1) Complete the following activity to solve the simultaneous equations.

$5x + 3y = 9$ -----(I)

$2x + 3y = 12$ ----- (II)

Let's add eqations (I) and (II).

$$\begin{array}{r} 5x + 3y = 9 \\ + \quad 2x - 3y = 12 \\ \hline \square \; x = \square \end{array}$$

$x = \dfrac{\square}{\square} \qquad x = \square$

Place $x = 3$ in equation (I).

$5 \times \square + 3y = 9$

$3y = 9 - \square$

$3y = \square$

$y = \dfrac{\square}{3}$

$y = \square$

$\therefore$ Solution is $(x, y) = (\square, \square)$.

2. Solve the following simultaneous equations.

(1) $3a + 5b = 26$; $a + 5b = 22$ (2) $x + 7y = 10$; $3x - 2y = 7$

(3) $2x - 3y = 9$; $2x + y = 13$ (4) $5m - 3n = 19$; $m - 6n = -7$

(5) $5x + 2y = -3$; $x + 5y = 4$ (6) $\frac{1}{3}x + y = \frac{10}{3}$; $2x + \frac{1}{4}y = \frac{11}{4}$

(7) $99x + 101y = 499$; $101x + 99y = 501$

(8) $49x - 57y = 172$; $57x - 49y = 252$

Let's recall.

Graph of a linear equation in two variables

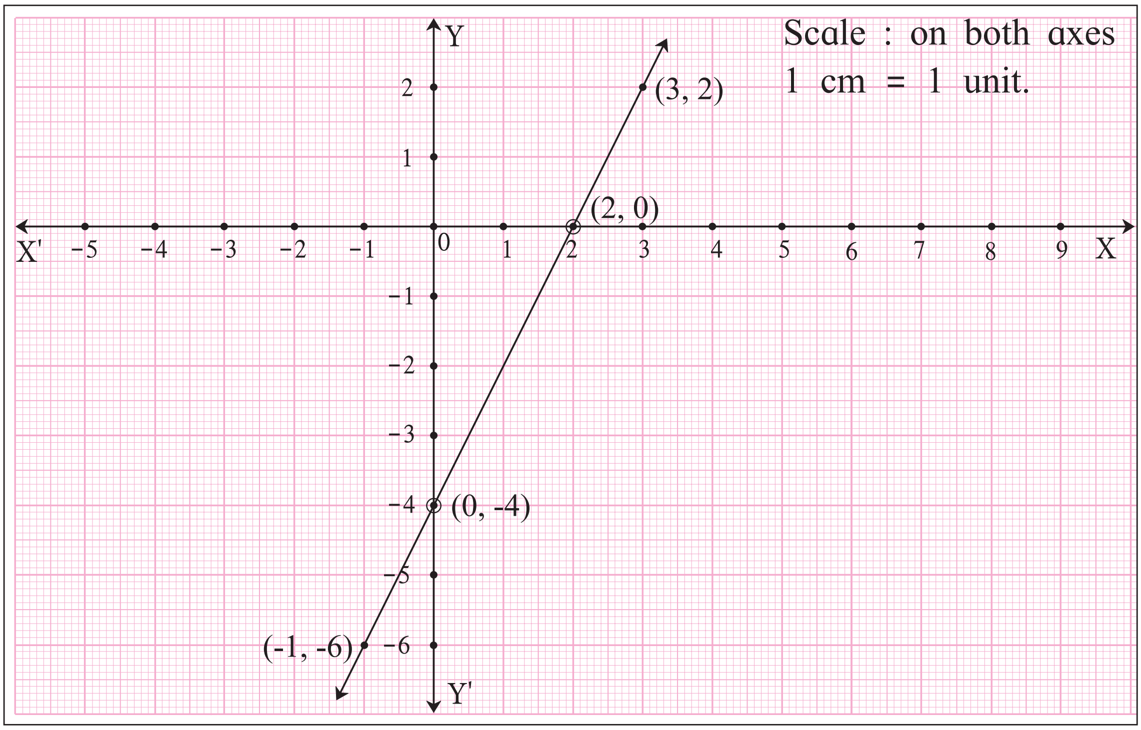
In the 9th standard we learnt that the graph of a linear equation in two variables is a straight line. The ordered pair which satisfies the equation is a solution of that equation. The ordered pair represents a point on that line.

Ex. Draw graph of $2x - y = 4$.

Solution : To draw a graph of the equation let's write 4 ordered pairs.

x	0	2	3	-1
y	-4	0	2	-6
(x, y)	$(0, -4)$	$(2, 0)$	$(3, 2)$	$(-1, -6)$

To obtain ordered pair by simple way let's take $x = 0$ and then $y = 0$.



Steps to follow for drawing a graph of linear equation in two variables.

Find at least 4 ordered pairs for given equation

↓

Draw X- axis, Y-axis on graph paper and plot the points

↓

See that all 4 points lie on one line

Two points are sufficient to represent a line, but if co-ordinates of one of the two points are wrong then you will not get a correct line.

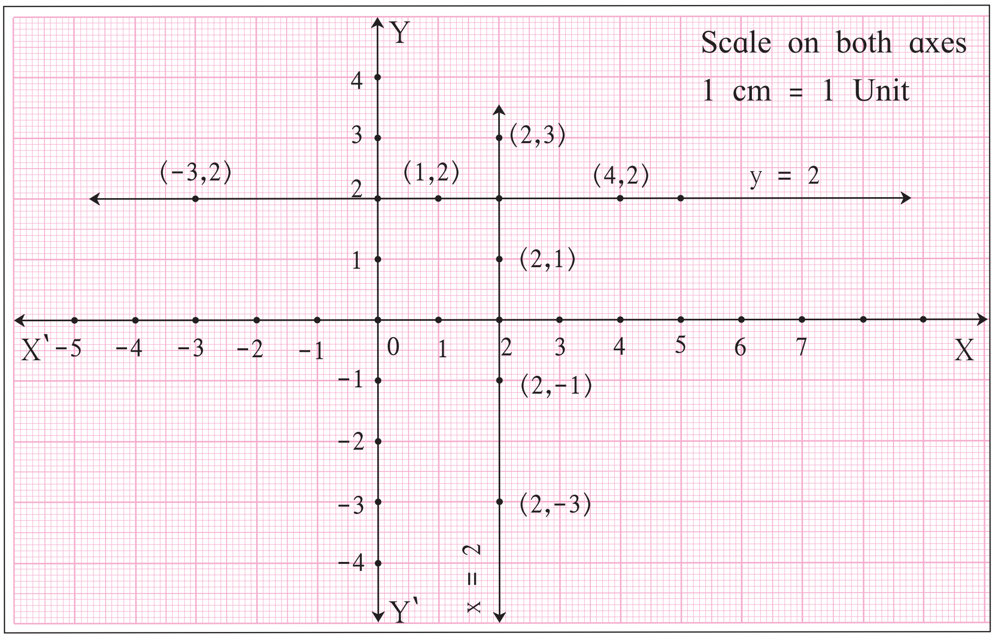
If you plot three points and if they are non collinear then it is understood that one of the points is wrongly plotted. But it is not easy to identify the incorrect point.

If we plot four points, it is almost certain that three of them will be collinear.

A linear equation $y = 2$ is also written as $0x + y = 2$. The graph of this line is parallel to X- axis; as for any value of x, y is always 2.

x	1	4	-3
y	2	2	2
(x, y)	(1, 2)	(4, 2)	(-3, 2)

Similarly equation $x = 2$ is written as $x + 0y = 2$ and its graph is parallel to Y-axis.



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Let's learn.

Graphical method

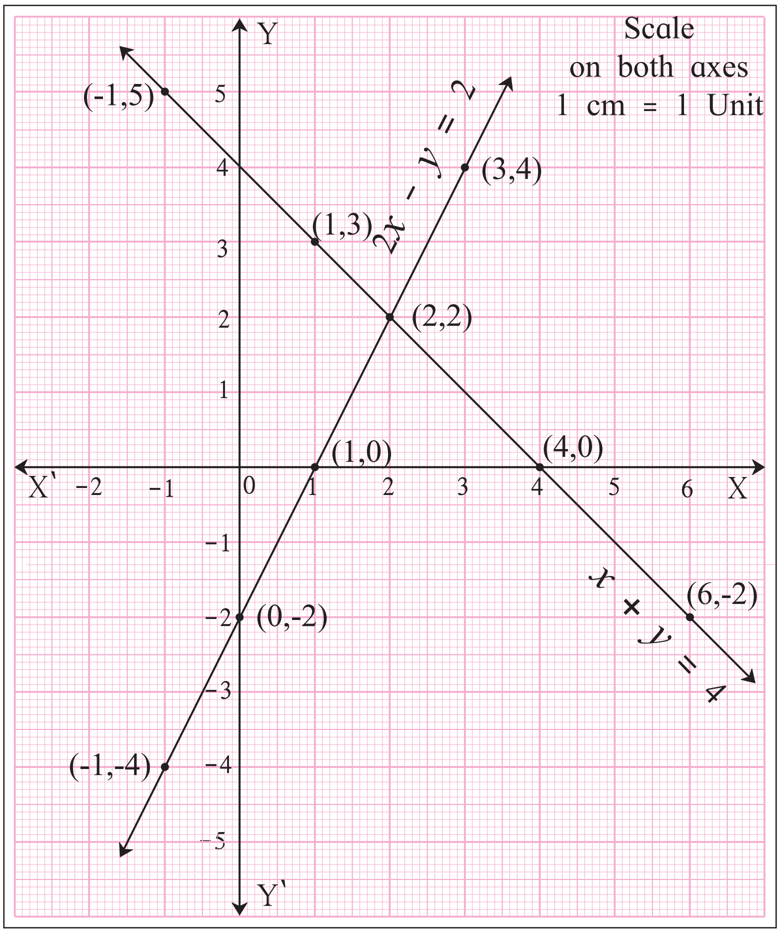
Ex. Let's draw graphs of $x + y = 4$, $2x - y = 2$ and observe them.

$x + y = 4$

x	-1	4	1	6
y	5	0	3	-2
(x,y)	$(-1,5)$	$(4,0)$	$(1,3)$	$(6,-2)$

$2x - y = 2$

x	0	1	3	-1
y	-2	0	4	-4
(x,y)	$(0,-2)$	$(1,0)$	$(3,4)$	$(-1,-4)$



Each point on the graph satisfies the equation. The two lines intersect each other at (2, 2).

Hence ordered pair (2, 2) i.e. $x = 2$, $y = 2$ satisfies the equations $x + y = 4$ and $2x - y = 2$.

The values of variables that satisfy the given equations, give the solution of given equations.

$\therefore$ the solution of given equations $x + y = 4$, $2x - y = 2$ is $x = 2$, $y = 2$.

Let's solve these equations by method of elimination.

$x + y = 4$ . . . (I)

$2x - y = 2$ . . . (II)

Adding equations (I) and (II) we get,

$3x = 6 \therefore x = 2$

substituting this value in equation (I)

$x + y = 4$

$\therefore 2 + y = 4$

$\therefore y = 2$

Activity (I) : Solve the following simultaneous equations by graphical method.

- Complete the following tables to get ordered pairs.

$x - y = 1$

x	0		3	
y		0		-3
(x, y)				

$5x - 3y = 1$

x	2			-4
y		8	-2	
(x, y)				

- Plot the above ordered pairs on the same co-ordinate plane.
- Draw graphs of the equations. • Note the co-ordinates of the point of intersection of the two graphs. Write solution of these equations.

Activity II : Solve the above equations by method of elimination. Check your solution with the solution obtained by graphical method.

Let's think.

The following table contains the values of x and y co-ordinates for ordered pairs to draw the graph of $5x - 3y = 1$

x	0	$\frac{1}{5}$	1	-2
y	$-\frac{1}{3}$	0	$\frac{4}{3}$	$-\frac{11}{3}$
(x, y)	$\left(0, -\frac{1}{3}\right)$	$\left(\frac{1}{5}, 0\right)$	$\left(1, \frac{4}{3}\right)$	$\left(-2, -\frac{11}{3}\right)$

- Is it easy to plot these points ? • Which precaution is to be taken to find ordered pairs so that plotting of points becomes easy ?

Practice Set 1.2

1. Complete the following table to draw graph of the equations –

(I) $x + y = 3$ (II) $x - y = 4$

$x + y = 3$

x	3	□	□
y	□	5	3
(x, y)	(3, 0)	□	(0, 3)

$x - y = 4$

x	□	-1	0
y	0	□	-4
(x, y)	□	□	(0, -4)

2. Solve the following simultaneous equations graphically.

(1) $x + y = 6$; $x - y = 4$
(2) $x + y = 5$; $x - y = 3$
(3) $x + y = 0$; $2x - y = 9$
(4) $3x - y = 2$; $2x - y = 3$
(5) $3x - 4y = -7$; $5x - 2y = 0$
(6)★ $2x - 3y = 4$; $3y - x = 4$

Let's discuss.

To solve simultaneous equations $x + 2y = 4$; $3x + 6y = 12$ graphically, following are the ordered pairs.

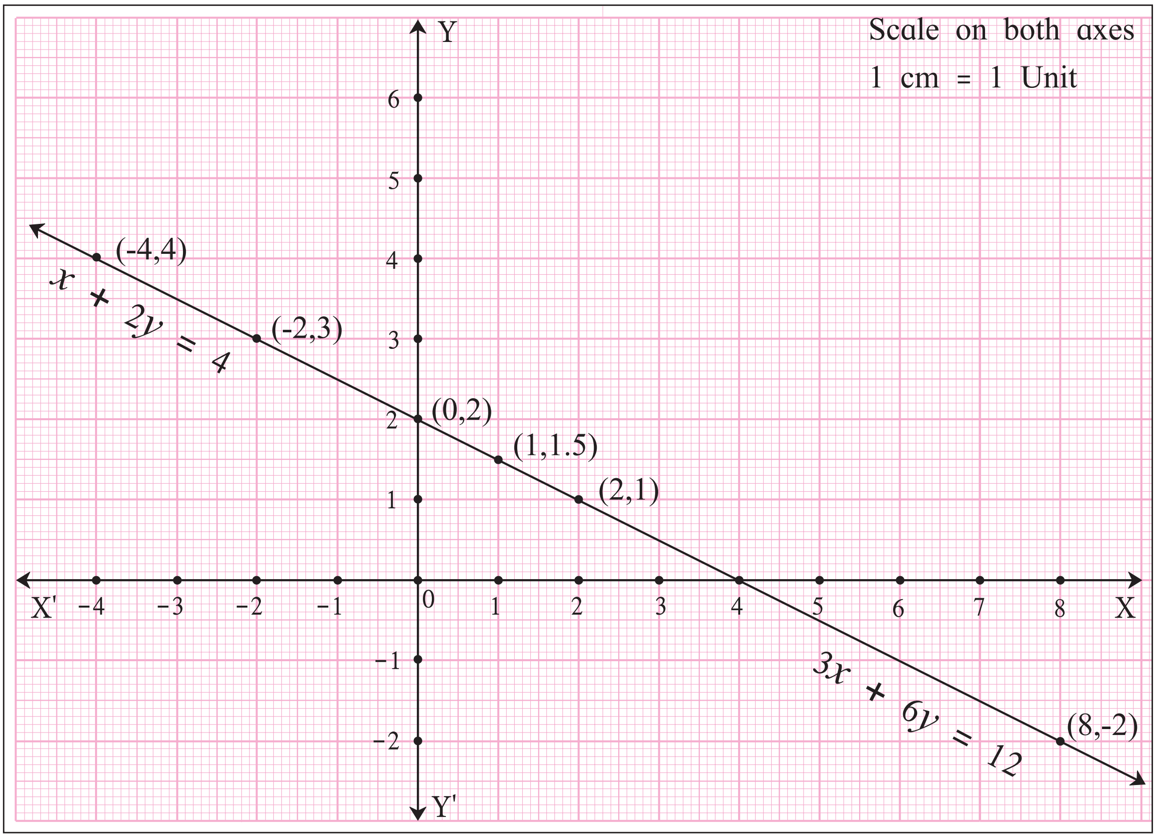
$x + 2y = 4$

x	-2	0	2
y	3	2	1
(x, y)	$(-2, 3)$	$(0, 2)$	$(2, 1)$

$3x + 6y = 12$

x	-4	1	8
y	4	1.5	-2
(x, y)	$(-4, 4)$	$(1, 1.5)$	$(8, -2)$

Plotting the above ordered pairs, graph is drawn. Observe it and find answers of the following questions.



(1) Are the graphs of both the equations different or same ?

(2) What are the solutions of the two equations $x + 2y = 4$ and $3x + 6y = 12$? How many solutions are possible ?

(3) What are the relations between coefficients of x, coefficients of y and constant terms in both the equations ?

(4) What conclusion can you draw when two equations are given but the graph is only one line ?

Now let us consider another example.

Draw graphs of $x - 2y = 4$, $2x - 4y = 12$ on the same co-ordinate plane. Observe it. Think of the realation between the coefficients of x, coefficients of y and the constant terms and draw the inference.

ICT Tools or Links.

Use Geogebra software, draw X- axis, Y-axis. Draw graphs of simultaneous equations.

Let's learn.

Determinant

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is a determinant. (a, b), (c, d) are rows and $\begin{pmatrix} a \\ c \end{pmatrix}$, $\begin{pmatrix} b \\ d \end{pmatrix}$ are columns.

Degree of this determinant is 2, because there are 2 elements in each column and 2 elements in each row. Determinant represents a number which is $(ad\text{-}bc)$.

i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad\text{-}bc$

$ad\text{-}bc$ is the value of determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinants, usually, are represented with capital letters as A, B, C, D, ... etc.

Solved Examples

Ex. Find the values of the following determinants.

(1) $A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix}$ (2) $N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix}$ (3) $B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$

Solution :

(1) $A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} = (5 \times 9) - (3 \times 7) = 45 - 21 = 24$

(2) $N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix} = [(-8) \times (4)] - [(-3\ ) \times 2)] = -32 - (-6)$

$= -32 + 6 = -26$

(3) $B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix} = [2\sqrt{3} \times 3\sqrt{3})] - [2 \times 9)] = 18 - 18 = 0$

Let's learn.

Determinant method (Cramer's Rule)

Using determinants, simultaneous equaions can be solved easily and in less space. This method is known as determinant method. This method was first given by a Swiss mathematician Gabriel Cramer, so it is also known as Cramer's method.

To use Cramer's method, the equations are written as $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

$a_1x + b_1y = c_1 \quad . . . \quad$ (I)

$a_2x + b_2y = c_2 \quad . . . \quad$ (II)

Here x and y are variables, a_1, b_1, c_1 and a_2, b_2, c_2 are real numbers, $a_1b_2 - a_2b_1 \neq 0$

Now let us solve these equations.

Multiplying equation (I) by b_2.

$a_1b_2x + b_1b_2y = c_1b_2 \quad . . . \quad$ (III)

Multiplying equation (II) by b_1.

$a_2b_1x + b_2b_1y = c_2b_1 \quad . . . \quad$ (IV)

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Subtracting equation (III) from (IV)

$$\begin{array}{rl} a_1 b_2 x + b_1 b_2 y &= c_1 b_2 \\ -\ a_2 b_1 x\ -\ b_2 b_1 y &= -\ c_2 b_1 \\ \hline (a_1 b_2 - a_2 b_1)\ x &= c_1 b_2 - c_2 b_1 \end{array}$$

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} \quad . . . \quad (V)$$

Similarly $y = \dfrac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$ . . . (VI)

To remember and write the expressions

$c_1 b_2 - c_2 b_1$, $a_1 b_2 - a_2 b_1$, $a_1 c_2 - a_2 c_1$ we use the determinants.

Now $a_1 x + b_1 y = c_1$

and $a_2 x + b_2 y = c_2$

We can write 3 columns. $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

The values x, y in equation (V), (VI) are written using determinants as follows

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}},$$

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

To remember let us denote $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$$\therefore\ x = \frac{D_x}{D},\quad y = \frac{D_y}{D}$$

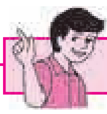
For writting D, Dx, Dy remember the order of columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

From the equations,

$a_1 x + b_1 y = c_1$

and $a_2 x + b_2 y = c_2$ we get the columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

- In D the column of constants $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ is omitted.
- In D_x the column of the coefficients of x, $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.
- In D_y the column of the coefficients of y, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.



Let's remember!

Cramer's method to solve simultaneous equations.

Write given equations in the form $ax + by = c$.

↓

Find the values of determinants D, D_x and D_y

↓

Using, $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$

find values of x, y.

Gabriel Cramer

(31 July, 1704 to 4 January, 1752)

This Swiss mathematician was born in Geneva. He was very well versed in mathematics, since childhood. At the age of eighteen, he got a doctorate. He was a professor in Geneva.

Solved Example

Ex. (1) Solve the following simultaneous equations using Cramer's Rule.

$5x + 3y = -11$; $2x + 4y = -10$

Solution : Given equations

$5x + 3y = -11$

$2x + 4y = -10$

$$D = \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} = (5 \times 4) - (2 \times 3) = 20 - 6 = 14$$

$$D_x = \begin{vmatrix} -11 & 3 \\ -10 & 4 \end{vmatrix} = (-11) \times 4 - (-10) \times 3 = -44 - (-30)$$

$$= -44 + 30 = -14$$

$$D_y = \begin{vmatrix} 5 & -11 \\ 2 & -10 \end{vmatrix} = 5 \times (-10) - 2 \times (-11) = -50 - (-22)$$

$$= -50 + 22 = -28$$

$$x = \frac{D_x}{D} = \frac{-14}{14} = -1 \qquad \text{and } y = \frac{D_y}{D} = \frac{-28}{14} = -2$$

$\therefore$ $(x, y) = (-1, -2)$ is the solution.

Activity 1 : To solve the simultaneous equations by determinant method, fill in the blanks

$y + 2x - 19 = 0$; $2x - 3y + 3 = 0$

Solution : Write the given equations in the form $ax + by = c$

$2x + y = 19$

$2x - 3y = -3$

$$D = \begin{vmatrix} \square & \square \\ 2 & -3 \end{vmatrix} = [\square \times (-3)] - [2 \times (\square)] = \square - (\square)$$

$$= \square - \square = \square$$

$$D_x = \begin{vmatrix} 19 & \square \\ \square & -3 \end{vmatrix} = [19 \times (\square)] - [(\square) \times (\square)] = \square - \square$$

$$= \square$$

$$D_y = \begin{vmatrix} \square & 19 \\ 2 & \square \end{vmatrix} = [(\square) \times (\square)] - [(\square) \times (\square)]$$

$$= \square - \square = \square$$

By Cramer's Rule –

$x = \dfrac{D_x}{D}$ | $y = \dfrac{D_y}{D}$

$\therefore\ x = \dfrac{\square}{\square} = \square$ | $y = \dfrac{\square}{\square} = \square$

$\therefore\ (x,\ y) = (\square,\ \square)$ is the solution of the given equations.

Activity 2 : Complete the following activity –

$3x - 2y = 3$ $2x + y = 16$

Find the values of determinants in the given equations

$D = \begin{vmatrix} & \\ & \end{vmatrix} = \square$ $D_x = \begin{vmatrix} & \\ & \end{vmatrix} = \square$ $D_y = \begin{vmatrix} & \\ & \end{vmatrix} = \square$

Values according to Cramer's Rule

$x = \dfrac{\square}{\square} = \square$ $y = \dfrac{\square}{\square} = \square$

$\therefore\ (x,\ y) = (\square,\ \square)$ is the solution.

Let's think.

- What is the nature of solution if D = 0 ?
- What can you say about lines if common solution is not possible?

Practice Set 1.3

1. Fill in the blanks with correct number

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \square - \square \times 4 = \square - 8 = \square$$

2. Find the values of following determinants.

(1) $\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix}$ (2) $\begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix}$ (3) $\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$

3. Solve the following simultaneous equations using Cramer's rule.

(1) $3x - 4y = 10$; $4x + 3y = 5$ (2) $4x + 3y - 4 = 0$; $6x = 8 - 5y$

(3) $x + 2y = -1$; $2x - 3y = 12$ (4) $6x - 4y = -12$; $8x - 3y = -2$

(5) $4m + 6n = 54$; $3m + 2n = 28$ (6) $2x + 3y = 2$; $x - \frac{y}{2} = \frac{1}{2}$

Let's learn.

Equations reducible to a pair of linear equations in two variables

Activity : Complete the following table.

Equation	No. of variables	whether linear or not
$\frac{3}{x} - \frac{4}{y} = 8$	2	Not linear
$\frac{6}{x-1} + \frac{3}{y-2} = 0$	$\square$	$\square$
$\frac{7}{2x+1} + \frac{13}{y+2} = 0$	$\square$	$\square$
$\frac{14}{x+y} + \frac{3}{x-y} = 5$	$\square$	$\square$

Let's think.

In the above table the equations are not linear. Can you convert the equations into linear equations ?

Let's remember!

We can create new variables making a proper change in the given variables. Substituting the new variables in the given non-linear equations, we can convert them in linear equations.

Also remember that the denominator of any fraction of the form $\frac{m}{n}$ cannot be zero.

Solved examples

Solve:

Ex. (1) $\frac{4}{x} + \frac{5}{y} = 7;\ \frac{3}{x} + \frac{4}{y} = 5$

Solution : $\frac{4}{x} + \frac{5}{y} = 7;\ \frac{3}{x} + \frac{4}{y} = 5$

$$4\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = 7 \quad . . . \quad \text{(I)}$$

$$3\left(\frac{1}{x}\right) + 4\left(\frac{1}{y}\right) = 5 \quad . . . \quad \text{(II)}$$

Replacing $\left(\frac{1}{x}\right)$ by m and $\left(\frac{1}{y}\right)$ by n in equations (I) and (II), we get

$4m + 5n = 7$ . . . (III)

$3m + 4n = 5$ . . . (IV)

On solving these equations we get

$m = 3,\ n = -1$

Now, $m = \frac{1}{x}$ $\quad \therefore 3 = \frac{1}{x}$ $\quad \therefore x = \frac{1}{3}$

$n = \frac{1}{y}$ $\quad \therefore -1 = \frac{1}{y}$ $\quad \therefore y = -1$

$\therefore$ Solution of given simultaneous equations is $(x, y) = (\frac{1}{3}, -1)$

Ex.(2) $\frac{4}{x-y} + \frac{1}{x+y} = 3$; $\frac{2}{x-y} - \frac{3}{x+y} = 5$

Solution : $\frac{4}{x-y} + \frac{1}{x+y} = 3$; $\frac{2}{x-y} - \frac{3}{x+y} = 5$

$$4\left(\frac{1}{x-y}\right) + 1\left(\frac{1}{x+y}\right) = 3 \quad . . . \quad \text{(I)}$$

$$2\left(\frac{1}{x-y}\right) - 3\left(\frac{1}{x+y}\right) = 5 \quad . . . \quad \text{(II)}$$

Replacing $\left(\frac{1}{x-y}\right)$ by a and $\left(\frac{1}{x+y}\right)$ by b we get

$$4a + b = 3 \quad . . . \quad \text{(III)}$$

$$2a - 3b = 5 \quad . . . \quad \text{(IV)}$$

On solving these equations we get, $a = 1$ $b = -1$

But $a = \left(\frac{1}{x-y}\right)$, $b = \left(\frac{1}{x+y}\right)$

$\therefore \left(\frac{1}{x-y}\right) = 1, \left(\frac{1}{x+y}\right) = -1$

$\therefore \quad x - y = 1 \quad . . . \quad \text{(V)}$

$\quad x + y = -1 \quad . . . \quad \text{(VI)}$

Solving equation (V) and (VI) we get $x = 0$, $y = -1$

$\therefore$ Solution of the given equations is $(x, y) = (0, -1)$

Let's think.

In the above examples the simultaneous equations obtained by transformation are solved by elimination method.

If you solve these equations by graphical method and by Cramer's rule will you get the same answers ? Solve and check it.

Activity : To solve given equations fill the boxes below suitably.

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \; ; \; \frac{6}{x-1} - \frac{3}{y-2} = 1$$

Replacing $\left(\frac{1}{x-1}\right)$ by m, $\left(\frac{1}{y-2}\right)$ by n

New equations

$\square$; $6m - 3n = 1$

On solving

$m = \square$, $n = \square$

Replacing m, n by their original values.

$\left(\frac{1}{x-1}\right) = \frac{1}{3}$; $\square$

On solving

$x = \square$, $y = \square$

$\therefore$ $(x, y) = (\;\; , \;\;)$ is the solution of the given simultaneous equations.

Practice Set 1.4

1. Solve the following simultaneous equations.

(1) $\frac{2}{x} - \frac{3}{y} = 15 \; ; \; \frac{8}{x} + \frac{5}{y} = 77$

(2) $\frac{10}{x+y} + \frac{2}{x-y} = 4 \; ; \; \frac{15}{x+y} - \frac{5}{x-y} = -2$

(3) $\frac{27}{x-2} + \frac{31}{y+3} = 85 \; ; \; \frac{31}{x-2} + \frac{27}{y+3} = 89$

(4) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} \; ; \; \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

Let's learn.

Application of Simultaneous equations

Activity : There are some instructions given below. Frame the equations from the information and write them in the blank boxes shown by arrows.

Sarthak's age is less by 8 than double the age of Sakshi

I am Sarthak — My Present age is x years.

My Present age is y years. — I am Sakshi

4 years ago Sakshi's age was 3 years less than Sarthak's age at that time.

The sum of present ages of Sarthak and Sakshi is 25.

Ex. (1) The perimeter of a rectangle is 40 cm. The length of the rectangle is more than double its breadth by 2. Find length and breadth.

Solution : Let length of rectangle be x cm and breadth be y cm.

From first condition –

$$2(x + y) = 40$$

$$x + y = 20 \quad \ldots \quad \text{(I)}$$

From 2^{nd} condition –

$$x = 2y + 2$$

$$\therefore\ x - 2y = 2 \quad \ldots \quad \text{(II)}$$

Let's solve eq. (I), (II) by determinant method

$$x + y = 20$$

$$x - 2y = 2$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = [1 \times (-2)] - (1 \times 1) = -2 - 1 = -3$$

$$D_x = \begin{vmatrix} 20 & 1 \\ 2 & -2 \end{vmatrix} = [20 \times (-2)] - (1 \times 2) = -40 - 2 = -42$$

$$D_y = \begin{vmatrix} 1 & 20 \\ 1 & 2 \end{vmatrix} = (1 \times 2) - (20 \times 1) = 2 - 20 = -18$$

$$x = \frac{D_x}{D} \text{ and } y = \frac{D_y}{D}$$

$$\therefore\ x = \frac{-42}{-3} \text{ and } y = \frac{-18}{-3}$$

$$\therefore\ x = 14,\quad y = 6$$

$\therefore$ Length of the rectangle is 14 cm and breadth is 6 cm.

Ex. (2)

Sale ! Sale !! Sale !!! only for 2 days

I have some analogue wrist watches and some digital wrist watches. I am going to sell them at a discount

Sale of 1st day

Analogue watch = 11

Digital watch = 6

Received amount = ₹ 4330

Sale of the 2nd day

Analogue watch = 22

Digital watch = 5

Received amount = ₹ 7330

Find selling price of wrist watch of each type.

Solution : Let selling price of each analogue watch be ₹ x

Selling price of each digital watch be ₹ y

From first condition –

$$11x + 6y = 4330 \quad . . . \quad (I)$$

from 2[nd] condition –

$$22x + 5y = 7330 \quad . . . \quad (II)$$

multiplying equation (I) by 2 we get,

$$22x + 12y = 8660 \quad . . . \quad (III)$$

subtract equation (III) from equation (II).

$$22x + 5y = 7330 \quad . . . \quad (II)$$

$$-\ 22x + 12y = 8660 \quad . . . \quad (III)$$

$$- \quad - \quad -$$

$$-7y = -1330$$

$$\therefore \quad y = 190$$

Substitute this value of y in equation (I)

$$11x + 6y = 4330$$

$$\therefore \quad 11x + 6(190) = 4330$$

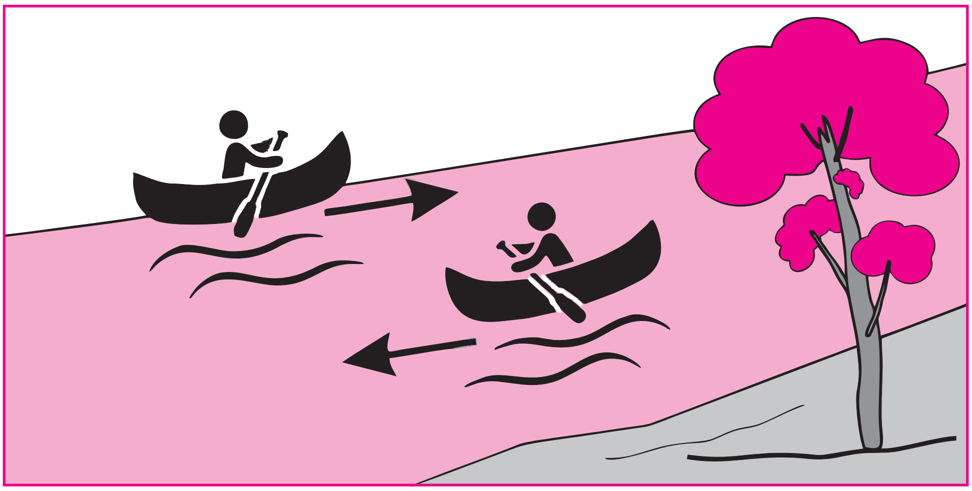
$$\therefore \quad 11x + 1140 = 4330$$

$$\therefore \quad 11x = 3190$$

$$\therefore \quad x = 290$$

$\therefore$ selling price of each analogue watch is ₹ 290 and of each digital watch is ₹ 190.

Ex. (3)



A boat travels 16 km upstream and 24 km downstream in 6 hours.

The same boat travels 36 km upstream and 48 km downstream in 13 hours.

Find the speed of water current and speed of boat in still water.

Solution : Let the speed of the boat in still water be x km/hr and the speed of water current be y km/hr

$\therefore$ speed of boat in downstream = $(x + y)$ km/hr.

and that in upstream = $(x - y)$ km/hr.

Now distance = speed × time $\qquad \therefore$ time = $\dfrac{\text{distance}}{\text{speed}}$

Time taken by the boat to travel 16 km upstream = $\dfrac{16}{x-y}$ hours and it takes $\dfrac{24}{x+y}$ hours to travel 24 km downstream.

from first condition –

$$\frac{16}{x-y}+\frac{24}{x+y} = 6 \quad . . . \quad \text{(I)}$$

from 2nd condition

$$\frac{36}{x-y}+\frac{48}{x+y} = 13 \quad . . . \quad \text{(II)}$$

By replacing $\dfrac{1}{x-y}$ by m and $\dfrac{1}{x+y}$ by n we get

$16m + 24n = 6$ . . . (III)

$36m + 48n = 13$ . . . (IV)

Solving equations (III) and (IV) $m = \frac{1}{4}$, $n = \frac{1}{12}$

Repalcing m, n by their original values we get

$x - y = 4$ . . . (V) $x + y = 12$ . . . (VI)

Solving equations (V), (VI) we get $x = 8$, $y = 4$

$\therefore$ speed of the boat in still water is 8 km/hr. and speed of water current is 4 km/hr.

Ex. (4) A certain amount is equally distributed among certain number of students. Each would get ₹ 2 less if 10 students were more and each would get ₹ 6 more if 15 students were less. Find the number of students and the amount distributed.

Solution : Let the number of students be x and amount given to each student be ₹ y.

$\therefore$ Total amount distributed is xy

From the first condition we get,

$(x + 10)(y - 2) = xy$

$\therefore xy - 2x + 10y - 20 = xy$

$\therefore - 2x + 10y = 20$

$\therefore - x + 5y = 10$ . . . (I)

From the 2nd condition we get,

$(x - 15)(y + 6) = xy$

$\therefore xy + 6x - 15y - 90 = xy$

$\therefore 6x - 15y = 90$

$\therefore 2x - 5y = 30$ . . . (II)

Adding equations (I) and (II)

$$\begin{array}{r} -x + 5y = 10 \\ + \quad 2x - 5y = 30 \\ \hline x = 40 \end{array}$$

Substitute this value of x in equation (I)

$-x + 5y = 10$

$\therefore - 40 + 5y = 10$

$\therefore 5y = 50$

$\therefore \quad y = 10$

Total amount distributed is $= xy = 40 \times 10 =$ ₹ 400.

$\therefore$ ₹ 400 distributed equally among 40 students.

Ex. (5) A three digit number is equal to 17 times the sum of its digits; If the digits are reversed, the new number is 198 more than the old number ; also the sum of extreme digits is less than the middle digit by unity. Find the original number.

Solution : Let the digit in hundreds place be x and that in unit place be y.

H	T	unit
x	$x + y + 1$	y

$\therefore$ the three digit number is $100x + 10(x + y + 1) + y$

$= 100x + 10x + 10y + 10 + y = 110x + 11y + 10$

the sum of the digits in the given number $= x + (x + y + 1) + y = 2x + 2y + 1$

$\therefore$ From first condition

Given number $= 17 \times$ (sum of the digits)

$\therefore 110x + 11y + 10 = 17 \times (2x + 2y + 1)$

$\therefore 110x + 11y + 10 = 34x + 34y + 17$

$\therefore 76x - 23y = 7 \; . \; . \; . \quad$ (I)

The number obtained by reversing the digits

$= 100y + 10(x + y + 1) + x = 110y + 11x + 10$

Given number $= 110x + 11y + 10$

From 2^{nd} condition, Given number + 198 = new number.

$110x + 11y + 10 + 198 = 110y + 11x + 10$

$99x - 99y = -198$

$x - y = -2$

$\therefore x = y - 2 \; . \; . \; . \quad$ (II)

Substitute this value of x in equation (I).

$\therefore 76(y - 2) - 23y = 7$

$\therefore 76y - 152 - 23y = 7$

$53y = 159$

$\therefore \quad y = 3 \qquad \therefore$ the digit in units place is $= 3$

Substitute this value in equation (II)

$x = y - 2$

$\therefore \quad x = 3 - 2 \quad = 1$

$\therefore \qquad x = 1 \qquad \therefore$ The digit in hundred's place is 1

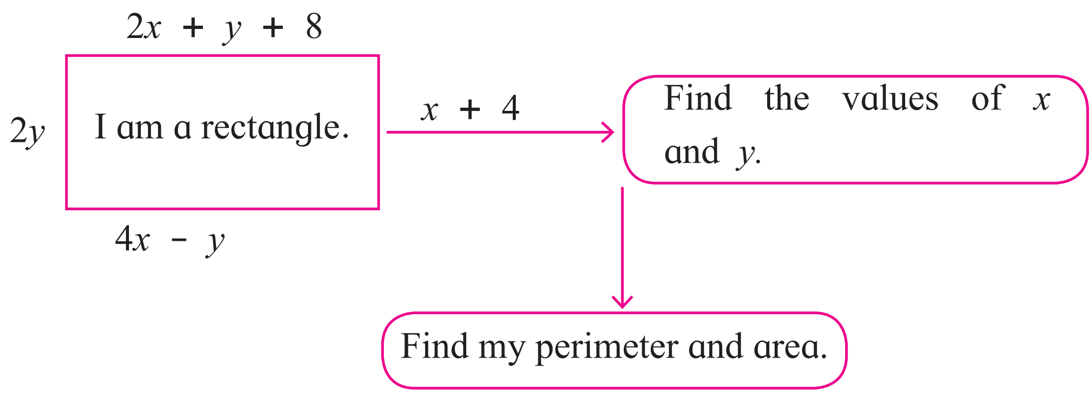
the digit in ten's place is $3 + 1 + 1 = 5$

$\therefore$ the number is 153.

Practice Set 1.5

(1) Two numbers differ by 3. The sum of twice the smaller number and thrice the greater number is 19. Find the numbers.

(2) Complete the following.



(3) The sum of father's age and twice the age of his son is 70. If we double the age of the father and add it to the age of his son the sum is 95. Find their present ages.

(4) The denominator of a fraction is 4 more than twice its numerator. Denominator becomes 12 times the numerator, if both the numerator and the denominator are reduced by 6. Find the fraction.

(5) Two types of boxes A, B are to be placed in a truck having capacity of 10 tons. When 150 boxes of type A and 100 boxes of type B are loaded in the truck, it weighes 10 tons. But when 260 boxes of type A are loaded in the truck, it can still accommodate 40 boxes of type B, so that it is fully loaded. Find the weight of each type of box.

★(6) Out of 1900 km, Vishal travelled some distance by bus and some by aeroplane. Bus travels with average speed 60 km/hr and the average speed of aeroplane is 700 km/hr. It takes 5 hours to complete the journey. Find the distance, Vishal travelled by bus.

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Problem Set - 1

1. Choose correct alternative for each of the following questions

(1) To draw graph of $4x + 5y = 19$, Find y when $x = 1$.

(A) 4 (B) 3 (C) 2 (D) −3

(2) For simultaneous equations in variables x and y, $D_x = 49$, $D_y = -63$, $D = 7$ then what is x ?

(A) 7 (B) −7 (C) $\frac{1}{7}$ (D) $\frac{-1}{7}$

(3) Find the value of $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$

(A) −1 (B) −41 (C) 41 (D) 1

(4) To solve $x + y = 3$; $3x - 2y - 4 = 0$ by determinant method find D.

(A) 5 (B) 1 (C) −5 (D) −1

(5) $ax + by = c$ and $mx + ny = d$ and $an \neq bm$ then these simultaneous equations have -

(A) Only one common solution. (B) No solution.

(C) Infinite number of solutions. (D) Only two solutions.

2. Complete the following table to draw the graph of $2x - 6y = 3$

x	−5	☐
y	☐	0
(x, y)	☐	☐

3. Solve the following simultaneous equations graphically.

(1) $2x + 3y = 12$; $x - y = 1$

(2) $x - 3y = 1$; $3x - 2y + 4 = 0$

(3) $5x - 6y + 30 = 0$; $5x + 4y - 20 = 0$

(4) $3x - y - 2 = 0$; $2x + y = 8$

(5) $3x + y = 10$; $x - y = 2$

4. Find the values of each of the following determinants.

(1) $\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$ (2) $\begin{vmatrix} 5 & -2 \\ -3 & 1 \end{vmatrix}$ (3) $\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}$

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5. Solve the following equations by Cramer's method.

(1) $6x - 3y = -10$; $3x + 5y - 8 = 0$

(2) $4m - 2n = -4$; $4m + 3n = 16$

(3) $3x - 2y = \frac{5}{2}$; $\frac{1}{3}x + 3y = -\frac{4}{3}$

(4) $7x + 3y = 15$; $12y - 5x = 39$

(5) $\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x-y}{4}$

6. Solve the following simultaneous equations.

(1) $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$; $\frac{3}{x} + \frac{2}{y} = 0$ (2) $\frac{7}{2x+1} + \frac{13}{y+2} = 27$; $\frac{13}{2x+1} + \frac{7}{y+2} = 33$

(3) $\frac{148}{x} + \frac{231}{y} = \frac{527}{xy}$; $\frac{231}{x} + \frac{148}{y} = \frac{610}{xy}$ (4) $\frac{7x-2y}{xy} = 5$; $\frac{8x+7y}{xy} = 15$

(5) $\frac{1}{2(3x+4y)} + \frac{1}{5(2x-3y)} = \frac{1}{4}$; $\frac{5}{(3x+4y)} - \frac{2}{(2x-3y)} = -\frac{3}{2}$

7. Solve the following word problems.

(1) A two digit number and the number with digits interchanged add up to 143. In the given number the digit in unit's place is 3 more than the digit in the ten's place. Find the original number.

Let the digit in unit's place is x

and that in the ten's place is y

$\therefore$ the number = $\square\, y + x$

The number obtained by interchanging the digits is $\square\, x + y$

According to first condition two digit number + the number obtained by interchanging the digits = 143

$\therefore$ $\boxed{10y + x} + \square = 143$

$\therefore$ $\square\, x + \square\, y = 143$

$x + y = \square$ . . . . . (I)

From the second condition,

digit in unit's place = digit in the ten's place + 3

$\therefore$ $x = \square + 3$

$\therefore$ $x - y = 3$ . . . . . (II)

Adding equations (I) and (II)

$2x = \square$

$\boxed{x = 8}$

Putting this value of x in equation (I)

$x + y = 13$

$8 + \square = 13$

$\therefore \quad y = \square$

The original number is $10y + x$

$= \square + 8$

$= 58$

(2) Kantabai bought $1\frac{1}{2}$ kg tea and 5 kg sugar from a shop. She paid ₹ 50 as return fare for rickshaw. Total expense was ₹ 700. Then she realised that by ordering online the goods can be bought with free home delivery at the same price. So next month she placed the order online for 2 kg tea and 7 kg sugar. She paid ₹ 880 for that. Find the rate of sugar and tea per kg.

(3) To find number of notes that Anushka had, complete the following activity.

Suppose that Anushka had $\boxed{x}$ notes of ₹ 10 and $\boxed{y}$ notes of ₹ 50 each

Anushka got ₹ 2500/- from Anand as denominations mentioned above
$\therefore$ .... equation I

If Anand would have given her the amount by interchanging number of notes, Anushka would have received ₹ 500 less than the previous amount
$\therefore$ .... equation II

$\therefore$ The No. of notes ($\square$, $\square$)

(4) Sum of the present ages of Manish and Savita is 31. Manish's age 3 years ago was 4 times the age of Savita. Find their present ages.

★ (5) In a factory the ratio of salary of skilled and unskilled workers is 5 : 3. Total salary of one day of both of them is ₹ 720. Find daily wages of skilled and unskilled workers.

★ (6) Places A and B are 30 km apart and they are on a straight road. Hamid travels from A to B on bike. At the same time Joseph starts from B on bike, travels towards A. They meet each other after 20 minutes. If Joseph would have started from B at the same time but in the opposite direction (instead of towards A) Hamid would have caught him after 3 hours. Find the speed of Hamid and Joseph.

N51ZCL

2 Quadratic Equations

Let's study.

- Quadratic equation : Introduction
- Methods of solving quadratic equation
- Nature of roots of quadratic equation
- Relation between roots and coefficients
- Applications of quadratic equations

Let's recall.

You have studied polynomials last year. You know types of polynomials according to their degree. When the degree of polynomial is 1 it is called a linear polynomial and if degree of a polynomial is 2 it is called a quadratic polynomial.

Activity : Classify the following polynomials as linear and quadratic.

$5x + 9$, $x^2 + 3x - 5$, $3x - 7$, $3x^2 - 5x$, $5x^2$

Linear polynomials

Quadratic polynomials

Now equate the quadratic polynomial to 0 and study the equation we get. Such type of equation is known as quadratic equation. In practical life we may use quadratic equations many times.

Ex. Sanket purchased a rectangular plot having area 200 m^2. Length of the plot was 10 m more than its breadth. Find the length and the breadth of the plot.

Let the breadth of the plot be x metre.

$\therefore$ Length = $(x + 10)$ metre

Area of rectangle = length × breadth

$\therefore\ 200 = (x + 10) \times x$

$\therefore\ 200 = x^2 + 10x$

That is $x^2 + 10x = 200$

$\therefore\ x^2 + 10x - 200 = 0$

Now, solving equation $x^2 + 10x - 200 = 0$, we will decide the dimensions of the plot.

Let us study how to solve the quadratic equation.

Let's recall.

Activity : $x^2 + 3x - 5$, $3x^2 - 5x$, $5x^2$; Write the polynomials in the index form.

Observe the coefficients and fill in the boxes.

$x^2 + 3x - 5$, $3x^2 - 5x + 0$, $5x^2 + 0x + 0$

- Coefficients of x^2 are [1], [3] and [5] these coefficients are non zero.
- Coefficients of x are 3, [ ] and [ ] respectively.
- Constants terms are [ ], [ ] and [ ] respectively.

Here constant term of second and third polynomial is zero.

Let's learn.

Standard form of quadratic equation

The equation involving one variable and having 2 as the maximum index of the variable is called the quadratic equation.

General form is $ax^2 + bx + c = 0$

In $ax^2 + bx + c = 0$, a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is the general form of quadratic equation.

Activity : Complete the following table

Quadratic Equation	General form	a	b	c
$x^2 - 4 = 0$	$x^2 + 0x - 4 = 0$	1	0	-4
$y^2 = 2y - 7$	. . . .	. . .	. . .	. . .
$x^2 + 2x = 0$	. . . .	. . .	. . .	. . .

Solved Examples

Ex. (1) Decide which of the following are quadratic equations ?

(1) $3x^2 - 5x + 3 = 0$ (2) $9y^2 + 5 = 0$ (3) $m^3 - 5m^2 + 4 = 0$ (4) $(l + 2)(l - 5) = 0$

Solution : (1) In the equation $3x^2 - 5x + 3 = 0$, x is the only variable and maximum index of the variable is 2

$\therefore$ It is a quadratic equation.

(2) In the equation $9y^2 + 5 = 0$, ☐ is the only variable and maximum index of the variable is ☐

$\therefore$ It ☐ a quadratic equation.

(3) In the equation $m^3 - 5m^2 + 4 = 0$, ☐ is the only variable but maximum index of the variable is not 2.

$\therefore$ It ☐ a quadratic equation.

(4) $(l + 2)(l - 5) = 0$

$\therefore l(l - 5) + 2(l - 5) = 0$

$\therefore l^2 - 5l + 2l - 10 = 0$

$\therefore l^2 - 3l - 10 = 0$, In this equation ☐ is the only variable and maximum index of the variable is ☐ .

$\therefore$ It ☐ a quadratic equation.

Let's learn.

Roots of a quadratic equation

In the previous class you have studied that if value of the polynomial is zero for $x = a$ then $(x - a)$ is a factor of that polynomial. That is if p(x) is a polynomial and p(a) = 0 then $(x - a)$ is a factor of p(x). In this case 'a' is the root or solution of p(x) = 0

For Example ,

Let $x = -6$ in the polynomial $x^2 + 5x - 6$

$x^2 + 5x - 6 = (-6)^2 + 5 \times (-6) - 6$

$= 36 - 30 - 6 \quad = 0$

$\therefore x = -6$ is a solution of the equation.

Hence -6 is one root of the equation

$x^2 + 5x - 6 = 0$

Let $x = 2$ in polynomial $x^2 + 5x - 6$

$x^2 + 5x - 6 = 2^2 + 5 \times 2 - 6$

$= 4 + 10 - 6$

$= 8 \neq 0$

$\therefore \quad x = 2$ is not a solution of the equation $x^2 + 5x - 6 = 0$

Solved Example

Ex. $2x^2 - 7x + 6 = 0$ check whether (i) $x = \frac{3}{2}$, (ii) $x = -2$ are solutions of the equations.

Solution : (i) Put $x = \frac{3}{2}$ in the polynomial $2x^2 - 7x + 6$

$$2x^2 - 7x + 6 = 2\left(\frac{3}{2}\right)^2 - 7\left(\frac{3}{2}\right) + 6$$

$$= 2 \times \frac{9}{4} - \frac{21}{2} + 6$$

$$= \frac{9}{2} - \frac{21}{2} + \frac{12}{2} = 0$$

$\therefore\ x = \frac{3}{2}$ is a solution of the equation.

(ii) Let $x = -2$ in $2x^2 - 7x + 6$

$$2x^2 - 7x + 6 = 2(-2)^2 - 7(-2) + 6$$

$$= 2 \times 4 + 14 + 6$$

$$= 28 \neq 0$$

$\therefore\ x = -2$ is not a solution of the equation.

Activity : If $x = 5$ is a root of equation $kx^2 - 14x - 5 = 0$ then find the value of k by completing the following activity.

Solution : One of the roots of equation $kx^2 - 14x - 5 = 0$ is $\square$.

$\therefore$ Now Let $x = \square$ in the equation.

$k\square^2 - 14\square - 5 = 0$

$\therefore\ 25k - 70 - 5 = 0$

$25k - \square = 0$

$25k = \square$

$\therefore\ k = \frac{\square}{\square} = 3$

Let's remember!

(1) $ax^2 + bx + c = 0$ is the general form of equation where a, b, c are real numbers and 'a' is non zero.

(2) The values of variable which satisfy the equation [or the value for which both the sides of equation are equal] are called solutions or roots of the equation.

Practice Set 2.1

1. Write any two quadratic equations.

2. Decide which of the following are quadratic equations.

(1) $x^2 + 5x - 2 = 0$ (2) $y^2 = 5y - 10$ (3) $y^2 + \frac{1}{y} = 2$

(4) $x + \frac{1}{x} = -2$ (5) $(m + 2)(m - 5) = 0$ (6) $m^3 + 3m^2 - 2 = 3m^3$

3. Write the following equations in the form $ax^2 + bx + c = 0$, then write the values of a, b, c for each equation.

(1) $2y = 10 - y^2$ (2) $(x - 1)^2 = 2x + 3$ (3) $x^2 + 5x = -(3 - x)$

(4) $3m^2 = 2m^2 - 9$ (5) $P(3 + 6p) = -5$ (6) $x^2 - 9 = 13$

4. Determine whether the values given against each of the quadratic equation are the roots of the equation.

(1) $x^2 + 4x - 5 = 0$, $x = 1, -1$ (2) $2m^2 - 5m = 0$, $m = 2, \frac{5}{2}$

5. Find k if $x = 3$ is a root of equation $kx^2 - 10x + 3 = 0$.

6. One of the roots of equation $5m^2 + 2m + k = 0$ is $\frac{-7}{5}$. Complete the following activity to find the value of 'k'.

Solution : $\square$ is a root of quadratic equation $5m^2 + 2m + k = 0$

$\therefore$ Put $m = \square$ in the equation.

$5 \times \square^2 + 2 \times \square + k = 0$

$\square + \square + k = 0$

$\square + k = 0$

$k = \square$

Let's recall.

Last year you have studied the methods to find the factors of quadratic polynomials like $x^2 - 4x - 5$, $2m^2 - 5m$, $a^2 - 25$. Try the following activity and revise the same.

Activity : Find the factors of the following polynomials.

(1) $x^2 - 4x - 5$

$= \underline{x^2 - 5x} + \underline{1x - 5}$

$= x(\ldots\ldots) + 1(\ldots\ldots)$

$= (\ldots\ldots)(\ldots\ldots)$

(2) $2m^2 - 5m$

$= \ldots\ldots\ \ldots\ldots$

(3) $a^2 - 25$

$= a^2 - 5^2$

$= (\ldots\ldots)(\ldots\ldots)$

Let's learn.

Solutions of a quadratic equation by factorisation

By substituting arbitrary values for the variable and deciding the roots of quadratic equation is a time consuming process. Let us learn to use factorisation method to find the roots of the given quadratic equation.

$x^2 - 4x - 5 = (x - 5)(x + 1)$

$(x - 5)$ and $(x + 1)$ are two linear factors of quadratic polynomial $x^2 - 4x - 5$.

So the quadratic equation obtained from $x^2 - 4x - 5$ can be written as

$(x - 5)(x + 1) = 0$

If product of two numbers is zero then at least one of them is zero.

$\therefore\ x - 5 = 0$ or $x + 1 = 0$

$\therefore\ x = 5$ or $x = -1$

$\therefore$ 5 and the -1 are the roots of the given quadratic equation.

While solving the equation first we obtained the linear factors. So we call this method as 'factorization method' of solving quadratic equation.

Solved Examples

Ex. Solve the following quadratic equations by factorisation.

(1) $m^2 - 14m + 13 = 0$ (2) $3x^2 - x - 10 = 0$

(3) $3y^2 = 15y$ (4) $x^2 = 3$ (5) $6\sqrt{3}x^2 + 7x = \sqrt{3}$

(1) $m^2 - 14m + 13 = 0$

$\therefore\ m^2 - 13m - 1m + 13 = 0$

$\therefore\ \overline{m(m - 13)}\ \overline{-1(m - 13)} = 0$

$\therefore\ (m - 13)(m - 1) = 0$

$\therefore\ m - 13 = 0$ or $m - 1 = 0$

$\therefore\ m = 13$ or $m = 1$

$\therefore$ 13 and 1 are the roots of the given quadratic equation.

(2) $3x^2 - x - 10 = 0$

$\therefore\ \underline{3x^2 - 6x} + \underline{5x - 10} = 0$

$\therefore\ 3x(x - 2) + 5(x - 2) = 0$

$\therefore\ (3x + 5)(x - 2) = 0$

$\therefore\ (3x + 5) = 0$ or $(x - 2) = 0$

$\therefore\ x = -\frac{5}{3}$ or $x = 2$

$\therefore\ -\frac{5}{3}$, and 2 are the roots of the given quadratic equation.

(3) $3y^2 = 15y$

$\therefore 3y^2 - 15y = 0$

$\therefore 3y(y-5) = 0$

$\therefore 3y = 0$ or $(y-5) = 0$

$\therefore y = 0$ or $y = 5$

$\therefore$ 0 and 5 are the roots of quadratic equation.

(4) $x^2 = 3$

$\therefore x^2 - 3 = 0$

$\therefore x^2 - (\sqrt{3})^2 = 0$

$\therefore (x+\sqrt{3})(x-\sqrt{3}) = 0$

$\therefore (x+\sqrt{3}) = 0$ or $(x-\sqrt{3}) = 0$

$\therefore x = -\sqrt{3}$ or $x = \sqrt{3}$

$\therefore -\sqrt{3}$ and $\sqrt{3}$ are the roots of given quadratic equation.

(5) $6\sqrt{3}x^2 + 7x = \sqrt{3}$

$\therefore 6\sqrt{3}x^2 + 7x - \sqrt{3} = 0$

$\therefore 6\sqrt{3}x^2 + 9x - 2x - \sqrt{3} = 0$

$\therefore 3\sqrt{3}x(2x+\sqrt{3}) - 1(2x+\sqrt{3}) = 0$

$\therefore (2x+\sqrt{3})(3\sqrt{3}x - 1) = 0$

$\therefore 2x + \sqrt{3} = 0$ or $3\sqrt{3}x - 1 = 0$

$\therefore 2x = -\sqrt{3}$ or $3\sqrt{3}x = 1$

$\therefore x = -\frac{\sqrt{3}}{2}$ or $x = \frac{1}{3\sqrt{3}}$

$\therefore -\frac{\sqrt{3}}{2}$ and $\frac{1}{3\sqrt{3}}$ are the roots of the given quadratic equation.

$6\sqrt{3} \times -\sqrt{3} = -18$

-18 → 9, -2

$9 = 3\sqrt{3} \times \sqrt{3}$

Practice Set 2.2

1. Solve the following quadratic equations by factorisation.

(1) $x^2 - 15x + 54 = 0$ (2) $x^2 + x - 20 = 0$ (3) $2y^2 + 27y + 13 = 0$

(4) $5m^2 = 22m + 15$ (5) $2x^2 - 2x + \frac{1}{2} = 0$ (6) $6x - \frac{2}{x} = 1$

(7) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ to solve this quadratic equation by factorisation, complete the following activity.

Solution : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + \square + \square + 5\sqrt{2} = 0$

$x(.....) + \sqrt{2}(.....) = 0$

$(\ldots\ldots)(x + \sqrt{2}) = 0$

$(\ldots\ldots) = 0$ or $(x + \sqrt{2}) = 0$

$\therefore x = \square$ or $x = -\sqrt{2}$

$\therefore \square$ and $-\sqrt{2}$ are roots of the equation.

(8)★ $3x^2 - 2\sqrt{6}\,x + 2 = 0$ (9) $2m\,(m - 24) = 50$

(10) $25m^2 = 9$ (11) $7m^2 = 21m$ (12) $m^2 - 11 = 0$

Let's learn.

Solution of a quadratic equation by completing the square

Teacher : Is $x^2 + 10x + 2 = 0$ a quadratic equation or not ?

Yogesh : Yes Sir, because it is in the form $ax^2 + bx + c = 0$, maximum index of the variable x is 2 and 'a' is non zero.

Teacher : Can you solve this equation ?

Yogesh : No Sir, because it is not possible to find the factors of 2 whose sum is 10.

Teacher : Right, so we have to use another method to solve such equations. Let us learn the method.

Let us add a suitable term to $x^2 + 10x$ so that the new expression would be a complete square.

If $x^2 + 10x + k = (x + a)^2$

then $x^2 + 10x + k = x^2 + 2ax + a^2$

$\therefore 10 = 2a$ and $k = a^2$

by equating the coefficients for the variable x and constant term

$\therefore a = 5 \quad \therefore k = a^2 = (5)^2 = 25$

$\therefore x^2 + 10x + 2 = (x + 5)^2 - 25 + 2 = (x + 5)^2 - 23$

Now can you solve the equation $x^2 + 10x + 2 = 0$?

Rehana : Yes Sir, left side of the equation is now difference of two squares and we can factorise it.

$(x + 5)^2 - (\sqrt{23})^2 = 0$

$\therefore (x + 5 + \sqrt{23})(x + 5 - \sqrt{23}) = 0$

$\therefore x + 5 + \sqrt{23} = 0$ or $x + 5 - \sqrt{23} = 0$

$\therefore x = -5 - \sqrt{23}$ or $x = -5 + \sqrt{23}$

Hameed : Sir, May I suggest another way ?

$(x + 5)^2 - (\sqrt{23})^2 = 0$

$\therefore (x + 5)^2 = (\sqrt{23})^2$

$\therefore x + 5 = \sqrt{23}$ or $x + 5 = -\sqrt{23}$

$\therefore x = -5 + \sqrt{23}$ or $x = -5 - \sqrt{23}$

Solved Examples

Ex. (1) Solve : $5x^2 - 4x - 3 = 0$

Solution : It is convenient to make coefficient of x^2 as 1 and then convert the equation as the of difference of two squares, so dividing the equation by 5,

we get, $x^2 - \frac{4}{5}x - \frac{3}{5} = 0$

now if $x^2 - \frac{4}{5}x + k = (x - a)^2$ then $x^2 - \frac{4}{5}x + k = x^2 - 2ax + a^2$.

compare the terms in $x^2 - \frac{4}{5}x$ and $x^2 - 2ax$.

$-2ax = -\frac{4}{5}x \quad \therefore a = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$

$\therefore k = a^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

Now, $x^2 - \frac{4}{5}x - \frac{3}{5} = 0$

$\therefore x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{3}{5} = 0$

$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{4}{25} + \frac{3}{5}\right) = 0$

$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{19}{25}\right) = 0$

$\therefore \left(x - \frac{2}{5}\right)^2 = \left(\frac{19}{25}\right)$

$\therefore x - \frac{2}{5} = \frac{\sqrt{19}}{5}$ or $x - \frac{2}{5} = -\frac{\sqrt{19}}{5}$

$\therefore x = \frac{2}{5} + \frac{\sqrt{19}}{5}$ or $x = \frac{2}{5} - \frac{\sqrt{19}}{5}$

$\therefore x = \frac{2+\sqrt{19}}{5}$ or $x = \frac{2-\sqrt{19}}{5}$

$\therefore \frac{2+\sqrt{19}}{5}$ and $\frac{2-\sqrt{19}}{5}$ are roots of the equation.

When equation is in the form $x^2 + bx + c = 0$, it can be written as

$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ that is,

$\left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$

Ex. (2) Solve : $x^2 + 8x - 48 = 0$

Method I : Completing the square.

$x^2 + 8x - 48 = 0$

$\therefore x^2 + 8x + 16 - 16 - 48 = 0$

$\therefore (x + 4)^2 - 64 = 0$

$\therefore (x + 4)^2 = 64$

$\therefore x + 4 = 8$ or $x + 4 = -8$

$\therefore x = 4$ or $x = -12$

Method II : Factorisation

$x^2 + 8x - 48 = 0$

$\therefore x^2 + 12x - 4x - 48 = 0$

$\therefore x(x + 12) - 4(x + 12) = 0$

$\therefore (x + 12)(x - 4) = 0$

$\therefore x + 12 = 0$ or $x - 4 = 0$

$\therefore x = -12$ or $x = 4$

Practice Set 2.3

Solve the following quadratic equations by completing the square method.

(1) $x^2 + x - 20 = 0$ (2) $x^2 + 2x - 5 = 0$ (3) $m^2 - 5m = -3$

(4) $9y^2 - 12y + 2 = 0$ (5) $2y^2 + 9y + 10 = 0$ (6) $5x^2 = 4x + 7$

Let's learn.

Formula for solving a quadratic equation

$ax^2 + bx + c$, Divide the polynomial by a ($\because a \neq 0$) to get $x^2 + \frac{b}{a}x + \frac{c}{a}$.

Let us write the polynomial $x^2 + \frac{b}{a}x + \frac{c}{a}$ in the form of difference of two square numbers. Now we can obtain roots or solutions of equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ which is equivalent to $ax^2 + bx + c = 0$.

$ax^2 + bx + c = 0$ . . . (I)

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ . . . . . dividing both sides by a

$$\therefore x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

$$\therefore \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$\therefore \left(x+\frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a^2} = 0$ $\quad\quad$ $\therefore \left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\therefore\ x+\frac{b}{2a} = \sqrt{\frac{b^2-4ac}{4a^2}}$ or $x+\frac{b}{2a} = -\sqrt{\frac{b^2-4ac}{4a^2}}$

$\therefore\ x = -\frac{b}{2a}+\sqrt{\frac{b^2-4ac}{4a^2}}$ or $x = -\frac{b}{2a}-\sqrt{\frac{b^2-4ac}{4a^2}}$

$\therefore x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$

In short the solution is written as $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and these values are denoted by α, β.

$\therefore\ \alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$, $\quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$ . . . . (I)

The values of a, b, c from equation $ax^2 + bx + c = 0$ are substituted in $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and further simplified to obtain the roots of the equation. So

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is the formula used to solve quadratic equation. Out of the two roots any one can be represented by α and the other by β.

That is, instead (I) we can write $\alpha = \frac{-b-\sqrt{b^2-4ac}}{2a}$, $\beta = \frac{-b+\sqrt{b^2-4ac}}{2a}$ . . . . (II)

Note that : If $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ then $\alpha > \beta$, if $\alpha = \frac{-b-\sqrt{b^2-4ac}}{2a}$ then $\alpha < \beta$

Solved Examples

Solve quadratic equations using formula.

Ex. (1) $m^2 - 14m + 13 = 0$

Solution : $m^2 - 14m + 13 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1$, $b = -14$, $c = 13$,

$\therefore b^2 - 4ac = (-14)^2 - 4 \times 1 \times 13$

$= 196 - 52$

$= 144$

$m = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-(-14)\pm\sqrt{144}}{2\times 1}$

$= \frac{14\pm 12}{2}$

$\therefore\ m = \frac{14+12}{2}$ or $m = \frac{14-12}{2}$

$\therefore\ m = \frac{26}{2}$ or $m = \frac{2}{2}$

$\therefore\ m = 13$ or $m = 1$

$\therefore$ 13 and 1 are roots of the equation.

Ex. (2) : $x^2 + 10x + 2 = 0$

Solution : $x^2 + 10x + 2 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1,\ b = 10,\ c = 2,$

$\therefore\ b^2 - 4ac = (10)^2 - 4 \times 1 \times 2$

$= 100 - 8$

$= 92$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{92}}{2 \times 1}$$

$$x = \frac{-10 \pm \sqrt{4 \times 23}}{2}$$

$$= \frac{-10 \pm 2\sqrt{23}}{2}$$

$$= \frac{2(-5 \pm \sqrt{23})}{2}$$

$\therefore\ x = -5 \pm \sqrt{23}$

$\therefore\ x = -5 + \sqrt{23}$ or $x = -5 - \sqrt{23}$

$\therefore$ the roots of the given quadratic equation are $-5 + \sqrt{23}$ and $-5 - \sqrt{23}$.

Ex. (3) : $x^2 - 2x - 3 = 0$

Solution : comparing with $ax^2 + bx + c = 0$

we get $a = 1,\ b = -2,\ c = -3,$

$\therefore\ b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-3) = 4 + 12 = 16$

$$\therefore x = \frac{-(-2) + \sqrt{16}}{2} \text{ or } x = \frac{-(-2) - \sqrt{16}}{2}$$

$$= \frac{2 + 4}{2} \text{ or } \frac{2 - 4}{2}$$

$= 3$ or -1

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For more information :

Let us understand the solution of equation $x^2 - 2x - 3 = 0$ when solved graphically.

$x^2 - 2x - 3 = 0 \quad \therefore \quad x^2 = 2x + 3$ The values which satisfy the equation are the roots of the equation.

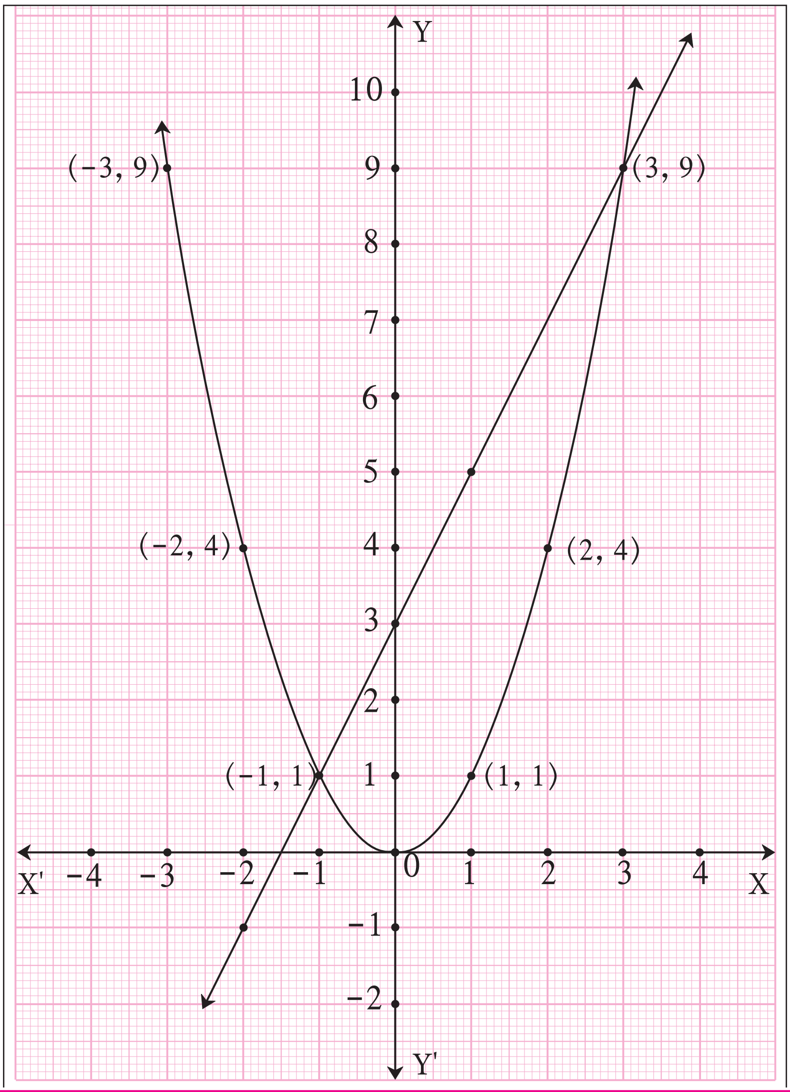
Let $y = x^2 = 2x + 3$. Let us draw graph of $y = x^2$ and $y = 2x + 3$

$y = x^2$

x	3	2	1	0	−1	−2	−3
y	9	4	1	0	1	4	9

$y = 2x + 3$

x	−1	0	1	−2
y	1	3	5	−1



These graphs intersect each other at (-1, 1) and (3, 9).

$\therefore$ The solutions of $x^2 = 2x + 3$ i.e $x^2 - 2x - 3 = 0$ are $x = -1$ or $x = 3$.

In the adjacent diagram the graphs of equations $y = x^2$ and $y = 2x + 3$ are given. From their points of intersection, observe and understand how you get the solutions of $x^2 = 2x + 3$ i.e solutions of $x^2 - 2x - 3 = 0$.

Ex. (4) $25x^2 + 30x + 9 = 0$

Solution : $25x^2 + 30x + 9 = 0$ comparing the equation with $ax^2 + bx + c = 0$ we get $a = 25,\ b = 30,\ c = 9$,

$\therefore\ b^2 - 4ac = (30)^2 - 4 \times 25 \times 9$

$= 900 - 900 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-30 \pm \sqrt{0}}{2 \times 25}$$

$\therefore\ x = \frac{-30+0}{50}$ or $x = \frac{-30-0}{50}$

$\therefore\ x = -\frac{30}{50}$ or $x = -\frac{30}{50}$

$\therefore\ x = -\frac{3}{5}$ or $x = -\frac{3}{5}$

that is both the roots are equal.

Also note that $25x^2 + 30x + 9 = 0$ means $(5x + 3)^2 = 0$

Ex. (5) $x^2 + x + 5 = 0$

Solution : $x^2 + x + 5 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1,\ b = 1,\ c = 5$,

$\therefore\ b^2 - 4ac = (1)^2 - 4 \times 1 \times 5$

$= 1 - 20$

$= -19$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{-19}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{-19}}{2}$$

But $\sqrt{-19}$ is not a real number. Hence roots of the equation are not real.

Activity : Solve the equation $2x^2 + 13x + 15 = 0$ by factorisation method, by completing the square method and by using the formula. Verify that you will get the same roots every time.

Practice Set 2.4

1. Compare the given quadratic equations to the general form and write values of *a, b, c.*

(1) $x^2 - 7x + 5 = 0$ (2) $2m^2 = 5m - 5$ (3) $y^2 = 7y$

2. Solve using formula.

(1) $x^2 + 6x + 5 = 0$ (2) $x^2 - 3x - 2 = 0$ (3) $3m^2 + 2m - 7 = 0$

(4) $5m^2 - 4m - 2 = 0$ (5) $y^2 + \frac{1}{3}y = 2$ (6) $5x^2 + 13x + 8 = 0$

(3) With the help of the flow chart given below solve the equation $x^2 + 2\sqrt{3}x + 3 = 0$ using the formula.

Solution : compare equations $x^2 + 2\sqrt{3}x + 3 = 0$ and $ax^2 + bx + c = 0$ find the values of a,b,c. → Find value of $b^2 - 4ac$ → Write formula to solve quadratic equation. → Substitute values of a,b,c and find roots.

Let's learn.

Nature of roots of a quadratic equation

You know that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are roots of quadratic equation $ax^2 + bx + c = 0$

(1) If $b^2 - 4ac = 0$ then, $x = \frac{-b \pm \sqrt{0}}{2a}$ $\therefore$ $x = \frac{-b + 0}{2a}$ or $x = \frac{-b - 0}{2a}$

$\therefore$ the roots of the quadratic equation are real and equal.

(2) If $b^2 - 4ac > 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i.e. $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ roots of the quadratic equation are real and unequal.

(3) If $b^2 - 4ac < 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are not real numbers $\therefore$ the roots of quadratic equations are not real.

Nature of roots of quadratic equation is determined by the value of $b^2 - 4ac$. $b^2 - 4ac$ is called discriminant of a quadratic equation and is denoted by Greek letter Δ (Delta)

Activity - Fill in the blanks.

	Value of discriminant		Nature of roots
(1)	50	→	
(2)	−30	→	
(3)	0	→	

Solved examples

Ex. (1) Find the value of the discriminant of the equation $x^2 + 10x - 7 = 0$

Solution : Comparing $x^2 + 10x - 7 = 0$ with $ax^2 + bx + c = 0$.

$a = 1,\ b = 10,\ c = -7,$

$\therefore\ b^2 - 4ac = 10^2 - 4 \times 1 \times -7$

$= 100 + 28$

$= 128$

Ex. (2) Determine nature of roots of the quadratic equations.

(i) $2x^2 - 5x + 7 = 0$

Solution : Compare $2x^2 - 5x + 7 = 0$ with

$ax^2 + bx + c = 0$

$a = 2,\ b = -5,\ c = 7,$

$\therefore\ b^2 - 4ac = (-5)^2 - 4 \times 2 \times 7$

$D = 25 - 56$

$D = -31$

$\therefore\ b^2 - 4ac < 0$

$\therefore$ the roots of the equation are not real.

(ii) $x^2 + 2x - 9 = 0$

Solution : Compare $x^2 + 2x - 9 = 0$ with

$ax^2 + bx + c = 0$.

$a = \square,\ b = 2,\ c = \square,$

$\therefore\ b^2 - 4ac = 2^2 - 4 \times \square \times \square$

$D = 4 - \square$

$D = 40$

$\therefore\ b^2 - 4ac > 0$

$\therefore$ the roots of the equation are real and unequal.

Ex. (3) $\sqrt{3}x^2 + 2\sqrt{3}x + \sqrt{3} = 0$

Solution : Compare $\sqrt{3}x^2 + 2\sqrt{3}x + \sqrt{3} = 0$ with $ax^2 + bx + c = 0$

We get $a = \sqrt{3},\ b = 2\sqrt{3},\ c = \sqrt{3},$

$\therefore\ b^2 - 4ac = (2\sqrt{3})^2 - 4 \times \sqrt{3} \times \sqrt{3}$

$= 4 \times 3 - 4 \times 3$

$= 12 - 12$

$= 0$

$\therefore\ b^2 - 4ac = 0$

$\therefore$ Roots of the equation are real and equal.

Let's learn.

The relation between roots of the quadratic equation and coefficients

α and β are the roots of the equation $ax^2 + bx + c = 0$ then,

$$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$

$$= -\frac{2b}{2a}$$

$$\therefore \boxed{\alpha + \beta = -\frac{b}{a}}$$

$$\alpha \times \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{\left(-b+\sqrt{b^2-4ac}\right)\times\left(-b-\sqrt{b^2-4ac}\right)}{4a^2}$$

$$= \frac{b^2-\left(b^2-4ac\right)}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$

$$\therefore \boxed{\alpha\beta = \frac{c}{a}}$$

Activity : Fill in the empty boxes below properly

For $10x^2 + 10x + 1 = 0$,

$\alpha + \beta = \square$ and $\alpha \times \beta = \square$

Solved examples

Ex. (1) If α and β are the roots of the quadratic equation $2x^2 + 6x - 5 = 0$, then find $(\alpha + \beta)$ and $\alpha \times \beta$.

Solution : Comparing $2x^2 + 6x - 5 = 0$ with $ax^2 + bx + c = 0$.

$\therefore\ a = 2,\ b = 6,\ c = -5$

$$\therefore\ \alpha + \beta = -\frac{b}{a} = -\frac{6}{2} = -3$$

$$\text{and } \alpha \times \beta = \frac{c}{a} = \frac{-5}{2}$$

Ex. (2) The difference between the roots of the equation $x^2 - 13x + k = 0$ is 7 find k.

Solution : Comparing $x^2 - 13x + k = 0$ with $ax^2 + bx + c = 0$

$a = 1, \ b = -13, \ c = k$

Let α and β be the roots of the equation.

$\alpha + \beta = -\frac{b}{a} = -\frac{(-13)}{1} = 13 \ldots$ (I)

But $\alpha - \beta = 7 \ldots\ldots\ldots\ldots$ (given) (II)

$2\alpha = 20 \ \ldots$ (adding (I) and (II))

$\therefore \alpha = 10$

$\therefore 10 + \beta = 13 \ldots$ (from (I))

$\therefore \beta = 13 - 10$

$\therefore \beta = 3$

But $\alpha \times \beta = \frac{c}{a}$

$\therefore 10 \times 3 = \frac{k}{1}$

$\therefore k = 30$

Ex. (3) If α and β are the roots of $x^2 + 5x - 1 = 0$ then find -

(i) $\alpha^3 + \beta^3$ (ii) $\alpha^2 + \beta^2$.

Solution : $x^2 + 5x - 1 = 0$

$a = 1, \ b = 5, \ c = -1$

$\alpha + \beta = -\frac{b}{a} = \frac{-5}{1} = -5$

$\alpha \times \beta = \frac{c}{a} = \frac{-1}{1} = -1$

(i) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (-5)^3 - 3 \times (-1) \times (-5)$

$= -125 - 15$

$\alpha^3 + \beta^3 = -140$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-5)^2 - 2 \times (-1)$

$= 25 + 2$

$\alpha^2 + \beta^2 = 27$

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Let's learn.

To obtain a quadratic equation having given roots

Let α and β be the roots of a quadratic equation in variable x

$\therefore x = \alpha$ or $x = \beta$

$\therefore x - \alpha = 0$ or $x - \beta = 0$

$\therefore (x - \alpha)(x - \beta) = 0$

$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$

When two roots of equation are given then quadratic equation can be obtained as x^2 – (addition of roots) x + product of the roots = 0.

Activity (I) : Write the quadratic equation if addition of the roots is 10 and product of the roots = 9

$\therefore$ Quadratic equation : $x^2 - \square\, x + \square = \square$

Activity (II) : What will be the quadratic equation if $\alpha = 2$, $\beta = 5$

It can be written as $x^2 - (\square + \square)\, x + \square \times \square = 0.$

that is $\square\, x^2 - \square\, x + \square = 0.$

Note that, if this equation is multiplied by any non zero number, the roots of the equation are not changed.

Solved examples

Ex. Obtain the quadratic equation if roots are –3, –7.

Solution : Let $\alpha = -3$ and $\beta = -7$

$\therefore \alpha + \beta = (-3) + (-7) = -10$ and $\alpha \times \beta = (-3) \times (-7) = 21$

$\therefore$ and quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (-10)x + 21 = 0$

$\therefore x^2 + 10x + 21 = 0$

Let's remember!

(1) If α and β are roots of quadratic equation $ax^2 + bx + c = 0$,

(i) $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

(ii) $\alpha + \beta = -\frac{b}{a}$ and $\alpha \times \beta = \frac{c}{a}$

(2) Nature of roots of quadratic equation $ax^2 + bx + c = 0$ depends on the value of $b^2 - 4ac$. Hence $b^2 - 4ac$ is called discriminant and is denoted by Greek letter Δ.

(3) If $\Delta = 0$ The roots of quadratic equation are real and equal.

If $\Delta > 0$ then the roots of quadratic equation are real and unequal.

If $\Delta < 0$ then the roots of quadratic equation are not real.

(4) The quadratic equation, whose roots are α and β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Practice Set 2.5

1. Activity : Fill in the gaps and complete.

(1) Quadratic equation $ax^2 + bx + c = 0$ → $b^2 - 4ac = 5$ / $b^2 - 4ac = -5$ → Nature of roots

(2) Sum of roots $= -7$ → Quadratic equation . . . . ← Product of roots $= 5$

(3) If α, β are roots of quadratic equation,

$2x^2 - 4x - 3 = 0$ → $\alpha + \beta = \ldots\ldots$; $\alpha \times \beta = \ldots\ldots$

2. Find the value of discriminant.

(1) $x^2 + 7x - 1 = 0$ (2) $2y^2 - 5y + 10 = 0$ (3) $\sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$

3. Determine the nature of roots of the following quadratic equations.

(1) $x^2 - 4x + 4 = 0$ (2) $2y^2 - 7y + 2 = 0$ (3) $m^2 + 2m + 9 = 0$

4. Form the quadratic equation from the roots given below.

(1) 0 and 4 (2) 3 and -10 (3) $\frac{1}{2}, -\frac{1}{2}$ (4) $2-\sqrt{5},\ 2+\sqrt{5}$

5.★ Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$

6.★ α , β are roots of $y^2 - 2y - 7 = 0$ find,

(1) $\alpha^2 + \beta^2$ (2) $\alpha^3 + \beta^3$

7. The roots of each of the following quadratic equations are real and equal, find k.

(1) $3y^2 + ky + 12 = 0$ (2) $kx\,(x - 2) + 6 = 0$

Let's learn.

Application of quadratic equation

Quadratic equations are useful in daily life for finding solutions of some practical problems. We are now going to learn the same.

Ex. (1) There is a rectangular onion storehouse in the farm of Mr. Ratnakarrao at Tivasa. The length of rectangular base is more than its breadth by 7 m and diagonal is more than length by 1 m. Find length and breadth of the storehouse.

Solution : Let breadth of the storehouse be x m.

$\therefore$ length $= (x + 7)$ m, diagonal $= x + 7 + 1 = (x + 8)$ m

By Pythagorus theorem

$x^2 + (x + 7)^2 = (x + 8)^2$

$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$

$\therefore x^2 + 14x - 16x + 49 - 64 = 0$

$\therefore x^2 - 2x - 15 = 0$

$\therefore \underline{x^2 - 5x} + \underline{3x - 15} = 0$

$\therefore x(x - 5) + 3(x - 5) = 0$

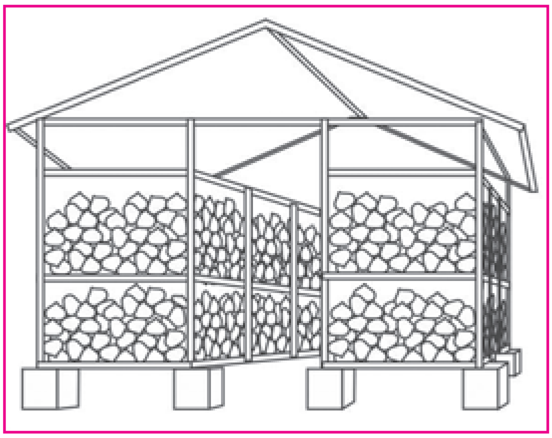
$\therefore (x - 5)(x + 3) = 0$

$\therefore x - 5 = 0$ or $x + 3 = 0$

$\therefore x = 5$ or $x = -3$

But length is never negative $\therefore x \neq -3$

$\therefore x = 5$ and $x + 7 = 5 + 7 = 12$



Onion Storehouse (Kandachal)

$\therefore$ Length of the base of storehouse is 12m and breadth is 5m.

Ex. (2) A train travels 360 km with uniform speed. The speed of the train is increased by 5 km/hr, it takes 48 minutes less to cover the same distance. Find the initial speed of the train.

Solution : Let initial speed of the train be x km/hr.

$\therefore$ New speed is $(x+5)$ km/hr.

time to cover 360 km = $\frac{\text{distance}}{\text{speed}} = \frac{360}{x}$ hours.

New time after increasing speed $= \frac{360}{x+5}$ hours.

from given condition

$\frac{360}{x+5} = \frac{360}{x} - \frac{48}{60}$ ------ (48 min = $\frac{48}{60}$ hrs)

$\therefore \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$

$\therefore \frac{1}{x} - \frac{1}{x+5} = \frac{48}{60\times 360}$ ------ (Dividing both sides by 360)

$\therefore \frac{x+5-x}{x(x+5)} = \frac{4}{5\times 360}$

$\therefore \frac{5}{x^2+5x} = \frac{1}{5\times 90}$

$\therefore \frac{5}{x^2+5x} = \frac{1}{450}$

$\therefore x^2 + 5x = 2250$

$\therefore x^2 + 5x - 2250 = 0$

$\therefore x^2 + 50x - 45x - 2250 = 0$

$\therefore x(x+50) - 45(x+50) = 0$

$\therefore (x+50)(x-45) = 0$

$\therefore x + 50 = 0$ or $x - 45 = 0$

$\therefore x = -50$ or $x = 45$

-2250 → +50, -45

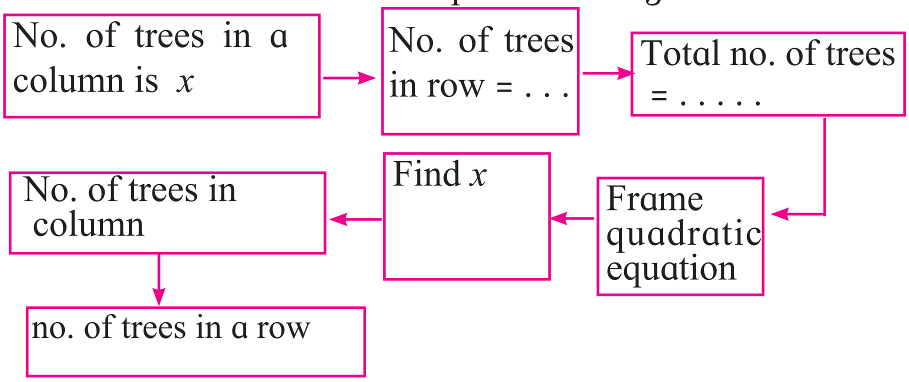
But speed is never negative $\therefore x \neq -50$

$\therefore x = 45$

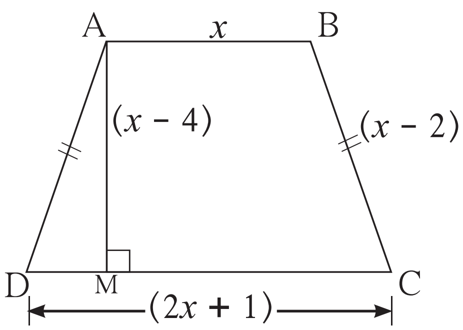
$\therefore$ Initial speed of the train is 45 km/hr.

Practice Set 2.6

1. Product of Pragati's age 2 years ago and 3 years hence is 84. Find her present age.
2. The sum of squares of two consecutive natural numbers is 244; find the numbers.
3. In the orange garden of Mr. Madhusudan there are 150 orange trees. The number of trees in each row is 5 more than that in each column. Find the number of trees in each row and each column with the help of following flow chart.



4. Vivek is older than Kishor by 5 years. The sum of the reciprocals of their ages is $\frac{1}{6}$. Find their present ages.
5. Suyash scored 10 marks more in second test than that in the first. 5 times the score of the second test is the same as square of the score in the first test. Find his score in the first test.
6. ★ Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹ 40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ₹ 600, find production cost of one pot and number of pots he makes per day.
7. ★ Pratik takes 8 hours to travel 36 km downstream and return to the same spot. The speed of boat in still water is 12 km. per hour. Find the speed of water current.
8. ★ Pintu takes 6 days more than those of Nishu to complete certain work. If they work together they finish it in 4 days. How many days would it take to complete the work if they work alone.
9. ★ If 460 is divided by a natural number, quotient is 6 more than five times the divisor and remainder is 1. Find quotient and diviser.
10. ★



In the adjoining fig. □ABCD is a trapezium AB ∥ CD and its area is 33 cm^2. From the information given in the figure find the lengths of all sides of the □ABCD. Fill in the empty boxes to get the solution.

Solution : □ABCD is a trapezium.

AB || CD

A (□ABCD) = $\frac{1}{2}$ (AB + CD) × ☐

33 = $\frac{1}{2}$ (x + 2x + 1) × ☐

∴ ☐ = (3x + 1) × ☐

∴ $3x^2$ + ☐ − ☐ = 0

∴ 3x (. . . .) +10(. . . .) = 0

∴ (3x +10)(----) = 0

∴ (3x +10) = 0 or ☐ = 0

∴ $x = -\frac{10}{3}$ or x = ☐

But length is never negative.

∴ $x \neq -\frac{10}{3}$ ∴ x = ☐

AB = ---, CD = --- , AD = BC = ---

Problem Set - 2

1. Choose the correct answers for the following questions.

(1) Which one is the quadratic equation ?

(A) $\frac{5}{x} - 3 = x^2$ (B) $x(x + 5) = 2$ (C) $n - 1 = 2n$ (D) $\frac{1}{x^2}(x + 2) = x$

(2) Out of the following equations which one is not a quadratic equation ?

(A) $x^2 + 4x = 11 + x^2$ (B) $x^2 = 4x$ (C) $5x^2 = 90$ (D) $2x - x^2 = x^2 + 5$

(3) The roots of $x^2 + kx + k = 0$ are real and equal, find k.

(A) 0 (B) 4 (C) 0 or 4 (D) 2

(4) For $\sqrt{2}x^2 - 5x + \sqrt{2} = 0$ find the value of the discriminant.

(A) −5 (B) 17 (C) $\sqrt{2}$ (D) $2\sqrt{2} - 5$

(5) Which of the following quadratic equations has roots 3, 5 ?

(A) $x^2 - 15x + 8 = 0$ (B) $x^2 - 8x + 15 = 0$

(C) $x^2 + 3x + 5 = 0$ (D) $x^2 + 8x - 15 = 0$

(6) Out of the following equations, find the equation having the sum of its roots −5.

(A) $3x^2 - 15x + 3 = 0$ (B) $x^2 - 5x + 3 = 0$

(C) $x^2 + 3x - 5 = 0$ (D) $3x^2 + 15x + 3 = 0$

(7) $\sqrt{5}m^2 - \sqrt{5}m + \sqrt{5} = 0$ which of the following statement is true for this given equation ?

(A) Real and uneual roots (B) Real and equal roots

(C) Roots are not real (D) Three roots.

(8) One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.

(A) −2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2

2. Which of the following equations is quadratic ?

(1) $x^2 + 2x + 11 = 0$ (2) $x^2 - 2x + 5 = x^2$ (3) $(x + 2)^2 = 2x^2$

3. Find the value of discriminant for each of the following equations.

(1) $2y^2 - y + 2 = 0$ (2) $5m^2 - m = 0$ (3) $\sqrt{5}x^2 - x - \sqrt{5} = 0$

4. One of the roots of quadratic equation $2x^2 + kx - 2 = 0$ is -2, find k.

5. Two roots of quadratic equations are given ; frame the equation.

(1) 10 and -10 (2) $1-3\sqrt{5}$ and $1+3\sqrt{5}$ (3) 0 and 7

6. Determine the nature of roots for each of the quadratic equation.

(1) $3x^2 - 5x + 7 = 0$ (2) $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$ (3) $m^2 - 2m + 1 = 0$

7. Solve the following quadratic equations.

(1) $\frac{1}{x+5} = \frac{1}{x^2}$ (2) $x^2 - \frac{3x}{10} - \frac{1}{10} = 0$ (3) $(2x + 3)^2 = 25$

(4) $m^2 + 5m + 5 = 0$ (5) $5m^2 + 2m + 1 = 0$ (6) $x^2 - 4x - 3 = 0$

8.★ Find m if $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ has real and equal roots.

9.★ The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

10.★ Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation $2x^2 + 2(p + q)x + p^2 + q^2 = 0$

11.★ Mukund possesses ₹ 50 more than what Sagar possesses. The product of the amount they have is 15,000. Find the amount each one has.

12.★ The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.

13.★ Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.

14.★ Mr. Dinesh owns an agricultural farm at village Talvel. The length of the farm is 10 meter more than twice the breadth. In order to harvest rain water, he dug a square shaped pond inside the farm. The side of pond is $\frac{1}{3}$ of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and of the pond

15.★ A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely ?

3 Arithmetic Progression

Let's study.

- Sequence
- Arithmetic Progression
- n^{th} term of an A.P.
- Sum of n terms of an A.P.

Let's learn.

Sequence

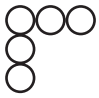
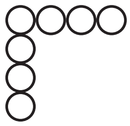
We write numbers 1, 2, 3, 4, . . . in an order. In this order we can tell the position of any number. For example, number 13 is at 13^{th} position. The numbers 1, 4, 9, 16, 25, 36, 49, . . . are also written in a particular order. Here $16 = 4^2$ is at 4^{th} position. similarly, $25 = 5^2$ is at the 5^{th} position; $49 = 7^2$ is at the 7^{th} position. In this set of numbers also, place of each number is detremined.

A set of numbers where the numbers are arranged in a definite order, like the natural numbers, is called a **sequence.**

In a sequence a particular number is written at a particular position. If the numbers are written as a_1, a_2, a_3, a_4 . . . then a_1 is first, a_2 is second, . . . and so on. It is clear that a_n is at the n^{th} place. A sequence of the numbers is also represented by alphabets f_1, f_2, f_3, . . . and we find that there is a definite order in which numbers are arranged.

When students stand in a row for drill on the playground they form a sequence. We have experienced that some sequences have a particular pattern.

Complete the given pattern

Pattern									
Number of circles	1	3	5	7					

Pattern	ΔΔ Δ ΔΔ	ΔΔΔ Δ Δ ΔΔΔ	ΔΔΔΔ Δ Δ Δ ΔΔΔΔ				
Number of triangles	5	8	11				

Look at the patterns of the numbers. Try to find a rule to obtain the next number from its preceding number. This helps us to write all the next numbers.

See the numbers 2, 11, −6, 0, 5, −37, 8, 2, 61 written in this order.

Here $a_1 = 2$, $a_2 = 11$, $a_3 = -6$, . . . This list of numbers is also a sequence. But in this case we cannot tell why a particular term is at a particular position ; similarly we cannot tell a definite relation between the consecutive terms.

In general, only those sequences are studied where there is a rule which determines the next term.

For example (1) 4, 8, 12, 16 . . . (2) 2, 4, 8, 16, 32, . . .

(3) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}$ . . .

Terms in a sequence

In a sequence, ordered terms are represented as $t_1, t_2, t_3, \ldots\ldots t_n \ldots$ In general sequence is written as $\{t_n\}$. If the sequence is infinite, for every positive integer *n*, there is a term t_n.

Activity I : Some sequences are given below. Show the positions of the terms by $t_1, t_2, t_3, \ldots$

(1) 9, 15, 21, 27, . . . Here $t_1 = 9$, $t_2 = 15$, $t_3 = 21$, . . .

(2) 7, 7, 7, 7, . . . Here $t_1 = 7$, $t_2 = \square$, $t_3 = \square$, . . .

(3) −2, −6, −10, −14, . . . Here $t_1 = -2$, $t_2 = \square$, $t_3 = \square$, . . .

Activity II : Some sequences are given below. Check whether there is any rule among the terms. Find the similarity between two sequences.

To check the rule for the terms of the sequence look at the arrangements on the next page, and fill the empty boxes suitably.

(1) 1, 4, 7, 10, 13, . . . (2) 6, 12, 18, 24, . . .

(3) 3, 3, 3, 3, . . . (4) 4, 16, 64, . . .

(5) −1, −1.5, −2, −2.5, . . . (6) $1^3, 2^3, 3^3, 4^3, \ldots$

Let's find the relation in these sequences. Let's understand the thought behind it.

(1) 1, 4, 7, 10, ☐, ☐, . . .

1 (+3), 4 (+3), 7 (+3), 10 (+3), ☐, ☐

(2) 6, 12, 18, 24, ☐, ☐, . . .

6 + 6, 12 + 6, 18 + 6, ☐, ☐

(3) 3, 3, 3, 3, ☐, ☐ . . .

3 + 0, 3 + 0, 3 + 0, ☐, ☐

(4) 4, 16, 64, ☐, ☐, ☐, . . .

4 × 4, 16 × 4, 64 × 4, ☐ × 4, ☐ × 4

(5) -1, -1.5, -2, -2.5, ☐ . . .

$(-1)+(-0.5)$, $-1.5 + (-0.5)$, $-2 + (-0.5)$, $-2.5 + (-0.5)$

(6) 1^3, 2^3, 3^3, . . .

Here in the sequences (1), (2), (3), (5), the similarity is that next term is obtained by adding a particular number to the previous number. Each ot these sequences is called an Arithmetic Progression.

Sequence (4) is not an arithmetic progression. In this sequence the next term is obtained by mutliplying the previous term by a particular number. This type of sequences is called a Geometric Progression.

Sequence (6) is neither arithmetic progression nor geometric progression.

This year we are going to study arithmetic progression.

Arithmetic Progression

Some sequences are given below. For every sequence write the next three terms.

(1) 100, 70, 40, 10, . . . (2) $-7, -4, -1, 2, \dots$ (3) 4, 4, 4, . . .

In the given sequences, observe how the next term is obtained.

1st term 2nd term 3rd term

(1) 100 70 40 10 -20 -50

100+(-30) 70+(-30) 40+(-30) 10+(-30) (-20)+(-30)

(2) -7 -4 -1 2 5 8

-7+3 -4+3 -1+3 2+3 5+3

(3) 4 4 4 4 4 . . . .

4 + 0 4 + 0 4 + 0 4 + 0

In each sequence above, every term is obtained by adding a particular number in the previous term. The difference between two consecutive terms is constant.

The diference in ex. (i) is negative, in ex. (ii) it is positive and in ex. (iii) it is zero.

If the difference between two consecutive terms is constant then it is called the common difference and is generally denoted by letter d.

In the given sequence if the difference between two consecutive terms $(t_{n+1} - t_n)$ is constant then the sequence is called Arithmetic Progression (A.P.). In this sequence $t_{n+1} - t_n = d$ is the common difference.

In an A.P. if first term is denoted by a and common difference is d then,

$t_1 = a$, $t_2 = a + d$

$t_3 = (a + d) + d = a + 2d$

A.P. having first term as a and common difference d is

$a, (a + d), (a + 2d), (a + 3d), \ldots\ldots$

Let's see some examples of A.P.

Ex.(1) Arifa saved ₹ 100 every month. In one year the total amount saved after every month is as given below.

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Saving (')	100	200	300	400	500	600	700	800	900	1000	1100	1200

The numbers showing the total saving after every month are in A.P.

Ex. **(2)** Pranav borrowed ₹ 10000 from his friend and agreed to repay ₹ 1000 per month. So the remaining amount to be paid in every month will be as follows.

No. of month	1	2	3	4	5	...	...	...
Amount to be paid (₹)	10,000	9,000	8,000	7,000	...	2,000	1,000	0

Ex. **(3)** Consider the table of 5, that is numbers divisible by 5.

5, 10, 15, 20, . . . 50, 55, 60, ... is an inifinite A.P.

Ex (1) and (2) are finite A.P. while (3) is an infinite A.P.

Let's remember!

(1) In a sequence if difference $(t_{n+1} - t_n)$ is constant then the sequence is called an arithmetic progression.
(2) In an A.P. the difference between two consecutive terms is constant and is denoted by d.
(3) Difference d can be positive, negative or zero.
(4) In an A.P. if the first term is a, and common difference is d then the terms in the sequence are a, $(a + d)$, $(a + 2d)$, . . .

Activity : Write one example of finite and infinite A.P. each.

Solved examples

Ex. **(1)** Which of the following sequences are A.P ? If it is an A.P, find next two terms.

(i) 5, 12, 19, 26, . . . (ii) 2, −2, −6, −10, . . .

(iii) 1, 1, 2, 2, 3, 3, . . . (iv) $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, \ldots$

Solution : (i) In this sequence 5, 12, 19, 26, . . . ,

First term = $t_1 = 5$, $t_2 = 12$, $t_3 = 19$, . . .

$t_2 - t_1 = 12 - 5 = 7$

$t_3 - t_2 = 19 - 12 = 7$

Here first term is 5 and common difference which is constant is $d = 7$

∴ This sequence is an A.P.

Next two terms in this A.P. are 26 + 7 = 33 and 33 + 7 = 40.

Next two terms in given A.P. are 33 and 40

(ii) In the sequence 2, −2, −6, −10, . . . ,

$t_1 = 2$, $t_2 = -2$, $t_3 = -6$, $t_4 = -10$ . . .

$t_2 - t_1 = -2 - 2 = -4$

$t_3 - t_2 = -6 - (-2) = -6 + 2 = -4$

$t_4 - t_3 = -10 - (-6) = -10 + 6 = -4$

From this difference between two consecutive terms that is $t_n - t_{n-1} = -4$

$\therefore$ $d = -4$, which is constant. $\therefore$ It is an A.P.

Next two terms in this A.P. are $(-10) + (-4) = -14$ and $(-14) + (-4) = -18$

(iii) In the sequence 1, 1, 2, 2, 3, 3, . . . ,

$t_1 = 1$, $t_2 = 1$, $t_3 = 2$, $t_4 = 2$, $t_5 = 3$, $t_6 = 3$ . . .

$t_2 - t_1 = 1 - 1 = 0$ $t_3 - t_2 = 2 - 1 = 1$

$t_4 - t_3 = 2 - 2 = 0$ $t_3 - t_2 \neq t_2 - t_1$

In this sequence difference between two consecutive terms is not constant.

$\therefore$ This sequence is not an A.P.

(iv) In the sequence $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots$,

$t_1 = \frac{3}{2}$, $t_2 = \frac{1}{2}$, $t_3 = -\frac{1}{2}$, $t_4 = -\frac{3}{2}$ . . .

$t_2 - t_1 = \frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$

$t_3 - t_2 = -\frac{1}{2} - \frac{1}{2} = -\frac{2}{2} = -1$

$t_4 - t_3 = -\frac{3}{2} - (-\frac{1}{2}) = -\frac{3}{2} + \frac{1}{2} = -\frac{2}{2} = -1$

Here the common difference $d = -1$ which is constant.

$\therefore$ Given sequence is an A.P. Let's find next two terms of this A.P.

$-\frac{3}{2} - 1 = -\frac{5}{2}$, $\frac{5}{2} - 1 = -\frac{7}{2}$

$\therefore$ Next two terms are $-\frac{5}{2}$ and $-\frac{7}{2}$

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Ex. **(2)** The first term a and common difference d are given. Find first four terms of A.P.

(i) $a = -3, d = 4$ (ii) $a = 200, d = 7$

(iii) $a = -1, d = -\frac{1}{2}$ (iv) $a = 8, d = -5$

Solution : (i) Given $a = -3, d = 4$

$t_1 = -3$

$t_2 = t_1 + d = -3 + 4 = 1$

$t_3 = t_2 + d = 1 + 4 = 5$

$t_4 = t_3 + d = 5 + 4 = 9$

$\therefore$ A.P. is = $-3, 1, 5, 9, \ldots$

(ii) Given $a = 200, d = 7$

$a = t_1 = 200$

$t_2 = t_1 + d = 200 + 7 = 207$

$t_3 = t_2 + d = 207 + 7 = 214$

$t_4 = t_3 + d = 214 + 7 = 221$

$\therefore$ A.P. is = $200, 207, 214, 221, \ldots$

(iii) $a = -1, d = -\frac{1}{2}$

$a = t_1 = -1$

$t_2 = t_1 + d = -1 + (-\frac{1}{2}) = -\frac{3}{2}$

$t_3 = t_2 + d = -\frac{3}{2} + (-\frac{1}{2}) = -\frac{4}{2} = -2$

$t_4 = t_3 + d = -2 + (-\frac{1}{2})$

$= -2 - \frac{1}{2} = -\frac{5}{2}$

$\therefore$ A.P. is = $-1, -\frac{3}{2}, -2, -\frac{5}{2}, \ldots$

(iv) $a = 8, d = -5$

$a = t_1 = 8$

$t_2 = t_1 + d = 8 + (-5) = 3$

$t_3 = t_2 + d = 3 + (-5) = -2$

$t_4 = t_3 + d = -2 + (-5) = -7$

$8, 3, -2, -7, \ldots$

$\therefore$ A.P. is = $8, 3, -2, -7, \ldots$

Practice Set 3.1

1. Which of the following sequences are A.P. ? If they are A.P. find the common difference .

(1) $2, 4, 6, 8, \ldots$ (2) $2, \frac{5}{2}, 3, \frac{7}{3}, \ldots$ (3) $-10, -6, -2, 2, \ldots$

(4) $0.3, 0.33, .0333, \ldots$ (5) $0, -4, -8, -12, \ldots$ (6) $-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \ldots$

(7) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$ (8) $127, 132, 137, \ldots$

2. Write an A.P. whose first term is a and common difference is d in each of the following.

(1) $a = 10, d = 5$ (2) $a = -3, d = 0$ (3) $a = -7, d = \frac{1}{2}$

(4) $a = -1.25, d = 3$ (5) $a = 6, d = -3$ (6) $a = -19, d = -4$

3. Find the first term and common difference for each of the A.P.

(1) 5, 1, −3, −7, . . .   (2) 0.6, 0.9, 1.2, 1.5, . . .

(3) 127, 135, 143, 151, . . .   (4) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$

Let's think.

- Is 5, 8, 11, 14, . . . an A.P. ? If so then what will be the 100th term? Check whether 92 is in this A.P.? Is number 61 in this A.P.?

Let's learn.

n^{th} term of an A. P.

In the sequence 5, 8, 11, 14, . . . the difference between two consecutive terms is 3. Hence, this sequence is an A.P.

Here the first term is 5. If 3 is added to 5 we get the second term 8. Similarly to find 100th term what should be done?

	First term	Second term	Third term	. . .
Number	5,	5 + 3 = 8	8 + 3 = 11	. . .

In this way reaching upto 100th term will be time consuming. Let's see if we can find any formula for it.

5	8	11	14	. . .	. . .	. . .	. . .
5	$5 + 1 \times 3$	$5 + 2 \times 3$	$5 + 3 \times 3$	. . .	$5 + (n - 1) \times 3$	$5 + n \times 3$	. . .
1st term t_1	2nd term t_2	3rd term t_3	4th term t_4	. . .	n^{th} term t_n	$(n + 1)^{th}$ term t_{n+1}	. . .

Generally in the A.P. $t_1, t_2, t_3, \ldots$ If first term is a and common difference is d,

$$t_1 = a$$

$$t_2 = t_1 + d = a + d = a + (2 - 1)\,d$$

$$t_3 = t_2 + d = a + d + d = a + 2d = a + (3 - 1)d$$

$$t_4 = t_3 + d = a + 2d + d = a + 3d = a + (4 - 1)d$$

We get $t_n = a + (n - 1)\,d$.

Using the above formula we can find the 100th term of the A.P. 5, 8, 11, 14, . . .

Here $a = 5 \quad d = 3$

$t_n = a + (n - 1)d$

$\therefore \; t_{100} = 5 + (100 - 1) \times 3$

$= 5 + 99 \times 3$

$= 5 + 297$

$t_{100} = 302$

100th tem of this A.P. is 302.

Let's check whether 61 is in this A.P. To find the answer we use the same formula.

$t_n = a + (n - 1)d$

$t_n = 5 + (n - 1) \times 3$

If 61 is n^{th} term means t_n, then

$\therefore \; 61 = 5 + 3n - 3$

$= 3n + 2$

$\therefore \; 3n = 59$

$\therefore \; n = \frac{59}{3}$

But then, n is not a natural number.

$\therefore$ 61 is not in this A.P.

Let's think.

Kabir's mother keeps a record of his height on each birthday. When he was one year old, his height was 70 cm, at 2 years he was 80 cm tall and 3 years he was 90 cm tall. His aunt Meera was studying in the 10th class. She said, ''it seems like Kabir's height grows in Arithmetic Progression''. Assuming this, she calculated how tall Kabir will be at the age of 15 years when he is in the 10th ! She was shocked to find it. You too assume that Kabir grows in A.P. and find out his height at the age of 15 years.

Solved examples

Ex. (1) Find t_n for following A.P. and then find 30th term of A.P.

3, 8, 13, 18, . . .

Solution :Given A.P. 3, 8, 13, 18, . . .

Here $t_1 = 3,\ t_2 = 8,\ t_3 = 13,\ t_4 = 18, \ldots$

$d = t_2 - t_1 = 8 - 3 = 5$

We know that $t_n = a + (n-1)d$

$\therefore\ t_n = 3 + (n-1) \times 5 \quad \because\ a = 3,\ d = 5$

$\therefore\ t_n = 3 + 5n - 5$

$\therefore\ t_n = 5n - 2$

$\therefore$ 30th term $= t_{30} = 5 \times 30 - 2$

$= 150 - 2 \quad = 148$

Ex. (2) Which term of the following A.P. is 560?

2, 11, 20, 29, . . .

Solution : Given A.P. 2, 11, 20, 29, . .

Here $a = 2,\ d = 11 - 2 = 9$

n^{th} term of this A.P. is 560.

$t_n = a + (n-1)d$

$\therefore\ 560 = 2 + (n-1) \times 9$

$= 2 + 9n - 9$

$\therefore\ 9n = 567$

$\therefore\ n = \frac{567}{9} = 63$

$\therefore$ 63rd term of given A.P. is 560.

Ex. (3) Check whether 301 is in the sequence 5, 11, 17, 23, . . . ?

Solution : In the sequence 5, 11, 17, 23,. . .

$t_1 = 5,\ t_2 = 11,\ t_3 = 17,\ t_4 = 23, \ldots$

$t_2 - t_1 = 11 - 5 = 6$

$t_3 - t_2 = 17 - 11 = 6$

$\therefore$ This sequence is an A.P.

First term $a = 5$ and $d = 6$

If 301 is n^{th} term, then.

$t_n = a + (n-1)d = 301$

$\therefore\ 301 = 5 + (n-1) \times 6$

$= 5 + 6n - 6$

$\therefore\ 6n = 301 + 1 = 302$

$\therefore\ n = \frac{302}{6}$. But it is not an integer.

$\therefore$ 301 is not in the given sequence.

Ex. (4) How many two digit numbers are divisible by 4?

Solution : List of two digit numbers divisible by 4 is

12, 16, 20, 24, . . . , 96.

Let's find how many such numbers are there.

$t_n = 96, \quad a = 12, \quad d = 4$

From this we will find the value of n.

$t_n = 96$, $\therefore$ By formula,

$96 = 12 + (n-1) \times 4$

$= 12 + 4n - 4$

$\therefore\ 4n = 88$

$\therefore\ n = 22$

$\therefore$ There are 22 two digit numbers divisible by 4.

Ex. (5) – The 10th term and the 18th term of an A.P. are 25 and 41 respectively then find 38th term of that A.P., similarly if n^{th} term is 99. Find the value of n.

Solution : In the given A.P. $t_{10} = 25$ and $t_{18} = 41$.

We know that, $t_n = a + (n - 1)d$

$\therefore \quad t_{10} = a + (10 - 1)\, d$

$\therefore \quad 25 = a + 9d \; . . . \; (I)$

Similarly $t_{18} = a + (18 - 1)\, d$

$\therefore \quad 41 = a + 17d \; . . . \; (II)$

$25 = a + 9d \; . . .$ From (I) .

$a = 25 - 9d.$

Substituting this value in equation II.

$\therefore$ Equation (II) $a + 17d = 41$

$\therefore \quad 25 - 9d + 17d = 41$

$8d = 41 - 25 = 16$

$d = 2$

Substituting $d = 2$ in equation I.

$a + 9d = 25$

$\therefore \; a + 9 \times 2 = 25$

$\therefore \; a + 18 = 25$

$\therefore \; a = 7$

Now, $t_n = a + (n - 1)d$

$\therefore \quad t_{38} = 7 + (38 - 1) \times 2$

$t_{38} = 7 + 37 \times 2$

$t_{38} = 7 + 74$

$t_{38} = 81$

If n^{th} term is 99, then to find value of n.

$t_n = a + (n - 1)d$

$99 = 7 + (n - 1) \times 2$

$99 = 7 + 2n - 2$

$99 = 5 + 2n$

$\therefore \; 2n = 94$

$\therefore \; n = 47$

$\therefore$ In the given progression 38th term is 81 and 99 is the 47th term.

Practice Set 3.2

1. Write the correct number in the given boxes from the following A. P.

(i) 1, 8, 15, 22, . . .

Here $a = \square$, $t_1 = \square$, $t_2 = \square$, $t_3 = \square$,

$t_2 - t_1 = \square - \square = \square$

$t_3 - t_2 = \square - \square = \square$ $\therefore d = \square$

(ii) 3, 6, 9, 12, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, $t_4 = \square$,

$t_2 - t_1 = \square$, $t_3 - t_2 = \square$ $\therefore d = \square$

(iii) −3, −8, −13, −18, . . .

Here $t_3 = \square$, $t_2 = \square$, $t_4 = \square$, $t_1 = \square$,

$t_2 - t_1 = \square$, $t_3 - t_2 = \square$ $\therefore a = \square$, $d = \square$

(iv) 70, 60, 50, 40, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, . . .

$\therefore a = \square$, $d = \square$

2. Decide whether following sequence is an A.P., if so find the 20th term of the progression.

−12, −5, 2, 9, 16, 23, 30, . . .

3. Given Arithmetic Progression 12, 16, 20, 24, . . . Find the 24th term of this progression.

4. Find the 19th term of the following A.P.

7, 13, 19, 25, . . .

5. Find the 27th term of the following A.P.

9, 4, −1, −6, −11, . . .

6. Find how many three digit natural numbers are divisible by 5.

7. The 11th term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41th term of that A.P.

8. 11, 8, 5, 2, . . . In this A.P. which term is number −151 ?

9. In the natural numbers from 10 to 250, how many are divisible by 4 ?

10. In an A.P. 17th term is 7 more than its 10th term. Find the common difference.

The Wise Teacher

Once upon a time, there lived a king. He appointed two teachers Tara and Meera to teach horse riding for one year to his children Yashwantraje and Geetadevi. He asked both of them how much salary they wanted.

Tara said, "Give me 100 gold coins in first month and every month increase the amount by 100 gold coins." Meera said, "Give me 10 gold coins in the first month and every month just double the amount of the previous month."

The king agreed. After three months Yashwantraje said to his sister, "My teacher is smarter than your teacher as she had asked for more money." Geetadevi said, "I also thought the same, I asked Meeratai about it. She only smiled and said compare the salaries after 8 months. I calculated their 9 months salaries. You can also check."

Months	1	2	3	4	5	6	7	8	9	10	11	12
Tara's salary	100	200	300	400	500	600	700	800	900	-	-	-
Meera's salary	10	20	40	80	160	320	640	1280	2560	-	-	-

Complete the above table.

Tara's salary 100, 200, 300, 400, . . . is in A.P.

$t_1 = 100$, $t_2 = 200$, $t_3 = 300$,. . . $t_2 - t_1 = 100 = d$

Common difference is 100.

Meera's salary 10, 20, 40, 80, . . . is not an A.P. Reason is $20 - 10 = 10$, $40 - 20 = 20$, $80 - 40 = 40$ So the common difference is not constant.

But here each term is double the preceding term.

Here $\frac{t_2}{t_1} = \frac{20}{10} = 2$, $\frac{t_3}{t_2} = \frac{40}{20} = 2$, $\frac{t_4}{t_3} = \frac{80}{40} = 2$

$\therefore \frac{t_{n+1}}{t_n}$ The ratio of a term and its preceding term is constant. This type of progression is called a geometric progression. Notice that if ratio $\frac{t_{n+1}}{t_n}$ is greater than 1, then geometric progression will increase faster than arithmetic progression.

If the ratio is smaller than 1, note how the geometric progression changes.

This year we are going to study Arithmetic Progression only. We have seen how to find the n^{th} term of an A.P. Now we are going to see how to find the sum of the first n terms.

Quick Addition

Three hundred years ago there was a single teacher school in Germany. The teacher was Buttner and he had an assistant Johann Martin Bortels. He used to teach alphabets to the children and sharpen their pencils. Buttner was a strict teacher. One day he wanted to do some work and wanted peace in the class, so he tried to occupy all students with a lengthy addition. They were asked to add all intergers from 1 to 100. In few minutes one slate was slammed on the floor. He looked at Carl Gauss and asked, ''I asked you to add all integers from 1 to 100. Why did you keep the slate down? Don't you want to do it ?''

Carl Gauss said, ''I have done the addition.''

The teacher asked, ''How did you do it so quickly? You wouldn't have written all the numbers ! What is the answer ?''

Carl Gauss said, ''Five thousand fifty''

Teacher was so surprised and asked him, 'How do you find the answer?''

Carl Gauss explained his quick addition method:

Nos. in increasing order	1	2	3	4	. . . .	100
	+	+	+	+		+
Nos. in decreasing order	100	99	98	97	. . . .	1
Sum	101	101	101	101		101

The sum of each pair is 101. This sum occurs 100 times so 101×100 is the product needed. It is 10100. In this 1 to 100 are counted two times. Therefore, half of 10100 is 5050 and sum of 1 to 100 is 5050. The teacher appreciated his work.

Now using this method of Gauss, let's find sum of n terms of an A.P.

Johann Friedrich Carl Gauss

30th April 1777 – 23rd February 1855.

Carl Gauss was a great German mathematician, He was born in Braunschweig, he was the only son of uneducated parents. He showed a glimpse of his intelligence in Buttner's school. After some years, Buttner's helper, Johann Martin Bartels and Gauss became friends. Together, they published a book on Algebra. Bartels made the other people realise the extra ordinary intelligence of Gauss.

Let's learn.

Sum of first n terms of an A. P.

Arithmetic Progression $a, a + d, a + 2d, a + 3d, \ldots \; a + (n - 1)d$

In this progression a is the first term and d is the common difference. Let's write the sum of first n terms as S_n.

$S_n = [a] + [a + d] + \ldots + [a+(n-2)d] + [a+(n-1)d]$

Reversing the terms and rewritting the expression again,

$S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots + [a + d] + [a]$

On adding,

$2S_n = [a+a+(n-1)d] + [a + d+a+(n-2)d] + \ldots + [a+(n-2)d+ a + d] + [a+(n-1)d+a]$

$2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d] \ldots n$ times.

$\therefore \; 2S_n = n\,[2a+(n-1)d]$

$\therefore \; S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = na + \frac{n(n-1)}{2}d$

Ex. Let's find the sum of first 100 terms of A.P. 14, 16, 18, ...

Here $a = 14$, $d = 2$, $n = 100$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\therefore \; S_{100} = \frac{100}{2}[2 \times 14+(100-1) \times 2]$$

$$= 50\,[28 + 198]$$

$$= 50 \times 226 = 11300$$

$\therefore$ Sum of first 100 terms of given A.P. is 11,300

Let's remember!

For the given Arithmetic Progression, if first term is a and common diffrence is d then

$$t_n = [a+(n-1)d]$$

$$S_n = \frac{n}{2}[2a+(n-1)d] = na + \frac{n(n-1)}{2}d$$

Let's find one more formula for sum of first n terms.

In the A.P. $a, a+d, a+2d, a+3d, \ldots\ a+(n-1)d$

First term = $t_1 = a$ and n^{th} term is $[a+(n-1)d]$

Now $S_n = \frac{n}{2}\left[a + a+(n-1)d\right]$

$\therefore\ S_n = \frac{n}{2}[t_1 + t_n] = \frac{n}{2}$ [First term+ last term]

Solved examples

Ex. (1) Find the sum of first n natural numbers.

Solution : First n natural numbers are $1, 2, 3, \ldots, n$.

Here $a = 1,\ d = 1,\ n^{th}$ term = n

$\therefore\ S_n = 1 + 2 + 3 + \ldots + n$

$S_n = \frac{n}{2}$ [First term+ last term] . . . . . (by the formula)

$= \frac{n}{2}[1+n]$

$= \frac{n(n+1)}{2}$

$\therefore$ Sum of first n natural number is $\frac{n(n+1)}{2}$.

Ex. (2) Find the sum of first n even natural numbers.

Solution : First n even natural numbers are $2, 4, 6, 8, \ldots, 2n$.

t_1 = First term = 2 , t_n = last term = $2n$

Method I

$= \frac{n}{2}[t_1 + t_n]$

$= \frac{n}{2}[2 + 2n]$

$= \frac{n}{2} \times 2(1+n)$

$= n(1+n)$

Method II

$S_n = 2 + 4 + 6 \ldots + 2n$

$= 2(1 + 2 + 3 + \ldots + n)$

$= \frac{2[n(n+1)]}{2}$

$= n(1+n)$

Method III

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2 \times 2+(n-1)2]$

$= \frac{n}{2}[4+2n-2]$

$= \frac{n}{2}[2+2n]$

$= \frac{n}{2} \times 2(1+n)$

$= n(1+n)$

$\therefore$ Sum of first n even natural numbers is $n(1+n)$.

Ex. (3) Find the sum of first n odd natural numbers.

Solution : First n natural numbers

$1, 3, 5, 7, \ldots, (2n - 1)$.

$a = t_1 = 1$ and $t_n = (2n - 1)$, $d = 2$

Method I

$S_n = \frac{n}{2}[t_1 + t_n]$

$= \frac{n}{2}[1 + (2n - 1)]$

$= \frac{n}{2}[1 + 2n - 1]$

$= \frac{n}{2} \times 2n$

$= n^2$

Method II

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1) \times 2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n$

$= n^2$

Method III

$1 + 3 + \ldots\ 2n-1$

$= (1 + 2 + 3 + \ldots + 2n)$

$\quad - (2 + 4 + 6 + \ldots + 2n)$

$= \frac{2n(2n+1)}{2} - \frac{2n(n+1)}{2}$

$= (2n^2 + n) - (n^2 + n)$

$= n^2$

Ex. (4) Find the sum of all odd numbers from 1 to 150.

Solution : 1 to 150 all odd numbers are $1, 3, 5, 7, \ldots, 149$.

Which is an A.P.

Here $a = 1$ and $d = 2$. First let's find how many odd numbers are there from 1 to 150, so find the value of n, if $t_n = 149$

$t_n = a + (n - 1)d$

$149 = 1 + (n - 1)2$ $\quad\therefore 149 = 1 + 2n - 2$

$\therefore n = 75$

Now let's find the sum of these 75 numbers $\quad 1 + 3 + 5 + \ldots + 149$.

$a = 1$ and $d = 2$, $n = 75$

Method I $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \boxed{}$

$S_n = \boxed{} \times \boxed{}$

$S_n = \boxed{}$

Method II $S_n = \frac{n}{2}[t_1 + t_n]$

$S_n = \frac{75}{2}[1 + 149]$

$S_n = \boxed{} \times \boxed{}$

$S_n = \boxed{}$

Practice Set 3.3

1. First term and common difference of an A.P. are 6 and 3 respectively ; find S_{27}.

$a = 6, \ d = 3, \ S_{27} = ?$

$S_n = \frac{n}{2}[\square + (n-1)\,d]$

$S_{27} = \frac{27}{2}[12 + (27-1)\,\square]$

$= \frac{27}{2} \times \square$

$= 27 \times 45 \quad = \square$

2. Find the sum of first 123 even natural numbers.

3. Find the sum of all even numbers from 1 to 350.

4. In an A.P. 19^{th} term is 52 and 38^{th} term is 128, find sum of first 56 terms.

5. Complete the following activity to find the sum of natural numbers from 1 to 140 which are divisible by 4.

From 1 to 140, natural numbers divisible by 4

↓

4, 8, ... , 136

↓

How many numbers ? ∴ $n = \square$

↓

$n = \square$, $a = \square$, $d = \square$

↓

$t_n = a+(n-1)d$

↓

$136 = \square + (n - 1) \times \square$

→ $n = \square$ → $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{\square} = \frac{\square}{2}[\qquad]$ $= \square$

Sum of numbers from 1 to 140, which are divisible by 4 = $\square$

6.★ Sum of first 55 terms in an A.P. is 3300, find its 28^{th} term.

7.★ In an A.P. sum of three consecutive terms is 27 and their product is 504, find the terms. (Assume that three consecutive terms in A.P. are $a-d$, a, $a+d$.)

8.★ Find four consecutive terms in an A.P. whose sum is 12 and sum of 3rd and 4th term is 14.
(Assume the four consecutive terms in A.P. are $a-d$, a, $a+d$, $a+2d$.)

9.★ If the 9th term of an A.P. is zero then show that the 29th term is twice the 19th term.

Let's learn.

Application of A.P.

Ex. (1) A mixer manufacturing company manufactured 600 mixers in 3rd year and in 7th year they manufactured 700 mixers. If every year there is same growth in the production of mixers then find (i) Production in the first year (ii) Production in 10th year (iii) Total production in first seven years.

Solution : Addition in the number of mixers manufactured by the company per year is constant therefore the number of production in successive years is in A.P. (i) Let's assume that company manufactured t_n mixers in the n^{th} year then as per given information,

$t_3 = 600,\ t_7 = 700$

We know that $t_n = a+(n-1)d$

$t_3 = a+(3-1)d$

$a + 2d = 600 \ldots$ (I)

$t_7 = a+(7-1)d$

$t_7 = a+6d = 700$

$a+2d = 600 \quad \therefore$ Substituting $a = 600 - 2d$ in equation (II),

$600 - 2d + 6d = 700$

$4d = 100 \quad \therefore\ d = 25$

$a+2d = 600 \quad \therefore\ a + 2 \times 25 = 600$

$a + 50 = 600 \quad \therefore\ a = 550$

$\therefore$ Production in first year was 550.

(ii) $t_n = a+(n-1)d$

$t_{10} = 550+(10-1) \times 25$

$= 550 + 225$

Production in 10th year was 775.

(iii) For finding total production in first 7 years let's use formula for S_n.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_n = \frac{7}{2}[1100 + 150] = \frac{7}{2}[1250] = 7 \times 625 = 4375$

Total production in first 7 years is 4375 mixers.

Ex. (2) Ajay sharma repays the borrowed amount of ₹ 3,25,000 by paying ₹ 30500 in the first month and then decreases the payment by ₹ 1500 every month. How long will it take to clear his amount?

Solution : Let the time required to clear the amount be n months. The monthly payment decreases by ₹ 1500. Therefore the payments are in A.P.

First term = a = 30500, d = −1500

Amount = S_n = 3,25,000

$S_n = \frac{n}{2}[2a+(n-1)d]$

$3,25,000 = \frac{n}{2}[2 \times 30500+(n-1)d]$

$= \frac{n}{2}[2 \times 30500 - 1500n + 1500]$

$3,25,000 = 30500n - 750n^2 + 750n$

$\therefore 750n^2 - 31250n + 325000 = 0$

divide both sides by 250.

$\therefore 3n^2 - 125n + 1300 = 0$

$\therefore 3n^2 - 60n - 65n + 1300 = 0$

$\therefore 3n(n-20) - 65(n-20) = 0$

$\therefore (n - 20)(3n - 65) = 0$

$\therefore n - 20 = 0 , 3n - 65 = 0$

$\therefore n = 20$ or $n = \frac{65}{3} = 21\frac{2}{3}$

In an A.P. n is a natural number.

$\therefore \quad n \neq \frac{65}{3} \qquad \therefore \quad n = 20$

(Or, after 20 months, S_{20} = 3,25,000 then the total amount will be repaid. It is not required to think about further period of time.)

$\therefore$ To clear the amount 20 months are needed.

Ex. **(3)** Anvar saves some amount every month. In first three months he saves ₹ 200, ₹ 250 and ₹ 300 respectively. In which month will he save ₹ 1000 ?

Solution: Saving in first month ₹ 200; Saving in second month ₹ 250; .....

200, 250, 300, . . . this is an A.P.

Here $a = 200$, $d = 50$, Let's find n using t_n formula and then find S_n.

$$t_n = a+(n-1)d$$
$$= 200 + (n-1)50$$
$$= 200 + 50n - 50$$
$$1000 = 150 + 50n$$
$$150 + 50n = 1000$$
$$50n = 1000 - 150$$
$$50n = 850$$
$$\therefore n = 17$$

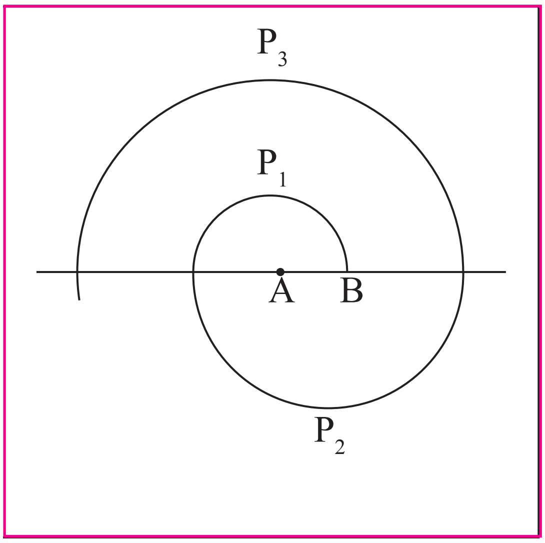
In the 17^{th} month he will save ₹ 1000.

Let's find that in 17 months how much total amount is saved.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{17}{2}[2 \times 200+(17-1) \times 50]$$
$$= \frac{17}{2}[400 + 800]$$
$$= \frac{17}{2}[1200]$$
$$= 17 \times 600$$
$$= 10200$$

In 17 months total saving is ₹ 10200.

Ex. (4) As shown in the figure, take point A on the line and draw a half circle P_1 of radius 0.5 with A as centre. It intersects given line in point B. Now taking B as centre draw a half circle P_2 of radius 1 cm which is on the other side of the line.



Now again taking A as centre draw a half circle P_3 of radius 1.5 cm. If we draw half circles like this having radius 0.5 cm, 1 cm, 1.5 cm, 2 cm, we get a figure of spiral shape. Find the length of such spiral shaped figure formed by 13 such half circles. ($\pi = \frac{22}{7}$)

Solution : Semi circumferences $P_1, P_2, P_3, \ldots$ are drawn by taking centres A, B, A, B,... It is given that radius of the first circle is 0.5 cm. The radius of the second circle is 1.0 cm,... From this information we will find $P_1, P_2, P_3, \ldots P_3$.

Length of the first semi circumference = $P_1 = \pi r_1 = \pi \times \frac{1}{2} = \frac{\pi}{2}$

$$P_2 = \pi r_2 = \pi \times 1 = \pi$$

$$P_3 = \pi r_3 = \pi \times 1.5 = \frac{3}{2}\pi$$

The lengths are $P_1, P_2, P_3, \ldots$, and the numbers $\frac{1}{2}\pi, 1\pi, \frac{3}{2}\pi, \ldots$ are in A.P.

Here $a = \frac{1}{2}\pi$, $d = \frac{1}{2}\pi$, From this let's find S_{13}.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{13} = \frac{13}{2}\left[2 \times \frac{\pi}{2} + (13-1) \times \frac{1}{2}\pi\right]$$

$$= \frac{13}{2}[\pi + 6\pi]$$

$$= \frac{13}{2} \times 7\pi =$$

$$= \frac{13}{2} \times 7 \times \frac{22}{7}$$

$$= 143 \text{ cm.}$$

$\therefore$ The total length of spiral shape formed by 13 semicircles is 143 cm.

Ex. (5) In the year 2010 in the village there were 4000 people who were literate. Every year the number of literate people increases by 400. How many people will be literate in the year 2020 ?

Solution :

Year	2010	2011	2012	. . .	2020
Literate People	4000	4400	4800	. . .	☐

$a = 4000, \quad d = 400 \quad n = 11$

$$t_n = a+(n-1)d$$
$$= 4000 + (11-1)400$$
$$= 4000 + 4000$$
$$= 8000$$

In year 2020, 8000 people will be literate.

Ex. (6) In year 2015, Mrs. Shaikh got a job with salary ₹ 1,80,000 per year. Her employer agreed to give ₹ 10,000 per year as increment. Then in how many years will her annual salary be ₹ 2,50,000 ?

Solution :

Year	First Year (2015)	Second Year (2016)	Third Year (2017)	. . .
Salary (₹)	[1,80,000]	[1,80,000 + 10,000]		. . .

$a = 1,80,000 \qquad d = 10,000 \qquad n = ? \qquad t_n = 2,50,000$ ₹

$t_n = a+(n-1)d$

$2,50,000 = 1,80,000 + (n-1) \times 10,000$

$(n-1) \times 10000 = 70,000$

$(n-1) = 7$

$n = 8$

In the 8^{th} year her annual salary will be ₹ 2,50,000.

Practice Set 3.4

1. On 1st Jan 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st Dec 2016 what would be her total saving?
2. A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly instalments. Each instalment being less than the preceding one by ₹ 40. Find the amount of the first and last instalment.
3. Sachin invested in a national saving certificate scheme. In the first year he invested ₹ 5000, in the second year ₹ 7000, in the third year ₹ 9000 and so on. Find the total amount that he invested in 12 years.
4. There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the 15th row and also find how many total seats are there in the auditorium?
5. Kargil's temperature was recorded in a week from Monday to Saturday. All readings were in A.P.The sum of temperatures of Monday and Saturday was $5°$ C more than sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was $-30°$ celsius then find the temperature on the other five days.
6. On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Find the total number of trees in the 25 rows.

Problem Set - 3

1. Choose the correct alternative answer for each of the following sub questions.

(1) The sequence $-10, -6, -2, 2, \ldots$

(A) is an A.P., Reason $d = -16$ (B) is an A.P., Reason $d = 4$

(C) is an A.P., Reason $d = -4$ (D) is not an A.P.

(2) First four terms of an A.P. are ....., whose first term is -2 and common difference is -2.

(A) $-2, 0, 2, 4$ (B) $-2, 4, -8, 16$

(C) $-2, -4, -6, -8$ (D) $-2, -4, -8, -16$

(3) What is the sum of the first 30 natural numbers ?

(A) 464 (B) 465 (C) 462 (D) 461

(4) For an given A.P. $t_7 = 4$, $d = -4$ then $a = \ldots$

(A) 6 (B) 7 (C) 20 (D) 28

(5) For an given A.P. $a = 3.5$, $d = 0$, $n = 101$, then $t_n = \ldots$

(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

(6) In an A.P. first two terms are -3, 4 then 21st term is . . .

(A) -143 (B) 143 (C) 137 (D) 17

(7) If for any A.P. $d = 5$ then $t_{18} - t_{13} = \ldots$

(A) 5 (B) 20 (C) 25 (D) 30

(8) Sum of first five multuiples of 3 is. . .

(A) 45 (B) 55 (C) 15 (D) 75

(9) 15, 10, 5, . . . In this A.P. sum of first 10 terms is . . .

(A) -75 (B) -125 (C) 75 (D) 125

(10) In an A.P. 1st term is 1 and the last term is 20. The sum of all terms is = 399 then $n = \ldots$

(A) 42 (B) 38 (C) 21 (D) 19

2. Find the fourth term from the end in an A.P. $-11, -8, -5, \ldots, 49$.

3. In an A.P. the 10th term is 46, sum of the 5th and 7th term is 52. Find the A.P.

4. The A.P. in which 4th term is -15 and 9th term is -30. Find the sum of the first 10 numbers.

5. Two A.P.'s are given 9, 7, 5, . . . and 24, 21, 18, ... If n^{th} term of both the progressions are equal then find the value of n and n^{th} term.

6. If sum of 3rd and 8th terms of an A.P. is 7 and sum of 7th and 14th terms is -3 then find the 10th term.

7. In an A.P. the first term is -5 and last term is 45. If sum of all numbers in the A.P. is 120, then how many terms are there? What is the common difference?

8. Sum of 1 to n natural numbers is 36, then find the value of n.

9. Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623.

10. There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three terms is 429. Write the A.P.

11★ If first term of an A.P. is a, second term is b and last term is c, then show that sum of all terms is $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

12.★ If the sum of first p terms of an A.P. is equal to the sum of first q terms then show that the sum of its first (p + q) terms is zero. (p $\neq$ q)

13.★ If m times the m^{th} term of an A.P. is eqaul to n times n^{th} term then show that the $(m + n)^{th}$ term of the A.P. is zero.

14. ₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity.

$$\text{Simple interest} = \frac{P \times R \times N}{100}$$

$$\text{Simple interest after 1 year} = \frac{1000 \times 10 \times 1}{100} = \square$$

$$\text{Simple interest after 2 year} = \frac{1000 \times 10 \times 2}{100} = \square$$

$$\text{Simple interest after 3 year} = \frac{\square \times \square \times \square}{100} = 300$$

According to this the simple interest for 4, 5, 6 years will be 400, $\square$, $\square$ respectively.

From this $d = \square$, and $a = \square$

Amount of simple interest after 20 years

$$t_n = a + (n-1)d$$

$$t_{20} = \square + (20-1)\square$$

$$t_{20} = \square$$

Amount of simple interest after 20 years is = $\square$

N5JRFH

4 Financial Planning

Let's study.

- GST - Introduction
- GST - Computation and ITC
- GST - Tax Invoice
- Shares, Mutual Funds and SIP

Let's discuss.

Teacher : Dear students, in our country which tax system is in practice for business ?

Ayush : GST system is in practice.

Teacher : Very good ! What do you know about GST ?

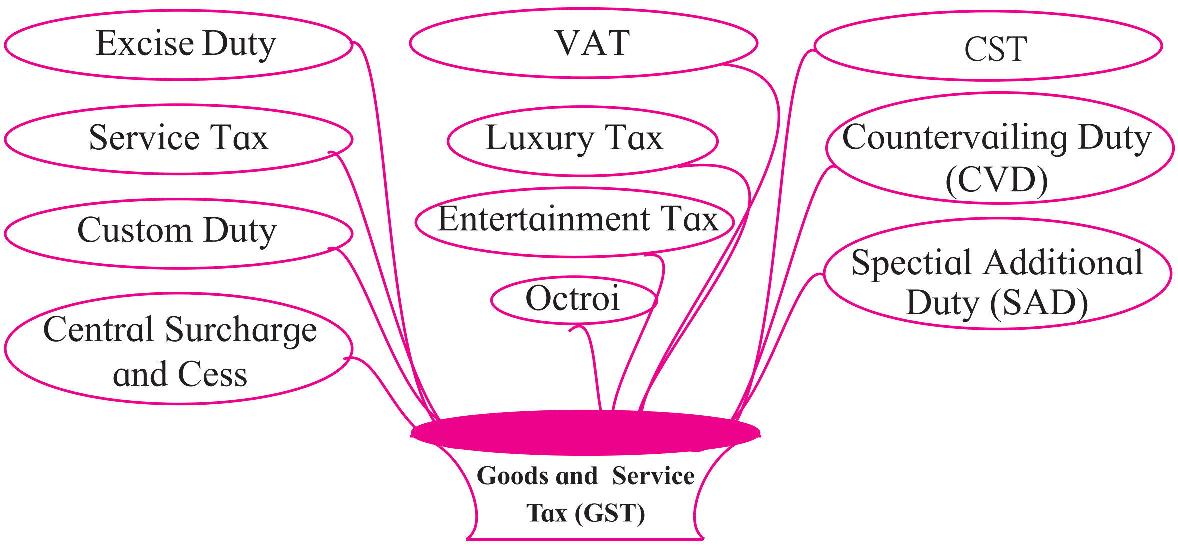
Ayan : **GST** stands for **G**oods and **S**ervice **T**ax.

Aisha : Yes, the whole country follows the same tax levy system.

Teacher : Correct, before GST every state had variety of taxes levied at different stages of trading. Observe the picture given below and tell which taxes existed before GST and are now subsumed in GST ?

Shafik : Taxes that existed before were Excise Duty, Custom Duty, VAT, Entertainment tax, Central sales tax, Service tax, Octroi etc.

Teacher : All these taxes are subsumed under GST, that is why GST is One nation, One tax, One market. GST is in effect from 1st of July 2017.



Let's learn.

Tax Invoice

Tax Invoice of goods purchase (Sample)

SUPPLIER : A to Z SWEET MART GSTIN : 27ABCDE1234H1Z5
143, Shivaji Rasta, Mumbai : 400001, Maharashtra.
Mo No. 92636 92111 e-mail - atoz@gmail.com

Invoice No. GST/110 **Invoice Date**: 31-Jul-2017

S. No.	HSN code	Name of Product	Rate	Quantity	Taxable Amount	CGST Rate	CGST Tax	SGST Rate	SGST Tax	Total ₹
1	210690	Pedhe	₹ 400 per kg.	500 gm.	200.00	2.5%	5.00	2.5%	5.00	210.00
2	210691	Chocolate	₹ 80	1Bar	80.00	14%	11.20	14%	11.20	102.40
3	2105	Ice-cream	₹ 200 per pack	1 pack (500 gm.)	200.00	9%	18.00	9%	18.00	236.00
4	1905	Bread	₹ 35	1 pack	35.00	0%	0.00	0%	0.00	35.00
5	210690	Butter	₹ 500 per kg.	250 gm	125.00	6%	7.50	6%	7.50	140.00
					Total Rupees		41.70		41.70	723.40

Ved : In the invoice we see some new words, please explain them.

Teacher : CGST and SGST are two components of GST. CGST is **C**entral **G**oods and **S**ervice **T**ax which is to be paid to the central government. Whereas SGST is **S**tate **G**oods and **S**ervice **T**ax which is to be paid to the state government.

Ria : What is in the right most corner with a long queue of numbers and alphabets?

Teacher : It is GSTIN, dealer's indentification number. (**GSTIN**- **G**oods and **S**ervice **T**ax **I**dentification **N**umber). GSTIN is mandatory for the dealer whose annual turn over in previous financial year exceeds rupees 20 lacs. You know that PAN has 10 alpha-numerals, similarly GSTIN has 15 alpha-numerals. It includes 10 digit PAN of the dealer.

e.g. : 27 A B C D E 1 2 3 4 H 1 Z 5

- 27 → 2 digits for state code
- ABCDE1234H → 10 digit PAN of the firm
- 1 → 1 represents one registration
- Z → Uniform for all (By default)
- 5 → Check sum digit (confirms the validity of GSTIN when entered on GST portal)

Note : Here 27 is the state code of Maharashtra. From 27, one can understand that a person or a firm is registered in Maharashtra.

Jennie : There is a word HSN code in the tax invoice.

Teacher : All Goods are classified by giving numerical code called HSN code. It is to be quoted in the tax invoice. Full form of **HSN** is **H**armonized **S**ystem of **N**omenclature.

Joseph : As usual there is name of the shop, address, state, date, invoice number, mobile number and e-mail ID also in the tax invoice.

Teacher : Now we will see how the GST is charged for each product (Goods) in the bill. Observe the given bill and fill in the boxes with the appropriate number. Price of 1 kg of Pedhe is ₹ 400, therefore cost of 500 gm. of Pedhe is ₹ 200.

- CGST at the rate of 2.5% is ₹ ☐ and SGST at the rate of ☐ % is ₹ 5.00.
- It means that the rate of GST on Pedhe is 2.5+2.5=5% and hence the total GST is ₹10.
- The rate of GST on chocolate is ☐ % and hence the total GST is ₹ ☐
- Rate of GST on Ice-cream is ☐ %, hence the total cost of ice-cream is ₹ ☐
- On butter CGST rate is ☐ % and SGST rate is also ☐ %. So GST rate on butter is ☐ %.

Aditya : Rate of GST on bread is 0 %. The rate of CGST and SGST is same for each product.

Ninad : Rates of GST are different for diferent products such as 0%, 5%, 12%, 18% and 28%.

Teacher : These rates are fixed and prescribed by the government. Now let us observe the tax invoice of services provided. Fill in the blanks with the help of given information.

Tax invoice of services provided (Sample)

Food Junction, Khed-Shivapur, Pune   Invoice No. 58
Mo. No. 7588580000 email - ahar.khed@yahoo.com
GSTIN : 27 AAAAA5555B1ZA   Invoice Date 25-Dec-2017

SAC	Food items	Qty	Rate (in ₹)	Taxable amount	CGST		SGST	
9961	Coffee	1	20	20.00	2.5%	. . .	2.5%	. . .
9963	Masala Tea	1	10	10.00	. . . .	. . .	2.5%	. . .
9962	Masala Dosa	2	60	. . . .	2.5%	. . .	. . .	. . .
			Total	. . .		. . .	. . .	. . .
					Grand Total = ₹ --------			

Teacher : Compare both, Goods and Service Tax invoices and find the difference in codes.

Patrick : In the tax invoice for Goods, there is HSN code while in service invoice there is SAC.

Teacher : Services are also classified and special code numbers are given. These are called SAC or **S**ervice **A**ccounting **C**ode.

Sr. No.	Types	Rate of GST	Goods and services items list
I	Zero rated	0%	**Goods** - Essential Commodities like food grains, fruits, vegetables, milk, salt, earthen pots etc. **Services** - Charitable trust activities, transport of water, use of roads and bridges, public library, agriculture related services, Education and Health care services etc.
II	Low rated	5%	**Goods**- Commonly used items- LPG cylinder, Tea, coffee, oil, Honey, Frozen vegetables, spices, sweets etc. **Services** - Railway transport services, bus transport services, taxi services, Air transport (economy class), Hotels providing food and beverages etc.
III	Standard rated (I slab)	12%	**Goods**- Consumer goods: Butter, Ghee, Dry fruits, Jam, Jelly, Sauces, Pickles. Mobile phone etc. **Services** - Printing job work, Guest house, Services related to construction business.
IV	Standard rated (II slab)	18% (Most of the Goods and services are included)	**Goods** - Marble, Granite, Perfumes, Metal items, Computer, Printer, Monitor, CCTV etc. **Services** - Courier services, Outdoor catering, Circus, Drama, Cinema, Exhibitions, Currency exchange, Broker Services in share trading etc.
V	Highly rated	28%	**Goods** - Luxury items, Motor Cycles and spare parts, Luxury cars, Pan-masala, Vacuum cleaner, Dish washer, AC, Washing machine, Fridge, Tobacco products, Aerated water etc. **Services** - Five star Hotel accommodation Amusement parks, Water parks, Theme parks, Casino, Race course, IPL games, Air transport (business class) etc.

Reference : www.cbec.gov.in (Central Board of Excise & Customs)
Besides these rates, find on which goods are the GST rates levied between 0 and 5 ?

Note : – The rates and types of GST are as prescribed by the government at the time of writing this chapter. GST rates are subject to change. Electricity, petrol, diesel etc are not under purview of GST.

Activity I : Make a list of ten things you need in your daily life. Find the GST rates with the help of GST rate chart given here, News papers or books, internet, or the bills of purchases. Verify these rates with the list prepared by your friends.

Goods	Rate of GST	Goods	Rate of GST
1. Sketch book	- - - - -	6. - - - --	- - - - -
2. Compass-box	- - - - -	7.- - - - -	- - - - -
3. - - - - -	- - - - -	8. - - - - -	- - - - -
4. - - - - -	- - - - -	9. - - - - -	- - - - -
5. - - - - -	- - - - -	10. - - - - -	- - - - -

Activity II : Make a list of ten services and their GST rates as per actvity I. (e.g. Railway and ST bus booking services etc.) You can also collect service bills and complete the given information

Services	Rate of GST	Services	Rate of GST
1.Railway booking	- - - - -	6. - - - --	- - - - -
2.Courier Services	- - - - -	7.- - - - -	- - - - -
3.- - - --	- - - - -	8. - - - - -	- - - - -
4.- - - - -	- - - - -	9. - - - - -	- - - - -
5.- - - - -	- - - - -	10. - - - - -	- - - - -

Activity III : Complete the given table by writing remaining SAC and HSN codes with rates and add some more items in the list.

Services	SAC	GST rate	Goods	HSN	GST rate
Railway transport services	996511	--	Dulux paint	3208	28%
Airways services (economy)	996411	--	Ball bearing	84821011	28%
Currency exchange services	997157	--	Speedometer	8714	28%
Brokerage services	997152	--	Potatoes	0701	0%
Taxi services	996423	--	--	--	--
Five-star Hotel services	9963	--	--	--	--
--	--	--	--	--	--

Activity IV : Prepare a chart of Goods and Services as in activity III with codes and GST rates. Stick or draw the pictures of Goods and services to enhance your activity.

Note : Rates on goods and services, SAC and HSN codes are only for information, no need to remember them.

Project : Collect various Goods and Service tax invoices. Study these invoices with reference to GST and discuss with your classmates.

Solved Examples

Ex. (1) Arati Gas Agency supplied LPG cylinder to the consumer for taxable value of ₹ 545. GST charged is 5%. What is the amount of CGST and SGST in the tax invoice ? What is the total amount paid by the consumer ? Find the amount of GST to be paid by Arati Gas Agency.

Solution : Rate of GST = 5% $\therefore$ Rate of CGST 2.5%, and Rate of SGST = 2.5%.

$$\text{CGST} = \frac{2.5}{100} \times 545 = 13.625 = ₹\ 13.63$$

$\therefore$ SGST = CGST = ₹ 13.63

Amount paid by the consumer = Taxable value + CGST + SGST

= 545 + 13.63 + 13.63

= 572.26

Arati Gas Agency has to pay CGST = ₹ 13.63. and SGST = ₹ 13.63

$\therefore$ Total GST to be paid = 13.63 × 2 = ₹ 27.26.

Ex. (2) Courier service agent charged total ₹ 590 to courier a parcel from Nashik to Nagpur. In the tax invoice taxable value is ₹ 500 on which CGST is ₹ 45 and SGST is ₹ 45. Find the rate of GST charged for this service.

Solution : Total GST = CGST + SGST = 45 + 45 = ₹ 90.

$$\text{Rate of GST} = \frac{90}{500} \times 100 = 18\%$$

$\therefore$ Rate of GST charged by agent is 18%.

Ex. (3) Shreekar bought a Laptop with 10% discount on printed price. The printed price of that Laptop was ₹ 50,000. 18% GST was charged on discounted price. Find the amount of CGST and SGST. What amount did Shreekar pay ?

Solution : Discount = 10% of 50,000 = ₹ 5,000

$\therefore$ Taxable value of Laptop = 50,000 - 5,000 = ₹ 45,000.

$\therefore$ Rate of GST = 18% (Given) $\therefore$ Rate of CGST = 9%

$\therefore$ CGST = 9% of 45,000 = $\frac{9}{100} \times 45000$ = ₹ 4050.

$\therefore$ SGST = ₹ 4050.

$\therefore$ Amount paid = 45000 + 4050 + 4050 = ₹ 53,100.

Ans. Shreekar paid ₹ 53,100 for the Laptop.

Note : Value of Goods on which GST is levied is called taxable value. Total value or Invoice value is the value with GST. If not mentioned take the selling prices as taxable price. Remember that in tax invoice CGST amount is always equal to SGST amount.

Practice Set 4.1

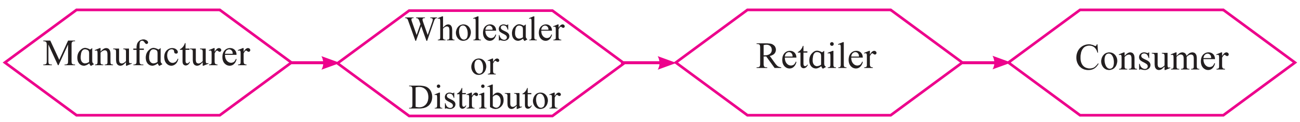
1. 'Pawan Medical' supplies medicines. On some medicines the rate of GST is 12%, then what is the rate of CGST and SGST?
2. On certain article if rate of CGST is 9% then what is the rate of SGST? and what is the rate of GST?
3. 'M/s. Real Paint' sold 2 tins of lustre paint and taxable value of each tin is ₹ 2800. If the rate of GST is 28%, then find the amount of CGST and SGST charged in the tax invoice.
4. The taxable value of a wrist watch belt is ₹ 586. Rate of GST is 18%. Then what is price of the belt for the customer ?
5. The total value (with GST) of a remote-controlled toy car is ₹ 1770. Rate of GST is 18% on toys. Find the taxable value, CGST and SGST for this toy-car.
6. 'Tiptop Electronics' supplied an AC of 1.5 ton to a company. Cost of the AC supplied is ₹ 51,200 (with GST). Rate of CGST on AC is 14%. Then find the following amounts as shown in the tax invoice of Tiptop Electronics.

   (1) Rate of SGST (2) Rate of GST on AC (3) Taxable value of AC

   (4) Total amount of GST (5) Amount of CGST (6) Amount of SGST
7. Prasad purchased a washing-machine from 'Maharashtra Electronic Goods'. The discount of 5% was given on the printed price of ₹ 40,000. Rate of GST charged was 28%. Find the purchase price of washing machine. Also find the amount of CGST and SGST shown in the tax invoice.

Let's learn.

GST in trading chain



Trading Chain (within state)

Downloaded from https:// www.studiestoday.com

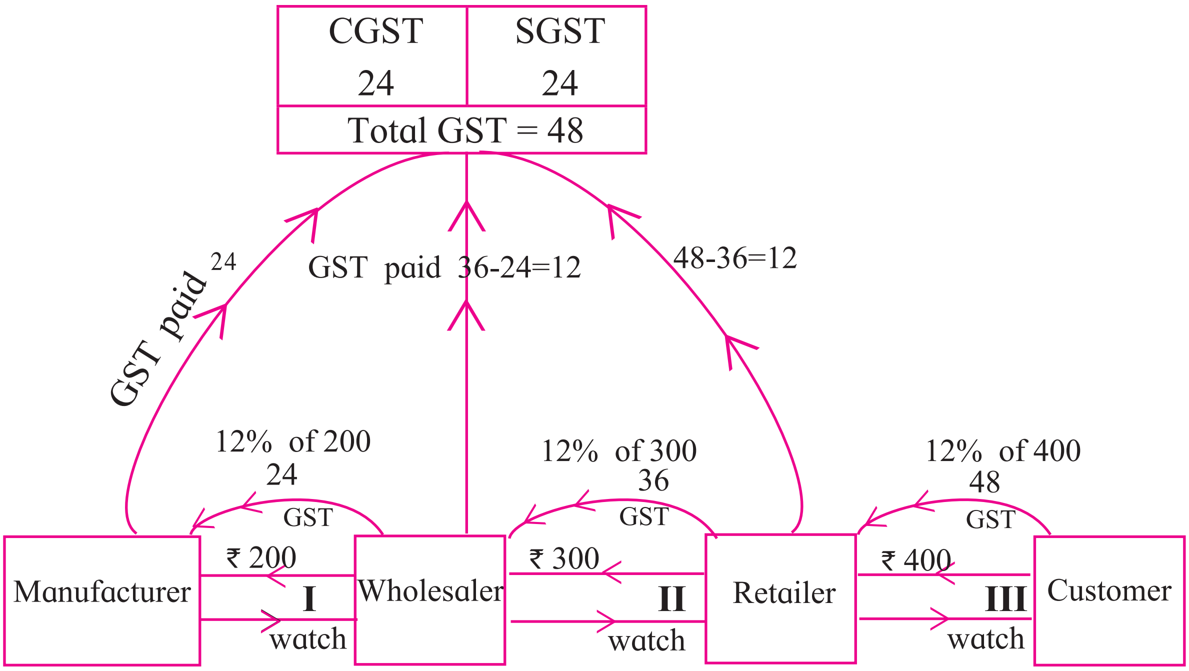
Let's learn through an example how GST is charged and paid to the government at every stage of trading.

Illustration : Suppose manufacturer of a watch has sold one watch for

₹ 200. (including profit) to the wholsaler. Wholesaler sold that watch for ₹ 300 to the retailer. Retailer sold it to the customer for ₹ 400. Rate of GST charged at every stage is 12%. Then how each trader pays GST and takes his input tax credit (ITC) at every stage of transaction is shown in the following flow-chart. Observe and study it.

Explanation :

Here three financial transactions took place till the watch from manufacturer reaches to the customer. How the taxes are charged, collected and paid to the central government and state government at each stage is shown below. The statement of taxes paid is given in the table thereafter.



Here all the three financial transactions took place in one state. Therefore three tax invoices were generated as follows. Each tax invoice shows the brief computation of GST.

GST in Tax Invoice I		
Price of watch	=	₹ 200
CGST 6%	=	₹ 12
SGST 6%	=	₹ 12
Total	=	₹ 224

Tax invoice of manufacturer

B2B

GST in Tax Invoice II		
Price of watch	=	₹ 300
CGST 6%	=	₹ 18
SGST 6%	=	₹ 18
Total	=	₹ 336

Tax invoice of wholesaler

B2B

GST in Tax Invoice III		
Price of watch	=	₹ 400
CGST 6%	=	₹ 24
SGST 6%	=	₹ 24
Total	=	₹ 448

Tax invoice of relailer

B2C

Let's remember!

- Trading between GSTIN holders is known as Business to Business, in short B2B. Trading between GSTIN holder and consumer is known as Business to Consumer, in short B2C. This is the last link in the trading chain.

Bifurcation of taxes paid to the government by the traders at each stage.

	CGST		SGST		Total GST paid
• By the manufacturer	₹ 12	+	₹ 12	=	₹ 24
• By the wholesaler	₹ 6	+	₹ 6	=	₹ 12
• By the retailer	₹ 6	+	₹ 6	=	₹ 12
	₹ 24	+	₹ 24	=	₹ 48

Note : Observe that at every stage, a trader has paid GST after subtracting the tax he paid at the time of purchase from the tax he collected at the time of sale.

At the end the customer paid ₹ 448 for the watch. So the total tax paid by the traders was indirectly paid by the customer. So GST is a type of indirect tax. In this case, the wholesaler and retailer used their input tax as credit and got back all the GST paid by them.

What is Input Tax Credit ? (ITC)

GST is levied and collected at every stage of trading from manufacturer to consumer. When trader pays GST at the time of purchase, it is called **'Input tax'** and he collects GST at the time of sale which is called **'Output tax'.** At the time of paying GST to the government a trader deducts the input tax from the output tax and pays the remaining tax. This deduction of input tax is called **Input Tax Credit**.

GST payable = Output tax - ITC

In short, while paying taxes to the government each trader in the trading chain subtracts the tax paid at the time of purchase from the tax collected at the time of sale and pays the remaining tax.

Solved Examples

Ex. (1) Mr. Rohit is a retailer. He paid GST of ₹ 6500 at the time of purchase. He collected GST of ₹ 8000 at the time of sale. (i) Find his input tax and out put tax. (ii) What is his Input tax credit ? (iii) Find his payable GST. (iv) Hence find the payable CGST and payable SGST.

Solution : Mr. Rohit's payable GST means, GST to be paid to the government by Mr. Rohit.

(i) Output tax (tax collected at the time of sale) = ₹ 8000.

(ii) Input tax (tax paid at the time of purchase) = ₹ 6500

$\therefore$ ITC = ₹ 6500.

(iii) GST payable = Output tax - ITC

= ₹ 8000 - ₹ 6500 = ₹1500

(iv) $\therefore$ payable CGST = $\frac{1500}{2}$ = ₹ 750 and payable SGST = ₹ 750.

Ex. (2) M/s. Jay Chemicals purchased a liquid soap having taxable value ₹ 8000 and sold it to the consumers for the taxable value ₹ 10,000. Rate of GST is 18%. Find the CGST and SGST payable by M/s. Jay Chemicals.

Solution :

Input Tax = 18% of 8000

$= \frac{18}{100} \times 8000 =$ ₹ 1440.

Output Tax = 18% of 10,000

$= \frac{18}{100} \times 10000 =$ ₹ 1800

$\therefore$ GST payable = Output tax - ITC

= 1800 - 1440

= ₹ 360

$\therefore$ payable CGST = ₹ 180 and payable SGST = ₹ 180 by M/s. Jay Chemicals

Ex. (3) M/s. Jay Chemicals purchased a liquid soap for ₹ 8000 (with GST) and sold it to the consumers for ₹ 10,000 (with GST). Rate of GST is 18%. Find the amount of CGST and SGST to be paid by Jay Chemicals

Solution : Note that here the prices are including GST.

Total value (value with GST) = Taxable value + GST

If the taxable value of liquid soap is ₹ 100, then the total value is ₹ 118.

The ratio of $\frac{\text{Total value}}{\text{Taxable Value}}$ is constant as the rate of GST is same.

i) For total value of ₹ 118, the taxable value is ₹ 100 and for total value of ₹ 8000, let the taxable value be ₹ x .

$\therefore \frac{x}{8000} = \frac{100}{118}$

$\therefore x = \frac{8000}{118} \times 100 =$ ₹ 6779.66

$\therefore$ GST paid at the time of purchase = 8000 - 6779.66

Input tax = ₹ 1220.34 $\quad \therefore$ ITC = ₹ 1220.34 . . . . . (I)

ii) For total value of ₹ 10,000 let the taxable value be ₹ y.

$\therefore \frac{y}{10000} = \frac{100}{118}$

$\therefore y = \frac{10,00,000}{118} =$ ₹ 8474.58 .

$\therefore$ Output tax (tax collected) = 10000.00 - 8474.58

= ₹1525.42. . . . . (II)

$\therefore$ GST payable = Output tax - Input tax = 1525.42 - 1220.34

= ₹ 305.08.

$\therefore$ payable CGST = payable SGST = 305.08 ÷ 2 = ₹152.54

Ans. : Jay Chemicals has to pay ₹ 152.54 CGST and ₹152.54 SGST.

Note : Observe Ex. 2 and Ex. 3 carefully. Both the types of 'Tax Invoices' are commonly used. While purchasing goods, ask the shopkeeper whether the printed price includes GST.

ICT Tools or Links.

Note : A trader (tax payer) has to pay the GST within the prescribed time limit. He has to submit and file the GST returns as per the rules. All these can be done online. You can learn more about GST returns on www.gst.gov.in. (GST offline utility is also available to prepare returns)

Ex. (4) Suppose a manufacturer sold a cycle for a taxable value of ₹ 4000 to the wholesaler. Wholesaler sold it to the retailer for ₹ 4800 (taxable value). Retailer sold it to a customer for ₹ 5200 (taxable value). Rate of GST is 12%. Complete the following activity to find the payable CGST and SGST at each stage of trading.

Solution : Trading chain

[ ] —₹4000, 12%→ [ ] —₹4800, 12%→ [ ] —₹5200, 12%→ [ ]

Output tax of manufacturer = 12% of 4000 = [. . . × $\frac{\dots}{\dots}$] = [ ]

GST payable by manufacturer = ₹480

Output tax of wholesaler = 12% of 4800 = [₹ 576]

∴ GST payable by wholesaler = Output tax - Input tax

= [576] - [480]

= [₹ 96]

Output tax of retailer = 12% of 5200 = [ ]

∴ GST payable by Retailer = Output tax of retailer - ITC of retailer

= [ ] – [ ]

= [ ]

Statement of GST payable at each stage of trading

Individual	GST payable	CGST payable	SGST payable
Manufacturer	₹ 480	₹ 240	₹ [ ]
Wholesaler	₹ 96	₹ [ ]	₹ [ ]
Retailer	₹ [ ]	₹ [ ]	₹ [ ]
Total	₹ [ ]	₹ [ ]	₹ [ ]

Let's think.

- Suppose in the month of July the output tax of a trader is equal to the input tax, then what is his payable GST?
- Suppose in the month of July output tax of a trader is less than the input tax then how to compute his GST?

Practice Set 4.2

1. 'Chetana Store' paid total GST of ₹ 1,00,500 at the time of purchase and collected GST ₹ 1,22,500 at the time of sale during 1st of July 2017 to 31st July 2017. Find the GST payable by Chetana Stores.

2. Nazama is a proprietor of a firm, registered under GST. She has paid GST of ₹ 12,500 on purchase and collected ₹ 14,750 on sale. What is the amount of ITC to be claimed ? What is the amount of GST payable ?

3. Amir Enterprise purchased chocolate sauce bottles and paid GST of ₹ 3800. He sold those bottles to Akbari Bros. and collected GST of ₹ 4100. Mayank Food Corner purchased these bottles from Akabari Bros and paid GST of ₹ 4500. Find the amount of GST payable at every stage of trading and hence find payable CGST and SGST.

4. Malik Gas Agency (Chandigarh Union Territory) purchased some gas cylinders for industrial use for ₹ 24,500, and sold them to the local customers for ₹ 26,500. Find the GST to be paid at the rate of 5% and hence the CGST and UTGST to be paid for this transaction. (for Union Territories there is UTGST instead of SGST.)

5. M/s Beauty Products paid 18% GST on cosmetics worth ₹ 6000 and sold to a customer for ₹ 10,000. What are the amounts of CGST and SGST shown in the tax invoice issued ?

6. Prepare Business to Consumer (B2C) tax invoice using given information. Write the name of the supplier, address, state, Date, invoice number, GSTIN etc. as per your choice. Supplier : M/s - - -- - Address- - - - - State - - - - - Date - - - - - - -

   Invoice No. - - - - - GSTIN - - - - - - - - - - - - - -

   Particulars - Rate of Mobile Battery - ₹ 200 Rate of GST 12% HSN 8507, 1 pc.

   Rate of Headphone - ₹ 750 Rate of GST 18% HSN 8518, 1 pc.

(7) Prepare Business to Business (B2B) Tax Invoice as per the details given below. name of the supplier, address, Date etc. as per your choice.

Supplier - Name, Address, State, GSTIN, Invoice No., Date

Recipient - Name, Address, State, GSTIN,

Items : (1) Pencil boxes 100, HSN - 3924, Rate - ₹ 20, GST 12%

(2) Jigsaw Puzzles 50, HSN 9503, Rate - ₹ 100 GST 12%.

For More Information

Composition Scheme

The person whose annual turn over in the previous financial year is less than 1.5 crore can opt for composition scheme under GST rules. GST rates applicable to composition dealers are as follows.

GST rates for composition Scheme

Sr. No.	Supplier	GST rate	(CGST + SGST)
1.	Restaurants	5%	2.5% + 2.5%
2.	Manufacturers & traders	1%	0.5% + 0.5%

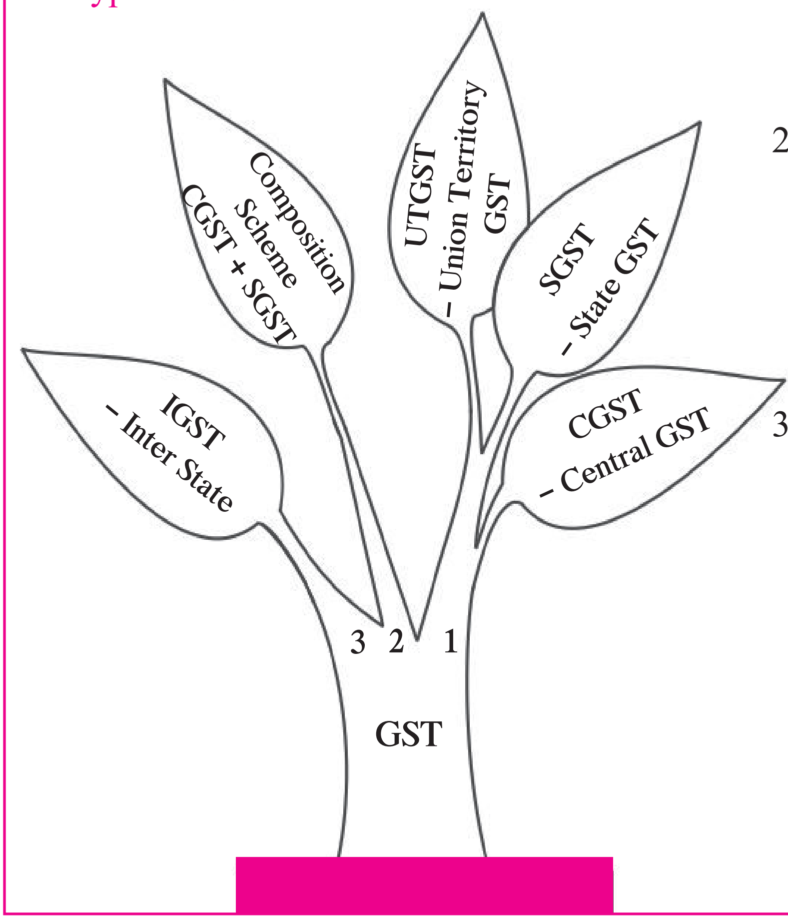
Some rules for composition dealers

- Composition dealers cannot collect tax from the customers, hence they can not issue tax invoice. They have to give '**bill of supply**'.
- Composition dealers should file the return quarterly (i.e. every 3 months.)
- Composition dealers cannot sell goods outside the state (Inter-state sale is not allowed) But they can purchase goods from other states.
- Composition dealers cannot avail the benefits of ITC.
- On the signboard of the shop, he should mention 'Composition taxable person'.
- On the Bill of supply it is mandatory to print 'Composition taxable person not eligible to collect tax on supplies' in bold letters.

Features of GST

- Many Indirect Taxes are subsumed **under GST.**
- No dispute between Goods and Services.
- Statewise Registration for traders.
- **GSTIN holder needs to keep all the records and should pay** GST in time.
- Transparency in transactions.
- This tax system is simple and easy to understand .
- Removal of cascading effect of taxes hence the prices are controlled
- Increase in Quality of Goods and Services as they are globally competetive.
- Boost to 'Make in India' project.
- Technology driven tax system leads to speedy decisions.
- Goods and Service Tax system is a Dual model, as equal amount of tax is levied by Central and State governments.

Types of taxes under GST



1. CGST-SGST (UTGST): Tax levied for trading within state (Intra state).
2. Composition Scheme : For those GSTIN holders whose annual turnover is between 20 lacs to 1.5 crore. They pay CGST and SGST with special rates.
3. IGST : Tax levied by central government for Inter state trading.

For More Information

IGST- Integrated GST (for Inter state trade)

When trading of goods and services takes place between two or more states, the GST is levied only by the Central Government, and it is termed as IGST, hence the total amount is paid to the Central Government.

Suppose if a trader buys goods from another state and sells them in his state, then let us see how he can avail of the ITC, which he has paid as IGST at the time of purchase.

For example : Trader 'M' (of Maharashtra) purchased scooter parts for ₹ 20,000 from trader 'P' (of Punjab) and paid tax of ₹ 5600 as IGST (GST rate 28%) to the trader 'P'.

Trader 'M' sold these parts to local consumers for ₹ 25,000 and collected ₹ 7000 GST at the rate of 28%, bifurcated as CGST ₹ 3500 + SGST ₹ 3500

At the time of paying taxes to the Government, see, how to take ITC of ₹ 5600.

Note : For taking credit of IGST first preference to be given to pay the liability of IGST then CGST and remaining amount can be utilised to pay SGST. Here there is no IGST during the sale for trader 'M', so first the credit is used for CGST and then for SGST.

CGST payable = 3500 - 3500 = 0 ₹

So out of ₹ 5600 credit of ₹ 3500 is utilised for CGST and the remaining amount 5600 - 3500 = 2100 is the credit available for SGST

∴ SGST payable = 3500 - 2100 = 1400 ₹

Trader 'M' has to pay ₹ 1400 as SGST.

Note that, trader 'M' got full credit of ₹ 5600. (so that ITC is completely utilised)

Rule for availing ITC

ITC paid at the time of purchase — Tax collected (output liabilities)

While taking credit of IGST (₹ 5600)
- Firstly ₹ (0) → utilised for IGST
- Secondly (₹ 3500) → utilised for CGST
- At the end (₹ 2100) → utilised for SGST

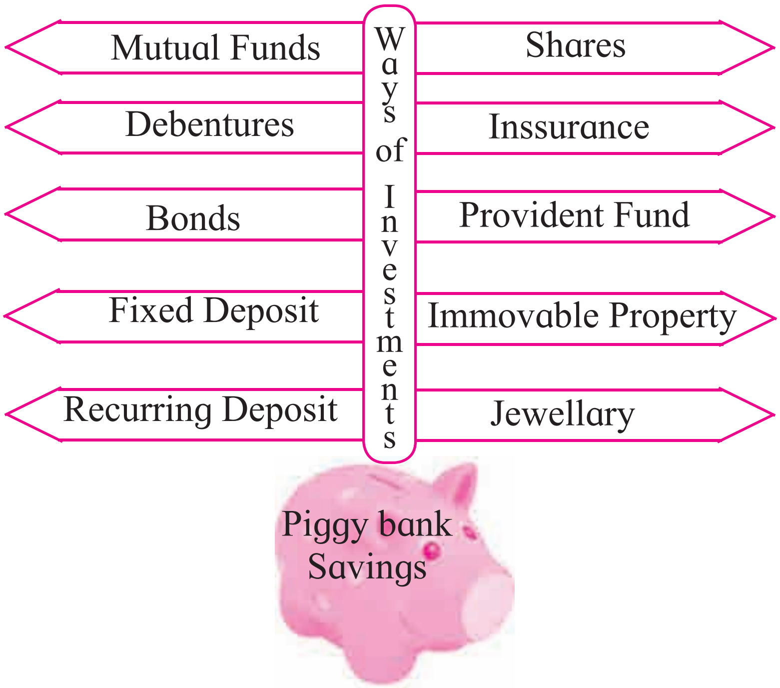
Hence payable SGST is ₹ 1400

Let's recall.

In the previous class we have learnt the importance of savings and investments, which you might have started practising whenever possible. We develop good habits to maintain physical health, in the same way we should develop a habit of saving and investing regularly to maintain financial health. There are many different ways of investments. So deep study and experience both are essential.

Let's discuss.

Shweta is working in a company. From this month her salary increased by 5% and in the next month she will also get bonus. She is thinking of investing this increment. Her friend Neha is working in the office of a financial advisor, so she can advise Shweta in this matter. Neha told, 'It is important to have diversification in one's investments. e.g. you should think of investing in life insurance, health insurance, owning a house, FD's and recurring account in the bank etc...' Shweta said, 'I have insurance and FD in the bank. Even Provident Fund is deducted regularly from my salary. What are the other ways?' Neha answered, ''Investing in shares, Mutual Funds (MF), Debentures, Bonds etc is more popular these days. Inclination of people towards SIP is also increasing. Well I think, you are getting salary increment every month, so **S**ystematic **I**nvestment **P**lan (SIP) is suitable for you.''



We hear such dialogues every now and then. So we all must have the current information beneficial for all as it says '*बहुजन हिताय, बहुजन सुखाय*'.

In this chapter we shall learn about shares, mutual funds and SIP before actually investing in them.

Let's learn.

Shares

To own a shop is proprietorship. When two or more individuals coming together to carry out a business is a partnership, which requires small capital. To establish a company, desiring persons come together and form a company. Company is to be registered under the Indian Companies' Act, 1956. Persons who form a company are called Promoters and the company is called Limited Company.

Amount required to start a company is called **Capital**. This capital is divided into small equal parts, each part is of ₹ 1, ₹ 2, ₹ 5, ₹ 10 or ₹ 100 etc. This small part is called share of the company. These shares are sold in the sharemarket to raise the capital.

Share : A share is the smallest unit of the capital. The value of a share is printed on the company's certificate with other details and it is called a share certificate.

Share Holder : A person who owns the share is called a share holder. The shareholder is a part owner of the company in the proportion of number of shares he/she holds.

Stock Exchange : It is a place where buying and selling of shares take place. It is also known as share market or stock market, equity market or capital market. Companies should be listed in the stock market for trading.

Face Value (FV) : The value printed on the share certificate is called the Face value of the share. It is also called Nominal value or Printed value or par value.

Market Value (MV) : The price at which the shares are sold or purchased in the stock market is called Market value (MV) of the share.

In the live sharemarket the Market Value changes frequently.

If the company's performance is better than expected, then those shares are in demand. The number of shares is fixed, therefore share supply could not be increased and hence the share price increases. If the company is not doing well, the share price falls. [Increase in price is shown by ▲ (green triangle upward), and decrease in price is shown by ▼ (Red triangle downward).] This is the reason for increase or decrease in SENSEX and NIFTY index.

Dividend : The part of annual profit of a company which is distributed per share among shareholders is called dividend. If the company is performing well then the value of share capital increases hence the price of the share goes up. As a result company gives good devidend. For the sharehlders the devidend income is taxfree.

Let's remember!

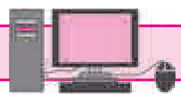
Whatever may be the market value, the dividend is always reckoned on the Face Value of a share.

For more Information :

There are two main stock exchanges, of India, **BSE (Bombay Stock Exchange**) and **NSE (National Stock Exchange).** BSE is the oldest in Asia while NSE is the India's largest stock Exchange.

There are two share indices namely - SENSEX and NIFTY which reflect the overall market sentiments. SENSEX is **SENS**itive + ind**EX**. Which was introduced by BSE on 1-1-1986. SENSEX is detemined from 30 stocks. They are the stocks of well established and financially sound companies from the main sectors.

NIFTY as the name suggests is made up of two words that is **N**SE and F**IFTY** which was introduced by NSE. It depends on India's topmost outperforming 50 companies.



ICT Tools or Links.

Visit the website of SEBI. Also get information of share market from TV channels, BSE, NSE websites or watch videos on internet. There you can see two strips continuously flashing advances and declines in the market value of shares. Generally the upper strip shows BSE shares while lower strip shows NSE shares. Also find out what is the book value of shares from the available resources.

Comparison of FV and MV

(1) If MV > FV then the share is at premium.

(2) If MV = FV then the share is at par.

(3) If MV < FV then the share is at discount.

For example : (1) suppose FV = ₹ 10, MV = ₹ 15 and 15 - 10 = ₹ 5

∴ The share is at premium of ₹ 5, as MV > FV

(2) suppose FV = ₹ 10, MV = ₹ 10 and 10 - 10 = 0

∴ The share is at par. As MV = FV

(3) suppose FV = ₹ 10, MV = ₹ 7 and 10 - 7 = 3

∴ The share is at discount. As MV < FV.

Sum Invested : Total amount required to purchase the shares is sum invested.

Sum invested = Number of shares × MV

Ex. (1) If 50 shares of FV ₹ 100 each are purchased for MV ₹ 120. Find the sum invested.

Solution : Sum invested = number of shares × MV

= 50 × 120 = ₹ 6000

Ex. (2) If you want to purchase 50 shares of MV ₹ 50 each. What is the total amount to be paid ?

Solution : Sum invested = Number of shares × MV = 50 × 50 = ₹ 2500

Rate of Return - RoR

When we invest in shares, it is important to know the return on investment. Observe the following example.

Ex. (1) Shriyash purchased a share of FV ₹ 100 for MV of ₹ 120. Company declared 15% dividend on the share. Find the rate of return.

Solution : FV = ₹ 100, MV = ₹ 120 D = Dividend = 15% per share.

Remember here, that on investment of ₹ 120 Shriyash got ₹ 15.

Let the rate of return be x%

$\therefore \frac{15}{120} = \frac{x}{100} \quad \therefore x = \frac{15 \times 100}{120} = \frac{25}{2} = 12.5\%$

If	120	:	15
then	100	:	x

Ans. The rate of return for Shriyash is 12.5%.

Ex. (2) FV = ₹ 100, premium = ₹ 65 then MV = ?

Solution : MV = FV + Premium = 100 + 65 = ₹ 165.

Market value is ₹ 165 per share.

Ex. (3) Complete the following table using given information.

Sr.No.	FV	Share is at	MV
(i)	₹ 10	Premium of ₹ 7	
(ii)	₹ 25		₹ 16
(iii)		at par	₹ 5

Solution : (i) MV=10+7=₹17 (ii) at discount of 25 – 16=₹9 (iii) FV=₹5.

Ex. (4) Neel has invested in shares as follows. Find his total investment.

Company A : 350 shares, FV = ₹ 10, premium = ₹ 7

Company B : 2750 shares, FV = ₹ 5, Discount = ₹ 1.

Company C : 50 shares, FV = ₹ 100, MV = ₹ 150.

Solution : Company A : Premium = ₹ 7 MV = FV + Premium

= 10 + 7 = ₹ 17.

∴ Investment in company A = Number of shares × MV

= 350 × 17 = ₹ 5950.

Company B : FV = ₹ 5, MV = ₹ 4.

∴ Investment in company B = Number of shares × MV

= 2750 × 4 = ₹ 11,000.

Company C : FV = ₹ 100, MV = ₹ 150.

∴ Investment in company C = Number of shares × MV

= 50 × 150 = ₹ 7500.

Ans. Neel has invested 5950 + 11000 + 7500 = ₹ 24,450.

Ex. (5) Smita has invested ₹ 12,000 and purchased shares of FV ₹ 10 at a premium of ₹ 2. Find the number of shares she purchased. complete the given activity to get the answer.

Solution : FV = ₹ 10, Premium = ₹ 2,.

∴ MV = FV + ☐ = ☐ + ☐ = ☐

$$\therefore \text{ Number of shares } = \frac{\text{Total investment}}{\text{MV}} = \frac{12000}{\square} = \square \text{ shares}$$

Ans : Smita has purchased ☐ shares.

Ex. (6) If 50 shares of FV ₹10 were purchased for MV of ₹25. Company declared 30% dividend on the shares then find (1) Sum investment (2) Dividend received (3) Rate of return.

Solution : FV = ₹10, MV = ₹25, Number of shares = 50.

(1) ∴ Sum investment = 25 × 50 = ₹1250.

(2) Dividend per share = $10 \times \frac{30}{100}$ = ₹3

∴ Total dividend received = 50 × 3 = ₹ 150.

(3) Rate of return = $\frac{\text{Dividend income}}{\text{Sum invested}} \times 100$

$= \frac{150}{1250} \times 100 = 12\%$

Ans : (1) Sum invested is ₹ 1250 (2) Dividend received is ₹ 150 (3) Rate of return is 12%.

Practice Set 4.3

(1) Complete the following table by writting suitable numbers and words.

Sr.No	FV	Share is at	MV
(1)	₹ 100	par	. . .
(2)	. . .	premium ₹ 500	₹ 575
(3)	₹ 10	. . .	₹ 5

(2) Mr.Amol purchased 50 shares of Face Value ₹ 100 when the Market value of the share was ₹ 80. Company had given 20% dividend. Find the rate of return on investment.

(3) Joseph purchased following shares, Find his total investment.
Company A : 200 shares, FV = ₹ 2 Premium = ₹ 18.
Company B : 45 shares, MV = ₹ 500
Company C : 1 share, MV = ₹ 10,540.

(4) Smt. Deshpande purchased shares of FV ₹ 5 at a premium of ₹ 20. How many shares will she get for ₹ 20,000 ?

(5) Shri Shantilal has purchased 150 shares of FV ₹ 100, for MV of ₹ 120. Company has paid dividend at 7%. Find the rate of return on his investment.

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(6) If the face value of both the shares is same, then which investment out of the following is more profitable ?

Company A : dividend 16%, MV = ₹ 80, Company B : dividend 20%, MV = ₹ 120.

ICT Tools or Links.

Select any five shares of your choice, find their Face Values and Market Values using internet or TV or news papers. Draw the joint bar diagram and compare the difference in FV and MV of each share. Take both the types ▲, ▼ of shares.

Let's learn.

Brokerage and taxes on share trading

Brokerage : We directly can't go to the stock market and buy or sell shares, only the registered members or organization (agency) of the stock market can buy or sell on our behalf. These members are called **'Share Brokers'**. For catering the service of buying and selling of shares they charge some amount which is called **'Brokerage'** Brokerage is paid on the Market value of the share.

Ex. (1) Suppose if the face value of the share is ₹ 100 and market value is ₹ 150. Let the rate of brokerage be 0.5%. What amount should one pay for purchasing 100 such shares ? What amount should one receive after selling 100 such shares ?

Situation (I) At the time of buying shares:

Buying price of 1 share = MV + Brokerage

= 150 + 0.5% of 150

= 150 + 0.75

= ₹ 150.75

If someone purchases 100 such shares the total cost is 100×150.75 = ₹ 15075.
Here ₹ 15000 is the share price and ₹ 75 is the brokerage paid.

Situation (II) At the time of selling shares.

Selling price per share = MV - Brokerage

= 150-5% of 150 = 150 - 0.75

= ₹ 149.25.

If someone sells 100 such shares, he will get,

100×150 - ₹ 75 = ₹ 14925 after selling 100 such shares.

Let's remember!

- Brokerage is always calculated on Market value of shares.
- In the contract note of sale-purchase of shares, price of one share is shown with brokerage and taxes.

Project I : Visit the office of a share broker or agency in your area. Collect the information of brokerage charges, other charges and facilities given to the investors and compare.

Project II : Obtain a statement of 'Demat Account' and 'Trading Account'. Consult a share broker or elders in the house or use internet. Try to learn all the terms in the statement. Discuss with your friends in the class.

For more information :

Every broker is registered and governed by SEBI (Securities and Exchange Board of India) Act 1992.

For keeping records of shares, bonds, mutual funds one must have Demat account (Dematerialised Account). For sale and purchase of shares, a trading account is a must. These accounts can be opened with banks or share brokers. They are known as DP-Depository Participants. These DPs are under the control of two main depositories namely NSDL and CDSL. Demat account is same as bank saving account where shares bought are credited and shares sold are debited just like bank pass book. The statement of holding is given to the account holder with nominal charges when requested. The shares held in Demat A/c are in electronic form. Saving account is to be linked with these two accounts so that the money can be transfered as and when required. In the same way money gets credited when shares are sold. For opening these accounts share broker or bank representative gives guidance.

Let's learn.

GST on Brokerage Services

Share brokers provide services for purchase and sale of shares for their clients. These services are charged under GST. Rate of GST is 18% on brokerage. You can find the SAC for brokerage services.

Note :- For the safety of the investors, there are other nominal charges besides GST on brokerage. These are Security Transaction Tax (STT), SEBI charge, stamp duty etc. Here we will only consider GST on brokerage.

Ex. (2) As per Ex. (1) suppose a person has paid ₹ 15,075 for buying 100 shares. In that ₹ 75 is the brokerage. So the buyer has to pay 18% GST on ₹ 75. Let us find the amount of GST he paid to the broker and prepare the contract note.

Solution : GST = 18% of 75 = $\frac{18}{100} \times 75$ = ₹ 13.50.

For the above share trading the contract note is as follows. (B means Buy)

No. of shares	MV	Total price	brokerage 0.5%	CGST 9% on brokerage	SGST 9% on brokerage	Total value.
100 (B)	150	₹ 15000	₹ 75	₹ 6.75	₹ 6.75	₹ 15088.50

Ex. (3) Bashirkhan purchased 100 shares of MV ₹ 40. Brokerage paid at the rate of 0.5% and rate of GST on brokerage is 18%. Find the total amount he paid for the share purchase.

Solution : Value of 100 shares = 40 × 100 = ₹ 4000.

Brokerage per share = $\frac{0.5}{100} \times 40$ = ₹ 0.20.

∴ Cost of one share = MV + Brokerage

= 40 + 0.20 = ₹ 40.20.

∴ Cost of 100 shares = 40.20 × 100 = ₹ 4020

∴ Brokerage on 100 share = 0.20 × 100 = ₹ 20

∴ GST = $\frac{18}{100} \times 20$

= ₹ 3.60.

Ans. : Bashirkhan paid ₹ 4020 + ₹ 3.60 = ₹ 4023.60 for 100 shares.

Ex. (4) Pankajrao invested ₹ 1,25,295 in shares of FV ₹ 10 when MV is ₹ 125. Rate of brokerage is 0.2% and GST is 18%. Then find (1) How many shares were purchased. (2) the amount of brokerage paid and (3) GST paid for the trading.

Solution : Sum invested = ₹ 1,25,250, brokerage = 0.2%, GST rate = 18%

∴ Brokerage per share = 125 × $\frac{0.2}{100}$ = ₹ 0.25.

GST per share on brokerage = 18% of 0.25 = ₹ 0.045

∴ Cost of 1 share = MV + Brokerage + GST

$= 125 + 0.25 + 0.045 = ₹\ 125.295.$

$\therefore$ No. of shares $= \dfrac{125250}{125.25} = 1000$

Total brokerage = brokerage per share × No. of shares

$\therefore$ Total brokerage $= 0.25 \times 1000 = ₹\ 250.$

Total GST $= 1000 \times 0.045 = ₹\ 45.$

Ans. (1) 1000 shares were purchased.

(2) Brokerage paid was ₹ 250.

(3) GST paid was ₹ 45.

Ex. (5) Nalinitai invested ₹ 6024 in the shares of FV ₹ 10 when the Market Value was ₹ 60. She sold all the shares at MV of ₹ 50 after taking 60% dividend. She paid 0.4% brokerage at each stage of transactions. What was the total gain or loss in this transaction ?

Solution : Rate of GST is not given in the example, so it is not considered.

Shares Purchased : FV = ₹ 10, MV = ₹ 60

Brokerage per share $= \dfrac{0.4}{100} \times 60 = ₹\ \square$

$\therefore$ Cost of one share $= 60 + 0.24 = ₹\ \square$

$\therefore$ Number of shares $= \dfrac{6024}{60.24} = 100$

Shares sold : FV ₹ 10, MV = ₹ 50

$\therefore$ Brokerage per share $= \dfrac{0.4}{100} \times 50 = ₹\ 0.20$

$\therefore$ Selling price per share $= 50 - 0.20 = ₹\ \square$

$\therefore$ Selling price of 100 shares $= 100 \times 49.80 = ₹\ \square$

Dividend received 60%

$\therefore$ Dividend per share $= \dfrac{60}{100} \times 10 = ₹\ 6$

$\therefore$ Dividend on 100shares $= 6 \times 100 = ₹\ \square$

$\therefore$ Nalinitai's income $= \square + \square = ₹\ 5580$

Sum invested = ₹ 6024

$\therefore$ Loss $= \square - \square = ₹\ \square$

Ans. Nalinitai's loss is ₹ 444

Activity : In example (5) if GST was paid at 18% on brokerage, then the loss is ₹ 451.92. Verify whether you get the same answer.

Let's learn.

Mutual Fund - MF

We have learnt that a group of persons come together to form a company. They raise capital from the society by issuing shares. If company performs well, then the investors of the company get benefits in terms of dividend, bonus shares and increase in the market value gives more profit on investments. Company's market capitalization rises. All this totality helps for the progress of the country. In short, principle of sociology 'together we can progress' works here. But every coin has two sides. sometimes it might happen that instead of profit an investor may incur a loss. Can we reduce this loss ? Is there a way to reduce the risk in investments? Yes, to overcome this more people invest in Mutual Funds.

In Mutual Fund, many investors with common objectives give their money to the professional experts. They not only invest in one type of shares but also invest in various other schemes. As a result, investment is diversified which reduces risk factor and total dividend or profit is divided equally among the investors. How to invest in Mutual Fund ? What is the rate of return ? What is the locking period ? What are the different types of investment schemes ? All these questions could be answered by a Financial advisor or financial planner.

> You may have heard or read this sentence that, 'Investments in Mutual Funds are subject to Market risks. Read all scheme related documents carefully before invesing.' Interpret the meaning correctly. Sometimes instead of profit, investment in Mutual Fund might give loss which investors have to bear.

Mutual Fund is a professionally managed investment scheme, usually run by an AMC i.e. Asset Management Company. They invest the money given by the investors in different schemes e.g. equity fund (in shares), debt fund (in debentures, bonds etc.) or balanced funds as per the investor's choice.

As we get 'shares' for the investment in sharemarket, we get '**units**' when we invest in mutual fund.

The market value of 'a unit' is called '**NAV**' (Net Asset Value)

NAV of one unit × Number of units = Total fund value.

Note : As the market value of share changes frequently NAV of a unit also changes. One can redeem the units when needed.

Investments in FDs of nationalised bank or Indian Postal services are more secured and safe, in comparison with other investments, but the rate of return is low. It hardly helps to overcome the rate of inflation. One must remember always that if the money is invested wisely it generates more money. For this the knowledge of financial planning is of great help.

Investments in shares and mutual funds should be made carefully because risk and returns always go hand in hand. So the habit of regular and deep study is the only key.

Systematic Investment Plan

Suppose, one does not want to invest a big amount at once, then one could invest small amounts at regular time intervals e.g. ₹ 500 per month could be invested in mutual fund. Investment could be done monthly or quarterly. This way of investment is called SIP. SIP develops discipline of savings. SIP is a good option which in long term can achieve one's financial goals. Investment in mutual funds through SIP for a long term is beneficial. It protects investor from market fluctuations. One should invest in mutual fund for minimum of 3 to 5 years to get better returns and it is best if investment is for 10 to 15 years.

Benefits of Mutual Funds

- Professional fund managers.
- Diversifications of funds.
- Transparency and sufficiently safe investment.
- Liquidity - redemption of units can be done.
- Limited risks.
- Advantage of long term and short term gain.
- Investments in funds like ELSS are admissible for deduction under section 80C of income tax.

Solved Examples

Ex. (1) If the total value of the mutual fund scheme is ₹ 200 crores and 8 crore units are issued then find the NAV of one unit.

Solution : NAV = ₹ 200 crore / 8 crore units = ₹ 25 per unit.

Ex. (2) If NAV of one unit is ₹ 25, then how many units will be alloted for the investment of ₹ 10,000 ?

Solution : Number of units = sum invested / NAV = 10,000/25 = 400 units.

Practice Set 4.4

1. Market value of a share is ₹ 200. If the brokerage rate is 0.3% then find the purchase value of the share.
2. A share is sold for the market value of ₹ 1000. Brokerage is paid at the rate of 0.1%. What is the amount received after the sale ?
3. Fill in the blanks given in the contract note of sale-purchase of shares. (B - buy S - sell)

No. of shares	MV of shares	Total value	Brokerage 0.2%	9% CGST on brokerage	9% SGST on brokerage	Total value of shares
100 B	₹ 45					
75 S	₹ 200					

4. Smt. Desai sold shares of face value ₹ 100 when the market value was ₹ 50 and received ₹ 4988.20. She paid brokerage 0.2% and GST on brokerage 18%, then how many shares did she sell ?
5. Mr. D'souza purchased 200 shares of FV ₹ 50 at a premium of ₹ 100. He received 50% dividend on the shares. After receiving the dividend he sold 100 shares at a discount of ₹ 10 and remaining shares were sold at a premium of ₹ 75. For each trade he paid the brokerage of ₹ 20. Find whether Mr. D'souza gained or incurred a loss ? by how much ?

Problem Set 4A

1. Write the correct alternative for each of the following.

(1) Rate of GST on essential commodities is ...

(A) 5% (B) 12% (C) 0% (D) 18%

(2) The tax levied by the central government for trading within state is...

(A) IGST (B) CGST (C) SGST (D) UTGST

(3) GST system was introduced in our country from ...

(A) 31st March 2017 (B) 1st April 2017

(C) 1st January 2017 (D) 1st July 2017

(4) The rate of GST on stainless steel utensils is 18%, then the rate of State GST is . . .

(A) 18% (B) 9% (C) 36% (D) 0.9%

(5) In the format of GSTIN there are ... alpha-numerals.

(A) 15 (B) 10 (C) 16 (D) 9

(6) When a registered dealer sells goods to another registered dealer under GST, then this trading is termed as . . .

(A) BB (B) B2B (C) BC (D) B2C

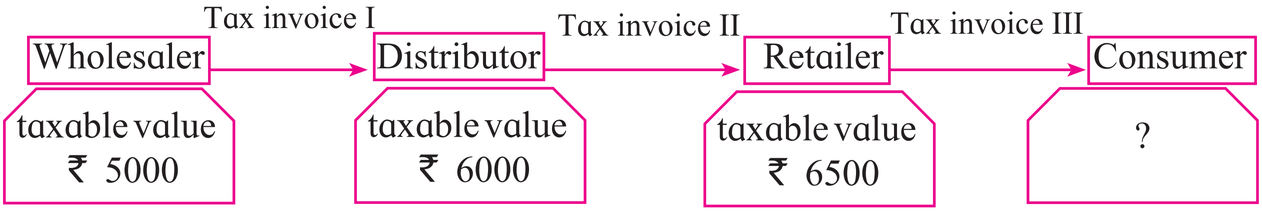
2. A dealer has given 10% discount on a showpiece of ₹ 25,000. GST of 28% was charged on the discounted price. Find the total amount shown in the tax invoice. What is the amount of CGST and SGST ?
3. A ready-made garment shopkeeper gives 5% discount on the dress of ₹ 1000 and charges 5% GST on the remaining amount, then what is the purchase price of the dress for the customer ?
4. A trader from Surat, Gujarat sold cotton clothes to a trader in Rajkot, Gujarat. The taxable value of cotton clothes is ₹ 2.5 lacs. What is the amount of GST at 5% paid by the trader in Rajkot ?
5. Smt. Malhotra purchased solar panels for the taxable value of ₹ 85,000. She sold them for ₹ 90,000. The rate of GST is 5%. Find the ITC of Smt. Malhotra. What is the amount of GST payable by her ?
6. A company provided Z-security services for the taxable value of ₹ 64,500. Rate of GST is 18%. Company had paid GST of ₹ 1550 for laundry services and uniforms etc. What is the amount of ITC (input Tax Credit) ? Find the amount of CGST and SGST payable by the company.
7. A dealer supplied Walky-Talky set of ₹ 84,000 (with GST) to police control room. Rate of GST is 12%. Find the amount of state and central GST charged by the dealer. Also find the taxable value of the set.
8. ★ A wholesaler purchased electric goods for the taxable amount of ₹ 1,50,000. He sold it to the retailer for the taxable amount of ₹ 1,80,000. Retailer sold it to the customer for the taxable amount of ₹ 2,20,000. Rate of GST is 18%. Show the computation of GST in tax invoices of sales. Also find the payable CGST and payable SGST for wholesaler and retailer.
9. ★ Anna Patil (Thane, Maharashtra) supplied vacuum cleaner to a shopkeeper in Vasai (Mumbai) for the taxable value of ₹ 14,000, and GST rate of 28%. Shopkeeper sold it to the customer at the same GST rate for ₹ 16,800 (taxable value) Find the following -

(1) Amount of CGST and SGST shown in the tax invoice issued by Anna Patil.

(2) Amount of CGST and SGST charged by the shopkeeper in Vasai.

(3) What is the CGST and SGST payable by shopkeeper in Vasai at the time of filing the return.

10.★ For the given trading chain prepare the tax invoice I, II, III. GST at the rate of 12% was charged for the article supplied.



(1) Prepare the statement of GST payable under each head by the wholesaler, distributor and retailer at the time of filing the return to the government.

(2) At the end what amount is paid by the consumer ?

(3) Write which of the invoices issued are B2B and B2C ?

Problem Set 4B

1. Write the correct alternative for the following questions.

(1) If the Face Value of a share is ₹ 100 and Market value is ₹ 75, then which of the following statements is correct ?

(A) The share is at premium of ₹ 175 (B) The share is at discount of ₹ 25

(C) The share is at premium of ₹ 25 (D) The share is at discount of ₹ 75

(2) What is the amount of dividend received per share of face value ₹ 10 and dividend declared is 50%.

(A) ₹ 50 (B) ₹ 5 (C) ₹ 500 (D) ₹ 100

(3) The NAV of a unit in mutual fund scheme is ₹ 10.65 then find the amount required to buy 500 such units.

(A) 5325 (B) 5235 (C) 532500 (D) 53250

(4) Rate of GST on brokerage is . . .

(A) 5% (B) 12% (C) 18% (D) 28%

(5) To find the cost of one share at the time of buying the amount of Brokerage and GST is to be . . . the MV of share .

(A) added to (B) substracted from

(C) Multiplied with (D) divided by

2. Find the purchase price of a share of FV ₹ 100 if it is at premium of ₹ 30. The brokerage rate is 0.3%.

3. Prashant bought 50 shares of FV ₹ 100, having MV ₹ 180. Company gave 40% dividend on the shares. Find the rate of return on investment.

4. Find the amount received when 300 shares of FV ₹ 100, were sold at a discount of ₹ 30.

5. Find the number of shares received when ₹ 60,000 was invested in the shares of FV ₹ 100 and MV ₹ 120.

6. Smt. Mita Agrawal invested ₹ 10,200 when MV of the share is ₹ 100. She sold 60 shares when the MV was ₹ 125 and sold remaining shares when the MV was ₹ 90. She paid 0.1% brokerage for each trading. Find whether she made profit or loss ? and how much ?

7. Market value of shares and dividend declared by the two companies is given below. Face Value is same and it is ₹ 100 for both the shares. Investment in which company is more profitable ?

   (1) Company A - ₹ 132 , 12%   (2) Company B - ₹ 144, 16%

8.★ Shri. Aditya Sanghavi invested ₹ 50,118 in shares of FV ₹ 100, when the market value is ₹ 50. Rate of brokerage is 0.2% and Rate of GST on brokerage is 18%, then How many shares were purchased for ₹ 50,118 ?

9.★ Shri. Batliwala sold shares of ₹ 30,350 and purchased shares of ₹ 69,650 in a day. He paid brokerage at the rate of 0.1% on sale and purchase. 18% GST was charged on brokerage. Find his total expenditure on brokerage and tax.

10.★ Smt. Aruna Thakkar purchased 100 shares of FV 100 when the MV is ₹ 1200. She paid brokerage at the rate of 0.3% and 18% GST on brokerage. Find the following -

   (1) Net amount paid for 100 shares.

   (2) Brokerage paid on sum invested.

   (3) GST paid on brokerage.

   (4) Total amount paid for 100 shares.

11.★ Smt. Anagha Doshi purchased 22 shares of FV ₹ 100 for Market Value of ₹ 660. Find the sum invested. After taking 20% dividend, she sold all the shares when market value was ₹ 650. She paid 0.1% brokerage for each trading done. Find the percent of profit or loss in the share trading.

   (Write your answer to the nearest integer.)

5 Probability

Let's study.

- Probability : Introduction
- Random experiment and its outcome
- Sample space and event
- Probability of an event

Let's discuss.

Teacher : Friends, this box contains folded chits. The number of chits is exactly the same as the number of students in our class. Each student should pick one chit. Names of different plants are written on the chits. No two chits bear the same name of the plant. Let us see who gets the chit having the name 'Basil'. Make a line in the order of your roll numbers. No one will unfold the chit until the last student takes his chit.

Aruna : Sir, I am the first one in a line, but I do not want to pick a chit first, as the possibility of getting 'basil' chit from all the chits is very low.

Zarina : Sir, I am the last student in the row, I do not want to pick the chit at last as the chit containing the name 'basil' will most likely be picked up by some one else before my turn.

The first and the last student feel that for each of them, the possibility of getting the chit having the name 'basil' is very low. The above conversation indicates the thinking of less or more possibility.

We use the following words to express the possibility in our daily conversation.

- Probable
- may be
- impossible
- sure
- nearly
- 50 – 50

Read the following statements regarding predictions (possibilities for the future).

- Most probably the rain will start from today.
- The inflation is likely to rise.
- It is impossible to defeat Indian cricket team in the next match.
- I will surely get first class.
- There is no possibility of Polio infection if a child is given the polio vaccine in time.

The adjoining picture shows a 'toss' before a cricket match.

What are the possibilities ?

☐ or ☐

So here there are ☐ possibilities.

Activity 1 : Let each student in the class toss a coin once. What will you get?
(Teacher writes the observations on the board in a table.)

Possibilities	(H)	(T)
Number of students	. . .	. . .

Activity 2 : Ask each student to toss the same coin twice. What are the possibilities?

Possibilities	H H	HT	TH	TT
Number of students				

Activity 3 : Now throw a die, once. What are the different possibilities of getting dots on the upper face ?

Each of these is a possible result of throwing a die.

Let's learn.

Random Experiment

The experiment in which all possible results are known in advance but none of them can be predicted with certainty and there is equal possibility for each result is known as a 'Random experiment'.

For example, Tossing a coin, throwing a die, picking a card from a set of cards bearing numbers from 1 to 50, picking a card from a pack of well shuffled playing cards, etc.

Outcome

Result of a random experiment is known as an 'Outcome'.

Ex. (1) In a random experiment of tossing a coin - there are only two outcomes.

Head (H) or Tail (T)

(2) In a random experiment of throwing a die, there are 6 outcomes, according to the number of dots on the six faces of the die.

1 or 2 or 3 or 4 or 5 or 6.

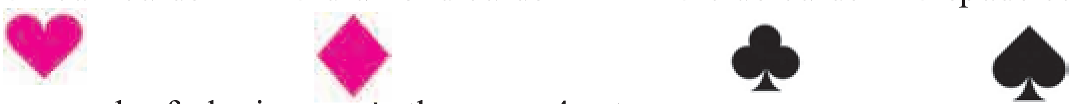
(3) In a random experiment of picking a card bearing numbers from 1 to 50, there are 50 outcomes.

(4) A card is drawn randomly from a pack of well shuffled playing cards.

There are 52 cards in a pack as shown below.

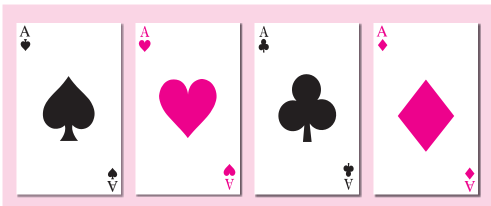
Total cards 52

- 26 red cards
  - 13 heart cards
  - 13 diamond cards
- 26 black cards
  - 13 club cards
  - 13 spade cards



In a pack of playing cards there are 4 sets, namely heart, diamond, club and spade. In each set there are 13 cards as King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2 and Ace.

King, Queen and Jack are known as face cards. In each pack of cards there are 4 cards of king, 4 cards of Queen and 4 cards of Jack. Thus total face cards are 12.



Ace of spade | Ace of heart | Ace of club | Ace of diamond

Equally Likely Outcomes

If a die is thrown, any of the numbers from 1, 2, 3, 4, 5, 6 may appear on the upper face. It means that each number is equally likely to occur. However, if a die is so formed that a particular face come up most often, then that die is biased. In this case the outcomes are not likely to occur equally.

Here, we assume that objects used for random experiments are fair or unbiased.

A given number of outcomes are said to be equally likely if none of them occurs in preferance to others. For example if a coin is tossed, possibilities of getting head or tail are equal. A die, having numbers from 1 to 6 on its different faces, is thrown. Check the possibility of getting one of the numbers. Here all the outcomes are eqully likely.

Practice Set 5.1

1. How many possibilities are there in each of the following?

(1) Vanita knows the following sites in Maharashtra. She is planning to visit one of them in her summer vacation.

Ajintha, Mahabaleshwar, Lonar Sarovar, Tadoba wild life sanctuary, Amboli, Raigad, Matheran, Anandavan.

(2) Any day of a week is to be selected randomly.

(3) Select one card from the pack of 52 cards.

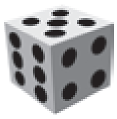
(4) One number from 10 to 20 is written on each card. Select one card randomly.

Let's think.

In which of the following experiments possibility of expected outcome is more?

(1) Getting 1 on the upper face when a die is thrown.

(2) Getting head by tossing a coin.



Let's learn.

Sample Space

The set of all possible outcomes of a random experiment is called the sample space. It is denoted by 'S' or 'Ω' (A greek letter 'Omega'). Each element of sample space is called a 'sample point'. The number of elements in the set 'S' is denoted by n(S). If n(S) is finite, then the sample space is said to be a finite sample space.

Following are some examples of finite sample spaces.

S. No.	Random experiment	Sample space	Number of sample points in S
1	One coin is tossed	S = {H, T}	n(S) = 2
2	Two coins are tossed	S = { HH, HT, TH, TT}	n(S) = ☐
3	Three coins are tossed	S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}	n(S) = 8
4	A die is thrown	S = {1, 2, 3, 4, 5, 6}	n(S) = ☐
5	Two dice are thrown	S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}	n(S) = 36
6	A card is drawn from a pack bearing numbers from 1 to 25	S = {1, 2, 3, 4, ..., 25}	n(S) = ☐
7	A card is drawn from a well shuffled pack of 52 playing cards.	Diamond : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Spade : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Heart : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Club : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King	n(S) = 52

Let's remember!

(i) The sample space for a coin tossed twice is the same as that of two coins tossed simultaneously. The same is true for three coins.

(ii) The sample space for a die rolled twice is the same as two dice rolled simultaneously.

Practice Set 5.2

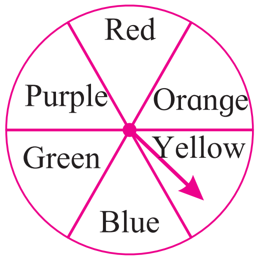
(1) For each of the following experiments write sample space 'S' and number of sample points n(S).

(1) One coin and one die are thrown simultaneously.

(2) Two digit numbers are formed using digits 2, 3 and 5 without repeating a

digits.

2. The arrow is rotated and it stops randomly on the disc. Find out on which colour it may stop.



MARCH - 2019

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

3. In the month of March 2019, find the days on which the date is a multiple of 5. (see the given page of the calender)

4. Form a 'Road safety commitee' of two, from 2 boys (B_1, B_2) and 2 girls(G_1, G_2). Complete the following activity to write the sample space.

(a) Committee of 2 boys = ☐ (b) Committee of 2 girls = ☐

(c) Committee of one boy and one girl = $B_1 G_1$ ☐ ☐ ☐

∴ Sample space = {..., ..., ..., ..., ..., ...}

Let's learn.

Event

The outcomes satisfying particular condition are called favourable outcomes.

A set of favourable outcomes of a given sample space is an '**event**'. Event is a subset of the sample space.

Events are generally denoted by capital letteres A, B, C, D etc. For example, if two coins are tossed and A is the event of getting at least one tail, then the favourable outcomes are as follows.

A = {TT, TH, HT}

The number of elements in the event A is denoted by n(A). Here n(A) = 3.

For more information

Types of event.

(i) Certain event/Sure event

(ii) Impossible event

(iii) Simple/Elementary event

(iv) Complement of an event

(v) Mutually exclusive events

(vi) Exhaustive event

Solved Examples

Ex. (1) Two coins are tossed simultaneously. Write the sample space (S) and number of sample points n(S). Also write the following events in the set form and write the number of sample points in each event.

(i) Condition for event A : to get at least one tail.

(ii) Condition for event B : to get only one head.

(iii) Condition for event C : to get at most one tail.

(iv) Condition for event D : to get no head.

Solution : If two coins are tossed simultaneously,

S = {HH, HT, TH, TT} n(S) = 4

(i) Condition for event A : at least one head.

A = {HH, HT, TH} n(A) = 3

(ii) Condition for event B : only one head.

B = { HT, TH} n(B) = 2

(iii) Condition for event C : at most one tail.

C = {HH, HT, TH} n(C) = 3

(iv) Condition for event D : No head.

D = {TT} n(D) = 1

Ex. (2) A bag contains 50 cards. Each card bears only one number from 1 to 50. One card is drawn at randomfrom the bag. Write the sample space. Also write the events A, B and find the number of sample points in them.

(i) Condition for event A : the number on the card is divisible by 6.

(ii) Condition for event B : the number on the card is a complete square.

Solution : S = {1, 2, 3, . . . 49, 50}, n(S) = 50

(i) Condition for event A : number is divisible by 6.

A = {6, 12, 18, 24, 30, 36, 42, 48} n(A) = 8

(ii) Condition for event B : the number on the card is a complete square.

B = {1, 4, 9, 16, 25, 36, 49} n(B) = 7

Ex. (3) A sanitation committee of 2 members is to be formed from 3 boys and 2 girls. Write sample space 'S' and number of sample points n(S). Also write the following events in set form and number of sample points in the event.

(i) Condition for event A : at least one girl must be a member of the committee.

(ii) Condition for event B : Committee must be of one boy and one girl.

(iii) Condition for event C : Committee must be of boys only.

(iv) Condition for event D : At the most one girl should be a member of the committee.

Solution : Suppose B_1, B_2, B_3 are three boys and G_1, G_2 are two girls

Out of these boys and girls, a sanitation committee of two members is to be formed.

$\therefore$ S = $\{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$ $\therefore$ n(S) = 10

(i) Condition for event A is that at least one girl should be in the committee.

A = $\{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$ $\therefore$ n(A) = 7

(ii) Condition for event B is that one boy and one girl should be there in the committee.

B = $\{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$ $\therefore$ n(B) = 6

(iii) Condition for event C is that there should be only boys in the committee.

C = $\{B_1B_2, B_1B_3, B_2B_3\}$ n(C) = 3

(iv) Condition for event D is that there can be at most one girl in the committee.

D = $\{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$ $\therefore$ n(D) = 9

Ex. (4) Two dice are rolled, write the sample space 'S' and number of sample points n(S). Also write events and number of sample points in the event according to the given condition.

(i) Sum of the digits on upper face is a prime number.

(ii) Sum of the digits on the upper face is multiple of 5.

(iii) Sum of the digits on the upper face is 25.

(iv) Digit on the upper face of the first die is less than the digit on the second die.

Solution : Sample space,

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}   n(S) = 36

(i) Event E : The sum of the digits on the upper face is a prime number.

E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}   ∴ n(E) = 15

(ii) Event F : The sum of the digits on the upper face is a multiple of 5.

F = { (1, 4), (2, 3), (3, 2), (4, 1), (4, 6), (5, 5), (6, 4) }   ∴ n(F) = 7

(iii) Event G : The sum of the digits on the upper face is 25.

G = { } = ϕ ∴ n (G) = 0

(iv) Event H : The number on upper face of first die is less than the digit on second die.

H = {(1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 3) (2, 4) (2, 5) (2, 6) (3, 4) (3, 5) (3, 6) (4, 5) (4, 6) (5, 6)}   ∴ n(H) = 15

Practice Set 5.3

1. Write sample space 'S' and number of sample point n(S) for each of the following experiments. Also write events A, B, C in the set form and write n(A), n(B), n(C).

(1) One die is rolled,

Event A : Even number on the upper face.

Event B : Odd number on the upper face.

Event C : Prime number on the upper face.

(2) Two dice are rolled simultaneously,

Event A : The sum of the digits on upper faces is a multiple of 6.

Event B : The sum of the digits on the upper faces is minimum 10.

Event C : The same digit on both the upper faces.

(3) Three coins are tossed simultaneously.

Condition for event A : To get at least two heads.

Condition for event B : To get no head.

Condition for event C : To get head on the second coin.

(4) Two digit numbers are formed using digits 0, 1, 2, 3, 4, 5 without repetition of the digits.

Condition for event A : The number formed is even

Condition for event B : The number formed is divisible by 3.

Condition for event C : The number formed is greater than 50.

(5) From three men and two women, environment committee of two persons is to be formed.

Condition for event A : There must be at least one woman member.

Condition for event B : One man, one woman committee to be formed.

Condition for event C : There should not be a woman member.

(6) One coin and one die are thrown simultaneously.

Condition for event A : To get head and an odd number.

Condition for event B : To get a head or tail and an even number.

Condition for event C : Number on the upper face is greater than 7 and tail on the coin.

Let's learn.

Probability of an event

Let us think of a simple experiment. A bag contains 4 balls of the same size. Three of them are white and the fourth is black. You are supposed to pick one ball at random without seeing it. Then obviously, possibility of getting a white ball is more.

In Mathematical language, when possibility of an expected event is expressed in number, it is called 'Probability' . It is expressed as a fraction or percentage using the following formula.

For a random experiment, if sample space is 'S'and 'A' is an expected event then probability of 'A' is P(A). It is given by following formula.

$$P(A) = \frac{\text{Number of sample points in event A}}{\text{Number of sample points in sample spaces}} = \frac{n(A)}{n(S)}$$

In the above experiment, getting a white ball is event A. As there are three white balls n(A) = 3, As the number of balls is 4, n(S) = 4

$\therefore$ probability of getting a white ball is, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$.

Similarly, if getting black ball is event B, then n(B) = 1 $\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$.

Solved Examples

Ex. (1) Find the probability of the following, when one coin is tossed.

(i) getting head (ii) getting tail

Solution : Let 'S' be the sample space.

S = {H, T} n(S) = 2

(i) Let event A be getting head

A = {H} $\therefore$ n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

(ii) Let event B be getting tail

B = {T} $\therefore$ n(B) = 1

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$$

Ex. (2) If one die is rolled then find the probability of each of the following events.

(i) Number on the upper face is prime

(ii) Number on the upper face is even.

Solution : 'S' is the sample space.

S = {1, 2, 3, 4, 5, 6} $\therefore$ n(S) = 6

(i) Event A : Prime number on the upper face.

A = {2, 3, 5} $\therefore$ n(A) = 3

$$P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$$

(ii) Event B : Even number on the upper face.

B = {2, 4, 6} $\quad\therefore$ n(B) = 3

$$P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{3}{6} = \frac{1}{2}$$

Ex. (3) A card is drawn from a well shuffled pack of 52 playing cards. Find the probability of each event. The card drawn is (i) a red card (ii) a face card

Solution : 'S' is the sample space. $\quad\therefore$ n(S) = 52

Event A : Card drawn is a red card.

Total red cards = 13 hearts + 13 diamonds = 26

$\therefore$ n(A) = 26

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Event B : Card drawn is a face card.

Total face cards = 12 $\quad\therefore$ n(B) = 12

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

Ex. (4) A box contains 5 strawberry chocolates, 6 coffee chocolates and 2 peppermint chocolates. Find the probability of each of the following events, if one of the chocolates is picked from the box at random. (i) it is a coffee chocolate. (ii) it is a peppermint chocolate.

Solution : Sample space is 'S' and n(S) = 5 + 6 + 2 = 13

Event A : it is a coffee chocolate

$\therefore$ n(A) = 6

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{13}$$

Event B : it is a peppermint chocolate

$\therefore$ n(B) = 2

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{2}{13}$$

Let's remember!

- The Probability is expressed as a fraction or a percentage.
- The probability of any event is from 0 to 1 or 0% to 100%.

  If E is any event, $0 \le P(E) \le 1$ or $0\ \% \le P(E) \le 100\ \%$.

  e.g. probability $\frac{1}{4}$ is written as 25 %.
- This lesson began with a discussion of 40 chits with names of plants and each of 40 students picking a chit. Only one chit had the name Basil on it. The probability of any student getting the chit of Basil is $\frac{1}{40}$. For a student standing first or last in the row, or anywhere in between, the probability is the same.

Practice Set 5.4

1. If two coins are tossed, find the probability of the following events.

   (1) Getting at least one head. (2) Getting no head.

2. If two dice are rolled simultaneously, find the probability of the following events.

   (1) The sum of the digits on the upper faces is at least 10.

   (2) The sum of the digits on the upper faces is 33.

   (3) The digit on the first die is greater than the digit on second die.

3. There are 15 tickets in a box, each bearing one of the numbers from 1 to 15. One ticket is drawn at random from the box. Find the probability of event that the ticket drawn -

   (1) shows an even number. (2) shows a number which is a multiple of 5.

4. A two digit number is formed with digits 2, 3, 5, 7, 9 without repetition. What is the probability that the number formed is

   (1) an odd number ? (2) a multiple of 5 ?

5. A card is drawn at random from a pack of well shuffled 52 playing cards. Find the probability that the card drawn is -

   (1) an ace. (2) a spade.

Problem Set 5

1. Choose the correct alternative answer for each of the following questions.

(1) Which number cannot represent a probability ?

(A) $\frac{2}{3}$ (B) 1.5 (C) 15 % D) 0.7

(2) A die is rolled. What is the probability that the number appearing on upper face is less than 3 ?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ D) 0

(3) What is the probability of the event that a number chosen from 1 to 100 is a prime number ?

(A) $\frac{1}{5}$ (B) $\frac{6}{25}$ (C) $\frac{1}{4}$ D) $\frac{13}{50}$

(4) There are 40 cards in a bag. Each bears a number from 1 to 40. One card is drawn at random. What is the probability that the card bears a number which is a multiple of 5 ?

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ D) $\frac{1}{3}$

(5) If $n(A) = 2$, $P(A) = \frac{1}{5}$, then $n(S) = ?$

(A) 10 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ D) $\frac{1}{3}$

2. Basketball players John, Vasim, Akash were practising the ball drop in the basket. The probabilities of success for John, Vasim and Akash are $\frac{4}{5}$, 0.83 and 58% respectively. Who had the greatest probability of success ?

3. In a hockey team there are 6 defenders , 4 offenders and 1 goalee. Out of these, one player is to be selected randomly as a captain. Find the probability of the selection that -

(1) The goalee will be selected. (2) A defender will be selected.

4. Joseph kept 26 cards in a cap, bearing one English alphabet on each card. One card is drawn at random. What is the probability that the card drawn is a vowel card ?

5. A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets,

(1) a red balloon (2) a blue balloon (3) a green balloon.

6. A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue?

7. Six faces of a die are as shown below.

A	B	C	D	E	A

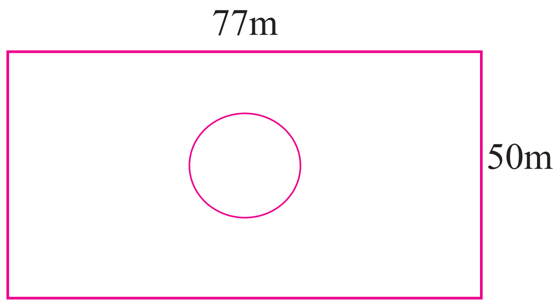
If the die is rolled once, find the probability of -

(1) 'A' appears on upper face.

(2) 'D' appears on upper face.

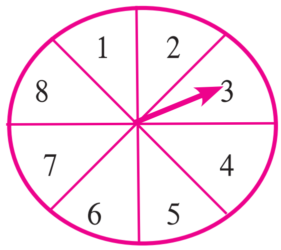
8. A box contains 30 tickets, bearing only one number from 1 to 30 on each. If one ticket is drawn at random, find the probability of an event that the ticket drawn bears (1) an odd number (2) a complete square number.

9. Length and breadth of a rectangular garden are 77 m and 50 m. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake.



10. In a game of chance, a spinning arrow comes to rest at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8.

All these are equally likely outcomes. Find the probability that it will rest at



(1) 8.

(2) an odd number.

(3) a number greater than 2.

(4) a number less than 9.

11. There are six cards in a box, each bearing a number from 0 to 5. Find the probability of each of the following events, that a card drawn shows,

(1) a natural number.

(2) a number less than 1.

(3) a whole number.

(4) a number is greater than 5.

12. A bag contains 3 red, 3 white and 3 green balls. One ball is taken out of the bag at random. What is the probability that the ball drawn is -

    (1) red. (2) not red (3) either red or white.

13. Each card bears one letter from the word 'mathematics' The cards are placed on a table upside down. Find the probability that a card drawn bears the letter 'm'.

14. Out of 200 students from a school, 135 like Kabbaddi and the remaining students do not like the game. If one student is selected at random from all the students, find the probability that the student selected dosen't like Kabbaddi.

15.★ A two digit number is to be formed from the digits 0, 1, 2, 3, 4. Repetition of the digits is allowed. Find the probability that the number so formed is a -

    (1) prime number (2) multiple of 4

    (3) multiple of 11.

16.★ The faces of a die bear numbers 0, 1, 2, 3, 4, 5. If the die is rolled twice, then find the probability that the product of digits on the upper face is zero.

17. Do the following activity -

Activity I : Total number of students in your class, n(S) = ☐

Number of students from your class, wearing spectacles, n(A) = ☐

Probability of a randomly selected student wearing spectacles, P(A) = ☐

Probability of a randomly selected student not wearing spectacles, P(B) = ☐

Activity II : Decide the sample space yourself and fill in the following boxes.

Sample space → S = { } → n(S) = ☐

The condition for event A is 'getting an even number'. → A = { } → n(A) = ☐

$\therefore$ P(A) = $\dfrac{\square}{\square}$ = ☐

N63IJ7

6 Statistics

Let's study.

- Measures of a central tendency-
  mean, median and mode from grouped frequency table.
- Graphical representation of statistical data -
  histogram, frequency polygon, pie diagram

Statistics is useful in many fields of life: for example, agriculture, economics, commerce, medicine, botany, biotechnology, physics, chemistry, education, sociology, administration etc. An experiment can have many outcomes. To assess the possibility of possible outcomes, one has to carry out the experiment on a large scale and keep the record meticulously. Possibilities of different outcomes can be assessed using the record. For this purpose, rules are formulated in statistics.

Francis Galton (1822-1911) has done much of fundamental work in statistics.He used to prepare questionnaires, distribute them among people and request them to fill them up. He collected information from a number of people and recorded their backgrounds, financial situations, likes and dislikes, health etc. on a large scale. By that time, it was known that the fingerprints of different people are different. He collected finger-prints of a large number of people and invented a method of their classification. Using statistical methods, he showed that the possibility of finger prints of two different people being identical is nearly zero. This result made it possible to identify a person from his finger-prints. This method of identifying criminals was accepted in the judiciary. He had done much work in the field of anthropology of humans and other animals also.

Francis Galton

Let's recall.

We usually find a specific property in the numerical data collected in a survey that the scores have a tendency to cluster around a particular score. This score is a representative number of the group. The number is called the measure of central tendency.

In the previous standards we have studied the measures of central tendency, namely the mean, median and mode, for ungrouped data.

Activity 1 : Measure the height in cm of all students in your class. We find that the heights of many students cluster near a specific number.

Activity 2 : Collect a number of fallen leaves of a peepal tree. Distribute the leaves among the students and ask them to measure the lengths of them. Record the lengths. We notice that their lengths tend to cluster around a number.

Now we are going to do some more study of the mean, median and mode. Let us know the symbols and the terminology required for it.

$$\text{The mean of statistical data} = \frac{\text{The sum of all scores}}{\text{Total no. of scores}} = \frac{\sum_{i=1}^{N} x_i}{N}$$

(Here x_i is the i th score)

Mean is denoted by $\overline{X}$ and it represents the average of the given data.

$$\overline{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Let's learn.

Mean from classified frequency distribution

When the number of scores in a data is large, it becomes tedious to write all numbers in the above formula and take their sum. So we use some different methods to find the sum.

Sometimes, the large data collected from an experiment is presented in a table in the grouped form. In such a case, we cannot find the exact mean of statistical data. Hence, let us study a method which gives the approximate mean, or a number nearby.

Direct method

Let us study the method by an example.

Ex. : The following table shows the frequency distribution of the time required for each worker to complete a work . From the table find the mean time required to complete the job for a worker.

Time (Hrs.) for each to complete the work	15-19	20-24	25-29	30-34	35-39
No. of workers	10	15	12	8	5

Solution :

(1) Vertical columns are drawn as shown in the table.

(2) Classes are written in the first column.

(3) The class mark x_i is in the second column.

(4) In the third column, the number of workers, that is frequency (f_i) is written.

(5) In the fourth column, the product ($x_i \times f_i$) for each class is written.

(6) Then $\sum_{i=1}^{N} x_i f_i$ is written.

(7) The mean is found using the formula

Class (Time-hours)	Class mark x_i	Frequency (No. of Workers) f_i	Class mark × Frequency $x_i f_i$
15-19	17	10	170
20-24	22	15	330
25-29	27	12	324
30-34	32	8	256
35-39	37	5	185
Total		$\sum f_i = 50$	$\sum x_i f_i = 1265$

$$\text{Mean} = \overline{X} = \frac{\sum x_i f_i}{N} = \frac{1265}{50} = 25.3 \quad \because \sum f_i = N$$

The mean time required to complete the work for a worker = 25.3 hrs. (Approx)

Solved Examples

Ex. **(1)** The percentage of marks of 50 students in a test is given in the following table. Find the mean of the percentage.

Percentage of marks	0-20	20-40	40-60	60-80	80-100
No. of students	3	7	15	20	5

Solution : The following table is prepared as per steps.

Class (Percentage of marks)	Class mark x_i	Frequency (No. of students) f_i	Class mark × frequency $x_i f_i$
0-20	10	3	30
20-40	30	7	210
40-60	50	15	750
60-80	70	20	1400
80-100	90	5	450
Total		$N = \sum f_i = 50$	$\sum x_i f_i = 2840$

$$\overline{X} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{2840}{50}$$

$$= 56.8$$

$\therefore$ The mean of the percentage = 56.8

Ex. (2) The maximum temperatures in °C of 30 towns, in the last summer, is shown in the following table. Find the mean of the maximum temperatures.

Max. temp.	24–28	28–32	32–36	36–40	40–44
No. of towns	4	5	7	8	6

Solution :

Class (Temp. °C)	Class mark x_i	Frequency (No. of towns) f_i	Class mark × frequency $x_i f_i$
24–28	26	4	104
28–32	30	5	150
32–36	34	7	238
36–40	38	8	304
40–44	42	6	252
Total		$N = \sum f_i = 30$	$\sum x_i f_i = 1048$

$$\text{Mean} = \overline{X} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1048}{30} = 34.9\ ^\circ C$$

Assumed mean method

In the examples solved above, we see that some times the product $x_i f_i$ is large. Hence it becomes difficult to calculate the mean by direct method. So let us study another method, called the 'assumed mean method'. Finding the mean becomes simpler if we use addition and division in this method.

For example, we have to find the mean of the scores 40, 42, 43, 45, 47 and 48.

The obeservation of the scores reveals that the mean of the data is more than 40. So let us assume that the mean is 40. 40–40 = 0, 42 – 40 = 2, 43–40 = 3, 45–40 = 5, 47 – 40 = 7, 48 – 40 = 8 These are called 'deviations'. Let us find their mean. Adding this mean to the assumed mean, we get the mean of the data.

That is, mean = assumed mean + mean of the deviations

$$\overline{X} = 40 + \left(\frac{0+2+3+5+7+8}{6}\right) = 40 + \frac{25}{6} = 40 + 4\frac{1}{6} = 44\frac{1}{6}$$

Using the symbols-

A– for assumed mean; d- for deviation and $\overline{d}$ – for the mean of the deviations, the formula for mean of the given data can be briefly written as $\overline{X} = A + \overline{d}$.

Let us solve the same example taking 43 as assumed mean. For this, let us find the deviations by subtracting 43 from each score.

$40 - 43 = -3,\ 42 - 43 = -1,\ 43 - 43 = 0,\ 45 - 43 = 2,\ 47 - 43 = 4,\ 48 - 43 = 5$

The sum of the deviations = $-3 -1 + 0 + 2 + 4 + 5 = 7$

Now, $\overline{X} = A + \overline{d}$

$= 43 + \left(\frac{7}{6}\right)$ (as the number of deviations is 6)

$= 43 + 1\frac{1}{6} = 44\frac{1}{6}$

Note that; use of assumed mean method reduces the work of calculations.

Also note that; taking any score, or any other convenient number as asssumed mean does not change the mean of the data.

Ex. : The daily sale of 100 vegetable vendors is given in the following table. Find the mean of the sale by assumed mean method.

Daily sale (Rupees)	1000-1500	1500-2000	2000-2500	2500-3000
No. of vendors	15	20	35	30

Solution : Assumed mean = A = 2250, $d_i = x_i$ - A is the deviation.

Class Daily sale (Rupees)	Class mark x_i	$d_i = x_i - A$ $= x_i - 2250$	Frequency (No. of vendors) f_i	Frequency × deviation $f_i d_i$
1000-1500	1250	-1000	15	-15000
1500-2000	1750	-500	20	-10000
2000-2500	2250→A	0	35	0
2500-3000	2750	500	30	15000
Total			$N = \sum f_i = 100$	$\sum f_i d_i = -10000$

The table is prepared according to the following steps :-

(1) Assumed mean, A is choosen as 2250. (Generally, the class mark of the class having maximum frequency is chosen as the assumed mean.)

(2) Classes of sale are written in the first column.

(3) Class marks are written in the second column.

(4) Values of $d_i = x_i - A = x_i - 2250$ are written in the third column.

(5) In the fourth column, the number of vendors and their sum is written as $\sum f_i$.

(6) In the fifth column, the product $(f_i \times d_i)$ and their sum is written as $\sum f_i d_i$.

$\overline{d}$ and $\overline{X}$ are calculated using the formulae.

$$\overline{d} = \frac{\sum f_i d_i}{\sum f_i} = -\frac{10000}{100} = -100 \qquad \therefore \text{ mean } \overline{X} = A + \overline{d} = 2250 - 100 = 2150$$

The mean of sale is ₹ 2150.

Activity :- Solve the above example by direct method.

Solved Examples

Ex. (1) The following table shows the frequency table of daily wages of 50 workers in a trading company. Find the mean wages of a worker, by assumed mean method.

Daily Wages (Rs)	200-240	240-280	280-320	320-360	360-400
Frequency (No. of workers)	5	10	15	12	8

Solution : Let us take the assumed mean A = 300.

Class (₹ Wage)	Class mark x_i	$d_i = x_i - A$ $d_i = x_i - 300$	Frequency (No. of workers) f_i	Frequency × Deviation $f_i d_i$
200-240	220	-80	5	-400
240-280	260	-40	10	-400
280-320	300→A	0	15	0
320-360	340	40	12	480
360-400	380	80	8	640
Total			$\sum f_i = 50$	$\sum f_i d_i = 320$

$$\overline{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{320}{50} = 6.4$$

Mean, $\overline{X} = A + \overline{d}$

$= 300 + 6.4$

$= 306.40$

The mean of daily wages = 306.40 ₹

Step deviation method

We studied the direct method and assumed mean method to find the mean. Now we study one more method which reduces the calculations still further.

- Find the values of d_i as $d_i = x_i$ - A and write in the column.
- If we can find g , the G.C.D. of all d_i easily, we create a column for all u_i where $u_i = \frac{d_i}{g}$
- Find the mean $\overline{u}$ of all u_i.
- Using the formula $\overline{X} = A + \overline{u}\, g$, find the mean of the data.

Example : The amount invested in health insurance by 100 families is given in the following frequency table. Find the mean of investments using step deviation method.

Amount invested (₹)	800–1200	1200–1600	1600–2000	2000–2400	2400–2800	2800–3200
No. of families	3	15	20	25	30	7

Solution : Assumed mean A = 2200 observing all 'd_i's g = 400.

Class (Insurance ₹)	Class Mark x_i	$d_i = x_i - A$ $= x_i - 2200$	$u_i = \frac{d_i}{g}$	Frequency (No. of families) f_i	$f_i u_i$
800–1200	1000	–1200	–3	3	–9
1200–1600	1400	–800	–2	15	–30
1600–2000	1800	–400	–1	20	–20
2000–2400	2200→A	0	0	25	0
2400–2800	2600	400	1	30	30
2800–3200	3000	800	2	7	14
Total				$\sum f_i = 100$	$\sum f_i u_i = -15$

The above table is made using the following steps.

(1) The classes of investment are written in the first column.

(2) The values of x_i are written in the second column.

(3) The values of $d_i = x_i - A$ are written in the third column.

(4) The G.C.D of all values of d_i is 400. Therefore $g = 400$.

(5) The corresponding frequencies are written in the fifth column.

(6) The product $f_i \times u_i$ for each class is written in the sixth column.

The mean of u_i is found by the following formula.

$$\overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-15}{100} = -0.15$$

$$\overline{X} = A + \overline{u}\, g$$

$$= 2200 + (-0.15)(400)$$

$$= 2200 + (-60.00)$$

$$= 2200 - 60 = 2140$$

∴ The mean of investments in health insurance = ₹ 2140.

Activity : Solve the above example by direct method and by assumed mean method and see that the mean found by any method is the same.

Solved Example

Ex. (1) The following table shows the funds collected by 50 students for flood affected people. Find the mean of the funds.

Fund (Rupees)	0-500	500-1000	1000-1500	1500-2000	2000-2500	2500-3000
No. of students	2	4	24	18	1	1

If the number of scores in two consecutive classes is very low, it is convenient to club them. So, in the above example, we club the classes 0 - 500, 500 - 1000 and 2000 - 2500, 2500 - 3000. Now the new table is as follows

Fund (Rupees)	0-1000	1000-1500	1500-2000	2000-3000
No. of students	6	24	18	2

Solution : Let A = 1250, examining all d_i, g = 250.

Class Fund (₹)	Class mark x_i	$d_i = x_i - A$ $= x_i - 1250$	$u_i = \frac{d_i}{g}$	Frequency f_i	$f_i u_i$
0-1000	500	-750	-3	6	-18
1000-1500	1250→A	0	0	24	0
1500 - 2000	1750	500	2	18	36
2000-3000	2500	1250	5	2	10
Total				$\sum f_i = 50$	$\sum f_i u_i = 28$

$$\overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{28}{50} = 0.56,$$

$$\overline{u}\, g = 0.56 \times 250 = 140$$

$$\overline{X} = A + g\,\overline{u} = 1250 + 140 = 1390$$

$\therefore$ the average of the funds is ₹ 1390.

Activity -

1. Solve the above example by direct method.
2. Verify that the mean calculated by assumed mean method is the same.
3. Find the mean in the above example by taking A = 1750.

Practice Set 6.1

1. The following table shows the number of students and the time they utilized daily for their studies. Find the mean time spent by students for their studies by direct method.

Time (hrs.)	0-2	2-4	4-6	6-8	8-10
No. of students	7	18	12	10	3

2. In the following table, the toll paid by drivers and the number of vehicles is shown. Find the mean of the toll by 'assumed mean' method.

Toll (Rupees)	300-400	400-500	500-600	600-700	700-800
No. of vehicles	80	110	120	70	40

3. A milk centre sold milk to 50 customers. The table below gives the number of customers and the milk they purchased. Find the mean of the milk sold by direct method.

Milk Sold (Litre)	1-2	2-3	3-4	4-5	5-6
No. of Customers	17	13	10	7	3

4. A frequency distribution table for the production of oranges of some farm owners is given below. Find the mean production of oranges by 'assumed mean' method.

Production (Thousand rupees)	25-30	30-35	35-40	40-45	45-50
No. of farm owners	20	25	15	10	10

5. A frequency distribution of funds collected by 120 workers in a company for the drought affected people are given in the following table. Find the mean of the funds by 'step deviation' method.

Fund (Rupees)	0-500	500-1000	1000-1500	1500-2000	2000-2500
No. of workers	35	28	32	15	10

6. The following table gives the information of frequency distribution of weekly wages of 150 workers of a company. Find the mean of the weekly wages by 'step deviation' method.

Weekly wages (Rupees)	1000-2000	2000-3000	3000-4000	4000-5000
No. of workers.	25	45	50	30

Let's recall.

There was a science exhibition in a city for two days. A school sent two boys and two girls to participate in the exhibition. There were ten hotels, within a distance of one kilometer, from the venue of exhibition. Their rates of meals, in the ascending order were rupees 40, 45, 60, 65, 70, 80, 90, 100 and 500. They had to choose one of them for the dinner.

The average of rates in all the hotels was ₹ $\frac{1130}{10} = 113$.

Which hotel do you think they chose? Except the rate ₹ 500, all others were less than ₹ 113. The students decided to choose a hotel having medium rate. The first day they chose the hotel with rate ₹ 70 and on the next day, the hotel with the rate ₹ 80/-.

This example shows that sometimes the median is used instead of the mean.

In the previous standard we have studied the concept of a median.

- If the numbers in a data are arranged in the ascending order, the number at the middle position is called the median of the data.
- The median divides the array of numbers in two equal parts, that is the number of scores below and above the median is equal.
- The scores are written as $k_1 \leq k_2 \leq k_3 \ldots \ldots \leq k_n$.
- If the number of scores is odd, then the $\frac{n+1}{2}$ th score is the median of the data. That is, the number of scores below as well as above $k_{\frac{n+1}{2}}$ is $\frac{n-1}{2}$; verify the fact by taking n = 2m + 1.
- If the number of the scores is even, then the mean of the middle two terms is the median. This is because the number of terms below $k_{\frac{n}{2}}$ and above $k_{\frac{n+2}{2}}$ is equal, which is $\frac{n-2}{2}$. Verify this by taking n = 2m.
- Hence the mean of $\frac{n}{2}$ th and $\frac{n+2}{2}$ th term is the median of the data.

Ex. (1) In 32, 33, 38, 40, 43, 48, 50; the fourth number is at the middle. Hence the median of the data is 40

Ex. (2) In 61, 62, 65, 66, 68, 70, 74, 75 ; the number of scores is 8, that is even. Therefore, the fourth and the fifth numbers are at the middle, which are 66 and 68. Hence the median = $\frac{66+68}{2}$ = 67

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Let's learn.

Median for grouped frequency distribution

When the number of scores in a data is large, it is difficult to arrange them in ascending order. In such case, the data is divided into groups. So let us study, with an example, how the median of grouped data is found.

Ex. The scores 6, 8, 10.4, 11, 15.5, 12, 18 are grouped in the following table.

Class	Tally Marks	Frequency
6−10	||	2
11−15	||	2
16−20	|	1

Class	Tally Marks	Frequency
5.5−10.5	|||	3
10.5−15.5	||	2
15.5−20.5	||	2

We could not record the scores 10.4 and 15.5 in the first table, as they cannot be placed in any of the classes 6−10, 11−15, 16−20. We know that in such a case the classes are made continuous.

For this, in the first table, the lower class limits are reduced by 0.5 and the upper class limits are increased by 0.5 and the second table is prepared. In the second table, the score 15.5 is placed in the class 15.5−20.5.

Note that if the method of making groups is changed, the frequency distribution may change.

Let's remember!

In the above table, the class mark of 6−10 is $= \frac{6+10}{2} = \frac{16}{2} = 8$;

Similarly, the class mark of 5.5−10.5 is $= \frac{5.5+10.5}{2} = \frac{16}{2} = 8$.

This shows that, if the classes are made continuous, the class marks do not change

Solved Example :

The following table shows frequency distribution of marks of 100 students of 10^{th} class which they obtained in a practice examination. Find the median of the marks.

Marks in exam	0−20	20−40	40−60	60−80	80−100
No. of students	4	20	30	40	6

Solution : N = 100

$\therefore \frac{N}{2} = 50$. Hence the 50th number will be the approximate median. Hence we have to find out the class which contains the 50th term. Writing the cumulative frequencies less than the upper limit, we can find it.

So, let us prepare less than cumulative frequency distribution table.

Class (Student's marks)	No. of students fi	Cumulaive frequency less than the upper limit cf
0–20	4	4
20–40	20	24
40–60	30	54
60–80	40	94
80–100	6	100

- From the table, the 50th score is in the class 40–60. The class which contains the median, is called the **median class**. So, here 40–60 is the median class.
- The lower class limit of 40–60 is 40. Its frequency is 30.
- Out of the first 50 scores, 24 scores are less than 40. The remaining 50 – 24 = 26 are in class (40–60). The 50th score in that class is estimated as follows.
- 26 out of 30 scores in the class 40–60, are upto the 50th score and the class interval is 20. So it is assumed that, the 50th score is more than 40 by $\frac{26}{30} \times 20$.

$\therefore$ it is approximately $40 + \frac{26}{30} \times 20 = 40 + \frac{52}{3} = 57\frac{1}{3}$.

$\therefore$ median $= 57\frac{1}{3}$

- We can formulate this as follows,

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

In the formula,

L = Lower class limit of the median class,

N = Sum of frequencies

h = Class interval of the median class,

f = Frequency of the median class

cf = Cumulative frequency of the class preceding the median class.

In the above example; $\frac{N}{2} = 50$, $cf = 24$, $h = 20$, $f = 30$, $L = 40$,

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h \quad \ldots\ldots\ (\text{Formula})$$

$$= 40 + \left(\frac{50 - 24}{30}\right) \times 20$$

$$= 40 + \frac{26 \times 20}{30}$$

$$= 40 + 17\frac{1}{3}$$

$$= 57\frac{1}{3}$$

Let's remember!

✦ If the given classes are not continuous, we have to make them continuous to find out the median.

✦ It is difficult to write the scores in the asscending order when the number of scores is large. So the data is classified into groups. It is not possible to find the exact median of a classified data, but the approximate median is found by the formula.

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

Solved Examples

Ex. **(1)** Observe the following frequency distribution table. It shows the distances travelled by 60 public transport buses in a day. Find the median of the distance travelled.

Daily distance travelled (in Km)	200-209	210-219	220-229	230-239	240-249
No. of buses	4	14	26	10	6

Solution : (1) The classes in the table are not continuous.

The upper class limit of a class and the lower class of its succeeding class differ by 1.

∴ Let us subtract 1 ÷ 2 = 0.5 from the lower class limit of each class and add to the upper class limit of each class, and make the classes continuous.

(2) Make a column of cumulative frequency 'less than' in the new table showing the continuous classes.

Given Class	Continuous classes	Frequency f_i	Cumulative frequency less than
200-209	199.5-209.5	4	4
210-219	209.5-219.5	14	18 → cf
220-229	219.5-229.5	26 → f	44
230-239	229.5-239.5	10	54
240-249	239.5-249.5	6	60

Here, total of frequencies = Σf_i = N = 60 ∴ $\frac{N}{2}$ = 30.

∴ 30th score is the approximate median.

First 18 scores are less than 219.5 and the remaining, 30 - 18 = 12 scores are in the class 219.5 - 229.5. Therefore, 219.5 - 229.5 is the median class. The cumulative frequency of the class 219.5-229.5 is 44.

In the formula,

L = Lower class limit = 219.5, h = Class interval of the median class = 10

cf = The frequency of the class preceding the median class = 18,

f = The frequency of the median class = 26

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

$\therefore$ Median $= 219.5 + \left(\frac{30-18}{26}\right) \times 10$

$= 219.5 + \left(\frac{12 \times 10}{26}\right)$

$= 219.50 + 4.62$

$= 224.12$

$\therefore$ The median of the distance travelled is = 224.12 Km

Ex. (2) The following table shows the ages of persons who visited a museum on a certain day. Find the median age of the persons visiting the museum.

Age (Years)	No. of persons
Less than 10	3
Less than 20	10
Less than 30	22
Less than 40	40
Less than 50	54
Less than 60	71

Solution : The given cumulative frequency table is of the 'less than' form. So, we will have to decide the true class limits first. We know that, the 'less than' cumulative frequency is associated with the upper class limits. The upper class limit of the first class is 10. The age of any person is a positive number, so the first class must be 0–10. The upper class limit of the next class is 20, so the second class must be 10–20. In this way, make the classes of interval 10. In this way the last class is 50–60. So the given table can now be rewritten as follows.

Age (years)	Class	No. of persons (Frequency)	Cumulative frequency Less than
Less than 10	0–10	3	3
Less than 20	10–20	10 – 3 = 7	10
Less than 30	20–30	22 – 10 = 12	22 $\rightarrow cf$
Less than 40	30–40	40 – 22 = 18 $\rightarrow f$	40
Less than 50	40–50	54 – 40 = 14	54
Less than 60	50–60	71 – 54 = 17	71

Here N = 71 $\therefore \frac{N}{2}$ = 35.5 and h = 10

The number 35.5 is in the class 30−40, hence it is the median class. The cumulative frequency of its precending class is 22, $\therefore$ cf = 22, L = 30, f = 18.

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

$$= 30 + (35.5 - 22)\frac{10}{18}$$

$$= 30 + (13.5)\frac{10}{18}$$

$$= 30 + 7.5$$

$$= 37.5$$

$\therefore$ the median age of the persons visiting the museum is = 37.5 years

Practice Set 6.2

1. The following table shows classification of number of workers and the number of hours they work in a software company. Find the median of the number of hours they work.

Daily No. of hours	8−10	10−12	12−14	14−16
Number of workers	150	500	300	50

2. The frequency distribution table shows the number of mango trees in a grove and their yield of mangoes. Find the median of data.

No. of Mangoes	50−100	100−150	150−200	200−250	250−300
No. of trees	33	30	90	80	17

3. The following table shows the classification of number of vehicles and their speeds on Mumbai-Pune express way. Find the median of the data.

Average Speed of Vehicles(Km/hr)	60−64	64−69	70−74	75−79	79−84	84−89
No. of vehicles	10	34	55	85	10	6

4. The production of electric bulbs in different factories is shown in the following table. Find the median of the productions.

No. of bulbs produced (Thousands)	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of factories	12	35	20	15	8	7	8

Let's learn.

Mode for grouped frequency distribution

We know that the score repeating maximum number of times in a data is called the mode of the data.

For example, a company manufactures bicycles of different colours. To know which colour is most wanted, the company needs to know the mode. If a company manufactures many items, it may want to know which item sells most. In such cases, the mode is needed.

We have learnt the method of finding the mode of an ungrouped data.

Now let us study the method of estimation of mode of grouped data.

The following formula is used for the purpose.

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

In the above formula,

L = Lower class limit of the modal class.

f_1 = Frequency of the modal class.

f_0 = Frequency of the class preceding the modal class.

f_2 = Frequency of the class succeeding the modal class.

h = Class interval of the modal class.

Let us see, with an example, how the mode is estimated using the above formula.

Solved Examples

Ex.(1) The classification of children according to their ages, playing on a ground is shown in the following table. Find the mode of ages of the children.

Age-group of children (Yrs)	6−8	8−10	10−12	12−14	14−16
No. of children	43	58 $\rightarrow f_0$	70 $\rightarrow f_1$	42 $\rightarrow f_2$	27

From the table, we note that the maximum number of children is of the age-group 10−12. So the modal class is 10−12.

Solution : Here f_1 = 70, and modal class is 10-12.

$\therefore$ in the given example,

L = Lower class limit of the modal class = 10

h = Class interval of the modal class = 2

f_1 = Frequency of the modal class = 70

f_0 = Frequency of the class preceding the modal class = 58

f_2 = Frequency of the class succeeding the modal class = 42

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 10 + \left[\frac{70-58}{2(70)-58-42}\right] \times 2$$

$$= 10 + \left[\frac{12}{140-100}\right] \times 2$$

$$= 10 + \left[\frac{12}{40}\right] \times 2$$

$$= 10 + \frac{24}{40}$$

$$= 10 + 0.6$$

$$= 10.6$$

$\therefore$ the mode of the ages of children playing on the ground is 10.6 Years.

Ex. **(2)** The following frequency distribution table shows the classification of the number of vehicles and the volume of petrol filled in them. Find the mode of the volume.

Petrol filled (Litre)	1–3	4–6	7–9	10–12	13–15
No. of vehicle	33	40	27	18	12

Solution : The given classes are not continuous. So, let us make them continuous and rewrite the table.

Class	Continuous classes	Frequency
1–3	0.5–3.5	33 $\rightarrow f_0$
4–6	3.5–6.5	40 $\rightarrow f_1$
7–9	6.5–9.5	27 $\rightarrow f_2$
10–12	9.5–12.5	18
13–15	12.5–15.5	12

From the above table, the modal class is 3.5–6.5

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$\text{Mode} = 3.5 + \left[\frac{40 - 33}{2(40) - 33 - 27}\right] \times h$$

$$= 3.5 + \left[\frac{7}{80 - 60}\right] \times 3$$

$$= 3.5 + \frac{21}{20}$$

$$= 3.5 + 1.05$$

$$= 4.55$$

$\therefore$ The mode of the volume of petrol filled is = 4.55 litre.

Practice Set 6.3

1. The following table shows the information regarding the milk collected from farmers on a milk collection centre and the content of fat in the milk, measured by a lactometer. Find the mode of fat content.

Content of fat (%)	2-3	3-4	4-5	5-6	6-7
Milk collected (Litre)	30	70	80	60	20

2. Electricity used by some families is shown in the following table. Find the mode for use of electricity.

Use of electricity (Unit)	0-20	20-40	40-60	60-80	80-100	100-120
No. of families	13	50	70	100	80	17

3. Grouped frequency distribution of supply of milk to hotels and the number of hotels is given in the following table. Find the mode of the supply of milk.

Milk (Litre)	1-3	3-5	5-7	7-9	9-11	11-13
No. of hotels	7	5	15	20	35	18

4. The following frequency distribution table gives the ages of 200 patients treated in a hospital in a week. Find the mode of ages of the patients.

Age (years)	Less than 5	5-9	10-14	15-19	20-24	25-29
No. of patients	38	32	50	36	24	20

Activity :-

- Find the mean weight of 20 students in your class.
- Find the mode of sizes of shirts of students in your class
- Every student in your class should measure his/her own pulse rate, note the pulse rates of all students and find the mode of the pulse rate.
- Measure the height of every student in the class, prepare a grouped frequency distribution table and find the median of the heights.

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Let's remember!

We have studied the central tendencies mean, median and mode. Before selecting any of these measures, we have to know the purpose of its selection clearly.

Suppose, we have to judge the performance of five divisions of standard 10 in the internal examination. For the purpose, we have to find the 'mean' of marks of students in each division.

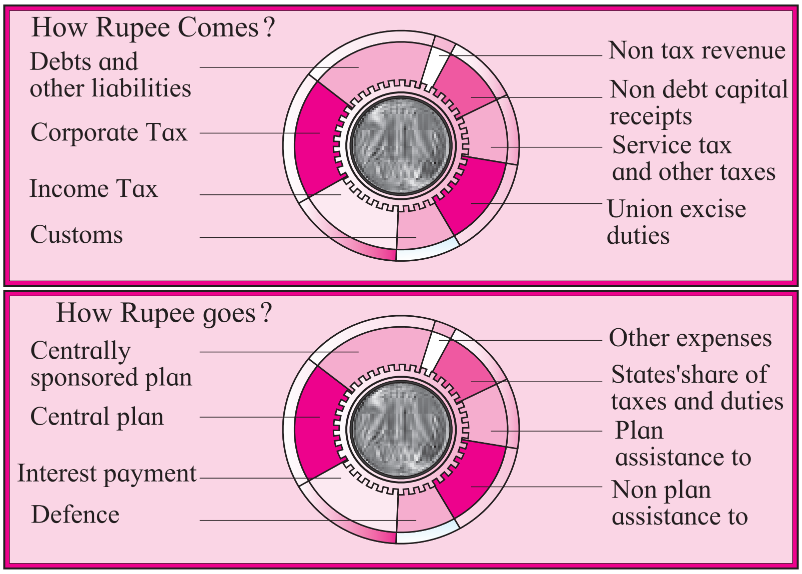
If we have to make two groups of students in a division based on their marks in the examination, we have to find the 'median' of their marks

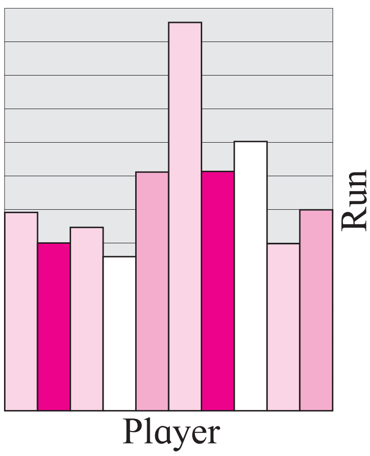
If a 'bachat' group producing chalks wants to know about the colour of chalks having maximum demand, it will have to choose the 'mode'.

Pictorial representation of statistical data

The mean, median or mode of a numerical data or analysis of the data is useful to draw some useful inferences.

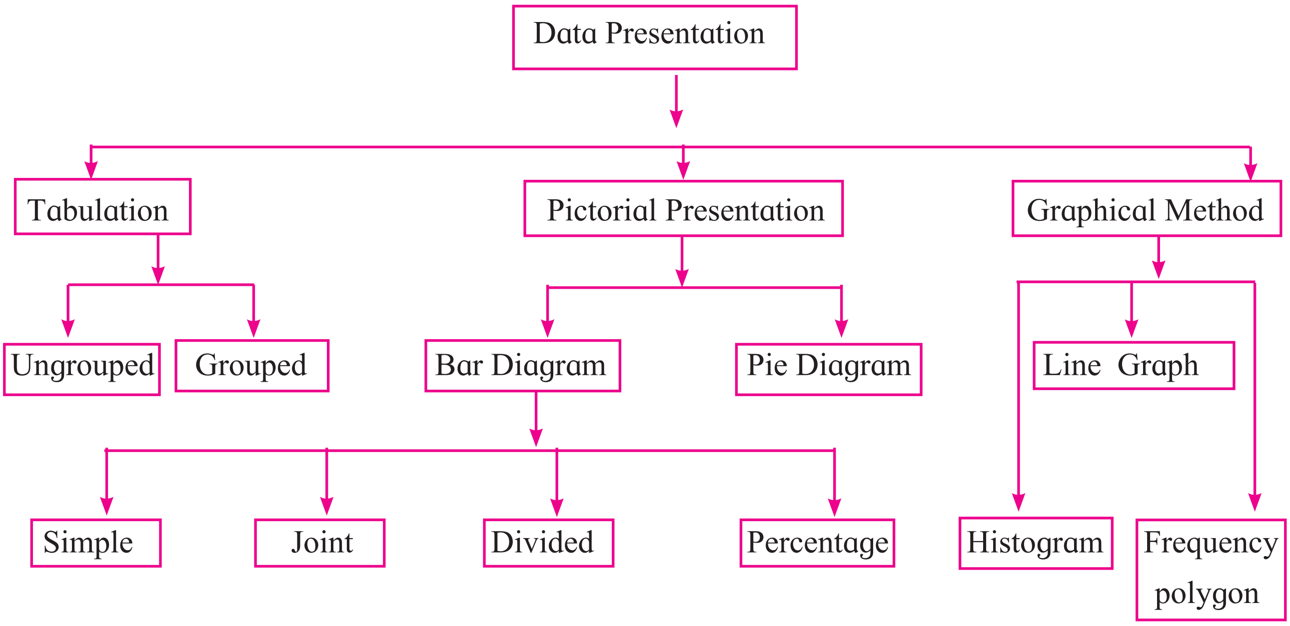
We know that tabulation is one of the methods of representing numerical data in brief. But a table does not quickly reveal some aspects of the data. A common man is interested in the important aspects of a data. For example, annual budget, information about a game, etc. Let us think of another way of data representation for the purpose.





Presentation of data

Pictorial and graphical presentation are attractive methods of data interpretation. The tree chart below shows different methods of data interpretation.



We have studied some of these methods and graphs in previous standards. Now we will learn a histogram, a frequency polygon and a pie diagram.

Florence Nightingale (1820-1910) The lady is considered as an idol in the field of nursing. She was devoted to the work of caring for the wounded and the sick. In the crimean war, she nursed wounded soldiers and saved their lives. She is also known for her fundamental work is the field of statistics. She kept a systematic record of the conditions of wounded soldiers, treatments given to them and the results of the treatments and deduced important conclusions. The cause of the death of soldiers was more often a disease like typhoid or cholera and not the wounds in the war. The causes of the diseases were lack of cleanliness of the surronding, polluted water and crowded dwelling of the patients. Florence exhibited the information in the form of graphs, and pie charts to convince the people. She showed that proper treatments and observing the rules of cleanliness decreases the death rate considerably. The municipalities accepted her observations, that to maintain the hygiene of town, good drainage system and clean drinking water for everyone are necessary. Her work established that systematic records and the statistical methods are useful in drawing reliable inferences.

Let's learn.

Histogram Study the following example to know about a histogram and how to draw it.

Ex : The table below shows the net asset value (NAV) per unit of mutual funds of some companies.

Draw a histogram representing the information.

NAV (₹)	8-9	10-11	12-13	14-15	16-17
No. of mutual funds	20	40	30	25	15

Solution : The given classes are not continuous. Lets make the classes continuous.

Continuous Classes	7.5-9.5	9.5-11.5	11.5-13.5	13.5-15.5	15.5-17.5
Frequency	20	40	30	25	15

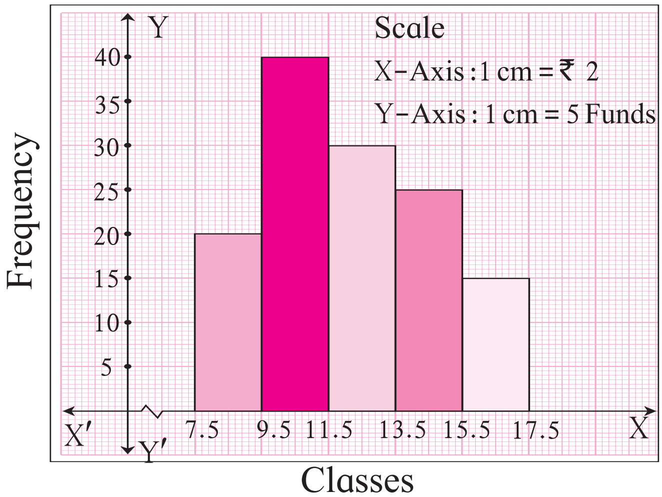


fig 6.1

Method of drawing a histogram :

1. If the given classes are not continuous, make them continuous. Such classes are called extended class intervals.
2. Show the classes on the X- axis with a proper scale.
3. Show the frequencies of the Y- axis with a proper scale.
4. Taking each class as the base, draw rectangles with heights proportional to the frequencies.

Note :

On the X-axis, a mark ‘—⋀⋁—’ is called the krink mark and it is shown between the origin and the first class. It means, there are no observations upto the first class. The mark can be used on the Y- axis also, if needed. This enables us to draw a graph of optimum size.

Practice Set 6.4

1. Draw a histogram of the following data.

Height of student (cm)	135–140	140–145	145–150	150–155
No. of students	4	12	16	8

2. The table below shows the yield of jowar per acre. Show the data by histogram.

Yield per acre (quintal)	2–3	4–5	6–7	8–9	10–11
No. of farmers	30	50	55	40	20

3. In the following table, the investment made by 210 families is shown. Present it in the form of a histogram.

Investment (Thousand Rupees)	10–15	15–20	20–25	25–30	30–35
No. of families	30	50	60	55	15

4. Time alloted for the preparation of an examination by some students is shown in the table. Draw a histogram to show the information.

Time (minutes)	60–80	80–100	100–120	120–140	140–160
No. of students	14	20	24	22	16

Let’s learn.

Frequency polygon

The information in a frequency table can be presented in various ways. We have studied a histogram. A frequency polygon is another way of presentation.

Let us study two methods of drawing a frequency polygon.

(1) With the help of a histogram (2) Without the help of a histogram.

(1) We shall use the histogram in figure 6.1 to learn the method of drawing a frequency polygon.

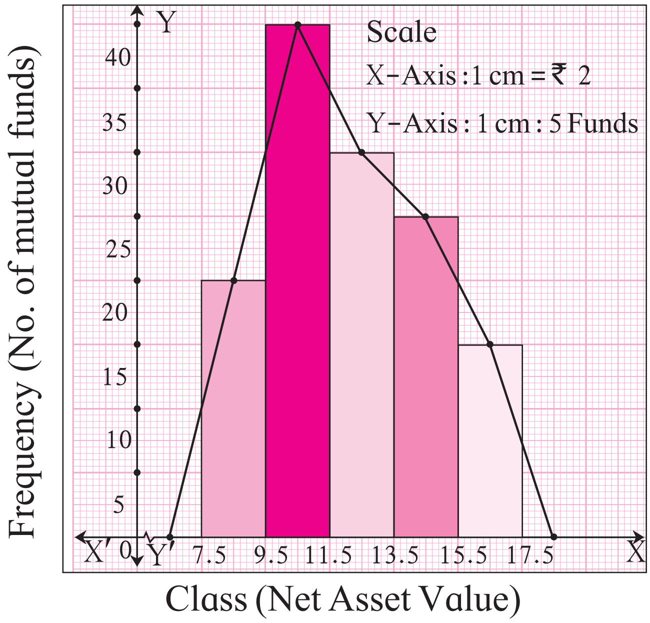


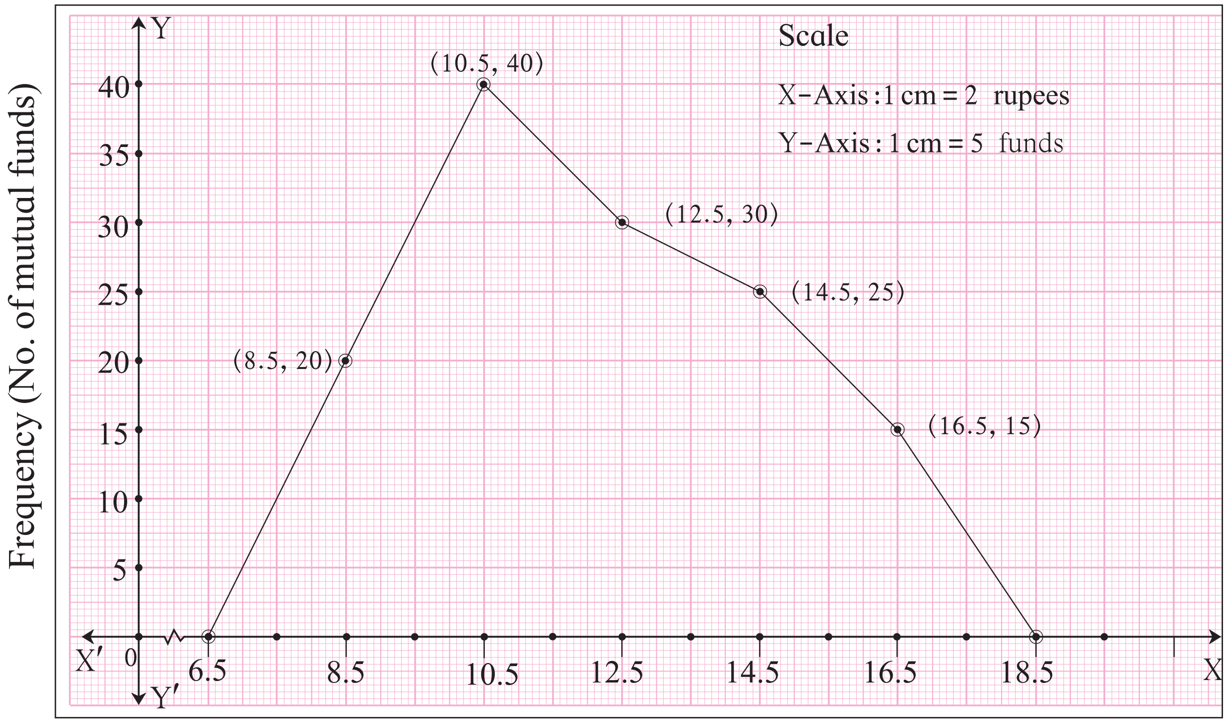
Fig. 6.2

1. Mark the mid-point of upper side of each rectangle in the histogram.
2. Assume that a rectangle of zero height exists preceeding the first rectangle and mark its mid-point. Similarly, assume a rectangle succeeding the last rectangle and mark its mid-point.
3. Join all mid-points in order by line segments.
4. The closed figure so obtained is the frequency polygon.

(2) Observe the following table. It shows how the coordinates of points are decided to draw a frequency polygon, without drawing a histogram.

Class	Continuous class	Class mark	Frequency	Coordinates of points
6 - 7	5.5 - 7.5	6.5	0	(6.5, 0)
8 - 9	7.5 - 9.5	8.5	20	(8.5, 20)
10 - 11	9.5 - 11.5	10.5	40	(10.5, 40)
12 -13	11.5 - 13.5	12.5	30	(12.5, 30)
14 - 15	13.5 - 15.5	14.5	25	(14.5, 25)
16 - 17	15.5 - 17.5	16.5	15	(16.5, 15)
18 - 19	17.5 - 19.5	18.5	0	(18.5, 0)

The points corresponding to the coordinates in the fifth column are plotted. Joining them in order by line segments, we get a frequency polygon. The polygon is shown in figure 6.3. Observe it.



Class (Net Asset Value)

fig 6.3

Solved Examples

Ex. **(1)** Answer the following questions based on the frequency polygon given in the adjacent figure.

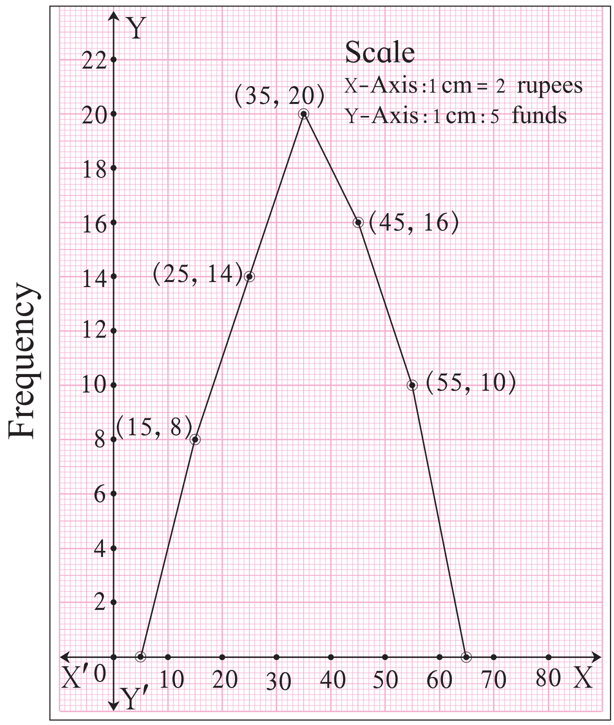
(1) Write frequency of the class 50-60.

(2) State the class whose frequency is 14.

(3) State the class whose class mark is 55.

(4) Write the class in which the frequency is maximum.

(5) Write the classes whose frequencies are zero.



Class

fig 6.4

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Solution :

(1) The class marks are on the X- axis. The point whose x- coordinate is 55 (as the mid-point of the class 50−60 is 55.) y-coordinate is 10. So, the frequency of the class 50−60 is 10.

(2) The frequencies are shown on the Y-axis. The x-coordinate of the point whose y- coordinate is 14, is 25. Note the mark 14 on the Y-axis . The class mark of the class 20−30 is 25. Hence, the frequency of the class 20−30 is 14.

(3) The class mark of the class 50−60 is 55.

(4) The frequency is shown on the Y-axis. On the polygon the maximum value of the y- coordinate is 20. Its corresponding x− coordinate is 35, which is the mark of the class 30−40. Therefore, the maximum frequency is in the class 30−40.

(5) The frequencies of the classes 0−10 and 60−70 are zero.

Ex. **(2)** The following table shows the weights of children and the number of children. Draw a frequency polygon showing the information.

Weight of children (kg)	18−19	19−20	20−21	21−22	22−23	23−24
No. of children	4	13	15	19	17	6

Let us prepare a table showing the co-ordinates necessary to draw a frequency polygon.

Class	18−19	19−20	20−21	21−22	22−23	23−24
Class mark	18.5	19.5	20.5	21.5	22.5	23.5
Frequency	4	13	15	19	17	6
Coordinates of points	(18.5, 4)	(19.5,13)	(20.5,15)	(21.5,19)	(22.5,17)	(23.5,6)

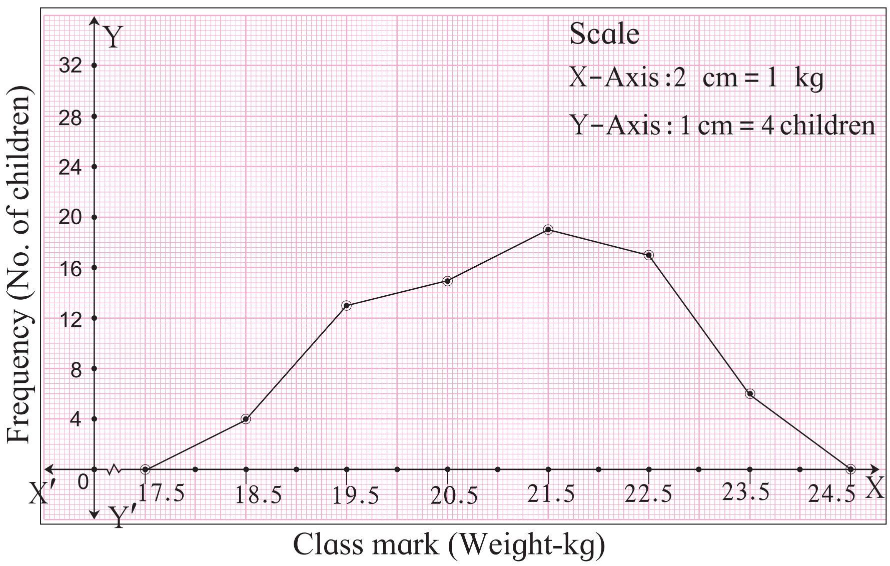


Fig. 6.5

Practice Set 6.5

1. Observe the following frequency polygon and write the answers of the questions below it.

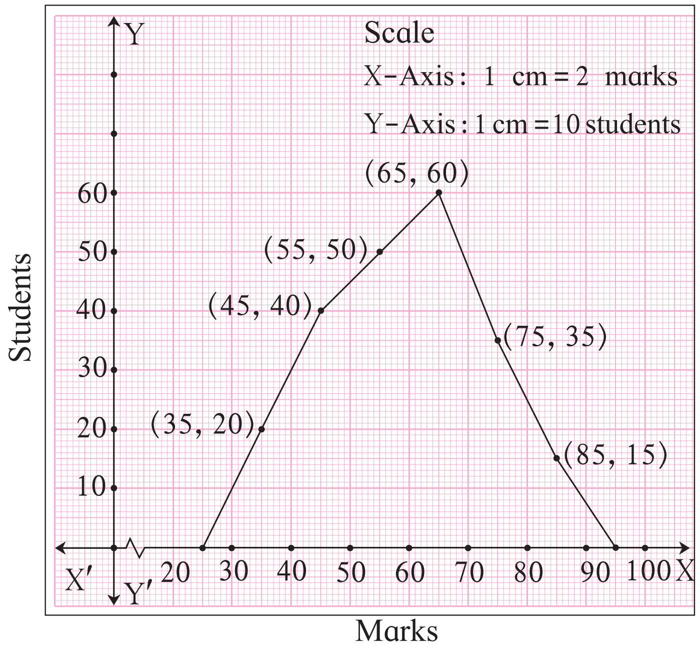


fig 6.6

(1) Which class has the maximum number of students ?

(2) Write the classes having zero frequency.

(3) What is the class-mark of the class, having frequency of 50 students ?

(4) Write the lower and upper class limits of the class whose class mark is 85.

(5) How many students are in the class 80-90 ?

2. Show the following data by a frequency polygon.

Electricity bill (₹)	0-200	200-400	400-600	600-800	800-1000
Families	240	300	450	350	160

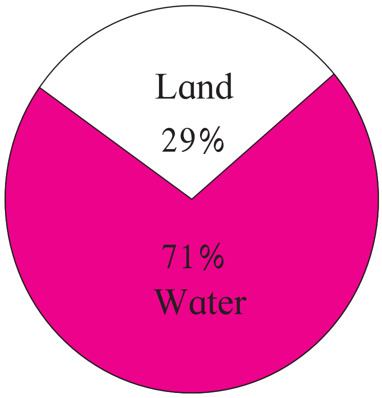
3. The following table shows the classification of percentages of marks of students and the number of students. Draw a frequency polygon from the table.

Result (Percentage)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	7	33	45	65	47	18	5

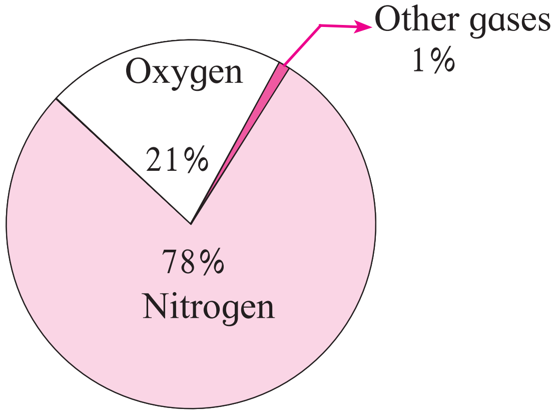
Let's recall.

Pie diagram

In the previous standards, we have seen the following figures in Geography and Science. Such graphs are called pie diagrams.



Proportion of land and water on the earth



Proportion of constituents of air

fig 6.7

In a pie diagram, the numerical data is shown in a circle. Different components of a data are shown by proportional sectors of the circle.

In figure 6.8, seg OA and seg OB are radii of a circle with centre O.

$\angle$ AOB is the central angle.

The shaded region O – AXB is a sector of the circle.

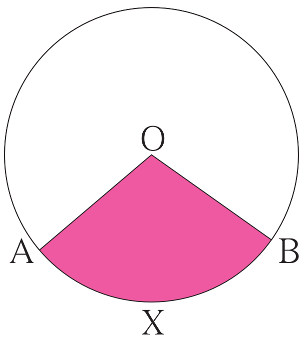


fig 6.8

Let's learn.

Reading of Pie diagram

The following example illustrates how a pie chart gives information at a glance. 120 students of standard 10 were asked which game they like. The information obtained is shown in the adjacent pie diagram. Answers to the question as-

'Which game is liked the most'

'What percentage of students like kho-kho?'

'What percentage of students like kabaddi?'

can be obtained from the pie diagram at a glance.

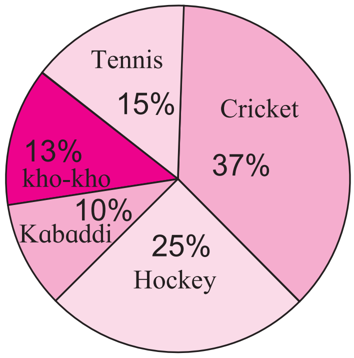


fig 6.9

Observe one more pie diagram.

Figure 6.10 shows the annual financial planning of a school. From the pie diagram we see that

- 45% of the amount is reserved for educational equipment.
- 35% of the amount is shown for games.
- 10% of the amount is kept for sanitation.
- 10% of the amount is reserved for environment.

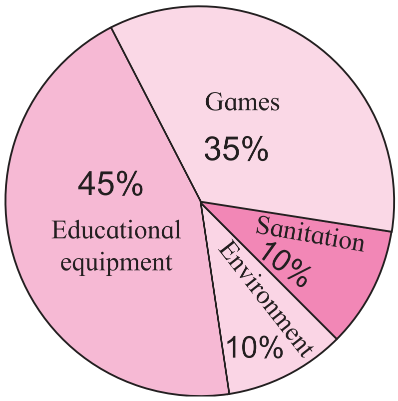


fig 6.10

In this way, we get information at a glance from a pie diagram.

Let us have more information about a pie diagram.

Many times we find information of different types in newspapers given in the form of pie diagrams. For example, the annual budget, performance of different nations in olympic games, etc.

Now we shall see, by examples, how to interpret the information from a pie diagram.

Example :

As deduced from a survey, the classification of skilled workers is shown in the pie diagram (fig 6.11). If the number of workers in the production sector is 4500, answer the following questions.

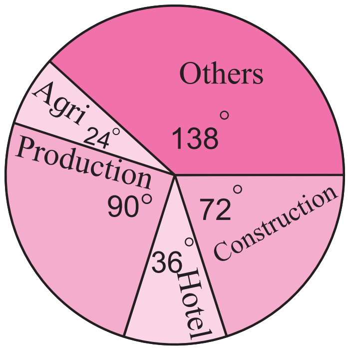


fig 6.11

(i) What is the total number of skilled workers in all fields?

(ii) What is the number of skilled workers in the field of constructions?

(iii) How many skilled workers are in agriculture?

(iv) Find the difference between the numbers of workers in the field of production and construction.

Solution : (i) Suppose, the total number of skilled workers in all fields is x.

$\therefore$ the central angle for x persons is = 360°

Central angle for number of persons in production field

$$= \frac{\text{Number of persons in production field}}{x} \times 360$$

$$\therefore 90 = \frac{4500}{x} \times 360$$

$$\therefore x = 18000$$

$\therefore$ total number of skilled workers in all the fields together = 18000.

(ii) The angle shown for construction sector = 72°.

$$\therefore 72 = \frac{\text{Number of persons in construction}}{18000} \times 360$$

$$\therefore \text{number of persons in construction field} = \frac{72 \times 18000}{360}$$

$$= 3600$$

(iii) The central angle for agriculture field is 24°.

$$\therefore 24 = \frac{\text{Number of workers in agriculture}}{\text{total skilled workers}} \times 360$$

$$24 = \frac{\text{Number of workers in agriculculture}}{18000} \times 360$$

$$\therefore \text{number of workers in agriculture} = \frac{24 \times 18000}{360}$$

$$= 1200$$

(iv) The difference between angles relating fields of production and construction

$= 90° - 72° = 18°.$

$\therefore$ The difference between the central angles =

$$\frac{\text{Difference between numbres of workers in the fields}}{\text{Total number of skilled workers}} \times 360$$

$$18 = \frac{\text{Difference between the numbers of workers in the fields}}{18000} \times 360$$

Difference between the numbers of workers in the two fields $= \frac{18 \times 18000}{360}$

$= 900$

Let's remember!

- Every component of a data is shown by a sector associated with it.
- The measure of the central angle of the sector is in proportion with the number of scores in that component.
- The measure of central angle $(\theta) = \frac{\text{Number of scores in component}}{\text{Total number of scores}} \times 360^0$
- A circle of suitable radius should be drawn. Divide the circle in sectors such that the measure of central angle of each sector is proportional to the number of scores in its corresponding component in the data.

Let's learn.

To draw a Pie diagram

We have seen how to read a pie diagram. Now let us learn to draw it.

1. To draw a pie diagram, the whole circle is divided into sectors proportional to the components of the data
2. The measure of central angle of each sector is found by the following formula.

The measure of central angle of sector θ

$$= \frac{\text{Number of scores in the components}}{\text{Total number of scores}} \times 360$$

A circle of a suitable radius is drawn. Then it is divided into sectors such that, the number of sectors is equal to the number of components in the data.

Let us understand the method through examples.

Solved Examples

Ex. (1) In a bicycle shop, number of bicycles purchased and choice of their colours was as follows. Find the measures of sectors of a circle to show the information by a pie diagram.

Solution : In all 36 bicycles were purchased. Out of them 10 bicycles were white coloured.

$\therefore$ the measure of sector showing white coloured bicycles

$$= \frac{\text{Number of white bicycles}}{\text{Total number of bicycles}} \times 360$$

$$= \frac{10}{36} \times 360 = 100$$

The measures of angles of sector relating to bicycles of other colours can be calculated similarly which are shown in the adjacent table.

Colour	Number of bicycles	Central angle of the sector
White	10	$\frac{10}{36} \times 360^\circ = 100^\circ$
Black	9	$\frac{9}{36} \times 360^\circ = 90^\circ$
Blue	6	60°
Grey	7	70°
Red	4	40°
Total	36	360°

Ex. (2) The following table shows the daily supply of electricity to different places in a town. Show the information by a pie diagram.

Places	Factories	Houses	Roads	Shops	Offices	Others
Supply of electricity (Thousand units)	24	14	7	5	6	4

Solution : The total supply of electricity is 60,000 units. Let us find the measures of central angles and show in the table.

Supply of electricity	Unit	Measure of central angle
Factories	24	$\frac{24}{60} \times 360 = 144^\circ$
Houses	14	$\frac{14}{60} \times 360 = 84^\circ$
Roads	7	$\frac{7}{60} \times 360 = 42^\circ$
Shops	5	$\frac{5}{60} \times 360 = 30^\circ$
Offices	6	$\frac{6}{60} \times 360 = 36^\circ$
Others	4	$\frac{4}{60} \times 360 = 24^\circ$
Total	60	360°

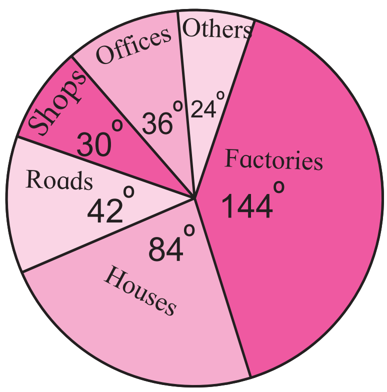


fig 6.12

Steps of drawing pie chart :

(1) As shown in the figure, a circle and a radius is drawn. Then the sectors having measures of angles in the table, (144°,84°, 42°, 30°, 36°, and 24°) were drawn one by one, in the clockwise direction. (While drawing the sectors one by one, we can change their order.)

(2) The components of the data were recorded in the sectors.

Activity :

The monthly expenditure of a family on different items is shown in the following table. Calculate the related central angles and draw a pie chart.

Different items	Percentage of expenditure	Measure of central angle
Food	40	$\frac{40}{100} \times 360 = \square$
Cloting	20	$\square \times \square = \square$
House rent	15	$\square \times \square = \square$
Education	20	$\square \times \square = \square$
Expenditure	05	$\square \times \square = \square$
Total	100	360°

Practice Set 6.6

1. The age group and number of persons, who donated blood in a blood donation camp is given below. Draw a pie diagram from it.

Age group (Yrs)	20-25	25-30	30-35	35-40
No. of persons	80	60	35	25

2. The marks obtained by a student in different subjects are shown. Draw a pie diagram showing the information.

Subject	English	Marathi	Science	Mathematics	Social science	Hindi
Marks	50	70	80	90	60	50

3. In a tree plantation programme, the number of trees planted by students of different classes is given in the following table. Draw a pie diagram showing the information.

Standard	5 th	6 th	7 th	8 th	9 th	10 th
No. of trees	40	50	75	50	70	75

4. The following table shows the percentages of demands for different fruits registered with a fruit vendor. Show the information by a pie diagram.

Fruits	Mango	Sweet lime	Apples	Cheeku	Oranges
Percentages of demand	30	15	25	20	10

5. The pie diagram in figure 6.13 shows the proportions of different workers in a town. Answer the following questions with its help.

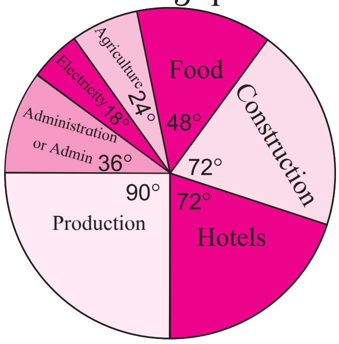


fig 6.13

(1) If the total workers is 10,000; how many of them are in the field of construction?

(2) How many workers are working in the administration?

(3) What is the percentage of workers in production?

6. The annual investments of a family are shown in the adjacent pie diagram. Answer the following questions based on it.

(1) If the investment in shares is ₹ 2000/, find the total investment.

(2) How much amount is deposited in bank?

(3) How much more money is invested in immovable property than in mutual fund?

(4) How much amount is invested in post?

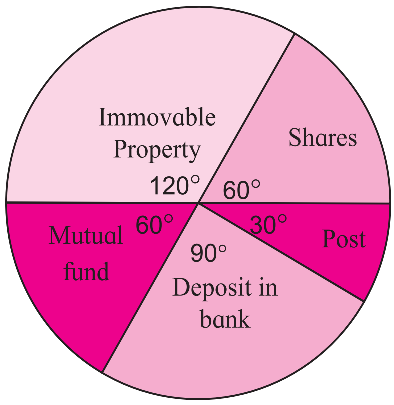


fig 6.14

Miscellaneous Problems – 6

1. Find the correct answer from the alternatives given.

(1) The persons of O– blood group are 40%. The classification of persons based on blood groups is to be shown by a pie diagram. What should be the measures of angle for the persons of O– blood group?

(A) 114° (B) 140° (C) 104° (D) 144°

(2) Different expenditures incurred on the construction of a building were shown by a pie diagram. The expenditure ₹ 45,000 on cement was shown by a sector of central angle of 75°. What was the total expenditure of the construction ?

(A) 2,16,000 (B) 3,60,000 (C) 4,50,000 (D) 7,50,000

(3) Cumulative frequencies in a grouped frequency table are useful to find . . .

(A) Mean (B) Median (C) Mode (D) All of these

(4) The formula to find mean from a grouped frequency table is $\overline{X} = A + \frac{\sum f_i u_i}{\sum f_i} \times h\ g$

In the formula u_i = . . .

(A) $\frac{x_i + A}{g}$ (B) $(x_i - A)$ (C) $\frac{x_i - A}{g}$ (D) $\frac{A - x_i}{g}$

(5)

Distance Covered per litre (km)	12-14	14-16	16-18	18-20
No. of cars	11	12	20	7

The median of the distances covered per litre shown in the above data is in the group . . . . . .

(A) 12-14 (B) 14-16 (C) 16-18 (D) 18-20

(6)

No. of trees planted by each student	1-3	4-6	7-9	10-12
No. of students	7	8	6	4

The above data is to be shown by a frequency polygon. The coordinates of the points to show number of students in the class 4-6 are . . . .

(A) (4, 8) (B) (3, 5) (C) (5, 8) (D) (8, 4)

2. The following table shows the income of farmers in a grape season. Find the mean of their income.

Income (Thousand Rupees)	20-30	30-40	40-50	50-60	60-70	70-80
Farmers	10	11	15	16	18	14

3. The loans sanctioned by a bank for construction of farm ponds are shown in the following table. Find the mean of the loans.

Loan (Thousand rupees)	40-50	50-60	60-70	70-80	80-90
No. of farm ponds	13	20	24	36	7

4. The weekly wages of 120 workers in a factory are shown in the following frequency distribution table. Find the mean of the weekly wages.

Weekly wages (Rupees)	0-2000	2000-4000	4000-6000	6000-8000
No. of workers	15	35	50	20

5. The following frequency distribution table shows the amount of aid given to 50 flood affected families. Find the mean of the amount of aid.

Amount of aid (Thosand rupees)	50-60	60-70	70-80	80-90	90-100
No. of families	7	13	20	6	4

6. The distances covered by 250 public transport buses in a day is shown in the following frequency distribution table. Find the median of the distances.

Distance (km)	200-210	210-220	220-230	230-240	240-250
No. of buses	40	60	80	50	20

7. The prices of different articles and demand for them is shown in the following frequency distribution table. Find the median of the prices.

Price (Rupees)	20 less than	20-40	40-60	60-80	80-100
No. of articles	140	100	80	60	20

8. The following frequency table shows the demand for a sweet and the number of customers. Find the mode of demand of sweet.

Weight of sweet (gram)	0-250	250-500	500-750	750-1000	1000-1250
No. of customers	10	60	25	20	15

9. Draw a histogram for the following frequency distribution.

Use of electricity (Unit)	50-70	70-90	90-110	110-130	130-150	150-170
No. of families	150	400	460	540	600	350

10. In a handloom factory different workers take different periods of time to weave a saree. The number of workers and their required periods are given below. Present the information by a frequency polygon.

No. of days	8-10	10-12	12-14	14-16	16-18	18-20
No. of workers	5	16	30	40	35	14

11. The time required for students to do a science experiment and the number of students is shown in the following grouped frequency distribution table. Show the information by a histogram and also by a frequency polygon.

Time required for experiment (minutes)	20-22	22-24	24-26	26-28	28-30	30-32
No. of students	8	16	22	18	14	12

12. Draw a frequency polygon for the following grouped frequency distribution table.

Age of the donor (Yrs.)	20-24	25-29	30-34	35-39	40-44	45-49
No. of blood doners	38	46	35	24	15	12

13. The following table shows the average rainfall in 150 towns. Show the information by a frequency polygon.

Average rainfall (cm)	0-20	20-40	40-60	60-80	80-100
No. of towns	14	12	36	48	40

14. Observe the adjacent pie diagram. It shows the percentages of number of vehicles passing a signal in a town between 8 am and 10 am

(1) Find the central angle for each type of vehicle.

(2) If the number of two-wheelers is 1200, find the number of all vehicles.

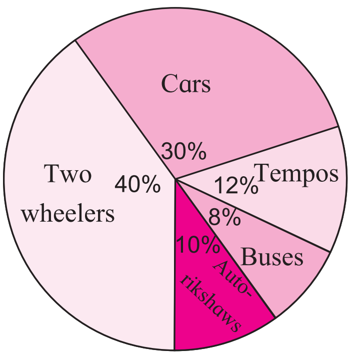


fig 6.15

15. The following table shows causes of noise pollution. Show it by a pie diagram.

Construction	Traffic	Aircraft take offs	Industry	Trains
10%	50%	9%	20%	11%

16. A survey of students was made to know which game they like. The data obtained in the survey is presented in the adjacent pie diagram. If the total number of students are 1000,

(1) How many students like cricket ?

(2) How many students like football ?

(3) How many students prefer other games ?

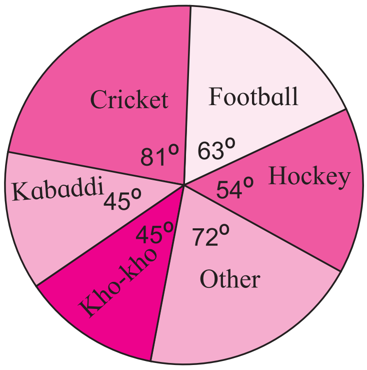


fig 6.16

17. Medical check up of 180 women was conducted in a health centre in a village. 50 of them were short of haemoglobin, 10 suffered from cataract and 25 had respiratory disorders. The remaining women were healthy. Show the information by a pie diagram.

18. On an environment day, students in a school planted 120 trees under plantation project. The information regarding the project is shown in the following table. Show it by a pie diagram.

Tree name	Karanj	Behada	Arjun	Bakul	Kadunimb
No. of trees	20	28	24	22	26

Answers

1. Linear Equations In Two Variables

Practice Set 1.1

2. (1) (2, 4) (2) (3, 1) (3) (6,1) (4) (5, 2)
(5) (-1, 1) (6) (1, 3) (7) (3, 2) (8) (7, 3)

Practice Set 1.2

1. (1)

x	3	-2	0
y	**0**	5	3
(x, y)	(3, 0)	**(−2, 5)**	(0, 3)

(2)

x	**4**	-1	0
y	0	**−5**	-4
(x, y)	**(4, 0)**	**(−1,−5)**	(0,−4)

2. (1) (5, 1) (2) (4, 1) (3) (3, -3) (4) (-1, -5) (5) (1, 2.5) (6) (8, 4)

Practice Set 1.3

1. $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{5} - \boxed{2} \times 4 = \boxed{15} - 8 = \boxed{7}$

2. (1) -18 (2) 21 (3) $-\frac{4}{3}$

3. (1) (2, −1) (2) (−2, 4) (3) (3, −2) (4) (2, 6) (5) (6, 5) (6) $(\frac{5}{8}, \frac{1}{4})$

Practice Set 1.4

1. (1) $(\frac{1}{9}, 1)$ (2) (3, 2) (3) $(\frac{5}{2}, -2)$ (4) (1, 1)

Practice Set 1.5

1. The numbers are 5 and 2

2. $x = 12$, $y = 8$, Area = 640 sq. unit, Perimeter = 112 unit

3. Son's age is 15 years, father's age is 40 years

4. $\frac{7}{18}$

5. A – 30 kg, B – 55 kg

6. 150 km.

Problem Set 1

1. (1) B (2) A (3) D (4) C (5) A

2.

x	-5	$\frac{3}{2}$
y	$-\frac{13}{6}$	0
(x, y)	$(-5, -\frac{13}{6})$	$(\frac{3}{2}, 0)$

3. (1) (3, 2) (2) (-2, -1) (3) (0, 5) (4) (2, 4) (5) (3, 1)

4. (1) 22 (2) −1 (3) 13

5. (1) $(-\frac{2}{3}, 2)$ (2) (1, 4) (3) $(\frac{1}{2}, -\frac{1}{2})$ (4) $(\frac{7}{11}, \frac{116}{33})$ (5) (2, 6)

6. (1) (6, −4) (2) $(-\frac{1}{4}, -1)$ (3) (1, 2) (4) (1, 1) (5) (2, 1)

7. (2) Tea; ₹300 per kg.
   sugar; ₹ 40 per kg.
   (3) ₹100 notes 20
   ₹50 notes 10
   (4) Manisha's age 23 years
   Savita's age 8 years.
   (5) Skilled worker's wages ₹ 450.
   unskilled worker's wages ₹ 270.
   (6) Hamid's speed 50 km/hr.
   Joseph's speed 40 km/hr.

2. Quadratic Equations

Practice Set 2.1

1. Any equations of the type $m^2 + 5m + 3 = 0$, $y^2 - 3 = 0$

2. (1), (2), (4), (5) are quadratic equations.

3. (1) $y^2 + 2y - 10 = 0$, $a = 1, b = 2, c = -10$
   (2) $x^2 - 4x - 2 = 0$, $a = 1, b = -4, c = -2$
   (3) $x^2 + 4x + 3 = 0$, $a = 1, b = 4, c = 3$
   (4) $m^2 + 0m + 9 = 0$, $a = 1, b = 0, c = 9$
   (5) $6p^2 + 3p + 5 = 0$, $a = 6, b = 3, c = 5$
   (6) $x^2 + 0x - 22 = 0$, $a = 1, b = 0, c = -22$

4. (1) 1 is a root, −1 is not. (2) $\frac{5}{2}$ is a root, 2 is not.

5. k = 3 6. k = −7

Practice Set 2.2

1. (1) 9, 6 (2) −5, 4 (3) $-13, -\frac{1}{2}$ (4) $5, -\frac{3}{5}$
   (5) $\frac{1}{2}, \frac{1}{2}$ (6) $\frac{2}{3}, -\frac{1}{2}$ (7) $-\frac{5}{\sqrt{2}}, -\sqrt{2}$ (8) $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$
   (9) 25, −1 (10) $-\frac{3}{5}, \frac{3}{5}$ (11) 0, 3 (12) $-\sqrt{11}, \sqrt{11}$

Practice Set 2.3

1. (1) 4, −5 (2) $(\sqrt{6}-1), (-\sqrt{6}-1)$ (3) $\frac{\sqrt{13}+5}{2}, \frac{-\sqrt{13}+5}{2}$

(4) $\frac{\sqrt{2}+2}{3}, \frac{-\sqrt{2}+2}{3}$ (5) $-\frac{5}{4}, -\frac{5}{2}$ (6) $\frac{2+\sqrt{39}}{5}, \frac{2-\sqrt{39}}{5}$

Practice Set 2.4

1. (1) 1, −7, 5 (2) 2, −5, 5 (3) 1, −7, 0

2. (1) −1, −5 (2) $\frac{3+\sqrt{17}}{2}, \frac{3-\sqrt{17}}{2}$ (3) $\frac{-1+\sqrt{22}}{3}, \frac{-1-\sqrt{22}}{3}$

(4) $\frac{2+\sqrt{14}}{5}, \frac{2-\sqrt{14}}{5}$ (5) $\frac{-1+\sqrt{73}}{6}, \frac{-1-\sqrt{73}}{6}$ (6) $-1, -\frac{8}{5}$

3. $-\sqrt{3}, -\sqrt{3}$

Practice Set 2.5

1. (1) Roots are distinct and real when $b^2 - 4ac = 5$, not real when $b^2 - 4ac = -5$.

(2) $x^2 + 7x + 5 = 0$ (3) $\alpha + \beta = 2$, $\alpha \times \beta = -\frac{3}{2}$

2. (1) 53 (2) −55 (3) 0

3. (1) Real and equal. (2) Real and unequal. (3) Not real.

4. (1) $x^2 - 4x = 0$ (2) $x^2 + 7x - 30 = 0$

(3) $x^2 - \frac{1}{4} = 0$ (4) $x^2 - 4x - 1 = 0$

5. k = 3 **6.** (1) 18 (2) 50

7. (1) k = 12 or k = −12 (2) k = 6

Practice Set 2.6

1. 9 years **2.** 10 and 12 **3.** In vertical row 10, in horizontal row 15.

4. Kishor's present age is 10 years and Vivek's present age is 15 years

5. 10 marks **6.** No. of pots 6, production cost of each is ₹ 100.

7. 6 km/hr **8.** For Nishu 6 days, for Pintu 12 days.

9. Divisor = 9, quotient = 51 **10.** AB = 7 cm, CD = 15 cm, AD = BC = 5 cm.

Problem Set 2

1. (1) B (2) A (3) C (4) B (5) B (6) D (7) C (8) C

2. (1) and (3) are quadratic equations.

3. (1) −15 (2) 1 (3) 21

4. k = 3 5. (1) $x^2 - 100 = 0$ (2) $x^2 - 2x - 44 = 0$ (3) $x^2 - 7x = 0$

6. (1) Not real. (2) Real and unequal (3) Real and equal

7. (1) $\frac{1+\sqrt{21}}{2}$, $\frac{1-\sqrt{21}}{2}$ (2) $\frac{1}{2}, -\frac{1}{5}$ (3) 1, −4
(4) $\frac{-5+\sqrt{5}}{2}$, $\frac{-5-\sqrt{5}}{2}$ (5) Roots are not real. (6) $(2+\sqrt{7}), (2-\sqrt{7})$

8. $m = 14$ 9. $x^2 - 5x + 6 = 0$ 10. $x^2 - 4pqx - (p^2 - q^2)^2 = 0$

11. ₹ 100 with Sagar, ₹ 150 with Mukund.

12. 12 and $\sqrt{24}$ or 12 and $-\sqrt{24}$ 13. No. of students 60

14. Breadth 45 m. length 100 m, side of the pond 15 m.

15. For larger tap 3 hours and for smaller tap 6 hours.

3. Arithmetic Progression

Practice Set 3.1

1. (1) Yes, $d = 2$ (2) Yes, $d = \frac{1}{2}$ (3) Yes, $d = 4$ (4) No
(5) Yes, $d = -4$ (6) Yes, $d = 0$ (7) Yes, $d = \sqrt{2}$ (8) Yes, $d = 5$

2. (1) 10, 15, 20, 25, . . . (2) −3, −3, −3, −3, . . . (3) −7, −6.5, −6, −5.5, . .
(4) −1.25, 1.75, 4.75, 7.75, . . .(5) 6, 3, 0, −3 . . . (6) −19, −23, −27, −31

3. (1) $a = 5, d = -4$ (2) $a = 0.6, d = 0.3$ (3) $a = 127, d = 8$ (4) $a = \frac{1}{4}, d = \frac{1}{2}$

Practice Set 3.2

1. (1) $d = 7$ (2) $d = 3$ (3) $a = -3, d = -5$ (4) $a = 70, d = -10$

2. Yes. 121 3. 104 4. 1115 5. -121 6. 180

7. 55 8. 55th 9. 60 10. 1

Practice Set 3.3

1. 1215 2. 15252 3. 30450 5. 5040

5. 2380 6. 60 7. 4, 9, 14 or 14, 9, 4 8. −3, 1, 5, 9

Practice Set 3.4

1. ₹ 70455 2. First instalment ₹ 1000, last instalment ₹ 560. 3. ₹ 1,92,000

4. 48, 1242 5. −20°, −25°, −30°, −35°, −40°, −45° 6. 325

Problem Set 3

1. (1) B (2) C (3) B (4) D (5) B (6) C (7) C (8) A (9) A (10) B

2. 40 3. 1, 6, 11, . . . 4. −195 5. 16, -21 6. -1 7. 6, 10

8. 8 9. 67, 69, 71 10. 3, 7, 11, ... , 147 14. ₹ 2000.

4. Financial Planning

Practice Set 4.1

1. CGST 6%, SGST 6% 2. SGST 9%, GST 18%
3. CGST ₹ 784 and SGST ₹ 784
4. The customer gets the belt for ₹ 691.48.
5. Taxable value of toy car is ₹ 1500, CGST ₹ 135, SGST ₹ 135
6. (1) Rate of SGST 14% (2) Rate of GST on AC 28%
   (3) Taxable value of AC ₹ 40,000. (4) Total GST ₹ 11,200.
   (5) CGST ₹ 5600. (6) SGST ₹ 5600.
7. Prasad gets the washing machine for ₹ 48,640 and CGST ₹ 5320, SGST ₹ 5320.

Practice Set 4.2

1. Payable GST ₹ 22,000.
2. Input Tax Credit for Nazama is ₹ 12,500 and her payable GST is ₹ 2250.
3. Ameer Enterprises : Payable GST ₹ 300, payable CGST ₹ 150, payable SGST ₹ 150.
   Akabari Brothers : payable GST ₹ 400, payable CGST ₹ 200, payable SGST ₹ 200.
4. Payable GST ₹ 100 so CGST ₹ 50 and UTGST ₹ 50. 5. CGST = SGST = ₹ 900

Practice Set 4.3

1. (1) MV ₹ 100 (2) FV ₹ 75 (3) At discount of ₹ 5.
2. 25% 3. ₹ 37,040 4. 800 shares
5. Rate of return 5.83% 6. Company A– more profitable.

Practice Set 4.4

1. ₹ 200.60 2. ₹ 999
3.

No. of shares	MV of shares	Total value	Brokerage 0.2%	9% CGST on brokerage	9% SGST on brokerage	Total value of shares
100 B	₹ 45	₹ 4500	₹ 9	₹ 0.81	₹ 0.81	₹ 4510.62
75 S	₹ 200	₹15000	₹ 30	₹ 2.70	₹ 2.70	₹ 14964.60

4. No. of shares sold = 100. 5. Loss of ₹ 8560.

Problem Set 4A

1. (1) C (2) B (3) D (4) B (5) A (6) B
2. Total bill ₹ 28,800 , CGST ₹ 3150, SGST ₹ 3150.

3. ₹ 997.50 4. ₹ 12,500 5. ITC ₹ 4250, payable tax ₹ 250
6. ITC ₹ 1550, payable CGST ₹ 5030, payable SGST ₹ 5030.
7. Taxable value ₹ 75,000, CGST ₹ 4500, SGST ₹ 4500
8. (1) Wholesaler's tax invoice : CGST ₹ 16200; SGST ₹ 16200.
Retailer's tax invoice: CGST ₹ 19,800; SGST ₹ 19,800.
(2) Wholesaler : payable CGST ₹ 2700 and payable SGST ₹ 2700,
Retailer : payable CGST ₹ 3600 and payable SGST ₹ 3600
9. (1) Anna Patil's invoice : CGST ₹ 1960, SGST ₹ 1960
(2) Trader in Vasai : CGST ₹ 2352 and SGST ₹ 2352
(3) Trader in Vasai : payable CGST ₹ 392 and payable SGST ₹ 392
10.
(1)

Person	Payable CGST (₹)	Payable SGST (₹)	Payable GST (₹)
Manufacturer	300	300	600
Distributor	360−300 = 60	60	120
Retailer	390−360 = 30	30	60
Total Tax	390	390	780

(2) Finally, the customer will get the article for ₹ 7280.
(3) Manufacturer to distributor B2B, distributor to retailer B2B, retailer to customer B2C

Problem Set 4B

1. (1) B (2) B (3) A (4) C (5) A
2. ₹ 130.39 3. 22.2% 4. will get ₹ 21,000 .
5. Will get 500 shares. 6. Profit ₹ 1058.52 7. Company B, as returns are more
8. Will get 1000 shares. 9. ₹ 118.
10. (1) ₹ 1,20,000 (2) ₹ 360 (3) ₹ 64.80 (4) ₹ 120424.80.
11. 1% profit

5. Probability

Practice Set 5.1

1. (1) 8 (2) 7 (3) 52 (4) 11

Practice Set 5.2

1. (1) S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T} n(S) = 12

(2) S = {23, 25, 32, 35, 52, 53}   n(S) = 6

2. S = {Red, Purple, Orange, Yellow, Blue, Green}   n(S) = 6

3. S = {Tuesday, Sunday, Friday, Wednesday, Monday, Saturday}   n(S) = 6

4. (1) B_1B_2 (2) G_1G_2 (3) B_1G_1 B_2G_1 B_1G_2 B_2G_2

(4) S = {B_1B_2, B_1G_1, B_1G_2, B_2G_1, B_2G_2, G_1G_2}

Practice Set 5.3

1. (1) S = {1, 2, 3, 4, 5, 6}   n(S) = 6

A = {2, 4, 6} n(A) = 3, B = {1, 3, 5} n(B) = 3,   C = {2, 3, 5}   n(C) = 3

(2) S = {(1,1), . . ., (1, 6), (2,1), . . ., (2, 6), (3, 1), . . .,(3, 6),
(4, 1), . . .,(4,6),(5, 1), . . .,(5, 6),(6, 1), . . .,(6, 6)}   n(S) = 36

A = {(1, 5) (2, 4) (3, 3) (4, 2) (5, 1) (6, 6)}   n(A) = 6

B = {(4, 6) (5, 5) (5, 6) (6, 4) (6, 5) (6, 6)}   n(B) = 6

C = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}   n(C) = 6

(3) S = {HHH, HHT, HTT, HTH, THT, TTH, THH, TTT} n(S) = 8

A = {HHH, HHT, HTH, THH}   n(A) = 4

B = {TTT}   n(B) = 1

C = {HHH, HHT, THH}   n(C) = 3

(4) S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43,
45, 50, 51, 52, 53, 54}   n(S) = 25

A = {10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52, 54}   n(A) = 13

B = {12, 15, 21, 24, 30, 42, 45, 51, 54}   n(B) = 9

C = {51, 52, 53, 54}   n(C) = 4

(5) S = {M_1M_2, M_1M_3, M_1F_1, M_1F_2, M_2M_3, M_2F_1, M_2F_2, M_3F_1, M_3F_2, F_1F_2}
n(S) = 10

A = {M_1F_1, M_1F_2, M_2F_1, M_2F_2, M_3F_1, M_3F_2, F_1F_2}   n(A) = 7

B = {M_1F_1, M_1F_2, M_2F_1, M_2F_2, M_3F_1, M_3F_2}   n(B) = 6

C = {M_1M_2, M_1M_3, M_2M_3}   n(C) = 3

(6) S = {H1, H2, H3, H4, H5, H6 T1, T2, T3, T4, T5, T6}   n(S) = 12

A = {H1, H3, H5}   n(A) = 3

B = {H2, H4, H6, T2, T4, T6}   n(B) = 6

C = { }   n(C) = 0

Practice Set 5.4

1. (1) $\frac{3}{4}$, (2) $\frac{1}{4}$ 2. (1) $\frac{1}{6}$ (2) 0   (3) $\frac{5}{12}$

3. (1) $\frac{7}{15}$ (2) $\frac{1}{5}$ 4. (1) $\frac{4}{5}$ (2) $\frac{1}{5}$ 5. (1) $\frac{1}{13}$ (2) $\frac{1}{4}$

Problem Set – 5

1. (1) B (2) B (3) C (4) A (5) A 2. Vasim's 3. (1) $\frac{1}{11}$ (2) $\frac{6}{11}$

4. $\frac{5}{26}$ 5. (1) $\frac{4}{9}$ (2) $\frac{1}{3}$ (3) $\frac{4}{9}$ 6. $\frac{1}{2}$ 7. (1) $\frac{1}{3}$ (2) $\frac{1}{6}$

8. (1) $\frac{1}{2}$ (2) $\frac{1}{6}$ 9. $\frac{1}{25}$ 10. (1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) 1

11. (1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) 1 (4) 0 12. (1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{2}{3}$ 13. $\frac{2}{11}$

14. $\frac{13}{40}$ 15. (1) $\frac{3}{10}$ (2) $\frac{3}{10}$ (3) $\frac{1}{5}$ 16. $\frac{11}{36}$

6. Statistics

Practice Set 6.1

(1) 4.36 hrs (2) ₹ 521.43. (3) 2.82 litre (4) ₹ 35310

(5) ₹ 985 or ₹ 987.5. (6) ₹ 3070 or ₹ 3066.67.

Practice Set 6.2

(1) 11.4 hrs (2) 184.4 means 184 mangoes approximately (3) 74.558 ≈ 75 vehicles

(4) 52750 lamps

Practice Set 6.3

1. 4.33 litre 2. 72 unit 3. 9.94 litre 4. 12.31 years

Practice Set 6.5

1. (1) 60–70 (2) 20–30 and 90–100 (3) 55 (4) 80 and 90 (5) 15

Practice Set 6.6

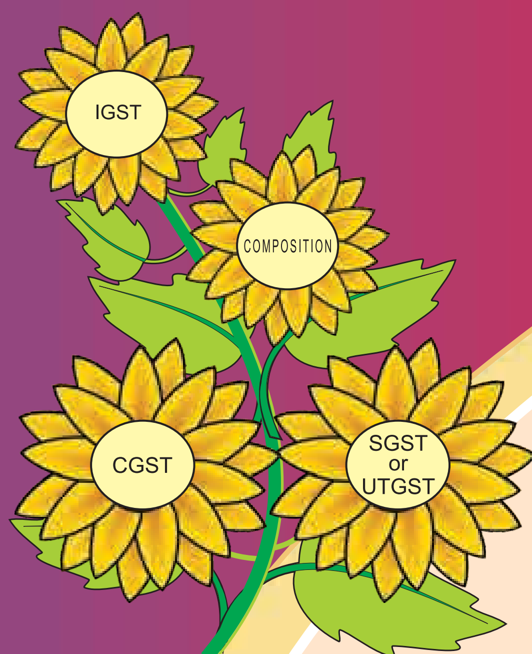
5. (1) 2000 (2) 1000 (3) 25%
6. (1) ₹ 12000 (2) ₹ 3000 (3) ₹ 2000 (4) ₹ 1000.

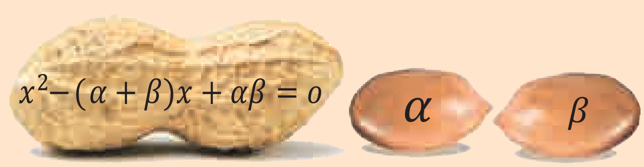
Problem Set – 6

1. (1) D (2) A (3) B (4) C (5) C (6) C
2. ₹ 52,500 3. ₹ 65,400 4. ₹ 4250
5. ₹ 72,400 6. 223.13 km. 7. ₹ 32 8. 397.06 gm

14. (1) Cars – 108°, Tempos – 43°, Buses – 29°, Auto-rickshaws – 36°, Two wheelers – 144°
    (2) Total number of vehicles – 3000

16. (1) Cricket – 225, (2) Football – 175 (3) Other games – 200.





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MATHEMATICS

Part - II

STANDARD X

The Coordination Committee formed by GR No. Abhyas - 2116/(Pra.Kra.43/16) SD - 4 Dated 25.4.2016 has given approval to prescribe this textbook in its meeting held on 29.12.2017 and it has been decided to implement it from the educational year 2018-19.

Mathematics

Part II

STANDARD X

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune - 411 004

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The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

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NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

Preface

Dear Students,

Welcome to the tenth standard !

This year you will study two text books - Mathematics Part-I and Mathematics Part-II

The main areas in the book Mathematics part - II are Geometry, Trigonometry, Coordinate geometry and Mensuration. All of these topics were introduced in the ninth standard. This year you will study some more details of the same. Their utility will be clear from the given examples. Wherever a new unit, formula or application is introduced, its lucid explanation is given. Each chapter contains illustrative solved examples and sets of questions for practice. Moreover, some questions in practice sets are star-marked, indicating that they are challenging for talented students.

After Tenth standard, some students do not opt for mathematics. They too need the basic concepts and the knowledge necessary for working in other fields.The matter under the head 'For more Information' is useful for those students who wish to study mathematics after tenth standard and achieve proficiency in it. So they are earnestly advised to study this part. Read the book thoroughly at least once and grasp the concepts.

Additional audio visual material regarding each lesson will be available to you by Q.R. Code through 'App'. It will definitely be useful to you for your studies.

Much importance is given to the tenth standard examination. You are advised not to take the stress and study to the best of your ability to achieve expected success.

Best wishes for it !

(Dr. Sunil Magar)
Director
Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

Pune
Date : 18 March 2018, Gudhipadva
Indian Solar Year : 27 Falgun 1939

It is expected that students will develop the following competencies after studying Mathematics- Part II syllabus in standard X

Area	Topic	Competency Statements
1. Geometry	1.1 Similar triangles 1.2 Circle	The students will be able to - • solve examples using properties of similar trianlges, properties of congruent traingles and Pythagoras theorem. • construct similar triangles. • be able to use properties of chords and tangents. • be able to construct tangents to a circle.
2. Co-ordinate Geometry	2.1 Co-ordinate geometry	• find distance between two points. • find the co-ordinates of a point dividing a segment in given ratio. • find slope of a line.
3. Mensuration	3.1 Surface area and volume	• find length of arc of a circle. • find areas of sector of a circle and segment of a circle. • compute surface areas and volumes of some three dimensional objects.
4. Trigonometry	4.1 Trigonometry	• solve examples using trigionometric identities • solve problems like measuring height of a tree, width of a river bed etc., using trigonometry.

Instructions for Teachers

Read the book in detail and grasp the content thoroughly.Take the help of activities to explain different topics, to verify the formulae etc.

Practicals is also a means of evaluation. Activities given can be used for this purpose. Encourage the students to think independently.Compliment a student if he solves an example by a different and logically correct method.

Suitable activities, other than those given in the text book, can be planned to understand the statements of the theorems and to develop the skill to solve problems.

List of some practicals (Specimen)

1. Cut out a triangular piece of card-board. Place a lit up candle or a small lamp on a table. Hold the triangle between a wall and the candle/ lamp. Observe the shadow of the triangle. Decide if the triangle and its shadow are similar. (What care will you take so that the triangle and its shadow are similar?)
2. Cut out two identical right angled triangles. Name the vertices of the triangles as A, B, C on both sides. Draw the altitude on the hypotenuse of one of them. Name the foot of the perpendicular as D. Cut the triangle on its altitude and obtain two triangles. State the correspondences by which the three triangles are similar with one another.
3. Draw a circle. Take three points - one on the circle, one in its interior and one in its exterior. Prepare a table, showing rough figures and stating how many tangents can be drawn to the circle through each of the three points.
4. Draw at least five different circles passing through two given distinct points indicating that innumerable circles can be drawn passing through them.
5. Take a geoboard on which nails are suitably fixed to verify properties of a circle. Prepare a figure using rubber bands for any one of the following theorems.

   (i) Inscribed angle theorem (ii) Tangent secant theorem of angles

   (iii) Theorem of angles inscribed in opposite arcs of a circle.
6. Prepare a model of a circle and an angle. Show different arcs intercepted by the angle in different situations. Draw the corresponding figures in your note book.
7. Draw an angle and divide it into four equal parts using compass and ruler.
8. Take a beaker. Measure its height and radius of base. Calculate its capacity using the formula. Fill it fully with water. Measure the volume of the water with a measuring cylinder. Compare the two results and draw inference.
9. Take a paper cup of the shape of frustum of a cone. Measure the radii of its base and top and also its height. Using formula, calculate its capacity. Fill it fully with water and then measure the volume of the water. Compare the measured and the calculated volumes and verify the formula.
10. Cut two similar triangles out of a card-board. Decide by actual measurements -

    (i) Are their areas proportional to the squares of their perimeters ?

    (ii) Are their areas proportional to the squares of their medians ?

INDEX

1 Similarity

Let's study.

- Ratio of areas of two triangles
- Basic proportionality theorem
- Converse of basic proportionality theorem
- Tests of similarity of triangles
- Property of an angle bisector of a triangle
- Property of areas of similar triangles
- The ratio of the intercepts made on the transversals by three parallel lines

Let's recall.

We have studied Ratio and Proportion. The statement, 'the numbers a and b are in the ratio $\frac{m}{n}$' is also written as, 'the numbers a and b are in proportion m:n.'
For this concept we consider positive real numbers. We know that the lengths of line segments and area of any figure are positive real numbers.
We know the formula of area of a triangle.

Area of a triangle = $\frac{1}{2}$ Base × Height

Let's learn.

Ratio of areas of two triangles

Let's find the ratio of areas of any two triangles.

Ex. In Δ ABC, AD is the height and BC is the base.
In Δ PQR, PS is the height and QR is the base

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

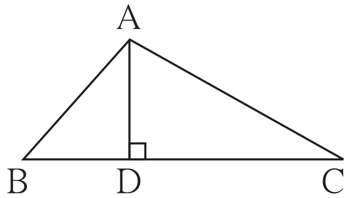

Fig. 1.1

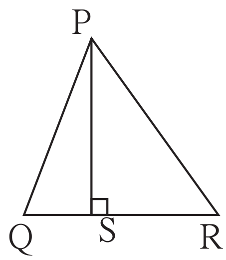

Fig. 1.2

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$

Hence the ratio of the areas of two triangles is equal to the ratio of the products of their bases and corrosponding heights.

Base of a triangle is b_1 and height is h_1. Base of another triangle is b_2 and height is h_2. Then the ratio of their areas $= \frac{b_1 \times h_1}{b_2 \times h_2}$

Suppose some conditions are imposed on these two triangles,

Condition 1: If the heights of both triangles are equal then-

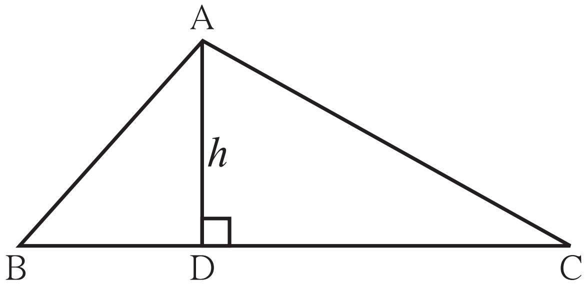

Fig. 1.3

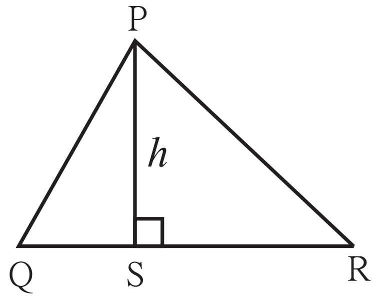

Fig. 1.4

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times h}{QR \times h} = \frac{BC}{QR}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{b_1}{b_2}$$

Property: The ratio of the areas of two triangles with equal heights is equal to the ratio of their corresponding bases.

Condition 2: If the bases of both triangles are equal then -

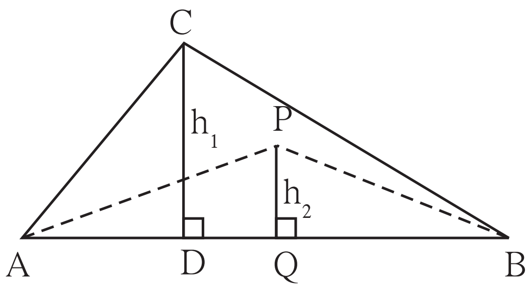

Fig. 1.5

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{AB \times h_1}{AB \times h_2}$$

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{h_1}{h_2}$$

Property: The ratio of the areas of two triangles with equal bases is equal to the ratio of their corresponding heights.

Activity :

Fill in the blanks properly.

(i)

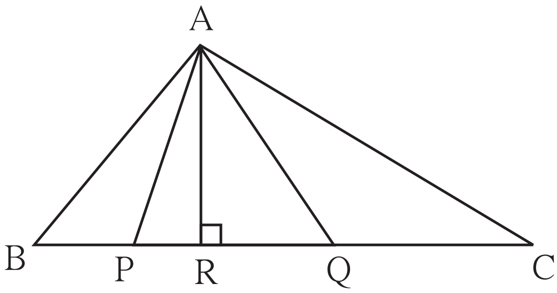

Fig. 1.6

$$\frac{A(\Delta ABC)}{A(\Delta APQ)} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$$

(ii)

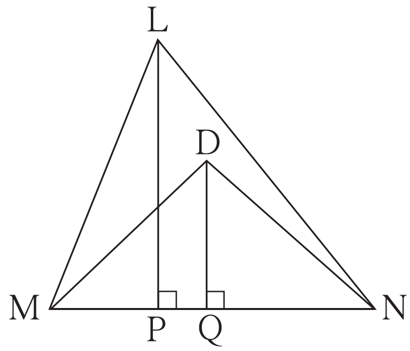

Fig.1.7

$$\frac{A(\Delta LMN)}{A(\Delta DMN)} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$$

(iii) M is the midpoint of seg AB and seg CM is a median of Δ ABC

$$\therefore \frac{A(\Delta AMC)}{A(\Delta BMC)} = \frac{\square}{\square}$$

$$= \frac{\square}{\square} = \square$$

State the reason.

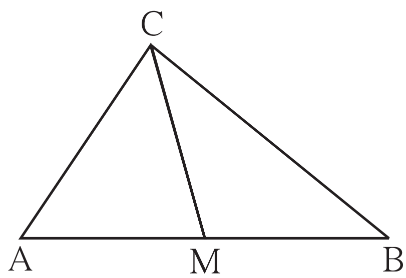

Fig. 1.8

Solved Examples

Ex. (1)

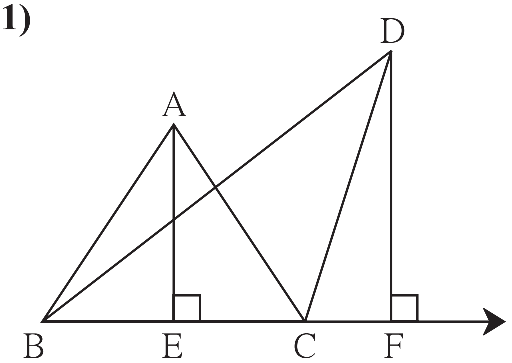

Fig.1.9

In adjoining figure

AE $\perp$ seg BC, seg DF $\perp$ line BC,

AE = 4, DF = 6 , then find $\frac{A(\Delta ABC)}{A(\Delta DBC)}$.

Solution : $\frac{A(\Delta ABC)}{A(\Delta DBC)} = \frac{AE}{DF}$ .... bases are equal, hence areas proportional to heights.

$$= \frac{4}{6} = \frac{2}{3}$$

Ex. (2) In Δ ABC point D on side BC is such that DC = 6, BC = 15. Find A(Δ ABD) : A(Δ ABC) and A(Δ ABD) : A(Δ ADC) .

Solution : Point A is common vertex of Δ ABD, Δ ADC and Δ ABC and their bases are collinear. Hence, heights of these three triangles are equal

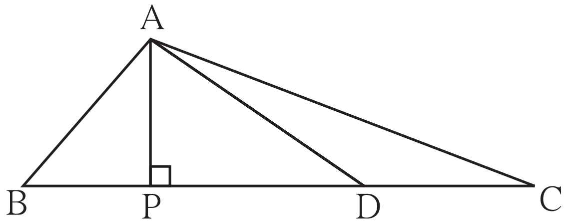

Fig. 1.10

BC = 15, DC = 6 $\therefore$ BD = BC − DC = 15 − 6 = 9

$\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{BD}{BC}$ .... heights equal, hence areas proportional to bases.

$= \frac{9}{15} = \frac{3}{5}$

$\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{BD}{DC}$ .... heights equal, hence areas proportional to bases.

$= \frac{9}{6} = \frac{3}{2}$

Ex. (3)

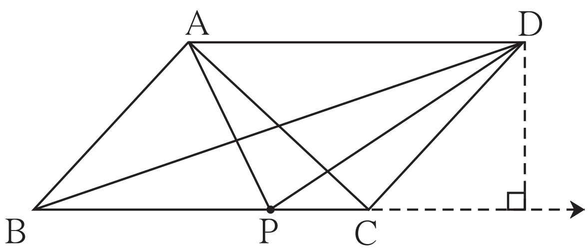

Fig. 1.11

□ ABCD is a parallelogram. P is any point on side BC. Find two pairs of triangles with equal areas.

Solution : □ ABCD is a parallelogram.

$\therefore$ AD || BC and AB || DC

Consider Δ ABC and Δ BDC.

Both the triangles are drawn in two parallel lines. Hence the distance between the two parallel lines is the height of both triangles.

In Δ ABC and Δ BDC, common base is BC and heights are equal.

Hence, A(Δ ABC) = A(Δ BDC)

In Δ ABC and Δ ABD, AB is common base and and heights are equal.

$\therefore$ A(Δ ABC) = A(Δ ABD)

Ex.(4)

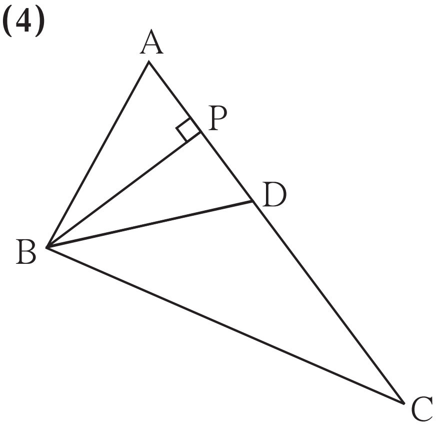

Fig. 1.12

In adjoining figure in Δ ABC, point D is on side AC. If AC = 16, DC = 9 and BP $\perp$ AC, then find the following ratios.

(i) $\frac{A(\Delta ABD)}{A(\Delta ABC)}$ (ii) $\frac{A(\Delta BDC)}{A(\Delta ABC)}$

(iii) $\frac{A(\Delta ABD)}{A(\Delta BDC)}$

Solution : In Δ ABC point P and D are on side AC, hence B is common vertex of Δ ABD, Δ BDC, Δ ABC and Δ APB and their sides AD, DC, AC and AP are collinear. Heights of all the triangles are equal. Hence, areas of these triangles are proportinal to their bases. AC = 16, DC = 9

$\therefore$ AD = 16 − 9 = 7

$\therefore \frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{AD}{AC} = \frac{7}{16}$ . . . . triangles having equal heights

$\frac{A(\Delta BDC)}{A(\Delta ABC)} = \frac{DC}{AC} = \frac{9}{16}$ . . . . triangles having equal heights

$\frac{A(\Delta ABD)}{A(\Delta BDC)} = \frac{AD}{DC} = \frac{7}{9}$ . . . . triangles having equal heights

Remember this !

- Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
- Areas of triangles with equal heights are proportional to their corresponding bases.
- Areas of triangles with equal bases are proportional to their corresponding heights.

Practice set 1.1

1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

2. In figure 1.13 BC ⊥ AB, AD ⊥ AB, BC = 4, AD = 8, then find $\frac{A(\Delta ABC)}{A(\Delta ADB)}$.

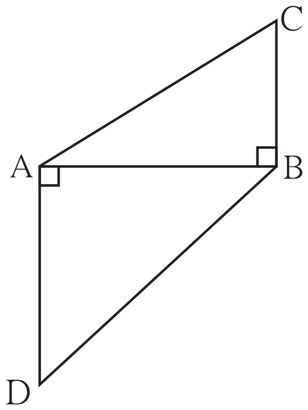

Fig. 1.13

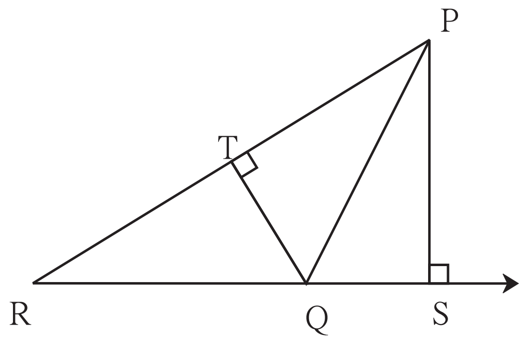

Fig. 1.14

3. In adjoining figure 1.14 seg PS ⊥ seg RQ seg QT ⊥ seg PR. If RQ = 6, PS = 6 and PR = 12, then find QT.

4. In adjoining figure, AP ⊥ BC, AD || BC, then find A(Δ ABC) : A(Δ BCD).

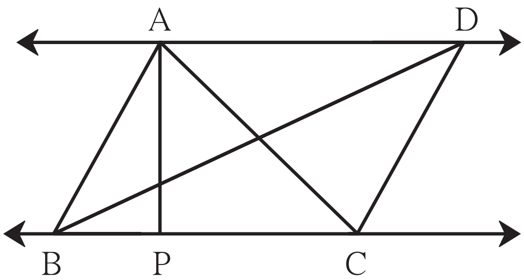

Fig. 1.15

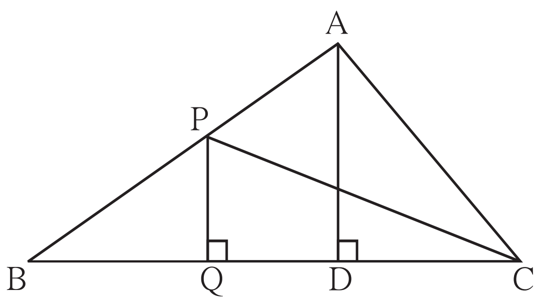

Fig. 1.16

5. In adjoining figure PQ ⊥ BC, AD ⊥ BC then find following ratios.

(i) $\frac{A(\Delta PQB)}{A(\Delta PBC)}$ (ii) $\frac{A(\Delta PBC)}{A(\Delta ABC)}$

(iii) $\frac{A(\Delta ABC)}{A(\Delta ADC)}$ (iv) $\frac{A(\Delta ADC)}{A(\Delta PQC)}$

Let's learn.

Basic proportionality theorem

Theorem : **If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.**

Given : In Δ ABC line l || line BC and line l intersects AB and AC in point P and Q respectively

To prove : $\frac{AP}{PB} = \frac{AQ}{QC}$

Construction: Draw seg PC and seg BQ

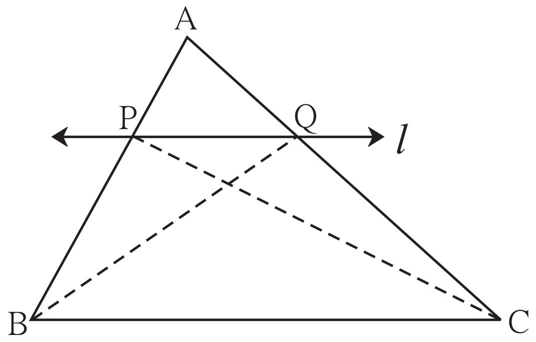

Fig. 1.17

Proof : Δ APQ and Δ PQB have equal heights.

$\therefore \frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{AP}{PB}$ .... (I) (areas proportionate to bases)

and $\frac{A(\Delta APQ)}{A(\Delta PQC)} = \frac{AQ}{QC}$ .... (II) (areas proportionate to bases)

seg PQ is common base of Δ PQB and Δ PQC. seg PQ || seg BC, hence Δ PQB and Δ PQC have equal heights.

$A(\Delta PQB) = A(\Delta PQC)$ .... (III)

$\frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{A(\Delta APQ)}{A(\Delta PQC)}$ .... [from (I), (II) and (III)]

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$ .... [from (I) and (II)]

Converse of basic proportionality theorem

Theorem : **If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.**

In figure 1.18, line l interesects the side AB and side AC of Δ ABC in the points P and Q respectively and $\frac{AP}{PB} = \frac{AQ}{QC}$, hence line l || seg BC.

This theorem can be proved by indirect method.

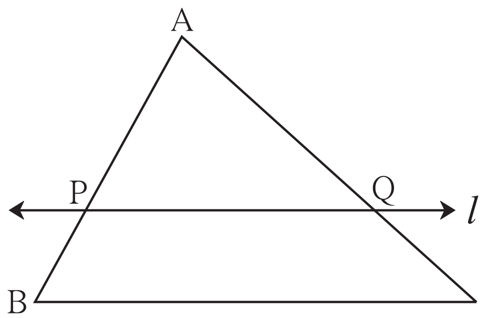

Fig. 1.18

Activity :

- Draw a Δ ABC.
- Bisect $\angle$ B and name the point of intersection of AC and the angle bisector as D.
- Measure the sides.

  AB = ☐ cm BC = ☐ cm

  AD = ☐ cm DC = ☐ cm
- Find ratios $\frac{AB}{BC}$ and $\frac{AD}{DC}$.

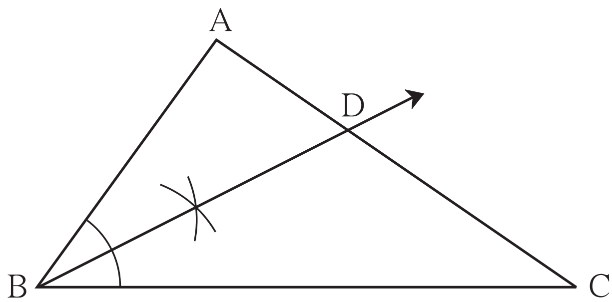

Fig. 1.19

- You will find that both the ratios are almost equal.
- Bisect remaining angles of the triangle and find the ratios as above. You can verify that the ratios are equal.

Let's learn.

Property of an angle bisector of a triangle

Theorem : The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Given : In Δ ABC, bisector of $\angle$C interesects seg AB in the point E.

To prove : $\frac{AE}{EB} = \frac{CA}{CB}$

Construction : Draw a line parallel to ray CE, passing through the point B. Extend AC so as to intersect it at point D.

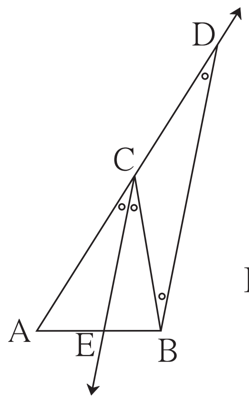

Fig. 1.20

Proof : ray CE || ray BD and AD is transversal,

$\therefore \angle ACE = \angle CDB$ .... (corresponding angles) ...(I)

Now taking BC as transversal

$\angle ECB = \angle CBD$ .... (alternate angle) ...(II)

But $\angle ACE \cong \angle ECB$ .... (given) ...(III)

$\therefore \angle CBD \cong \angle CDB$ .... [from (I), (II) and (III)]

In Δ CBD, side CB $\cong$ side CD ....(sides opposite to congruent angles)

$\therefore$ CB = CD ...(IV)

Now in Δ ABD, seg EC || seg BD .... (construction)

$\therefore \frac{AE}{EB} = \frac{AC}{CD}$ ....(Basic proportionality theorem)..(V)

$\therefore \frac{AE}{EB} = \frac{AC}{CB}$ .... [from (IV) and (V)]

For more information :

Write another proof of the theorem yourself.

Draw DM $\perp$ AB and DN $\perp$ AC. Use the following properties and write the proof.

(1) The areas of two triangles of equal heights are proportional to their bases.

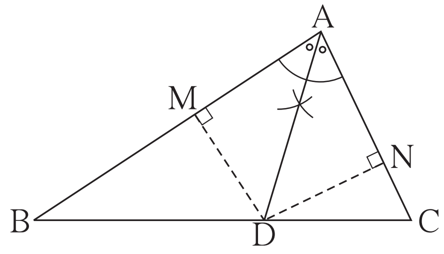

Fig. 1.21

(2) Every point on the bisector of an angle is equidistant from the sides of the angle.

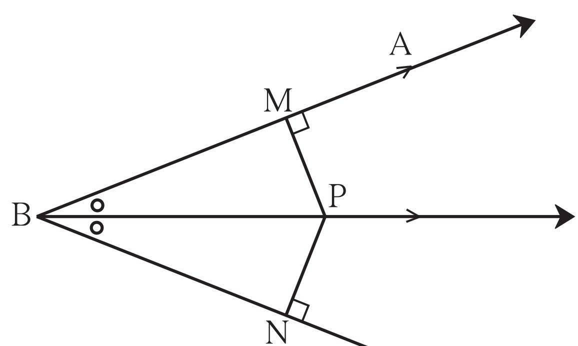

Fig. 1.22

Converse of angle bisector theorem

If in Δ ABC, point D on side BC such that $\frac{AB}{AC} = \frac{BD}{DC}$, then ray AD bisects $\angle$ BAC.

Property of three parallel lines and their transversals

Activity:

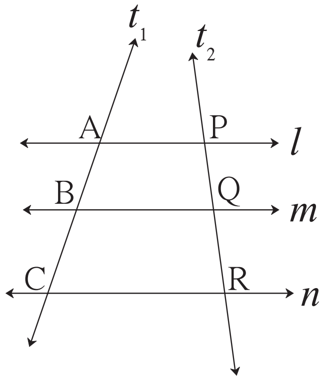

Fig. 1.23

- Draw three parallel lines.
- Label them as *l*, *m*, *n*.
- Draw transversals t_1 and t_2.
- AB and BC are intercepts on transversal t_1.
- PQ and QR are intercepts on transversal t_2.
- Find ratios $\frac{AB}{BC}$ and $\frac{PQ}{QR}$. You will find that they are almost equal.

Theorem : The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corrosponding intercepts made on any other transversal by the same parallel lines.

Given : line l || line m || line n

t_1 and t_2 are transversals.
Transversal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

To prove : $\frac{AB}{BC} = \frac{PQ}{QR}$

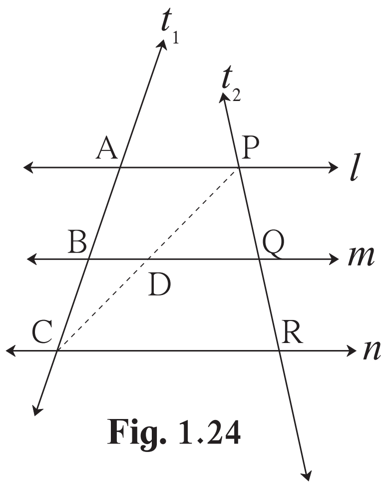

Fig. 1.24

Proof : Draw seg PC , which intersects line m at point D.

In Δ ACP, BD || AP

$\therefore \frac{AB}{BC} = \frac{PD}{DC}$. . . . . (I) (Basic proportionality theorem)

In Δ CPR, DQ || CR

$\therefore \frac{PD}{DC} = \frac{PQ}{QR}$. . . . . (II) (Basic proportionality theorem)

$\therefore \frac{AB}{BC} = \frac{PD}{DC} = \frac{PQ}{QR}$. . . . . from (I) and (II). $\therefore \frac{AB}{BC} = \frac{PQ}{QR}$

Remember this !

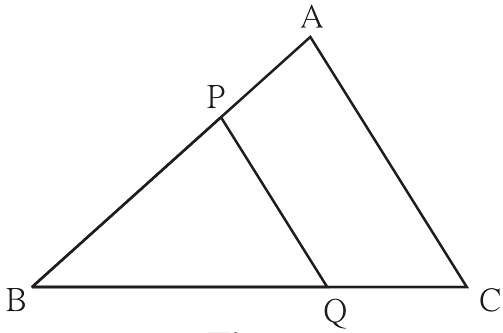

Fig. 1.25

(1) Basic proportionality theorem.

In Δ ABC, if seg PQ || seg AC

then $\dfrac{AP}{BP} = \dfrac{QC}{BQ}$

(2) Converse of basic proportionality theorem.

In ΔPQR, if $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

then seg ST || seg QR.

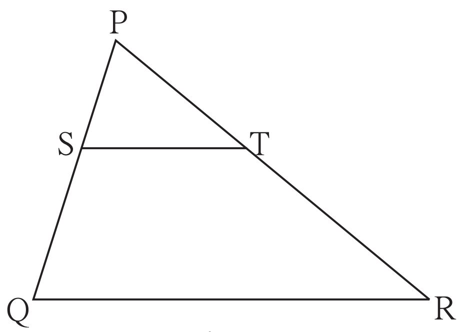

Fig. 1.26

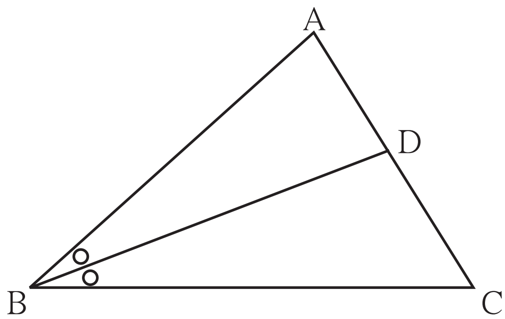

Fig. 1.27

(3) Theorem of bisector of an angle of a triangle.

If in Δ ABC, BD is bisector of $\angle$ ABC,

then $\dfrac{AB}{BC} = \dfrac{AD}{DC}$

(4) Property of three parallel lines and their transversals.

If line AX || line BY || line CZ and line l and line m are their transversals then $\dfrac{AB}{BC} = \dfrac{XY}{YZ}$

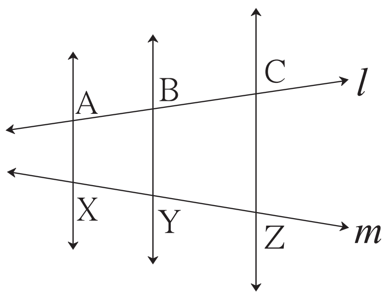

Fig. 1.28

Solved Examples

Ex. (1) In Δ ABC, DE || BC

If DB = 5.4 cm, AD = 1.8 cm

EC = 7.2 cm then find AE.

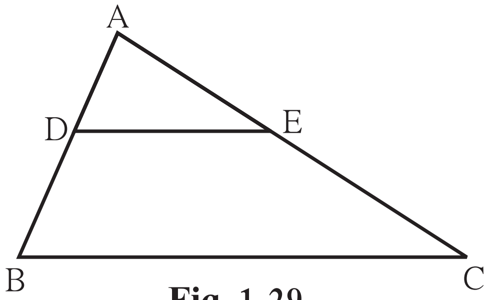

Fig. 1.29

Solution : In Δ ABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ ..... Basic proportionality theorem

$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$

$\therefore$ AE $\times$ 5.4 = 1.8 $\times$ 7.2

$\therefore$ AE = $\frac{1.8 \times 7.2}{5.4}$ = 2.4

AE = 2.4 cm

Ex. (2) In Δ PQR, seg RS bisects $\angle$ R.

If PR = 15, RQ = 20 PS = 12

then find SQ.

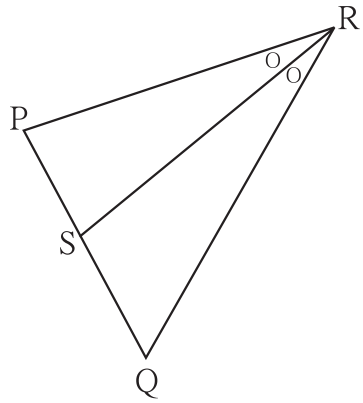

Fig. 1.30

Solution : In Δ PRQ, seg RS bisects $\angle$ R.

$\frac{PR}{RQ} = \frac{PS}{SQ}$ . . . . . . property of angle bisector

$\frac{15}{20} = \frac{12}{SQ}$

SQ = $\frac{12 \times 20}{15}$

$\therefore$ SQ = 16

Activity :

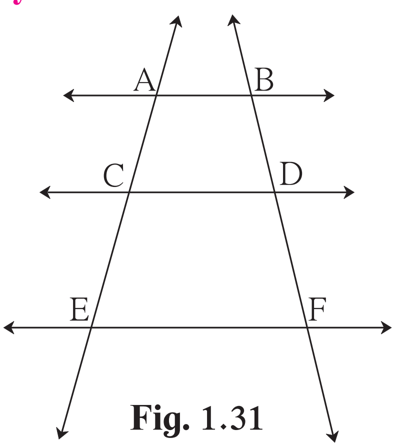

Fig. 1.31

In the figure 1.31, AB || CD || EF

If AC = 5.4, CE = 9, BD = 7.5

then find DF

Solution : AB || CD || EF

$\frac{AC}{\square} = \frac{\square}{DF}$ . . . . . (   )

$\frac{5.4}{9} = \frac{\square}{DF}$ $\therefore$ DF = $\square$

Activity :

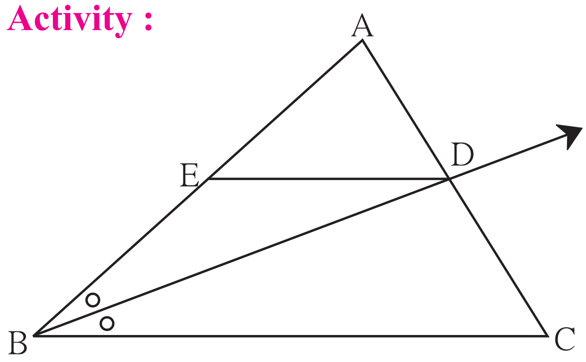

Fig. 1.32

In ΔABC, ray BD bisects $\angle$ABC. A–D–C, side DE || side BC, A–E–B then prove that, $\frac{AB}{BC} = \frac{AE}{EB}$

Proof : In Δ ABC, ray BD bisects $\angle$ B.

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$ . . (I)(Angle bisector theorem)

In Δ ABC, DE || BC

$\frac{AE}{EB} = \frac{AD}{DC}$ . . . . . (II) (. . . . .)

$\frac{AB}{\square} = \frac{\square}{EB}$ . . . . . . from (I) and (II)

Practice set 1.2

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of $\angle$ QPR.

(1)

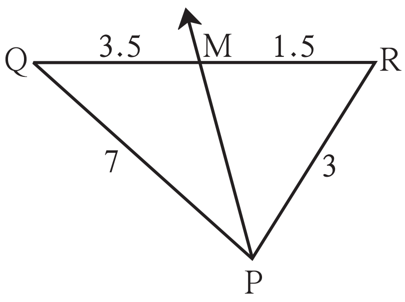

Fig. 1.33

(2)

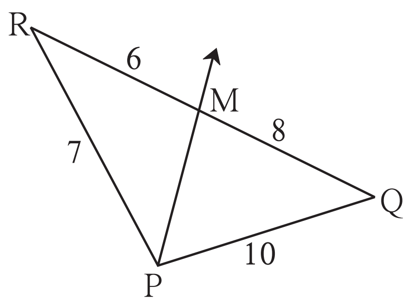

Fig. 1.34

(3)

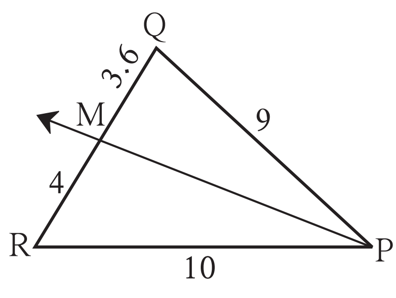

Fig. 1.35

2. In Δ PQR, PM = 15, PQ = 25 PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.

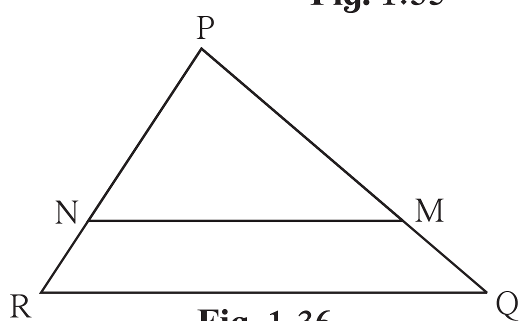

Fig. 1.36

3. In Δ MNP, NQ is a bisector of $\angle$ N. If MN = 5, PN = 7 MQ = 2.5 then find QP .

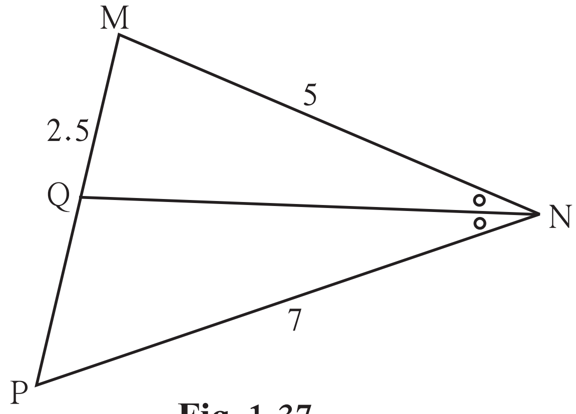

Fig. 1.37

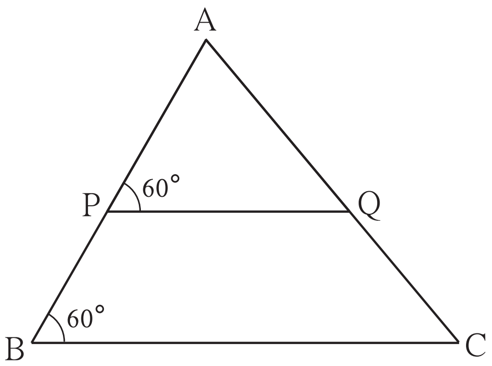

Fig. 1.38

4. Measures of some angles in the figure are given. Prove that

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

5. In trapezium ABCD, side AB || side PQ || side DC, AP = 15, PD = 12, QC = 14, find BQ.

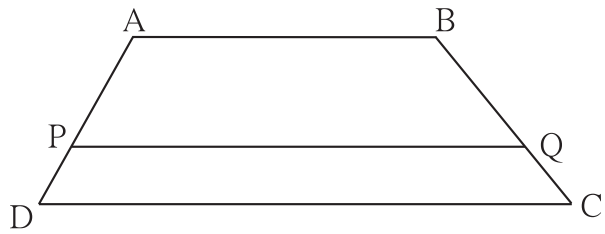

Fig. 1.39

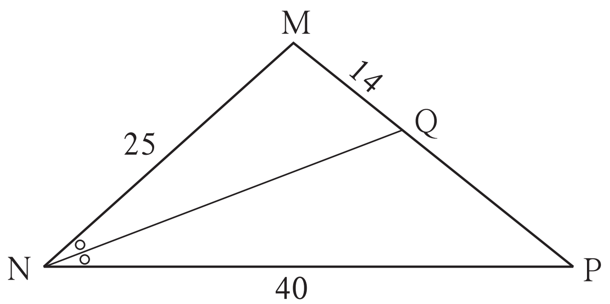

Fig. 1.40

6. Find QP using given information in the figure.

7. In figure 1.41, if AB || CD || FE then find x and AE.

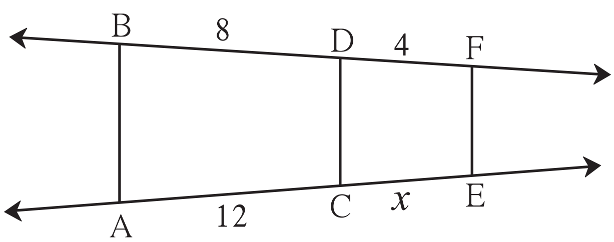

Fig. 1.41

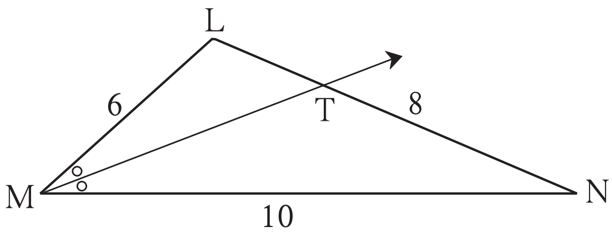

Fig. 1.42

8. In Δ LMN, ray MT bisects $\angle$ LMN

If LM = 6, MN = 10, TN = 8,

then find LT.

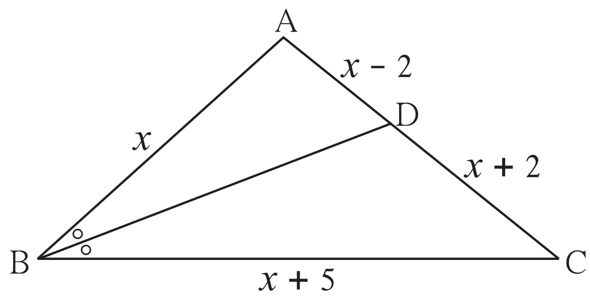

Fig. 1.43

9. In Δ ABC, seg BD bisects $\angle$ ABC.

If AB = x, BC = $x + 5$,

AD = $x - 2$, DC = $x + 2$, then find

the value of x.

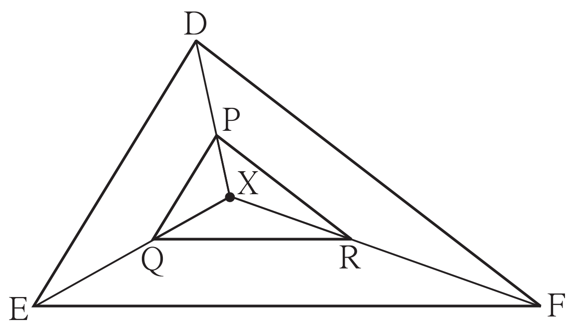

Fig. 1.44

10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ || seg DE, seg QR || seg EF. Fill in the blanks to prove that, seg PR || seg DF.

Proof : In Δ XDE, PQ || DE .... ____

$\therefore \frac{XP}{\_\_\_\_} = \frac{\_\_\_\_}{QE}$ .... (I) (Basic proportionality theorem)

In Δ XEF, QR || EF .... ____

$\therefore \frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$ ....(II) ____

$\therefore \frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$ .... from (I) and (II)

$\therefore$ seg PR || seg DE .... (converse of basic proportionality theorem)

11★. In Δ ABC, ray BD bisects $\angle$ ABC and ray CE bisects $\angle$ ACB. If seg AB $\cong$ seg AC then prove that ED || BC.

Let's recall.

Similar triangles

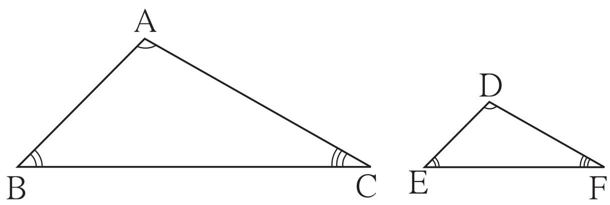

Fig. 1.45

In Δ ABC and Δ DEF, if $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

then ΔABC and ΔDEF are similar triangles.

'Δ ABC and Δ DEF are similar' is expressed as '$\Delta ABC \sim \Delta DEF$'

Let's learn.

Tests of similarity of triangles

For similarity of two triangles, the necessary conditions are that their corresponding sides are in same proportion and their corresponding angles are congruent. Out of these conditions; when three specifc conditions are fulfilled, the remaining conditions are automatically fulfilled. This means for similarity of two triangles, only three specific conditions are sufficient. Similarity of two triangles can be confirmed by testing these three conditions. The groups of such sufficient conditions are called tests of similarity, which we shall use.

AAA test for similarity of triangles

For a given correspondence of vertices, when corresponding angles of two triangles are congruent, then the two triangles are similar.

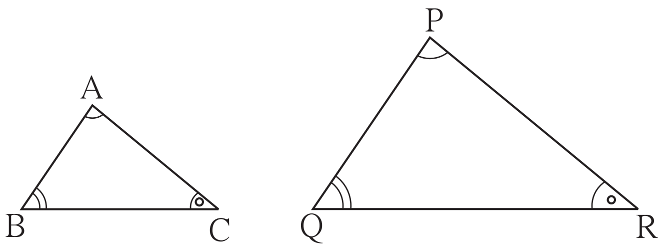

Fig. 1.46

In Δ ABC and Δ PQR, in the correspondence ABC $\leftrightarrow$ PQR if

$\angle A \cong \angle P$, $\angle B \cong \angle Q$ and $\angle C \cong \angle R$

then $\Delta ABC \sim \Delta PQR$.

For more information :

Proof of AAA test

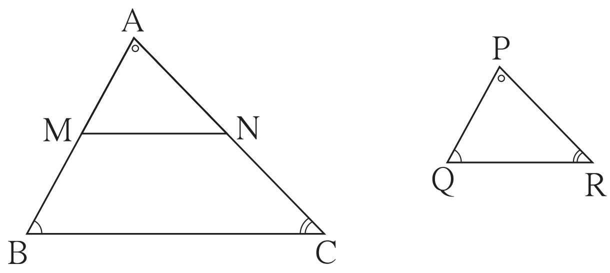

Fig. 1.47

Given : In Δ ABC and Δ PQR, $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$.

To prove : $\Delta ABC \sim \Delta PQR$

Let us assume that Δ ABC is bigger than Δ PQR. Mark point M on AB, and point N on AC such that AM = PQ and AN = PR.

Show that $\Delta AMN \cong \Delta PQR$. Hence show that MN || BC.

Now using basic proportionality theorem, $\frac{AM}{MB} = \frac{AN}{NC}$

That is $\frac{MB}{AM} = \frac{NC}{AN}$ .... (by invertendo)

$\frac{MB+AM}{AM} = \frac{NC+AN}{AN}$ .... (by componendo)

$\therefore \frac{AB}{AM} = \frac{AC}{AN}$

$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$

Similarly it can be shown that $\frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\quad\quad \therefore \Delta ABC \sim \Delta PQR$

A A test for similarity of triangles:

We know that for a given correspondence of vertices, when two angles of a triangle are congruent to two corresponding angles of another triangle, then remaining angle of first triangle is congruent to the remaining angle of the second triangle.

This means, when two angles of one triangle are congruent to two corresponding angles of another triangle then this condition is sufficient for similarity of two triangles. This condition is called AA test of similarity.

SAS test of similarity of triangles

For a given correspondence of vertices of two triangles, if two pairs of corresponding sides are in the same proportion and the angles between them are congruent, then the two triangles are similar.

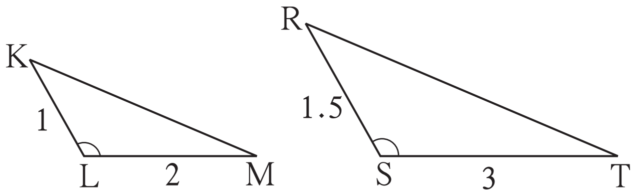

Fig. 1.48

For example, if in Δ KLM and Δ RST,

$\angle KLM \cong \angle RST$

$$\frac{KL}{RS} = \frac{LM}{ST} = \frac{2}{3}$$

Therefore, Δ KLM $\sim$ Δ RST

SSS test for similarity of triangles

For a given correspondence of vertices of two triangles, when three sides of a triangle are in proportion to corresponding three sides of another triangle, then the two triangles are similar.

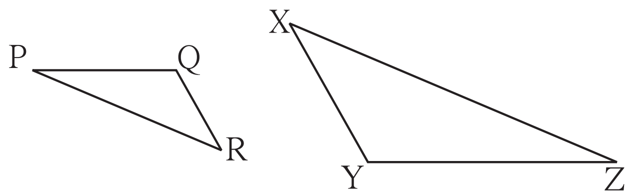

Fig. 1.49

For example, if in Δ PQR and Δ XYZ,

If $\frac{PQ}{YZ} = \frac{QR}{XY} = \frac{PR}{XZ}$

then Δ PQR $\sim$ Δ ZYX

Properties of similar triangles :

(1) Δ ABC $\sim$ Δ ABC – Reflexivity

(2) If Δ ABC $\sim$ Δ DEF then Δ DEF $\sim$ Δ ABC – Symmetry

(3) If Δ ABC $\sim$ Δ DEF and Δ DEF $\sim$ Δ GHI, then Δ ABC $\sim$ Δ GHI – Transitivity

Solved Examples

Ex. (1) In Δ XYZ,

$\angle Y = 100°$, $\angle Z = 30°$,

In Δ LMN,

$\angle M = 100°$, $\angle N = 30°$,

Are Δ XYZ and Δ LMN similar? If yes, by which test?

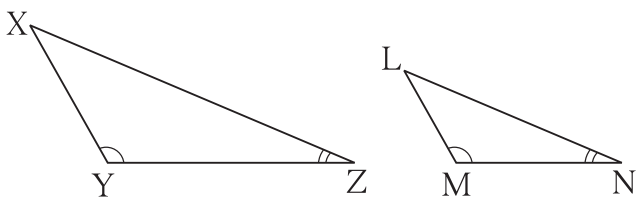

Fig. 1.50

Solution : In Δ XYZ and Δ LMN,

$\angle Y = 100°$, $\angle M = 100°$, $\therefore \angle Y \cong \angle M$

$\angle Z = 30°$, $\angle N = 30°$, $\therefore \angle Z \cong \angle N$

$\therefore \Delta XYZ \sim \Delta LMN$ ..... by AA test.

Ex. (2) Are two triangles in figure 1.51 similar, according to the information given? If yes, by which test?

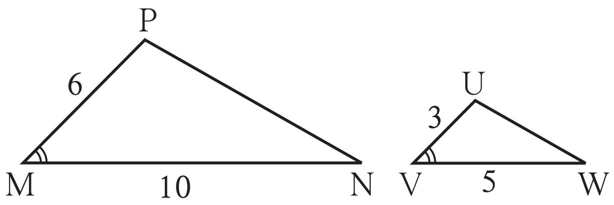

Fig. 1.51

Solution : In Δ PMN and Δ UVW

$\frac{PM}{UV} = \frac{6}{3} = \frac{2}{1}$, $\frac{MN}{VW} = \frac{10}{5} = \frac{2}{1}$

$\therefore \frac{PM}{UV} = \frac{MN}{VW}$

and $\angle M \cong \angle V$ .... Given

$\Delta PMN \sim \Delta UVW$ .... SAS test of similarity

Ex. (3) Can we say that the two triangles in figure 1.52 similar, according to information given? If yes, by which test ?

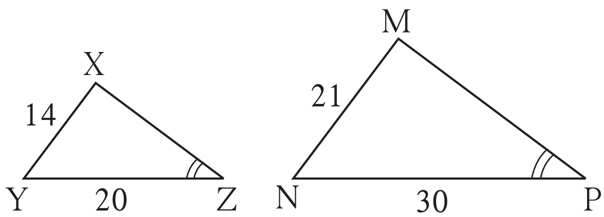

Fig. 1.52

Solution : Δ XYZ and Δ MNP,

$\frac{XY}{MN} = \frac{14}{21} = \frac{2}{3}$,

$\frac{YZ}{NP} = \frac{20}{30} = \frac{2}{3}$

It is given that $\angle Z \cong \angle P$.

But $\angle Z$ and $\angle P$ are not included angles by sides which are in proportion.

$\therefore \Delta$ XYZ and Δ MNP can not be said to be similar.

Ex. (4) In the adjoining figure $BP \perp AC$, $CQ \perp AB$, A – P – C, A – Q – B, then prove that Δ APB and Δ AQC are similar.

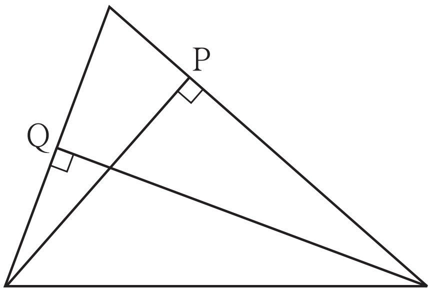

Fig. 1.53

Solution : In Δ APB and Δ AQC

$\angle$ APB = ☐° (I)

$\angle$ AQC = ☐° (II)

$\therefore$ $\angle$ APB $\cong$ $\angle$ AQC ....from(I) and (II)

$\angle$ PAB $\cong$ $\angle$ QAC .... (☐)

$\therefore$ Δ APB ~ Δ AQC ..... AA test

Ex. (5) Diagonals of a quadrilateral ABCD intersect in point Q. If 2QA = QC, 2QB = QD, then prove that DC = 2AB.

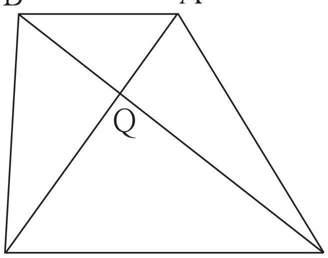

Fig. 1.54

Given : 2QA = QC

2QB = QD

To prove : CD = 2AB

Proof : 2QA = QC $\therefore \frac{QA}{QC} = \frac{1}{2}$ .... (I)

2QB = QD $\therefore \frac{QB}{QD} = \frac{1}{2}$ .... (II)

$\therefore \frac{QA}{QC} = \frac{QB}{QD}$ ....from (I) and (II)

In Δ AQB and Δ CQD,

$\frac{QA}{QC} = \frac{QB}{QD}$ .... proved

$\angle$ AQB $\cong$ $\angle$ DQC .... opposite angles

$\therefore$ Δ AQB ~ Δ CQD .... (SAS test of similarity)

$\therefore \frac{AQ}{CQ} = \frac{QB}{QD} = \frac{AB}{CD}$ .... correponding sides are proportional

But $\frac{AQ}{CQ} = \frac{1}{2}$ $\therefore \frac{AB}{CD} = \frac{1}{2}$

$\therefore$ 2AB = CD

Practice set 1.3

1. In figure 1.55, $\angle ABC = 75^\circ$, $\angle EDC = 75^\circ$ state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.

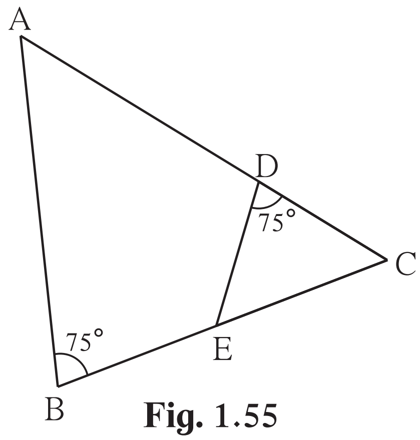

Fig. 1.55

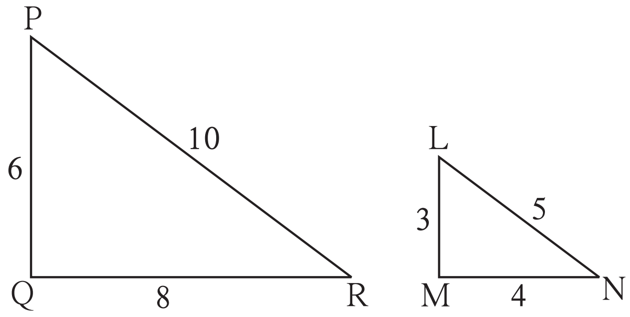

Fig. 1.56

2. Are the triangles in figure 1.56 similar? If yes, by which test ?

3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time ?

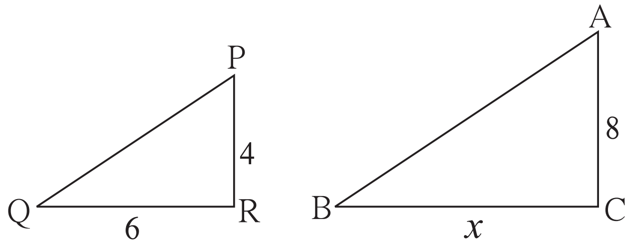

Fig. 1.57

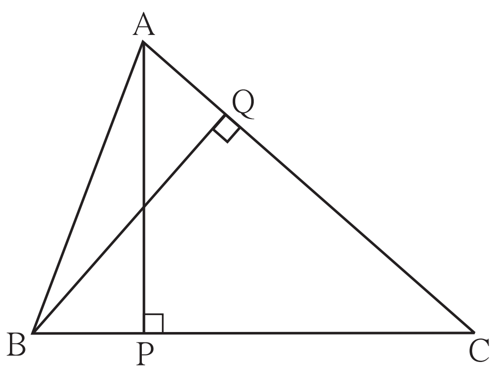

Fig. 1.58

4. In Δ ABC, AP $\perp$ BC, BQ $\perp$ AC B- P-C, A-Q - C then prove that, Δ CPA $\sim \Delta$ CQB.
 If AP = 7, BQ=8, BC=12 then find AC.

5. Given : In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ

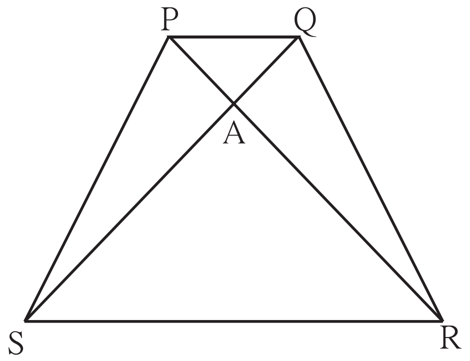

Fig. 1.59

6. In trapezium ABCD, (Figure 1.60) side AB || side DC, diagonals AC and BD intersect in point O. If AB = 20, DC = 6, OB = 15 then find OD.

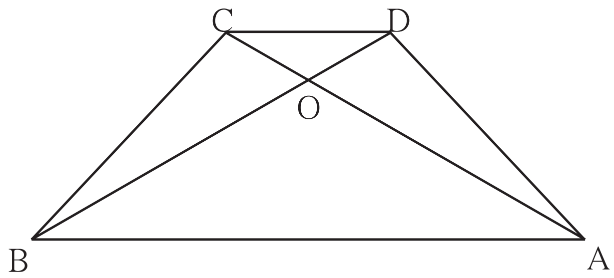

Fig. 1.60

7. □ABCD is a parallelogram point E is on side BC. Line DE intersects ray AB in point T. Prove that DE × BE = CE × TE.

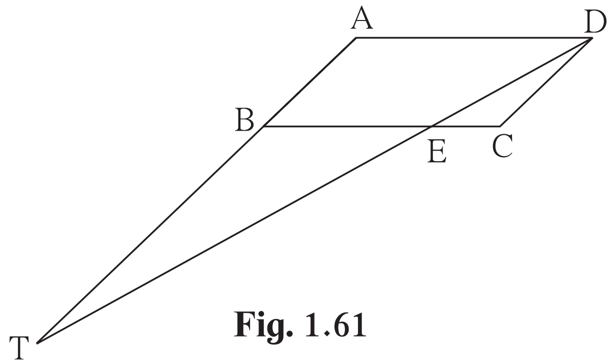

Fig. 1.61

8. In the figure, seg AC and seg BD intersect each other in point P and $\frac{AP}{CP} = \frac{BP}{DP}$. Prove that, Δ ABP ~ Δ CDP

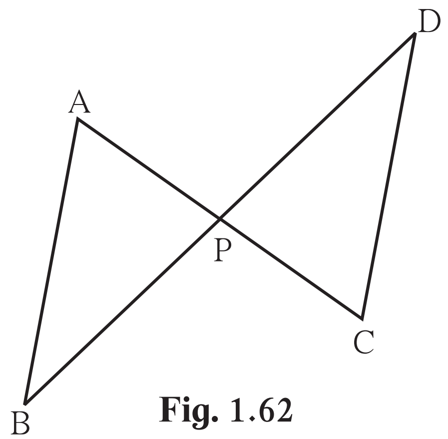

Fig. 1.62

9. In the figure, in Δ ABC, point D on side BC is such that, ∠BAC = ∠ADC.

Prove that, $CA^2 = CB \times CD$

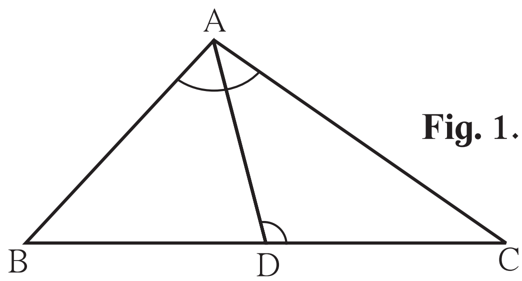

Fig. 1.63

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Let's learn.

Theorem of areas of similar triangles

Theorem : When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

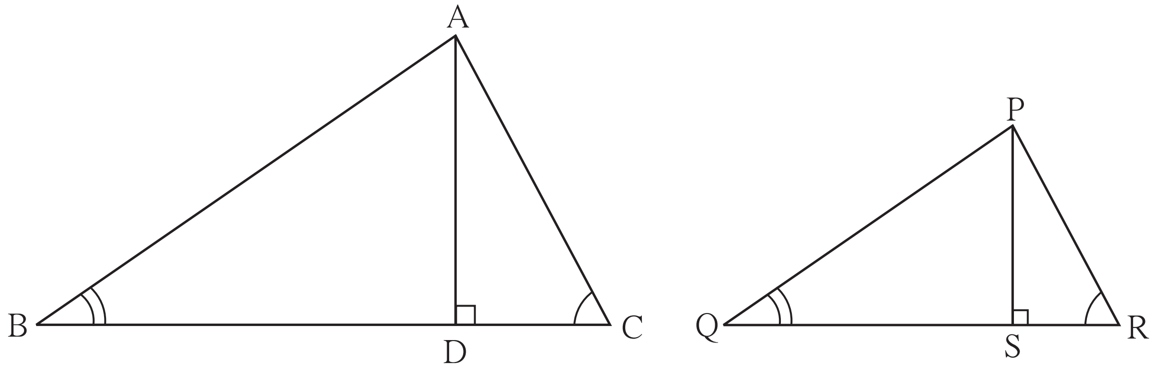

Fig. 1.64

Given : $\Delta ABC \sim \Delta PQR$, $AD \perp BC$, $PS \perp QR$

To prove: $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$

Proof : $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{BC \times AD}{QR \times PS} = \dfrac{BC}{QR} \times \dfrac{AD}{PS}$ .... (I)

In Δ ABD and Δ PQS,

$\angle B = \angle Q$ .... given

$\angle ADB = \angle PSQ = 90°$

$\therefore$ According to AA test $\Delta ABD \sim \Delta PQS$

$\therefore \dfrac{AD}{PS} = \dfrac{AB}{PQ}$ .... (II)

But $\Delta ABC \sim \Delta PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$ .... (III)

From (I), (II) and (III)

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2}$$

Solved Examples

Ex. (1) : $\Delta ABC \sim \Delta PQR$, A (ΔABC) = 16 , A (ΔPQP) = 25, then find the value of ratio $\frac{AB}{PQ}$.

Solution : $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$$ .... theorem of areas of similar triangles

$$\therefore \frac{16}{25} = \frac{AB^2}{PQ^2} \quad \therefore \frac{AB}{PQ} = \frac{4}{5}$$ .... taking square roots

Ex. (2) Ratio of corresponging sides of two similar triangles is 2:5, If the area of the small triangle is 64 sq.cm. then what is the area of the bigger triangle ?

Solution : Assume that $\Delta ABC \sim \Delta PQR$.

ΔABC is smaller and ΔPQR is bigger triangle.

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(2)^2}{(5)^2} = \frac{4}{25}$$ .... ratio of areas of similar triangles

$$\therefore \frac{64}{A(\Delta PQR)} = \frac{4}{25}$$

$4 \times A(\Delta PQR) = 64 \times 25$

$$A(\Delta PQR) = \frac{64 \times 25}{4} = 400$$

$\therefore$ area of bigger triangle = 400 sq.cm.

Ex. (3) In trapezium ABCD, side AB || side CD, diagonal AC and BD intersect each other at point P. Then prove that $\frac{A(\Delta ABP)}{A(\Delta CPD)} = \frac{AB^2}{CD^2}$.

Solution : In trapezium ABCD side AB || side CD

In ΔAPB and ΔCPD

$\angle PAB \cong \angle PCD$ ...... alternate angles

$\angle APB \cong \angle CPD$ .... opposite angles

$\therefore \Delta APB \sim \Delta CPD$ .... AA test of similarity

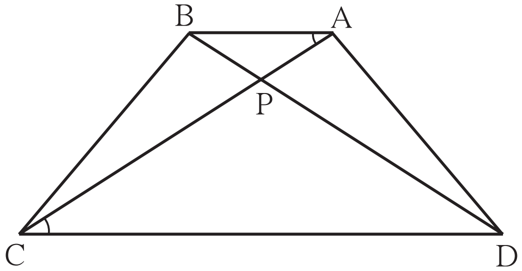

Fig. 1.65

$$\frac{A(\Delta APB)}{A(\Delta CPD)} = \frac{AB^2}{CD^2}$$ . . . . . . theorem of areas of similar triangles

Practice set 1.4

1. The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas .

2. If $\Delta ABC \sim \Delta PQR$ and AB: PQ = 2:3, then fill in the blanks.

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{\square} = \frac{2^2}{3^2} = \frac{\square}{\square}$$

3. If $\Delta ABC \sim \Delta PQR$, A ($\Delta$ ABC) = 80, A (Δ PQR) = 125, then fill in the blanks.

$$\frac{A(\Delta ABC)}{A(\Delta \ldots\ldots)} = \frac{80}{125} \qquad \therefore \frac{AB}{PQ} = \frac{\square}{\square}$$

4. $\Delta LMN \sim \Delta PQR$, $9 \times A(\Delta PQR) = 16 \times A(\Delta LMN)$. If QR = 20 then find MN.

5. Areas of two similar triangles are 225 sq.cm. 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of the bigger triangle .

6. ΔABC and Δ DEF are equilateral triangles. If A(ΔABC) : A (Δ DEF) = 1 : 2 and AB = 4, find DE.

7. In figure 1.66, seg PQ || seg DE, A(Δ PQF) = 20 units, PF = 2 DP, then find A($\square$ DPQE) by completing the following activity.

A(Δ PQF) = 20 units, PF = 2 DP, Let us assume DP = x. $\therefore$ PF = $2x$

DF = DP + $\square$ = $\square$ + $\square$ = $3x$

In Δ FDE and Δ FPQ,

$\angle$ FDE $\cong$ $\angle$ .... corresponding angles

$\angle$ FED $\cong$ $\angle$ .... corresponding angles

$\therefore$ Δ FDE $\sim$ Δ FPQ .... AA test

$$\therefore \frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{\square}{\square} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\Delta FDE) = \frac{9}{4} A(\Delta FPQ) = \frac{9}{4} \times \square = \square$$

A($\square$ DPQE) = A(Δ FDE) − A(Δ FPQ)

= $\square$ − $\square$

= $\square$

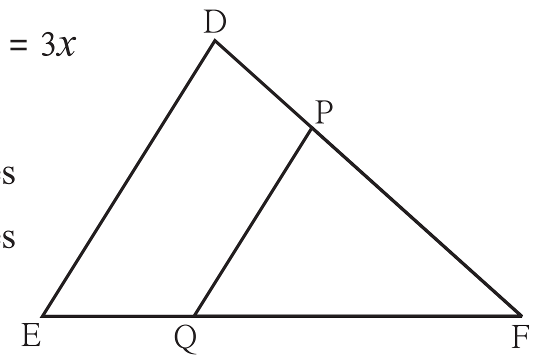

Fig. 1.66

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Problem set 1

1. Select the appropriate alternative.

(1) In Δ ABC and Δ PQR, in a one to one correspondence $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

(A) $\Delta PQR \sim \Delta ABC$
(B) $\Delta PQR \sim \Delta CAB$
(C) $\Delta CBA \sim \Delta PQR$
(D) $\Delta BCA \sim \Delta PQR$

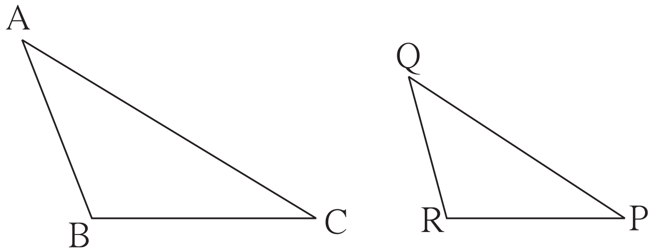


Fig. 1.67

(2) If in Δ DEF and Δ PQR, $\angle D \cong \angle Q$, $\angle R \cong \angle E$ then which of the following statements is false ?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$
(C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

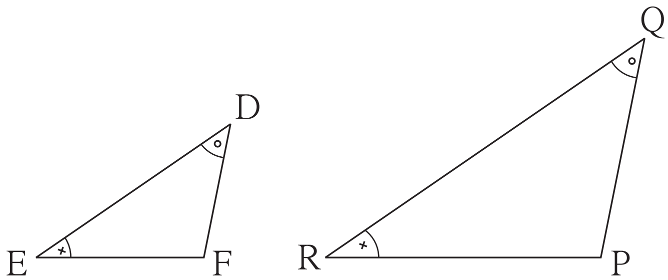


Fig. 1.68

(3) In Δ ABC and Δ DEF $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3DE then which of the statements regarding the two triangles is true ?

(A) The triangles are not congruent and not similar
(B) The triangles are similar but not congruent.
(C) The triangles are congruent and similar.
(D) None of the statements above is true.

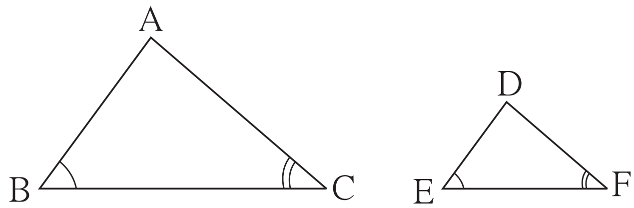


Fig. 1.69

(4) Δ ABC and Δ DEF are equilateral triangles, $A(\Delta ABC) : A(\Delta DEF) = 1:2$ If AB = 4 then what is length of DE ?

(A) $2\sqrt{2}$ (B) 4 (C) 8 (D) $4\sqrt{2}$

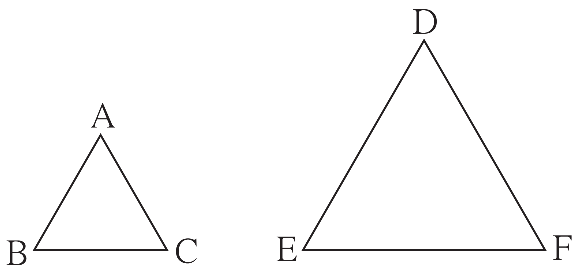


Fig. 1.70

(5) In figure 1.71, seg XY || seg BC, then which of the following statements is true?

(A) $\frac{AB}{AC} = \frac{AX}{AY}$ (B) $\frac{AX}{XB} = \frac{AY}{AC}$

(C) $\frac{AX}{YC} = \frac{AY}{XB}$ (D) $\frac{AB}{YC} = \frac{AC}{XB}$

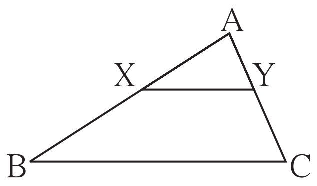


Fig. 1.71

2. In Δ ABC, B - D - C and BD = 7, BC = 20 then find following ratios.

(1) $\frac{A(\Delta ABD)}{A(\Delta ADC)}$

(2) $\frac{A(\Delta ABD)}{A(\Delta ABC)}$

(3) $\frac{A(\Delta ADC)}{A(\Delta ABC)}$

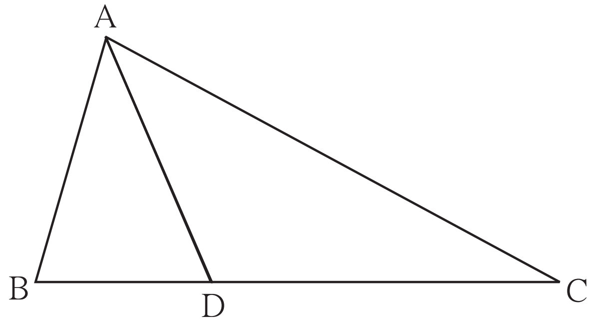


Fig. 1.72

3. Ratio of areas of two triangles with equal heights is 2 : 3. If base of the smaller triangle is 6 cm then what is the corresponding base of the bigger triangle ?

4.

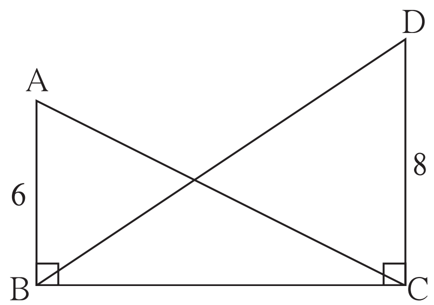


Fig. 1.73

In figure 1.73, $\angle ABC = \angle DCB = 90^\circ$

AB = 6, DC = 8

then $\frac{A(\Delta ABC)}{A(\Delta DCB)} = ?$

5. In figure 1.74, PM = 10 cm

$A(\Delta PQS)$ = 100 sq.cm

$A(\Delta QRS)$ = 110 sq.cm

then find NR.

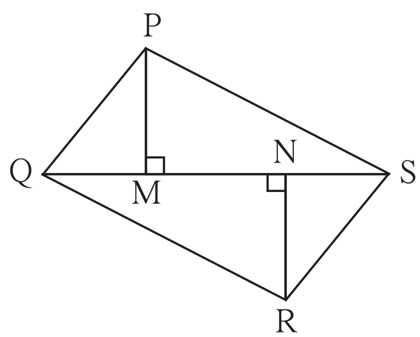


Fig. 1.74

6. Δ MNT ~ Δ QRS. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio $\frac{A(\Delta MNT)}{A(\Delta QRS)}$.

7. In figure 1.75, A–D–C and B–E–C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

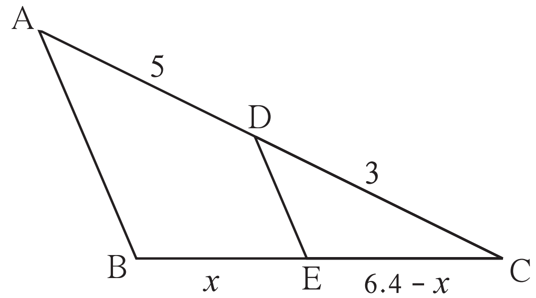


Fig. 1.75

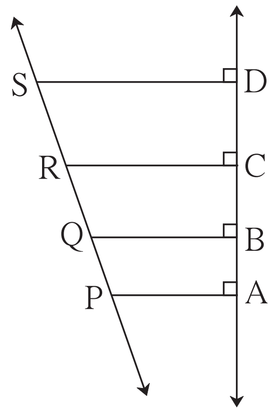


Fig. 1.76

8. In the figure 1.76, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD.

AB = 60, BC = 70, CD = 80, PS = 280 then find PQ, QR and RS.

9. In Δ PQR seg PM is a median. Angle bisectors of ∠PMQ and ∠PMR intersect side PQ and side PR in points X and Y respectively. Prove that XY || QR.

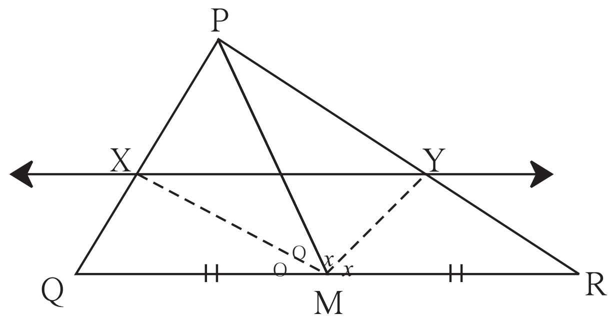


Fig. 1.77

Complete the proof by filling in the boxes.

In Δ PMQ, ray MX is bisector of ∠PMQ.

$\therefore \dfrac{\square}{\square} = \dfrac{\square}{\square}$ .... (I) theorem of angle bisector.

In Δ PMR, ray MY is bisector of ∠PMR.

$\therefore \dfrac{\square}{\square} = \dfrac{\square}{\square}$ .... (II) theorem of angle bisector.

But $\dfrac{MP}{MQ} = \dfrac{MP}{MR}$ .... M is the midpoint QR, hence MQ = MR.

$\therefore \dfrac{PX}{XQ} = \dfrac{PY}{YR}$

∴ XY || QR .... converse of basic proportionality theorem.

10. In fig 1.78, bisectors of $\angle B$ and $\angle C$ of Δ ABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\frac{AX}{XY}$.

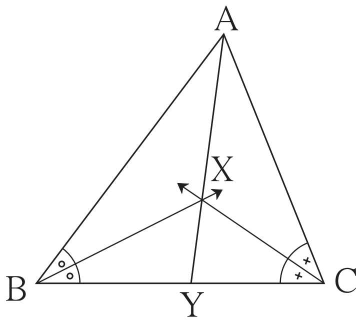


Fig. 1.78

11. In $\square$ ABCD, seg AD || seg BC. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$

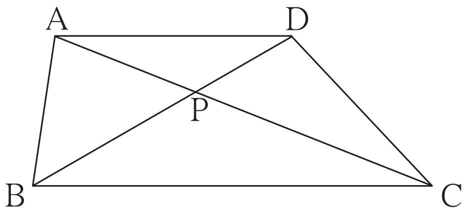


Fig. 1.79

12. In fig 1.80, XY || seg AC.
If 2AX = 3BX and XY = 9. Complete the activity to find the value of AC.

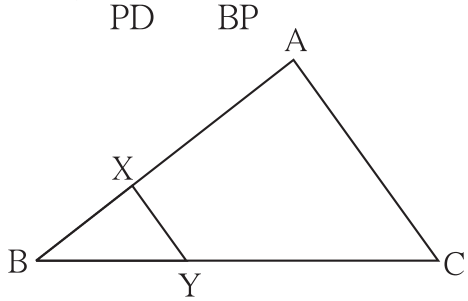


Fig. 1.80

Activity : 2AX = 3BX $\therefore \frac{AX}{BX} = \frac{\square}{\square}$

$\frac{AX+BX}{BX} = \frac{\square + \square}{\square}$ .... by componendo.

$\frac{AB}{BX} = \frac{\square}{\square}$ .... (I)

Δ BCA ~ Δ BYX .... $\square$ test of similarity.

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ .... corresponding sides of similar triangles.

$\therefore \frac{\square}{\square} = \frac{AC}{9}$ $\therefore$ AC = $\square$ ...from (I)

13*. In figure1.81, the vertices of square DEFG are on the sides of Δ ABC. $\angle A = 90°$. Then prove that $DE^2 = BD \times EC$
(Hint : Show that Δ GBD is similar to Δ CFE. Use GD = FE = DE.)

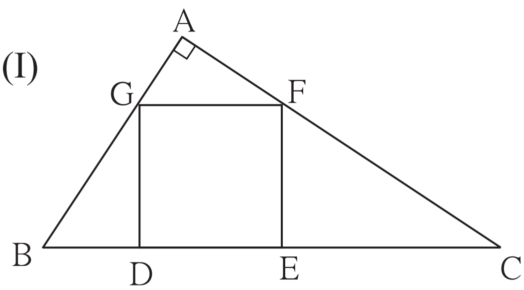


Fig. 1.81

2 Pythagoras Theorem

Let's study.

- Pythagorean triplet
- Theorem of geometric mean
- Application of Pythagoras theorem
- Similarity and right angled triangles
- Pythagoras theorem
- Apollonius theorem

Let's recall.

Pythagoras theorem :

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaning two sides.

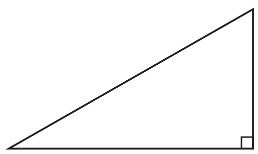


Fig. 2.1

In Δ PQR $\angle$ PQR = 90°

$l(PR)^2 = l(PQ)^2 + l(QR)^2$

We will write this as,

$PR^2 = PQ^2 + QR^2$

The lengths PQ, QR and PR of Δ PQR can also be shown by letters r, p and q. With this convention, refering to figure 2.1, Pythagoras theorem can also be stated as $q^2 = p^2 + r^2$.

Pythagorean Triplet :

In a triplet of natural numbers, if the square of the largest number is equal to the sum of the squares of the remaining two numbers then the triplet is called Pythagorean triplet.

For Example: In the triplet (11, 60, 61) ,

$11^2 = 121$, $60^2 = 3600$, $61^2 = 3721$ and $121 + 3600 = 3721$

The square of the largest number is equal to the sum of the squares of the other two numbers.

$\therefore$ 11, 60, 61 is a Pythagorean triplet.

Verify that (3, 4, 5), (5, 12, 13), (8, 15, 17), (24, 25, 7) are Pythagorean triplets.

Numbers in Pythagorean triplet can be written in any order.

For more information

Formula for Pythagorean triplet:

If a, b, c are natural numbers and $a > b$, then $[(a^2 + b^2),(a^2 - b^2),(2ab)]$ is Pythagorean triplet.

$\because\ (a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4$ .... (I)

$(a^2 - b^2) = a^4 - 2a^2b^2 + b^4$ .... (II)

$(2ab)^2 = 4a^2b^2$ .... (III)

$\therefore$ by (I), (II) and (III) , $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\therefore [(a^2 + b^2), (a^2 - b^2), (2ab)]$ is Pythagorean Triplet.

This formula can be used to get various Pythagorean triplets.

For example, if we take $a = 5$ and $b = 3$,

$a^2 + b^2 = 34$, $a^2 - b^2 = 16$, $2ab = 30$.

Check that (34, 16, 30) is a Pythagorean triplet.

Assign different values to a and b and obtain 5 Pythagorean triplet.

Last year we have studied the properties of right angled triangle with the angles $30° - 60° - 90°$ and $45° - 45° - 90°$.

(I) Property of $30°-60°-90°$ triangle.

If acute angles of a right angled triangle are $30°$ and $60°$, then the side opposite $30°$angle is half of the hypotenuse and the side opposite to $60°$ angle is $\frac{\sqrt{3}}{2}$ times the hypotenuse.

See figure 2.2. In Δ LMN, $\angle$ L = 30°, $\angle$ N = 60°, $\angle$ M = 90°

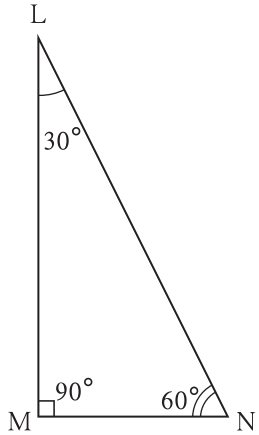


Fig. 2.2

$\therefore$ side opposite 30°angle = MN = $\frac{1}{2} \times$ LN

side opposite 60°angle = LM = $\frac{\sqrt{3}}{2} \times$ LN

If LN = 6 cm, we will find MN and LM.

MN = $\frac{1}{2} \times$ LN

$= \frac{1}{2} \times 6$

$= 3$ cm

LM = $\frac{\sqrt{3}}{2} \times$ LN

$= \frac{\sqrt{3}}{2} \times 6$

$= 3\sqrt{3}$ cm

(II) Property of $45^\circ-45^\circ-90^\circ$

If the acute angles of a right angled triangle are 45° and 45°, then each of the perpendicular sides is $\frac{1}{\sqrt{2}}$ times the hypotenuse.

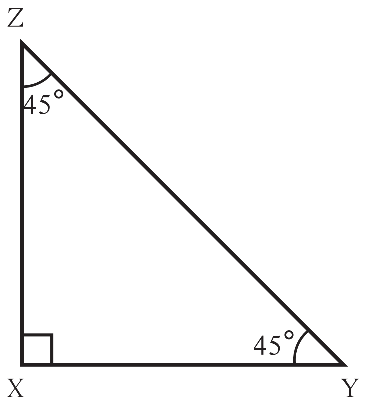


Fig. 2.3

See Figure 2.3. In Δ XYZ,

$$XY = \frac{1}{\sqrt{2}} \times ZY$$

$$XZ = \frac{1}{\sqrt{2}} \times ZY$$

$$\therefore \quad XY = XZ = \frac{1}{\sqrt{2}} \times ZY$$

If $ZY = 3\sqrt{2}$ cm then we will find XY and ZX

$$XY = XZ = \frac{1}{\sqrt{2}} \times 3\sqrt{2}$$

$$XY = XZ = 3\text{cm}$$

In 7th standard we have studied theorem of Pythagoras using areas of four right angled triangles and a square. We can prove the theorem by an alternative method.

Activity:

Take two congruent right angled triangles. Take another isosceles right angled triangle whose congruent sides are equal to the hypotenuse of the two congruent right angled triangles. Join these triangles to form a trapezium

Area of the trapezium = $\frac{1}{2}$ $\times$ (sum of the lengths of parallel sides) $\times$ height

Using this formula, equating the area of trapezium with the sum of areas of the three right angled triangles we can prove the theorem of Pythagoras.

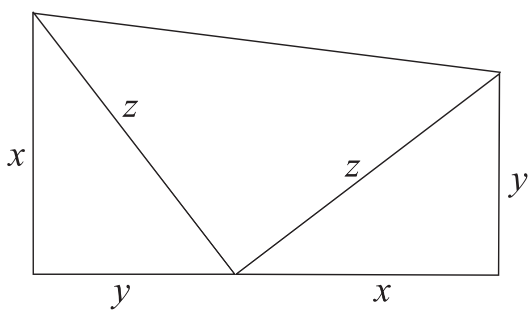


Fig. 2.4

Let's learn.

Now we will give the proof of Pythagoras theorem based on properties of similar triangles. For this, we will study right angled similar triangles.

Similarity and right angled triangle

Theorem : In a right angled triangle, if the altitude is drawn to the hypotenuse, then the two triangles formed are similar to the original triangle and to each other.

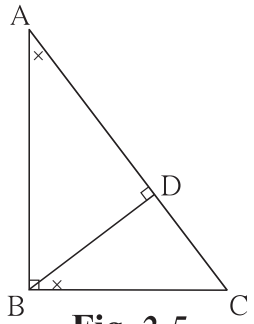


Fig. 2.5

Given : In Δ ABC, $\angle$ ABC = 90°,

seg BD $\perp$ seg AC, A-D-C

To prove: Δ ADB ~ Δ ABC

Δ BDC ~ Δ ABC

Δ ADB ~ Δ BDC

Proof : In Δ ADB and Δ ABC

$\angle$DAB $\cong$ $\angle$BAC ...(common angle)

$\angle$ ADB $\cong$ $\angle$ ABC ... (each 90°)

Δ ADB ~ Δ ABC ... (AA test)... (I)

In Δ BDC and Δ ABC

$\angle$ BCD $\cong$ $\angle$ ACB .....(common angle)

$\angle$ BDC $\cong$ $\angle$ ABC ..... (each 90°)

Δ BDC ~ Δ ABC ..... (AA test) ... (II)

$\therefore$ Δ ADB ~ Δ BDC from (I) and (II) ....(III)

$\therefore$ from (I), (II) and (III), Δ ADB ~ Δ BDC ~ Δ ABC ....(transitivity)

Theorem of geometric mean

In a right angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided.

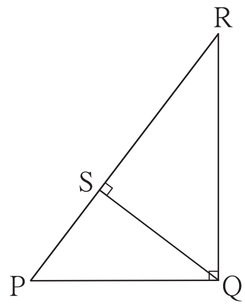


Fig. 2.6

Proof : In right angled triangle PQR, seg QS $\perp$ hypotenuse PR

Δ QSR ~ Δ PSQ .... (similarity of right triangles)

$$\frac{QS}{PS} = \frac{SR}{SQ}$$

$$\frac{QS}{PS} = \frac{SR}{QS}$$

QS^2 = PS × SR

$\therefore$ seg QS is the 'geometric mean' of seg PS and SR.

Pythagoras Theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.

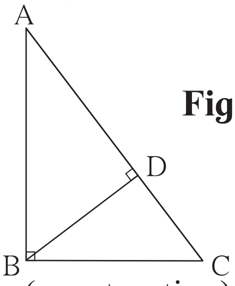


Fig. 2.7

Given : In Δ ABC, $\angle ABC = 90°$

To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw perpendicular seg BD on side AC.

A-D-C.

Proof : In right angled Δ ABC, seg BD $\perp$ hypotenuse AC ..... (construction)

$\therefore \Delta ABC \sim \Delta ADB \sim \Delta BDC$ ..... (similarity of right angled triangles)

$\Delta ABC \sim \Delta ADB$

$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB}$ - corresponding sides

$\frac{AB}{AD} = \frac{AC}{AB}$

$AB^2 = AD \times AC$ .... (I)

Similarly, $\Delta ABC \sim \Delta BDC$

$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC}$ -corresponding sides

$\frac{BC}{DC} = \frac{AC}{BC}$

$BC^2 = DC \times AC$ .... (II)

Adding (I) and (II)

$AB^2 + BC^2 = AD \times AC + DC \times AC$

$= AC (AD + DC)$

$= AC \times AC$ .... (A-D-C)

$\therefore AB^2 + BC^2 = AC^2$

$\therefore AC^2 = AB^2 + BC^2$

Converse of Pythagoras theorem

In a triangle if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle.

Given : In Δ ABC, $AC^2 = AB^2 + BC^2$

To prove : $\angle ABC = 90°$

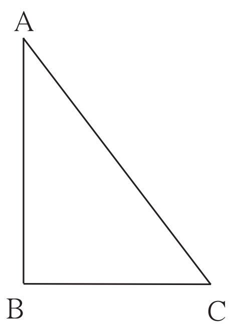


Fig. 2.8

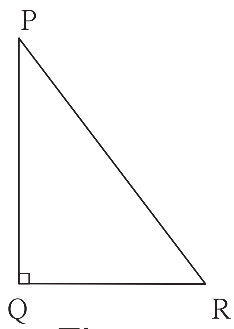


Fig. 2.9

Construction : Draw Δ PQR such that, AB = PQ, BC = QR , $\angle$ PQR = 90°.

Proof : In Δ PQR, $\angle$ Q = 90°

$PR^2 = PQ^2 + QR^2$ .... (Pythagoras theorem)

$= AB^2 + BC^2$ .... (construction) ......(I)

$= AC^2$ .... (given) ......(II)

$\therefore PR^2 = AC^2$

$\therefore$ PR = AC .... (III)

$\therefore \Delta ABC \cong \Delta PQR$ .... (SSS test)

$\therefore \angle ABC = \angle PQR = 90°$

Remember this !

(1) (a) Similarity and right angled triangle

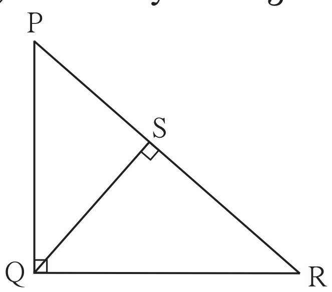


Fig. 2.10

In ΔPQR $\angle$ Q = 90° , seg QS $\perp$ seg PR, $\Delta PQR \sim \Delta PSQ \sim \Delta QSR$. Thus all the right angled triangles in the figure are similar to one another.

(b) Theorem of geometric mean

In the above figure, $\Delta PSQ \sim \Delta QSR$

$\therefore QS^2 = PS \times SR$

$\therefore$ seg QS is the geometric mean of seg PS and seg SR

(2) Pythagoras Theorem:

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.

(3) Converse of Pythagoras Theorem:

In a triangle, if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle

(4) Let us remember one more very useful property.

In a right angled triangle, if one side is half of the hypotenuse then the angle opposite to that side is 30°.

This property is the converse of 30°–60°–90° theorem.

Solved Examples

Ex. (1) See fig 2.11. In Δ ABC, $\angle$ B= 90°, $\angle$ A= 30°, AC=14, then find AB and BC

Solution :

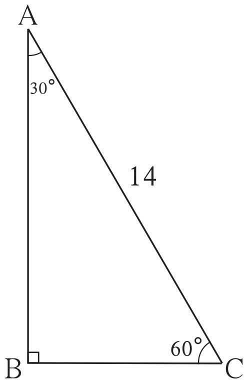


Fig. 2.11

In Δ ABC,

$\angle B = 90°$, $\angle A = 30°$, $\therefore \angle C = 60°$

By 30°– 60°– 90° theorem,

$BC = \frac{1}{2} \times AC$

$BC = \frac{1}{2} \times 14$

$BC = 7$

$AB = \frac{\sqrt{3}}{2} \times AC$

$AB = \frac{\sqrt{3}}{2} \times 14$

$AB = 7\sqrt{3}$

Ex. (2) See fig 2.12, In Δ ABC, seg AD $\perp$ seg BC, $\angle C = 45°$, BD = 5 and AC = $8\sqrt{2}$ then find AD and BC.

Solution : In Δ ADC

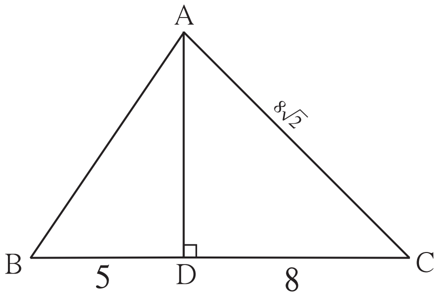


Fig. 2.12

$\angle ADC = 90°$, $\angle C = 45°$, $\therefore \angle DAC = 45°$

$AD = DC = \frac{1}{\sqrt{2}} \times 8\sqrt{2}$ ... by 45°–45°–90° theorem

$DC = 8 \quad \therefore AD = 8$

$BC = BD + DC$

$= 5 + 8$

$BC = 13$

Ex. (3) In fig 2.13, $\angle$ PQR = 90°,seg QN $\perp$ seg PR, PN = 9, NR = 16. Find QN.

Solution : In Δ PQR, seg QN $\perp$ seg PR

$NQ^2 = PN \times NR$ ... theorem of geometric mean

$\therefore NQ = \sqrt{PN \times NR}$

$= \sqrt{9 \times 16}$

$= 3 \times 4$

$= 12$

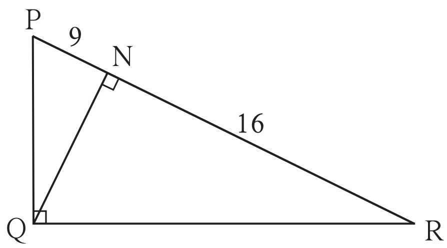


Fig. 2.13

Ex. (4) See figure 2.14.In Δ PQR, $\angle$ PQR = 90°, seg QS $\perp$ seg PR then find x, y, z.

Solution : In Δ PQR, $\angle$ PQR = 90°, seg QS $\perp$ seg PR

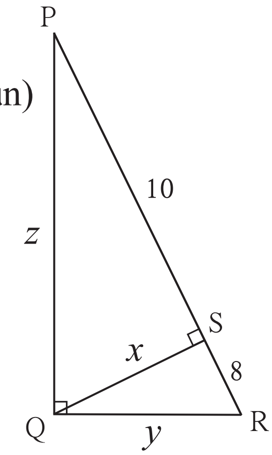


Fig. 2.14

$QS = \sqrt{PS \times SR}$ .... (theorem of geometric mean)

$= \sqrt{10 \times 8}$

$= \sqrt{5 \times 2 \times 8}$

$= \sqrt{5 \times 16}$

$= 4\sqrt{5}$

$\therefore \; x = 4\sqrt{5}$

In Δ QSR, by Pythagoras theorem

$QR^2 = QS^2 + SR^2$

$= \left(4\sqrt{5}\right)^2 + 8^2$

$= 16 \times 5 + 64$

$= 80 + 64$

$= 144$

$\therefore$ QR = 12

In Δ PSQ, by Pythagoras theorem

$PQ^2 = QS^2 + PS^2$

$= \left(4\sqrt{5}\right)^2 + 10^2$

$= 16 \times 5 + 100$

$= 80 + 100$

$= 180$

$= 36 \times 5$

$\therefore$ PQ = $6\sqrt{5}$

Hence $x = 4\sqrt{5}$, $y = 12$, $z = 6\sqrt{5}$

Ex. (5) In the right angled triangle, sides making right angle are 9 cm and 12 cm. Find the length of the hypotenuse

Solution: In Δ PQR ,$\angle$ Q = 90°

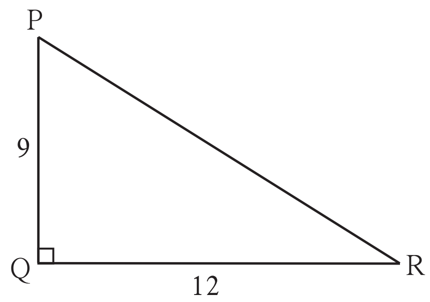


Fig. 2.15

$PR^2 = PQ^2 + QR^2$ (Pythagoras theorem)

$= 9^2 + 12^2$

$= 81 + 144$

$PR^2 = 225$

PR = 15

Hypotenuse= 15 cm

Ex. (6) In Δ LMN , $l = 5$, $m = 13$, $n = 12$. State whether ΔLMN is a right angled triangle or not.

Solution : $l = 5$, $m = 13$, $n = 12$

$l^2 = 25$, $m^2 = 169$, $n^2 = 144$

$\therefore m^2 = l^2 + n^2$

$\therefore$ by converse of Pythagoras theorem Δ LMN is a right angled triangle.

Ex. (7) See fig 2.16. In Δ ABC, seg AD $\perp$ seg BC. Prove that:

$AB^2 + CD^2 = BD^2 + AC^2$

Solution : According to Pythagoras theorem, in Δ ADC

$AC^2 = AD^2 + CD^2$

$\therefore AD^2 = AC^2 - CD^2$ ... (I)

In Δ ADB

$AB^2 = AD^2 + BD^2$

$\therefore AD^2 = AB^2 - BD^2$ ... (II)

$\therefore AB^2 - BD^2 = AC^2 - CD^2$ ....from I and II

$\therefore AB^2 + CD^2 = AC^2 + BD^2$

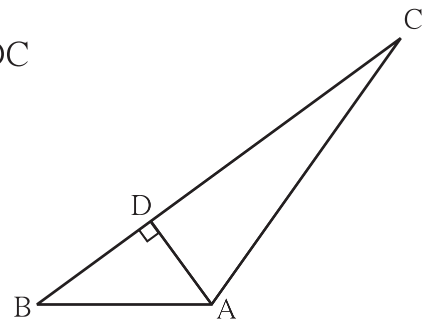


Fig. 2.16

Practice set 2.1

1. Identify, with reason, which of the following are Pythagorean triplets.

(i)(3, 5, 4) (ii)(4, 9, 12) (iii)(5, 12, 13)

(iv) (24, 70, 74) (v)(10, 24, 27) (vi)(11, 60, 61)

2. In figure 2.17, $\angle$ MNP = 90°, seg NQ $\perp$seg MP, MQ = 9, QP = 4, find NQ.

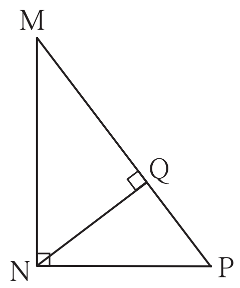


Fig. 2.17

3. In figure 2.18, $\angle$ QPR = 90°, seg PM $\perp$ seg QR and Q-M-R, PM = 10, QM = 8, find QR.

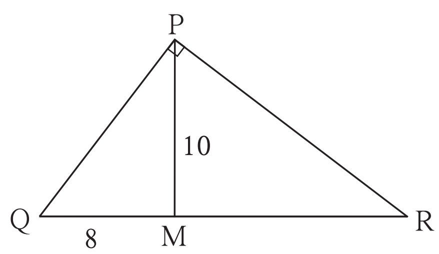


Fig. 2.18

4. See figure 2.19. Find RP and PS using the information given in Δ PSR.

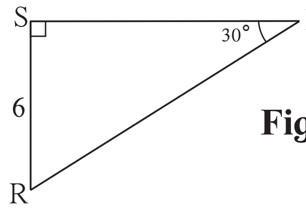


Fig. 2.19

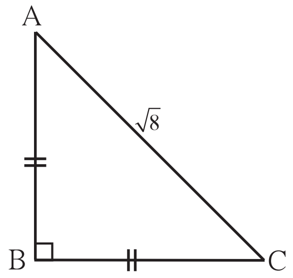


Fig. 2.20

5. For finding AB and BC with the help of information given in figure 2.20, complete following activity.

AB = BC .... $\boxed{}$

$\therefore$ $\angle$ BAC = $\boxed{}$

$\therefore$ AB = BC = $\boxed{}$ $\times$ AC

= $\boxed{}$ $\times$ $\sqrt{8}$

= $\boxed{}$ $\times$ $2\sqrt{2}$

= $\boxed{}$

6. Find the side and perimeter of a square whose diagonal is 10 cm.

7. In figure 2.21, $\angle$ DFE = 90°, FG $\perp$ ED, If GD = 8, FG = 12, find (1) EG (2) FD and (3) EF

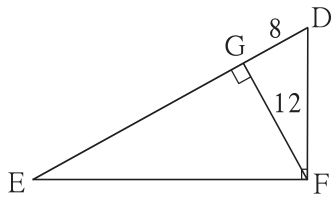


Fig. 2.21

8. Find the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

9★. In the figure 2.22, M is the midpoint of QR. $\angle$ PRQ = 90°. Prove that, $PQ^2 = 4PM^2 - 3PR^2$

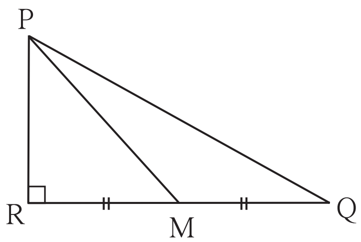


Fig. 2.22

10★. Walls of two buildings on either side of a street are parellel to each other. A ladder 5.8 m long is placed on the street such that its top just reaches the window of a building at the height of 4 m. On turning the ladder over to the other side of the street , its top touches the window of the other building at a height 4.2 m. Find the width of the street.

Let's learn.

Application of Pythagoras theorem

In Pythagoras theorem, the relation between hypotenuse and sides making right angle i.e. the relation between side opposite to right angle and the remaining two sides is given.

In a triangle, relation between the side opposite to acute angle and remaining two sides and relation of the side opposite to obtuse angle with remaining two sides can be determined with the help of Pythagoras theorem. Study these relations from the following examples.

Ex. (1) In Δ ABC, $\angle$ C is an acute angle, seg AD $\perp$ seg BC. Prove that:

$AB^2 = BC^2 + AC^2 - 2BC \times DC$

In the given figure let AB = c, AC = b, AD = p, BC = a, DC = x,

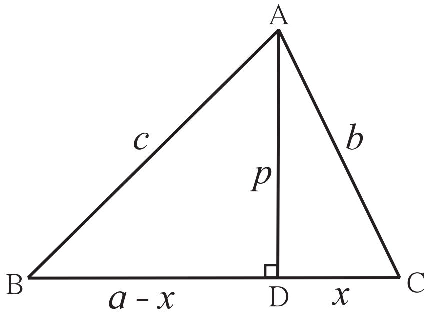


Fig. 2.23

$\therefore$ BD = $a - x$

In Δ ADB , by Pythagoras theorem

$c^2 = (a-x)^2 + \square$

$c^2 = a^2 - 2ax + x^2 + \square$ .... (I)

In Δ ADC, by Pythagoras theorem

$b^2 = p^2 + \square$

$p^2 = b^2 - \square$ .... (II)

Substituting value of p^2 from (II) in (I),

$c^2 = a^2 - 2ax + x^2 + b^2 - x^2$

$\therefore c^2 = a^2 + b^2 - 2ax$

$\therefore AB^2 = BC^2 + AC^2 - 2BC \times DC$

Ex. (2) In Δ ABC, $\angle$ ACB is obtuse angle, seg AD $\perp$ seg BC. Prove that:

$AB^2 = BC^2 + AC^2 + 2BC \times CD$

In the figure seg AD $\perp$ seg BC

Let AD = p, AC = b, AB = c,

BC = a and DC = x.

DB = $a + x$

In Δ ADB, by Pythagoras theorem,

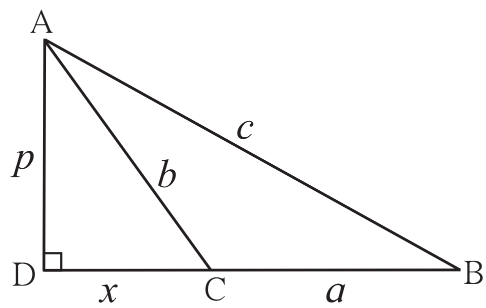


Fig. 2.24

$c^2 = (a + x)^2 + p^2$

$c^2 = a^2 + 2ax + x^2 + p^2$ .... (I)

Similarly, in Δ ADC

$b^2 = x^2 + p^2$

$\therefore \quad p^2 = b^2 - x^2$ .... (II)

$\therefore$ substituting the value of p^2 from (II) in (I)

$\therefore \quad c^2 = a^2 + 2ax + b^2$

$\therefore$ $AB^2 = BC^2 + AC^2 + 2BC \times CD$

Apollonius theorem

In Δ ABC, if M is the midpoint of side BC, then $AB^2 + AC^2 = 2AM^2 + 2BM^2$

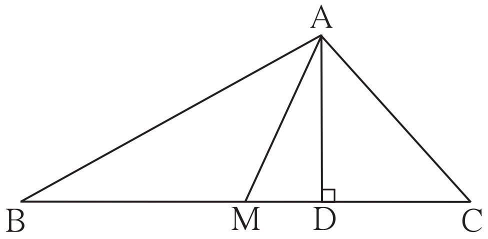


Fig. 2.25

Given : In Δ ABC, M is the midpoint of side BC.

To prove : $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Construction: Draw seg AD $\perp$ seg BC

Proof : If seg AM is not perpendicular to seg BC then out of $\angle$AMB and $\angle$AMC one is obtuse angle and the other is acute angle

In the figure, $\angle$AMB is obtuse angle and $\angle$AMC is acute angle.

From examples (1) and (2) above,

$AB^2 = AM^2 + MB^2 + 2BM \times MD$ ..... (I)

and $AC^2 = AM^2 + MC^2 - 2MC \times MD$

$\therefore$ $AC^2 = AM^2 + MB^2 - 2BM \times MD$ ($\because$ BM = MC) ....(II)

$\therefore$ adding (I) and (II)

$AB^2 + AC^2 = 2AM^2 + 2BM^2$

Write the proof yourself if seg AM $\perp$ seg BC.

From this example we can see the relation among the sides and medians of a triangle.

This is known as Apollonius theorem.

Solved Examples

Ex. (1) In the figure 2.26, seg PM is a median of ΔPQR. PM = 9 and $PQ^2 + PR^2 = 290$, then find QR.

Solution : In Δ PQR , seg PM is a median.

M is the midpoint of seg QR .

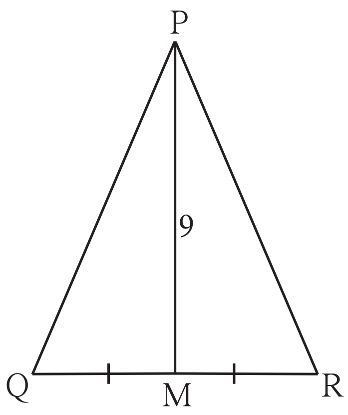


Fig. 2.26

$$QM = MR = \frac{1}{2}QR$$

$PQ^2 + PR^2 = 2PM^2 + 2QM^2$ (by Apollonius theorem)

$$290 = 2 \times 9^2 + 2QM^2$$
$$290 = 2 \times 81 + 2QM^2$$
$$290 = 162 + 2QM^2$$
$$2QM^2 = 290 - 162$$
$$2QM^2 = 128$$
$$QM^2 = 64$$
$$QM = 8$$
$$\therefore \quad QR = 2 \times QM$$
$$= 2 \times 8$$
$$= 16$$

Ex. (2) Prove that, the sum of the squares of the diagonals of a rhombus is equal to the sum of the squares of the sides.

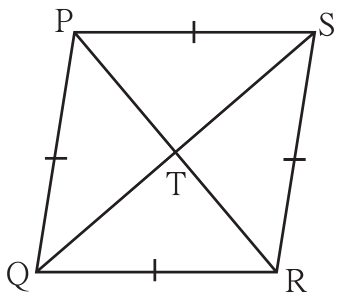


Fig. 2.27

Given : □PQRS is a rhombus. Diagonals PR and SQ intersect each other at point T

To prove : $PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2$

Proof : Diagonals of a rhombus bisect each other .

$\therefore$ by Apollonius' theorem,

$PQ^2 + PS^2 = 2PT^2 + 2QT^2$ .... (I)

$QR^2 + SR^2 = 2RT^2 + 2QT^2$ .... (II)

$\therefore$ adding (I) and (II) ,

$PQ^2 + PS^2 + QR^2 + SR^2 = 2(PT^2 + RT^2) + 4QT^2$

$= 2(PT^2 + PT^2) + 4QT^2$ .... (RT = PT)

$= 4PT^2 + 4QT^2$

$= (2PT)^2 + (2QT)^2$

$= PR^2 + QS^2$

(The above proof can be written using Pythagoras theorem also.)

Practice set 2.2

1. In Δ PQR, point S is the midpoint of side QR.If PQ = 11,PR = 17, PS =13, find QR.

2. In Δ ABC, AB = 10, AC = 7, BC = 9 then find the length of the median drawn from point C to side AB

3. In the figure 2.28 seg PS is the median of Δ PQR and PT $\perp$ QR.
Prove that,

(1) $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

ii) $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$

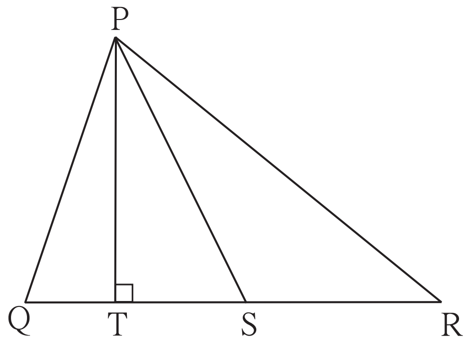


Fig. 2.28

4. In Δ ABC, point M is the midpoint of side BC.
If, $AB^2 + AC^2 = 290 \text{ cm}^2$,
AM = 8 cm, find BC.

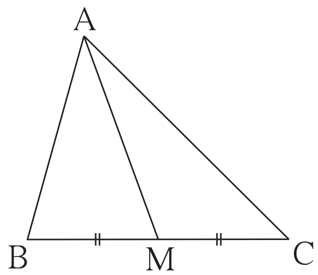


Fig. 2.29

5★. In figure 2.30, point T is in the interior of rectangle PQRS,
Prove that, $TS^2 + TQ^2 = TP^2 + TR^2$
(As shown in the figure, draw seg AB || side SR and A-T-B)

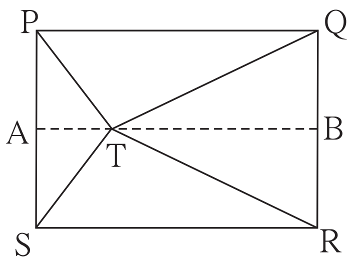


Fig. 2.30

Problem set 2

1. Some questions and their alternative answers are given. Select the correct alternative.

(1) Out of the following which is the Pythagorean triplet?

(A) (1, 5, 10) (B) (3, 4, 5) (C) (2, 2, 2) (D) (5, 5, 2)

(2) In a right angled triangle, if sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?

(A) 15 (B) 13 (C) 5 (D) 12

(3) Out of the dates given below which date constitutes a Pythagorean triplet ?

(A) 15/08/17 (B) 16/08/16 (C) 3/5/17 (D) 4/9/15

(4) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle.

(A) Obtuse angled triangle (B) Acute angled triangle

(C) Right angled triangle (D) Equilateral triangle

(5) Find perimeter of a square if its diagonal is $10\sqrt{2}$ cm.

(A) 10 cm (B) $40\sqrt{2}$ cm (C) 20 cm (D) 40 cm

(6) Altitude on the hypotenuse of a right angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.

(A) 9 cm (B) 4 cm (C) 6 cm (D) $2\sqrt{6}$ cm

(7) Height and base of a right angled triangle are 24 cm and 18 cm find the length of its hypotenus

(A) 24 cm (B) 30 cm (C) 15 cm (D) 18 cm

(8) In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm, BC = 6 cm. Find measure of $\angle$A.

(A) 30° (B) 60° (C) 90° (D) 45°

2. Solve the following examples.

(1) Find the height of an equilateral triangle having side $2a$.

(2) Do sides 7 cm , 24 cm, 25 cm form a right angled triangle ? Give reason.

(3) Find the length a diagonal of a rectangle having sides 11 cm and 60cm.

(4) Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm.

(5) A side of an isosceles right angled triangle is x. Find its hypotenuse.

(6) In Δ PQR; PQ = $\sqrt{8}$, QR = $\sqrt{5}$, PR = $\sqrt{3}$. Is Δ PQR a right angled triangle ? If yes, which angle is of 90° ?

3. In Δ RST, $\angle$ S = 90°, $\angle$ T = 30°, RT = 12 cm then find RS and ST.

4. Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm.

5★. Find the length of the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.

6. In Δ ABC seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$ Find AP.

7*. Δ ABC is an equilateral triangle. Point P is on base BC such that PC = $\frac{1}{3}$ BC, if AB = 6 cm find AP.

8. From the information given in the figure 2.31, prove that
PM = PN = $\sqrt{3} \times a$

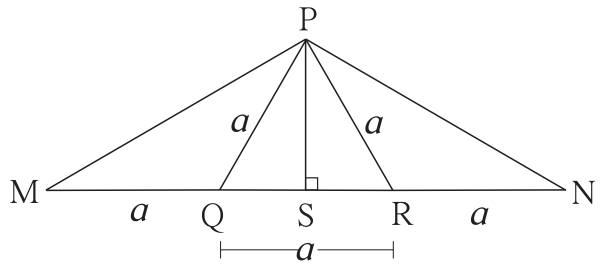


Fig. 2.31

9. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

10. Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was $15\sqrt{2}$ km. Find their speed per hour.

11*. In Δ ABC, $\angle$ BAC = 90°,
seg BL and seg CM are medians of Δ ABC. Then prove that:
$4(BL^2 + CM^2) = 5\,BC^2$

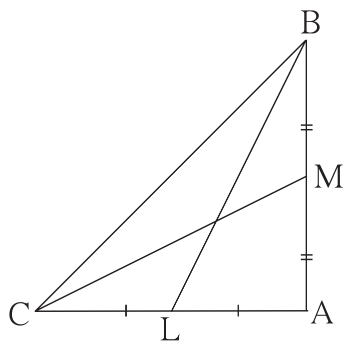


Fig. 2.32

12. Sum of the squares of adjacent sides of a parallelogram is 130 sq.cm and length of one of its diagonals is14 cm. Find the length of the other diagonal.

13. In Δ ABC, seg AD $\perp$ seg BC
DB = 3CD. Prove that :
$2AB^2 = 2AC^2 + BC^2$

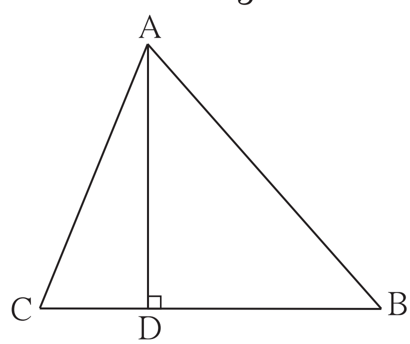


Fig. 2.33

14*. In an isosceles triangle, length of the congruent sides is 13 cm and its base is 10 cm. Find the distance between the vertex opposite the base and the centroid.

15. In a trapezium ABCD,
seg AB || seg DC
seg BD ⊥ seg AD,
seg AC ⊥ seg BC,
If AD = 15, BC = 15
and AB = 25. Find A(□ ABCD)

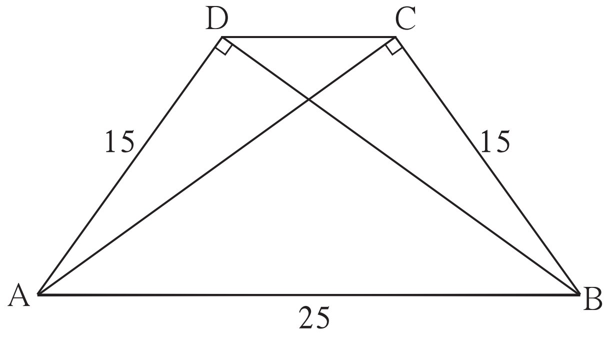


Fig. 2.34

16★. In the figure 2.35, Δ PQR is an equilatral triangle. Point S is on seg QR such that
$QS = \frac{1}{3} QR$.
Prove that : $9\ PS^2 = 7\ PQ^2$

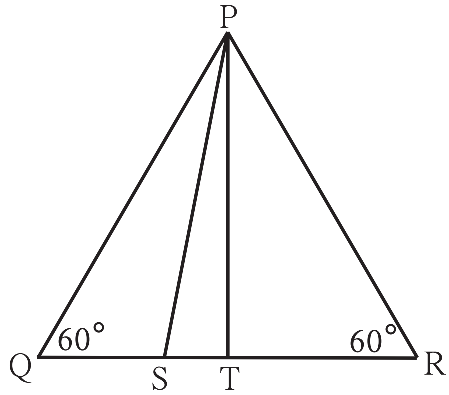


Fig. 2.35

17★. Seg PM is a median of Δ PQR. If PQ = 40, PR = 42 and PM = 29, find QR.

18. Seg AM is a median of Δ ABC. If AB = 22, AC = 34, BC = 24, find AM

ICT Tools or Links

Obtain information on 'the life of Pythagoras' from the internet. Prepare a slide show.

PU5EHH

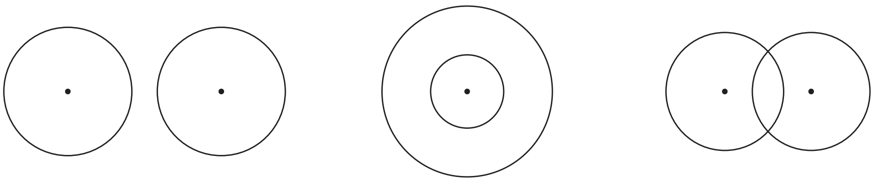
3 Circle

Let's study.

- Circles passing through one, two,three points
- Circles touching each other
- Inscribed angle and intercepted arc
- Secant tangent angle theorem
- Secant and tangent
- Arc of a circle
- Cyclic quadrilateral
- Theorem of intersecting chords

Let's recall.

You are familiar with the concepts regarding circle, like - centre, radius, diameter, chord, interior and exterior of a circle. Also recall the meanings of - congruent circles, concentric circles and intersecting circles.



congruent circles concentric circles intersecting circles

Recall the properties of chord studied in previous standard and perform the activity below.

Activity I : In the adjoining figure, seg DE is a chord of a circle with centre C. seg CF ⊥ seg DE. If diameter of the circle is 20 cm, DE =16 cm find CF.

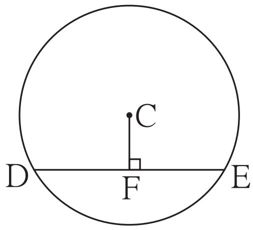


Fig. 3.1

Recall and write theorems and properties which are useful to find the solution of the above problem.

(1) The perpendicular drawn from centre to a chord ________________

(2) ________________

(3) ________________

Using these properties, solve the above problem.

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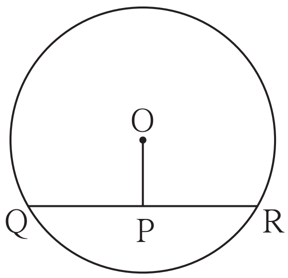

Fig. 3.2

Activity II : In the adjoining figure, seg QR is a chord of the circle with centre O. P is the midpoint of the chord QR. If QR = 24, OP = 10 , find radius of the circle.

To find solution of the problem, write the theorems that are useful.

(1) ____________________

(2) ____________________

Using these theorems solve the problems.

Activity III : In the adjoining figure, M is the centre of the circle and seg AB is a diameter.

seg MS ⊥ chord AD

seg MT ⊥ chord AC

∠DAB ≅ ∠CAB.

Prove that : chord AD ≅ chord AC.

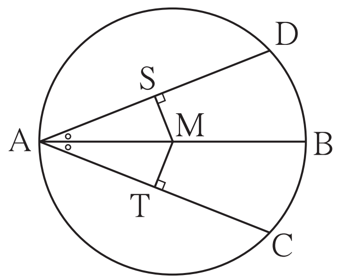

Fig. 3.3

To solve this problem which of the following theorems will you use ?

(1) The chords which are equidistant from the centre are equal in length.

(2) Congruent chords of a circle are equidistant from the centre.

Which of the following tests of congruence of triangles will be useful?

(1) SAS, (2) ASA, (3) SSS, (4) AAS, (5) hypotenuse-side test.

Using appropriate test and theorem write the proof of the above example.

Let's learn.

Circles passing through one, two, three points

In the adjoining figure, point A lies in a plane. All the three circles with centres P, Q, R pass through point A. How many more such circles may pass through point A?

If your answer is many or innumerable, it is correct.

Infinite number of circles pass through a point.

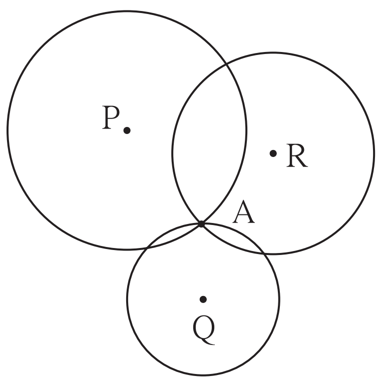

Fig. 3.4

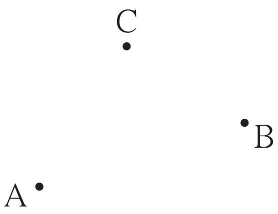

Fig. 3.5

In the adjoining figure, how many circles pass through points A and B?

How many circles contain all the three points A, B, C?

Perform the activity given below and try to find the answer.

Activity I: Draw segment AB. Draw perpendicular bisector l of the segment AB. Take point P on the line l as centre, PA as radius and draw a circle. Observe that the circle passes through point B also. Find the reason. (Recall the property of perpendicular bisector of a segment.)

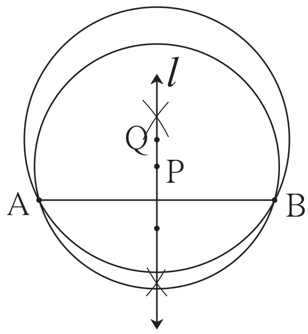

Fig. 3.6

Taking any other point Q on the line l, if a circle is drawn with centre Q and radius QA, will it pass through B ? Think.

How many such circles can be drawn, passing through A and B ? Where will their centres lie?

Activity II : Take any three non-collinear points. What should be done to draw a circle passing through all these points ? Draw a circle passing through these points.
Is it possible to draw one more circle passing through these three points ? Think of it.

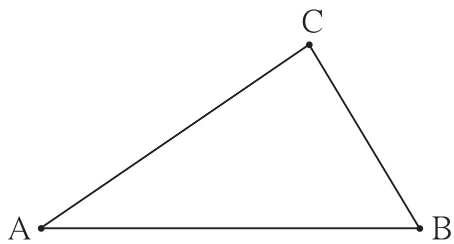

Fig. 3.7

Activity III : Take 3 collinear points D, E, F. Try to draw a circle passing through these points. If you are not able to draw a circle, think of the reason.

Let's recall.

(1) Infinite circles pass through one point.

(2) Infinite circles pass through two distinct points.

(3) There is a unique circle passing through three non-collinear points.

(4) No circle can pass through 3 collinear points.

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Let's learn.

Secant and tangent

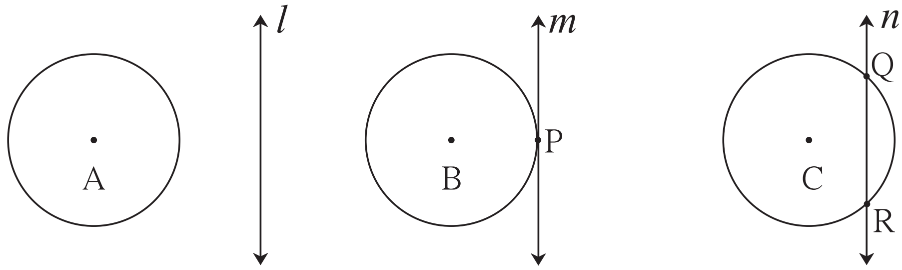


Fig. 3.8

In the figure above, not a single point is common in line *l* and circle with centre A.

Point P is common to both, line *m* and circle with centre B. Here, line *m* is called *a tangent* of the circle and point P is called the point of contact.

Two points Q and R are common to both, the line *n* and the circle with centre C.

Q and R are intersecting points of line *n* and the circle. Line *n* is called *a secant* of the circle .

Let us understand an important property of a tangent from the following activity.

Activity :

Draw a sufficiently large circle with centre O. Draw radius OP. Draw a line AB ⊥ seg OP. It intersects the circle at points A, B. Imagine the line slides towards point P such that all the time it remains parallel to its original position. Obviously, while the line slides, points A and B approach each other along the circle. At the end, they get merged in point P, but the angle between the radius OP and line AB will remain a right angle.

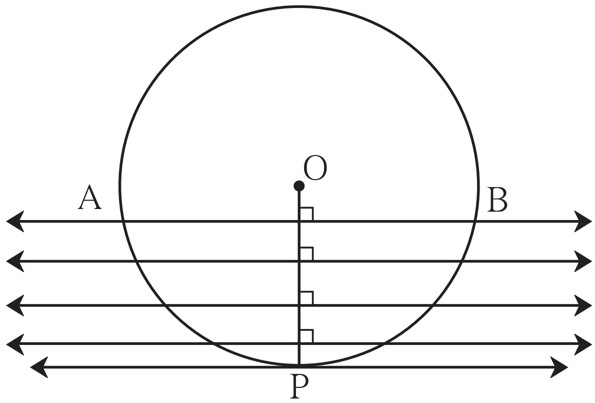


Fig. 3.9

At this stage the line AB becomes a tangent of the circle at P.

So it is clear that, the tangent at any point of a circle is perpendicular to the radius at that point.

This property is known as 'tangent theorem'.

Tangent theorem

Theorem : A tangent at any point of a circle is perpendicular to the radius at the point of contact.

There is an indirect proof of this theorem.

For more information

Given : Line l is a tangent to the circle with centre O at the point of contact A.

To prove : line $l \perp$ radius OA.

Proof : Assume that, line l is not perpendicular to seg OA.

Suppose, seg OB is drawn perpendicular to line l.

Of course B is not same as A.

Now take a point C on line l such that A–B–C and BA = BC .

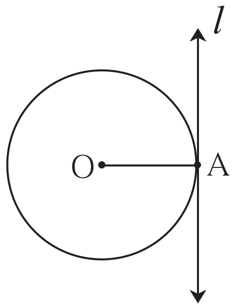


Fig. 3.10

Now in, Δ OBC and Δ OBA

seg BC $\cong$ seg BA .... (construction)

$\angle$ OBC $\cong$ $\angle$ OBA .... (each right angle)

seg OB $\cong$ seg OB

$\therefore$ Δ OBC $\cong$ Δ OBA .... (SAS test)

$\therefore$ OC = OA

But seg OA is a radius.

$\therefore$ seg OC must also be radius.

$\therefore$ C lies on the circle.

That means line l intersects the circle in two distinct points A and C.

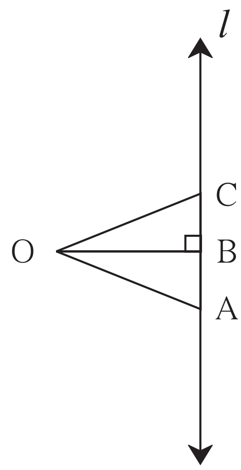


Fig. 3.11

But line l is a tangent. .... (given)

$\therefore$ it intersects the circle in only one point.

Our assumption that line l is not perpendicular to radius OA is wrong.

$\therefore$ line $l \perp$ radius OA.

Let's recall.

Which theorems do we use in proving that hypotenuse is the longest side of a right angled triangle?

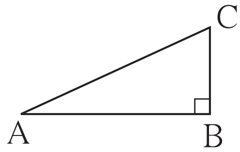


Fig. 3.12

Let's learn.

Converse of tangent theorem

Theorem: A line perpendicular to a radius at its point on the circle is a tangent to the circle.

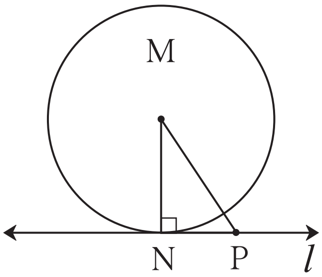


Fig. 3.13

Given : M is the centre of a circle seg MN is a radius.
Line $l \perp$ seg MN at N.

To prove : Line l is a tangent to the circle.

Proof : Take any point P, other that N, on the line l. Draw seg MP.

Now in Δ MNP, $\angle$ N is a right angle.

$\therefore$ seg MP is the hypotenuse.

$\therefore$ seg MP > seg MN.

As seg MN is radius, point P can't be on the circle.

$\therefore$ no other point, except point N, of line l is on the circle.

$\therefore$ line l intersects the circle in only one point N.

$\therefore$ line l is a tangent to the circle.

Let's discuss.

In figure 3.14 , B is a point on the circle with centre A. The tangent of the circle passing through B is to be drawn. There are infinite lines passing through the point B. Which of them will be the tangent ? Can the number of tangents through B be more than one ?

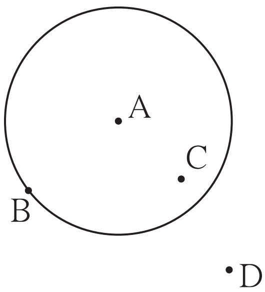


Fig. 3.14

Point C lies in the interior of the circle. Can you draw tangents to the circle through C ?

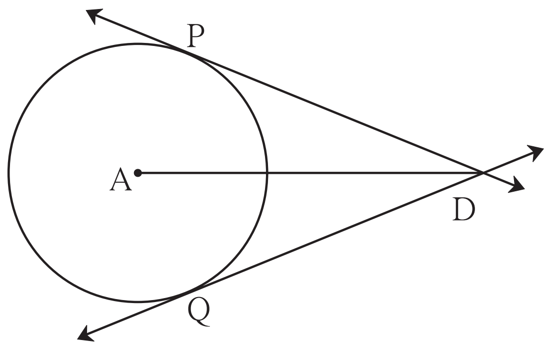


Fig. 3.15

Point D is in the exterior of the circle. Can you draw a tangent to the circle through D? If yes, how many such tangents are possible? From the discussion you must have understood that two tangents can be drawn to a circle from the point outside the circle as shown in the figure.

In the adjoing figure line DP and line DQ. touch the circle at points P and Q. Seg DP and seg DQ are called tangent segments.

Tangent segment theorem

Theorem : Tangent segments drawn from an external point to a circle are congruent.

Observe the adjoining figure. Write 'given' and 'to prove.'

Draw radius AP and radius AQ and complete the following proof of the theorem.

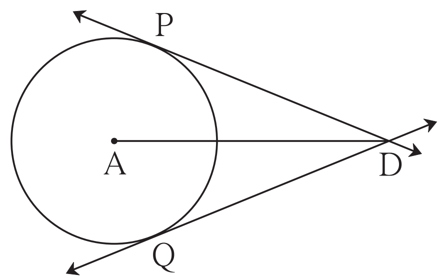


Fig. 3.16

Proof : In Δ PAD and Δ QAD,

seg PA $\cong$ ________ radii of the same circle.

seg AD $\cong$ seg AD ____________

$\angle APD = \angle AQD = 90^\circ$ ...... tangent theorem

$\therefore$ Δ PAD $\cong$ Δ QAD ____________

$\therefore$ seg DP $\cong$ seg DQ ____________

Solved Examples

Ex. (1) In the adjoining figure circle with centre D touches the sides of $\angle$ ACB at A and B. If $\angle ACB = 52^\circ$, find measure of $\angle$ ADB.

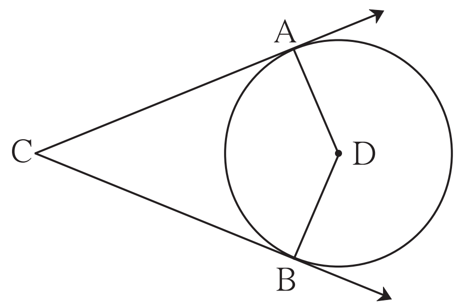


Fig. 3.17

Solution : The sum of all angles of a quadrilateral is 360°.

$\therefore \angle ACB + \angle CAD + \angle CBD + \angle ADB = 360^\circ$

$\therefore 52^\circ + 90^\circ + 90^\circ + \angle ADB = 360^\circ$ .... Tangent theorem

$\therefore \angle ADB + 232^\circ = 360^\circ$

$\therefore \angle ADB = 360^\circ - 232^\circ = 128^\circ$

Eg. (2) Point O is the centre of a circle. Line a and line b are parallel tangents to the circle at P and Q. Prove that segment PQ is a diameter of the circle.

Solution : Draw a line c through O which is parallel to line a. Draw radii OQ and OP.

Now, $\angle$ OPT = 90° ..... Tangent theorem

$\therefore \angle$SOP = 90°... Int. angle property ... (I)

line a || line c ..... construction

line a || line b ..... given

$\therefore$ line b || line c

$\therefore \angle$SOQ = 90°... Int. angle property ...(II)

$\therefore$ From (I) and (II),

$\angle$ SOP + $\angle$ SOQ = 90° + 90° = 180°

$\therefore$ ray OP and ray OQ are opposite rays.

$\therefore$ P, O, Q are collinear points.

$\therefore$ seg PQ is a diameter of the circle.

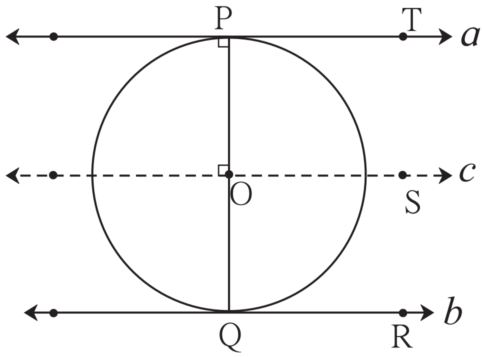
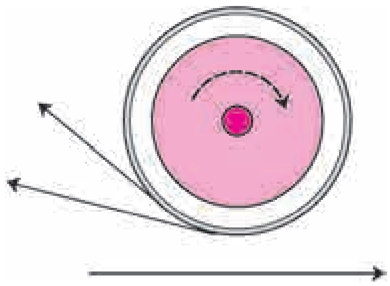


Fig. 3.18

When a motor cycle runs on a wet road in rainy season, you may have seen water splashing from its wheels. Those splashes are like tangents of the circle of the wheel. Find out the reason from your science teacher.

Observe the splinters escaping from a splintering wheel in Diwali fire works and while sharpning a knife. Do they also look like tangents ?



Direction of running wheel

Remember this !

(1) Tangent theorem : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(2) A line perpendicular to a radius at its point on the circle, is a tangent to the circle.

(3) Tangent segments drawn from an external point to a circle are congruent.

Practice set 3.1

1. In the adjoining figure the radius of a circle with centre C is 6 cm, line AB is a tangent at A. Answer the following questions.

(1) What is the measure of ∠CAB ? Why ?

(2) What is the distance of point C from line AB? Why ?

(3) d(A,B)= 6 cm, find d(B,C).

(4) What is the measure of ∠ABC ? Why ?

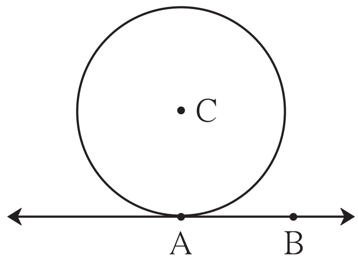


Fig. 3.19

2. In the adjoining figure, O is the centre of the circle. From point R, seg RM and seg RN are tangent segments touching the circle at M and N. If (OR) = 10 cm and radius of the circle = 5 cm, then

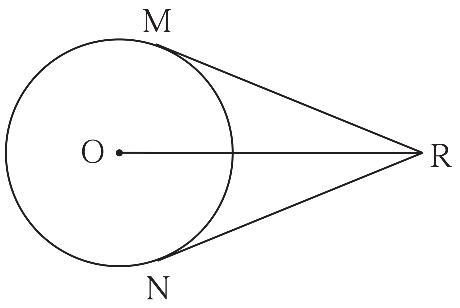


Fig. 3.20

(1) What is the length of each tangent segment ?

(2) What is the measure of ∠MRO ? (3) What is the measure of ∠ MRN ?

3. Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR bisects ∠MRN as well as ∠MON.

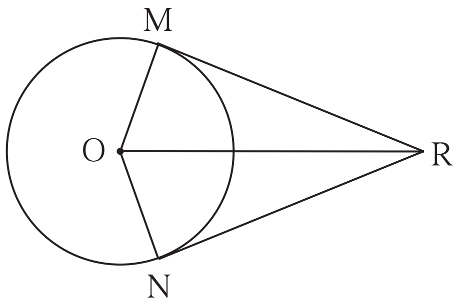


Fig. 3.21

4. What is the distance between two parallel tangents of a circle having radius 4.5 cm ? Justify your answer.

ICT Tools or Links

With the help of Geogebra software, draw a circle and its tangents from a point in its exterior. Check that the tangent segments are congrurent.

Let's learn.

Touching circles

Activity I :

Take three collinear points X-Y-Z as shown in figure 3.22. Draw a circle with centre X and radius XY.

Draw another circle with centre Z and radius YZ.

Note that both the circles intersect each other at the single point Y.

Draw a line through point Y and perpendicular to seg XZ.

Note that this line is a common tangent of the two circles.

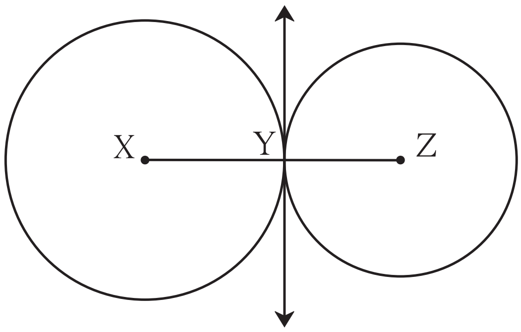


Fig. 3.22

Activity II :

Take points Y-X-Z as shown in the figure 3.23.

Draw a circle with centre Z and radius ZY.

Also draw a circle with centre X and radius XY.

Note that both the circles intersect each other at the point Y.

Draw a line perpendicular to seg YZ through point Y, that is the common tangent for the circles.

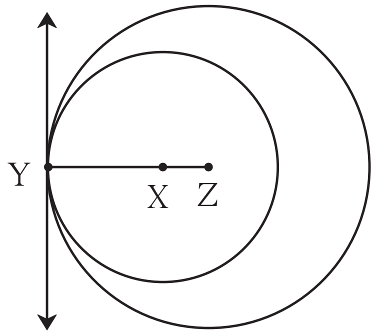


Fig. 3.23

You must have understood, the circles in both the figures are coplaner and intersect at one point only. Such circles are said to be circles touching each other.

Touching circles can be defined as follows.

If two circles in the same plane intersect with a line in the plain in only one point, they are said to be touching circles and the line is their common tangent. The point common to the circles and the line is called their common point of contact.

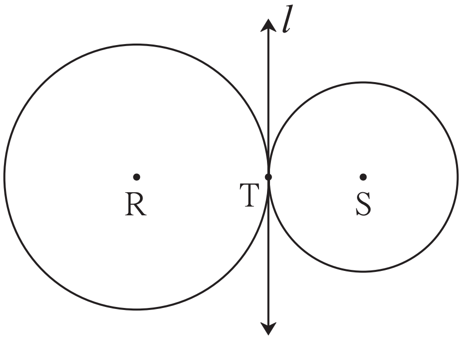


Fig. 3.24

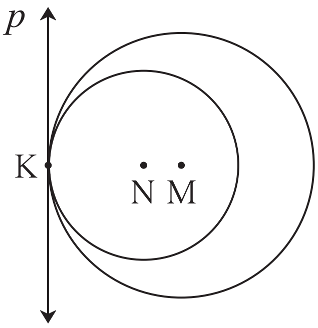


Fig. 3.25

In figure 3.24, the circles with centres R and S touch the line l in point T. So they are two touching circles with l as common tangent. They are touching externally.

In figure 3.25 the circles with centres M, N touch each other internally and line p is their common tangent.

Let's think.

(1) The circles shown in figure 3.24 are called externally touching circles. why ?

(2) The circles shown in figure 3.25 are called internally touching circles. why ?

(3) In figure 3.26, the radii of the circles with centers A and B are 3 cm and 4 cm respectively. Find -

(i) d(A,B) in figure 3.26 (a) (ii) d(A,B) in figure 3.26 (b)

Theorem of touching circles

Theorem : If two circles touch each other, their point of contact lies on the line joining their centres.

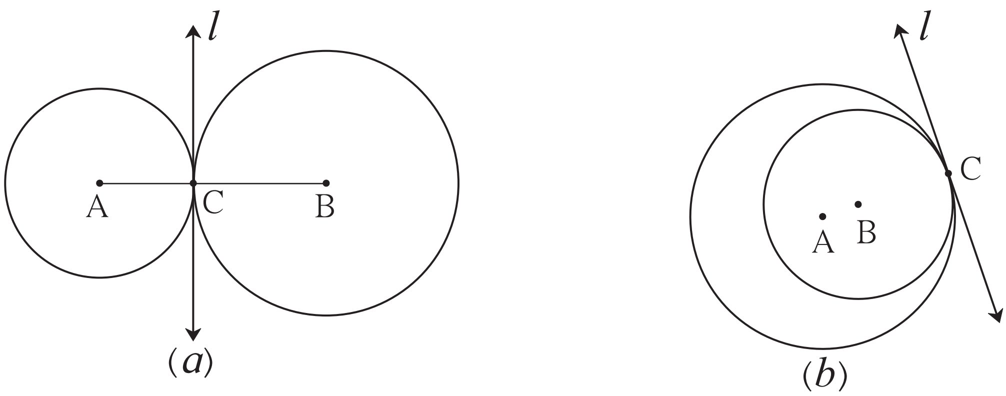


Fig. 3.26

Given : C is the point of contact of the two circles with centers A, B.

To prove : Point C lies on the line AB.

Proof : Let line l be the common tangent passing through C, of the two touching circles. line $l \perp$ seg AC, line $l \perp$ seg BC. $\therefore$ seg AC $\perp$ line l and seg BC $\perp$ line l. Through C, only one line perpendicular to line l can be drawn. $\therefore$ points C, A, B are collinear.

Remember this!

(1) The point of contact of the touching circles lies on the line joining their centres.

(2) If the circles touch each other externally, distance between their centres is equal to the sum of their radii.

(3) The distance between the centres of the circles touching internally is equal to the difference of their radii.

Practice set 3.2

1. Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres.
2. Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres.
3. If radii of two circles are 4 cm and 2.8 cm. Draw figure of these circles touching each other - (i) externally (ii) internally.
4. In fig 3.27, the circles with centres P and Q touch each other at R. A line passing through R meets the circles at A and B respectively. Prove that -

   (1) seg AP || seg BQ,

   (2) Δ APR $\sim$ Δ RQB, and

   (3) Find $\angle$ RQB if $\angle$ PAR = 35°

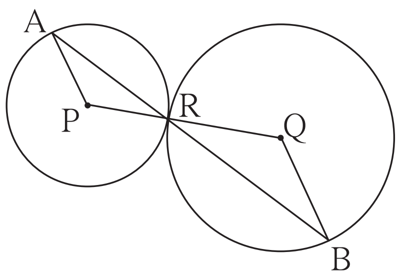


Fig. 3.27

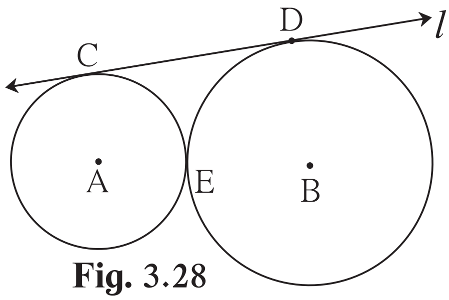


Fig. 3.28

5*. In fig 3.28 the circles with centres A and B touch each other at E. Line l is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii of the circles are 4 cm, 6 cm.

Arc of a circle

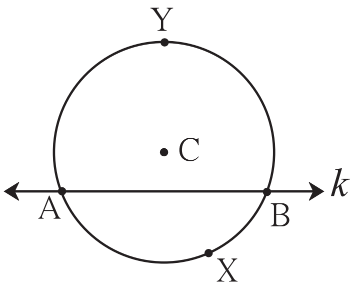


Fig. 3.29

A secant divides a circle in two parts. Any one of these two parts and the common points of the circle and the secant constitute an **arc of the circle.**

The points of intersection of circle and secant are called the end points of the arcs.

In figure 3.29, due to secant k we get two arcs of the circle with centre C– arc AYB, arc AXB.

If the centre of a circle is on one side of the secant then the arc on the side of the centre is called **'major arc'** and the arc which is on the other side of the centre is called **'minor arc'**. In the figure 3.29 arc AYB is a major arc and arc AXB is a minor arc. If there is no confusion then the name of a minor arc is written using its end points only. For example, the arc AXB in figure 3.29, is written as arc AB.

Here after, we are going to use the same convention for writing the names of arcs.

Central angle

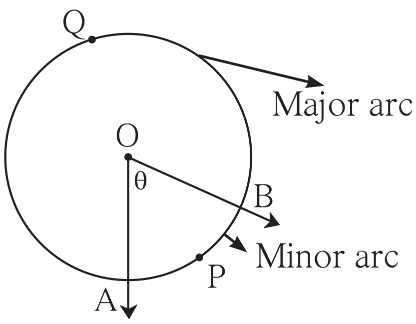


Fig. 3.30

When the vertex of an angle is the centre of a circle, it is called a central angle. In the figure 3.30, O is the centre of a circle and $\angle$AOB is a central angle.

Like secant, a central angle also divides a circle into two arcs.

Measure of an arc

To compare two arcs, we need to know their measures. Measure of an arc is defined as follows.

(1) Measure of a minor arc is equal to the measure of its corresponding central angle. In figure 3.30 measure of central $\angle$ AOB is θ.
$\therefore$ measure of minor arc APB is also θ.

(2) Measure of major arc = $360°$ – measure of corresponding minor arc.
In figure 3.30 measure of major arc AQB = $360°$ – measure of minor arc APB
= $360° - \theta$

(3) Measure of a semi circular arc, that is of a semi circle is $180°$.

(4) Measure of a complete circle is $360°$.

Let's learn.

Congruence of arcs

When two coplanar figures coincide with each other, they are called congruent figures. We know that two angles of equal measure are congruent.

Similarily, are two arcs of the same measure congruent ?
Find the answer of the question by doing the following activity.

Activity :

Draw two circles with centre C, as shown in the figure. Draw $\angle$ DCE, $\angle$ FCG of the same measure and $\angle$ ICJ of different measure.

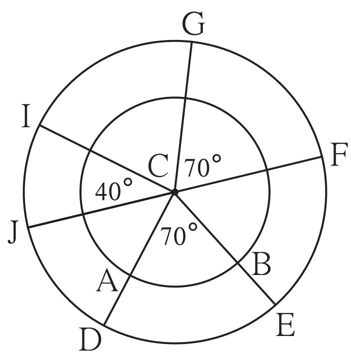


Fig. 3.31

Arms of $\angle$ DCE intersect inner circle at A and B.

Do you notice that the measures of arcs AB and DE are the same ? Do they coincide ? No, definitely not.

Now cut and seperate the sectors C-DE; C-FG and C-IJ. Check whether the arc DE, arc FG and arc IJ coincide with each other.

Did you notice that equality of measures of two arcs is not enough to make the two arcs congruent ? Which additional condition do you think is neccessary to make the two arcs congruent ?

From the above activity -

Two arcs are congruent if their measures and radii are equal.

'Arc DE and arc GF are congruent' is written in symbol as arc DE $\cong$ arc GF.

Property of sum of measures of arcs

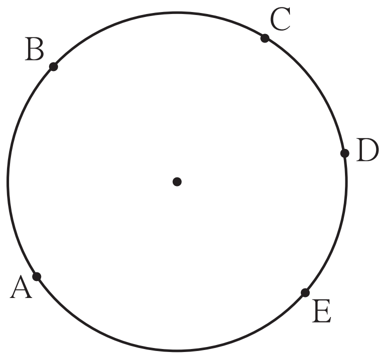


Fig. 3.32

In figure 3.32, the points A, B, C, D, E are concyclic. With these points many arcs are formed. There is one and only one common point C to arc ABC and arc CDE. So measure of arc ACE is the sum of measures of arc ABC and arc CDE.

m(arc ABC) + m(arc CDE) = m(arc ACE)

But arc ABC and arc BCE have many points in common. [All points on arc BC.]

So m(arc ABE) $\neq$ m(arc ABC) + m(arc BCE).

Theorem : The chords corresponding to congruent arcs of a circle (or congruent circles) are congruent.

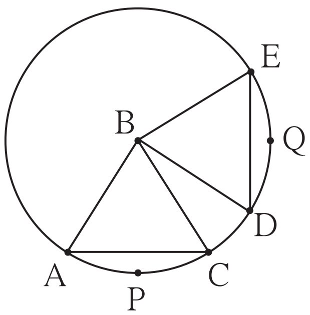


Fig. 3.33

Given : In a circle with centre B arc APC $\cong$ arc DQE

To Prove : Chord AC $\cong$ chord DE

Proof : (Fill in the blanks and complete the proof.)

In Δ ABC and Δ DBE,

side AB $\cong$ side DB (........)

side .... $\cong$ side .... (........)

$\angle$ ABC $\cong$ $\angle$ DBE measures of congruent arcs

$\therefore$ Δ ABC $\cong$ Δ DBE (.......)

$\therefore$ chord AC $\cong$ chord DE (......)

Theorem : Corresponding arcs of congruent chords of a circle (or congruent circles) are congruent.

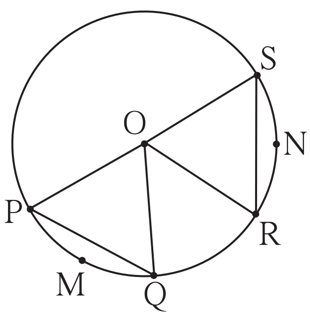


Fig. 3.34

Given : O is the centre of a circle chord PQ $\cong$ chord RS.

To prove : Arc PMQ $\cong$ arc RNS

Proof : Consider the following statements and write the proof.

Two arcs are congruent if their measures and radii are equal. Arc PMQ and arc RNS are arcs of the same circle, hence have equal radii.

Their measures are same as the measures of their central angles. To obtain central angles we have to draw radii OP, OQ, OR, OS.

Can you show that Δ OPQ and Δ ORS are congruent ?

Prove the above two theorems for congruent circles.

Let's think.

- While proving the first theorem of the two, we assume that the minor arc APC and minor arc DQE are congruent. Can you prove the same theorem by assuming that corresponding major arcs congruent ?
- In the second theorem, are the major arcs corresponding to congruent chords congruent ? Is the theorem true, when the chord PQ and chord RS are diameters of the circle ?

Solved Examples

Ex. (1) A, B, C are any points on the circle with centre O.

(i) Write the names of all arcs formed due to these points.

(ii) If *m* arc (BC) = 110° and *m* arc (AB) = 125°, find measures of all remaining arcs.

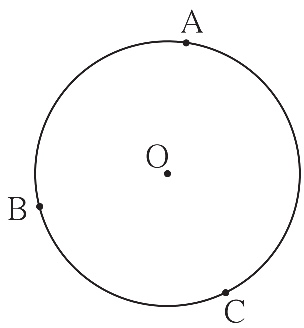


Fig. 3.35

Solution : (i) Names of arcs -

arc AB, arc BC, arc AC, arc ABC, arc ACB, arc BAC

(ii) m(arc ABC) = m(arc AB) + m(arc BC)

$= 125° + 110° = 235°$

m (arc AC) = 360° − m (arc ACB)

$= 360° - 235° = 125°$

Similarly, m(arc ACB) = 360° − 125° = 235°

and m(arc BAC) = 360° − 110° = 250°

Ex. (2) In the figure 3.36 a rectangle PQRS is inscribed in a circle with centre T. Prove that, (i) arc PQ ≅ arc SR

(ii) arc SPQ ≅ arc PQR

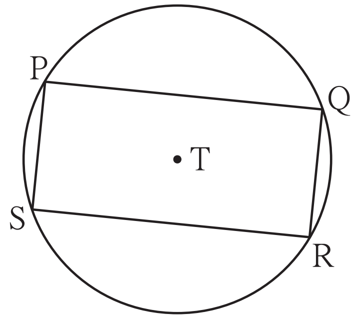


Fig. 3.36

Solution : (i) □ PQRS in a rectangle.

∴ chord PQ ≅ chord SR .... opposite sides of a rectangle

∴ arc PQ ≅ arc SR ...... arcs corresponding to congruent chords.

(ii) chord PS ≅ chord QR ..... Opposite sides of a rectangle

∴ arc SP ≅ arc QR ..... arcs corresponding to congruent chords.

∴ measures of arcs SP and QR are equal

Now, m(arc SP) + m(arc PQ) = m(arc PQ) + m(arc QR)

∴ m(arc SPQ) = m(arc PQR)

∴ arc SPQ ≅ arc PQR

Remember this !

(1) An angle whose vertex is the centre of a circle is called a central angle.

(2) Definition of measure of an arc – (i) The measure of a minor arc is the measure of its central angle. (ii) Measure of a major arc = 360° – measure of its corresponding minor arc. (iii) measure of a semicircle is 180°.

(3) When two arcs are of the same radius and same measure, they are congruent.

(4) When only one point C is common to arc ABC, and arc CDE of the same circle, m(arc ABC) + m(arc CDE) = m(arc ACE)

(5) Chords of the same or congruent circles are equal if the related arcs are congruent.

(6) Arcs of the same or congruent circles are equal if the related chords are congruent.

Practice set 3.3

1. In figure 3.37, points G, D, E, F are concyclic points of a circle with centre C.

∠ ECF = 70°, m(arc DGF) = 200°

find m(arc DE) and m(arc DEF).

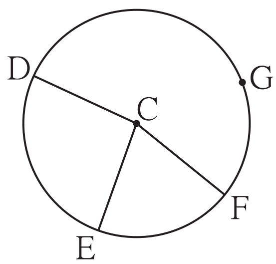


Fig. 3.37

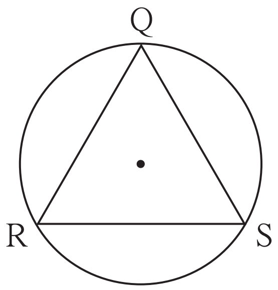


Fig. 3.38

2*. In fig 3.38 Δ QRS is an equilateral triangle. Prove that,

(1) arc RS ≅ arc QS ≅ arc QR

(2) m(arc QRS) = 240°.

3. In fig 3.39 chord AB ≅ chord CD, Prove that,

arc AC ≅ arc BD

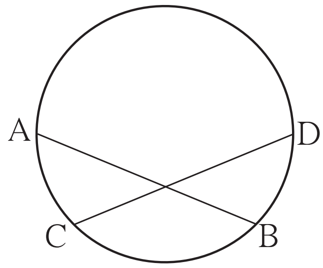


Fig. 3.39

Let's learn.

We have learnt some properties relating to a circle and points as well as lines (tangents). Now let us learn some properties regarding circle and angles with the help of some activities.

Activity I :

Draw a sufficiently large circle of any radius as shown in the figure 3.40. Draw a chord AB and central ∠ ACB. Take any point D on the major arc and point E on the minor arc.

(1) Measure ∠ ADB and ∠ ACB and compare the measures.

(2) Measure ∠ ADB and ∠ AEB. Add the measures.

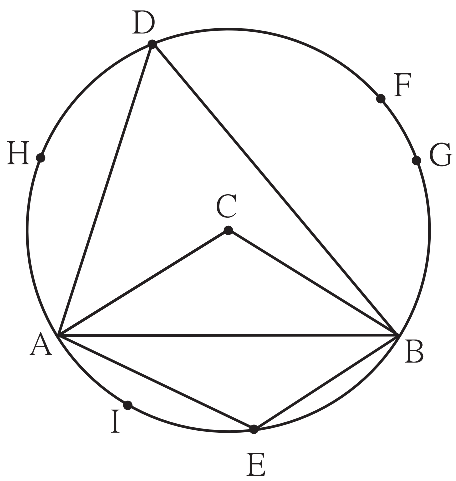


Fig. 3.40

(3) Take points F, G, H on the arc ADB. Measure $\angle$AFB, $\angle$AGB, $\angle$AHB. Compare these measures with each other as well as with measure of $\angle$ADB.

(4) Take any point I on the arc AEB. Measure $\angle$AIB and compare it with $\angle$AEB.

From the activity you must have noticed-

(1) The measure $\angle$ ACB is twice the measure of $\angle$ ADB.

(2) The sum of the measures of $\angle$ ADB and $\angle$ AEB is $180°$.

(3) The angles $\angle$ AHB, $\angle$ ADB, $\angle$ AFB and $\angle$ AGB are of equal measure.

(4) The measure of $\angle$ AEB and $\angle$ AIB are equal.

Activity II :

Draw a sufficiently large circle with centre C as shown in the figure 3.41. Draw any diameter PQ. Now take points R,S,T on both the semicircles. Measure $\angle$PRQ, $\angle$ PSQ, $\angle$PTQ. Note that each is a right angle.

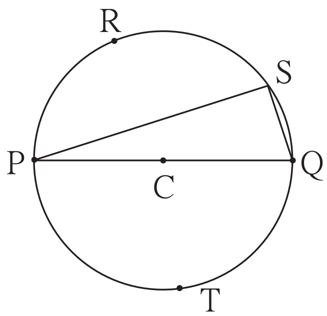


Fig. 3.41

The properties you saw in the above activities are theorems that give relations between circle and angles.

Let us learn some definitions required to prove the theorems.

Inscribed angle

In figure 3.42, C is the centre of a circle. The vertex D, of $\angle$PDQ lies on the circle. The arms of $\angle$PDQ intersect the circle at A and B. Such an angle is called an angle inscribed in the circle or in the arc.

In figure 3.42, $\angle$ADB is inscribed in the arc ADB.

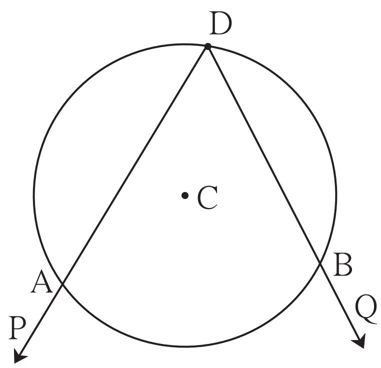


Fig. 3.42

Intercepted arc

Observe all figures (i) to (vi) in the following figure 3.43.

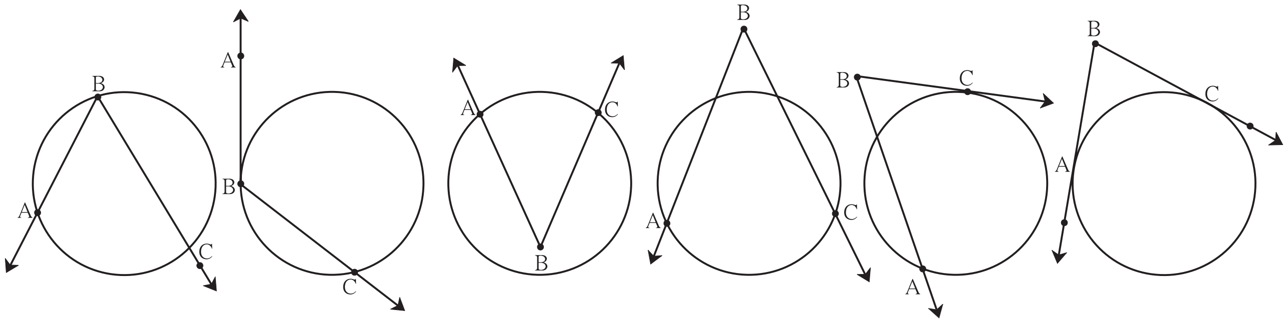


Fig. 3.43

In each figure, the arc of a circle that lies in the interior of the ∠ ABC is an arc intercepted by the ∠ ABC. The points of intersection of the circle and the angle are end points of that intercepted arc. Each side of the angle has to contain an end point of the arc.

In figures 3.43 (i), (ii) and (iii) only one arc is intercepted by that angle; and in (iv), (v) and (vi), two arcs are intercepted by the angle.

Also note that, only one side of the angle touches the circle in (ii) and (v), but in (vi) both sides of the angle touch the circle.

In figure 3.44, the arc is not intercepted arc, as arm BC does not contain any end point of the arc.

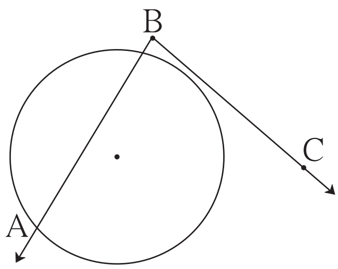


Fig. 3.44

Inscribed angle theorem

The measure of an inscribed angle is half of the measure of the arc intercepted by it.

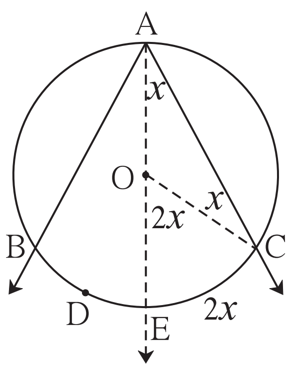


Fig. 3.45

Given : In a circle with centre O, ∠ BAC is inscribed in arc BAC. Arc BDC is intercepted by the angle.

To prove: $\angle BAC = \frac{1}{2}\ m(\text{arc BDC})$

Construction : Draw ray AO. It intersects the circle at E. Draw radius OC.

Proof : In Δ AOC,

side OA $\cong$ side OC ...... radii of the same circle.

$\therefore$ $\angle$ OAC = $\angle$ OCA ..... theorem of isosceles triangle.

Let $\angle$ OAC = $\angle$ OCA = x ...... (I)

Now, $\angle$ EOC = $\angle$ OAC + $\angle$ OCA .... exterior angle theorem of a triangle.

$= x° + x° = 2x°$

But $\angle$ EOC is a central angle.

$\therefore$ m(arc EC) = $2x°$ .... definition of measure of an arc ..... (II)

$\therefore$ from (I) and (II).

$\angle$ OAC = $\angle$ EAC = $\frac{1}{2}$ m(arc EC) ..... (III)

Similarly, drawing seg OB, we can prove $\angle$EAB = $\frac{1}{2}$ m(arc BE)...... (IV)

$\therefore$$\angle$EAC + $\angle$EAB = $\frac{1}{2}$ m(arc EC) + $\frac{1}{2}$ m(arc BE).... from (III) and (IV)

$\therefore$ $\angle$BAC = $\frac{1}{2}$ [m(arc EC) + m(arc BE)]

= $\frac{1}{2}$ [m(arc BEC)] = $\frac{1}{2}$ [m(arc BDC)] ..... (V)

Note that we have to consider three cases regarding the position of the centre of the circle and the inscribed angle. The centre of the circle lies (i) on one of the arms of the angle (ii) in the interior of the angle (iii) in the exterior of the angle. Out of these, first two are proved in (III) and (V). We will prove now the third one.

In figure 3.46,

$\angle$ BAC = $\angle$ BAE – $\angle$ CAE

= $\frac{1}{2}$ m(arc BCE) – $\frac{1}{2}$ m(arc CE)

...... from (III)

= $\frac{1}{2}$ [m(arc BCE) – m(arc CE)]

= $\frac{1}{2}$ [m(arc BC)] ...... (VI)

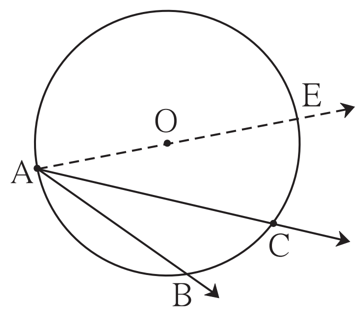


Fig. 3.46

The above theorem can also be stated as follows.

The measure of an angle subtended by an arc at a point on the circle is half of the measure of the angle subtended by the arc at the centre.

The corollaries of the above theorem can also be stated in similar language.

Corollaries of inscribed angle theorem

1. Angles inscribed in the same arc are congruent.

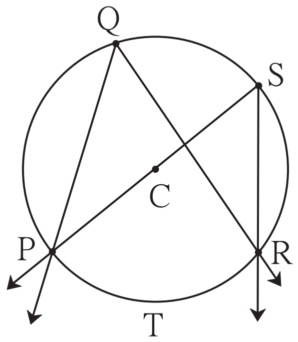


Fig. 3.47

Write 'given' and 'to prove' with the help of the figure 3.47.

Think of the answers of the following questions and write the proof.

(1) Which arc is intercepted by ∠ PQR ?

(2) Which arc is intercepted by ∠ PSR ?

(3) What is the relation between an inscribed angle and the arc intersepted by it ?

2. Angle inscribed in a semicircle is a right angle.

With the help of figure 3.48 write 'given', 'to prove' and 'the proof'.

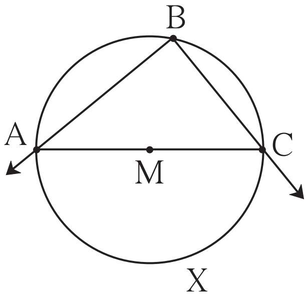


Fig. 3.48

Cyclic quadrilateral

If all vertices of a quadrilateral lie on the same circle then it is called a cyclic quadrilateral.

Theorem of cyclic quadrilateral

Theorem: **Opposite angles of a cyclic quadrilateral are supplementry.**

Fill in the blanks and complete the following proof.

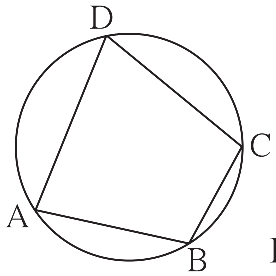


Fig. 3.49

Given : □ [ ] is cyclic.

To prove : ∠ B + ∠ D = [ ]

[ ] + ∠ C = 180°

Proof : Arc ABC is intercepted by the inscribed angle ∠ ADC.

∴ $\angle ADC = \frac{1}{2}$ [ ] .... (I)

Similarly, [ ] is an inscribed angle. It intercepts arc ADC.

$\therefore$ $\boxed{}$ $= \frac{1}{2}\, m(\text{arc ADC})$ ..... (II)

$\therefore m\angle ADC +$ $\boxed{}$ $= \frac{1}{2}$ $\boxed{}$ $+ \frac{1}{2}\, m(\text{arc ADC})$ ..... from (I) & (II)

$= \frac{1}{2}\, [$ $\boxed{}$ $+ m(\text{arc ADC})]$

$= \frac{1}{2} \times 360°$ .... arc ABC and arc ADC constitute a complete circle.

$=$ $\boxed{}$

Similarly we can prove, $\angle A + \angle C =$ $\boxed{}$.

Corollary of cyclic quadrilateral theorem

An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle.

Write the proof of the theorem yourself.

Let's think.

In the above theorem, after proving $\angle B + \angle D = 180°$, can you use another way to prove $\angle A + \angle C = 180°$?

Converse of cyclic quadrilateral theorem

Theorem : If a pair of opposite angles of a quadrilateral is supplementary, the quadrilateral is cyclic.

Try to prove this theorem by 'indirect method'. From the above converse, we know that if opposite angles of a quadrilateral are supplementary then there is a circumcircle for the quadrilateral.

For every triangle there exists a circumcircle but there may not be a circumcircle for every quadrilateral.

The above converse gives us the condition to ensure the existence of circumcircle of a quadrilateral.

With one more condition four non-collinear points are concyclic. It is stated in the following theorem.

Theorem : If two points on a given line subtend equal angles at two distinct points which lie on the same side of the line, then the four points are concyclic.

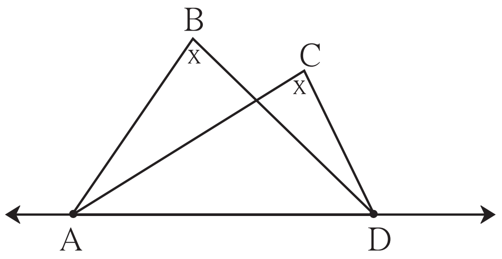


Fig. 3.50

Given : Points B and C lie on the same side of the line AD. $\angle$ ABD $\cong$ $\angle$ ACD

To prove: Points A, B, C, D are concyclic. (That is, $\square$ABCD is cyclic.)

This theorem can be proved by 'indirect method'.

Let's think.

The above theorem is converse of a certain theorem. State it.

Solved Examples

Ex. (1) In figure 3.51, chord LM $\cong$ chord LN

$\angle$ L = 35° find

(i) m(arc MN)

(ii) m(arc LN)

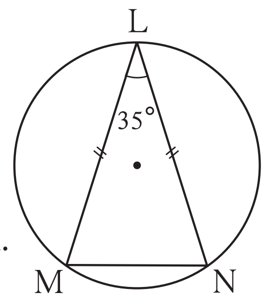


Fig. 3.51

Solution : (i) $\angle L = \frac{1}{2}\, m(\text{arc MN})$ ......inscribed angle theorem.

$\therefore\ 35 = \frac{1}{2}\, m(\text{arc MN})$

$\therefore\ 2 \times 35 = m(\text{arc MN}) = 70°$

(ii) m(arc MLN) = 360° − m(arc MN) ...... definition of measure of arc

= 360° − 70° = 290°

Now, chord LM $\cong$ chord LN

$\therefore$ arc LM $\cong$ arc LN

but m(arc LM) + m(arc LN) = m(arc MLN) = 290° ..... arc addition property

$m(\text{arc LM}) = m(\text{arc LN}) = \frac{290°}{2} = 145°$

or, (ii) chord LM $\cong$ chord LN

$\therefore\ \angle M = \angle N$ ...... isosceles trinangle theorem.

$\therefore\ 2\angle M = 180° - 35° = 145°$

$\therefore\ \angle M = \frac{145°}{2}$

Now, $m(\text{arc LN}) = 2 \times \angle M$

$$= 2 \times \frac{145^\circ}{2} = 145^\circ$$

Ex. (2) In figure 3.52, chords PQ and RS intersect at T.

(i) Find $m(\text{arc SQ})$ if $m\angle STQ = 58^\circ$, $m\angle PSR = 24^\circ$.

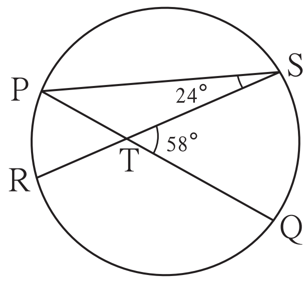


Fig. 3.52

(ii) Verify,

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$

(iii) Prove that :

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$

for any measure of $\angle STQ$.

(iv) Write in words the property in (iii).

Solution : (i) $\angle SPQ = \angle SPT = 58^\circ - 24^\circ = 34^\circ$ .... exterior angle theorem.

$m(\text{arc QS}) = 2\angle SPQ = 2 \times 34^\circ = 68^\circ$

(ii) $m(\text{arc PR}) = 2\angle PSR = 2 \times 24^\circ = 48^\circ$

Now, $\frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})] = \frac{1}{2}[48 + 68]$

$= \frac{1}{2} \times 116 = 58^\circ$

$= \angle STQ$

(iii) Fill in the blanks and complete the proof of the above property.

$\angle STQ = \angle SPQ + \square$ .... exterior angle theorem of a triangle

$= \frac{1}{2} m(\text{arc SQ}) + \square$ .... inscribed angle theorem

$= \frac{1}{2}[\square + \square]$

(iv) If two chords of a circle intersect each other in the interior of a circle then the measure of the angle between them is half the sum of measures of arcs intercepted by the angle and its opposite angle.

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Ex. (3) Prove that, if two lines containing chords of a circle intersect each other outside the circle, then the measure of angle between them is half the difference in measures of the arcs intercepted by the angle.

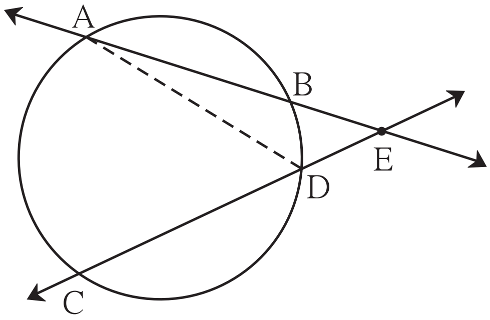

Fig. 3.53

Given : Chords AB and CD intersect at E in the exterior of the circle.

To prove: $\angle AEC = \frac{1}{2}[m(\text{arc } AC) - m(\text{arc } BD)]$

Construction: Draw seg AD.

Consider angles of Δ AED and its exterior angle and write the proof.

Remember this !

(1) The measure of an inscribed angle is half the measure of the arc intercepted by it.

(2) Angles inscribed in the same arc are congruent.

(3) Angle inscribed in a semicircle is a right angle.

(4) If all vertices of a quadrilateral lie on the same circle then the quadrilateral is called a cyclic quadrilateral.

(5) Opposite angles of a cyclic quadrilateral are supplementary.

(6) An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle.

(7) If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

(8) If two points on a given line subtend equal angles at two different points which lie on the same side of the line, then those four points are concyclic.

(9) In figure 3.54,

(i) $\angle AEC = \frac{1}{2}[m(\text{arc } AC) + m(\text{arc } DB)]$

(ii) $\angle CEB = \frac{1}{2}[m(\text{arc } AD) + m(\text{arc } CB)]$

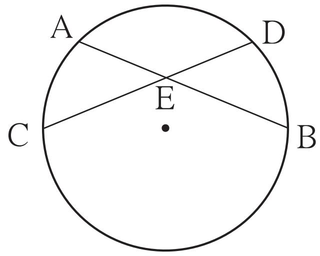

Fig. 3.54

(10) In figure 3.55,

$\angle BED = \frac{1}{2}[m(\text{arc BD}) - m(\text{arc AC})]$

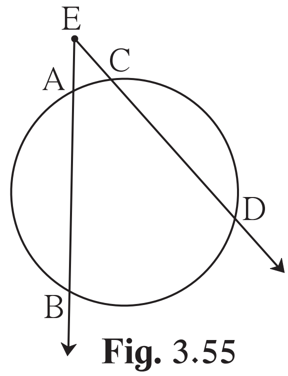

Fig. 3.55

Practice set 3.4

1. In figure 3.56, in a circle with centre O, length of chord AB is equal to the radius of the circle. Find measure of each of the following.

(1) $\angle AOB$ (2) $\angle ACB$

(3) arc AB (4) arc ACB.

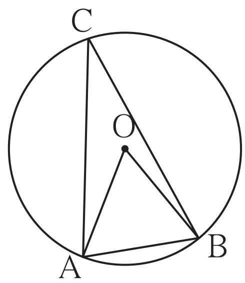

Fig. 3.56

2. In figure 3.57, $\square$ PQRS is cyclic. side PQ $\cong$ side RQ. $\angle PSR = 110°$, Find–

(1) measure of $\angle PQR$

(2) m(arc PQR)

(3) m(arc QR)

(4) measure of $\angle PRQ$

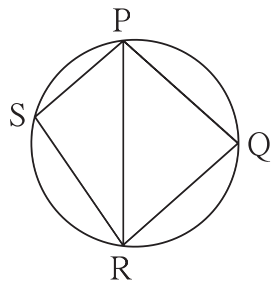

Fig. 3.57

3. $\square$ MRPN is cyclic, $\angle R = (5x - 13)°$, $\angle N = (4x + 4)°$. Find measures of $\angle R$ and $\angle N$.

4. In figure 3.58, seg RS is a diameter of the circle with centre O. Point T lies in the exterior of the circle. Prove that $\angle RTS$ is an acute angle.

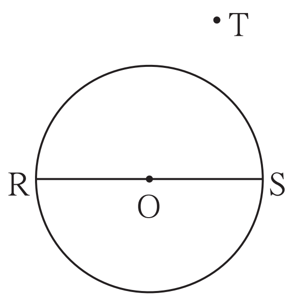

Fig. 3.58

5. Prove that, any rectangle is a cyclic quadrilateral.

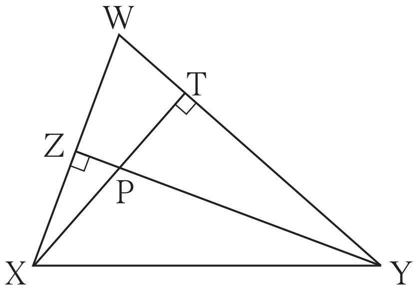

Fig. 3.59

6. In figure 3.59, altitudes YZ and XT of Δ WXY intersect at P. Prove that,

(1) $\square$ WZPT is cyclic.

(2) Points X, Z, T, Y are concyclic.

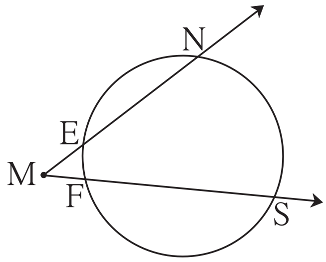

Fig. 3.60

7. In figure 3.60, m(arc NS) = 125°, m(arc EF) = 37°, find the measure $\angle$NMS.

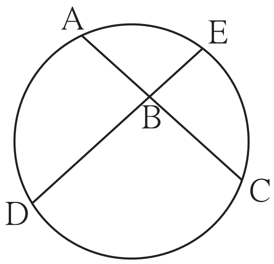

Fig. 3.61

8. In figure 3.61, chords AC and DE intersect at B. If $\angle$ ABE = 108°, m(arc AE) = 95°, find m(arc DC) .

Let's learn.

Activity :

Draw a circle as shown in figure 3.62. Draw a chord AC. Take any point B on the circle. Draw inscribed $\angle$ABC, measure it and note the measure.

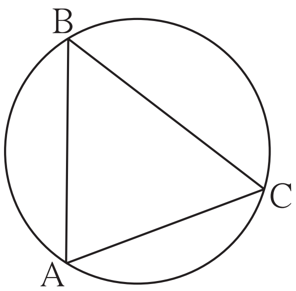

Fig. 3.62

Now as shown in figure 3.63, draw a tangent CD of the same circle, measure angle $\angle$ACD and note the measure.

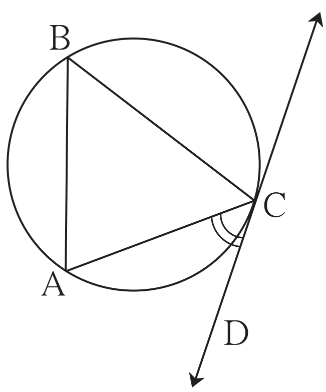

Fig. 3.63

You will find that $\angle ACD = \angle ABC$.

You know that $\angle ABC = \frac{1}{2}\ m(\text{arc } AC)$

From this we get $\angle ACD = \frac{1}{2}\ m(\text{arc } AC)$.

Now we will prove this property.

Theorem of angle between tangent and secant

If an angle has its vertex on the circle, its one side touches the circle and the other intersects the circle in one more point, then the measure of the angle is half the measure of its intercepted arc.

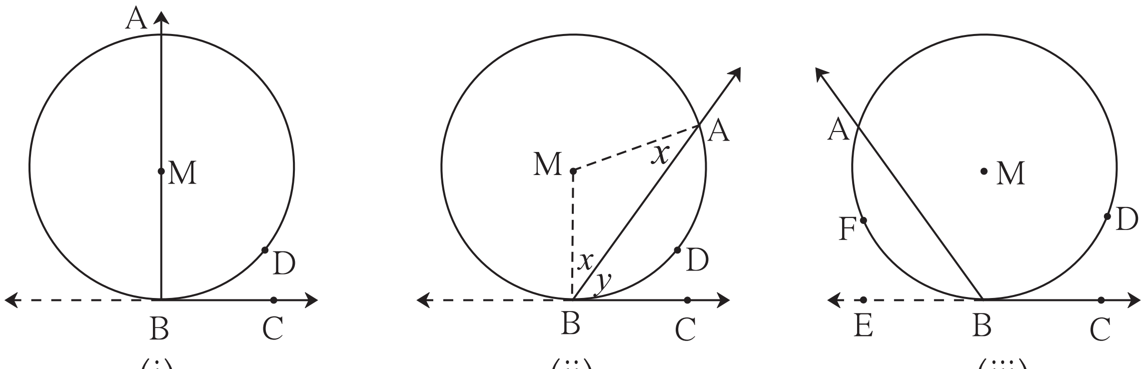

Fig. 3.64

Given : Let $\angle ABC$ be an angle, where vertex B lies on a circle with centre M. Its side BC touches the circle at B and side BA intersects the circle at A. Arc ADB is intercepted by $\angle ABC$.

To prove: $\angle ABC = \frac{1}{2}\ m(\text{arc } ADB)$

Proof : Consider three cases.

(1) In figure 3.64 (i), the centre M lies on the arm BA of $\angle ABC$,

$\angle ABC = \angle MBC = 90°$ ..... tangent theorem (I)

arc ADB is a semicircle.

$\therefore\ m(\text{arc } ADB) = 180°$ ..... definition of measure of arc (II)

From (I) and (II)

$\angle ABC = \frac{1}{2}\ m(\text{arc } ADB)$

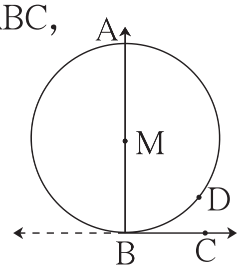

Fig. 3.64(i)

(2) In figure 3.64 (ii) centre M lies in the exterior of $\angle ABC$,

Draw radii MA and MB.

Now, $\angle MBA = \angle MAB$ ..... isosceles triangle theorem

$\angle MBC = 90°$ ..... tangent theorem..... (I) let $\angle$ MBA = $\angle$ MAB = x and $\angle$ ABC = y.

$\angle$ AMB = 180 − (x + x) = 180 − 2x

$\angle$ MBC = $\angle$ MBA + $\angle$ ABC = x + y

$\therefore$ $x + y = 90°$ $\quad \therefore$ $2x + 2y = 180°$

In Δ AMB, $2x$ + $\angle$ AMB = 180°

$\therefore$ $2x + 2y = 2x$ + $\angle$ AMB

$\therefore$ $2y$ = $\angle$ AMB

$\therefore$ y = $\angle$ ABC = $\frac{1}{2}$ $\angle$ AMB = $\frac{1}{2}$ m(arc ADB)

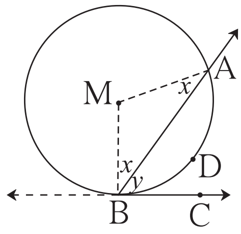

Fig. 3.64(ii)

(3) With the help of fig 3.64 (iii),

Fill in the blanks and write proof.

Ray [ ] is the opposite ray of ray BC.

Now, $\angle$ABE = $\frac{1}{2}$ m( ) ...... proved in (ii).

$\therefore$ 180 − [ ] = $\frac{1}{2}$ m(arc AFB) ..... linear pair

= $\frac{1}{2}$ [360 − m([ ])]

$\therefore$ 180 − $\angle$ABC = 180 − $\frac{1}{2}$ m(arc ADB)

$\therefore$ − $\angle$ABC = − $\frac{1}{2}$ m([ ])

$\therefore$ $\angle$ABC = $\frac{1}{2}$ m(arc ADB)

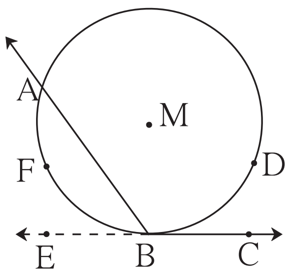

Fig. 3.64(iii)

Alternative statement of the above theorem.

In the figure 3.65, line AB is a secant and line BC is a tangent. The arc ADB is intercepted by $\angle$ ABC. Chord AB divides the circle in two parts. These are opposite arcs of each other.

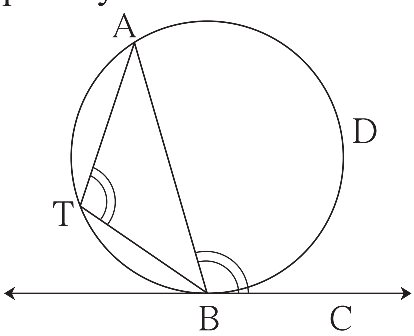

Fig. 3.65

Now take any point T on the arc opposite to arc ADB .

From the above theorem,

$\angle$ ABC = $\frac{1}{2}$ m (arc ADB) = $\angle$ ATB.

$\therefore$ **the angle between a tangent of a circle and a chord drawn from the point of contact is congruent to the angle inscribed in the arc opposite to the arc intercepted by that angle.**

Converse of theorem of the angle between tangent and secant

A line is drawn from one end point of a chord of a circle and if the angle between the chord and the line is half the measure of the arc intercepted by that angle then that line is a tangent to the circle.

In figure 3.66,

If $\angle PQR = \frac{1}{2} m(\text{arc } PSQ)$,

[or $\angle PQT = \frac{1}{2} m(\text{arc } PUQ)$]

then line TR is a tangent to the circle.

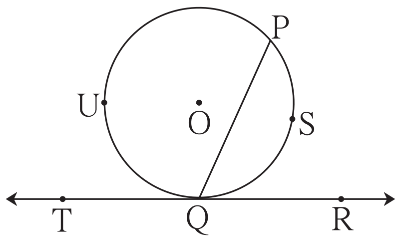

Fig. 3.66

This property is used in constructing a tangent to the given circle.

An indirect proof of this converse can be given.

Theorem of internal division of chords

Suppose two chords of a circle intersect each other in the interior of the circle, then the product of the lengths of the two segments of one chord is equal to the product of the lengths of the two segments of the other chord.

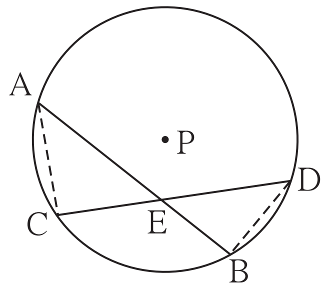

Fig. 3.67

Given : Chords AB and CD of a circle with centre P intersect at point E.

To prove: AE × EB = CE × ED

Construction : Draw seg AC and seg DB.

Proof : In Δ CAE and Δ BDE,

$\angle AEC \cong \angle DEB$ ..... opposite angles

$\angle CAE \cong \angle BDE$ ..... angles inscribed in the same arc

$\therefore \Delta CAE \sim \Delta BDE$ ..... AA test

$\therefore \frac{AE}{DE} = \frac{CE}{BE}$ ..... corresponding sides of similar triangles

$\therefore AE \times EB = CE \times ED$

Let's think.

We proved the theorem by drawing seg AC and seg DB in figure 3.67, Can the theorem be proved by drawing seg AD and segCB, instead of seg AC and seg DB?

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For more information

In figure 3.67 point E divides the chord AB into segments AE and EB. AE × EB is the area of a rectangle having sides AE and EB. Similarly E divides CD into segments CE and ED. CE × ED is the area of a rectangle of sides CE and ED. We have proved that AE × EB = CE × ED.

So the above theorem can be stated as, 'If two chords of a circle intersect in the interior of a circle then the area of the rectangle formed by the segments of one chord is equal to the area of similar rectangle formed by the other chord.'

Theorem of external division of chords

If secants containing chords AB and CD of a circle intersect outside the circle in point E, then AE × EB = CE × ED.

Write 'given' and 'to prove' with the help of the statement of the theorem and figure 3.68.

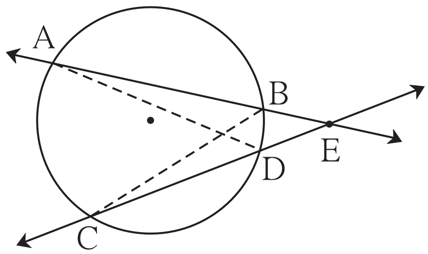

Fig. 3.68

Construction : Draw seg AD and seg BC.

Fill in the blanks and complete the proof.

Proof : In Δ ADE and Δ CBE,

∠ AED ≅ ☐ .... common angle

∠ DAE ≅ ∠ BCE ....(☐)

∴ Δ ADE ~ ☐ ....(☐)

$\therefore \frac{(AE)}{\square} = \frac{\square}{\square}$ .... corresponding sides of similar triangles

∴ ☐ = CE × ED

Tangent secant segments theorem

Point E is in the exterior of a circle. A secant through E intersects the circle at points A and B, and a tangent through E touches the circle at point T, then $EA \times EB = ET^2$.

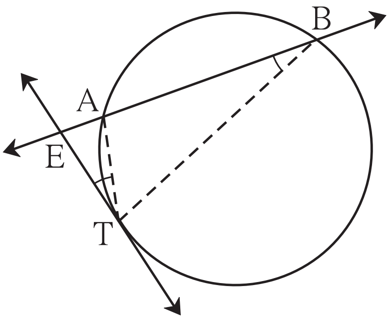

Fig. 3.69

Write 'given' and 'to prove' with reference to the statement of the theorem.

Construction : Draw seg TA and seg TB.

Proof : In Δ EAT and Δ ETB,

$\angle$ AET $\cong$ $\angle$ TEB .... common angle

$\angle$ ETA $\cong$ $\angle$ EBT... tangent secant theorem

$\therefore$ Δ EAT ~ Δ ETB ..... AA similarity

$\therefore \dfrac{ET}{EB} = \dfrac{EA}{ET}$ ..... corresponding sides

$\therefore EA \times EB = ET^2$

Remember this !

(1) In figure 3.70,

$AE \times EB = CE \times ED$

This property is known as theorem of chords intersecting inside the circle.

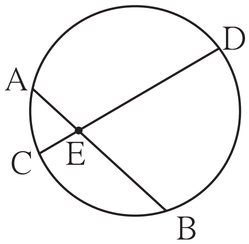

Fig. 3.70

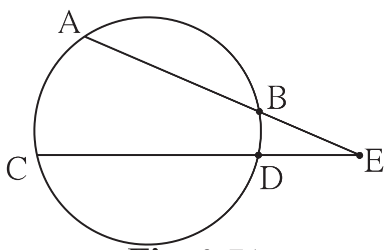

Fig. 3.71

(2) In figure 3.71,

$AE \times EB = CE \times ED$

This property is known as theorem of chords intersecting outside the circle.

(3) In figure 3.72,

$EA \times EB = ET^2$

This property is known as tangent secant segments theorem.

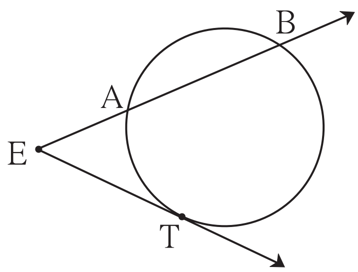

Fig. 3.72

Solved Examples

Ex. (1) In figure 3.73, seg PS is a tangent segment. Line PR is a secant.
If PQ = 3.6,
QR = 6.4, find PS.

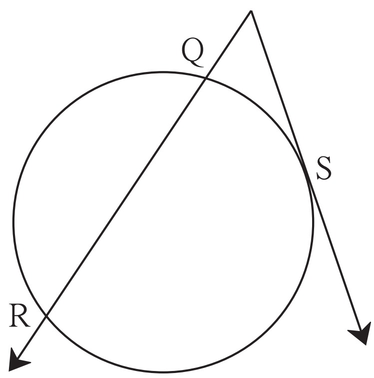

Fig. 3.73

Solution : $PS^2 = PQ \times PR$ .... tangent secant segments theorem

$= PQ \times (PQ + QR)$

$= 3.6 \times [3.6 + 6.4]$

$= 3.6 \times 10$

$= 36$

$\therefore PS = 6$

Ex. (2) In figure 3.74, chord MN and chord RS intersect each other at point P.
If PR = 6, PS = 4, MN = 11
find PN.

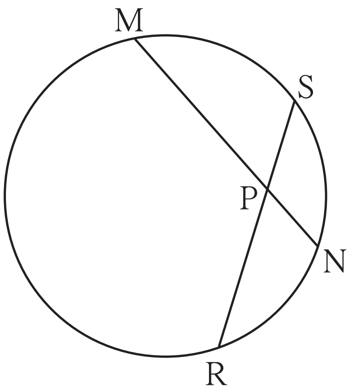

Fig. 3.74

Solution : By theorem on interesecting chords,

$PN \times PM = PR \times PS$... (I)

let PN = x. $\therefore$ PM = $11 - x$

substituting the values in (I),

$x\,(11 - x) = 6 \times 4$

$\therefore\; 11x - x^2 - 24 = 0$

$\therefore\; x^2 - 11x + 24 = 0$

$\therefore\; (x - 3)\,(x - 8) = 0$

$\therefore\; x - 3 = 0$ or $x - 8 = 0$

$\therefore\; x = 3$ or $x = 8$

$\therefore$ PN = 3 or PN = 8

Ex. (3) In figure 3.75, two circles intersect each other in points X and Y. Tangents drawn from a point M on line XY touch the circles at P and Q.

Prove that, seg PM ≅ seg QM.

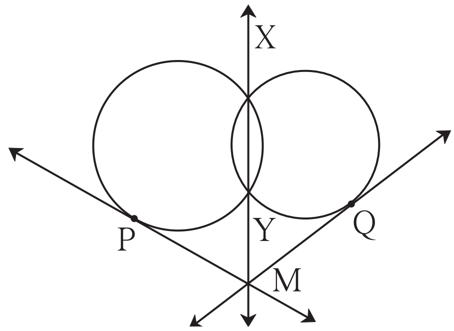

Fig. 3.75

Solution : Fill in the blanks and write proof.

Line MX is a common .... of the two circles.

$\therefore PM^2 = MY \times MX$ ..... (I)

Similarly .... = ... × .... , tangent secant segment theorem ......(II)

∴ From (I) and (II) ...... = QM^2

∴ PM = QM

seg PM ≅ seg QM

Ex. (4)

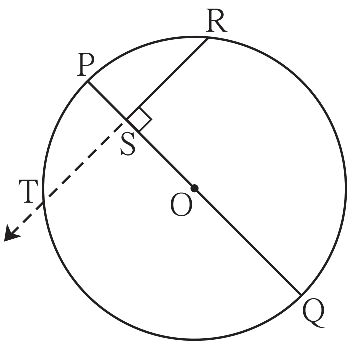

Fig. 3.76

In figure 3.76, seg PQ is a diameter of a circle with centre O. R is any point on the circle.

seg RS ⊥ seg PQ.

Prove that, SR is the geometric mean of PS and SQ.

[That is, $SR^2 = PS \times SQ$]

Solution : Write the proof with the help of the following steps.

(1) Draw ray RS. It intersects the circle at T.

(2) Show that RS = TS.

(3) Write a result using theorem of intersection of chords inside the circle.

(4) Using RS = TS complete the proof.

Let's think.

(1) In figure 3.76, if seg PR and seg RQ are drawn, what is the nature of Δ PRQ ?

(2) Have you previously proved the property proved in example (4) ?

Practice set 3.5

1. In figure 3.77, ray PQ touches the circle at point Q. PQ = 12, PR = 8, find PS and RS.

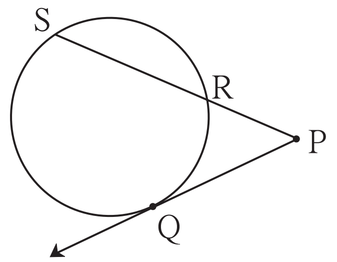

Fig. 3.77

2. In figure 3.78, chord MN and chord RS intersect at point D.

(1) If RD = 15, DS = 4, MD = 8 find DN

(2) If RS = 18, MD = 9, DN = 8 find DS

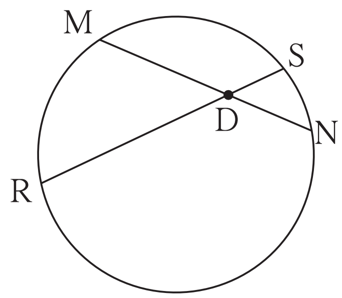

Fig. 3.78

3. In figure 3.79, O is the centre of the circle and B is a point of contact. seg OE ⊥ seg AD, AB = 12, AC = 8, find

(1) AD (2) DC (3) DE.

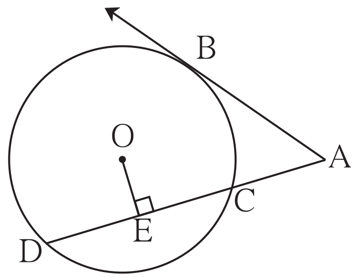

Fig. 3.79

4. In figure 3.80, if PQ = 6, QR = 10, PS = 8 find TS.

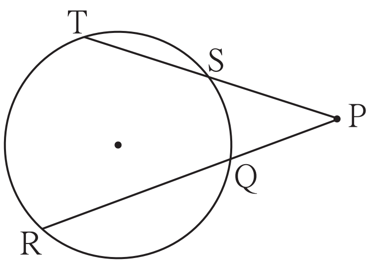

Fig. 3.80

5. In figure 3.81, seg EF is a diameter and seg DF is a tangent segment. The radius of the circle is r. Prove that, DE × GE = $4r^2$

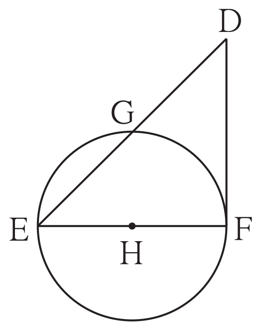

Fig. 3.81

Problem set 3

1. Four alternative answers for each of the following questions are given. Choose the correct alternative.

(1) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other. What is the distance between their centers ?

(A) 4.4 cm (B) 8.8 cm (C) 2.2 cm (D) 8.8 or 2.2 cm

(2) Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle ?

(A) 6 cm (B) 12 cm (C) 24 cm (D) can't say

(3) A circle touches all sides of a parallelogram. So the parallelogram must be a, ...

(A) rectangle (B) rhombus (C) square (D) trapezium

(4) Length of a tangent segment drawn from a point which is at a distance 12.5 cm from the centre of a circle is 12 cm, find the diameter of the circle.

(A) 25 cm (B) 24 cm (C) 7 cm (D) 14 cm

(5) If two circles are touching externally, how many common tangents of them can be drawn?

(A) One (B) Two (C) Three (D) Four

(6) $\angle$ACB is inscribed in arc ACB of a circle with centre O. If $\angle$ACB = 65°, find m(arc ACB).

(A) 65° (B) 130° (C) 295° (D) 230°

(7) Chords AB and CD of a circle intersect inside the circle at point E. If AE = 5.6, EB = 10, CE = 8, find ED.

(A) 7 (B) 8 (C) 11.2 (D) 9

(8) In a cyclic $\square$ABCD, twice the measure of $\angle$ A is thrice the measure of $\angle$C. Find the measure of $\angle$C ?

(A) 36 (B) 72 (C) 90 (D) 108

9)★ Points A, B, C are on a circle, such that m(arc AB) = m(arc BC) = 120°. No point, except point B, is common to the arcs. Which is the type of Δ ABC ?

(A) Equilateral triangle (B) Scalene triangle

(C) Right angled triangle (D) Isosceles triangle

(10) Seg XZ is a diameter of a circle. Point Y lies in its interior. How many of the following statements are true ?

(i) It is not possible that ∠ XYZ is an acute angle.

(ii) ∠ XYZ can't be a right angle.

(iii) ∠ XYZ is an obtuse angle.

(iv) Can't make a definite statement for measure of ∠ XYZ.

(A) Only one (B) Only two (C) Only three (D) All

2. Line l touches a circle with centre O at point P. If radius of the circle is 9 cm, answer the following.

(1) What is d(O, P) = ? Why ?

(2) If d(O, Q) = 8 cm, where does the point Q lie ?

(3) If d(PQ) = 15 cm, How many locations of point R are line on line l ? At what distance will each of them be from point P ?

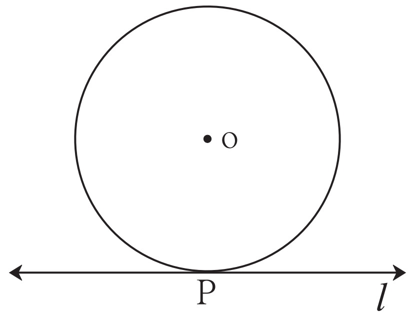

Fig. 3.82

3. In figure 3.83, M is the centre of the circle and seg KL is a tangent segment.

If MK = 12, KL = $6\sqrt{3}$ then find -

(1) Radius of the circle.

(2) Measures of ∠ K and ∠ M.

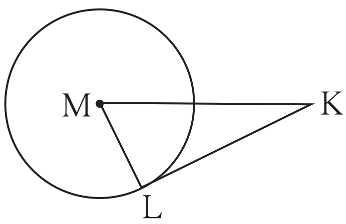

Fig. 3.83

4. In figure 3.84, O is the centre of the circle. Seg AB, seg AC are tangent segments. Radius of the circle is r and l(AB) = r, Prove that, □ABOC is a square.

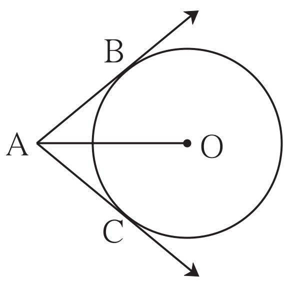

Fig. 3.84

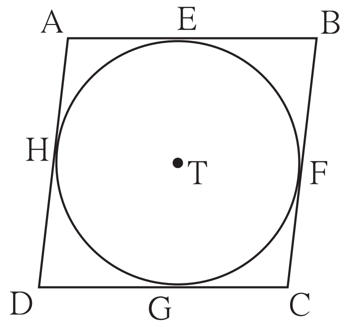

Fig. 3.85

5. In figure 3.85, □ABCD is a parallelogram. It circumscribes the circle with cnetre T. Point E, F, G, H are touching points. If AE = 4.5, EB = 5.5, find AD.

6. In figure 3.86, circle with centre M touches the circle with centre N at point T. Radius RM touches the smaller circle at S. Radii of circles are 9 cm and 2.5 cm. Find the answers to the following questions hence find the ratio MS:SR.

(1) Find the length of segment MT

(2) Find the length of seg MN

(3) Find the measure of ∠ NSM.

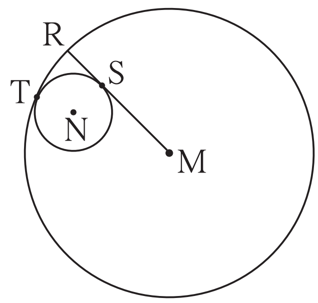

Fig. 3.86

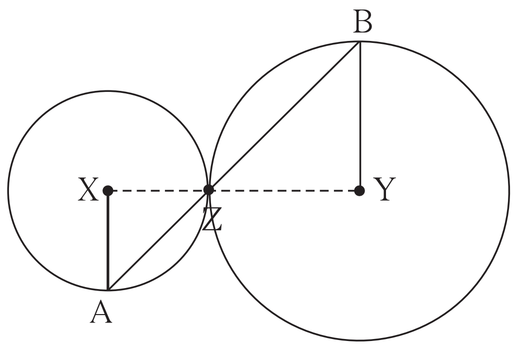

Fig. 3.87

7. In the adjoining figure circles with centres X and Y touch each other at point Z. A secant passing through Z intersects the circles at points A and B respectively. Prove that, radius XA || radius YB.

Fill in the blanks and complete the proof.

Construction : Draw segments XZ and ....

Proof :By theorem of touching circles, points X, Z, Y are ....

∴ ∠ XZA ≅ .... opposite angles

Let ∠ XZA = ∠ BZY = a ..... (I)

Now, seg XA ≅ seg XZ .... (..........)

∴ ∠ XAZ = ... = a .... (isosceles triangle theorem) (II)

similarly, seg YB ≅ .... (..........)

∴ ∠ BZY = ... = a .... (..........) (III)

∴ from (I), (II), (III),

∠ XAZ = ...

∴ radius XA || radius YB .... (..........)

8. In figure 3.88, circles with centres X and Y touch internally at point Z . Seg BZ is a chord of bigger circle and it itersects smaller circle at point A. Prove that, seg AX || seg BY.

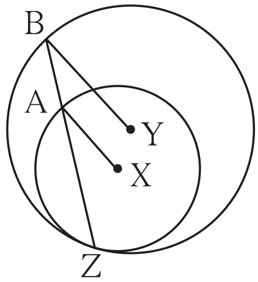

Fig. 3.88

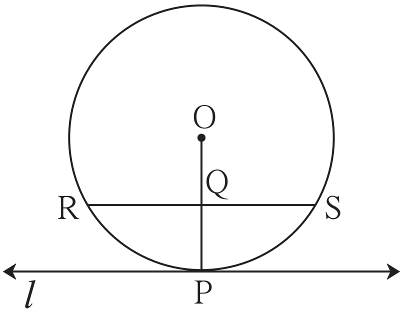

Fig. 3.89

9. In figure 3.89, line l touches the circle with centre O at point P. Q is the mid point of radius OP. RS is a chord through Q such that chords RS || line l. If RS = 12 find the radius of the circle.

10★. In figure 3.90, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, which touches the circle at point T.
seg AP ⊥ line PQ and seg BQ ⊥ line PQ.
Prove that, seg CP ≅ seg CQ.

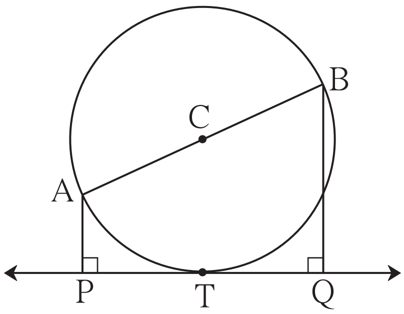

Fig. 3.90

11★. Draw circles with centres A, B and C each of radius 3 cm, such that each circle touches the other two circles.

12★. Prove that any three points on a circle cannot be collinear.

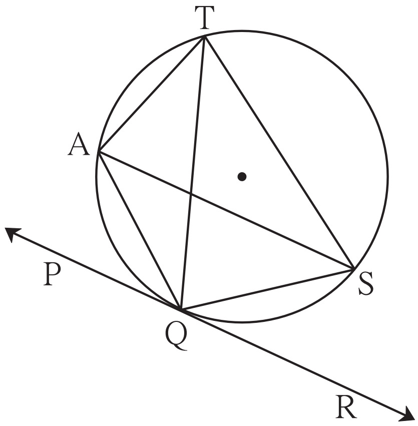

Fig. 3.91

13. In figure 3.91, line PR touches the circle at point Q. Answer the following questions with the help of the figure.

(1) What is the sum of ∠TAQ and ∠TSQ ?

(2) Find the angles which are congruent to ∠AQP.

(3) Which angles are congruent to ∠QTS ?

(4) ∠TAS = 65°, find the measure of ∠TQS and arc TS.

(5) If ∠AQP = 42°and ∠SQR = 58° find measure of ∠ATS.

14. In figure 3.92, O is the centre of a circle, chord PQ ≅ chord RS

If ∠ POR = 70°

and (arc RS) = 80°, find –

(1) m(arc PR)

(2) m(arc QS)

(3) m(arc QSR)

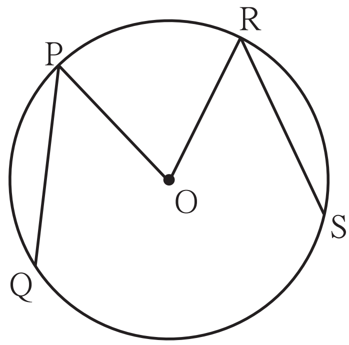

Fig. 3.92

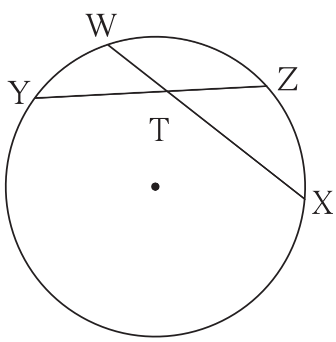

Fig. 3.93

15. In figure 3.93, m(arc WY) = 44°, m(arc ZX) = 68°, then

(1) Find the measure of ∠ ZTX.

(2) If WT = 4.8, TX = 8.0, YT = 6.4, find TZ.

(3) If WX = 25, YT = 8, YZ = 26, find WT.

16. In figure 3.94,

(1) m(arc CE) = 54°,

m(arc BD) = 23°, find measure of ∠CAE.

(2) If AB = 4.2, BC = 5.4,

AE = 12.0, find AD

(3) If AB = 3.6, AC = 9.0,

AD = 5.4, find AE

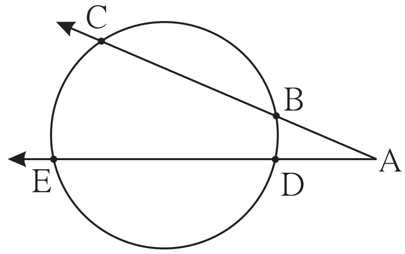

Fig. 3.94

17. In figure 3.95, chord EF || chord GH. Prove that, chord EG ≅ chord FH.

Fill in the blanks and write the proof.

Proof : Draw seg GF.

∠ EFG = ∠ FGH .... ☐ (I)

∠ EFG = ☐ .... inscribed angle theorem (II)

∠ FGH = ☐ .... inscribed angle theorem (III)

∴ m(arc EG) = ☐ from (I), (II), (III).

chord EG ≅ chord FH .... ☐

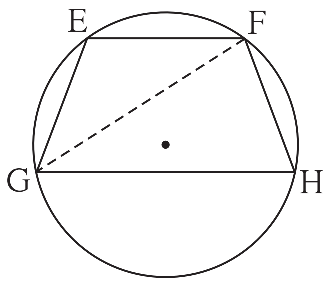

Fig. 3.95

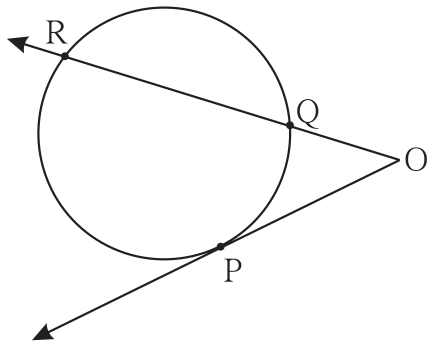

Fig. 3.96

18. In figure 3.96 P is the point of contact.

(1) If m(arc PR) = 140°,

∠ POR = 36°,

find m(arc PQ)

(2) If OP = 7.2, OQ = 3.2,

find OR and QR

(3) If OP = 7.2, OR = 16.2,

find QR.

19. In figure 3.97, circles with centres C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at point A. Prove that, seg EA ≅ seg AB.

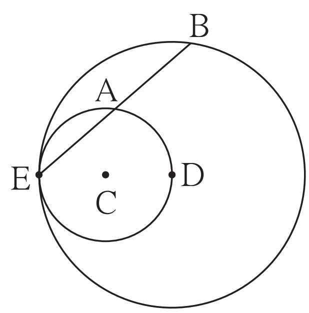

Fig. 3.97

20. In figure 3.98, seg AB is a diameter of a circle with centre O . The bisector of $\angle$ACB intersects the circle at point D. Prove that, seg AD $\cong$ seg BD. Complete the following proof by filling in the blanks.

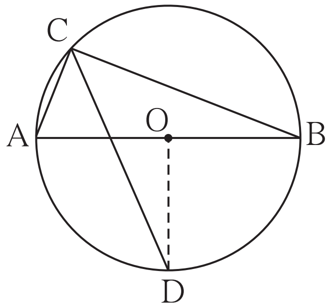

Fig. 3.98

Proof : Draw seg OD.

$\angle$ ACB = [ ] .... angle inscribed in semicircle

$\angle$ DCB = [ ] .... CD is the bisector of $\angle$ C

m(arc DB) = [ ] .... inscribed angle theorem

$\angle$ DOB = [ ] .... definition of measure of an arc (I)

seg OA $\cong$ seg OB .... [ ] (II)

$\therefore$ line OD is [ ] of seg AB .... From (I) and (II)

$\therefore$ seg AD $\cong$ seg BD

21. In figure 3.99, seg MN is a chord of a circle with centre O. MN = 25, L is a point on chord MN such that ML = 9 and d(O,L) = 5. Find the radius of the circle.

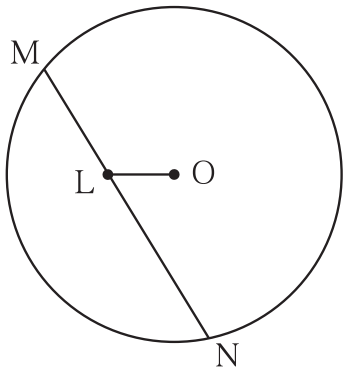

Fig. 3.99

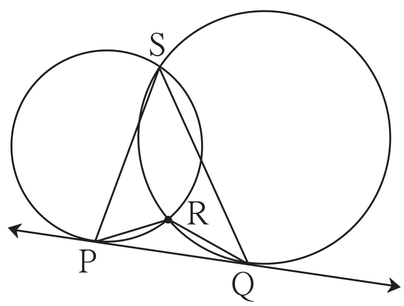

Fig. 3.100

22*. In figure 3.100, two circles intersect each other at points S and R. Their common tangent PQ touches the circle at points P, Q.
Prove that, $\angle$ PRQ + $\angle$ PSQ = 180°

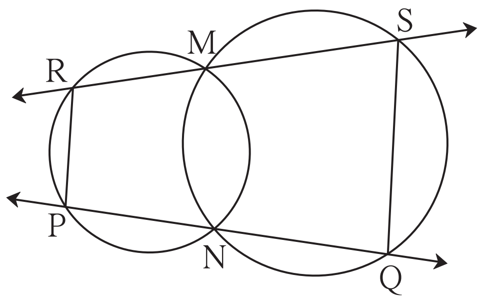

Fig. 3.101

23★. In figure 3.101, two circles intersect at points M and N. Secants drawn through M and N intersect the circles at points R, S and P, Q respectively.
Prove that : seg SQ || seg RP.

24★. In figure 3.102, two circles intersect each other at points A and E. Their common secant through E intersects the circles at points B and D. The tangents of the circles at points B and D intersect each other at point C.
Prove that □ ABCD is cyclic.

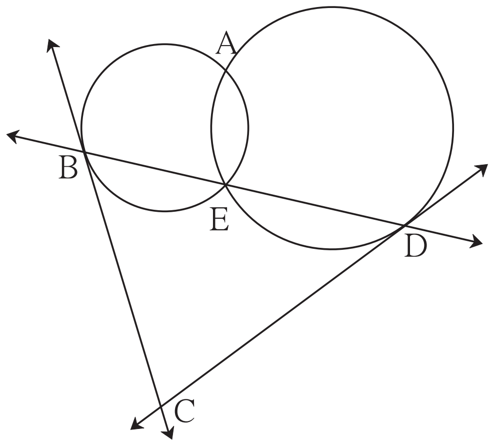

Fig. 3.102

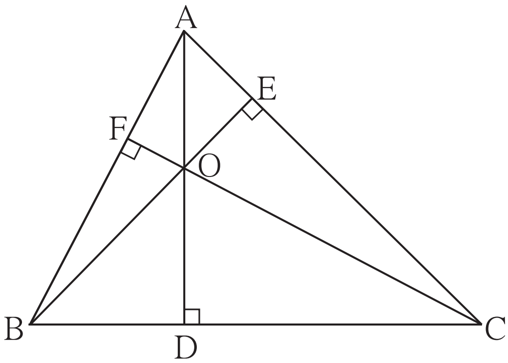

Fig. 3.103

25★. In figure 3.103, seg AD ⊥ side BC, seg BE ⊥ side AC, seg CF ⊥ side AB. Ponit O is the orthocentre. Prove that , point O is the incentre of Δ DEF.

ICT Tools or Links

Use the geogebra to verify the properties of chords and tangents of a circle.

PUEAJJ

Downloaded from https:// www.studiestoday.com

4 Geometric Constructions

Let's study.

- Construction of a triangle similar to the given triangle
  - ✻ To construct a triangle, similar to the given triangle, bearing the given ratio with the sides of the given triangle.
    - (i) When vertices are distinct
    - (ii) When one vertex is common
- Construction of a tangent to a circle.
  - ✻ To construct a tangent at a point on the circle.
    - (i) Using centre of the circle.
    - (ii) Without using the centre of the circle.
  - ✻ To construct tangents to the given circle from a point outside the circle.

Let's recall.

In the previous standard you have learnt the following constructions. Let us recall those constructions.

- To construct a line parallel to a given line and passing through a given point outside the line.
- To construct the perpendicular bisector of a given line segment.
- To construct a triangle whose sides are given.
- To divide a given line segment into given number of equal parts
- To divide a line segment in the given ratio.
- To construct an angle congruent to the given angle.

In the ninth standard you have carried out the activity of preparing a map of surroundings of your school. Before constructing a building we make its plan. The surroundings of a school and its map, the building and its plan are similar to each other. We need to draw similar figures in Geography, architecture, machine drawing etc. A triangle is the simplest closed figure. We shall learn how to construct a triangle similar to the given triangle.

Let's learn.

Construction of Similar Triangle

To construct a triangle similar to the given triangle, satisfying the condition of given ratio of corresponding sides.

The corresponding sides of similar triangles are in the same proportion and the corresponding angles of these triangles are equal. Using this property, a triangle which is similar to the given triangle can be constructed.

Ex. (1) Δ ABC ~ Δ PQR, in Δ ABC, AB = 5.4 cm, BC = 4.2 cm, AC = 6.0 cm. AB: PQ = 3:2. Construct Δ ABC and Δ PQR.

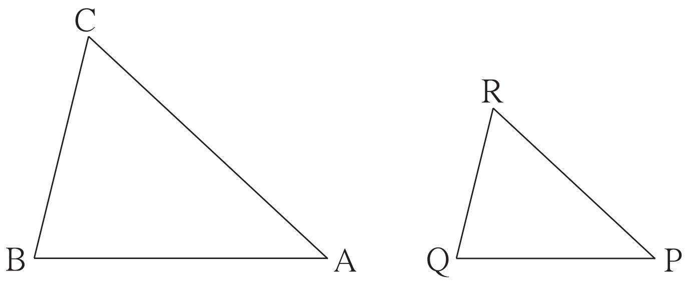

Fig. 4.1
Rough Figure

Construct Δ ABC of given measure.

Δ ABC and Δ PQR are similar.

$\therefore$ their corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2} \quad \text{..........} \quad \text{(I)}$$

As the sides AB, BC, AC are known, we can find the lengths of sides PQ, QR, PR.

Using equation [I]

$$\frac{5.4}{PQ} = \frac{4.2}{QR} = \frac{6.0}{PR} = \frac{3}{2}$$

$\therefore$ PQ = 3.6 cm, QR = 2.8 cm and PR = 4.0 cm

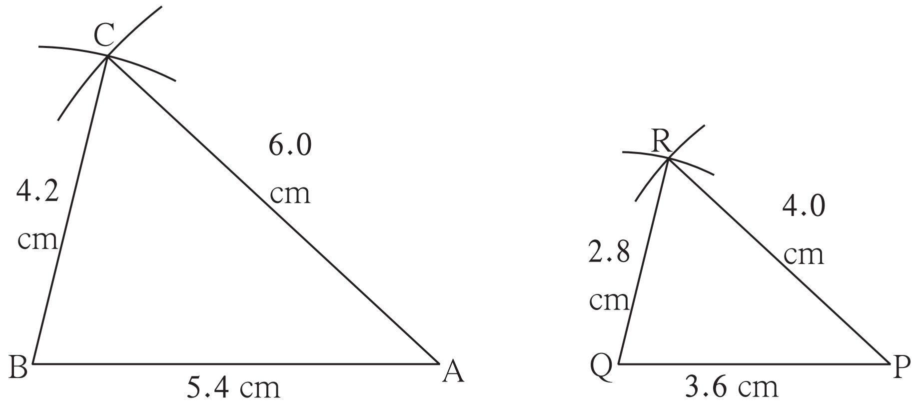

Fig. 4.2

> **For More Information**
>
> While drawing the triangle similar to the given triangle, sometimes the lengths of the sides we obtain by calculation are not easily measureable by a scale. In such a situation we can use the construction 'To divide the given segment in the given number of eqaul parts'.
>
> For example, if length of side AB is $\frac{11.6}{3}$ cm, then by dividing the line segment of length 11.6 cm in three equal parts, we can draw segment AB.

If we know the lengths of all sides of Δ PQR, we can construct Δ PQR.

In the above example (1) there was no common vertex in the given triangle and the triangle to be constructed. If there is a common vertex, it is convenient to follow the method in the following example.

Ex.(2) Construct any Δ ABC. Construct Δ A′BC′ such that AB : A′B = 5:3 and Δ ABC ~ Δ A′BC′

Analysis : As shown in fig 4.3 , let the points B, A, A′ and B, C, C′ be collinear.

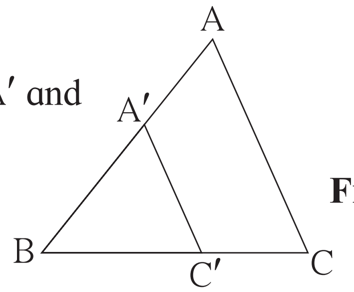

Fig. 4.3

Rough Figure

Δ ABC ~ Δ A′BC′ $\therefore$ $\angle$ ABC = $\angle$A′BC′

$$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{5}{3}$$

$\therefore$ sides of Δ ABC are longer than corresponding sides of Δ A′BC′ .

$\therefore$ the length of side BC′ will be equal to 3 parts out of 5 equal parts of side BC. So if we construct Δ ABC, point C′ will be on the side BC, at a distance equal to 3 parts from B. Now A′ is the point of intersection of AB and a line through C′, parallel to CA.

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{3}{5} \text{ i.e, } \frac{BA}{BA'} = \frac{BC}{BC'} = \frac{5}{3} \text{ ...... Taking inverse}$$

Steps of construction :

(1) Construct any Δ ABC.

(2) Divide segment BC in 5 equal parts.

(3) Name the end point of third part of seg BC as C′ $\therefore$ BC′ = $\frac{3}{5}$ BC

(4) Now draw a line parallel to AC through C′. Name the point where the parallel line intersects AB as A′.

(5) Δ A′BC′ is the required trinangle similar to Δ ABC

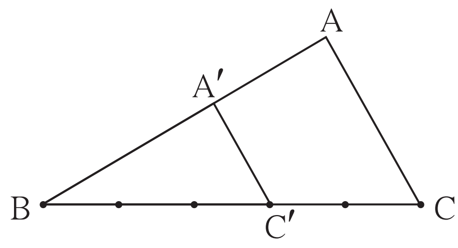

Fig. 4.4

Note : To divide segment BC, in five equal parts, it is convenient to draw a ray from B, on the side of line BC in which point A does not lie.

Take points T_1, T_2, T_3, T_4, T_5 on the ray such that

$BT_1 = T_1T_2 = T_2T_3 = T_3T_4 = T_4T_5$

Join T_5C and draw lines parallel to T_5C through T_1, T_2, T_3, T_4 .

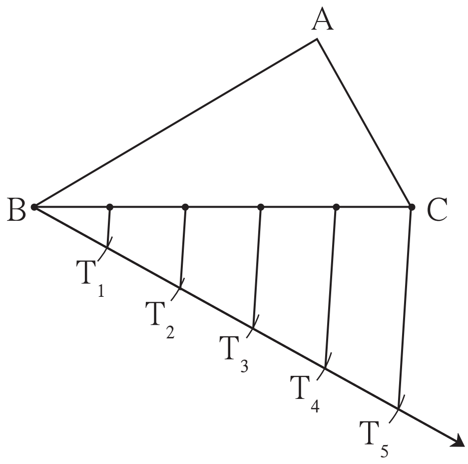

Fig. 4.5

Let's think

Δ A′BC′ can also be constructed as shown in the adjoining figure.

What changes do we have to make in steps of construction in that case ?

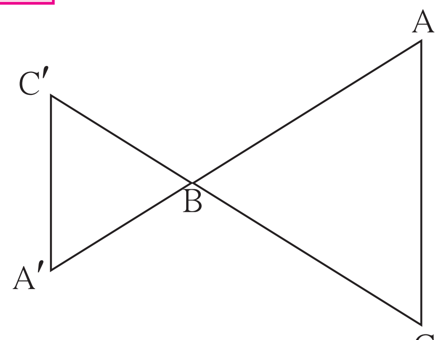

Fig. 4.6

Ex. (3) Construct Δ A′BC′ similar to Δ ABC such that AB:A′B = 5:7

Analysis : Let points B, A, A′ as well as points B, C, C′ be collinear.

Δ ABC ~ Δ A′BC′ and AB : A′B = 5:7

$\therefore$ sides of Δ ABC are smaller than sides of Δ A′BC′

and $\angle ABC \cong \angle A'BC'$

Let us draw a rough figure with these considerations. Now $\frac{BC}{BC'} = \frac{5}{7}$

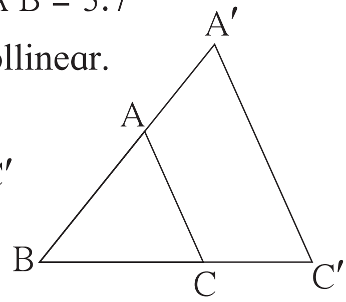

Fig. 4.7
Rough Figure

$\therefore$ If seg BC is divided into 5 equal parts, then seg BC′ will be 7 times each part of seg BC.

$\therefore$ let us divide side BC of Δ ABC in 5 equal parts and locate point C′ at a distance equal to 7 such parts from B on ray BC. A line through point C′ parallel to seg AC is drawn it will intersect ray BA at point A′. According to the basic proportionality theorem we will get Δ A′BC′ as described.

Steps of construction :

(1) Construct any Δ ABC.

(2) Divide segment BC into 5 five equal parts. Fix point C′ on ray BC such that length of BC′ is seven times of each equal part of seg BC

(3) Draw a line parallel to side AC, through C′. Name the point of intersection of the line and ray BA as A′.

We get the required Δ A′BC′ similar to Δ ABC.

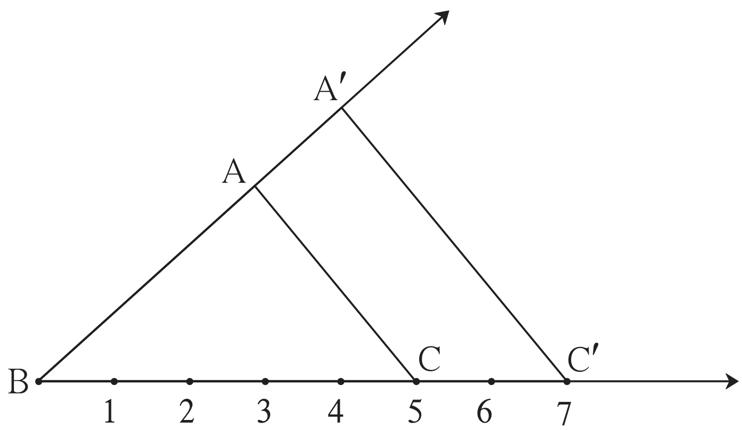

Fig. 4.8

Practice set 4.1

1. Δ ABC ~ Δ LMN. In Δ ABC, AB = 5.5 cm, BC = 6 cm, CA = 4.5 cm. Construct Δ ABC and Δ LMN such that $\frac{BC}{MN} = \frac{5}{4}$.
2. Δ PQR ~ Δ LTR. In Δ PQR, PQ = 4.2 cm, QR = 5.4 cm, PR = 4.8 cm. Construct Δ PQR and Δ LTR, such that $\frac{PQ}{LT} = \frac{3}{4}$.
3. Δ RST ~ ΔXYZ. In Δ RST, RS = 4.5 cm, $\angle$RST = 40°, ST = 5.7 cm Construct Δ RST and Δ XYZ, such that $\frac{RS}{XY} = \frac{3}{5}$.
4. Δ AMT ~ Δ AHE. In Δ AMT, AM = 6.3 cm, $\angle$TAM = 50°, AT = 5.6 cm. $\frac{AM}{AH} = \frac{7}{5}$. Construct Δ AHE.

Construction of a tangent to a circle at a point on the circle

(i) Using the centre of the circle.

Analysis :

Suppose we want to construct a tangent l passing through a point P on the circle with centre C. We shall use the property that a line perpendicular to the radius at its outer end is a tangent to the circle. If CP is a radius with point P on the circle, line l through P and perpendicular to CP is the tangent at P. For this we will use the construction of drawing a perpendicular to a line through a point on it.

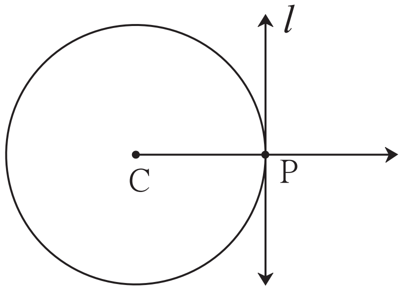

Fig. 4.9

For convenience we shall draw ray CP

Steps of construction

(1) Draw a circle with centre C. Take any point P on the circle.

(2) Draw ray CP.

(3) Draw line l perpendicular to ray CX through point P. Line l is the required tangent to the circle at point 'P'.

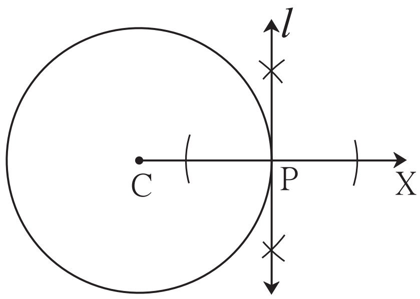

Fig. 4.10

ii) Without using the centre of the circle.

Example: Construct a circle of any radius. Take any point C on it. Construct a tangent to the circle without using centre of the circle.

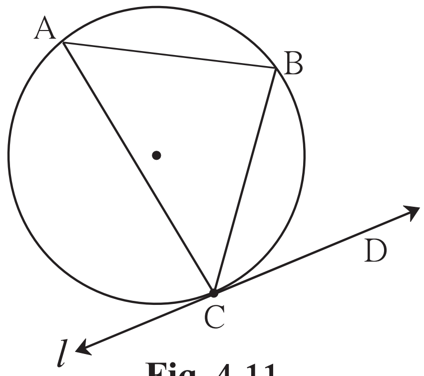

Fig. 4.11

Analysis :

As shown in the figure, let line l be the tangent to the circle at point C. Line CB is a chord and ∠ CAB is an inscribed angle. Now by tangent- secant angle theorem, ∠CAB ≅ ∠BCD.

By converse of tangent- secant theorem, if we draw the line CD such that, ∠CAB ≅ ∠BCD, then it will be the required tangent.

Steps of Construction :

(1) Draw a circle of a suitable radius. Take any point C on it.

(2) Draw chord CB and an inscribed ∠CAB .

(3) With the centre A and any convenient radius draw an arc intersecting the sides of ∠BAC in points M and N.

(4) Using the same radius and centre C, draw an arc intersecting the chord CB at point R.

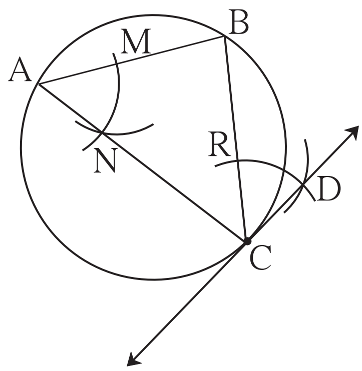

Fig. 4.12

(5) Taking the radius equal to d(MN) and centre R, draw an arc intersecting the arc drawn in the previous step. Let D be the point of intersection of these arcs. Draw line CD. Line CD is the required tangent to the circle.

Note that ∠ MAN and ∠ BCD in the above figure are congruent. If we draw seg MN and seg RD, then Δ MAN and Δ RCD are congruent by SSS test.

∴ ∠ MAN ≅ ∠ BCD

To construct tangents to a circle from a point outside the circle.

Analysis :

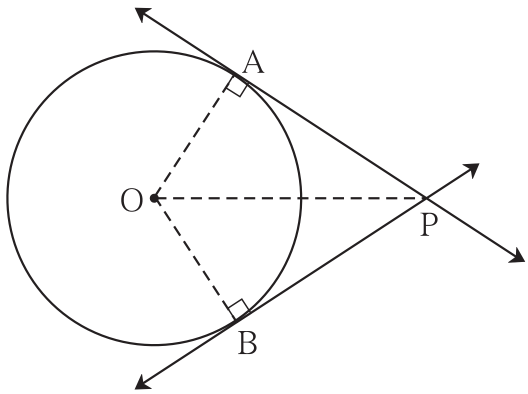

Fig. 4.13

As shown in the figure let P be a point in the exterior of the circle. Let PA and PB be the tangents to the circle with the centre O, touching the circle in points A and B respectively. So if we find points A and B on the circle, we can construct the tangents PA and PB. If OA and OB are the radii of the circle, then OA $\perp$ line PA and seg OB $\perp$ line PB.

Δ OAP and OBP are right angled triangles and seg OP is their common hypotenuse. If we draw a circle with diameter OP, then the points where it intersects the circle with centre O, will be the positions of points A and B respectively, because angle inscribed in a semicircle is a right angle.

Steps of Construction

(1) Construct a circle of any radius with centre O.

(2) Take any point P in the exterior of the circle.

(3) Draw segment OP. Draw perpendicular bisector of seg OP to get its midpoint M.

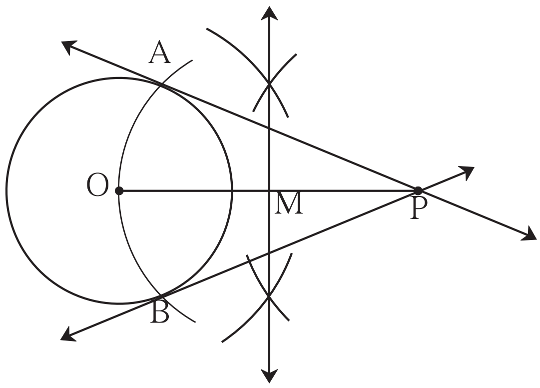

Fig. 4.14

(4) Draw a circle with radius OM and centre M

(5) Name the points of intersection of the two circles as A and B.

(6) Draw line PA and line PB.

Practice set 4.2

1. Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it.

2. Draw a circle of radius 2.7 cm. Draw a tangent to the circle at any point on it.

3. Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre.

4. Draw a circle of radius 3.3 cm Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q. Write your obeservation about the tangents.

5. Draw a circle with radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. construct tangents at point M and N to the circle.

6. Draw a circle with centre P and radius 3.4 cm. Take point Q at a distance 5.5 cm from the centre. Construct tangents to the circle from point Q.

7. Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

Problem set 4

1. Select the correct alternative for each of the following questions.

(1) The number of tangents that can be drawn to a circle at a point on the circle is ....

(A) 3 (B) 2 (C) 1 (D) 0

(2) The maximun number of tangents that can be drawn to a circle from a point out side it is ....

(A) 2 (B) 1 (C) one and only one (D) 0

(3) If $\Delta ABC \sim \Delta PQR$ and $\frac{AB}{PQ} = \frac{7}{5}$, then ....

(A) Δ ABC is bigger. (B) Δ PQR is bigger.

(C) Both triangles will be equal. (D) Can not be decided.

2. Draw a circle with centre O and radius 3.5 cm. Take point P at a distance 5.7 cm from the centre. Draw tangents to the circle from point P.

3. Draw any circle. Take any point A on it and construct tangent at A without using the centre of the circle.

4. Draw a circle of diameter 6.4 cm. Take a point R at a distance equal to its diameter from the centre. Draw tangents from point R.

5. Draw a circle with centre P. Draw an arc AB of 100° measure. Draw tangents to the circle at point A and point B.

6. Draw a circle of radius 3.4 cm and centre E. Take a point F on the circle. Take another point A such that E-F-A and FA = 4.1 cm. Draw tangents to the circle from point A.

7. Δ ABC $\sim$ ΔLBN. In Δ ABC, AB = 5.1cm, $\angle B = 40^\circ$, BC=4.8 cm, $\frac{AC}{LN} = \frac{4}{7}$. Construct Δ ABC and ΔLBN.

8. Construct Δ PYQ such that, PY = 6.3 cm, YQ = 7.2 cm, PQ = 5.8 cm. If $\frac{YZ}{YQ} = \frac{6}{5}$, then construct Δ XYZ similar to Δ PYQ.

PUN6L7

5 Co-ordinate Geometry

Let's study.

- Distance formula
- Section formula
- Slope of a line

Let's recall.

We know how to find the distance between any two points on a number line.

If co-ordinates of points P,Q and R are $-1, -5$ and 4 respectively then find the length of seg PQ, seg QR.

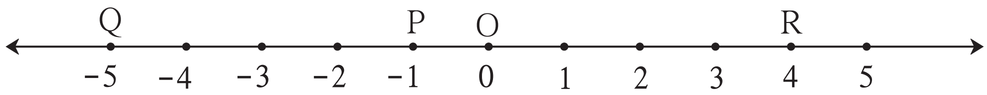

Fig. 5.1

If x_1 and x_2 are the co-ordinates of points A and B and $x_2 > x_1$ then length of seg AB = d(A,B) = $x_2 - x_1$

As shown in the figure, co-ordinates of points P,Q and R are $-1, -5$ and 4 respectively.

$\therefore$ d(P, Q) = $(-1)-(-5) = -1 + 5 = 4$

and d(Q, R) = $4 - (-5) = 4 + 5 = 9$

Using the same concept we can find the distance between two points on the same axis in XY-plane.

Let's learn.

(1) To find distance between any two points on an axis .

Two points on an axis are like two points on the number line. Note that points on the X-axis have co-ordinates such as $(2, 0)$, $(\frac{-5}{2}, 0)$, $(8, 0)$. Similarly points on the Y-axis have co-ordinates such as $(0, 1)$, $(0, \frac{17}{2})$, $(0, -3)$. Part of the X-axis which shows negative co-ordinates is OX′ and part of the Y-axis which shows negative co-ordinates is OY′.

i) To find distance between two points on X-axis.

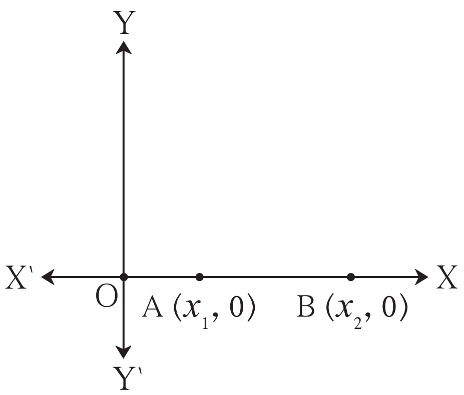

Fig. 5.2

In the above figure, points $A(x_1, 0)$ and $B(x_2, 0)$ are on X- axis such that, $x_2 > x_1$

$\therefore$ d(A, B) = $x_2 - x_1$

ii) To find distance between two points on Y-axis.

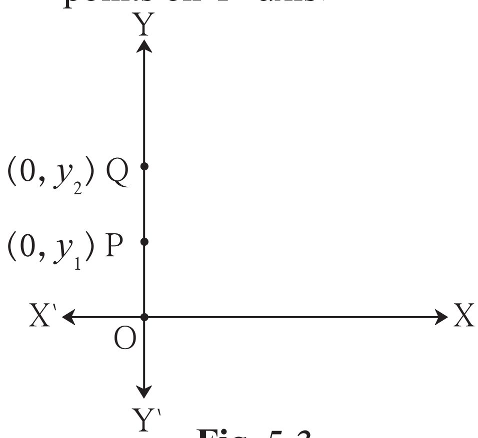

Fig. 5.3

In the above figure, points $P(0, y_1)$ and $Q(0, y_2)$ are on Y- axis such that, $y_2 > y_1$

$\therefore$ d(P,Q) = $y_2 - y_1$

(2) To find the distance between two points if the segment joining these points is parallel to any axis in the XY plane.

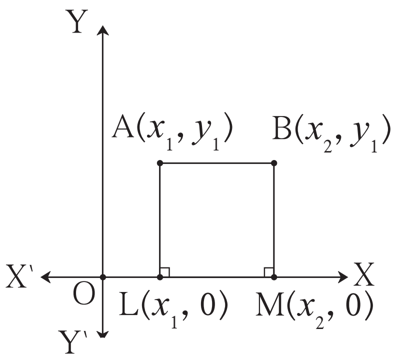

Fig. 5.4

i) In the figure, seg AB is parallel to X- axis.

$\therefore$ y co-ordinates of points A and B are equal

Draw seg AL and seg BM perpendicular to X-axis

$\therefore$ □ ABML is a rectangle.

$\therefore$ AB = LM

But, LM = $x_2 - x_1$

$\therefore$ d(A,B) = $x_2 - x_1$

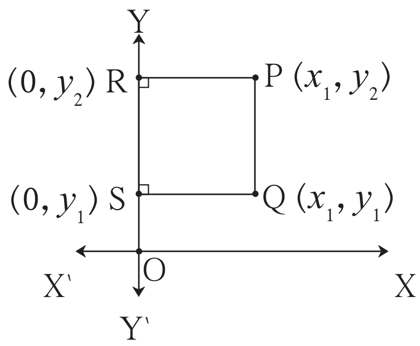

Fig. 5.5

ii) In the figure seg PQ is parallel to Y- axis.

$\therefore$ x co-ordinates of points P and Q are equal

Draw seg PR and seg QS perpendicular to Y-axis.

$\therefore$ □ PQSR is a rectangle

$\therefore$ PQ = RS

But, RS = $y_2 - y_1$

$\therefore$ d(P,Q) = $y_2 - y_1$

Activity:

In the figure, seg AB || Y-axis and seg CB || X-axis. Co-ordinates of points A and C are given.

To find AC, fill in the boxes given below.

Δ ABC is a right angled triangle.

According to Pythagoras theorem,

$(AB)^2 + (BC)^2 = \square$

We will find co-ordinates of point B to find the lengths AB and BC,

CB || X- axis ∴ y co-ordinate of B = $\square$

BA || Y- axis ∴ x co-ordinate of B = $\square$

AB = $\boxed{3} - \square = \square$ BC = $\square - \square = \boxed{4}$

∴ $AC^2 = \square + \square = \square$ ∴ AC = $\boxed{\sqrt{17}}$

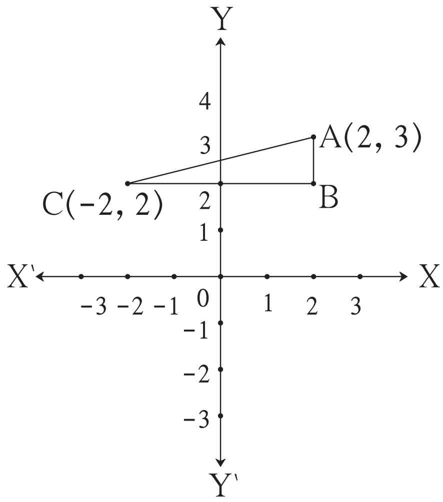

Fig. 5.6

Let's learn.

Distance formula

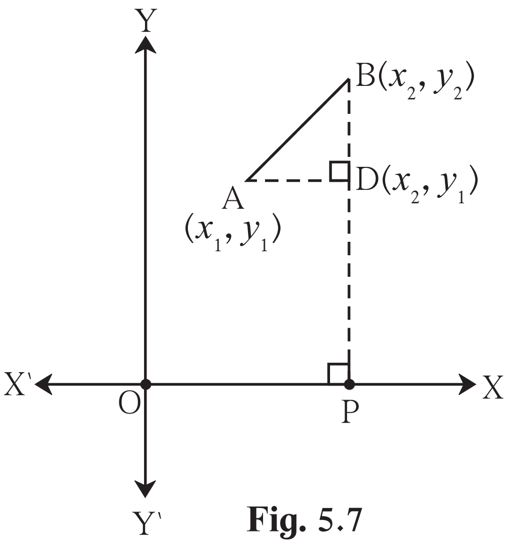

Fig. 5.7

In the figure 5.7, A(x_1, y_1) and B(x_2, y_2) are any two points in the XY plane.

From point B draw perpendicular BP on X−axis. Similarly from point A draw perpendicular AD on seg BP.

seg BP is parallel to Y−axis.

∴ the x co-ordinate of point D is x_2.

seg AD is parallel to X−axis.

∴ the y co-ordinate of point D is y_1.

∴ AD = d(A, D) = $x_2 - x_1$; BD = d(B, D) = $y_2 - y_1$

In right angled triangle Δ ABD, $AB^2 = AD^2 + BD^2$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is known as distance formula.

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Note that, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

In the previous activity, we found the lengths of seg AB and seg AC and then used Pythagoras theorem to find the length of seg AC.

Now we will use distance formula to find AC.

A(2, 3) and C(−2, 2) is given

Let $A(x_1, y_1)$ and $C(x_2, y_2)$.

$x_1 = 2, y_1 = 3, x_2 = -2, y_2 = 2$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-2)^2+(2-3)^2}$$
$$= \sqrt{(-4)^2+(-1)^2}$$
$$= \sqrt{16+1}$$
$$= \sqrt{17}$$

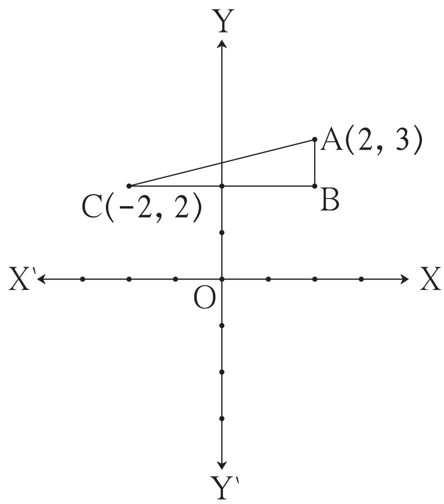


Fig. 5.8

seg AB || Y−axis and seg BC || X−axis.

∴ co-ordinates of point B are (2, 2).

$$\therefore AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-2)^2+(2-3)^2} = \sqrt{0+1} = 1$$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-2-2)^2+(2-2)^2} = \sqrt{(-4)^2+0} = 4$$

In the Figure 5.1, distance between points P and Q is found as (−1) − (−5) = 4. In XY- plane co-ordinates of these points are (−1, 0) and (−5, 0). Verify that, using the distance formula we get the same answer.

Remember this!

- Co-ordinates of origin are (0, 0). Hence if co-ordinates of point P are (x, y) then $d(O, P) = \sqrt{x^2+y^2}$.
- If points $P(x_1, y_1)$, $Q(x_2, y_2)$ lie on the XY plane then

  $d(P, Q) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

  that is, $PQ^2 = (x_2-x_1)^2+(y_2-y_1)^2 = (x_1-x_2)^2+(y_1-y_2)^2$

Solved Examples

Ex. (1) Find the distance between the points P(−1, 1) and Q (5,−7) .

Solution :Suppose co-ordinates of point P are (x_1, y_1) and of point Q are (x_2, y_2).

$x_1 = -1, \quad y_1 = 1, \quad x_2 = 5, \quad y_2 = -7$

According to distance formula, $d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{[5-(-1)]^2 + [(-7)-1]^2}$$

$$= \sqrt{(6)^2 + (-8)^2}$$

$$= \sqrt{36+64}$$

$$d(P, Q) = \sqrt{100} = 10$$

$\therefore$ distance between points P and Q is 10.

Ex. (2) Show that points A(−3, 2), B(1, −2) and C(9, −10) are collinear.

Solution : If the sum of any two distances out of d(A, B), d(B, C) and d(A, C) is equal to the third , then the three points A, B and C are collinear.

$\therefore$ we will find d(A, B), d(B, C) and d(A, C).

Co-ordinates of A	Co-ordinates of B	Distance formula
(−3, 2)	(1, −2)	$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(x_1, y_1)	(x_2, y_2)	

$\therefore d(A, B) = \sqrt{[1-(-3)]^2 + [(-2)-2]^2}$ .... from distance formula

$$= \sqrt{(1+3)^2 + (-4)^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32} = 4\sqrt{2} \quad \text{..........(I)}$$

$$d(B, C) = \sqrt{(9-1)^2 + (-10+2)^2}$$

$$= \sqrt{64+64} = 8\sqrt{2} \quad \text{..........(II)}$$

and $d(A, C) = \sqrt{(9+3)^2 + (-10-2)^2}$

$$= \sqrt{144+144} = 12\sqrt{2} \quad \text{..........(III)}$$

$\therefore$ from(I), (II) and (III) $4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2}$

$\therefore$ d(A, B) + d(B, C) = d(A, C)

$\therefore$ Points A, B, C are collinear.

Ex. (3) Verify, whether points P(6, −6), Q(3, −7) and R(3, 3) are collinear.

Solution : $PQ = \sqrt{(6-3)^2+(-6+7)^2}$ ....by distance formula

$= \sqrt{(3)^2+(1)^2} = \sqrt{10}$ .... (I)

$QR = \sqrt{(3-3)^2+(-7-3)^2}$

$= \sqrt{(0)^2+(-10)^2} = \sqrt{100}$ .... (II)

$PR = \sqrt{(3-6)^2+(3+6)^2}$

$= \sqrt{(-3)^2+(9)^2} = \sqrt{90}$ .... (III)

From I, II and III out of $\sqrt{10}$, $\sqrt{100}$ and $\sqrt{90}$, $\sqrt{100}$ is the largest number.

Now we will verify whether $\left(\sqrt{100}\right)$ and $\left(\sqrt{10}+\sqrt{90}\right)$ are equal.

For this compare $\left(\sqrt{100}\right)^2$ and $\left(\sqrt{10}+\sqrt{90}\right)^2$.

You will see that $\left(\sqrt{10}+\sqrt{90}\right) > \left(\sqrt{100}\right)$ $\therefore$ PQ + PR ≠ QR

$\therefore$ points P(6, −6), Q(3, −7) and R(3, 3) are not collinear.

Ex. (4) Show that points (1, 7), (4, 2), (−1, −1) and (−4, 4) are vertices of a square.

Solution : In a quadrilateral, if all sides are of equal length and both diagonals are of equal length, then it is a square.

$\therefore$ we will find lengths of sides and diagonals by using the distance formula.

Suppose, A(1, 7), B(4, 2), C(−1, −1) and D(−4,4) are the given points.

$AB = \sqrt{(1-4)^2+(7-2)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(4+1)^2+(2+1)^2} = \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2+(-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(1+4)^2+(7-4)^2} = \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1+1)^2+(7+1)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(4+4)^2+(2-4)^2} = \sqrt{64+4} = \sqrt{68}$

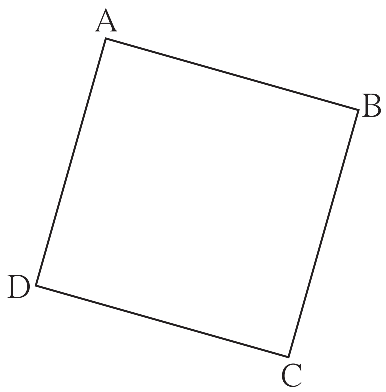


Fig. 5.9

$\therefore$ AB = BC = CD = DA and AC = BD

So, the lengths of four sides are equal and two diagonals are equal.

$\therefore$ (1,7), (4,2), (−1,−1) and (−4,4) are the vertices of a square.

Ex. (5) Find the co-ordinates of a point on Y- axis which is equidistant from M (−5,−2) and N(3,2)

Solution : Let point P(0, y) on Y- axis be equidistant from M(−5,−2) and N(3,2).

$$\therefore PM = PN \quad \therefore PM^2 = PN^2$$

$$\therefore [0-(-5)]^2 + [y-(-2)]^2 = (0-3)^2 + (y-2)^2$$

$$\therefore 25 + (y+2)^2 = 9 + y^2 - 4y + 4$$

$$\therefore 25 + y^2 + 4y + 4 = 13 + y^2 - 4y$$

$$\therefore 8y = -16 \quad \therefore y = -2$$

$\therefore$ the co-ordinates of the point on the Y-axis which is equidistant from M and N are M (0, −2).

Ex. (6) A(−3, −4), B(−5, 0), C(3, 0) are the vertices of Δ ABC. Find the co-ordinates of the circumcentre of Δ ABC.

Solution : Let, P(a, b) be the circumcentre of Δ ABC.

$\therefore$ point P is equidistant from A,B and C.

$\therefore PA^2 = PB^2 = PC^2$ .... (I) $\quad \therefore PA^2 = PB^2$

$$(a+3)^2 + (b+4)^2 = (a+5)^2 + (b-0)^2$$

$$\therefore a^2 + 6a + 9 + b^2 + 8b + 16 = a^2 + 10a + 25 + b^2$$

$$\therefore -4a + 8b = 0$$

$$\therefore a - 2b = 0 \text{ .... (II)}$$

Similarly $PA^2 = PC^2$ .... (I) From

$$\therefore (a+3)^2 + (b+4)^2 = (a-3)^2 + (b-0)^2$$

$$\therefore a^2 + 6a + 9 + b^2 + 8b + 16 = a^2 - 6a + 9 + b^2$$

$$\therefore 12a + 8b = -16$$

$$\therefore 3a + 2b = -4 \text{ .... (III)}$$

Solving (II) and (III) we get $a = -1$, $b = -\frac{1}{2}$

$\therefore$ co-ordinates of circumcentre are $(-1, -\frac{1}{2})$.

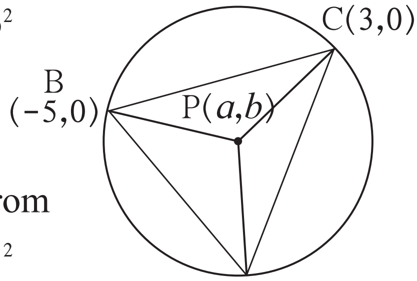


Fig. 5.10

Ex. (7) If point (x, y) is equidistant from points (7, 1) and (3, 5), show that $y = x-2$.

Solution : Let point P (x, y) be equidistant from points A(7, 1) and B(3, 5)

$$\therefore AP = BP$$

$$\therefore AP^2 = BP^2$$

$$\therefore (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$$

$$\therefore x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$

$$\therefore - 8x + 8y = -16$$

$$\therefore x - y = 2$$

$$\therefore y = x - 2$$

Ex. (8) Find the value of y if distance between points A (2,−2) and B (−1, y)is 5.

Solution : $AB^2 = [(-1) - 2]^2 + [y - (-2)]^2$ .... by distance formula

$\therefore 5^2 = (-3)^2 + (y + 2)^2$

$\therefore 25 = 9 + (y + 2)^2$

$\therefore 16 = (y + 2)^2$

$\therefore y + 2 = \pm\sqrt{16}$

$\therefore y + 2 = \pm 4$

$\therefore y = 4 - 2$ or $y = -4 - 2$

$\therefore y = 2$ or $y = -6$

$\therefore$ value of y is 2 or −6.

Practice set 5.1

1. Find the distance between each of the following pairs of points.

(1) A(2, 3), B(4, 1) (2) P(−5, 7), Q(−1, 3) (3) R(0, −3), S(0, $\frac{5}{2}$)

(4) L(5, −8), M(−7, −3) (5) T(−3, 6), R(9, −10) (6) W($\frac{-7}{2}$, 4), X(11, 4)

2. Determine whether the points are collinear.

(1) A(1, −3), B(2, −5), C(−4, 7) (2) L(−2, 3), M(1, −3), N(5, 4)

(3) R(0, 3), D(2, 1), S(3, −1) (4) P(−2, 3), Q(1, 2), R(4, 1)

3. Find the point on the X−axis which is equidistant from A(−3, 4) and B(1, −4).

4. Verify that points P(−2, 2), Q(2, 2) and R(2, 7) are vertices of a right angled triangle.

5. Show that points P(2, -2), Q(7, 3), R(11, -1) and S (6, -6) are vertices of a parallelogram.
6. Show that points A(-4, -7), B(-1, 2), C(8, 5) and D(5, -4) are vertices of a rhombus ABCD.
7. Find x if distance between points L(x, 7) and M(1, 15) is 10.
8. Show that the points A(1, 2), B(1, 6), C($1 + 2\sqrt{3}$, 4) are vertices of an equilateral triangle.

Let's recall.

Property of intercepts made by three parallel lines :

In the figure line $l \parallel$ line $m \parallel$ line n,

line p and line q are transversals,

then $\frac{AB}{BC} = \frac{DE}{EF}$

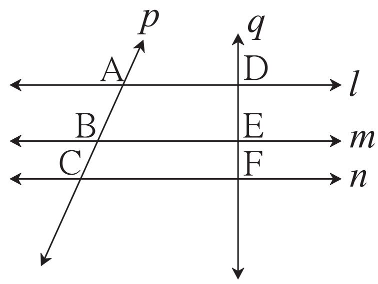


Fig. 5.11

Let's learn.

Division of a line segment

Fig. 5.12

In the figure, AP = 6 and PB = 10.

$\therefore \frac{AP}{PB} = \frac{6}{10} = \frac{3}{5}$

In other words it is said that 'point P divides the line segment AB in the ratio 3:5.

Let us see how to find the co-ordinates of a point on a segment which divides the segment in the given ratio.

Let's learn.

Section formula

In the figure 5.13, point P on the seg AB in XY plane, divides seg AB in the ratio $m : n$.

Assume $A(x_1, y_1)$ $B(x_2, y_2)$ and $P(x, y)$

Draw seg AC, seg PQ and seg BD perpendicular to X- axis.

$\therefore$ $C(x_1, 0)$; $Q(x, 0)$ and $D(x_2, 0)$.

$$\left.\begin{aligned}\therefore \quad CQ &= x - x_1 \\ \text{and } QD &= x_2 - x\end{aligned}\right\} \ldots\ldots\ldots\ (I)$$

seg AC || seg PQ || seg BD.

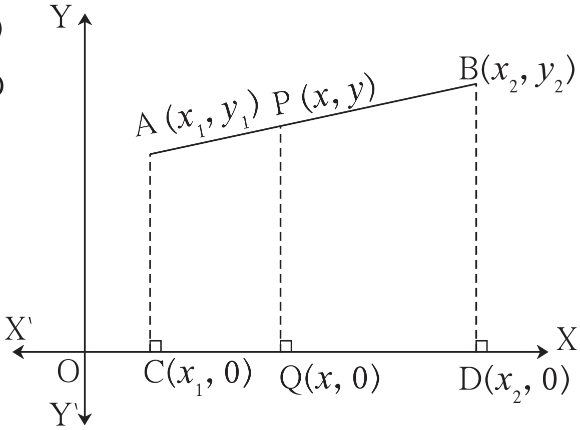


Fig. 5.13

$\therefore$ By the property of intercepts of three parallel lines, $\dfrac{AP}{PB} = \dfrac{CQ}{QD} = \dfrac{m}{n}$

Now $CQ = x - x_1$ and $QD = x_2 - x$ .... from (I)

$$\therefore \frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

$$\therefore n(x - x_1) = m(x_2 - x)$$

$$\therefore nx - nx_1 = mx_2 - mx$$

$$\therefore mx + nx = mx_2 + nx_1$$

$$\therefore x(m + n) = mx_2 + nx_1$$

$$\therefore x = \frac{mx_2 + nx_1}{m + n}$$

Similarly drawing perpendiculars from points A, P and B to Y- axis,

we get, $y = \dfrac{my_2 + ny_1}{m + n}$.

$\therefore$ co-ordinates of the point, which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ are given by

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right).$$

Co-ordinates of the midpoint of a segment

If A(x_1, y_1)and B(x_2, y_2) are two points and P (x, y) is the midpoint of seg AB then $m = n$.

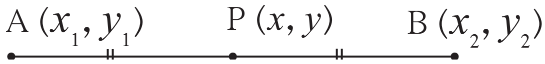


Fig. 5.14

$\therefore$ values of x and y can be written as

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{mx_2 + mx_1}{m+m} \quad \because \ m = n$$

$$= \frac{m(x_1 + x_2)}{2m}$$

$$= \frac{x_1 + x_2}{2}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{my_2 + my_1}{m+m} \quad \because \ m = n$$

$$= \frac{m(y_1 + y_2)}{2m}$$

$$= \frac{y_1 + y_2}{2}$$

$\therefore$ co-ordinates of midpoint P are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

This is called as **midpoint formula**.

In the previous standard we have shown that $\frac{a+b}{2}$ is the midpoint of the segment joining two points indicating rational numbers a and b on a number line. Note that it is a special case of the above midpoint formula.

Solved Examples

Ex. (1) If A(3,5), B(7,9) and point Q divides seg AB in the ratio 2:3 then find co-ordinates of point Q.

Solution : In the given example let (x_1, y_1)= (3, 5)

and (x_2, y_2)= (7, 9) .

$m : n = 2:3$

According to section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{2\times7+3\times3}{2+3} = \frac{23}{5} \qquad y = \frac{my_2 + ny_1}{m+n} = \frac{2\times9+3\times5}{2+3} = \frac{33}{5}$$

$\therefore$ Co-ordinates of Q are $\left(\frac{23}{5}, \frac{33}{5}\right)$

Ex. (2) Find the co-ordinates of point P if P is the midpoint of a line segment AB with A(–4,2) and B(6,2).

Solution : In the given example, suppose

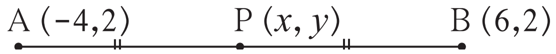


Fig. 5.15

$(-4, 2) = (x_1, y_1)$; $(6, 2) = (x_2, y_2)$ and co-ordinates of P are (x, y)

$\therefore$ according to midpoint theorem,

$$x = \frac{x_1 + x_2}{2} = \frac{-4+6}{2} = \frac{2}{2} = 1$$

$$y = \frac{y_1 + y_2}{2} = \frac{2+2}{2} = \frac{4}{2} = 2$$

$\therefore$ co-ordinates of midpoint P are (1,2) .

Let's recall.

We know that, medians of a triangle are concurrent .

The point of concurrence (centroid) divides the median in the ratio 2:1.

Let's learn.

Centroid formula

Suppose the co-ordinates of vertices of a triangle are given. Then we will find the co-ordinates of the centroid of the triangle.

In Δ ABC, $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices. Seg AD is a median and $G(x, y)$ is the centroid.

D is the mid point of line segment BC.

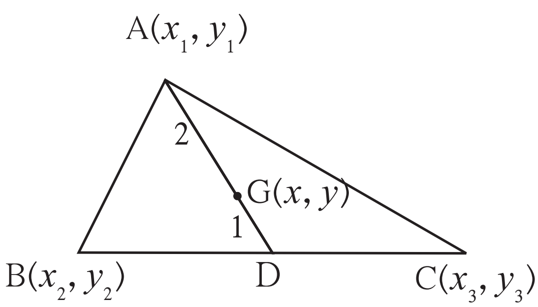


Fig. 5.16

$\therefore$ co-ordinates of point D are $x = \frac{x_2 + x_3}{2}$, $y = \frac{y_2 + y_3}{2}$ .... midpoint theorem

Point G(x, y) is centroid of triangle Δ ABC. $\therefore$ AG : GD = 2 : 1

$\therefore$ according to section formula,

$$x = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1 \times x_1}{2 + 1} = \frac{x_2 + x_3 + x_1}{3} = \frac{x_1 + x_2 + x_3}{3}$$

$$y = \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1 \times y_1}{2 + 1} = \frac{y_2 + y_3 + y_1}{3} = \frac{y_1 + y_2 + y_3}{3}$$

Thus if (x_1, y_1), (x_2, y_2) and $(x_3, y_3)$are the vertices of a triangle then the co-ordinates of the centroid are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

This is called the **centroid formula.**

Remember this!

- Section formula

  The co-ordinates of a point which divides the line segment joined by two distinct points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ are $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$.

- Midpoint formula

  The co-ordinates of midpoint of a line segment joining two distinct points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

- Centroid formula

  If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle then co-ordinates of the centroid are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Solved Examples

Ex. (1) If point T divides the segment AB with A(−7,4) and B(−6,−5) in the ratio 7:2, find the co-ordinates of T.

Solution : Let the co-ordinates of T be (x, y).

∴ by the section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{7\times(-6)+2\times(-7)}{7+2}$$

$$= \frac{-42-14}{9} = \frac{-56}{9}$$

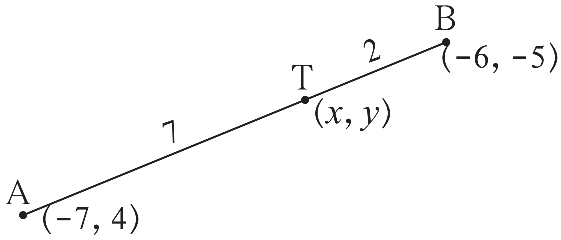


Fig. 5.17

$$y = \frac{my_2 + ny_1}{m+n} = \frac{7\times(-5)+2\times(4)}{7+2}$$

$$= \frac{-35+8}{9} = \frac{-27}{9} = -3$$

∴ co-ordinates of point T are $\left(\frac{-56}{9}, -3\right)$.

Ex. (2) If point P(−4, 6) divides the line segment AB with A(−6, 10) and B(r, s) in the ratio 2:1, find the co-ordinates of B.

Solution : By section formula

$$-4 = \frac{2 \times r + 1 \times (-6)}{2 + 1}$$

$$\therefore \quad -4 = \frac{2r - 6}{3}$$

$$\therefore \quad -12 = 2r - 6$$

$$\therefore \quad 2r = -6$$

$$\therefore \quad r = -3$$

$$6 = \frac{2 \times s + 1 \times 10}{2 + 1}$$

$$\therefore \quad 6 = \frac{2s + 10}{3}$$

$$\therefore \quad 18 = 2s + 10$$

$$\therefore \quad 2s = 8$$

$$\therefore \quad s = 4$$

∴ co-ordinates of point B are (−3, 4).

Ex. (3) A(15,5), B(9,20) and A−P−B. Find the ratio in which point P(11,15) divides segment AB.

Solution : Suppose, point P(11,15) divides segment AB in the ratio $m : n$

∴ by section formula,

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$\therefore\ 11 = \frac{9m+15n}{m+n}$$

$$\therefore\ 11m + 11n = 9m + 15n$$

$$\therefore\ 2m = 4n$$

$$\therefore\ \frac{m}{n} = \frac{4}{2} = \frac{2}{1}$$

$\therefore$ The required ratio is 2 : 1.

Similarly, find the ratio using y co-ordinates. Write the conclusion.

Ex. (4) Find the co-ordinates of the points of trisection of the segment joining the points A (2,−2) and B(−7,4) .

(The two points that divide the line segment in three equal parts are called as points of trisection of the segment.)

Solution : Let points P and Q be the points of trisection of the line segment joining the points A and B.

Point P and Q divide line segment AB into three parts.

AP = PQ = QB .... (I)

$$\frac{AP}{PB} = \frac{AP}{PQ+QB} = \frac{AP}{AP+AP} = \frac{AP}{2AP} = \frac{1}{2} \quad \text{.......... From (I)}$$

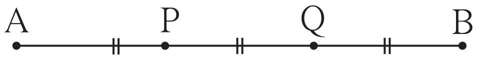


Fig. 5.18

Point P divides seg AB in the ratio 1:2.

$$x \text{ co-ordinate of point P} = \frac{1\times(-7)+2\times 2}{1+2} = \frac{-7+4}{3} = \frac{-3}{3} = -1$$

$$y \text{ co-ordinate of point P} = \frac{1\times 4+2\times(-2)}{1+2} = \frac{4-4}{3} = \frac{0}{3} = 0$$

Point Q divides seg AB in the ratio 2:1. $\therefore\ \frac{AQ}{QD} = \frac{2}{1}$

$$x \text{ co-ordinate of point Q} = \frac{2\times(-7)+1\times 2}{2+1} = \frac{-14+2}{3} = \frac{-12}{3} = -4$$

$$y \text{ co-ordinate of point Q} = \frac{2\times 4+1\times -2}{2+1} = \frac{8-2}{3} = \frac{6}{3} = 2$$

$\therefore$ co-ordinates of points of trisection are (−1, 0) and (−4, 2).

For more information :

See how the external division of the line segment joining points A and B takes place.

Let us see how the co-ordinates of point P can be found out if P divides the line segment joining points A(−4, 6) and B(5, 10) in the ratio 3:1 externally.

$\frac{AP}{PB} = \frac{3}{1}$ that is AP is larger than PB and A−B−P.

$\frac{AP}{PB} = \frac{3}{1}$ that is AP = $3k$, BP = k, then AB = $2k$

$$\therefore \frac{AB}{BP} = \frac{2}{1}$$

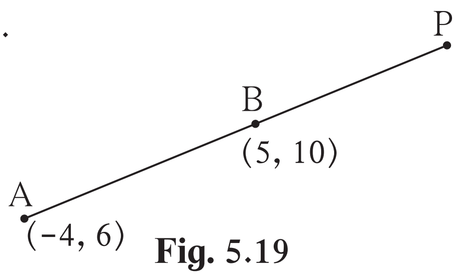


Fig. 5.19

Now point B divides seg AP in the ratio 2 : 1.

We have learnt to find the coordinates of point P if co-ordinates of points A and B are known.

Practice set 5.2

1. Find the coordinates of point P if P divides the line segment joining the points A(−1,7) and B(4,−3) in the ratio 2 : 3.

2. In each of the following examples find the co-ordinates of point A which divides segment PQ in the ratio $a : b$.

(1) P(−3, 7), Q(1, −4), $a : b = 2 : 1$

(2) P(−2, −5), Q(4, 3), $a : b = 3 : 4$

(3) P(2, 6), Q(−4, 1), $a : b = 1 : 2$

3. Find the ratio in which point T(−1, 6)divides the line segment joining the points P(−3, 10) and Q(6, −8).

4. Point P is the centre of the circle and AB is a diameter . Find the coordinates of point B if coordinates of point A and P are (2, −3) and (−2, 0) respectively.

5. Find the ratio in which point P(k, 7) divides the segment joining A(8, 9) and B(1, 2). Also find k .

6. Find the coordinates of midpoint of the segment joining the points (22, 20) and (0, 16).

7. Find the centroids of the triangles whose vertices are given below.

(1)(−7, 6), (2, −2), (8, 5)

(2) (3, −5), (4, 3), (11, −4)

(3) (4, 7), (8, 4), (7, 11)

8. In Δ ABC, G (-4, -7) is the centroid. If A (-14, -19) and B(3, 5) then find the co-ordinates of C.

9. A(h, -6), B(2, 3) and C(-6, k) are the co-ordinates of vertices of a triangle whose centroid is G (1, 5). Find h and k.

10. Find the co-ordinates of the points of trisection of the line segment AB with A(2, 7) and B(-4, -8).

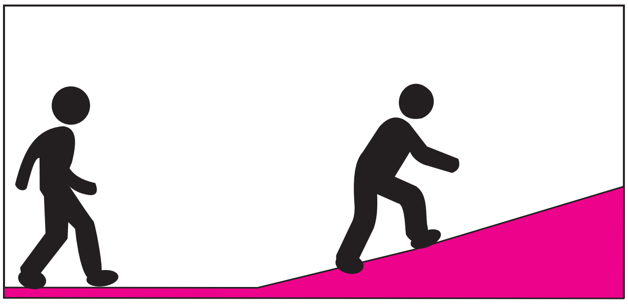
11. If A (-14, -10), B(6, -2) is given, find the coordinates of the points which divide segment AB into four equal parts.

12. If A (20, 10), B(0, 20) are given, find the coordinates of the points which divide segment AB into five congruent parts.

Let's learn.

Slope of a line

When we walk on a plane road we need not exert much effort but while climbing up a slope we need more effort. In science, we have studied that while climbing up a slope we have to work against gravitational force.



In co-ordinate geometry, slope of a line is an important concept. We will learn it through the following activity.

Activity I :

In the figure points A(-2, -5), B(0,-2), C(2,1), D(4,4), E(6,7) lie on line l. Observe the table which is made with the help of coordinates of these points on line l.

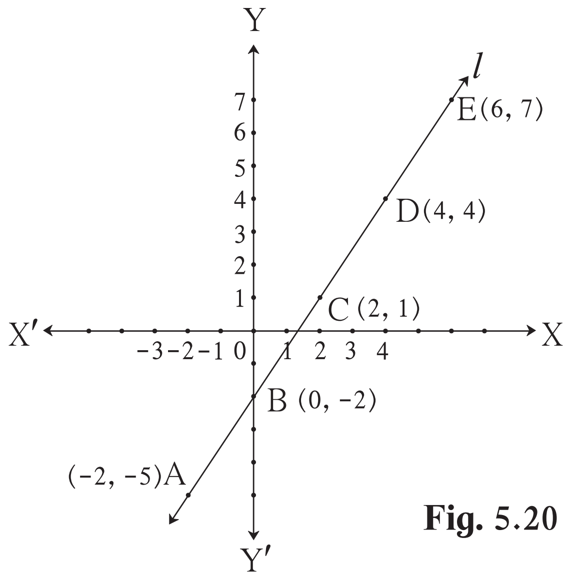


Fig. 5.20

Sr. No.	First point	Second point	Co-ordinates of first point (x_1, y_1)	Co-ordinates of second point (x_2, y_2)	$\frac{y_2 - y_1}{x_2 - x_1}$
1	C	E	(2, 1)	(6, 7)	$\frac{7-1}{6-2} = \frac{6}{4} = \frac{3}{2}$
2	A	D	(-2, -5)	(4, 4)	$\frac{4-(-5)}{4-(-2)} = \frac{9}{6} = \frac{3}{2}$
3	D	A	(4, 4)	(-2, -5)	$\frac{-5-4}{-2-4} = \frac{-9}{-6} = \frac{3}{2}$
4	B	C	--	--	--
5	C	A	--	--	--
6	A	C	--	--	--

Fill in the blank spaces in the above table. Similarly take some other pairs of points on line l and find the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ for each pair.

From this activity, we understand that for any two points (x_1, y_1) and (x_2, y_2) on line l, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is constant.

If (x_1, y_1) and (x_2, y_2) are any two points on line l, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is called the slope of the line l.

Generally slope is shown by letter m.

$$\therefore \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Activity II : In the figure, some points on line l, t and n are given. Find the slopes of those lines.

Now you will know,

(1) Slopes of line l and line t are positive.

(2) Slope of line n is negative.

(3) Slope of line t is more than slope of line l.

(4) Slopes of lines l and t which make acute angles with X- axis, are positive.

(5) Slope of line n making obtuse angle with X- axis is negative.

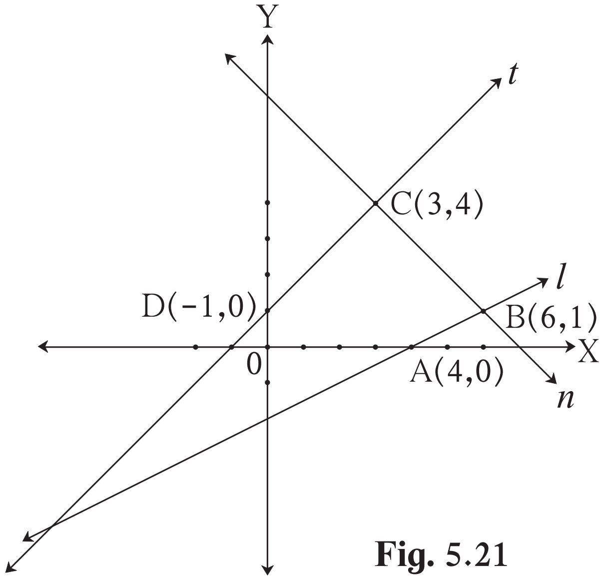


Fig. 5.21

Slopes of X-axis, Y-axis and lines parallel to axes.

In the figure 5.22, $(x_1, 0)$ and $(x_2, 0)$ are two points on the X- axis.

Slope of X- axis $= \dfrac{0 - 0}{x_2 - x_1} = 0$

In the same way $(0, y_1)$ and $(0, y_2)$ are two points on the Y- axis.

Slope of Y- axis $= \dfrac{y_2 - y_1}{0 - 0} = \dfrac{y_2 - y_1}{0}$,

But division by 0 is not possible.

$\therefore$ slope of Y- axis can not be determined.

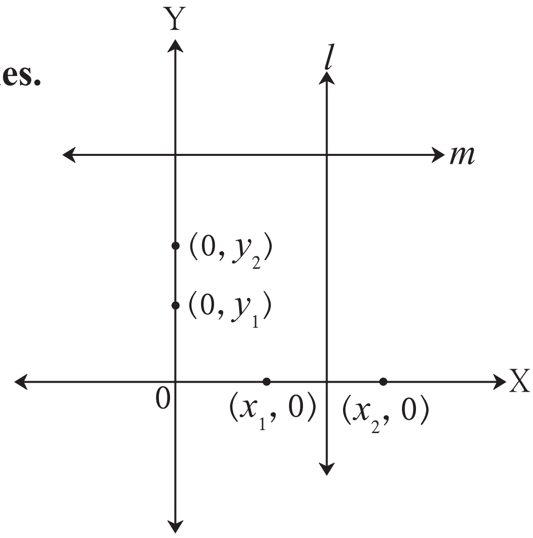


Fig. 5.22

Now try to find the slope of any line like line m which is parallel to X- axis. It will come out to be 0.

Similarly we cannot determine the slope of a line like line l which is parallel to Y- axis

Slope of line – using ratio in trigonometry

In the figure 5.23, points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are on line l.

Line l intersects X axis in point T.

seg QS $\perp$ X- axis, seg PR $\perp$ seg QS $\therefore$ seg PR || seg TS .... corresponding angle test

$\therefore$ QR $= y_2 - y_1$ and PR $= x_2 - x_1$

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$\therefore \quad \frac{QR}{PR} = \frac{y_2 - y_1}{x_2 - x_1}$ .... (I)

Line TQ makes an angle θ with the X- axis .

$\therefore \quad \frac{QR}{PR} = \tan\theta$ .... (II)

$\therefore$ From (I) and (II), $\frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$

$\therefore \quad m = \tan\theta$

Now seg PR || seg TS, line l is transversal

$\therefore \quad \angle QPR = \angle QTS$ ......corresponding angles

From this, we can define slope in another way. The tan ratio of an angle made by the line with the positive direction of X-axis is called the slope of that line.

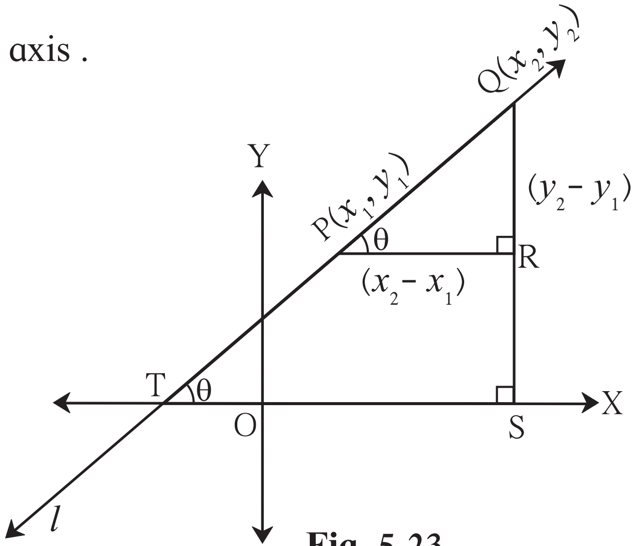

Fig. 5.23

When any two lines have same slope, these lines make equal angles with the positive direction of X- axis.

$\therefore$ These two lines are parallel.

Slope of Parallel Lines

Activity :

In the figure 5.22 both line l and line t make angle θ with the positive direction of X- axis.

$\therefore$ line l || line t .... corresponding angle test

Consider, point A(-3, 0) and point B(0, 3) on line l

Find the slope of line l.

Slope of line $l = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\square - \square}{\square - \square} = \frac{\square}{\square}$

$= \square$

In the similar way, consider suitable points on the line t and find the slope of line t.

From this, you can verify that parallel lines have equal slopes.

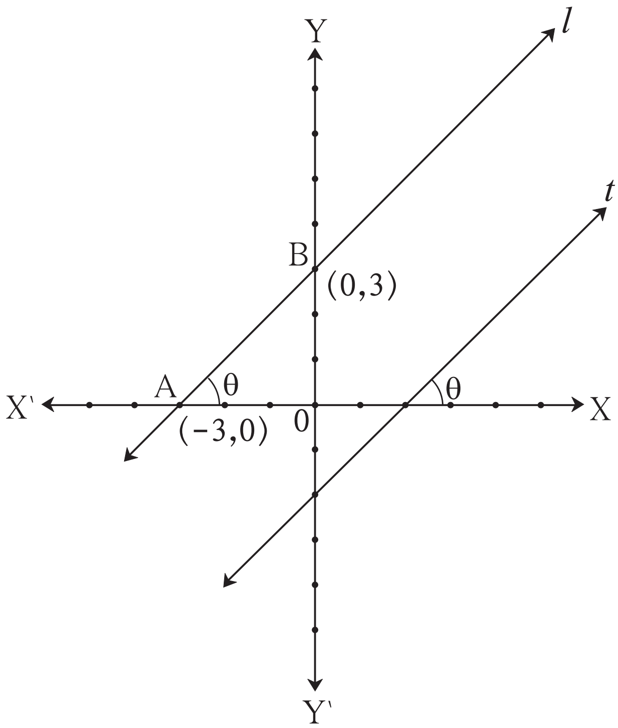

Fig. 5.24

Here $\theta = 45°$.

Use slope, $m = \tan\theta$ and verify that slopes of parallel lines are equal.

Similarly taking $\theta = 30°$, $\theta = 60°$ verify that slopes of parallel lines are equal.

Remember this!

The slope of X- axis and of any line parallel to X- axis is zero.

The slope of Y- axis and of any line parallel to Y- axis cannot be determined.

Solved Examples

EX. (1) Find the slope of the line passing through the points A (-3, 5), and B (4, -1)

Solution : Let, $x_1 = -3$, $x_2 = 4$, $y_1 = 5$, $y_2 = -1$

$$\therefore \text{ Slope of line AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{4-(-3)} = \frac{-6}{7}$$

EX. (2) Show that points P(-2, 3), Q(1, 2), R(4, 1) are collinear.

Solution : P(-2, 3), Q(1, 2) and R(4, 1) are given points

$$\text{slope of line PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{1-(-2)} = -\frac{1}{3}$$

$$\text{Slope of line QR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{4-1} = -\frac{1}{3}$$

Slope of line PQ and line QR is equal.

But point Q lies on both the lines.

$\therefore$ Point P, Q, R are collinear.

EX. (3) If slope of the line joining points P(k, 0) and Q(-3, -2) is $\frac{2}{7}$ then find k.

Solution : P(k, 0) and Q(-3, -2)

$$\text{Slope of line PQ} = \frac{-2-0}{-3-k} = \frac{-2}{-3-k}$$

But slope of line PQ is given to be $\frac{2}{7}$.

$$\therefore \frac{-2}{-3-k} = \frac{2}{7} \qquad \therefore k = 4$$

EX. (4) If A (6, 1), B (8, 2), C (9, 4) and D (7, 3) are the vertices of □ ABCD , show that □ ABCD is a parallelogram.

Solution : You know that Slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope of line AB $= \frac{2-1}{8-6} = \frac{1}{2}$ .... (I)

Slope of line BC $= \frac{4-2}{9-8} = 2$ .... (II)

Slope of line CD $= \frac{3-4}{7-9} = \frac{1}{2}$ .... (III)

Slope of line DA $= \frac{3-1}{7-6} = 2$ .... (IV)

Slope of line AB = Slope of line CD .... From (I) and (III)

$\therefore$ line AB || line CD

Slope of line BC = Slope of line DA .... From (II) and (IV)

$\therefore$ line BC || line DA

Both the pairs of opposite sides of the quadrilateral are parallel

$\therefore$ □ ABCD is a parallelogram.

Practice set 5.3

1. Angles made by the line with the positive direction of X–axis are given. Find the slope of these lines.

(1) 45° (2) 60° (3) 90°

2. Find the slopes of the lines passing through the given points.

(1) A (2, 3) , B (4, 7) (2) P (–3, 1) , Q (5, –2)

(3) C (5, –2) , D (7, 3) (4) L (–2, –3) , M (–6, –8)

(5) E(–4, –2) , F (6, 3) (6) T (0, –3) , S (0, 4)

3. Determine whether the following points are collinear.

(1) A(–1, –1), B(0, 1), C(1, 3) (2) D(–2, –3), E(1, 0), F(2, 1)

(3) L(2, 5), M(3, 3), N(5, 1) (4) P(2, –5), Q(1, –3), R(–2, 3)

(5) R(1, –4), S(–2, 2), T(–3, 4) (6) A(–4, 4), K(–2, $\frac{5}{2}$), N(4, –2)

4. If A (1, –1),B (0, 4),C (–5, 3) are vertices of a triangle then find the slope of each side.

5. Show that A (–4, –7),B (–1, 2), C (8, 5) and D (5, –4) are the vertices of a parallelogram.

6. Find k, if R(1, −1), S (−2, k) and slope of line RS is −2.

7. Find k, if B(k, −5), C (1, 2) and slope of the line is 7.

8. Find k, if PQ || RS and P(2, 4), Q (3, 6), R(3, 1), S(5, k) .

Problem set 5

1. Fill in the blanks using correct alternatives.

(1) Seg AB is parallel to Y−axis and coordinates of point A are (1,3) then co-ordinates of point B can be ....

(A) (3,1) (B) (5,3) (C) (3,0) (D) (1,−3)

(2) Out of the following, point .... lies to the right of the origin on X− axis.

(A) (−2,0) (B) (0,2) (C) (2,3) (D) (2,0)

(3) Distance of point (−3,4) from the origin is ....

(A) 7 (B) 1 (C) 5 (D) −5

(4) A line makes an angle of 30° with the positive direction of X− axis. So the slope of the line is ....

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

2. Determine whether the given points are collinear.

(1) A(0,2) , B(1,−0.5), C(2,−3)

(2) P(1, 2) , Q(2, $\frac{8}{5}$) , R(3, $\frac{6}{5}$)

(3) L(1,2) , M(5,3) , N(8,6)

3. Find the coordinates of the midpoint of the line segment joining P(0,6) and Q(12,20).

4. Find the ratio in which the line segment joining the points A(3,8) and B(−9, 3) is divided by the Y− axis.

5. Find the point on X−axis which is equidistant from P(2,−5) and Q(−2,9).

6. Find the distances between the following points.

(i) A(a, 0), B(0, a) (ii) P(−6, −3), Q(−1, 9) (iii) R(−3a, a), S(a, −2a)

7. Find the coordinates of the circumcentre of a triangle whose vertices are (−3,1), (0,−2) and (1,3)

8. In the following examples, can the segment joining the given points form a triangle ? If triangle is formed, state the type of the triangle considering sides of the triangle.

(1) L(6,4) , M(–5,–3) , N(–6,8)

(2) P(–2,–6) , Q(–4,–2), R(–5,0)

(3) A($\sqrt{2}$, $\sqrt{2}$), B($-\sqrt{2}$, $-\sqrt{2}$), C($-\sqrt{6}$, $\sqrt{6}$)

9. Find k if the line passing through points P(–12,–3) and Q(4, k)has slope $\frac{1}{2}$.

10. Show that the line joining the points A(4, 8) and B(5, 5) is parallel to the line joining the points C(2,4) and D(1,7).

11. Show that points P(1,–2), Q(5,2), R(3,–1), S(–1,–5) are the vertices of a parallelogram

12. Show that the □ PQRS formed by P(2,1), Q(–1,3), R(–5,–3) and S(–2,–5) is a rectangle

13. Find the lengths of the medians of a triangle whose vertices are A(–1, 1), B(5, –3) and C(3, 5) .

14★. Find the coordinates of centroid of the triangles if points D(–7, 6), E(8, 5) and F(2, –2) are the mid points of the sides of that triangle.

15. Show that A(4, –1), B(6, 0), C(7, –2) and D(5, –3)are vertices of a square.

16. Find the coordinates of circumcentre and radius of circumcircle of Δ ABC if A(7, 1), B(3, 5) and C(2, 0) are given.

17. Given A(4,–3), B(8,5). Find the coordinates of the point that divides segment AB in the ratio 3:1.

18★. Find the type of the quadrilateral if points A(–4, –2), B(–3, –7) C(3, –2) and D(2, 3) are joined serially.

19★. The line segment AB is divided into five congruent parts at P, Q, R and S such that A-P-Q-R-S-B. If point Q(12, 14) and S(4, 18) are given find the coordinates of A, P, R,B.

20. Find the coordinates of the centre of the circle passing through the points P(6,–6), Q(3,–7)and R(3,3).

21★. Find the possible pairs of coordinates of the fourth vertex D of the parallelogram, if three of its vertices are A(5,6), B(1,–2)and C(3,–2).

22. Find the slope of the diagonals of a quadrilateral with vertices A(1,7), B(6,3), C(0,–3) and D(–3,3).

PUX2HU

6 Trigonometry

Let's study.

- Trigonometric ratios
- Angle of elevation and angle of depression
- Trigonometric identities
- Problems based on heights and distances

Let's recall.

1. Fill in the blanks with reference to figure 6.1 .

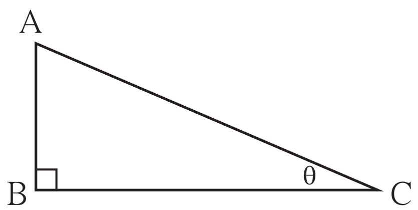

Fig. 6.1

$\sin\theta = \frac{\square}{\square}$, $\cos\theta = \frac{\square}{\square}$,

$\tan\theta = \frac{\square}{\square}$

2. Complete the relations in ratios given below .

(i) $\frac{\sin\theta}{\cos\theta} = \square$ (ii) $\sin\theta = \cos(90 - \square)$

(iii) $\cos\theta = \sin(90 - \square)$ (iv) $\tan\theta \times \tan(90 - \theta) = \square$

3. Complete the equation.

$\sin^2\theta + \cos^2\theta = \square$

4. Write the values of the following trigonometric ratios.

(i) $\sin 30° = \frac{1}{\square}$ (ii) $\cos 30° = \frac{\square}{\square}$ (iii) $\tan 30° = \frac{\square}{\square}$

(iv) $\sin 60° = \frac{\square}{\square}$ (v) $\cos 45° = \frac{\square}{\square}$ (vi) $\tan 45° = \square$

In std IX, we have studied some trigonometric ratios of some acute angles. Now we are going to study some more trigonometric ratios of acute angles.

Let's learn.

cosec, sec and cot ratios

Multiplicative inverse or the reciprocal of sine ratio is called cosecant ratio. It is written in brief as cosec. $\therefore$ cosec$\theta = \frac{1}{\sin\theta}$

Similarly, multiplicative inverses or reciprocals of cosine and tangent ratios are called "secant" and "cotangent" ratios respectively. They are written in brief as sec and cot.

$\therefore$ $\sec\theta = \frac{1}{\cos\theta}$ and $\cot\theta = \frac{1}{\tan\theta}$

In figure 6.2,

$$\sin\theta = \frac{AB}{AC}$$

$$\therefore \quad \text{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{AB}{AC}} = \frac{AC}{AB}$$

It means,

$$\text{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side}}$$

$$\tan\theta = \frac{AB}{BC}$$

$$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{AB}{BC}}$$

$$\cot\theta = \frac{BC}{AB} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\cos\theta = \frac{BC}{AC}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{BC}{AC}} = \frac{AC}{BC}$$

It means,

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

You know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$$

$$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta}$$

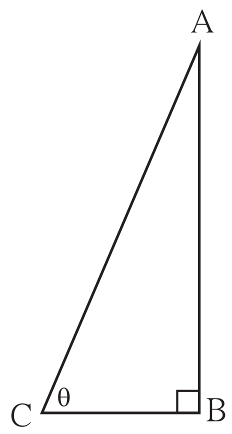

Fig. 6.2

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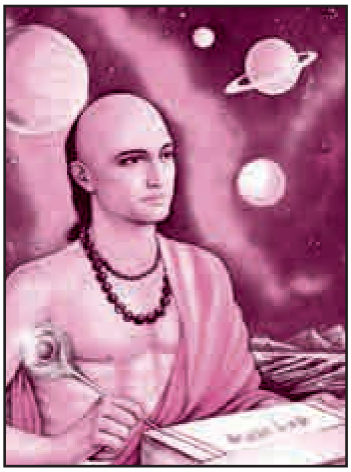
Remember this !

The relation between the trigonometric ratios, according to the definitions of cosec, sec and cot ratios

- $\frac{1}{\sin\theta} = \text{cosec}\ \theta \quad \therefore\ \sin\theta \times \text{cosec}\ \theta = 1$
- $\frac{1}{\cos\theta} = \sec\theta \quad \therefore\ \cos\theta \times \sec\theta = 1$
- $\frac{1}{\tan\theta} = \cot\theta \quad \therefore\ \tan\theta \times \cot\theta = 1$

For more information :



The great Indian mathematician Aryabhata was born in 476 A.D. in Kusumpur which was near Patna in Bihar. He has done important work in Arithmetic, Algebra and Geometry. In the book 'Aryabhatiya' he has written many mathematical formulae. For example,

(1) In an Arithmetic Progression, formulae for n^{th} term and the sum of first n terms.

(2) The formula to approximate $\sqrt{2}$

(3) The correct value of π upto four decimals, $\pi = 3.1416$.

In the study of Astronomy he used trigonometry and the sine ratio of an angle for the first time.

Comparing with the mathematics in the rest of the world at that time, his work was great and was studied all over India and was carried to Europe through Middle East.

Most observers at that time believed that the earth is immovable and the Sun, the Moon and stars move arround the earth. But Aryabhata noted that when we travel in a boat on the river, objects like trees, houses on the bank appear to move in the opposite direction. 'Similarly', he said 'the Sun, Moon and the stars are observed by people on the earth to be moving in the opposite direction while in reality the Earth moves !'

On 19 April 1975, India sent the first satellite in the space and it was named 'Aryabhata' to commemorate the great Mathematician of India.

✻ The table of the values of trigonometric ratios of angles 0°,30°,45°,60° and 90° .

Trigonometric ratio	Angle (θ)				
	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ $= \frac{1}{\sin\theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ $= \frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ $= \frac{1}{\tan\theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Let's learn.

Trigonometric identities

In the figure 6.3, Δ ABC is a right angled triangle, ∠B= 90°

(i) $\sin\theta = \frac{BC}{AC}$ (ii) $\cos\theta = \frac{AB}{AC}$

(iii) $\tan\theta = \frac{BC}{AB}$ (iv) $\operatorname{cosec}\theta = \frac{AC}{BC}$

(v) $\sec\theta = \frac{AC}{AB}$ (vi) $\cot\theta = \frac{AB}{BC}$

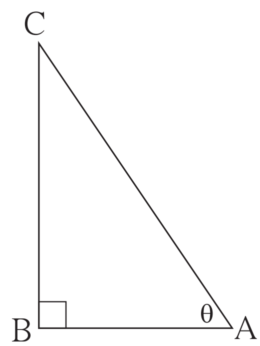


Fig. 6.3

By Pythagoras therom,

$BC^2 + AB^2 = AC^2$ . . . . .(I)

Dividing both the sides of (1) by AC^2

$$\frac{BC^2 + AB^2}{AC^2} = \frac{AC^2}{AC^2}$$

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$\therefore \dfrac{BC^2}{AC^2} + \dfrac{AB^2}{AC^2} = 1$

$\therefore \left(\dfrac{BC}{AC}\right)^2 + \left(\dfrac{AB}{AC}\right)^2 = 1$

$\therefore (\sin\theta)^2 + (\cos\theta)^2 = 1$ .... [$(\sin\theta)^2$ is written as $\sin^2\theta$ and $(\cos\theta)^2$ is written as $\cos^2\theta$.]

$\sin^2\theta + \cos^2\theta = 1$ .... (II)

Now dividing both the sides of equation (II) by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ .... (III)

Dividing both the sides of equation (II) by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \tan^2\theta = \sec^2\theta$ .... (IV)

Relations (II),(III), and (IV) are fundamental trigonometric identities.

Solved Examples

Ex. (1) If $\sin\theta = \dfrac{20}{29}$ then find $\cos\theta$

Solution : **Method I**

We have

$\sin^2\theta + \cos^2\theta = 1$

$\left(\dfrac{20}{29}\right)^2 + \cos^2\theta = 1$

$\dfrac{400}{841} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \dfrac{400}{841}$

$= \dfrac{441}{841}$

Taking square root of both sides.

$\cos\theta = \dfrac{21}{29}$

Method II

$\sin\theta = \dfrac{20}{29}$

from figure, $\sin\theta = \dfrac{AB}{AC}$

$\therefore$ AB = $20k$ and AC = $29k$

Let BC = x.

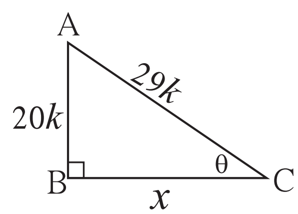


Fig. 6.4

According to Pythagoras therom,

$AB^2 + BC^2 = AC^2$

$(20k)^2 + x^2 = (29k)^2$

$400k^2 + x^2 = 841k^2$

$x^2 = 841k^2 - 400k^2$

$= 441k^2$

$\therefore \quad x = 21k$

$\therefore \cos\theta = \dfrac{BC}{AC} = \dfrac{21k}{29k} = \dfrac{21}{29}$

Ex. (2) If $\sec\theta = \frac{25}{7}$, find the value of $\tan\theta$.

Solution : Method I

we have ,

$1 + \tan^2\theta = \sec^2\theta$

$\therefore \quad 1 + \tan^2\theta = \left(\frac{25}{7}\right)^2$

$\therefore \quad \tan^2\theta = \frac{625}{49} - 1$

$= \frac{625 - 49}{49}$

$= \frac{576}{49}$

$\therefore \quad \tan\theta = \frac{24}{7}$

Method II

from the figure,

$\sec\theta = \frac{PR}{PQ}$

$\therefore \quad PQ = 7k,\ PR = 25k$

According to Pythagoras therom,

$PQ^2 + QR^2 = PR^2$

$\therefore \quad (7k)^2 + QR^2 = (25k)^2$

$\therefore \quad QR^2 = 625k^2 - 49k^2 = 576k^2$

$\therefore \quad QR = 24k$

Now, $\tan\theta = \frac{QR}{PQ} = \frac{24k}{7k} = \frac{24}{7}$

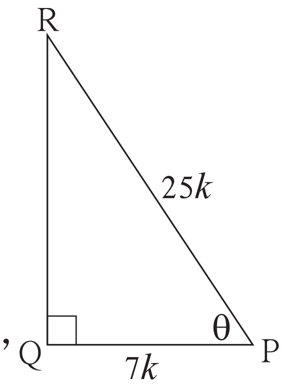


Fig. 6.5

Ex. (3) If $5\sin\theta - 12\cos\theta = 0$, find the values of $\sec\theta$ and $\text{cosec}\theta$.

Solution : $5\sin\theta - 12\cos\theta = 0$

$\therefore \ 5\sin\theta = 12\cos\theta$

$\therefore \ \frac{\sin\theta}{\cos\theta} = \frac{12}{5}$

$\therefore \ \tan\theta = \frac{12}{5}$

we have,

$1 + \tan^2\theta = \sec^2\theta$

$\therefore 1 + \left(\frac{12}{5}\right)^2 = \sec^2\theta$

$\therefore \ 1 + \frac{144}{25} = \sec^2\theta$

$\therefore \ \frac{25 + 144}{25} = \sec^2\theta$

$\therefore \quad \sec^2\theta = \frac{169}{25}$

$\therefore \quad \sec\theta = \frac{13}{5}$

$\therefore \cos\theta = \frac{5}{13}$

Now, $\sin^2\theta + \cos^2\theta = 1$

$\therefore \ \sin^2\theta = 1 - \cos^2\theta$

$\therefore \sin^2\theta = 1 - \left(\frac{5}{13}\right)^2$

$= 1 - \frac{25}{169}$

$= \frac{144}{169}$

$\therefore \ \sin\theta = \frac{12}{13}$

$\therefore \ \text{cosec}\theta = \frac{13}{12}$

Ex. (4) $\cos\theta = \frac{\sqrt{3}}{2}$ then find the value of $\frac{1-\sec\theta}{1+\operatorname{cosec}\theta}$.

Solution : Method I

$\cos\theta = \frac{\sqrt{3}}{2}$ $\therefore \sec\theta = \frac{2}{\sqrt{3}}$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

$\therefore \sin^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore \sin\theta = \frac{1}{2}$ $\therefore \operatorname{cosec}\theta = 2$

$$\therefore \frac{1-\sec\theta}{1+\operatorname{cosec}\theta} = \frac{1-\frac{2}{\sqrt{3}}}{1+2}$$

$$= \frac{\frac{\sqrt{3}-2}{\sqrt{3}}}{3}$$

$$= \frac{\sqrt{3}-2}{3\sqrt{3}}$$

Method II

$\cos\theta = \frac{\sqrt{3}}{2}$

we know that, $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

$\therefore \theta = 30^\circ$

$\therefore \sec\theta = \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\operatorname{cosec}\theta = \operatorname{cosec} 30^\circ = 2$

$$\therefore \frac{1-\sec\theta}{1+\operatorname{cosec}\theta} = \frac{1-\frac{2}{\sqrt{3}}}{1+2}$$

$$= \frac{\frac{\sqrt{3}-2}{\sqrt{3}}}{3}$$

$$= \frac{\sqrt{3}-2}{3\sqrt{3}}$$

Ex. (5) Show that $\sec x + \tan x = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Solution : $\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$$= \frac{1+\sin x}{\cos x}$$

$$= \sqrt{\frac{(1+\sin x)^2}{\cos^2 x}}$$

$$= \sqrt{\frac{(1+\sin x)(1+\sin x)}{1-\sin^2 x}}$$

$$= \sqrt{\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}}$$

$$= \sqrt{\frac{1+\sin x}{1-\sin x}}$$

Ex. (6) Eliminate θ from given equations.

$x = a\cot\theta - b\,\text{cosec}\,\theta$

$y = a\cot\theta + b\,\text{cosec}\,\theta$

Solution :

$x = a\cot\theta - b\,\text{cosec}\,\theta$ .... (I)

$y = a\cot\theta + b\,\text{cosec}\,\theta$ .... (II)

Adding equations (I) and (II).

$x + y = 2a\cot\theta$

$\therefore\ \cot\theta = \dfrac{x+y}{2a}$ .... (III)

Subtracting equation (II) from (I) ,

$y - x = 2b\,\text{cosec}\,\theta$

$\therefore\ \text{cosec}\,\theta = \dfrac{y-x}{2b}$ .... (IV)

Now, $\text{cosec}^2\theta - \cot^2\theta = 1$

$$\therefore\ \left(\frac{y-x}{2b}\right)^2 - \left(\frac{y+x}{2a}\right)^2 = 1$$

$$\therefore\ \frac{(y-x)^2}{4b^2} - \frac{(y+x)^2}{4a^2} = 1$$

$$\text{or}\ \left(\frac{y-x}{b}\right)^2 - \left(\frac{y+x}{a}\right)^2 = 4$$

Practice set 6.1

1. If $\sin\theta = \dfrac{7}{25}$, find the values of $\cos\theta$ and $\tan\theta$.

2. If $\tan\theta = \dfrac{3}{4}$, find the values of $\sec\theta$ and $\cos\theta$.

3. If $\cot\theta = \dfrac{40}{9}$, find the values of $\text{cosec}\,\theta$ and $\sin\theta$.

4. If $5\sec\theta - 12\,\text{cosec}\,\theta = 0$, find the values of $\sec\theta$, $\cos\theta$ and $\sin\theta$.

5. If $\tan\theta = 1$ then, find the values of $\dfrac{\sin\theta+\cos\theta}{\sec\theta+\text{cosec}\,\theta}$.

6. Prove that:

   (1) $\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

   (2) $\cos^2\theta(1 + \tan^2\theta) = 1$

(3) $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$ = secθ – tanθ

(4) (secθ – cosθ)(cotθ + tanθ) = tanθ secθ

(5) cotθ + tanθ = cosecθ secθ

(6) $\dfrac{1}{\sec\theta-\tan\theta}$ = secθ + tanθ

(7) $\sec^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

(8) secθ + tanθ = $\dfrac{\cos\theta}{1-\sin\theta}$

(9) If $\tan\theta + \dfrac{1}{\tan\theta} = 2$, then show that $\tan^2\theta + \dfrac{1}{\tan^2\theta} = 2$

(10) $\dfrac{\tan A}{\left(1+\tan^2 A\right)^2}+\dfrac{\cot A}{\left(1+\cot^2 A\right)^2}$ = sin A cos A

(11) $\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1$

(12) $\dfrac{\tan\theta}{\sec\theta-1} = \dfrac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}$

Let's learn.

Application of trigonometry

Many times we need to know the height of a tower, building, tree or distance of a ship from the lighthouse or width of a river etc.

We cannot measure them actually but we can find them with the help of trigonometric ratios.

For the purpose of computation, we draw a rough sketch to show the given information. 'Trees', 'hills' or 'towers' are vertical objects, so we shall represent them in the figure by segments which are perpendicular to the ground. We will not consider height of the observer and we shall normally regard observer's line of vision to be parallel to the horizontal level.

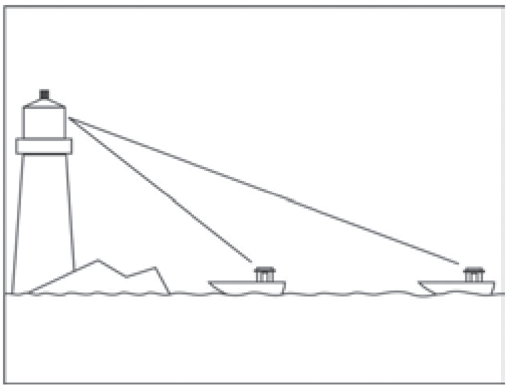


Fig. 6.6

Let us study a few related terms.

(i) Line of vision : If the observer is standing at the location 'A', looking at an object 'B' then the line AB is called line of the vision.

(ii) Angle of elevation :

If an observer at A, observes the point B which is at a level higher than A and AM is the horizontal line, then $\angle$ BAM is called the angle of elevation.

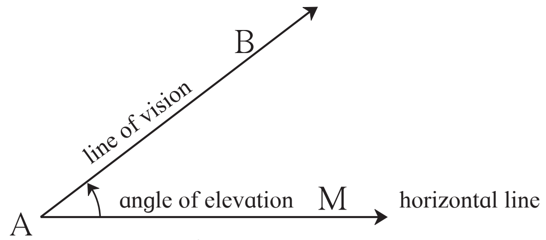


Fig. 6.7

(iii) Angle of depression :

If an observer at A, observes the point C which is at a level lower than A and AM is the horizontal line, the $\angle$ MAC is called the angle of depression.

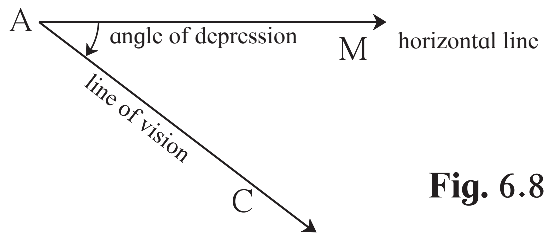


Fig. 6.8

When we see above the horizontal line, the angle formed is the angle of elevation. When we see below the horizontal line, the angle formed is the angle of depression.

Solved Examples

Ex. (1) An observer at a distance of 10 m from a tree looks at the top of the tree, the angle of elevation is 60°. What is the height of the tree ? ($\sqrt{3} = 1.73$)

Solution : In figure 6.9, AB = h = height of the tree.

BC = 10 m, distance of the observer from the tree .

Angle of elevation $(\theta) = \angle BCA = 60^\circ$

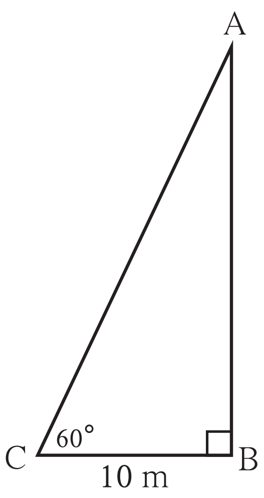


Fig. 6.9

from figure, $\tan\theta = \dfrac{AB}{BC}$ .... (I)

$\tan 60^\circ = \sqrt{3}$ .... (II)

$\therefore \dfrac{AB}{BC} = \sqrt{3}$ .... from equation (I)and (II)

$\therefore AB = BC\sqrt{3} = 10\sqrt{3}$

$\therefore AB = 10 \times 1.73 = 17.3$ m

$\therefore$ height of the tree is 17.3m.

Ex. (2) From the top of a building, an observer is looking at a scooter parked at some distance away, makes an angle of depression of 30°. If the height of the building is 40m, find how far the scooter is from the building. ($\sqrt{3}$ = 1.73)

Solution: In the figure 6.10, AB is the building. A scooter is at C which is 'x' m away from the building.
In figure, 'A' is the position of the observer.

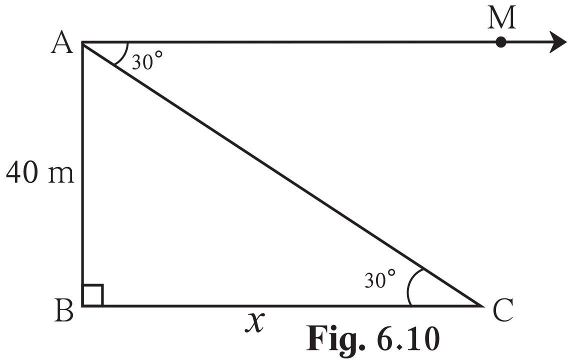


Fig. 6.10

AM is the horizontal line and ∠ MAC is the angle of depression.
∠ MAC and ∠ ACB are alternate angles.

from fig, $\tan 30° = \frac{AB}{BC}$

$$\therefore \quad \frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$\therefore \quad x = 40\sqrt{3}$$

$$= 40 \times 1.73$$

$$= 69.20 \text{ m.}$$

∴ the scooter is 69.20 m. away from the building.

Ex. (3) To find the width of the river, a man observes the top of a tower on the opposite bank making an angle of elevation of 61°. When he moves 50m backword from bank and observes the same top of the tower, his line of vision makes an angle of elevation of 35°. Find the height of the tower and width of the river. (tan61° = 1.8, tan35° = 0.7)

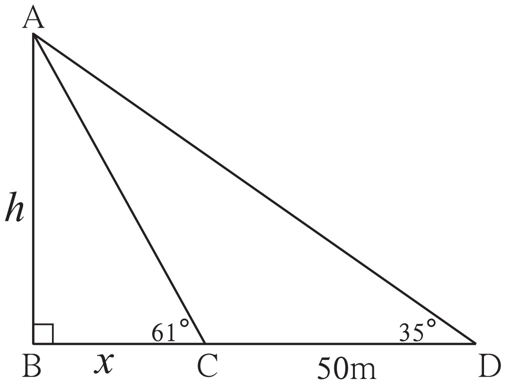


Fig. 6.11

Solution : seg AB shows the tower on the opposite bank. 'A' is the top of the tower and seg BC shows the width of the river. Let 'h' be the height of the tower and 'x' be the width of the river.

from figure, $\tan 61° = \frac{h}{x}$

$\therefore\ 1.8 = \dfrac{h}{x}$

$h = 1.8 \times x$

$10h = 18x$ .... (I)..... multipling by 10

In right angled Δ ABD,

$\tan 35 = \dfrac{h}{x + 50}$

$0.7 = \dfrac{h}{x + 50}$

$\therefore\ h = 0.7\,(x + 50)$

$\therefore\ 10h = 7\,(x + 50)$ .... (II)

$\therefore$ from equations (I) and (II) ,

$18x = 7(x + 50)$

$\therefore\ 18x = 7x + 350$

$\therefore\ 11x = 350$

$\therefore\ x = \dfrac{350}{11} = 31.82$

Now, $h = 1.8x = 1.8 \times 31.82$

$= 57.28$ m.

$\therefore$ width of the river = 31.82 m and height of tower = 57.28 m

Ex. (4) Roshani saw an eagle on the top of a tree at an angle of elevation of 61°, while she was standing at the door of her house. She went on the terrace of the house so that she could see it clearly. The terrace was at a height of 4m. While observing the eagle from there the angle of elevation was 52°. At what height from the ground was the eagle ? (Find the answer correct upto nearest integer)

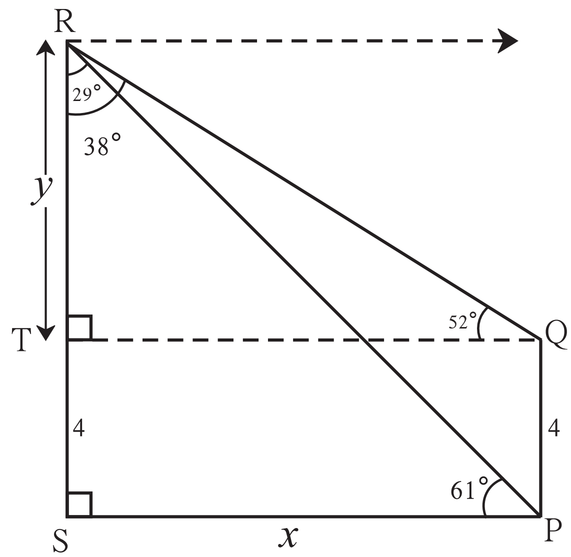


Fig. 6.12

($\tan 61° = 1.80$, $\tan 52° = 1.28$, $\tan 29° = 0.55$, $\tan 38° = 0.78$)

Solution : In figure 6.12, PQ is the house and SR is the tree. The eagle is at R.

Draw seg QT $\perp$ seg RS.

$\therefore$ $\square$ TSPQ is a rectangle.

Let SP = x and TR = y

Now in Δ RSP, $\angle$ PRS = $90^\circ - 61^\circ = 29^\circ$

and in Δ RTQ, $\angle$ QRT = $90^\circ - 52^\circ = 38^\circ$

$\therefore \tan \angle PRS = \tan 29^\circ = \frac{SP}{RS}$

$\therefore 0.55 = \frac{x}{y+4}$

$\therefore x = 0.55(y + 4)$ .... (I)

Similarly, $\tan \angle QRT = \frac{TQ}{RT}$

$\therefore \tan 38^\circ = \frac{x}{y}$ .... [$\because$ SP = TQ = x]

$\therefore 0.78 = \frac{x}{y}$

$\therefore x = 0.78y$ .... (II)

$\therefore 0.78y = 0.55(y + 4)$ .... from (I) and (II)

$\therefore 78y = 55(y + 4)$

$\therefore 78y = 55y + 220$

$\therefore 23y = 220$

$\therefore y = 9.565 = 10$ (upto nearest integer)

$\therefore RS = y + 4 = 10 + 4 = 14$

$\therefore$ the eagle was at a height of 14 metre from the ground.

Ex. (5) A tree was broken due to storm. Its broken upper part was so inclined that its top touched the ground making an angle of $30^\circ$with the ground.The distance from the foot of the tree and the point where the top touched the ground was 10 metre. What was the height of the tree.

Solution: As shown in figure 6.13, suppose AB is the tree. It was broken at 'C' and its top touched at 'D'.

$\angle$ CDB = 30°, BD = 10 m, BC = x m

CA= CD = y m

In right angled Δ CDB,

$$\tan 30° = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$x = \frac{10}{\sqrt{3}}$$

$$y = \frac{20}{\sqrt{3}}$$

$$x + y = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}}$$

$$x + y = 10\sqrt{3}$$

$\therefore$ height of the tree was $10\sqrt{3}$ m.

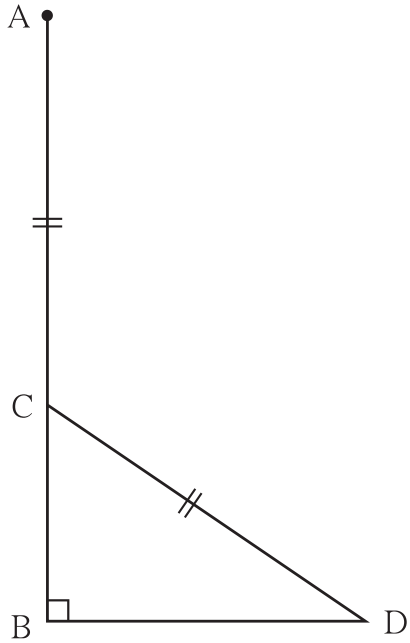


Fig. 6.13

Practice set 6.2

1. A person is standing at a distance of 80m from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.

2. From the top of a lighthouse, an observer looking at a ship makes angle of depression of 60°. If the height of the lighthouse is 90 metre, then find how far the ship is from the lighthouse. ($\sqrt{3}$ =1.73)

3. Two buildings are facing each other on a road of width 12 metre. From the top of the first building, which is 10 metre high, the angle of elevation of the top of the second is found to be 60°. What is the height of the second building?

4. Two poles of heights 18 metre and 7 metre are erected on a ground. The length of the wire fastened at their tops in 22 metre. Find the angle made by the wire with the horizontal.

5. A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree.

6. A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60° with the ground. Assuming that the string is straight, find the length of the string. ($\sqrt{3}$ =1.73)

Problem set 6

1. Choose the correct alternative answer for the following questions.

(1) $\sin\theta \ \mathrm{cosec}\theta$ = ?

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

(2) $\mathrm{cosec}45°$ = ?

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

(3) $1 + \tan^2\theta$ = ?

(A) $\cot^2\theta$ (B) $\mathrm{cosec}^2\theta$ (C) $\sec^2\theta$ (D) $\tan^2\theta$

(4) When we see at a higher level , from the horizontal line,angle formed is....... .

(A) angle of elevation. (B) angle of depression.

(C) 0 (D) straight angle.

2. If $\sin\theta = \frac{11}{61}$, find the values of $\cos\theta$ using trigonometric identity.

3. If $\tan\theta = 2$, find the values of other trigonometric ratios.

4. If $\sec\theta = \frac{13}{12}$, find the values of other trigonometric ratios.

5. Prove the following.

(1) $\sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta) = 1$

(2) $(\sec\theta + \tan\theta)\,(1 - \sin\theta) = \cos\theta$

(3) $\sec^2\theta + \mathrm{cosec}^2\theta = \sec^2\theta \times \mathrm{cosec}^2\theta$

(4) $\cot^2\theta - \tan^2\theta = \mathrm{cosec}^2\theta - \sec^2\theta$

(5) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

(6) $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

(7) $\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x$

(8) $\frac{\tan\theta}{\sec\theta + 1} = \frac{\sec\theta - 1}{\tan\theta}$

(9) $\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$

(10) $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$

6. A boy standing at a distance of 48 meters from a building observes the top of the building and makes an angle of elevation of 30°.Find the height of the building.

7. From the top of the light house, an observer looks at a ship and finds the angle of depression to be 30°.If the height of the light-house is 100 meters, then find how far the ship is from the light-house.

8. Two buildings are in front of each other on a road of width 15 meters. From the top of the first building, having a height of 12 meter, the angle of elevation of the top of the second building is 30°.What is the height of the second building?

9. A ladder on the platform of a fire brigade van can be elevated at an angle of 70° to the maximum. The length of the ladder can be extended upto 20m. If the platform is 2m above the ground, find the maximum height from the ground upto which the ladder can reach. (sin 70° = 0.94)

10 ⋆. While landing at an airport, a pilot made an angle of depression of 20°. Average speed of the plane was 200 km/hr. The plane reached the ground after 54 seconds. Find the height at which the plane was when it started landing. (sin 20° = 0.342)

PV6XPH

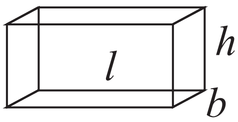
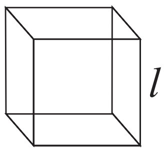
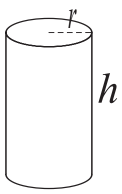
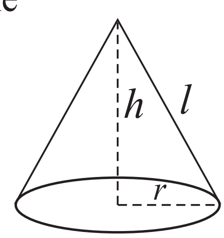
7 Mensuration

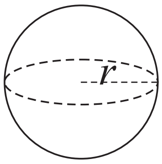
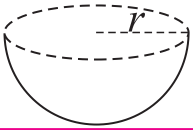
Let's study.

- Mixed examples on surface area and volume of different solid figures
- Arc of circle - length of arc
- Area of a sector
- Area of segment of a circle

Let's recall.

Last year we have studied surface area and volume of some three dimentional figures. Let us recall the formulae to find the surface areas and volumes.

No.	Three dimensional figure	Formulae
1 .	Cuboid 	Lateral surface area $= 2h\,(l + b)$ Total surface area $= 2\,(lb + bh + hl)$ Volume $= lbh$
2 .	Cube 	Lateral surface area $= 4l^2$ Total surface area $= 6l^2$ Volume $= l^3$
3 .	Cylinder 	Curved surface area $= 2\pi rh$ Total surface area $= 2\pi r\,(r + h)$ Volume $= \pi r^2 h$
4 .	Cone 	Slant height $(l) = \sqrt{h^2 + r^2}$ Curved surface area $= \pi rl$ Total surface area $= \pi r\,(r + l)$ Volume $= \frac{1}{3} \times \pi r^2 h$

No.	Three dimensional figure	Formulae
5.	Sphere 	Surface area = $4\ \pi r^2$ Volume = $\frac{4}{3}\ \pi r^3$
6.	Hemisphere 	Curved surface area = $2\pi r^2$ Total surface area of a solid hemisphere = $3\pi r^2$ Volume = $\frac{2}{3}\ \pi r^3$

Solve the following examples

Ex. (1)

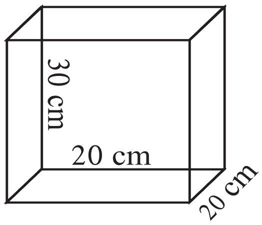


Fig 7.1

The length, breadth and height of an oil can are 20 cm, 20 cm and 30 cm respectively as shown in the adjacent figure.

How much oil will it contain ?

(1 litre = 1000 cm^3)

Ex. (2)

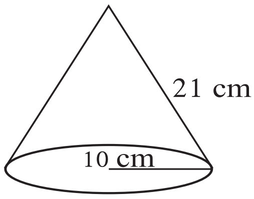


Fig 7.2

The adjoining figure shows the measures of a Joker's cap. How much cloth is needed to make such a cap ?

Let's think.

As shown in the adjacent figure, a sphere is placed in a cylinder. It touches the top, bottom and the curved surface of the cylinder. If radius of the base of the cylinder is 'r',

(1) What is the ratio of the radii of the sphere and the cylinder ?

(2) What is the ratio of the curved surface area of the cylinder and the surface area of the sphere ?

(3) What is the ratio of the volumes of the cylinder and the sphere ?

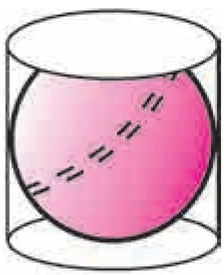


Fig. 7.3

Activity :

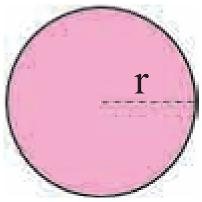


Fig. 7.4

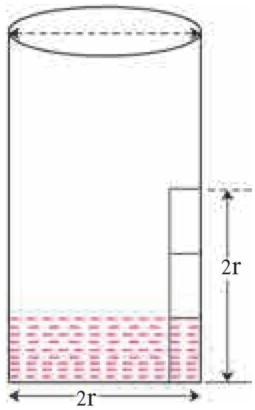


Fig. 7.5

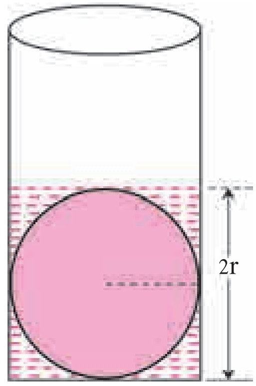


Fig. 7.6

As shown in the above figures, take a ball and a beaker of the same radius as that of the ball. Cut a strip of paper of length equal to the diameter of the beaker. Draw two lines on the strip dividing it into three equal parts. Stick it on the beaker straight up from the bottom. Fill water in the beaker upto the first mark of the strip from the bottom. Push the ball in the beaker slowly so that it touches its bottom. Observe how much the water level rises.

You will notice that the water level has risen exactly upto the total height of the strip. Try to understand how we get the formula for the volume of a the sphere. The shape of the beaker is cylindrical.

Therefore, the volume of the part of the beaker upto height 2r can be obtained by the formula of volume of a cylinder. Let us assume that the volume is V.

$\therefore$ $V = \pi r^2 h = \pi \times r^2 \times 2r = 2\pi r^3$ $\quad (\because h = 2r)$

But V = volume of the ball + volume of the water which was already in the beaker.

$= \text{volume of the ball} + \frac{1}{3} \times 2\pi r^3$

$\therefore$ volume of the ball $= V - \frac{1}{3} \times 2\pi r^3$

$$= 2\pi r^3 - \frac{2}{3}\pi r^3$$

$$= \frac{6\pi r^3 - 2\pi r^3}{3} \quad = \frac{4\pi r^3}{3}$$

Hence we get the formula of the volume of a sphere as $V = \frac{4}{3}\pi r^3$

(Now you can find the answer of the question number 3 relating to figure 7.3)

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Solved Examples

Ex. (1) The radius and height of a cylindrical water reservoir is 2.8 m and 3.5 m respectively. How much maximum water can the tank hold ? A person needs 70 litre of water per day. For how many persons is the water sufficient for a day ? ($\pi = \frac{22}{7}$)

Solution : (r) = 2.8 m, (h) = 3.5 m, $\pi = \frac{22}{7}$

Capacity of the water reservoir = Volume of the cylindrical reservoir

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 2.8 \times 2.8 \times 3.5$$

$$= 86.24 \text{ m}^3$$

$$= 86.24 \times 1000 \quad (\because 1m^3 = 1000 \text{ litre})$$

$$= 86240.00 \text{ litre.}$$

$\therefore$ the reservoir can hold 86240 litre of water.

The daily requirement of water of a person is 70 litre.

$\therefore$ water in the tank is sufficient for $\frac{86240}{70}$ = 1232 persons.

Ex. (2) How many solid cylinders of radius 10 cm and height 6 cm can be made by melting a solid sphere of radius 30 cm ?

Solution : Radius of a sphere, r = 30 cm

Radius of the cylinder, R = 10 cm

Height of the cylinder, H = 6 cm

Let the number of cylinders be n.

Volume of the sphere = n × volume of a cylinder

$$\therefore \; n = \frac{\text{Volume of the sphere}}{\text{Volume of a cylinder}}$$

$$= \frac{\frac{4}{3}\pi (r)^3}{\pi (R)^2 H}$$

$$= \frac{\frac{4}{3} \times (30)^3}{10^2 \times 6} = \frac{\frac{4}{3} \times 30 \times 30 \times 30}{10 \times 10 \times 6} = 60$$

$\therefore$ 60 cylinders can be made .

Ex. (3) A tent of a circus is such that its lower part is cylindrical and upper part is conical. The diameter of the base of the tent is 48 m and the height of the cylindrical part is 15 m. Total height of the tent is 33 m. Find area of canvas required to make the tent. Also find volume of air in the tent.

Solution : Total height of the tent = 33 m.

Let height of the cylindrical part be H

$\therefore$ H = 15 m.

Let the height of the conical part be h

$\therefore$ h = (33−15) = 18 m.

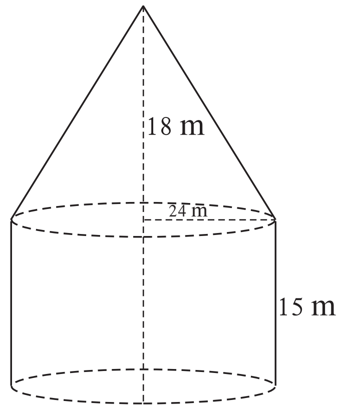

Fig 7.7

Slant height of cone, $l = \sqrt{r^2 + (h)^2}$

$= \sqrt{24^2 + 18^2}$

$= \sqrt{576 + 324}$

$= \sqrt{900}$

= 30 m.

Canvas required for tent = Curved surface area of the cylindrical part + Curved surface area of conical part

$= 2\pi rH + \pi rl$

$= \pi r\,(2H + l)$

$= \frac{22}{7} \times 24\,(2 \times 15 + 30)$

$= \frac{22}{7} \times 24 \times 60$

$= 4525.71\ m^2$

Volume of air in the tent = volume of cylinder + volume of cone

$= \pi r^2 H + \frac{1}{3}\pi r^2 h$

$= \pi r^2 \left(H + \frac{1}{3}h\right)$

$= \frac{22}{7} \times 24^2\,(15 + \frac{1}{3} \times 18)$

$= \frac{22}{7} \times 576 \times 21$

$= 38{,}016\ m^3$

$\therefore$ canvas required for the tent = $4525.71\ m^2$

$\therefore$ volume of air in the tent = $38{,}016\ m^3$.

Practice set 7.1

1. Find the volume of a cone if the radius of its base is 1.5 cm and its perpendicular height is 5 cm.
2. Find the volume of a sphere of diameter 6 cm.
3. Find the total surface area of a cylinder if the radius of its base is 5 cm and height is 40 cm.
4. Find the surface area of a sphere of radius 7 cm.
5. The dimensions of a cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.
6.

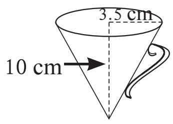

Fig 7.8
conical water jug

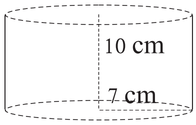

Fig 7.9
cylindrical water pot

Observe the measures of pots in figure 7.8 and 7.9. How many jugs of water can the cylindrical pot hold ?

7. A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100 cm^2.The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm^3. Find the total height of the figure.

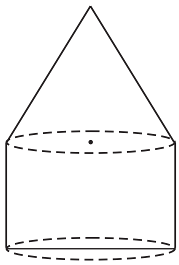
Fig 7.10

8. In figure 7.11, a toy made from a hemisphere, a cylinder and a cone is shown. Find the total area of the toy.

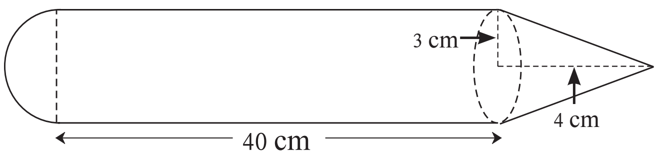

Fig. 7.11

9. In the figure 7.12, a cylindrical wrapper of flat tablets is shown. The radius of a tablet is 7 mm and its thickness is 5 mm. How many such tablets are wrapped in the wrapper ?

Fig. 7.12

10. Figure 7.13 shows a toy. Its lower part is a hemisphere and the upper part is a cone. Find the volume and the surface area of the toy from the measures shown in the figure.(π= 3.14)

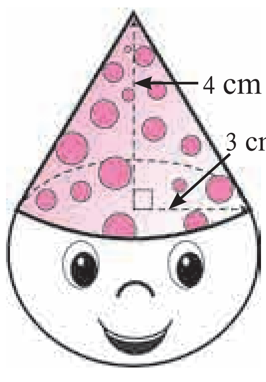

Fig. 7.13

11. Find the surface area and the volume of a beach ball shown in the figure.

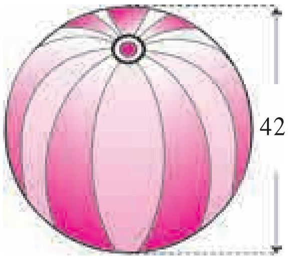

Fig. 7.14

12. As shown in the figure, a cylindrical glass contains water. A metal sphere of diameter 2 cm is immersed in it. Find the volume of the water.

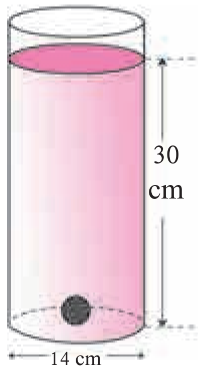

Fig. 7.15

Let's learn.

Frustum of a cone

The shape of glass used to drink water as well as the shape of water it contains, are examples of frustum of a cone.

Fig. 7.16

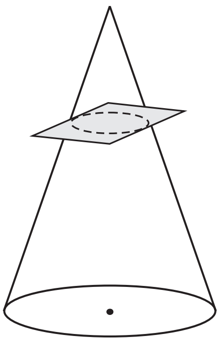

Fig. 7.17
A cone being cut

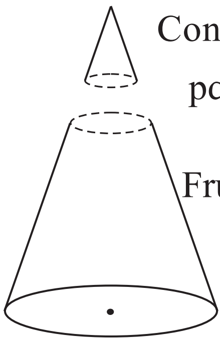

Fig. 7.18
Two parts of the cone

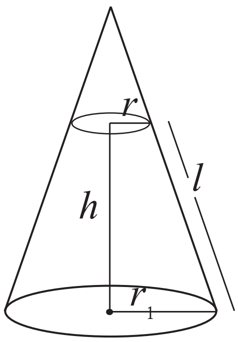

Fig. 7.19
Frustum

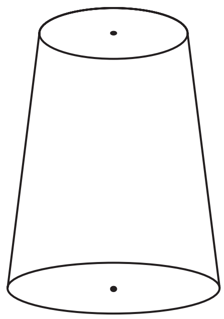

Fig. 7.20
A glass placed upside down

When a cone is cut parallel to its base we get two figures; one is a cone and the other is a frustum.

Volume and surface area of a frustum can be calculated by the formulae given below.

Remember this!

h = height of a frustum, l = slant height height of a frustum,

r_1 and r_2 = radii of circular faces of a frustum ($r_1 > r_2$)

Slant height of a frustum $= l = \sqrt{h^2 + (r_1 - r_2)^2}$

Curved surface area of a frustum $= \pi l\,(r_1 + r_2)$

Total surface area of a frustum $= \pi l\,(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Volume of a frustum $= \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 \times r_2)$

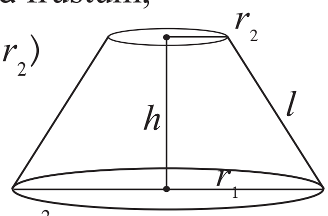

Fig. 7.21

Solved Examples

Ex. (1) A bucket is frustum shaped. Its height is 28 cm. Radii of circular faces are 12 cm and 15 cm. Find the capacity of the bucket. ($\pi = \frac{22}{7}$)

Solution : r_1 = 15 cm, r_2 = 12 cm, h = 28 cm

Capacity of the bucket = Volume of frustum

$$= \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 \times r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 28\,(15^2 \times 12^2 + 15 \times 12)$$

$$= \frac{22 \times 4}{3} \times (225 + 144 + 180)$$

$$= \frac{22 \times 4}{3} \times 549$$

$$= 88 \times 183$$

$$= 16104 \text{ cm}^3 = 16.104 \text{ litre}$$

$\therefore$ capacity of the bucket is 16.104 litre.

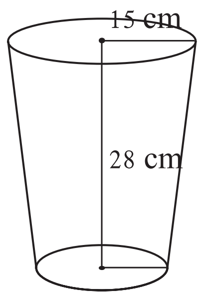

Fig. 7.22

Ex. (2) Radii of the top and the base of a frustum are 14 cm, 8 cm respectively. Its height is 8 cm. Find its

i) curved surface area ii) total surface area iii) volume.

Solution : r_1 = 14 cm , r_2 = 8 cm, h = 8 cm

Slant height of the frustum $= l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{8^2 + (14 - 8)^2}$$

$$= \sqrt{64 + 36} = 10 \text{ cm}$$

Curved surface area of the frustum = $\pi(r_1 + r_2)\, l$
= $3.14 \times (14 + 8) \times 10$
= $690.8\ cm^2$

Total surface area of frustum = $\pi\, l\, (r_1 + r_2) + \pi r_1^{\,2} + \pi r_2^{\,2}$
= $3.14 \times 10\,(14 + 8) + 3.14 \times 14^2 + 3.14 \times 8^2$
= $690.8 + 615.44 + 200.96$
= $690.8 + 816.4$
= $1507.2\ cm^2$

Volume of the frustum = $\frac{1}{3}\pi h(r_1^{\,2} + r_2^{\,2} + r_1 \times r_2)$
= $\frac{1}{3} \times 3.14 \times 8\,(14^2 + 8^2 + 14 \times 8)$
= $3114.88 cm^3$

Practice set 7.2

1. The radii of two circular ends of frustum shape bucket are 14 cm and 7 cm. Height of the bucket is 30 cm. How many liters of water it can hold ? (1 litre = $1000\ cm^3$)

2. The radii of ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its

i) curved surface area ii) total surface area. iii) volume ($\pi = 3.14$)

3. The circumferences of circular faces of a frustum are 132 cm and 88 cm and its height is 24 cm. To find the curved surface area of the frustum complete the following activity. ($\pi = \frac{22}{7}$).

circumference$_1$ = $2\pi r_1 = 132$

$r_1 = \frac{132}{2\pi} = \square$

circumference$_2$ = $2\pi r_2 = 88$

$r_2 = \frac{88}{2\pi} = \square$

slant height of frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{\square^2 + \square^2}$

$= \square$ cm

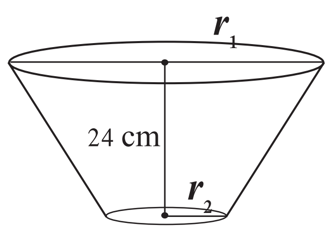

Fig. 7.23

curved surface area of the frustum = $\pi(r_1 + r_2)l$

$= \pi \times \square \times \square$

$= \square$ sq.cm.

Let's recall.

Complete the following table with the help of figure 7.24.

Type of arc	Name of the arc	Measure of the arc
Minor arc	arc AXB	....
....	arc AYB	....

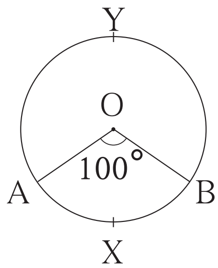

Fig. 7.24

Let's learn.

Sector of a circle

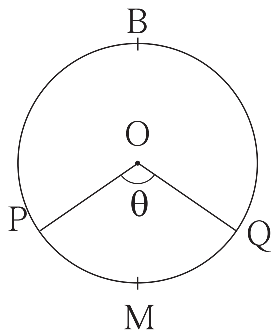

Fig. 7.25

In the adjacent figure, the central angle divides the circular region in two parts. Each of the parts is called a sector of the circle. Sector of a circle is the part enclosed by two radii of the circle and the arc joining their end points.

In the figure 7.25, O –PMQ and O–PBQ are two sectors of the circle.

Minor Sector :

Sector of a circle enclosed by two radii and their corresponding minor arc is called a 'minor sector'.

In the above figure O–PMQ is a minor sector.

Major Sector :

Sector of a circle that is enclosed by two radii and their corresponding major arc is called a 'major sector'.

In the above figure, O–PBQ is a major sector.

Area of a sector

Observe the figures below. Radii of all circles are equal. Observe the areas of the shaded regions and complete the following table.

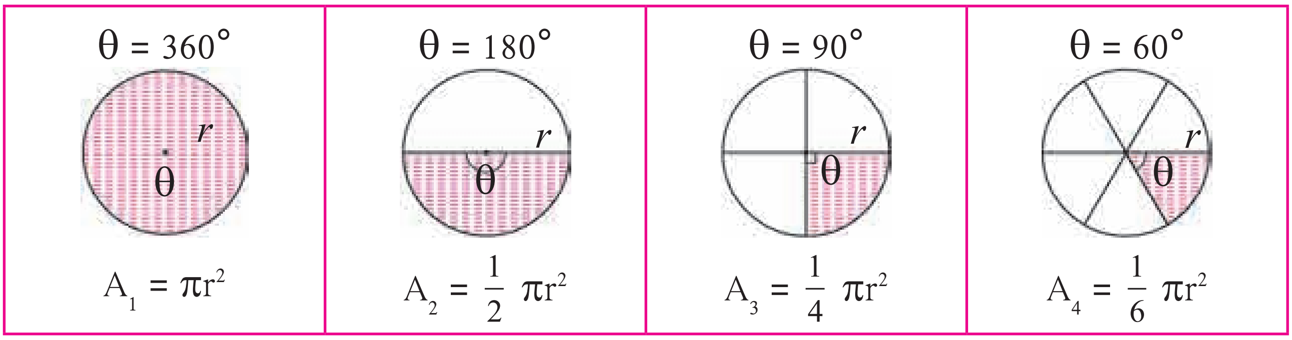

Fig. 7.26

Central angle of a circle is = 360° = complete angle

Central angle of a circle is = 360°, Area of a circle = πr^2			
Sector of circle	Measure of arc of the sector	$\frac{\theta}{360}$	Area of the sector A
A_1	360°	$\frac{360}{360}=1$	$1 \times \pi r^2$
A_2	180°	$\frac{1}{2}$	$\frac{1}{2} \times \pi r^2$
A_3	90°	$\frac{1}{4}$	$\frac{1}{4} \times \pi r^2$
A_4	60°	....	....
A	θ	$\frac{\theta}{360}$	$\frac{\theta}{360} \times \pi r^2$

From the above table we see that, if measure of an arc of a circle is θ, then the area of its corresponding sector is obtained by multiplying area of the circle by $\frac{\theta}{360}$.

$$\text{Area of sector (A)} = \frac{\theta}{360} \times \pi r^2$$

From the formula,

$$\frac{A}{\pi r^2} = \frac{\theta}{360} \text{ ; that is } \frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\theta}{360}$$

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Length of an arc

In the following figures, radii of all circles are equal. Observe the length of arc in each figure and complete the table.

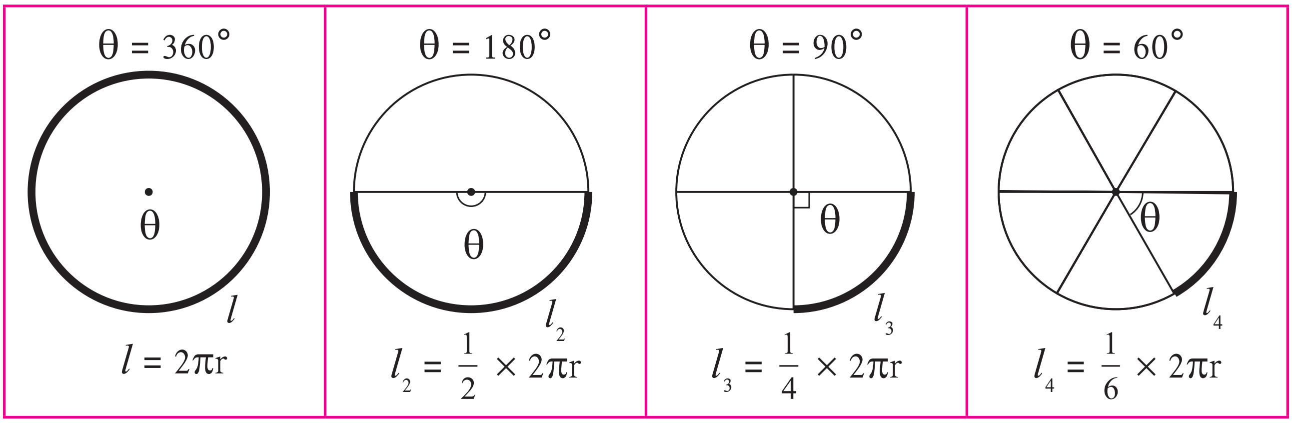

Fig. 7.27

Circumference of a circle = $2\pi r$			
Length of the arc	Measure of the arc (θ)	$\frac{\theta}{360}$	Length of the arc (l)
l_1	360°	$\frac{360}{360} = 1$	$1 \times 2\pi r$
l_2	180°	$\frac{180}{360} = \frac{1}{2}$	$\frac{1}{2} \times 2\pi r$
l_3	90°	$\frac{90}{360} = \frac{1}{4}$	$\frac{1}{4} \times 2\pi r$
l_4	60°	....	....
l	θ	$\frac{\theta}{360}$	$\frac{\theta}{360} \times 2\pi r$

The pattern in the above table shows that, if measure of an arc of a circle is θ, then its length is obtained by multiplying the circumference of the circle by $\frac{\theta}{360}$.

$$\text{Length of an arc } (l) = \frac{\theta}{360} \times 2\pi r$$

From the formula, $\frac{l}{2\pi r} = \frac{\theta}{360}$

that is, $\frac{\text{Length of an arc}}{\text{Circumference}} = \frac{\theta}{360}$

A relation between length of an arc and area of the sector

Area of a sector, (A) = $\frac{\theta}{360} \times \pi r^2$ I

Length of an arc, (l) = $\frac{\theta}{360} \times 2\pi r$

$\therefore \frac{\theta}{360} = \frac{l}{2\pi r}$ II

$\therefore A = \frac{l}{2\pi r} \times \pi r^2$ ....From I and II

$A = \frac{1}{2}\, lr = \frac{lr}{2}$

$\therefore$ Area of a sector = $\frac{\text{Length of the arc} \times \text{Radius}}{2}$

Similarly, $\frac{A}{\pi r^2} = \frac{l}{2\pi r} = \frac{\theta}{360}$

Solved Examples

Ex. (1) The measure of a central angle of a circle is 150 ° and radius of the circle is 21 cm. Find the length of the arc and area of the sector associated with the central angle.

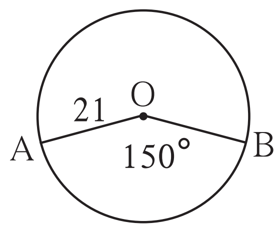

Fig. 7.28

Solution : r = 21 cm, θ = 150, π = $\frac{22}{7}$

Area of the sector, A $= \frac{\theta}{360} \times \pi r^2$

$= \frac{150}{360} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1155}{2} = 577.5 \text{ cm}^2$

Length of the arc, $l = \frac{\theta}{360} \times 2\pi r$

$= \frac{150}{360} \times 2 \times \frac{22}{7} \times 21$

= 55 cm

ex. (2) In figure 7.29, P is the centre of the circle of radius 6 cm. Seg QR is a tangent at Q. If PR = 12, find the area of the shaded region. ($\sqrt{3}$ = 1.73)

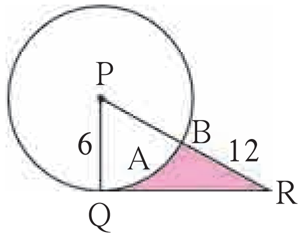

Fig. 7.29

Solution Radius joining point of contact of the tangent is perpendicular to the tangent.

$\therefore$ in Δ PQR, $\angle$ PQR = 90°, PQ = 6 cm, PR = 12 cm $\therefore$ PQ = $\frac{PR}{2}$

If one side of a right anglesd triangle is half the hypoteneus then angle opposite to, that side is of 30° measure

$\therefore \angle R = 30°$ and $\angle P = 60°$

$\therefore$ by 30°-60°-90° theorem, QR = $\frac{\sqrt{3}}{2} \times PR = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$

$\therefore QR = 6\sqrt{3}$ cm

$\therefore A(\Delta PQR) = \frac{1}{2} QR \times PQ$

$= \frac{1}{2} \times 6\sqrt{3} \times 6$

$= 18\sqrt{3} = 18 \times 1.73$

$= 31.14 \text{ cm}^2$

Area of a sector = $\frac{\theta}{360} \times \pi r^2$

$A(P-QAB) = \frac{60}{360} \times 3.14 \times 6^2$

$= \frac{1}{6} \times 3.14 \times 6 \times 6 = 3.14 \times 6$

$= 18.84 \text{ cm}^2$

Area of shaded region = A(PQR) − A(P-QAB)

$= 31.14 - 18.84$

$= 12.30 \text{ cm}^2$

Area of the shaded region = 12.30 cm^2

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Activity In figure 7.30, side of square ABCD is 7 cm. With centre D and radius DA, sector D – AXC is drawn. Fill in the following boxes properly and find out the area of the shaded region.

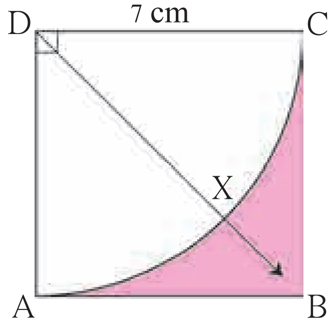

Fig. 7.30

Solution : Area of a square = ☐ (Formula)

= ☐

= 49 cm^2

Area of sector (D- AXC) = ☐ (Formula)

$= \frac{\square}{360} \times \frac{22}{7} \times \square$

= 38.5 cm^2

A (shaded region) = A ☐ – A ☐

= ☐ cm^2 – ☐ cm^2

= ☐ cm^2

Practice set 7.3

1. Radius of a circle is 10 cm. Measure of an arc of the crcleis 54°. Find the area of the sector associated with the arc. (π = 3.14)

2. Measure of an arc of a circle is 80 cm and its radius is 18 cm. Find the length of the arc. (π = 3.14)

3. Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.

4. Radius of a circle is 10 cm. Area of a sector of the sector is 100 cm^2. Find the area of its corresponding major sector. (π = 3.14)

5. Area of a sector of a circle of radius 15 cm is 30 cm^2. Find the length of the arc of the sector.

6. In the figure 7.31, radius of the circle is 7 cm and m(arc MBN) = 60°, find (1) Area of the circle .

(2) A(O - MBN) .

(3) A(O - MCN) .

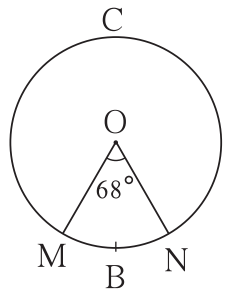

Fig. 7.31

7. In figure 7.32, radius of circle is 3.4 cm and perimeter of sector P-ABC is 12.8 cm. Find A(P-ABC).

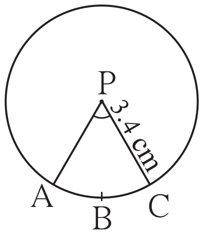

Fig. 7.32

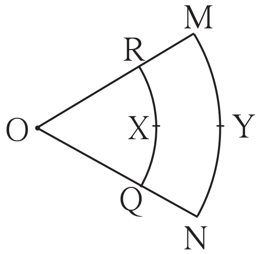

Fig. 7.33

8. In figure 7.33 O is the centre of the sector. $\angle$ ROQ = $\angle$ MON = 60°. OR = 7 cm, and OM = 21 cm. Find the lengths of arc RXQ and arc MYN. ($\pi = \frac{22}{7}$)

9. In figure 7.34, if A(P-ABC) = 154 cm^2 radius of the circle is 14 cm,

find (1) $\angle$ APC.

(2) l(arc ABC) .

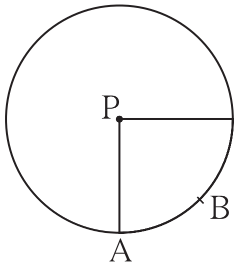

Fig. 7.34

10. Radius of a sector of a circle is 7 cm. If measure of arc of the sector is –

(1) 30° (2) 210° (3) three right angles;

find the area of the sector in each case.

11. The area of a minor sector of a circle is 3.85 cm^2 and the measure of its central angle is 36°. Find the radius of the circle.

12. In figure 7.35, □ PQRS is a rectangle. If PQ = 14 cm, QR = 21 cm, find the areas of the parts x, y and z .

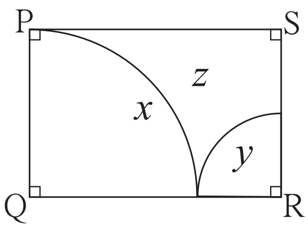

Fig. 7.35

13. Δ LMN is an equilateral triangle. LM = 14 cm. As shown in figure, three sectors are drawn with vertices as centres and radius7 cm. Find,

(1) A (Δ LMN)

(2) Area of any one of the sectors.

(3) Total area of all the three sectors.

(4) Area of the shaded region.

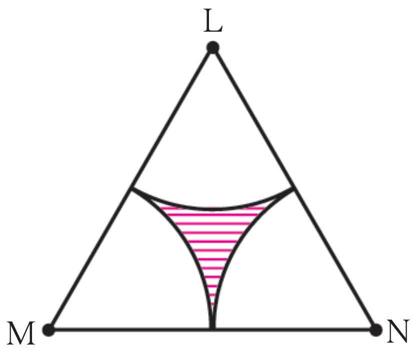

Fig. 7.36

Let's learn.

Segment of a circle

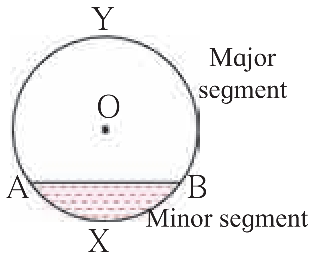

Fig. 7.37

Segment of a circle is the region bounded by a chord and its corresponding arc of the circle.

Minor segment : The area enclosed by a chord and its corresponding minor arc is called a minor segment. In the figure, segment AXB is a minor segment.

Major segment : The area enclosed by a chord and its corresponding major arc is called a major segment. In the figure, seg AYB is a major segment.

Semicircular segment : A segment formed by a diameter of a circle is called a semicircular segment.

Area of a Segment

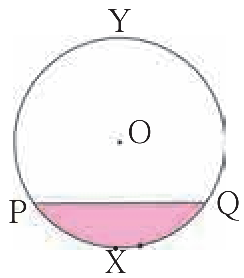

Fig. 7.38

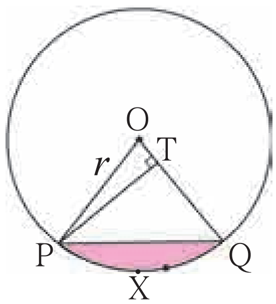

Fig. 7.39

In figure 7.38, PXQ is a minor segment and PYQ is a major segment.

How can we calculate the area of a minor segment?

In figure 7.39, draw radii OP and OQ. You know how to find the area of sector O–PXQ and Δ OPQ. We can get area of segment PXQ by substracting area of the triangle from the area of the sector.

A (segment PXQ) = A (O – PXQ) – A (Δ OPQ)

$$= \frac{\theta}{360} \times \pi r^2 - A(\Delta\ OPQ) \ldots\ldots\ldots\ldots (I)$$

In the figure, seg PT $\perp$ radius OQ.

Now, in Δ OTP $\sin\theta = \dfrac{PT}{OP}$

$\therefore$ PT = OP $\sin\theta$

$$PT = r \times \sin\theta \qquad (\because OP = r)$$

$$A(\Delta OPQ) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times OQ \times PT$$

$$= \frac{1}{2} \times r \times r \sin\theta$$

$$= \frac{1}{2} \times r^2 \sin\theta \ \ldots\ldots\ldots\ldots\ (II)$$

From (I) and (II) ,

$$A(\text{segment } PXQ) = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \times \sin\theta$$

$$= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right]$$

(Note that, we have studied the sine ratios of acute angles only. So we can use the above formula when $\theta \le 90°$.)

Solved Examples

Ex. (1) In the figure 7.40, $\angle AOB = 30°$, OA = 12 cm . Find the area of the segment. ($\pi = 3.14$)

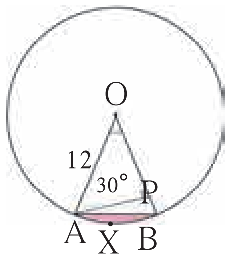

Fig. 7.40

Method I

$r = 12, \quad \theta = 30°, \quad \pi = 3.14$

$$A(O\text{-}AXB) = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{30}{360} \times 3.14 \times 12^2$$

$$= 3.14 \times 12$$

$$= 37.68 \text{ cm}^2$$

$$A(\Delta OAB) = \frac{1}{2} r^2 \times \sin\theta$$

$$= \frac{1}{2} \times 12^2 \times \sin 30$$

$$= \frac{1}{2} \times 144 \times \frac{1}{2}$$

$$\ldots\ldots(\because \sin 30° = \frac{1}{2})$$

$$= 36 \text{ cm}^2$$

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$$\text{A(segment AXB)} = \text{A (O-AXB)} - \text{A}(\Delta \text{ OAB})$$
$$= 37.68 - 36$$
$$= 1.68 \text{ cm}^2$$

Method II

$$\text{A(segment AXB)} = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$
$$= 12^2\left[\frac{3.14\times 30}{360} - \frac{\sin 30}{2}\right]$$
$$= 144\left[\frac{3.14}{12} - \frac{1}{2\times 2}\right]$$
$$= \frac{144}{4}\left[\frac{3.14}{3} - 1\right]$$
$$= 36\left[\frac{3.14-3}{3}\right]$$
$$= \frac{36}{3}\times 0.14$$
$$= 12 \times 0.14$$
$$= 1.68 \text{ cm}^2.$$

Ex. (2) The radius of a circle with centre P is 10 cm. If chord AB of the circle substends a right angle at P, find areas of the minor segment and the major segment. ($\pi = 3.14$)

Solution : $r = 10$ cm, $\theta = 90$, $\pi = 3.14$

$$\text{A(P-AXB)} = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{90}{360} \times 3.14 \times 10^2$$
$$= \frac{1}{4} \times 314$$
$$= 78.5 \text{ cm}^2$$

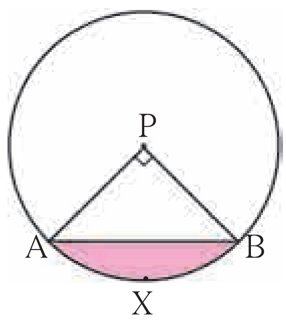


Fig. 7.41

$$\text{A}(\Delta\text{APB}) = \frac{1}{2} \text{ base} \times \text{height}$$
$$= \frac{1}{2} \times 10 \times 10$$
$$= 50 \text{ cm}^2$$

$$\text{A (minor segment)} = \text{A(P-AXB)} - \text{A}(\Delta\text{PAB})$$
$$= 78.5 - 50$$
$$= 28.5 \text{ cm}^2$$

$$A(\text{major segment}) = A(\text{circle}) - A(\text{minor segment})$$
$$= 3.14 \times 10^2 - 28.5$$
$$= 314 - 28.5$$
$$= 285.5 \text{ cm}^2$$

Ex. (3) A regular hexagon is inscribed in a circle of radius 14 cm. Find the area of the region between the circle and the hexagon. $(\pi = \frac{22}{7}, \sqrt{3} = 1.732)$

Solution : side of the hexagon = 14 cm

$$A(\text{hexagon}) = 6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2$$
$$= 6 \times \frac{\sqrt{3}}{4} \times 14^2$$
$$= 509.208 \text{ cm}^2$$

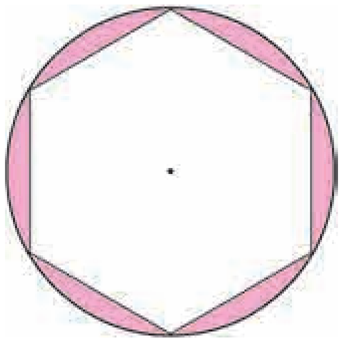


Fig. 7.42

$$A(\text{circle}) = \pi r^2$$
$$= \frac{22}{7} \times 14 \times 14$$
$$= 616 \text{ cm}^2$$

The area of the region between the circle and the hexagon

$$= A(\text{circle}) - A(\text{hexagon})$$
$$= 616 - 509.208$$
$$= 106.792 \text{ cm}^2$$

Practice set 7.4

1. In figure 7.43, A is the centre of the circle. $\angle ABC = 45°$ and $AC = 7\sqrt{2}$ cm. Find the area of segment BXC.

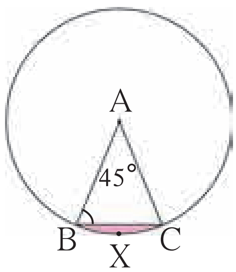


Fig. 7.43

2. In the figure 7.44, O is the centre of the circle. $m(\text{arc PQR}) = 60°$ OP = 10 cm. Find the area of the shaded region. $(\pi = 3.14, \sqrt{3} = 1.73)$

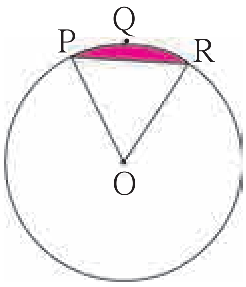


Fig. 7.44

3. In the figure 7.45, if A is the centre of the circle. $\angle$ PAR = 30°, AP = 7.5, find the area of the segment PQR

(π = 3.14)

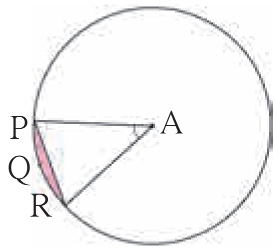


Fig. 7.45

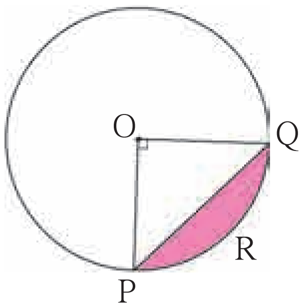
4. 

Fig. 7.46

In the figure 7.46, if O is the centre of the circle, PQ is a chord. $\angle$ POQ = 90°, area of shaded region is 114 cm^2, find the radius of the circle. (π = 3.14)

5. A chord PQ of a circle with radius 15 cm subtends an angle of 60° with the centre of the circle. Find the area of the minor as well as the major segment.

(π = 3.14, $\sqrt{3}$ = 1.73)

Problem set 7

1. Choose the correct alternative answer for each of the following questions.

(1) The ratio of circumference and area of a circle is 2:7. Find its circumference.

(A) 14π (B) $\frac{7}{\pi}$ (C) 7π (D) $\frac{14}{\pi}$

(2) If measure of an arc of a circle is 160° and its length is 44 cm, find the circumference of the circle.

(A) 66 cm (B) 44 cm (C) 160 cm (D) 99 cm

(3) Find the perimeter of a sector of a circle if its measure is 90 ° and radius is 7 cm.

(A) 44 cm (B) 25 cm (C) 36 cm (D) 56 cm

(4) Find the curved surface area of a cone of radius 7 cm and height 24 cm.

(A) 440 cm^2 (B) 550 cm^2 (C) 330 cm^2 (D) 110 cm^2

(5) The curved surface area of a cylinder is 440 cm^2 and its radius is 5 cm. Find its height.

(A) $\frac{44}{\pi}$ cm (B) 22π cm (C) 44π cm (D) $\frac{22}{\pi}$ cm

(6) A cone was melted and cast into a cylinder of the same radius as that of the base of the cone. If the height of the cylinder is 5 cm, find the height of the cone.

(A) 15 cm (B) 10 cm (C) 18 cm (D) 5 cm

(7) Find the volume of a cube of side 0.01 cm.

(A) 1 cm^3 (B) 0.001 cm^3 (C) 0.0001 cm^3 (D) 0.000001 cm^3

(8) Find the side of a cube of volume 1 m^3.

(A) 1 cm (B) 10 cm (C) 100 cm (D)1000 cm

2. A washing tub in the shape of a frustum of a cone has height 21 cm. The radii of the circular top and bottom are 20 cm and 15 cm respectively. What is the capacity of the tub ? ($\pi = \frac{22}{7}$)

3 ★. Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube ?

4. A metal parallelopiped of measures 16 cm × 11cm × 10 cm was melted to make coins. How many coins were made if the thickness and diameter of each coin was 2 mm and 2 cm respectively ?

5. The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of Rs. 10 per sq.m.

6. The diameter and thickness of a hollow metals sphere are 12 cm and 0.01 m respectively. The density of the metal is 8.88 gm per cm^3. Find the outer surface area and mass of the sphere.

7. A cylindrical bucket of diameter 28 cm and height 20 cm was full of sand. When the sand in the bucket was poured on the ground, the sand got converted into a shape of a cone. If the height of the cone was 14 cm, what was the base area of the cone ?

8. The radius of a metallic sphere is 9 cm. It was melted to make a wire of diameter 4 mm. Find the length of the wire.

9. The area of a sector of a circle of 6 cm radius is 15π sq.cm. Find the measure of the arc and length of the arc corresponding to the sector.

10.

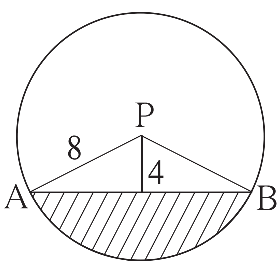


Fig.7.47

In the figure 7.47, seg AB is a chord of a circle with centre P. If PA = 8 cm and distance of chord AB from the centre P is 4 cm, find the area of the shaded portion.

($\pi = 3.14$, $\sqrt{3} = 1.73$)

11. In the figure 7.48, square ABCD is inscribed in the sector A-PCQ. The radius of sector C-BXD is 20 cm. Complete the following activity to find the area of shaded region.

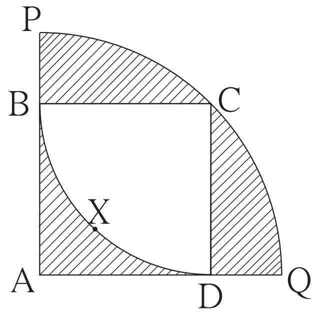


Fig. 7.48

Solution : Side of square ABCD = radius of sector C-BXD = ☐ cm

Area of square = $(\text{side})^2$ = $☐^2$ = ☐ ..... (I)

Area of shaded region inside the square

= Area of square ABCD − Area of sector C-BXD

$= ☐ - \frac{\theta}{360} \times \pi r^2$

$= ☐ - \frac{90}{360} \times \frac{3.14}{1} \times \frac{400}{1}$

$= ☐ - 314$

$= ☐$

Radius of bigger sector = Length of diagonal of square ABCD

$= 20\sqrt{2}$

Area of the shaded regions outside the square

= Area of sector A-PCQ − Area of square ABCD

= A(A-PCQ) − A(□ABCD)

$= \left(\frac{\theta}{360} \times \pi \times r^2\right) - ☐^2$

$= \frac{90}{360} \times 3.14\,(20\sqrt{2})^2 - (20)^2$

$= ☐ - ☐$

$= ☐$

$\therefore$ total area of the shaded region = 86 + 228 = 314 sq.cm.

12. In the figure 7.49, two circles with centres O and P are touching internally at point A. If BQ = 9, DE = 5, complete the following activity to find the radii of the circles.

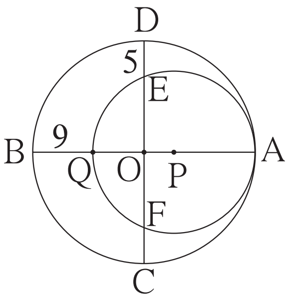


Fig. 7.49

Solution : Let the radius of the bigger circle be R and that of smaller circle be r.

OA, OB, OC and OD are the radii of the bigger circle

$\therefore$ OA = OB = OC = OD = R

PQ = PA = r

OQ = OB – BQ = $\square$

OE = OD – DE = $\square$

As the chords QA and EF of the circle with centre P intersect in the interior of the circle, so by the property of internal division of two chords of a circle,

OQ × OA = OE × OF

$\square \times R = \square \times \square$ .... ($\because$ OE = OF)

$R^2 - 9R = R^2 - 10R + 25$

R = $\square$

AQ = 2r = AB – BQ

2r = 50 – 9 = 41

r = $\square$ = $\square$

PVFTR5

ANSWERS

Chapter 1 Similarity

Practice set 1.1

1. $\frac{3}{4}$ **2.** $\frac{1}{2}$ **3.** 3 **4.** 1:1 **5.** (1) $\frac{BQ}{BC}$, (2) $\frac{PQ}{AD}$, (3) $\frac{BC}{DC}$, (4) $\frac{DC \times AD}{QC \times PQ}$

Practice set 1.2

1. (1) is a bisector. (2) is not a bisector. (3) is a bisector.

2. $\frac{PN}{NR} = \frac{PM}{MQ} = \frac{3}{2}$, therefore line NM || side RQ **3.** QP = 3.5 **5.** BQ = 17.5

6. QP = 22.4 **7.** x = 6 ; AE = 18 **8.** LT = 4.8 **9.** x = 10

10. Given, XQ, PD, Given, $\frac{\boxed{XR}}{\boxed{RF}} = \frac{\boxed{XQ}}{\boxed{QE}}$, Basic propotionality theorem, $\frac{\boxed{XP}}{\boxed{PD}} = \frac{\boxed{XR}}{\boxed{RF}}$

Practice set 1.3

1. Δ ABC ~ Δ EDC, AA test **2.** Δ PQR ~ Δ LMN ; SSS test of similarity

3. 12 metre **4.** AC = 10.5 **6.** OD = 4.5

Practice set 1.4

1. Ratio of areas = 9 : 25 **2.** $\boxed{PQ^2}$, $\frac{\boxed{4}}{\boxed{9}}$ **3.** $\boxed{A(\Delta PQR)}$, $\frac{4}{5}$

4. MN = 15 **5.** 20 cm **6.** $4\sqrt{2}$

7. $\boxed{PF}$; $\boxed{x}$ + $\boxed{2x}$; $\boxed{\angle FPQ}$; $\boxed{\angle FQP}$; $\frac{DF^2}{PF^2}$; $\boxed{20}$; $\boxed{45}$; $\boxed{45}$ – $\boxed{20}$; $\boxed{25 \text{ sq. unit}}$

Problem set 1

1. (1) (B), (2) (B), (3) (B), (4) (D), (5) (A)

2. $\frac{7}{13}$, $\frac{7}{20}$, $\frac{13}{20}$ **3.** 9 cm **4.** $\frac{3}{4}$ **5.** 11 cm **6.** $\frac{25}{81}$ **7.** 4

8. PQ = 80, QR = $\frac{280}{3}$, RS = $\frac{320}{3}$ **9.** $\frac{\boxed{PM}}{\boxed{MQ}} = \frac{\boxed{PX}}{\boxed{XQ}}$, $\frac{\boxed{PM}}{\boxed{MR}} = \frac{\boxed{PY}}{\boxed{YR}}$,

10. $\frac{AX}{XY} = \frac{3}{2}$ **12.** $\frac{\boxed{3}}{\boxed{2}}$, $\frac{\boxed{3}+\boxed{2}}{\boxed{2}}$, $\frac{\boxed{5}}{\boxed{3}}$, $\boxed{AA}$, $\frac{\boxed{5}}{\boxed{3}}$, $\boxed{15}$

Chapter 2 Pythagoras Theorem

Practice set 2.1

1. Pythagorean triplets ; (1), (3), (4), (6) **2.** NQ = 6 **3.** QR = 20.5

4. RP = 12, PS = $6\sqrt{3}$

5. | side opoosite to congruent angles | , | 45° |, | $\frac{1}{\sqrt{2}}$ |, | $\frac{1}{\sqrt{2}}$ |, | $\frac{1}{\sqrt{2}}$ |, | 2 |

6. side = $5\sqrt{2}$ cm, perimeter = $20\sqrt{2}$ cm **7.** (1) 18 (2) $4\sqrt{13}$ (3) $6\sqrt{13}$ **8.** 37 cm
10. 8.2 metre.

Practice set 2.2

1. 12 **2.** $2\sqrt{10}$ **4.** 18 cm

Problem set 2

1. (1) (B), (2) (B), (3) (A), (4) (C), (5) (D), (6) (C), (7) (B), (8) (A).
2. (1) $a\sqrt{3}$, (2) form a right angled triangle. (3) 61 cm, (4) 15 cm, (5) $x\sqrt{2}$, (6) ∠ PRQ.
3. RS = 6 cm, ST = $6\sqrt{3}$ cm **4.** 20 cm **5.** side = 2 cm, perimeter = 6 cm
6. 7 **7.** AP = $2\sqrt{7}$ cm **10.** 7.5 km / hr **12.** 8 cm **14.** 8 cm
15. 192 sq.unit **17.** 58 **18.** 26

Chapter 3 Circle

Practice set 3.1

1. (1) 90°, tangent-radius theorem (2) 6 cm ; perpendicular distance
(3) $6\sqrt{2}$ cm (4) 45°
2. (1) $5\sqrt{3}$ cm (2) 30° (3) 60° **4.** 9 cm

Practice set 3.2

1. 1.3 cm 2. 9.7 cm **4.** (3) 110° **5.** $4\sqrt{6}$ cm

Practice set 3.3

1. m(arc DE) = 90°, m(arc DEF) = 160°

Practice set 3.4

1. (1) 60° (2) 30° (3) 60° (4) 300° **2.** (1) 70° (2) 220° (3) 110° (4) 55°
3. ∠R = 92°; ∠N = 88° **7.** 44° **8.** 121°

Practice set 3.5

1. PS = 18; RS = 10, **2.** (1) 7.5 (2) 12 or 6
3. (1) 18 (2) 10 (3) 5 **4.** 4

Problem set 3

1. (1) D (2) B (3) B (4) C (5) B (6) D (7) A (8) B (9) A (10) C.
2. (1) 9 cm (2) in the interior of the circle (3) 2 locations, 12 cm
3. (1) 6 (2) ∠K = 30°; ∠M = 60° **5.** 10 **6.** (1) 9 cm (2) 6.5 cm

(3) 90° ; MS : SR = 2 : 1 **9.** $4\sqrt{3}$ cm

13. (1) 180° (2) $\angle AQP \cong \angle ASQ \cong \angle ATQ$

(3) $\angle QTS \cong \angle SQR \cong \angle SAQ$ (4) 65°, 130° (5) 100° **14.**(1) 70°

(2) 130° (3) 210° **15.** (1) 56° (2) 6 (3) 16 or 9 **16.** (1) 15.5°

(2) 3.36 (3) 6 **18.** (1) 68° (2) OR = 16.2, QR = 13 (3) 13 **21.** 13

Chapter 4 Geometric Constructions

Problem set 4

1. (1) C (2) A (3) A

Chapter 5 Co-ordinate Geometry

Practice set 5.1

1. (1) $2\sqrt{2}$ (2) $4\sqrt{2}$ (3) $\frac{11}{2}$ (4) 13 (5) 20 (6) $\frac{29}{2}$

2. (1) are collinear. (2) are not collinear. (3) are not collinear. (4) are collinear.

3. (−1, 0) **7.** 7 or −5

Practice set 5.2

1. (1, 3) **2.** (1) $\left(-\frac{1}{3},-\frac{1}{3}\right)$ (2) $\left(\frac{4}{7},-\frac{11}{7}\right)$ (3) $\left(0,\frac{13}{3}\right)$ **3.** 2:7 **4.** (−6, 3)

5. 2:5, k = 6 **6.** (11, 18) **7.** (1) (1, 3) (2) (6, −2) (3) $\left(\frac{19}{3},\frac{22}{3}\right)$

8. (−1, −7) **9.** h = 7, k = 18 **10.** (0, 2) ; (−2, −3)

11. (−9, −8), (−4, −6), (1, −4) **12.** (16, 12), (12, 14), (8, 16), (4, 18)

Practice set 5.3

1. (1) 1 (2) $\sqrt{3}$ (3) slope cannot be determined.

2. (1) 2 (2) $-\frac{3}{8}$ (3) $\frac{5}{2}$ (4) $\frac{5}{4}$ (5) $\frac{1}{2}$ (6) slope cannot be determined.

3. (1) are collinear. (2) are collinear. (3) are not collinear. (4) are collinear.

(5) are collinear. (6) are collinear.

4. $-5; \frac{1}{5}; -\frac{2}{3}$ 6. k = 5 7. k = 0 8. k = 5

Problem set 5

1. (1) D (2) D (3) C (4) C

2. (1) are collinear. (2) are collinear. (3) are not collinear. 3. (6, 13) 4. 3:1

5. (−7, 0) 6. (1) $a\sqrt{2}$ (2) 13 (3) $5a$ **7.** $\left(-\frac{1}{3},\frac{2}{3}\right)$

8. (1) Yes, scalene triangle (2) No. (3) Yes, equilateral triangle 9. $k = 5$

13. 5, $2\sqrt{13}$, $\sqrt{37}$ **14.** (1, 3) **16.** $\left(\frac{25}{6},\frac{13}{6}\right)$, radius = $\frac{13\sqrt{2}}{6}$ **17.** (7, 3)

18. Parallelogram **19.** A(20, 10), P(16, 12), R(8, 16), B(0, 20). **20.** (3, −2)

21. (7, 6) and (3, 6) **22.** 10 and 0

Chapter 6 Trigonometry

Practice set 6.1

1. $\cos\theta = \frac{24}{25}$; $\tan\theta = \frac{7}{24}$ **2.** $\sec\theta = \frac{5}{4}$; $\cos\theta = \frac{4}{5}$

3. $\text{cosec}\theta = \frac{41}{9}$; $\sin\theta = \frac{9}{41}$ **4.** $\sec\theta = \frac{13}{5}$; $\cos\theta = \frac{5}{13}$; $\sin\theta = \frac{12}{13}$

5. $\frac{\sin\theta + \cos\theta}{\sec\theta + \text{cosec}\,\theta} = \frac{1}{2}$

Practice set 6.2

1. Height of the church is 80 metre.
2. The ship is 51.90 metre away from the ligthhouse.
3. Height of the second building is $(10 + 12\sqrt{3})$ metre.
4. Angle made by the wire with the horizontal line is 30°.
5. Height of the tree is $(40 + 20\sqrt{3})$ metre.
6. The length of the string is 69.20 metre.

Problem set 6

1. (1) A (2) B (3) C (4) A

2. $\cos 60 = \frac{60}{61}$ **3.** $\sin\theta = \frac{2}{\sqrt{5}}$; $\cos\theta = \frac{1}{\sqrt{5}}$; $\text{cosec}\theta = \frac{\sqrt{5}}{2}$; $\sec\theta = \sqrt{5}$; $\cot\theta = \frac{1}{2}$

4. $\sin\theta = \frac{5}{13}$; $\cos\theta = \frac{12}{13}$; $\text{cosec}\theta = \frac{13}{5}$; $\tan\theta = \frac{5}{12}$; $\cot\theta = \frac{12}{5}$

6. Height of the building is $16\sqrt{3}$ metre.

7. The ship is $100\sqrt{3}$ metre away from the ligthhouse.

8. Height of the second buliding is $(12 + 15\sqrt{3})$ metre.

9. The maximum height that ladder can reach is 20.80 metre.

10. the plane was 1026 metre high at the time of landing.

Chapter 7 Mensuration

Practice set 7.1

1. 11.79 cm^3 **2.** 113.04 cm^3 **3.** 1413 sq.cm (by taking $\pi = 3.14$) **4.** 616 sq.cm
5. 21 cm **6.** 12 jugs **7.** 5 cm **8.** 273π sq.cm **9.** 20 tablets
10. 94.20 cm^3, 103.62 sq.cm **11.** 5538.96 sq.cm, 38772.72 cm^3
12. 1468.67π cm^3

Practice set 7.2

1. 10.780 litre **2.** (1) 628 sq.cm (2) 1356.48 sq.cm (3) 1984.48 cm^3

Practice set 7.3

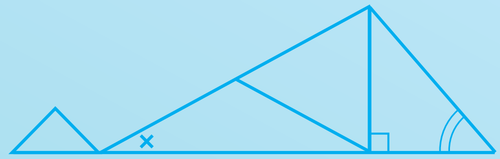
1. 47.1 sq.cm **2.** 25.12 cm **3.** 3.85 sq.cm **4.** 214 sq.cm **5.** 4 cm
6. (1) 154 sq.cm (2) 25.7 sq.cm (3) 128.3 sq.cm **7.** 10.2 sq.cm
8. 7.3 cm ; 22 cm **9.** (1) $90°$ (2) 22 cm
10. (1) 12.83 sq.cm (2) 89.83 sq.cm (3) 115.5 sq.cm **11.** 3.5 cm
12. x = 154 sq.cm ; y = 38.5 sq.cm ; z = 101.5 sq.cm
13. (1) 84.87 sq.cm (2) 25.67 sq.cm (3) 77.01 sq.cm (4) 7.86 sq.cm

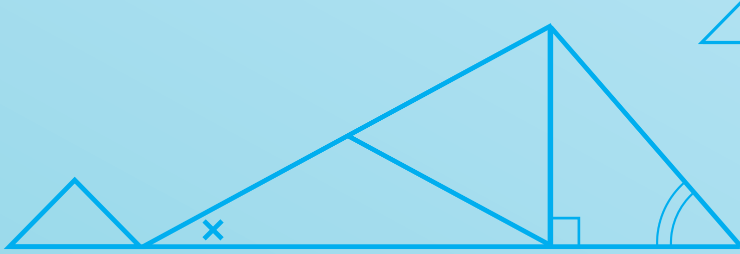
Practice set 7.4

1. 3.72 sq.cm **2.** 9.08 sq.cm **3.** 0.65625 sq.unit **4.** 20 cm
5. 20.43 sq.cm ; 686.07 sq.cm

Problem set 7

1. (1) A, (2) D, (3) B, (4) B, (5) A, (6) A, (7) D, (8) C.
2. 20.35 litre **3.** 7830 balls **4.** 2800 coins (by taking $\pi = \frac{22}{7}$) **5.** Rs. 6336
6. 452.16 sq.cm ; 3385.94 gm **7.** 2640 sq.cm **8.** 108 metre
9. $150°$; 5π cm **10.** 39.28 sq.cm





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