**Chapter** 

# **Refraction Through A Lens**

# **SYLLABUS**

Lenses (converging and diverging) including characteristics of the images formed (using ray diagrams only), magnifying glass, location of images using ray diagrams and thereby determining magnification.

Scope of syllabus : Types of lenses (converging and diverging), convex and concave action of a lens as a set of prism, technical terms: centre of curvature, radius of curvature, principal axis, foci, focal plane and focal length, detailed study of refraction of light in spherical lenses through ray diagrams, formation of images - principal rays or construction rays, location of images from ray diagrams for various positions of a small linear object on the principal axis, characteristics of images, sign convention and direct numerical problems using the lens formula are included (derivation of formula not required). Scale drawing or graphical representation of ray diagrams not required.

Power of a lens (concave and convex), simple direct numicrical problems. Magnifiying glass or simple microscope, location of image and magnification from ray diagram only (formula and numerical problems not included). Applications of lenses.

# (A) LENS AND REFRACTION OF LIGHT THROUGH A LENS

#### 5.1 LENS

We are all familiar with the use of lenses in spectacles. We define a lens as follows :

A lens is a transparent refracting medium bounded by either the two spherical surfaces or one surface spherical and other surface plane.

A plane surface can be treated as a spherical surface of infinite radius of curvature.

**Kind of lenses:** Lenses are of two kind:

- (1) converging or convex lens, and
- (2) diverging or concave lens.

#### (1) Converging or convex lens

A convex lens is thick in its middle and thin at the periphery. In other words, a lens which bulges out in the middle, is the convex lens. A light beam converges on passing through such a lens, so it is also called the converging lens.

A convex lens may be of the following three kinds:

- (i) bi-convex or double-convex or equi-convex,
- (ii) plano-convex, and

(iii) concavo-convex.

Fig. 5.1 shows the shape of different kind of convex lenses.



#### **Fig. 5.1 Convex lenses**

A biconvex lens has both its surfaces convex, a plano-convex lens has one surface plane and the other surface convex, while a concavo-convex lens has one surface convex and the other surface concave such that it is thicker in the middle as compared to its periphery.

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### (2) Diverging or concave lens

A concave lens is thick at its periphery and thin in the middle. In other words, a lens which is bent inwards in the middle, is the concave lens. Such a lens diverges the light rays incident on it, so it is also called the diverging lens.

A concave lens may be of the following three kinds:

(i) bi-concave or double-concave or equi-concave,

(ii) plano-concave, and

(iii) convexo-concave.

Fig. 5.2 shows the shape of different kind of concave lenses.



**Fig. 5.2 Concave lenses** 

A bi-concave lens has both its surfaces concave, a plano-concave lens has one of its surface plane and the other surface concave, while a convexo-concave lens has one surface concave and the other surface convex such that it is thinner in the middle as compared to its periphery.

**Note**: Both the concavo-convex and the convexo-concave lenses have one surface convex and the other surface concave, but they differ in their shape and action. A concavoconvex lens is thicker in the middle and has a converging action on a light beam, while a convexo-concave lens is thinner in the middle and has a *diverging action* on a light beam.

#### $5.2$ **ACTION OF A LENS AS A SET OF PRISMS**

We have read the refraction of light through a prism. The refraction of light through a lens can be understood in a simple way by considering a lens as being made up of a set of prisms as shown in Fig. 5.3.



To make it further simple, the prisms in the central portion of the lens, shown in Fig. 5.3, may be treated as a rectangular slab. Then the lens can be considered as being made up of a rectangular slab at the centre and one prism on either side of it as shown in Fig. 5.4.



(a) Convex lens (b) Concave lens Fig. 5.4 A lens being made up of a rectangular slab at the centre and one prism on either side of it

A convex lens in its upper part has a prism with its base downwards and in its lower part has a prism with its base upwards as shown in Fig.  $5.4(a)$ . On the other hand, a concave lens in its upper part has a prism with its base upwards and in its lower part has a prism with its base downwards as shown in Fig. 5.4(b).

### **Convergent action of a convex lens**

Let us consider the refraction of parallel rays of light A, B and C incident on the prisms in the upper, central and the lower parts of convex lens. We know that a ray of light incident on a prism, on refraction through it, always bends towards the base of prism. Therefore the prism in the upper part of convex lens bends the incident ray A downwards, while the prism in the lower part of convex lens bends the incident ray C upwards (Fig. 5.5). The central part which is a parallel sided glass slab passes the ray B normally incident on it, undeviated. Thus the set of prisms forming a convex lens converges the

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has a converging action on the incident light rays.



Fig. 5.5 Convergent action of a convex lens

#### Divergent action of a concave lens

In Fig. 5.6, the prism in the upper part of the concave lens bends the incident ray A upwards *i.e.*, towards its base, while the prism in the lower part of the concave lens bends the incident ray C downwards *i.e.*, towards its base. The central part, which is a parallel sided glass slab, passes the normally incident ray B undeviated. Thus, the set of prisms forming a concave lens *diverges* the parallel rays as if they are coming from a common point F situated on the side of rays incident on the lens. Therefore, a concave lens has a diverging action on the incident light rays.



Fig. 5.6 Divergent action of a concave lens

#### $5.3$ **TECHNICAL TERMS RELATED TO A LENS**

 $(1)$ Centre of curvature : A lens has two surfaces. Each surface of the lens is a part of a sphere. The centre of the sphere whose part is the lens surface, is called the centre of curvature of that surface of the lens. Since a lens has two spherical surfaces, so there are two centres of curvature of a lens. In Fig. 5.7,  $C_1$  and  $C_2$  are respectively the centres of curvature of the two surfaces 1 and 2 of the lens.



*Note*: For a convex lens,  $C_1$  is to the right of surface 1 and  $C_2$  is to the left of surface 2, while for a concave lens,  $C_1$  is to the left of surface 1 and  $C_2$  is to the right of surface 2.

- Radius of curvature : The radius of the  $(2)$ sphere whose part is the lens surface, is called the radius of curvature of that surface of the lens. In Fig. 5.7(a),  $PC_1$  and  $PC_2$  are the radii of curvature of the two surfaces 1 and 2 of the convex lens. If the lens is thin. then  $PC_1 = OC_1$  and  $PC_2 = OC_2$ . Similarly in Fig. 5.7(b),  $P_1C_1$  and  $P_2C_2$  are the radii of curvature of the two surfaces 1 and 2 of the concave lens. If the lens is thin, then  $P_1C_1 = OC_1$  and  $P_2C_2 = OC_2$ . The point O is called the *optical centre*. Thus for a thin lens, the radius of curvature of a surface of lens is equal to the distance of centre of curvature of that surface from the optical centre. For an equi-convex or equi-concave lens, the radii of curvature of both the surfaces are equal (i.e.,  $OC_1 = OC_2$ ). The lenses shown in Fig. 5.7 (a) and (b) are not equi-convex and equi-concave since the radii of curvature of the two surfaces  $OC_1$  and  $OC_2$  are not equal.
- Principal axis : It is the line joining the  $(3)$ centres of curvature of the two surfaces of *the lens.* In Fig. 5.7, the line  $C_1C_2$  is the principal axis. It can extend on either side of the lens.

(4) Optical centre : It is a point on the principal  $(5)$ axis of the lens such that a ray of light passing through this point emerges parallel to its direction of incidence. It is marked by the letter O in Fig. 5.7. The optical centre is thus the centre of lens.

Since the central portion of a thick lens can be considered to be a parallel sided glass slab, therefore a ray of light incident at the central portion of the lens, while passing through the optical centre O, is *slightly* displaced parallel to its original direction. In Fig. 5.8, the emergent ray is thus parallel to the incident ray.



(a) Convex lens (b) Concave lens Fig. 5.8 Optical centre (thick lens)

*Note*: In Fig 5.8, the lateral shift of the ray has been shown quite large, but actually it is very small.

Generally the lens is thin, so the lateral shift is small enough and it can be ignored. Therefore, a ray of light directed towards the optical centre of a thin lens can be considered to pass undeviated and undisplaced as shown in Fig. 5.9.



#### Thus,

Optical centre of a thin lens is the point on the principal axis of lens such that a ray of light directed towards it, passes undeviated through it.

Principal foci : A light ray can enter a lens from either side, therefore, a lens has two principal foci. If medium is same on either side of the lens, the two foci are situated at equal distances from the optical centre, one on either side of the lens. These are known as the first focal point (or first focus)  $F_1$  and the second focal point (or second focus)  $F_2$ .

## **First focal point**

For a convex lens, the first focal point is a point  $F<sub>1</sub>$  on the principal axis of the lens such that the rays of light coming from it, after refraction through the lens, become parallel to the principal axis of the lens [Fig. 5.10 (a)].



(b) Concave lens Fig. 5.10 First focus and focal length

For a concave lens, first focal point is a point F, on the principal axis of the lens such that the incident rays of light appearing to meet at it, after refraction from the lens become parallel to the principal axis of the lens [Fig.  $5.10$  (b)].

#### Downloaded from https:// www.studiestoday.com **Second focal point**  $(7)$

For a convex lens, the second focal point is a point  $F<sub>2</sub>$  on the principal axis of the lens such that the rays of light incident parallel to the principal axis, after refraction from the lens, pass through it [Fig. 5.11 (a)].



For a concave lens, second focal point is a point  $F<sub>2</sub>$  on the principal axis of the lens such that the rays of light incident parallel to the principal axis, after refraction from the lens, appear to be diverging from this point [Fig. 5.11 (b)].

- $(6)$ **Focal plane :** A plane normal to the principal axis, passing through the focus, is called the focal plane. A lens has two focal planes.
	- (i) First focal plane: A plane passing through the first focal point and normal to the principal axis of the lens, is called the first focal plane.
	- (ii) Second focal plane: A plane passing through the second focal point and normal to the principal axis of the lens, is called the second focal plane.

Focal length : The distance of focus (or focal point) from the optical centre of lens, is called its focal length. A lens has two focal lengths.

- (i) First focal length : The distance from the optical centre O of the lens to its first focal point  $F_1$  is called the first focal length  $f_1$  of the lens. In Fig. 5.10, it is shown as  $OF_1 = f_1$ .
- (ii) Second focal length : The distance from the optical centre O of the lens to the second focal point  $F<sub>2</sub>$  is called the second focal length  $f_2$  of the lens. In Fig. 5.11, it is shown as  $OF_2 = f_2$ .

Note :  $(1)$  If the medium on both sides of a lens is same, its first and second focal lengths are equal, i.e.,  $f_1 = f_2$  (numerically).

(2) Usually, when we say focus, we mean the second focal point. Hence the focal length of a lens implies the second focal length of the lens.

(3) A convex lens has a real focus (because the parallel rays incident on a convex lens actually pass through this point), while in a concave lens the focus is virtual (because the parallel rays incident on a concave lens do not actually pass through this point, but they appear to diverge from this point).

(4) Only a beam of light incident parallel to the principal axis converges to a single point  $F_2$ (the focus) on the principal axis after refraction through the convex lens. If the parallel beam of light is incident obliquely (*i.e.*, the rays are not parallel to the principal axis of the lens), it does not converge at the principal focus  $F_2$ , but it



Fig. 5.12 Refraction of an oblique parallel beam by a convex lens

# Downloaded from https:// www.studiestoday.com<br>converges at some point B in the second focal | 5.4 REFRACTION OF LIGHT THROUGH

plane of the lens as shown in Fig. 5.12. The point B lies on the second focal plane where the ray AO, through the optical centre O of the lens, meets the focal plane.

Similarly, if a parallel beam of light is incident obliquely on a concave lens, after refraction it appears to diverge from a point B in the second focal plane as shown in Fig. 5.13.





(5) The focal length of a lens depends on the following two factors :

- (i) The refractive index of the material of lens relative to its surrounding medium. If a lens is placed in water instead of air, its focal length increases.
- (ii) The radii of curvature of the two surfaces of lens. A thick lens has less focal length than a thin lens of same material.

(6) If a part of the lens is covered, its focal length remains unchanged, only the amount of light entering the lens decreases due to which the intensity of image decreases but the position, size and nature of image formed by it do not change.

#### Difference between a convex and a concave lens



#### **THE EQUI-CONVEX AND EQUI-CONCAVE LENSES**

Fig. 5.14 shows the refraction of a ray of light at the two surfaces of an equi-convex lens and an equi-concave lens. When a ray of light AB is incident on a lens, its path changes because it suffers refraction at two surfaces of the lens marked as I and II in the diagram. First the light ray AB suffers refraction at the first surface I from air to glass, so it bends towards the normal N<sub>1</sub>B at the point B. The refracted ray BC then falls on the second surface II of the lens where it now suffers refraction from glass to air, so it bends away from the normal  $N_2C$  at the point C and emerges out as CD. Thus the ray of light bends at both the surfaces of lens in the same direction and hence the total deviation produced by the lens is the sum of the deviations at the two surfaces of lens.



Fig. 5.14 Refraction through a lens

It is clear from Fig. 5.14, that a convex lens bends the ray of light towards its middle i.e., it converges the light, while a concave lens bends a ray of light towards its edges (or away from its middle) i.e., it diverges the light.

Note : For ray diagrams, we shall consider the lens to be thin and for simplicity we shall not show the bending of ray of light at each of the two surfaces of lens separately, but we shall show the net bending towards the central part in a convex lens or away from the central part in a concave lens, at the straight vertical line passing through the optical centre.

**EXERCISE-5(A)** 

- 1. What is a lens?
- 2. Name the two kinds of lens. Draw diagrams to illustrate them.
- 3. State difference between a convex and a concave lens in their (a) appearance, and (b) action on the incident light.
- 4. Which lens is converging : (i) an equiconcave lens or an equiconvex lens. (ii) a concavo-convex lens or a convexo-concave lens ?

Ans. (i) an equiconvex lens (ii) a concavo-convex lens

- 5. Out of the two lenses, one concave and the other convex, state which one will show the divergent action on a light beam. Draw diagram to illustrate **Ans.** Concave vou answer.
- 6. Show by a diagram the refraction of two light rays incident parallel to the principal axis on a convex lens by treating it as a combination of a glass slab and two triangular glass prisms.
- 7. Show by a diagram, the refraction of two light rays incident parallel to the principal axis on a concave lens by treating it as a combination of a glass slab and two triangular glass prisms.
- 8. How does the action of a convex lens differ from that of a concave lens on a parallel beam of light incident on them ? Draw diagrams to illustrate your answer.
- 9. Define the term principal axis of a lens.
- 10. Explain optical centre of a lens with the help of proper diagram(s).
- 11. A ray of light incident at a point on the principal axis of a convex lens passes undeviated through the lens. (a) What special name is given to this point on the principal axis ? (b) Draw a labelled diagram to support your answer in part (a).

Ans. (a) Optical centre, (b) Fig. 5.9(a)

12. State the condition when a lens is called an equiconvex or equi-concave.

> Ans. When radii of curvature of the two surfaces of lens are equal.

- 13. Define the term principal foci of a convex lens and illustrate your answer with the aid of proper diagrams.
- 14. Define the term principal foci of a concave lens and show them with the help of proper diagrams.
- 15. Draw a diagram to represent the second focus of a concave lens.
- 16. Draw a diagram to represent the second focus of a convex lens.
- 17. A ray of light, after refraction through a concave lens emerges parallel to the principal axis. (a) Draw a ray diagram to show the incident ray and its corresponding emergent ray. (b) The incident ray when produced meets the principal axis at a point F. Name the point F.

Ans. (a) See Fig.  $5.10$  (b); (b) first focus.

18. A ray of light after refraction through a convex lens emerges parallel to the principal axis. (a) Draw a ray diagram to show it. (b) The incident ray passes through a point F on the principal axis. Name the point F.

Ans. (a) See Fig. 5.10 (a); (b) first focus.

19. A beam of light incident on a convex lens parallel to its principal axis converges at a point F on the principal axis. Name the point F. Draw a ray diagram to show it.

Ans. Second focus, Fig. 5.11 (a)

20. A beam of light incident on a thin concave lens parallel to its principal axis diverges and appears to come from a point F on the principal axis. Name the point F. Draw a ray diagram to show it.

Ans. Second focus, Fig. 5.11 (b)

- 21. Define the term focal length of a lens.
- 22. What do you mean by focal plane of a lens?
- 23. State the condition for each of the following :
	- (a) a lens has both its focal lengths equal.
	- (b) a ray passes undeviated through the lens.

Ans. (a) medium is same on either side of lens,

- (b) it is incident towards the optical centre of the lens.
- 24. A parallel oblique beam of light falls on a (i) convex lens, (ii) concave lens. Draw a diagram in each case to show the refraction of light through the lens.

25. The diagram alongside shows a lens **as** a combination of a glass slab and two prisms.



- (ii) What is the line XX' called ?
- (iii) Complete the ray diagram and show the path of the incident ray AB after passing through the lens.
- (iv) The final emergent ray will either meet XX' at a point or appear to come from a point on XX'. Label the point as F. What is this point called ?
- 26. The diagram alongside shows a lens as a combination of a glass slab and two prisms.



- (i) Name the lens formed by the combination.
- (ii) What is the line XX' called ?
- (iii) Complete the path of the incident ray AB after passing through the lens.
- (iv) The final emergent ray either meets XX' at a point or appears to come from a point on XX'. Label it as F. What is this point called?
- 27. In Fig. 5.17,  $F_1$  and  $F_2$  are the positions of the two foci of the thin lenses shown in diagram (a) and (b). Draw the path taken by the light ray AB after it emerges from the lens in each diagram (a) and (b).



- Fig. 5.17
- 28. In Fig. 5.18,  $F_1$  and  $F_2$  are the two foci of the thin lenses shown in diagram (a) and (b) and AB is the incident ray. Complete the diagram to show the path of the ray AB after refraction through the lens in each diagram (a) and (b).



- 29. Complete the following sentences :
	- (a) If half part of a convex lens is covered, the focal of image ......................
	- (b) A convex lens is placed in water. Its focal length will .....................
	- (c) The focal length of a thin convex lens is ............... than that of a thick convex lens.

Ans. (a) does not, decreases (b) increase (c) more

#### **MULTIPLE CHOICE TYPE**

- 1. A ray of light after refraction through a lens emerges parallel to the principal axis of the lens. The incident ray passes through :
	- (a) its optical centre
	- (b) its first focus
	- (c) its second focus
	- (d) the centre of curvature of the first surface.

Ans. (b) its first focus

- 2. A ray of light incident on a lens parallel to its principal axis, after refraction passes through or appears to come from :
	- (a) its first focus
	- (b) its optical centre
	- (c) its second focus
	- (d) the centre of curvation of its second surface.

Ans. (c) its second focus

Downloaded from https:// www.studiestoday.com (B) FORMATION OF IMAGE BY A LENS

#### PRINCIPAL (OR CONSTRUCTION)  $5.5$ **RAYS FOR RAY DIAGRAMS**

The position, size and nature of the image of an object formed by a lens, can be determined by drawing a ray diagram. For this, we need to consider at least two rays, as per convenience, starting from a point on the object. The rays chosen are those for which the path after refraction from the lens is known to us.

Generally we use the following three principal rays for the construction of the ray diagrams.

 $(1)$  A ray of light incident at the optical centre O of the lens passes undeviated through the lens as shown in Fig. 5.19 (a) and (b) respectively for the convex and concave lens.



Fig. 5.19 A ray through the opticenter passes undenated

(2) A ray of light incident parallel to the principal axis of the lens, after refraction passes through the second focus  $F_2$  (in a convex lens) or appears to come from the second focus  $F_2$  (in a concave lens) as shown in Fig. 5.20 (a) and (b).



Fig. 5.20 A ray incident parallel to principal axis either passes or appears to pass through the second focus

(3) A ray of light passing through the first focus  $F_1$  (in a convex lens) or directed towards the first focus  $F_1$  (in a concave lens), emerges parallel to the principal axis after refraction as shown in Fig.  $5.21(a)$  and (b).



(b) Concave lens

#### Fig. 5.21 A ray either incident from or directed towards first focus becomes parallel to the principal axis

From a point of the object, although an infinite number of rays travel in all possible directions, but we choose only two convenient rays out of the above mentioned three rays from that point on the object. The point where the rays meet (or appear to meet) after refraction from the lens is the image of that point of the object. The image is obtained in a similar manner for each point of the object and all these image points together then form the full image of the object.

Kind of images : The images can be of two kind:  $(1)$  real, and  $(2)$  virtual.

- **Real image:** If the rays from a point of  $(1)$ object after refraction through the lens actually meet at a point, the image is real. If a screen is placed at this point, the image is obtained on it (i.e., a real image can be obtained on a screen).
- Virtual image: If the rays from a point of  $(2)$ object after refraction through the lens do not actually meet at a point, but they appear to diverge from a point, the image is virtual. A screen placed at this point will not show any image on it (i.e., a virtual image cannot be obtained on a screen). However, the eye kept between the diverging rays is able to see this image because the eye lens being convex, converges the diverging rays to form the image on the retina of eye.

### Distinction between a real and a virtual image



#### 5.6 **CONSTRUCTION OF RAY DIAGRAM FOR A LENS**

A ray diagram to determine the position and the characteristics of the image formed by a convex lens is drawn following the steps given below.

Draw a line PQ across the paper. Name it as  $(1)$ principal axis. Choose a point O on the principal axis as the *optical centre* (Fig. 5.22). Draw the *convex lens* and its principal section by a vertical line passing through O. For a thin lens, we shall show the light rays to bend at this line, instead of bending them at the two surfaces of lens.



Choose a proper scale and mark two principal  $(2)$ *foci*  $F_1$  and  $F_2$  on the principal axis PQ (Fig. 5.23).  $F_1$  is on the left of O, while  $F_2$ is on the right of O at an equal distance i.e.,  $OF<sub>1</sub> = OF<sub>2</sub>$ , since there is air on both sides of the lens.



 $(3)$ Draw a straight line AB on the principal axis to represent the linear object, of given height at the given position according to the scale choosen (Fig.  $5.24$ ).



Draw one ray of light AO from the top  $(4)$ point A of the object, passing straight through the optical centre O of the lens without any deviation (Fig. 5.25).



#### Fig. 5.25

Draw second ray of light AD from the same  $(5)$ top point A of the object, parallel to the principal axis PQ up to the lens. After refraction, this ray will pass through the focus  $F_2$  as DA' (Fig. 5.26).



For the second ray, the other choice can be to drawn a ray AD' from the top point A of the object passing through the first focus  $F_1$ and then up to the lens. After refraction, this ray becomes parallel to the principal axis PQ as D'A' (Fig. 5.27).



The point A' where the two refracted rays  $(6)$ meet (or appear to meet) is the image of the

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point A of the object. By drawing a perpendicular A'B' from A' on the line PO, we get the image A'B' of the object AB  $(Fig. 5.28).$ 



Fig. 5.28 Three construction rays in a convex lens

 $(7)$  Measure the height A'B' and the distance OB' of the image and then convert it on the chosen scale to get the actual height and position of the image.

The same procedure is adopted to draw the ray diagram for a *concave lens*. In a concave lens, the focus  $F_1$  is on the right of optical centre O and the focus  $F<sub>2</sub>$  is on its left. In Fig. 5.29, the ray of light AO incident at the optical centre passes undeviated as AOX, the ray of light AD incident parallel to the principal axis will appear to diverge as DY from  $F_2$  after refraction and the ray AD' appearing to meet at the focus F<sub>1</sub> will become parallel to the principal axis as D'Z after refraction. The refracted rays when produced backward, meet at A'. Thus A' is the virtual image of the point A of object. For the object AB, the virtual image is A'B'.



Fig. 5.29 Three construction rays in a concave lens.

#### **CHARACTERISTICS AND LOCATION**  $5.7$ OF IMAGES FOR A CONVEX LENS

Now we shall consider the ray diagrams for formation of images by a convex lens (focal length =  $f$ ) for different positions of the object. Let *u* denotes the distance of object from the lens. Case (i) : When the object is at infinity (*i.e.*,  $u = \infty$ ) In Fig. 5.30, the rays coming from an object at WWW.Studiestoday.com<br>infinity, are parallel to each other and they are incident on the convex lens parallel to the principal axis of the lens, which after refraction pass through the second focus  $F_2$  of the lens. Thus a real, inverted and diminished (almost to a point) image is formed at the focus on the other side of the lens.



Fig. 5.30 Image formation by a convex lens for an object at infinity

If the object AB is at a far distance  $(u \gg f)$ : The light rays reaching the lens from a point of the object will be parallel to each other, but will be incident obliquely on the lens. In Fig. 5.31, the object AB is not shown. The rays from the top point A of the object are obliquely incident on the convex lens. The incident ray passing through the optical centre O travels undeviated through the lens, as OA'. Another ray from the same point A of the object, incident on the lens through the first focus  $F_1$ , after refraction becomes parallel to the principal axis as DA'. The two refracted rays OA' and DA' meet at a point A'. Thus A' is the real image of the object point A which will lie in the focal plane of the lens passing through the second focus F<sub>2</sub>. Similarly, for the bottom point B of the object lying on the principal axis, the image will be B' at  $F_2$ . Thus B'A' is the real, inverted and highly diminished image of the object AB formed in the second focal plane.



Fig. 5.31 Image formation by a convex lens for a very distant object

#### Characteristics and location of the image formed

The image is at the *focus* (or in the focal plane) on the other side of the lens. It is

(a) real, (b) inverted, and (c) highly diminished.

**Application :** In this manner, a convex lens is used either as a burning glass (Fig. 5.30) or a camera lens (Fig. 5.31). To use the convex lens as a burning glass, the rays of light from sun (at infinity) are brought to

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focus on a piece of paper kept in the second focal plane of the lens. Due to sufficient heat of sun rays, the paper burns. In a camera, the object lies very far from the lens and the image is formed at the camera film which is at the second focus (or lies in the second focal plane) of the lens.

#### Case (ii) : When the object is beyond  $2F_1$  (i.e.,  $u > 2f$ )

In Fig. 5.32, AB is an object placed on the principal axis of a convex lens at a point beyond  $2F_1$ . From the point A of the object, the ray AD incident parallel to the principal axis, after refraction through the lens, passes through the second focus  $F_2$  as DA'. The other ray AO incident at the optical centre O of the lens travels undeviated as OA'. The two refracted rays DA' and OA' meet at a point A'. Thus A' is the real image of the point A of the object. Similarly B' is the image of the point B of the object. Thus A'B' is the real, inverted and diminished image of the object AB.



Fig. 5.32 Image formation by a convex lens for the object beyond 2F

#### Characteristics and location of the image formed

The image is between  $F_2$  and  $2F_2$  on the other side of the lens. It is

(a) real, (b) inverted, and (c) diminished.

**Application :** A convex lens is used in this manner as a camera lens, when the object, not very far, is to be photographed.

#### Case (iii) : When the object is at  $2F_1$  (i.e.,  $u = 2f$ )

In Fig. 5.33, AB is an object placed on the principal axis of a convex lens at a distance equal to twice the focal length of the lens *i.e.*, at  $2F_1$ . From the point A of the object, the ray AD incident parallel to the principal axis after refraction through the lens, passes through its second focus  $F_2$  as DA'. The other ray AO incident through the optical centre O of the lens, travels undeviated as OA'. The two refracted rays DA' and OA' meet at a point A' which is the *real image* of the point A of the object. Similarly for the point B of the object, the image is at B'. Thus A'B' is the real and inverted *image* of the object AB. This image is of same size as the object.



Fig. 5.33 Image formation by a convex lens for the object at 2F,

#### Characteristics and location of the image formed

The image is at  $2F_2$  on the other side of the lens. It is (a) real, (b) inverted, and (c) of the same size as the object.

**Application :** In this manner, a convex lens is used in a terrestrial telescope for erecting the inverted image formed by the objective lens.

#### Case (iv) : When the object is between  $F_1$  and  $2F_1$  $(i.e., f < u < 2f)$

In Fig. 5.34, AB is an object placed on the principal axis of a convex lens at a point between  $F_1$  and  $2F_1$ . From the point A of the object, a ray AD incident parallel to the principal axis, after refraction from the lens, passes through its second focus  $F_2$  as DA'. The other ray AO incident towards the optical centre O, passes undeviated through the lens as OA'. The two refracted rays DA' and OA' meet at a point A' which is the *real image* of the point A of the object. Similarly, for the point B of the object, the image is at B'. Thus A'B' is the real, inverted and magnified image of the object AB.



the object between F, and 2F,

#### Characteristics and location of the image formed

The image is beyond  $2F_2$  on the other side of the lens. It is (a) real, (b) inverted, and (c) magnified.

Application : In this manner, a convex lens is used in cinema and slide projectors. Here the magnified image is obtained on a screen placed at a large distance on

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the other side of the PAY Diggig ed trom https:// (or film *i.e.*, illuminated object) in front of the lens just beyond its focus in *inverted* position so as to obtain an erect and magnified image on the screen.

#### Case (v) : When the object is at  $F_1$  (*i.e.*,  $u = f$ )

In Fig. 5.35, AB is an object placed at the focus  $F_1$  on the principal axis of a convex lens. From the point A of the object, a ray AD incident parallel to the principal axis, after refraction from the lens, passes through the second focus  $F_2$  of the lens as  $DF_2$ . The other ray AO incident towards the optical centre O of the lens, passes undeviated through it as OA'. The two refracted rays  $DF_2$  and OA' being parallel to each other, do not converge at a point at finite distance. For point B, the image will be at B' at infinity on the principal axis. Thus a highly enlarged image A'B' is formed at infinity.



for the object at  $F<sub>1</sub>$ 

#### Characteristics and location of the image formed

The image is at infinity *i.e.*, at a very far distance, on the other side of the lens. It is (a) real, (b) inverted, and (c) highly magnified.

Application : In this manner, a convex lens is used in the *collimator* of a spectrometer to obtain a parallel beam of light by placing the source of light at the focus of convex lens.

#### Case (vi) : When the object is between the lens and focus (*i.e.*, between O and  $F_1$  or  $u < f$ )

In Fig. 5.36, AB is an object placed on the principal axis of a convex lens between its optical centre O and the first focus  $F_1$ . From point A of the object, a ray AD incident parallel to the principal axis, after refraction through the lens passes through the second focus  $F_2$  as  $DF_2$ . The other ray AO incident at the optical centre O of the lens, passes undeviated as OO'. The two refracted rays DF<sub>2</sub> and OO' do not meet each other, but they appear to diverge from a point A', *i.e.*, when they are WWW.ed Wallest.qdg Yncep an a point A'. Thus A' is the virtual image of the point A of the object. Similarly, for point B of the object, B' is the virtual image. Thus, A'B' is the virtual, erect and magnified image of the object AB which is formed on the same side and behind the object. The image can be distinctly seen by the eye by keeping it at the position shown in Fig. 5.36 so that the eye lens converges the diverging rays to form a real image on the retina of eye.



Fig. 5.36 Image formation by a convex lens for the object between the optical centre and the focus

#### Characteristics and location of the image formed

The image is on the same side and behind the object. It is (a) virtual, (b) erect or upright, and (c) magnified.

**Application :** In this manner, a convex lens is used as a reading lens (i.e., a magnifying glass or a simple *microscope)* to form a magnified virtual image of a tiny object (such as a small letter of a book or a small part of a watch etc.).

**Inference**: From above, we notice that both real as well as virtual images can be formed by a convex lens depending upon the position of object relative to the lens. The size of image (magnified, diminished or same) also depends on the position of object.

- When object is very far off from the convex  $(1)$ lens, a real, inverted and diminished image is formed at the focus.
- $(2)$ As the object moves towards the lens up to 2F, the image moves away from the focus of the lens up to 2F. However, the image formed will remain real, inverted and diminished, but its size will gradually increase.

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- As object comes at the position 2F, the image  $(3)$ is real, inverted and of the same size. It is obtained at the position 2F on the other side of the lens.
- On further moving the object from 2F  $(4)$ towards its focus F, the image will remain real and inverted, but it becomes magnified and is formed away from 2F on the other side of the lens. When object comes at focus F, the image moves to infinity.
- On further moving the object from F towards  $(5)$ the lens *i.e.*, if the object is brought between

the focus and the lens, the image becomes magnified, erect, and virtual. It is formed behind the object.

*Note*: The above observations will be same if the lens is moved towards the object instead of moving the object towards the lens.

The table below gives the position, size, nature of the image formed by a convex lens and its application corresponding to the different positions of the object.



#### Relative positions of the object and image in a convex lens

#### $5.8$ **CHARACTERISTICS AND LOCATION** OF IMAGES FOR A CONCAVE LENS

Let us now determine the position and characteristics of the image formed by a concave lens by ray diagrams for the different positions of the object.

#### Case (i) : When the object is at infinity

The light rays from an object at infinity are parallel to each other and in Fig. 5.37 they are incident parallel to the principal axis of the concave lens, so after refraction from the concave lens, they appear to diverge from the second focus  $F_2$ . Thus a virtual, erect and diminished image is formed at the second focus on the side of the object in front of the concave lens.



Fig. 5.37 Ray diagram for image formation by a concave lens when the object is at infinity

If the object is at a far distance on the principal axis of a concave lens (which is not shown in the diagram) : From the top point A of the object, rays reaching the lens will be parallel to each other and they will be obliquely incident on the lens. The ray PD incident towards the first focus  $F_1$  of the lens, after refraction from the lens, becomes parallel to the principal axis as DD'. The other ray QO from the same point A of object, incident at the optical centre O of the lens, passes undeviated through the lens as OO'. The two refracted rays DD' and OO' do not meet each other, but when produced backward, they meet at a point A' which is the virtual image of the point A of the object. Similarly, for the bottom point B of the object



Fig. 5.38 Ray diagram for image formation by a concave lens when the object is at a very large distance

lying on the principal axis, the virtual image is formed at B' at the second focus  $F_2$  of the lens. Thus A'B' is the virtual, erect (or upright) and highly diminished image of the object AB which is formed in the focal plane at second focus  $F_2$  of the lens.

#### Characteristics and location of the image

The image is at the second focus  $F_2$  (or in the focal plane at  $F_2$ ) on the side of the object. It is

(a) virtual, (b) erect, and (c) highly diminished.

Application : A concave lens is used in Galilean telescope in this manner.

#### Case (ii) : When the object is at any finite distance from the concave lens

Let AB be an upright object placed at a point on the principal axis at any finite distance, in front of a concave lens (Fig. 5.39). Two rays AD and AO from the point A of the object, after refraction through the lens, diverge from each other. The ray AD incident parallel to the principal axis is refracted as DD' so as to appear to come from the second focus  $F_2$ . The ray AO, incident towards the optical centre O, passes undeviated after refraction as OO'. The two refracted rays DD' and OO' do not meet at any point. But the



Fig. 5.39 Ray diagram for image formation by a concave lens when the object is between infinity and the lens

refracted ray DD', on being produced backward, meets the ray AO at a point A' which is the *virtual* image of the point A. Similarly B' is the virtual image of the point B. Thus A'B' is the virtual, erect and diminished image of the object AB formed between the optical centre and second focus of the lens.

#### Characteristics and location of the image

The image is between the lens and focus, on the side of object. It is (a) virtual, (b) erect, and (c) diminished.

**Application :** A concave lens is used in *spectacles* for the *short-sighted persons* in this manner.

Inference : From above, we notice that irrespective of the position of the object, the image formed by a thin concave (or divergent) lens is always virtual, upright, diminished, and it is situated on the side of the object between the focus and the lens.

- $(1)$ When the object moves from a large distance towards the lens, the image shifts from focus towards the optical centre of the lens and the size gradually increases, but it always remains smaller than the object.
- When the object is at a distance equal to the  $(2)$ focal length of the lens, the image is exactly at the mid-point between the optical centre and the second focus of the lens.

The table below gives the position, size, nature of the image formed by a concave lens and its application corresponding to the different positions of the object.



#### Relative positions of the object and image in a concave lens

#### Downloaded from https:// www.studiestoday.com 5.9 DIFFERENCE BETWEEN THE IMAGE FORMED BY A CONVEX AND A CONCAVE **LENS**



**EXAMPLES** 

Fig 5.40 (a) and (b) show the refracted ray BC 1. through a convex and concave lens respectively and their foci marked as  $F_1$  and  $F_2$ . Complete the diagram by drawing each corresponding incident ray.



Fig. 5.40 (a) and (b) show the refracted ray BC parallel to the principal axis. Therefore, the incident ray must be coming from first focus  $F_1$  in convex lens and must be travelling towards the first focus  $F_1$  in the concave lens. Thus, to find the incident ray,  $F_1$  is joined to the starting point B of the refracted ray and then produced backward as BA. Then AB is the required incident ray. The completed diagram is shown is Fig. 5.41(a) and (b).



2. Fig 5.42(a) and (b) show the refracted ray BC through a convex and concave lens respectively. Complete each diagram by draning the incident ray if  $F_1$  and  $F_2$  are in the principal foci of the lens.



#### Fig. 5.42

In Fig. 5.42, the refracted ray BC passes through the second focus  $F_2$  in the convex lens and appear to be coming from  $F<sub>2</sub>$  in the concave lens, so the incident ray must be parallel to the principal axis. For this, a line BA is drwn from the point B parallel to the principal axis in each case so that AB is the incident ray. The completed diagram is shown in Fig. 5.43 (a) and (b).



#### Fig. 5.43

The diagram ahead shows an object AB placed 3. on the principal axis of a lens L. The two foci of the lens are  $F_1$  and  $F_2$ . The image formed by the lens is erect, virtual and diminished.



Fig. 5.44

- (i) Draw the outline of the lens L used and name it.
- (ii) Draw a ray of light starting from B and passing through O. Show the same ray after refraction by the lens.
- (iii) Draw another ray from B which is incident parallel to the principal axis and show how does it emerge after refraction from the lens.
- (iv) Locate the final image formed.
- (i) Since the image formed by the lens is erect, virtual and diminished, the lens is concave. The outline of the lens L is shown in Fig. 5.45.



#### Fig. 5.45

- (ii) The light ray BO is incident at the optical centre O of the lens, it passes undeviated as OC after refraction through the lens.
- (iii) The light ray BP is incident parallel to the principal axis. It emerges as PQ after

refraction which appears to diverge from the second focus  $F_2$  of the lens.

(iv) The refracted rays OC and PQ do not actually meet, but they appear to diverge from a point  $B'$  (i.e., they meet at a point  $B'$  when they are produced backwards). Thus B' is the virtual image of the point B. The complete image A'B' is obtained by drawing perpendicular from B' on the line  $F_2OF_1$ . The image is formed between the optical centre O and focus  $F<sub>2</sub>$  of the lens.

The completed ray diagram is shown in Fig. 5.45.

4. Is it possible to burn a piece of paper using a convex lens in day light without using the matchbox or any direct flame? Draw a diagram to support your answer.

Yes, it is possible by converging the light rays coming from sun on the piece of paper placed in the focal plane of the convex lens. The ray diagram is. shown in Fig. 5.46.



Note: Oblique parallel rays can also be made incident on the convex lens as shown in Fig. 5.12.



- 1. What are the *three* principal rays that are drawn to construct the ray diagram for the image formed by a lens ? Draw diagrams to support your answer.
- In the diagrams below, XX' represents the principal  $2.$ axis, O the optical centre and F the focus of the lens. Complete the path of the rays A and B as they emerge out of the lens.



- Distinguish between a real and a virtual image. 3.
- 4. Study the diagram (Fig. 5.48) given below.



- (a) Name the lens LL'.
- (b) What are the points O and O' called ?
- (c) Complete the diagram to form the image of the object AB.

- (d) State the *three* characteristics of the image.
- (e) Name a device in which this action of lens is used.
- Study the diagram (Fig. 5.49) below. 5.



- (i) Name the lens LL'.
- (ii) What are the points O, O' called ?
- (iii) Complete the diagram to form the image of the object AB.
- (iv) State *three* characteristics of the image.
- 6. The diagram in Fig. 5.50 shows an object AB and a converging lens L with foci  $F_1$  and  $F_2$ .
	- (a) Draw two rays from the object AB and complete the diagram to locate the position of image CD. Also mark on the diagram the position of eye from where the image can be viewed.



Fig. 5.50

- (b) State three characteristics of the image in relation to the object.
- 7. The diagram in Fig. 5.51 shows the position of an object OA in relation to a converging lens L whose foci are at  $F_1$  and  $F_2$ .



(i) Draw two rays to locate the position of image.

- (ii) State the position of image with reference to the lens.
- (iii) Describe three characteristics of the image.
- (iv) Describe how the distance of image from the lens and its size change as the object is moved towards  $F_1$ .
- 8. A converging lens forms the image of an object placed in front of it, beyond 2F<sub>2</sub> of the lens. (a) Where is the object placed ? (b) Draw a ray diagram to show the formation of image. (c) State three characteristics of the image.
- 9. A convex lens forms an image of an object equal to the size of the object. (a) Where is the object placed in front of the lens ? (b) Draw a diagram to illustrate it. (c) State two more characterstics of the image.
- 10. A lens forms an erect, magnified, and virtual image of an object. (a) Name the kind of lens. (b) Where is the object placed in relation to the lens ? (c) Draw a ray diagram to show the formation of image. (d) Name the device which uses this principle.
- 11. A lens always forms an image between the object and the lens. (a) Name the lens. (b) Draw a ray diagram to shown the formation of such image. (c) State three characteristics of the image.
- 12. Where must a point source of light be placed in front of a convex lens so as to obtain a parallel Ans. At focus beam of light?
- 13. Classify as real or virtual, the image of a candle flame formed on a screen by a convex lens. Draw a ray diagram to illustrate how the image is formed.
- 14. Show by a ray diagram that a diverging lens cannot form a real image of an object placed anywhere on its principal axis.
- 15. Draw a ray diagram to show how a converging lens can form a real and enlarged image of an object.
- Where will the image be formed if an object is kept 16. in front of a concave lens at a distance equal to its focal length? Draw a ray diagram to illustrate your answer.
- 17. Draw a ray diagram to show how a converging lens is used as a magnifying glass to observe a small object. Mark on your diagram the foci of the lens and the position of the eye.
- 18. Draw a ray diagram to show how a converging lens can form an image of the sun. Hence give a reason for the term 'burning glass' for a converging lens used in this manner.

- 19. A lens forms an inverted image of an object.
	- (a) Name the kind of lens.
	- (b) State the nature of image whether real or virtual ? Ans. (a) Convex (b) real
- 20. A lens forms an upright and magnified image of an object.
	- (a) Name the lens.
	- (b) Draw a labelled ray diagram to show the image formation.
- 21. (a) Name the lens which always forms an erect and virtual image.
	- (b) State whether the image in part (a) is magnified or diminished.

Ans. (a) Concave, (b) diminished

22. Can a concave lens form an image of size two times that of the object? Give reason.

Ans. No. Reason : A concave lens diverges the rays incident on it, and the image is always diminished.

- 23. Give two characteristics of the image formed by a concave lens. Ans. Virtual, diminished
- 24. Give two characteristics of the virtual image formed by a convex lens. Ans. Erect, magnified
- In each of the following cases, where must an object  $25.$ be placed in front of a convex lens so that the image formed is
	- (a) at infinity,
	- (b) of same size as the object,
	- (c) inverted and enlarged,
	- (d) upright and enlarged?

Ans. (a) at focus, (b) at 2F, (c) between F and 2F (d) between optical centre and focus.

#### 26. Complete the following table :



- 27. State the changes in the position, size and nature of the image when the object is brought from infinity up to the convex lens. Illustrate your answer by drawing the ray diagrams.
- 28. State the changes in the position, size and nature of the image when the object is brought from infinity

up to the concave lens. Illustrate your answer by drawing the ray diagrams.

- 29. Complete the following sentences :
	- (a) An object is placed at a distance of more than 40 cm from a convex lens of focal length 20 cm. The image formed is real, inverted and......
	- (b) An object is placed at a distance  $2f$  from a convex lens of focal length  $f$ . The size of image formed is ................ that of the object.
	- (c) An object is placed at a distance 5 cm from a convex lens of focal length 10 cm. The image formed is virtual, upright and ...........

Ans. (a) diminished (b) equal to (or same as) (c) magnified

- State whether the following statements are 'true' or 30. 'false' by writing T/F against them.
	- (a) A convex lens has a divergent action and a concave lens has a convergent action.
	- (b) A concave lens, if kept at a proper distance from an object, can form its real image.
	- (c) A ray of light incident parallel to the principal axis of a lens, passes undeviated after refraction.
	- (d) A ray of light incident at the optical centre of lens, passes undeviated after refraction.
	- (e) A concave lens forms a magnified or diminished image depending on the distance of object from **Ans.** (a)  $F$  (b)  $F$  (c)  $F$  (d)  $T$  (e)  $F$ it.

#### **MULTIPLE CHOICE TYPE**

- 1. For an object placed at a distance 20 cm in front of a convex lens, the image is at a distance 20 cm behind the lens. The focal length of convex lens is: (a) 20 cm (b) 10 cm (c) 15 cm (d) 40 cm. Ans. (b) 10 cm
- 2. For the object placed between the optical centre and focus of a convex lens, the image is :
	- (a) real and enlarged
	- (b) real and diminished
	- (c) virtual and enlarged
	- (d) virtual and diminished.

Ans. (c) virtual and enlarged

- 3. A concave lens forms the image of an object which is :
	- (a) virtual, inverted, and diminished
	- (b) virtual, upright, and diminished
	- (c) virtual, inverted, and enlarged
	- (d) virtual, upright, and enlarged.

Ans. (b) virtual, upright, and diminished.

# (C) SIGN CONVENTION AND LENS FORMULA

# **5.10 SIGN CONVENTION OF MEASUREMENT OF DISTANCES**

We follow the cartesian sign convention to measure the distances in a lens according to which:

- (1) The optical centre of the lens is chosen as the origin of coordinate system.
- (2) The object is considered placed on the left of the lens (it makes the sign of distances measured identical to the sign of distances in cartesian coordinate system).
- (3) All the distances are measured along the principal axis from the optical centre of the lens. The distance of an object from the lens is denoted by  $u$ , the distance of image by  $v$  and the distance of second focus (*i.e.*, focal length of lens) by  $f$ .
- The distances measured in the direction of  $(4)$ incident ray are taken positive, while the distances opposite to the direction of incident ray are taken negative.



Fig. 5.52 Sign convention

 $(5)$ The length above the principal axis is taken positive, while the length below the principal axis is taken negative.

 $(6)$  Fig. 5.52 shows the distances for the convex and concave lens according to the sign convention. By sign convention, the focal length  $(=OF_2)$  of the convex lens is positive and that of concave lens is *negative*. The distance of object  $(u)$  in front of lens is always negative. The distance of image  $(v)$  is positive if it is real and formed behind the lens, while it is negative if the image is *virtual* and formed in front of the lens.

# 5.11 LENS FORMULA  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

The equation relating the distance of object  $(u)$ , distance of image  $(v)$  and focal length  $(f)$  of a lens is called the lens formula. It is same for both the convex and concave lens and is given as

$$
\frac{1}{v} - \frac{1}{u} = \frac{1}{f}
$$
 ....(5.1)

**Derivation**<sup>\*</sup>: Fig  $5.53$  (a) and (b) shows the formation of image  $A_1B_1$  of a linear object AB by a convex and concave lens respectively.



Derivation is not included in the syllabus.

In Fig. 5.53, the triangles AOB and  $A_1OB_1$ are identical

$$
\frac{\text{BA}}{\text{B}_1\text{A}_1} = \frac{\text{OB}}{\text{OB}_1} \qquad \qquad \dots (i)
$$

Similarly  $\triangle$  DOF, and  $\triangle$  A<sub>1</sub>B<sub>1</sub>F<sub>2</sub> are identical

 $\ddot{\cdot}$ 

 $\ddot{\cdot}$ 

$$
\frac{OD}{B_1A_1} = \frac{OF_2}{F_2B_1}
$$

 $\overline{a}$ 

But  $BA = OD$ 

 $\ddot{\cdot}$ 

$$
\frac{BA}{B_1A_1} = \frac{OF_2}{F_2B_1} \qquad \qquad \dots (ii)
$$

 $\sim$ 

From eqns. (i) and (ii),

$$
\frac{\text{OB}}{\text{OB}_1} = \frac{\text{OF}_2}{F_2B_1} \qquad \qquad \dots (iii)
$$

(a) For convex lens : In Fig.  $5.53(a)$ , by sign convention, distance of object from the optical centre OB =  $u$  (negative), focal length of lens  $OF_2 = f$  (positive), distance of image from the optical centre  $OB_1 = v$  (positive), and

$$
F_2B_1 = OB_1 - OF_2 = v - f
$$
 (positive)

 $\frac{-u}{v} = \frac{f}{v-f}$ 

From eqn. (iii),  $\dddot{\cdot}$ 

or

 $- uv + uf = vf$ 

or

 $uf - vf = uv$ 

Dividing both sides by  $uvf$ ,

$$
\frac{1}{v} - \frac{1}{u} = \frac{1}{f}
$$
 ....(iv)

(b) For concave lens : In Fig.  $5.53(b)$ , by sign convention,  $OB = u$  (negative),  $OF_2 = f$  (negative),  $OB_1 = v$  (negative),

and

 $F_2B_1 = OF_2 - OB_1$  $= f - v$  (negative)

Substituting these values in eqn. (iii),

$$
\frac{-u}{-v} = \frac{-f}{-(f-v)} \quad \text{or} \quad \frac{u}{v} = \frac{f}{f-v}
$$

or 
$$
uf - uv = vf
$$
  
or  $uf - vf = uv$ 

Dividing both sides by  $uvf$ ,

$$
\frac{1}{v} - \frac{1}{u} = \frac{1}{f}
$$
 ....(v)

From eqns. (iv) and (v), it is clear that the expressions are same for both the convex and concave lens.

*Note*: In numericals, the known values are substituted with their proper sign and then the unknown quantity is obtained with its proper sign. According to sign convention for a convex lens  $u$  is always negative,  $f$  is always positive,  $\nu$  is positive for the real image and  $\nu$  is negative for the virtual image. But for a concave lens  $u$ ,  $v$  and  $f$  all are negative and the numerical value of  $u$  is always greater than  $\nu$ .

### **5.12 LINEAR MAGNIFICATION**

It is not sufficient to know the postion of image of an object by a lens, but it is also required to know its size. When the position of object changes, the position as well as the size of image change. The ratio of length of image I perpendicular to the principal axis, to the length of object O, is called the linear *magnification*. It is generally denoted by the letter  $m$  and is related to the distance of image  $(v)$  and distance of object  $(u)$  for both the convex and concave lens as :

Linear magnification  
\n
$$
m = \frac{\text{length of image (I)}}{\text{length of object (O)}} = \frac{v}{u}
$$
\n...(5.2)

\*Derivation : Fig. 5.54 (a) and (b) shows the formation of image  $A_1B_1$  of an object AB by the convex and concave lens respectively.

\* Drivation is not included in the syllabus



Fig. 5.54 Magnification

In Fig. 5.54,  $\triangle ABO$  and  $\triangle A_1B_1O$  are similar triangles

$$
\frac{B_1 A_1}{BA} = \frac{OB_1}{OB}
$$
 ....(i)

- (a) For convex lens : In Fig.  $5.54(a)$ , by sign convention,  $B_1A_1 = I$  (negative), BA = O (positive),  $\overrightarrow{OB_1} = v$  (positive) and  $OB = u$  (negative).
- From eqn. (i), ∴.

$$
\frac{-I}{O} = \frac{\nu}{-u} \quad \text{or} \quad \frac{I}{O} = \frac{\nu}{u} \quad \text{....(ii)}
$$

In Fig.  $5.54$  (b), by sign convention,  $B_1A_1 = I$  (positive), BA = O (positive),  $OB_1 = v$  (negative) and OB =  $u$  (negative),

From eqn. (i), ∴.

$$
\frac{I}{O} = \frac{-\nu}{-u} \quad \text{or} \quad \frac{I}{O} = \frac{\nu}{u} \qquad \qquad \text{....(iii)}
$$

From eqns. (ii) and (iii), it is clear that the expressions for magnification are same for both the convex and concave lens.

*Note* : (1) For the real image (which is inverted), the magnification  $m$  is negative, while for the virtual image (which is erect), the magnification  $m$  is positive. Thus a convex lens can have the value of  $m$  positive as well as negative, but a concave lens always has the value of  $m$  positive.

(2) The numerical value of  $m$  is greater than 1 if image is magnified, is 1 for image of size same as of the object and is less than 1 for the diminished image. Thus the numerical value of  $m$  is always less than 1 for a concave lens, while it can be greater than, equal to or less than 1 for a convex lens depending on the position of the object.

### **5.13 POWER OF A LENS**

When a beam of light passes through a lens, it gets deviated from its path. The deviation produced by the lens is expressed in terms of its power. A lens which produces more deviation has more power. Thus,

The deviation of the incident light rays produced by a lens on refraction through it, is a measure of its power.

A thick lens i.e., a lens (having surfaces of more curvature) is of short focal length and it deviates the rays more, while a thin lens (i.e., a lens having surfaces of less curvature) is of large focal length and it deviates the rays less. Hence power of a lens is expressed (or measured) in terms of the reciprocal of focal length. Its unit is dioptre (symbol D). Thus

Power of lens (in D) = 
$$
\frac{1}{\text{focal length (in metre)}} \quad \text{...(5.3)}
$$

While giving prescription to a patient, an optician does not quote the focal length of lens, but he quotes the power of lens. A lens is of power 1 dioptre (or 1 D), if its focal length is 1 m (or 100 cm).

Depending on the direction in which a lens deviates the light ray, its power is either positive or negative. If a lens deviates a ray towards its centre, the power is positive and if it deviates the ray away from its centre, the power is negative. Therefore the power of a convex lens is positive and of a concave lens is negative. Thus power of a convex lens of focal length 20 cm is  $+ 5.0$  D

while that of a concave lens of same focal length  $is - 5.0 D$ .

If two thin lenses are placed in contact, the combination has a power equal to the algebraic sum of the powers of the individual lens. If a convex lens of power  $+2.0$  D is kept in contact with a concave lens of power  $-2.0$  D, the combination will have zero power and it will behave like a glass plate.

### **EXAMPLES**

1. An object of height 4.0 cm is placed at a distance 24 cm in front of a convex lens of focal length 8 cm. (a) Find the position and size of the image. (b) State the characteristics of the image.

Given :  $Q = 4.0$  cm,  $u = 24$  cm (negative),  $f = 8$  cm (positive)

- (a) From relation  $\frac{1}{v} \frac{1}{u} = \frac{1}{f}$ ,  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$ 
	- or  $\frac{1}{v} = \frac{1}{-24} + \frac{1}{8} = \frac{1}{12}$

or  $v = 12$  cm

The image is at distance 12 cm behind the lens.

From relation  $\frac{I}{O} = \frac{v}{u}$ ,  $\frac{I}{4.0} = \frac{12}{24}$ 

or 
$$
I = -\frac{12}{24} \times 4 = -2
$$
 cm

Thus the image is inverted of size 2 cm.

- (b) Characteristics of the image: The image is real, inverted and diminsihed (size 2.0 cm).
- $2.$ The focal length of a camera lens is 20 cm. Find how far away from the film must the lens be set in order to photograph an object located at a distance 100 cm from the lens.

Given :  $f = 20$  cm (positive),  $u = 100$  cm (negative)

From relation  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ,  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$ 

or 
$$
\frac{1}{v} = \frac{1}{-100} + \frac{1}{20} = \frac{1}{25}
$$
  
or  $v = 25$  cm

Thus the lens must be set at a distance 25 cm from the film towards the object.

- 3. A convex lens forms an image 16-0 cm long of an object 4.0 cm long kept at a distance 6 cm from the lens. The object and the image are on the same side of lens.
	- (a) What is the nature of image?
	- (b) Find : (i) the position of image, and (ii) the focal length of lens.
	- (a) Since the image is magnified and on same side of the lens as the object, so the image is virtual.
	- (b) Given :  $I = 16.0$  cm (positive),  $O = 4.0$  cm (positive),  $u = 6$  cm (negative)
	- (i) From relation,  $m = \frac{1}{\Omega} = \frac{v}{u}$ ,  $\frac{160}{40} = \frac{v}{-6}$

or  $v = -24$  cm

Thus the image is at distance 24 cm in front of lens.

(ii) From relation 
$$
\frac{1}{v} - \frac{1}{u} = \frac{1}{f}
$$

$$
\frac{1}{f} = \frac{1}{-24} - \frac{1}{(-6)} = \frac{-1}{24} + \frac{1}{6} = \frac{1}{8}
$$

or  $f = 8$  cm

Thus the focal length of lens =  $8 \text{ cm}$ .

**EXERCISE-5(C)** 

- 1. State the sign convention to measure the distances for a lens.
- The focal length of a lens is (i) positive, (ii) negative.  $2.$ In each case, state the kind of lens.

Ans. (i) convex (ii) concave

- 3. Write the lens formula explaining the meaning of the symbols used.
- 4. What do you understand by the term magnification? Write expression for it for a lens, explaining the meaning of the symbols used.

#### Downloaded from https:// www.studiestoday.com **NUMERICALS**

5. What information about the nature of image (i) real or virtual, (ii) erect or inverted, do you get from the sign of magnification + or  $-$ ?

> Ans. + sign : virtual and erect  $and - sign : real and inverted$

- 6. Define the term power of a lens. In what unit is it expressed?
- 7. How is the power of a lens related to its focal length?
- 8. How does the power of a lens change if its focal length is doubled? Ans. Power gets halved.
- 9. How is the sign  $(+ or -)$  of power of a lens related to its divergent or convergent action?
- 10. The power of a lens is negative. State whether it is convex or concave? Ans. concave

#### **MULTIPLE CHOICE TYPE**

- 1. If the magnification produced by a lens is  $-0.5$ , the correct statement is:
	- (a) the lens is concave
	- (b) the image is virtual
	- (c) the image is magnified
	- (d) the image is real and diminshed formed by a convex lens

Ans. (d) the image is real and diminshed formed by a convex lens

The correct lens formula is  $2.$ 

(a) 
$$
\frac{1}{u} + \frac{1}{v} = \frac{1}{f}
$$
 (b)  $\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$   
(c)  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  (d)  $f = \frac{u+v}{uv}$ 

**Ans.** (c) 
$$
\frac{1}{\nu} - \frac{1}{u} = \frac{1}{f}
$$

- 3. On reducing the focal length of a lens, its power :
	- (a) decreases
	- (b) increases
	- (c) does not change
	- (d) first increases then decreases.

Ans. (b) increases

- 4. The lens of power  $+1.0$  D is:
	- (a) convex of focal length 1.0 cm
	- (b) convex of focal length 1.0 m
	- (c) concave of focal length 1.0 cm
	- (d) concave of focal length  $1.0$  m.

Ans. (b) convex of focal length 1.0 m

- 1. (a) At what position a candle of length 3 cm be placed in front of a convex lens so that its image of length 6 cm be obtained on a screen placed at distance 30 cm behind the lens ?
	- (b) What is the focal length of lens in part (a)?

Ans. (a) 15 cm, (b) 10 cm

2. A concave lens forms the image of an object kept at a distance 20 cm in front of it, at a distance 10 cm on the side of the object. (a) What is the nature of the image ? (b) Find the focal length of the lens.

Ans. (a) virtual, erect, diminshed (b) 20 cm (negative)

- 3. The focal length of a convex lens is 25 cm. At what distance from the optical centre of the lens an object be placed to obtain a virtual image of twice the size ? Ans. 12.5 cm in front of the lens
- 4. Where should an object be placed in front of a convex lens of focal length 0.12 m to obtain a real image of size three times the size of the object, on the screen? Ans.  $0.16$  m in front of lens
- 5. An illuminated object lies at a distance 1.0 m from a screen. A convex lens is used to form the image of object on a screen placed at distance 75 cm from the lens. Find: (i) the focal length of lens. and (ii) the magnification. Ans. (i)  $18.75$  cm (ii)  $-3$
- 6. A lens forms the image of an object placed at a distance 15 cm from it, at a distance 60 cm in front of it. Find : (i) the focal length, (ii) the magnification, and (iii) the nature of image.

Ans. (i) 20 cm (ii)  $+4$ 

(iii) erect, virtual and magnified

7. A lens forms the image of an object placed at a distance of 45 cm from it on a screen placed at a distance 90 cm on other side of it. (a) Name the kind of lens. (b) Find  $:$  (i) the focal length of lens. and (ii) the magnification of image.

Ans. (a) convex (b) (i) 30 cm (ii)  $-2$ 

- 8. An object is placed at a distance of 20 cm in front of a concave lens of focal length 20 cm. Find : (a) the position of image, and (b) the magnification of image. Ans. (a) 10 cm in front of lens,  $(b) + 0.5$
- 9. The power of a lens is  $+2.0$  D. Find its focal length and state the kind of the lens.

Ans. 50 cm, convex

- 10. Express the power (with sign) of a concave lens of focal length 20 cm.  $Ans. - 5D$
- 11. The focal length of a convex lens is 25 cm. Express its power with sign. Ans.  $+4.0$  D

Downloaded from https:// www.studiestoday.com (D) MAGNIFYING GLASS AND APPLICATION OF LENSES

### **5.14 MAGNIFYING GLASS OR SIMPLE MICROSCOPE**

**Principle**: To observe a tiny object distinctly, we prefer to place it as near to our eye as possible, but to see an object by the naked eye, it is necessary to place it at least at a distance of 25 cm from the eye which is the least distance of distinct vision (D) for a normal eye. Thus an object is seen distinctly and of maximum size when it is placed at a distance D from the eye.

The size of image on retina is actually deternined by the angle subtended by the object at the eye. Smaller the angle subtended by the object at the eye, smaller is the size of image on the retina. The eye is not able to see an object distinctly if it subtends an angle less than 1'  $(=\frac{1}{60}$ °) at it. To observe a small object which subtends an angle smaller than 1' at the eye when placed at the least distance of distinct vision from the eye, we take the help of a *convex lens of* short focal length. We hold the convex lens in front of our eye at such a distance that the object lies within its focal length, so that the lens forms an erect, virtual, and magnified image on the same side and behind the object at a distance D. This magnified image now subtends on the eye an angle greater than 1', so it is distinctly seen by the eye. This is the principle of a simple microscope (or magnifying glass or reading glass).

*Construction* : A simple microscope is a convex lens of short focal length mounted in a lens holder as shown in Fig. 5.55.



Fig. 5.55 Magnifying glass (or simple microscope)

Way of using the microscope: The lens is placed near the eye and its distance from the object is adjusted such that the image is formed behind the object at a distance D (called the least distance of distinct vision  $= 25$  cm for the normal eye) from the lens.

#### Ray diagram and magnifying power

Ray diagram : In Fig 5.56, AB is an object (e.g. a word of a print or a part of a watch etc.) in front of a convex lens. The position of lens is so adjusted that the object AB lies in between its optical centre O and focus  $F_1$  and the image is formed at the least distance of distinct vision (D). A ray of light AD from the point A of the object, incident parallel to the principal axis, after refraction from the lens, passes through its focus  $F_2$  as DF<sub>2</sub>. The other incident ray AO passing through the optical centre O gets refracted as OO' without deviation. The two refracted rays  $DF<sub>2</sub>$  and OO' do not actually meet, but they meet at a point A' when they are produced backward. Thus A' is the *virtual* image of the point A of the object. Similarly, for the point B of the object, B' is the virtual image. Hence A'B' is the *virtual*, *magnified* and *erect image* of the object AB. To observe the image, the eye is kept very close to the lens on the other side of the object at the position shown in Fig. 5.56.



Fig. 5.56 Ray diagram for location of image in a magnifying glass

Magnifying power: The magnifying power of the microscope is given as :

Magnifying power  $m = 1 + \frac{D}{f}$ 

$$
...(5.4)
$$

where  $f$  is the focal length of the lens and  $D$  is the least distance of distinct vision  $(= 25 \text{ cm})$  for the normal eye.

The magnifying power of the microscope can be increased by using the lens of short focal length (i.e., shorter the focal length, more is the magnifying power). But it can not be increased indefinitely.

**Example**: The magnifying power of a convex lens of focal length 100 cm is 1.25, while that of a convex lens of focal length 5 cm is 6.

 $\mathbf{U}$ ses : A simple microscope (or magnifying glass) is used to see and read the small letters and figures. It is used by watch makers to see the small parts and screws of the watch. In optical instruments such as a travelling microscope, spectrometer, etc. a magnifying glass is provided above the vernier scale so as to read the scale accurately. Hence it is also called the reading lens.

# **5.15 APPLICATION OF LENSES**

Lenses are used in a large number of optical instruments. Some common applications are given below:

- (1) The objective lens of a telescope, camera, slide projector, etc., is a *convex lens* which forms the real and inverted image of the object.
- (2) Our eye lens is also a *convex lens*. The eye lens forms the inverted image of the object on retina.
- (3) When our eyesight weakens, we use spectacles having either the convex or the concave lens. A person suffering from long sightedness or hypermetropia (i.e., unable to see the near objects distinctly) wears spectacles having the convex lens, while a person suffering from short sightedness or myopia (i.e., unable to see

the far objects distinctly) wears spectacles having the *concave* lens. Some people are both short and long sighted. They use either the separate spectacles for seeing the near and far objects or use a spectacles with a bifocal lens. In the spectacles using the bifocal lens, the lower part is the convex lens which is used to see the near objects, while the upper part is the concave lens which is used to see the distant objects.

- (4) A magnifying glass is a convex lens of short focal length fitted in a steel (or plastic) frame provided with a handle.
- (5) In the collimator of a spectroscope, convex lens is used for obtaining a a parallel beam of light.
- (6) A *concave* lens is used as the objective lens in a Galilean telescope to obtain the final erect image of the object.
- (7) A single convex lens usually forms a coloured or blurred image due to chromatic aberration, therefore a combination of concave lens with a convex lens is used to overcome such defect.

# **5.16 EXPERIMENTAL DETERMINATION OF FOCAL LENGTH OF A CONVEX LENS**

# (1) Estimation of focal length by the distant object method

**Principle :** A beam of parallel rays from a distant object incident on a convex lens gets converged in the focal plane of the lens.

Method : In an open space, against a white (or light coloured) wall, place a metre rule horizontally with its 0 cm end touching the wall and its other end towards an illuminated object which is at a very large distance (such as a tree or an electric pole). Holding the given lens vertically on the metre rule, focus the object on the wall by moving the lens to and fro along the length of the metre rule (Fig. 5.57). Since the light rays incident from a distant object are nearly parallel, the image of it formed on the wall is almost at the focus of the lens, so read the distance of lens from the wall directly on the

Derivation of the formula is not included in the syllabus.



Fig. 5.57 Determination of focal length of a convex lens by the distant object method

metre rule. This gives the approximate focal length of the lens. Fig. 5.57 shows the formation of image on the wall of a distant tree situated on the left of the lens (which is not shown in diagram). The approximate focal length of the lens in Fig. 5.57 is 48 cm. The ray diagram for it is shown in Fig. 5.58.



Fig. 5.58 Ray diagram showing the focal length of a convex lens

Determination of exact focal length by an  $(2)$ auxillary plane mirror method : If we are given a vertical stand, a plane mirror, a lens and a pin, we can proceed as follows :

(i) Place the lens L on a plane mirror MM' kept on the horizontal surface of the vertical stand and arrange the pin P horizontally in the clamp so that its tip is vertically above the centre O of the lens L as shown in Fig. 5.59.



Fig. 5.59 Determination of focal length of a convex lens by an auxillary plane mirror

(ii) Adjust the height of the pin till it has no parallax with its inverted image as seen from vertically above the pin. To check for parallax, keep the eye vertically above the tip of the pin P at a distance nearly 25 cm from it and move it sideways. If the pin and its image shift together, then there is no parallax.

(iii) Measure the distance  $x$  of the pin P from the lens and the distance y of the pin from the mirror, using a metre rule and a plumb line. Calculate the average of the two distances. This gives the focal length of the lens, *i.e.*,

$$
f = \frac{x+y}{2} \qquad \qquad \dots (5.5)
$$

(iv) Take three different observations after removing the parallax each time and find the mean value of focal length  $f$  of the lens.

### **Alternative method**

The above experiment can also be performed by using a pin, a convex lens, and a plane mirror fixed on the separate vertical stands of an optical bench as shown in Fig. 5.60. We proceed as follows:

(i) Keep the lens L close to the plane mirror M. Adjust the height of the object pin O such that its tip lies at the centre of the convex lens L.



#### Fig. 5.69 Determination of focal length of a convex lens by one pin method using an optical bench

(ii) Move the object pin O away from the lens L on the optical bench till its inverted image is seen when viewed by keeping the eye behind the object pin O at a distance of about 25 cm from it.

(iii) Now adjust the position of the object pin O by gradual to and fro movement of it so that

the tip of the inverted image I coincides with the tip of the object pin O. In this situation, on moving the eye sideways, both the tip of the object pin and the tip of its image will appear to move together.

(iv) Note the position of the object pin (say,  $x_1$ ) and the position of lens (say,  $x_2$ ) on the optical bench. The difference  $(x_2 - x_1)$  gives the focal length of the convex lens, i.e.,

$$
f = x_2 - x_1 \tag{5.6}
$$

In Fig. 5.60,  $x_1 = 30.0$  cm,  $x_2 = 60.0$  cm, so  $f = 60.0 - 30.0 = 30.0$  cm.

Explanation : When the object pin O lies in the focal plane of lens L, the rays of light starting from O after refraction from lens, become parallel to its principal axis and therefore they strike normally on the plane mirror M kept behind the lens. Due to normal incidence  $(i = 0^{\circ})$  on the plane mirror, angle of reflection  $r = 0^{\circ}$ , *i.e.*, the rays are reflected back by the plane mirror on the same path and then they re-enter the lens as a



Fig. 5.61 Ray diagram for determination of focal length of a convex lens by using a pin and a plane mirror

parallel beam. The lens converges these rays in its focal plane, thus forming the image I just above the object pin O. The image I is inverted, real, and of size same as that of the object O. Fig. 5.61 shows the ray diagram. Thus, the distance  $OL$  (= IL) is equal to the focal length of lens.

*Note*: The position of plane mirror relative to the lens  $L$  (*i.e.*, the distance  $LM$ ) does not affect the position of image as long as the rays from the lens fall normally on the plane mirror M. However, the plane mirror M is kept close to the convex lens L so that the image I can be distinctly seen.

## 5.17 TO DIFFERENTIATE BETWEEN A **CONVEX AND A CONCAVE LENS**

- (1) By touching : If the lens is thick in the *middle* and thin at the edges, the lens is convex and if the lens is thin in the *middle* and thick at the edges, the lens is concave.
- (2) By seeing the image : (a) On keeping the lens near a printed page, if letters appear magnified, the lens is convex and if the letters appear *diminished*, the lens is concave.

(b) On seeing a distant object through the lens, if its *inverted* image is seen, the lens is convex and if the *upright* image is seen, the lens is *concave*.

# **EXERCISE-5(D)**

- 1. What is a magnifying glass ? State its two uses.
- 2. Draw a neat labelled ray diagram to show the formation of image by a magnifying glass. State three characteristics of the image.
- 3. Where is the object placed in reference to the principal focus of a magnifying glass, so as to see its enlarged image ? Where is the image obtained ?
- 4. Write expression for the magnifying power of a simple microscope. How can it be increased ?
- 5. State two applications each of a convex and concave lens.
- 6. Describe in brief how would you determine the approximate focal length of a convex lens.

7. The diagram in Fig. 5.62 shows the experimental set up for the determination of focal length of a lens using a plane mirror.



### Fig. 5.62

- (i) Draw two rays from the point O of the object to show the formation of image I at O itself.
- (ii) What is the size of the image I?
- (iii) State two more characteristics of the image I.
- (iv) Name the distance of the object O from the optical centre of the lens.
- (v) To what point will the rays return if the mirror is moved away from the lens by a distance equal to the focal length of the lens?
- 8. Describe how you would determine the focal length of a converging lens, using a plane mirror and one pin. Draw a ray diagram to illustrate your answer.
- 9. How will you differentiate between a convex and a concave lens by looking at (i) a distant object, (ii) a printed page?

#### **MULTIPLE CHOICE TYPE**

- 1. A magnifying glass forms :
	- (a) a real and diminished image
	- (b) a real and magnified image
	- (c) a virtual and magnified image
	- (d) a virtual and diminished image.

Ans. (c) a virtual and magnified image.

- 2. The maximum magnifying power of a convex lens of focal length 5 cm can be:
	- $(a)$  25  $(b) 10$
	- $(d)$  6  $(c)$  1 Ans. (d)  $6$