Chapter 3 Machines

SYLLABUS

Machines as force multipliers; load, effort, mechanical advantage, velocity ratio and efficiency; simple treatment of levers; pulley systems showing the utility of each type of machine.

Scope of syllabus : Functions and uses of simple machines. Terms : effort E, load L, mechanical advantage MA = L / E, velocity ratio VR = $V_E / V_L = d_E / d_L$, input (W_i), output (W_o), efficiency (η), relation between η and MA, VR (derivation included); for all practical machines $\eta < 1$; MA < VR.

Lever : Principle, First, second and third class of levers; examples; MA and VR in each case. Examples of each of these classes of levers as also found in the human body.

Pulley system : Single fixed, single movable, block and tackle; MA, VR and η in each case.

(A) MACHINES, TECHNICAL TERMS AND LEVERS

3.1 MACHINES

It is our common experience that it is easier to open a nut by the use of a wrench than to open it by hand. It is difficult to pull up a bucket of water directly from a well, but it becomes much easier to pull it up with the use of a pulley. It is difficult to shift a heavy block by pushing it, but it becomes easier to shift it by using a crow bar. We can find many such examples in our daily life where the use of a machine (such as the wrench, pulley, crow bar, etc.) makes the job easier.

Functions and uses of simple machines : The various functions of machines are useful to us in the following ***four*** ways :

(1) In lifting a heavy load by applying the less effort, *i.e.*, as a force multiplier.

Examples : A jack is used to lift a car, a bar is used to lift a heavy stone, a spade is used to turn the soil, pulleys are used to lift a heavy load. In all these examples, the effort is much less than the load, so the machine acts as a force multiplier.

(2) In changing the point of application of effort to a convenient point.

Example : The rear wheel of a cycle is rotated with the help of a chain joined to a toothed wheel by applying the effort on the pedal attached with it. Thus the point of application of effort is changed from rear wheel to the pedal.

(3) In changing the direction of effort to a convenient direction.

Example : If a bucket, full of water, is lifted up from a well without the use of a pulley, effort has to be applied upwards. By using a single fixed pulley, it becomes possible to lift the bucket from the well by applying the effort in the downward direction instead of applying it upwards so that the person lifting it up may also use his own weight as effort.

(4) For obtaining a gain in speed (*i.e.*, a greater movement of load by a smaller movement of effort).

Examples : (i) When a pair of scissors is used to cut the cloth, its blades move longer on cloth, while its handles move a little. (ii) The blade of a knife moves longer by a small displacement of its handle. Here the effort is more than the load.

Hence we define a machine as below :

A machine is a device by which we can either overcome a large resistive force (or load) at some point by applying a small force (or effort) at a convenient point and in a desired direction or by which we can obtain a gain in speed.

Note : A machine *can not be* used as a force multiplier as well as a speed multiplier simultaneously.

3.2 TECHNICAL TERMS RELATED TO A MACHINE

(1) Load

The resistive or opposing force to be overcome by a machine is called the load (L).

(2) Effort

The force applied on the machine to overcome the load is called the effort (E).

(3) Mechanical advantage (M.A.)

The ratio of the load to the effort is called the mechanical advantage of the machine, i.e.,

$$\text{Mechanical advantage (M.A.)} = \frac{\text{Load }(L)}{\text{Effort }(E)} \quad ..(3.1)$$

While using a machine to overcome a certain load, if the effort needed is less than the load, the machine has the mechanical advantage greater than 1, while if it needs an effort greater than the load, it has the mechanical advantage less than 1, A machine has the mechanical advantage equal to 1, if the effort needed is equal to the load. A machine having the mechanical advantage greater than 1, acts as a *force multiplier*, while the machine having the mechanical advantage less than 1, gives the *gain in speed*. The machine having the mechanical advantage equal to 1, is generally used to *change the direction of effort as there is no gain in force or speed*.

Unit : Since mechanical advantage is the ratio of two similar quantities, so **it has no unit.**

(4) Velocity ratio (V.R.)

The ratio of the velocity of effort to the velocity of load is called the velocity ratio of machine, i.e.,

$$\text{Velocity ratio (V.R.)} = \frac{\text{Velocity of effort } (V_E)}{\text{Velocity of load } (V_L)}$$

If d_L and d_E are the distances moved in same time t by the load and the effort respectively, then

$$\text{Velocity of load } (V_L) = \frac{d_L}{t};$$

$$\text{Velocity of effort } (V_E) = \frac{d_E}{t}$$

$$\therefore \text{Velocity ratio (V.R.)} = \frac{V_E}{V_L} = \frac{d_E/t}{d_L/t}$$

or

$$\text{V.R.} = \frac{d_E}{d_L} \quad ....(3.2)$$

Thus, *the velocity ratio is also defined as the ratio of the displacement of effort to the displacement of load.*

A machine in which the displacement of load is more than the displacement of effort, will have the velocity ratio less than 1 and such a machine gives the *gain in speed* because load is moving at a faster rate. On the other hand, if the velocity ratio of machine is more than 1, *i.e.*, the displacement of load is less than the displacement of effort, the machine acts as a *force multiplier*. The velocity ratio of a machine is 1 if the displacement of load is equal to the displacement of effort. Such a machine generally *changes the direction of effort.*

Unit : Since the velocity ratio is also the ratio of two similar quantities, so **it has no unit** just like M.A.

(5) Work input

The work done on the machine by the effort, is called the work input (W_{input}), i.e.,

Work input = work done by the effort. ..(3.4)

(6) Work output

The work done by the machine on the load, is called the work output (W_{output}), *i.e.,*

Work output = work done on the load. ..(3.5)

(7) Efficiency (η)

Efficiency of a machine is the ratio of the work done on load by the machine to the work done on the machine by the effort.

In other words, *efficiency is the ratio of the work output to the work input.* It is denoted by the symbol η (eta). Thus

$$\text{Efficiency } \eta = \frac{\text{Work output } (W_{output})}{\text{Work input } (W_{input})}$$

But efficiency is usually expressed as a percentage, so we may write

$$\text{Efficiency } \eta = \frac{\text{Work output } (W_{output})}{\text{Work input } (W_{input})} \times 100\% \quad ..(3.6)$$

Unit : It has no unit since it is also the ratio of two similar quantities (*i.e.*, work).

> *Note :* The presence of friction and weight of the moving parts of a machine of a given design, have no effect on its velocity ratio, but decreases both, its mechanical advantage and efficiency.

3.3 PRINCIPLE OF A MACHINE

When energy is supplied to a machine by applying the effort, it overcomes the load by doing some useful work on it.

The point at which the energy is supplied to a machine by applying the effort is called the ***effort point*** and the point where the energy is obtained by overcoming the load, is called the ***load point***.

Input energy = work done at the effort point
= effort × displacement of the point of application of effort.

Output energy = work obtained at the load point
= load × displacement of the point of application of load.

For an ideal machine,

Output energy = Input energy ...(3.7)

The useful work done by a machine (*i.e.*, output energy) can never be greater than the work done on the machine (*i.e.*, input energy), otherwise it will violate the principle of conservation of energy, therefore no machine can ever have efficiency greater than 1 (*i.e.*, more than 100%).

Ideal machine : An ideal machine is that in which there is no loss of energy in any manner. Here the work output is equal to the work input. *i.e.*, the efficiency of an ideal machine is 100%.

Actual machine : In an actual machine, the output energy is always less than the input energy indicating that there is some loss of energy during its operation. The loss in energy is mainly due to the following *three* reasons :

(i) the moving parts in it are neither weightless nor smooth (or frictionless),

(ii) the string in it (if any) is not perfectly elastic, and

(iii) its different parts are not perfectly rigid.

> *Note : The energy lost in overcoming the force of friction between the moving parts of a machine, is the most common type of loss of energy in it.*

If a machine is 80% efficient, it implies that 80% of the total energy supplied to the machine at the effort point is obtained as useful energy at the load point. The remaining 20% of the energy supplied is lost in overcoming the force of friction etc. and it appears as heat energy in its different parts as a result they get heated up.

3.4 RELATIONSHIP BETWEEN EFFICIENCY (η), MECHANICAL ADVANTAGE (MA) AND VELOCITY RATIO (VR)

Suppose a machine overcomes a load L by the application of an effort E, in time t. Let the displacement of effort be d_E and the displacement of load be d_L.

Work input = effort × displacement of effort

$$= E \times d_E \quad \text{....(i)}$$

Work output = load × displacement of load

$$= L \times d_L \quad \text{....(ii)}$$

By definition,

$$\text{Efficiency } \eta = \frac{\text{work output}}{\text{work input}}$$

From eqns. (i) and (ii),

$$\eta = \frac{L \times d_L}{E \times d_E} = \frac{L}{E} \times \frac{d_L}{d_E}$$

$$= \frac{L}{E} \times \frac{1}{d_E / d_L}$$

But $\frac{L}{E}$ = M.A. and $\frac{d_E}{d_L}$ = V.R.

$$\therefore \text{ Efficiency } \eta = \frac{\text{M.A.}}{\text{V.R.}}$$

or

$$\text{M.A.} = \text{V.R.} \times \eta \quad \text{.....(3.8)}$$

Thus, *the mechanical advantage of a machine is equal to the product of its efficiency and velocity ratio.*

Note : For an ideal machine (free from friction, etc.), work output is equal to the work input, so the efficiency is equal to 1 (or 100%) and the mechanical advantage is numerically equal to the velocity ratio.

In actual practice, the mechanical advantage for all practical machines is always less than its velocity ratio (*i.e.*, M.A. < V.R.) or the output work is always less than the input work, so the efficiency is less than 1 (*i.e.*, $\eta < 1$) due to some loss of input energy against friction, etc.

3.5 LEVERS

Levers are the simplest kind of machines used in our daily life.

A lever is a rigid, straight (or bent) bar which is capable of turning about a fixed axis.

The axis, about which the lever turns, passes through a point of the lever which is called the *fulcrum*. It is generally marked by the letter F. *This point does not move, but remains fixed when the lever is in use.*

Principle of a lever (M.A. of a lever)

Fig 3.1 shows a lever (or a straight rod) AB with the fulcrum at F. An effort *E*, applied at a point A of the lever, overcomes a load *L* at the point B. From the fulcrum F, the distance FA to the point A at which the effort is applied, is called the *effort arm* and the distance FB of point B at which the load acts, is called the *load arm*. For an *ideal lever*, it is assumed that the rod is *weightless* and there is *no friction* at the fulcrum.

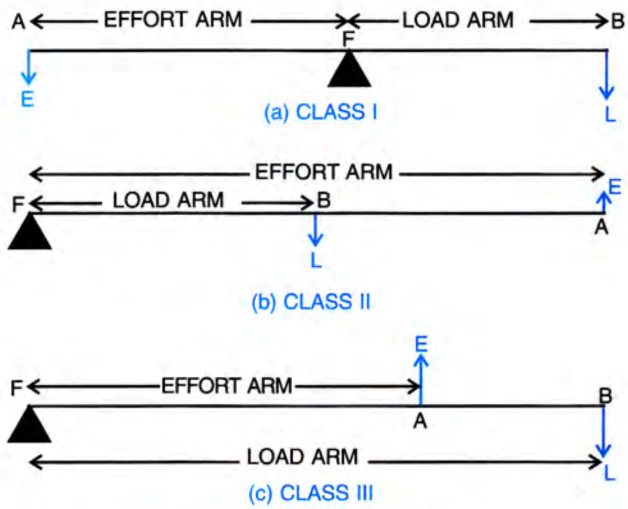


Fig. 3.1 Levers and their kinds

A lever works on the *principle of moments* according to which in the equilibrium position of lever, moment of load about the fulcrum must be equal to the moment of effort about the fulcrum and the two moments must always be in opposite direction. In Fig. 3.1, moment of load *L* about the fulcrum F is clockwise, while the moment of effort *E* about the fulcrum F is anticlockwise. Thus

Clockwise moment of load about the fulcrum

= Anticlockwise moment of effort about the fulcrum.

i.e., Load × load arm = Effort × effort arm

or $L \times \text{FB} = E \times \text{FA}$

or

$$\frac{L}{E} = \frac{\text{FA}}{\text{FB}} \quad \text{...(3.9)}$$

But from eqn. (3.1) $\frac{L}{E}$ = M.A.

$$\therefore \quad \text{M.A.} = \frac{\text{Effort arm FA}}{\text{Load arm FB}} \quad ..(3.10)$$

This relation is known as *the law of levers*. Thus

The mechanical advantage of a lever is equal to the ratio of the length of its effort arm to the length of its load arm.

From eqn. (3.10), it is clear that

(1) if effort arm = load arm, M.A. = 1,

(2) if effort arm > load arm, M.A. > 1, and

(3) if effort arm < load arm, M.A. < 1.

Obviously the mechanical advantage of a lever can be increased either by increasing its effort arm or by decreasing its load arm.

3.6 KINDS OF LEVERS

Depending upon the relative positions of the effort, load and fulcrum, there are following *three* types of levers : (1) Class I levers, (2) Class II levers, and (3) Class III levers.

(1) Class I levers

In this type of levers, *the fulcrum* F *is in between the effort* E *and the load* L as shown in Fig. 3.1 (a). Note that the fulcrum F *need not be* at the mid-point between the load *L* and the effort *E*, but both the load and effort are in same direction.

Examples : A seesaw, a pair of scissors, crowbar, handle of water pump, claw hammer, pair of pliers, beam of a physical balance, spade used for turning the soil, spoon used to open the lid of a tin can, catapult and nodding of the human head are the few examples of Class I levers. Some of these are shown in Fig. 3.2.

For class I levers, the mechanical advantage and velocity ratio can have any value either greater than 1, equal to 1 or less than 1.

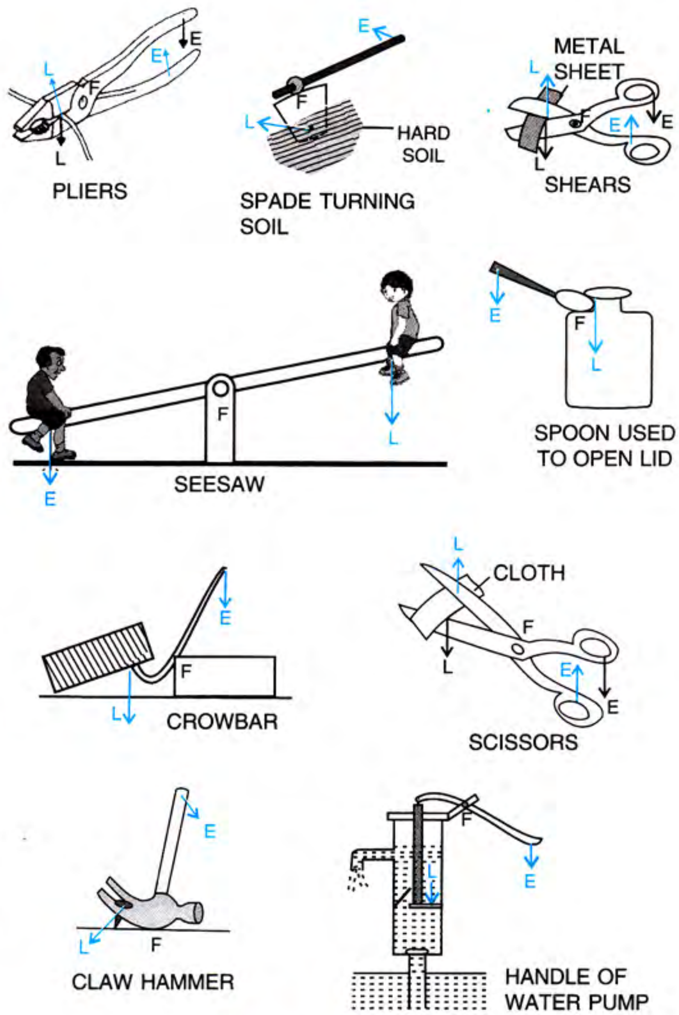


Fig. 3.2. Class I levers

When the effort arm is longer than the load arm, the mechanical advantage and the velocity ratio of the class I lever are greater than 1. Such a lever serves as a *force multiplier, i.e.*, it enables us to overcome a large resistive force (load) by a small effort. For example, shears (used for cutting the thin metal sheets) have much longer handles as compared to its blades. Similarly a crowbar, claw hammer, pliers and spoon (used to open the lid of a container) have long handles (or long effort arm).

If a class I lever has effort arm and load arm of equal lengths, then both its mechanical advantage and velocity ratio are equal to 1. For example, a physical balance with both arms equal in length has mechanical advantage and velocity ratio equal to 1.

When a lever of class I has the effort arm shorter than the load arm, both its mechanical advantage and velocity ratio are less than 1. Such levers are used to obtain the gain in speed because the velocity ratio less than 1 implies that $d_L > d_E$ *i.e.*, the displacement of load is more as compared to the displacement of effort. For example, a pair of scissors, whose blades are longer than its handles, is used to cut a piece of cloth (or paper) so that the blades move longer on the cloth (or paper) when the handles are moved a little.

A long handle oar used for rowing a boat by a single person acts as a lever of class I as shown in Fig. 3.3. The point at the edge of the boat at which the handle rests, acts as the fulcrum F. The boatman applies the effort *E* at its one end by keeping the effort arm shorter than the load arm so as to give a large movement to the blade of the oar to push water (*i.e.*, load *L*) back through a longer distance (*i.e.*, gain in speed). The force of reaction exerted by water on the boat moves the boat forward. Its mechanical advantage is less than 1.

Fig. 3.3 Rowing a boat with a long handle oar

(2) Class II levers

In this type of levers, *the fulcrum* F *and the effort* E are at the *two ends of the lever* and *the load L is somewhere in between the effort E and the fulcrum* F as shown in Fig. 3.1 (b). The load and effort are in opposite directions and *the effort arm is always longer than the load arm.* Therefore from eqn. (3.10), M.A. is always greater than 1 and since in an ideal machine M.A. is equal to V.R., hence V.R. is also greater than 1. Thus,

> *The mechanical advantage and velocity ratio of class II levers are always more than 1.*

In other words, a class II lever always acts as a *force multiplier i.e.,* a less effort is needed to overcome a large load. For example, in a nut cracker, a hard nut is broken by applying a small effort.

Examples : A nut cracker, a bottle opener, a wheel barrow, a lemon crusher, a paper cutter, a mango cutter, a bar used to lift a load, a door, raising the weight of the human body on toes, are the examples of Class II levers. Some of these are shown in Fig. 3.4.

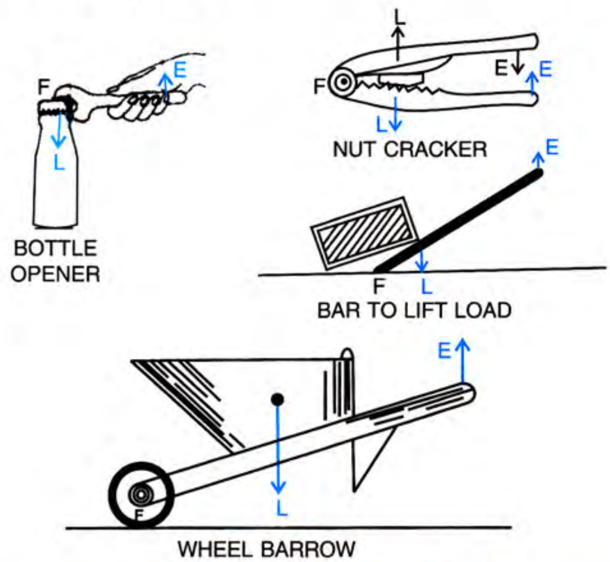


Fig. 3.4 Class II levers

> *Note :* If effort arm of a class II lever is 10 cm and load arm is 4 cm, in ideal case its M.A. $= \frac{10}{4} = 2{\cdot}5$, so its V.R. will also be 2·5. In actual lever, if the efficiency is 0·6, *i.e.*, M.A. < V.R., then the V.R. will remain 2·5, but its M.A. will become $2{\cdot}5 \times 0{\cdot}6 = 1{\cdot}5$ (*i.e.*, it will decrease from 2·5 to 1·5).

(3) Class III levers

In this type of levers, *the fulcrum* F *and the load L are at the two ends of the lever and the effort E is somewhere in between the fulcrum* F *and the load L* as shown in Fig. 3.1 (c). The effort and load are in opposite directions and *the* *effort arm is always smaller than the load arm.* Therefore from eqn. (3.10), M.A. < 1 and since for an ideal lever M.A. is equal to V.R., therefore, V.R. < 1 for these levers. Thus,

> *The mechanical advantage and velocity ratio of Class III levers are always less than 1.*

With levers of class III, we do not get gain in force, but we get *gain in speed, i.e.,* a larger displacement of load is obtained by a smaller displacement of effort. For example, the blade of a knife moves longer by a small displacement of its handle.

Examples : Sugar tongs, the forearm used for lifting a load (or action of the biceps muscle), fire tongs, foot treadle, knife, a spade used to lift coal (or soil), fishing rod, etc. are the examples of Class III levers. Some of these are shown in Fig. 3.5.

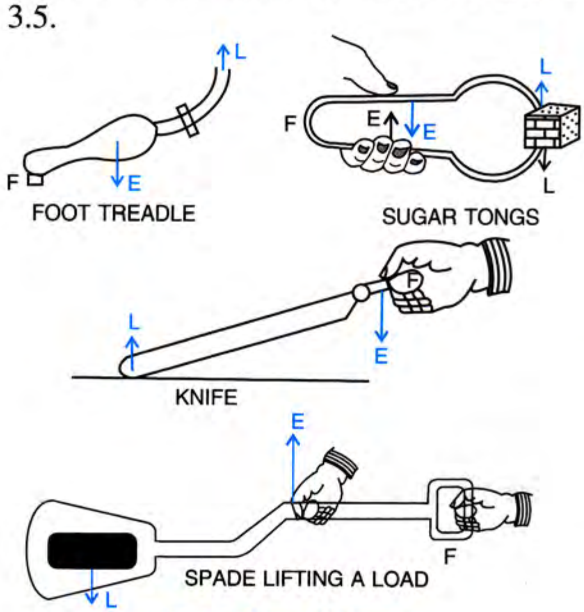


Fig. 3.5 Class III levers

The *short handle oar* used in a boat race by boatmen acts as a lever of class III. In Fig. 3.6, 'o' represents the positions of different boatmen rowing the boat, sitting near the edge of boat on both sides, one after the other, all facing towards the front. Each boatman sitting on the right side holds the top end of the oar stationary by his left hand. This end acts as the fulcrum F. He applies the effort *E* on the handle at some distance from the fulcrum, using his right hand. The blade of each oar pushes the water which acts as load *L* at the other end of the oar. The oar is raised out of water and the process is repeated. Since effort arm is shorter than the load arm, each oar provides gain in speed. All boatmen row the boat *in unison* so as to obtain more gain in speed and win the boat race.

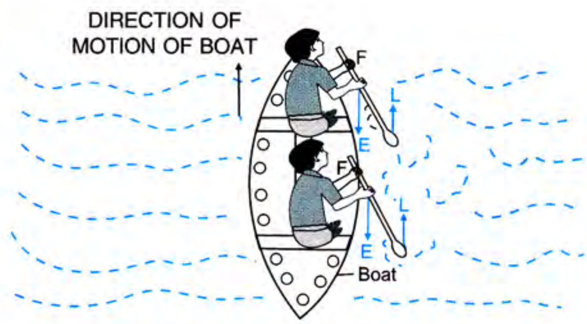


Fig. 3.6 In a boat race, rowing the boat by the short handle oars

> ***Note :*** If a class III lever has the effort arm 2·5 cm and load arm 10·0 cm, in ideal case its M.A. is $\frac{2{\cdot}5}{10{\cdot}0} = 0{\cdot}25$, so its V.R. will also be 0·25. In actual lever, if the efficiency is 0·6 (*i.e.,* M.A. < V.R.), its V.R. will remain 0·25, but its M.A. will become 0·6 × 0·25 = 0·15 (*i.e.,* it will reduce from 0·25 to 0·15).

3.7 EXAMPLES OF EACH CLASS OF LEVERS AS FOUND IN THE HUMAN BODY

In the human body, we can find the examples of all the three classes of levers. The muscles exert force (*i.e.,* effort) by contraction.

(1) Class I lever in the action of nodding of head : Fig. 3.7 shows the action of nodding of head. In this action, the spine acts as the fulcrum F, load *L* is at its front part, while effort *E* is at its rear part. Thus this is an example of Class I lever.

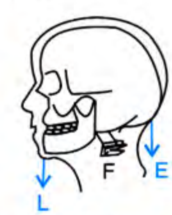


Fig. 3.7 Nodding action of head (Class I lever)

(2) Class II lever in raising the weight of the body on toes : Fig. 3.8 shows how the weight of the body is raised on toes. The fulcrum F is at toes at one end, the load L (*i.e.*, weight of the body) is in the middle and effort E by muscles is at the other end. Thus this is an example of Class II lever.

Fig. 3.8 Raising weight of the body on toes (Class II lever)

(3) Class III lever in raising a load by forearm : Fig. 3.9 shows the action of biceps. The elbow joint acts as fulcrum F at one end, biceps exerts the effort E in the middle and load L on the palm is at the other end. Thus this is an example of Class III lever.

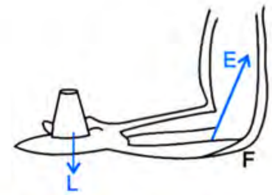


Fig. 3.9 Raising a load kept on a palm (Forearm as Class III lever)

EXAMPLES

1. A machine is driven by a 100 kg mass that falls 8·0 m in 4·0 s. It lifts a load of mass 500 kg vertically upwards. Taking $g = 10$ m s^{-2}, calculate :

(a) the force exerted by the falling mass,

(b) the work done by the falling mass in its displacement by 8·0 m,

(c) the power input to the machine,

(d) the power output of the machine if its efficiency is 60%, and

(e) the work done by the machine in 4·0 s.

Given : $m = 100$ kg, $d = 8{\cdot}0$ m, $t = 4{\cdot}0$ s, $L = 500$ kg

(a) Force exerted by the falling mass = mg
= 100 × 10 = **1000 N**

(b) Work done by the falling mass
= torce exerted by the falling mass × displacement
= $mg \times d$
= 1000 × 8·0 = **8000 J**

(c) Power input = $\dfrac{\text{work done on machine}}{\text{time}}$
= $\dfrac{8000\ \text{J}}{4{\cdot}0\ \text{s}}$ = **2000 W**

(d) Given, efficiency = 60% = 0·6,
power output = ?

Since efficiency = $\dfrac{\text{power output}}{\text{power input}}$

∴ Power output = power input × efficiency
= 2000 W × 0·6 = **1200 W**

(e) Work done by the machine in 4·0 s
= power output × time
= 1200 W × 4·0 s = **4800 J**

2. Calculate the ideal mechanical advantage of a lever in which the effort arm is 60 cm and the load arm is 4 cm.

Given : effort arm = 60 cm, load arm = 4 cm.

Ideal mechanical advantage

$$= \frac{\text{effort arm}}{\text{load arm}} = \frac{60}{4} = \mathbf{15}$$

3. Draw a simple diagram of a fire tongs and mark on it the fulcrum F and the points of application of load L and effort E. (a) Name the class of lever. (b) If load arm is 15 cm and effort arm is 5 cm, what is its mechanical advantage ?

(a) Fig. 3.10 shows a fire tongs in which the effort E is in between the load L and fulcrum F. This belongs to class III type of levers.

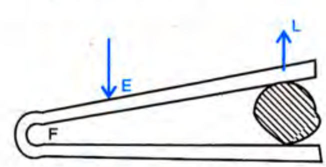


Fig. 3.10 Coal tongs.

(b) Given : load arm = 15 cm, effort arm = 5 cm

$$\text{Mechanical advantage} = \frac{\text{effort arm}}{\text{load arm}} = \frac{5}{15} = \mathbf{\frac{1}{3}}$$

4. A crowbar 2 m long is pivoted about a point 10 cm from its tip.

(i) Calculate the mechanical advantage of the crowbar.

(ii) What is the least force which must be applied at the other end to displace a load of 100 kgf?

Given : L = 100 kgf, since load is at tip which is at distance 10 cm from the fulcrum, so load arm = 10 cm,

Total length of crowbar = 2 m = 200 cm

Effort arm = 200 – 10 = 190 cm, E = ?

(i) Mechanical advantage = $\frac{\text{effort arm}}{\text{load arm}} = \frac{190}{10} = 19$

(ii) Taking moments about the pivot (fulcrum),

$$E \times 190 = 100 \times 10$$

$$\therefore \quad E = \frac{100 \times 10}{190} = 5{\cdot}26 \text{ kgf}$$

Alternative method :

$$\text{M.A.} = \frac{L}{E} \text{ or } 19 = \frac{100}{E} \text{ or } E = \frac{100}{19} = 5{\cdot}26 \text{ kgf}$$

5. A uniform seesaw, 5 m long, is supported at its centre. A boy weighing 40 kgf sits at a distance of 1 m from the centre of the seesaw.

(i) To which class of lever does it belong ?

(ii) Find where a girl of weight 20 kgf must sit on the seesaw so as to balance the weight of the boy.

(i) The seesaw has the fulcrum F at its centre *i.e.*, between the load and the effort. It is a class I lever.

(ii) To balance the weight of the boy, let the girl be sitting at a distance x metre from the centre on the opposite side of the boy, then

Effort arm = x metre, load arm = 1 metre, L = 40 kgf, E = 20 kgf

Taking moments about F,

$$40 \times 1 = 20 \times x$$

$$\therefore \quad x = \frac{40}{20} = 2 \text{ m.}$$

Hence the girl must sit at a distance 2 m from the centre on the opposite side of boy.

6. A cook uses the fire tongs of length 28 cm to lift a piece of burning coal of mass 250 g. If he applies the effort at a distance of 7 cm from the fulcrum, find the effort. Take g = 10 m s^{-2}.

The fire tongs is a lever of class III which has the fulcrum and load at the ends, with effort in between.

Given :

Load L = 250 gf = 0·25 kgf = 0·25 × 10 N = 2·5 N, load arm = 28 cm, effort arm = 7 cm.

By the principle of moments,

load × load arm = effort × effort arm

$$\therefore \text{ effort} = \text{load} \times \frac{\text{load arm}}{\text{effort arm}} = 2{\cdot}5 \times \frac{28}{7} = 10 \text{ N.}$$

7. The diagram below shows a lever in use.

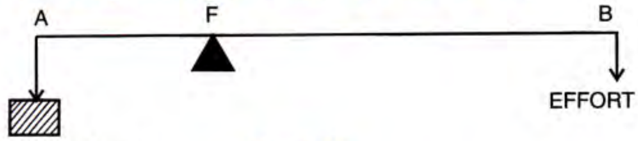


Fig. 3.11

(a) To which class of lever does it belong ? Give *one* example of this class.

(b) (i) State the principle of moments as applied to the above lever, and (ii) calculate its mechanical advantage if AB = 2·0 m and FA = 20 cm.

(c) Calculate the effort needed to lift the load.

(a) The fulcrum F is in between the load and the effort, so it is a class I lever. A pair of pliers is an example of this class.

(b) (i) By the principle of moments,

Load × FA = Effort × FB

(ii) Given : AB = 2·0 m, FA = 20 cm = 0·2 m,

FB = AB – FA = 2·0 – 0·2 = 1·8 m.

$$\therefore \quad \text{Mechanical advantage} = \frac{\text{effort arm FB}}{\text{load arm FA}} = \frac{1{\cdot}8 \text{ m}}{0{\cdot}2 \text{ m}} = 9$$

(c) Given : load = 18 kgf, M.A. = 9

Since $\quad \text{M.A.} = \frac{\text{load}}{\text{effort}}$

$$\therefore \quad \text{Effort} = \frac{\text{load}}{\text{M.A.}} = \frac{18}{9} = 2 \text{ kgf}$$

8. A nut which can be broken by applying a force of 40 kgf, is broken by using a nut cracker having its handle 20 cm long, by placing it at a distance 2 cm from the hinge. Calculate the minimum force needed to break the nut.

Given, load = 40 kgf, effort = ?,

load arm = 2 cm, effort arm = 20 cm

By the principle of moments,

load × load arm = effort × effort arm

40 kgf × 2 cm = E × 20 cm

$$\therefore E = \frac{40 \times 2}{20} \text{ kgf} = 4 \text{ kgf}$$

9. A man opens a nut by applying a force of 150 N by using a lever handle of length 0·4 m. What should be the length of the handle if he wants to open it by applying a force of 60 N ?

Given, when effort = 150 N, effort arm = 0·4 m. When effort = 60 N, effort arm = ?

Load and load arm remains same in both cases.

$\therefore \quad 150 \times 0{\cdot}4 = 60 \times$ effort arm

or Effort arm (*i.e.*, length of handle)

$$= \frac{150 \times 0{\cdot}4}{60} = \mathbf{1\ m.}$$

EXERCISE–3(A)

1. (a) What do you understand by a simple machine ?
   (b) State the principle of an ideal machine.
2. State *four* ways in which machines are useful to us.
3. Name a machine for each of the following use :
   (a) to multiply the force,
   (b) to change the point of application of force,
   (c) to change the direction of force,
   (d) to obtain the gain in speed.
4. What is the purpose of a jack in lifting a car by it ?
5. What do you understand by an ideal machine ? How does it differ from a practical machine ?
6. Explain the term mechanical advantage. State its unit.
7. Define the term velocity ratio. State its unit.
8. How is mechanical advantage related to the velocity ratio for (i) an ideal machine, (ii) a practical machine ? **Ans.** (i) M.A. = V.R. (ii) M.A. < V.R.
9. Define the term efficiency of a machine. Give *two* reasons for a machine not to be 100% efficient ?
10. When does a machine act as (a) a force multiplier, (b) a speed multiplier. Can a machine act as a force multiplier and a speed multiplier simultaneously ?
11. (a) State the relationship between mechanical advantage, velocity ratio and efficiency.
    (b) Name the term that will not change for a machine of a given design.
12. Derive a relationship between mechanical advantage, velocity ratio and efficiency of a machine.
13. How is the mechanical advantage related with the velocity ratio for an actual machine ? State whether the efficiency of such a machine is equal to 1, less than 1 or more than 1.
    Ans. M.A. < V.R., less than 1.
14. State *one* reason why is mechanical advantage less than the velocity ratio for an actual machine.
15. What is a lever ? State its principle.
16. Write down a relation expressing the mechanical advantage of a lever.
17. Name the *three* classes of levers and state how are they distinguished. Give *two* examples of each class.
18. Give *one* example each of a class I lever where mechanical advantage is (a) more than 1, and (b) less than 1.
19. What is the use of the lever if its mechanical advantage is (a) more than 1, (b) equal to 1, and (c) less than 1.
20. Both a pair of scissors and a pair of pliers belong to the same class of levers. Name the class of lever. Which one has the mechanical advantage less than 1 ?
21. Explain why scissors for cutting cloth may have blades longer than the handles, but shears for cutting metals have short blades and long handles.
22. Fig. 3.12, shows a uniform metre rule of weight W supported on a fulcrum at the 60 cm mark by applying the effort E at the 90 cm mark.
    (a) State with reason whether the weight W of the rule is greater than, less than or equal to the effort E.
    (b) Find the mechanical advantage in an ideal case.

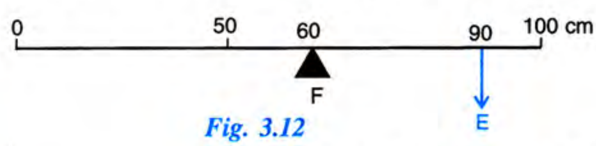


Fig. 3.12

Ans. (a) Greater than the effort E

Reason : The weight W of a uniform metre rule acts at the 50 cm mark. Since distance of weight of rule from the fulcrum F is less than that of the effort E, so the weight W of rule is greater than the effort E. (b) 3

23. Which type of lever has a mechanical advantage always more than 1 ? Give *one* example. What change can be made in this lever to increase its mechanical advantage ?

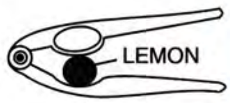
24. Draw a diagram of a lever which is always used as a force multiplier. How is the effort arm related to the load arm in such a lever ?

25. Explain why the mechanical advantage of a class II type of lever is always more than 1.

26. Draw a labelled diagram of a class II lever. Give *one* example of such a lever.

27. Fig. 3.13 shows a lemon crusher.

(a) In the diagram, mark the position of the fulcrum F and the line of action of load *L* and effort *E*.



(b) Name the class of lever.

28. The diagram below shows a rod lifting a stone. (a) Mark position of fulcrum F and draw arrows to show the directions of load *L* and effort *E*. (b) What class of lever is the rod ? (c) Give *one* more example of the same class of lever stated in part (b).

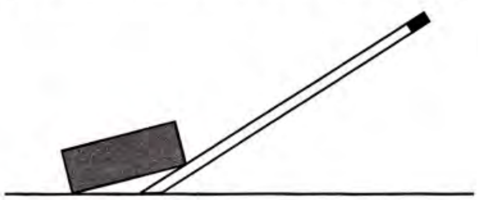


Fig. 3.14

29. State the kind of lever which always has the mechanical advantage less than 1. Draw a labelled diagram of such a lever.

30. Explain why the mechanical advantage of the class III type of lever is always less than 1.

31. Class III levers have the mechanical advantage less than 1. Why are they then used ?

32. Draw a labelled sketch of a class III lever. Give *one* example of this kind of lever.

33. State the class of levers and the relative positions of load (*L*), effort (*E*) and fulcrum (F) in (a) a bottle opener, and (b) sugar tongs.

34. Draw diagrams to illustrate the positions of fulcrum, load and effort, in each of the following :

(a) A seesaw (b) A common balance

(c) A nut cracker (d) Forceps.

35. Classify the following into levers as class I, class II or class III :

(a) a door (b) a catapult

(c) a wheel barrow (d) a fishing rod.

Ans. (a) class II, (b) class I, (c) class II, (d) class III

36. What type of lever is formed by the human body while (a) raising a load on the palm, and (b) raising the weight of body on toes ?

37. Indicate the positions of load *L*, effort *E* and fulcrum F in the forearm shown alongside in Fig. 3.15. Name the class of lever.

Fig. 3.15

38. Give example of each class of lever in a human body.

39. Complete the following sentences :

(a) Mechanical advantage = ... × velocity ratio

(b) In class II lever, effort arm is .... than the load arm.

(c) A scissors is a .... multiplier.

Ans. (a) efficiency (b) longer (c) speed

MULTIPLE CHOICE TYPE

1. Mechanical advantage (M.A.), load (*L*) and effort (*E*) are related as :

(a) M.A. = $L \times E$ (b) M.A. × $E = L$

(c) E = M.A. × L (d) none of these.

Ans. (b) M.A. × $E = L$

2. The correct relationship between the mechanical advantage (M.A.), the velocity ratio (V.R.) and the efficiency (η) is :

(a) M.A. = η × V.R. (b) V.R. = η × M.A.

(c) η = M.A. × V.R. (d) none of these.

Ans. (a) M.A. = η × V.R.

3. Select the incorrect statement :

(a) A machine always has the efficiency less than 100%.

(b) The mechanical advantage of a machine can be less than 1.

(c) A machine can be used as a speed multiplier.

(d) A machine can have the mechanical advantage greater than the velocity ratio.

Ans. (d) A machine can have the mechanical advantage greater than the velocity ratio.

4. The lever for which the mechanical advantage is less than 1 has the :

(a) fulcrum at mid point between the load and effort.

(b) load between the effort and fulcrum.

(c) effort between the fulcrum and load.

(d) load and effort acting at the same point.

Ans. (c) effort between the fulcrum and load.

5. Class II levers are designed to have :
(a) M.A. = V.R. (b) M.A. > V.R.
(c) M.A. > 1 (d) M.A. < 1

Ans. (c) M.A. > 1

NUMERICALS

1. A crowbar of length 120 cm has its fulcrum situated at a distance of 20 cm from the load. Calculate the mechanical advantage of the crowbar. **Ans.** 5

2. A pair of scissors has its blades 15 cm long, while its handles are 7·5 cm long. What is its mechanical advantage ? **Ans.** 0·5

3. A force of 5 kgf is required to cut a metal sheet. A shears used for cutting the metal sheet has its blades 5 cm long, while its handles are 10 cm long. What effort is needed to cut the sheet ? **Ans.** 2·5 kgf

4. Fig. 3.16 below shows a lever in use.

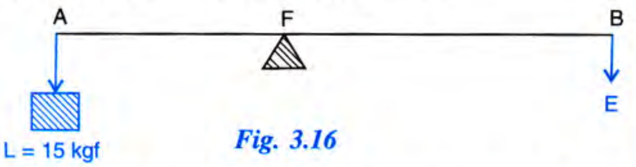


Fig. 3.16

(a) To which class of lever does it belong ?
(b) If AB = 1 m, AF = 0·4 m, find its mechanical advantage.
(c) Calculate the value of *E*.

Ans. (a) Class I (b) 1·5 (c) 10 kgf

5. A man uses a crowbar of length 1·5 m to raise a load of 75 kgf by putting a sharp edge below the bar at a distance of 1 m from his hand. (a) Draw a diagram of the arrangement showing the fulcrum (F), load *(L)* and effort *(E)* with their directions. (b) State the kind of lever. (c) Calculate : (i) load arm, (ii) effort arm, (iii) mechanical advantage, and (iv) the effort needed.

Ans. (b) class I. (c) (i) 0·5 m (ii) 1·0 m (iii) 2 (iv) 37·5 kgf.

6. A pair of scissors is used to cut a piece of a cloth by keeping it at a distance of 8·0 cm from its rivet and applying an effort of 10 kgf by fingers at a distance of 2·0 cm from the rivet. (a) Find : (i) the mechanical advantage of scissors, and (ii) the load offered by the cloth. (b) How does the pair of scissors act : as a force multiplier or as a speed multiplier ?

Ans. (a) (i) 0·25 (ii) 2·5 kgf. (b) speed multiplier

7. A 4 m long rod of negligible weight is supported at a point 125 cm from its one end and a load of 18 kgf is suspended at a point 60 cm from the support on the shorter arm.
(a) If a weight *W* is placed at a distance of 250 cm from the support on the longer arm, find *W*.
(b) If a weight 5 kgf is kept to balance the rod, find its position.
(c) To which class of lever does it belong ?

Ans. (a) 4·32 kgf (b) at distance 2·16 m from the support on longer arm (c) class I.

8. A lever of length 9 cm has its load arm 5 cm long and the effort arm is 9 cm long. (a) To which class does it belong ? (b) Draw diagram of the lever showing the position of fulcrum F and directions of both the load *L* and effort *E*. (c) What is the mechanical advantage and velocity ratio if the efficiency is 100%? (d) What will be the mechanical advantage and velocity ratio if the efficiency becomes 50% ?

Ans. (a) class II (c) M.A. = 1·8, V.R. = 1·8 (d) M.A. = 0·9, V.R. = 1·8

9. Fig. 3.17 below shows a lever in use.

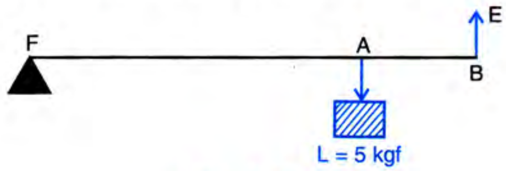


Fig. 3.17

(a) To which class of lever does it belong ?
(b) If FA = 80 cm, AB = 20 cm, find its mechanical advantage.
(c) Calculate the value of *E*.

Ans. (a) class II (b) 1·25 (c) 4 kgf

10. Fig. 3.18 below shows a wheel barrow of mass 15 kg carrying a load of 30 kgf with its centre of gravity at A. The points B and C are the centre of wheel and tip of the handle such that the horizontal distance AB = 20 cm and AC = 40 cm.

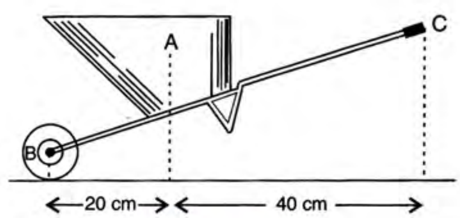


Fig. 3.18

Find : (a) the load arm, (b) the effort arm, (c) the mechanical advantage, and (d) the minimum effort required to keep the leg just off the ground.

Ans. (a) 20 cm (b) 60 cm, (c) 3 (d) 15 kgf

11. Fig. 3.19 below shows the use of a lever.

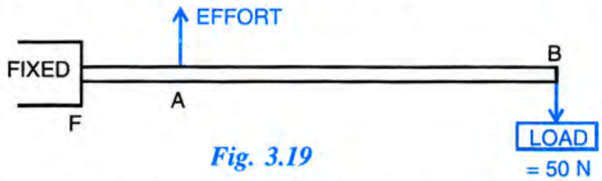


Fig. 3.19

(a) State the principle of moments as applied to the above lever.

(b) To which class of lever does it belong ? Give an example of this class of lever.

(c) If FA = 10 cm, AB = 490 cm, calculate : (i) the mechanical advantage, and (ii) the minimum effort required to lift the load (= 50 N).

Ans. (a) load × FB = effort × FA (b) class III, sugar tongs (c) (i) M.A. = $\frac{1}{50}$, (ii) Effort = 2500 N

12. A fire tongs has its arms 20 cm long. It is used to lift a coal of weight 1·5 kgf by applying an effort at a distance of 15 cm from the fulcrum. Find : (i) the mechancial advantage of the fire tongs, and (ii) the effort needed. **Ans.** (i) 0·75 (ii) 2·0 kgf

(B) PULLEY

3.8 PULLEY

Fig. 3.20 (a) and (b) shows a simple pulley. It is a metallic (or wooden) disc with a grooved rim. A string or rope is passed around the groove at the rim. The disc rotates about an axle passing through its centre. The axle is fixed rigidly to a frame by means of nails.

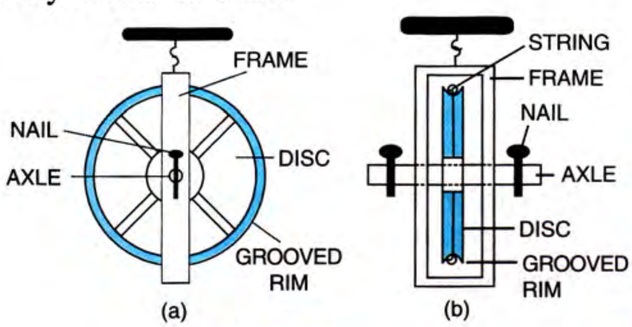


Fig. 3.20 Pulley

Either a single pulley or a combination of two or more pulleys fixed in a frame, is called a ***block,*** while a string (rope or chain) that winds around the pulleys in different blocks is known as ***tackle***.

A single pulley can be used in *two* ways : (1) as a *fixed pulley* by keeping its axis of rotation fixed in position, and (2) as a *movable pulley* without keeping its axis of rotation fixed *i.e.*, keeping the whole frame movable.

3.9 SINGLE FIXED PULLEY

A pulley which has its axis of rotation fixed in position, is called a fixed pulley.

Fig. 3.21 shows a single fixed pulley in which the axle is fixed to a rigid support and an inextensible strong string of negligible mass passes around the grooved rim of the pulley. One end of the string is connected to the load *L*, while the effort *E* is applied at the other (free) end of the string. Both the load *L* and the effort *E* act *downwards*. The tension throughout the string is *T* (same)* upwards. This type of pulley is used for lifting a small load (such as a water bucket or a basket).

Note : When the string is pulled, it is the friction between the string and surface of the rim of the pulley which rotates the pulley.

M.A., V.R. and η of a single fixed pulley

If we neglect (i) the mass of string, and (ii) the friction at the axle (or in the pulley bearings), then in the balanced position of load, we have

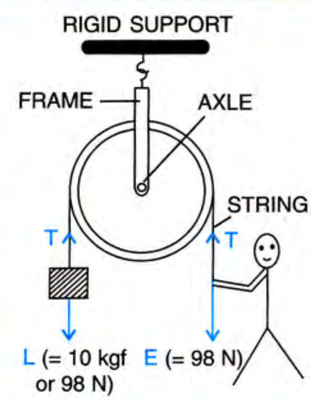


Fig. 3.21 Single fixed pulley

$L = T$ ...(i)

and $E = T$ (when pulley is not rotating) ...(ii)

From eqns. (i) and (ii), $\boxed{E = L}$ ...(3.11)

* If tension is not same throughout the string, the string will then move even when the pulley is not rotating.

Thus in case of a single fixed pulley, in the ideal case, the effort needed to lift a load is equal to the load itself. In Fig. 3.21, for L = 10 kgf (or 98 N), E = 98 N. In actual practice, there is always some friction at the axle or in the pulley bearings, so the effort needed is little more than the load to be lifted. Now

$$\text{Mechanical advantage} = \frac{\text{load } L}{\text{effort } E} = \frac{T}{T} = 1 \quad ..(3.12)$$

Thus, in this arrangement *there is no gain in the mechanical advantage*.

If the point of application of effort E moves a distance d downwards, the load L also moves the same distance d upwards, *i.e.*, if $d_E = d$ then $d_L = d$.

$$\therefore \text{ Velocity ratio} = \frac{d_E}{d_L} = \frac{d}{d} = 1 \quad ....(3.13)$$

$$\text{Hence efficiency } \eta = \frac{M.A.}{V.R.} = 1 \text{ or } 100\% \quad ...(3.14)$$

In actual practice, efficiency is always less than 100% because of some friction at the axle or in the pulley bearings.

Now the question arises : when there is no gain either in mechanical advantage or in speed, why is then a fixed pulley used ? *A fixed pulley is used only to change the direction of effort applied, i.e., with its use, the effort can be applied in a more convenient direction.*

It is difficult to apply the effort upwards to lift a load up directly, but it becomes easier with the help of a fixed pulley, because the effort can be applied in the downward direction to raise the load up. Further to apply the effort downwards, one can conveniently make use of his own weight also for the effort.

3.10 A SINGLE MOVABLE PULLEY

A pulley whose axis of rotation is movable (i.e., not fixed in position) is called a movable pulley.

Fig. 3.22 shows a single movable pulley suspended by passing an inextensible string of negligible mass around its grooved rim. The load L to be raised is suspended from its axle. One end of the string is tied to a hook H at a rigid support and the effort E is applied at its free end. The tension T acts on the string on both sides of the pulley as shown in Fig. 3.22.

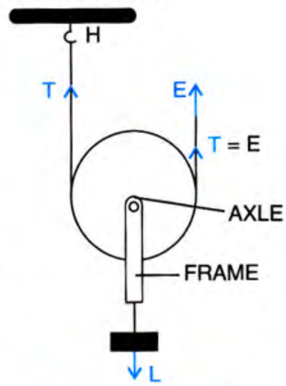


Fig. 3.22 Single movable pulley

M.A., V.R. and η of a single movable pulley

In Fig. 3.22, if we neglect (1) the friction in the pulley bearings or at the axle, and (2) the weight of the pulley and string, the load L is balanced by the tension T in *two* segments of the string and the effort E balances the tension T at the free end, so

$$L = T + T = 2T \quad ...(i)$$

and $$E = T \quad ...(ii)$$

From eqns. (i) and (ii), $$E = \frac{L}{2} \quad ...(3.15)$$

Thus, using a single movable pulley, the load can be lifted by applying an effort equal to half the load (in ideal situation), *i.e.*, the single movable pulley acts as a *force multiplier*. Now

$$\text{Mechanical advantage} = \frac{\text{load } L}{\text{effort } E}$$

or $$\text{M.A.} = \frac{2T}{T} = 2 \quad ..(3.16)$$

When the free end of string is pulled up by the effort through a distance $2d$, the load is raised up through a distance d. The reason is that the segment of string on both sides of the pulley moves up by a distance d. *i.e.*, if $d_E = 2d$, then $d_L = d$.

$$\therefore \text{ Velocity ratio} = \frac{\text{distance moved by the effort } d_E}{\text{distance moved by the load } d_L}$$

or $\boxed{\text{V.R.} = \frac{2d}{d} = 2}$ ...(3.17)

Hence $\boxed{\text{efficiency } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{2}{2} = 1 \text{ or } 100\%}$ .. (3.18)

For an *actual* single movable pulley, due to (1) friction in the pulley bearings or at the axle, and (2) the weight of the pulley and string, the effort needed to lift up a load L will be more than $\frac{L}{2}$, so the mechanical advantage will be less than 2. Since the velocity ratio will remain 2, hence the efficiency will be less than 100%

Way to change the direction of effort using a movable pulley : With a single movable pulley, the effort is to be applied in the upward direction. However, it is inconvenient to apply the effort in an upward direction, therefore a movable pulley A is used along with a fixed pulley B to change the direction of effort as shown in Fig. 3.23. The load is attached to the axle of movable pulley A and the effort is applied in the downward direction at the free end of the string passing over the fixed pulley B. One can also use his own weight as effort which will be quite convenient.

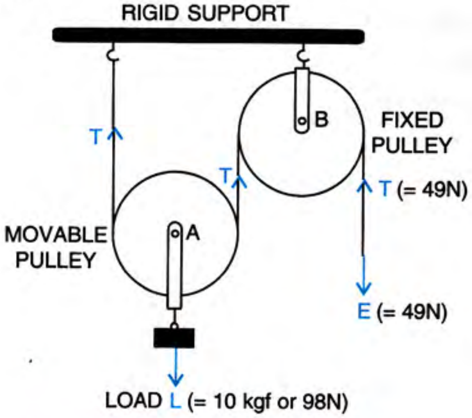


Fig. 3.23 Single movable pulley with a fixed pulley

> *Note :* The mechanical advantage and the velocity ratio of the arrangement shown in Fig. 3.23 remain same as that of a single movable pulley.

Comparison between a single fixed pulley and a single movable pulley

Single fixed pulley	Single movable pulley
1. It is fixed to a rigid support.	1. It is not fixed to a rigid support.
2. Its ideal mechanical advantage is 1.	2. Its ideal mechanical advantage is 2.
3. Its velocity ratio is 1.	3. Its velocity ratio is 2.
4. The weight of pulley itself does not affect its mechanical advantage.	4. The weight of pulley itself reduces its mechanical advantage.
5. It is used to change the direction of effort from upwards to downwards.	5. It is used as a force multiplier.
6. The load moves in a direction opposite to that of the effort.	6. The load moves in the direction of the effort.

3.11 COMBINATION OF PULLEYS

When a heavy load is to be lifted or shifted from one place to another, we require a pulley system of mechanical advantage much more than 2, so a single movable pulley is not enough. A combination of several pulleys with one (or more) strong thick string (or rope) is then used.

The combination can be made in *two* ways : (1) using *one* fixed pulley and *several* movable pulleys attached to the same rigid support, and (2) using *several* pulleys in *two* blocks (of which lower block is movable and upper block is fixed) known as *block and tackle system.*

(1) Using one fixed pulley and other movable pulleys

Fig. 3.24 shows a system of three movable pulleys A, B and C used with a fixed pulley D. Each movable pulley is attached with a separate string. The tension is same in one string, but it is different in different strings (being maximum in the string of pulley to which load is attached).

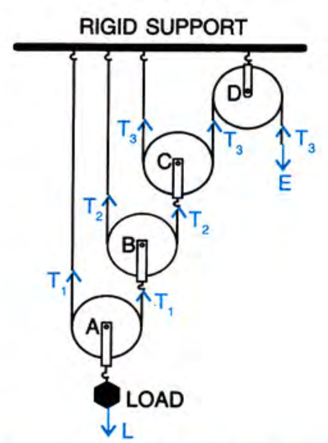


Fig. 3.24 System of one fixed pulley and three movable pulleys

Mechanical advantage : In equilibrium,

$$\text{Effort } E = T_3 \qquad .....(i)$$

The *two* segments of string passing over the pulley A supports the load L, therefore tension T_1 in this string is given as :

$$2T_1 = L \quad \text{or} \quad T_1 = \frac{L}{2} \qquad .....(ii)$$

Similarly, the *two* segments of string passing over the pulley B supports the tension T, so tension T_2 in this string is given as :

$$2T_2 = T_1 \text{ or } T_2 = \frac{T_1}{2} = \frac{L}{2^2} \text{ (using eqn. i)} \quad ..(iii)$$

Similarly, the tension T_3 in string passing over the pulley C is given as :

$$2T_3 = T_2 \text{ or } T_3 = \frac{T_2}{2} = \frac{L}{2^3} \text{ (using eqn. ii)} \quad ...(iv)$$

From eqn. (iv),

$$\text{Load } L = 2^3 \times T_3 \qquad ...(v)$$

From eqns. (i) and (v), $E = \frac{L}{2^3}$ ....(vi)

Hence mechanical advantage

$$\text{M.A.} = \frac{\text{load } L}{\text{effort } E} = \frac{2^3 \times T_3}{T_3} = 2^3 \quad ..(vii)$$

In general, if there are n movable pulleys with one fixed pulley, the mechanical advantage is then

$$\text{M.A.} = 2^n \qquad ....(3.19)$$

Velocity ratio : As one end of each string passing over a movable pulley is fixed, so the other end of string moves up twice the distance moved by the axle of the movable pulley. If the load L attached to the pulley A moves up by a distance x *i.e.* $d_L = x$, the string connected to the axle of pulley B moves up by a distance $2 \times x = 2x$, the string connected to the axle of pulley C moves up by a distance $2 \times 2x = 2^2x$ and the end of the string passing over the fixed pulley D moves up by a distance $2 \times 2^2x = 2^3x$, *i.e.* the effort E moves by a distance 2^3x or $d_E = 2^3x$.

$$\therefore \text{ Velocity ratio V.R.} = \frac{\text{distance moved by the effort } d_E}{\text{distance moved by the load } d_L}$$

$$= \frac{2^3x}{x} = 2^3 \qquad .....(viii)$$

In general if there are n movable pulleys connected to a fixed pulley, then velocity ratio is

$$\text{V.R.} = 2^n \qquad ....(3.20)$$

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{2^n}{2^n} = 1 \text{ or } 100\% \qquad ..(3.21)$$

Note that the efficiency of this arrangement is 100% only in an *ideal* situation. In actual practice both (1) the weight of the pulleys and string, and (2) the friction between the bearings of the pulleys, reduce the mechanical advantage and so the efficiency becomes less than 100%.

(2) Using several pulleys in two blocks (block and tackle system)

In this system of pulleys, two blocks of pulleys are used. One block (*upper*) having several pulleys is attached to a rigid support (*i.e., fixed*) and the other block (*lower*) having several pulleys is *movable*. This arrangement is called the block and tackle system.

The number of pulleys used in the movable lower block is either equal to or one less than the number of pulleys in the fixed upper block. A strong inextensible string (or rope) of negligible mass passes around all the pulleys. One end of the string is attached to the hook of the lower block (if the number of pulleys in the upper block is more than that in the lower block) or it is attached to the hook of the upper block (if the number of pulleys is equal in both the blocks) *so as to apply the effort in the downward direction.* Fig. 3.25 shows a block and tackle system of 5 pulleys in which one end of the string is attached to the hook of the lower block, while Fig. 3.26 shows a block and tackle system of 4 pulleys in which one end of the string is attached to the hook of the upper block.

The load L is attached to the movable lower block and the effort E is applied at the free end

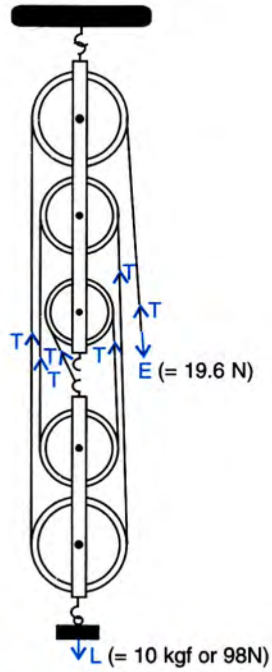


Fig. 3.25 Block and tackle for 5 pulleys

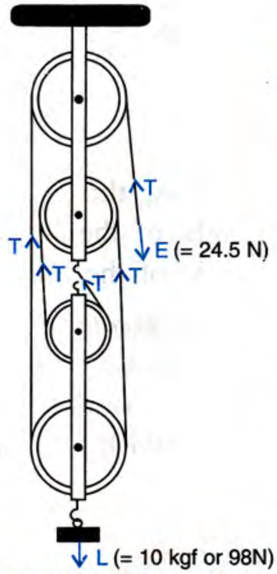


Fig. 3.26 Block and tackle for 4 pulleys

of the string. The tension along the entire length of the string is *same* and is denoted as T.

Mechanical advantage : In Fig. 3.25, the tension in the *five* segments of string supports the load L. Therefore, $L = 5T$ and $E = T$.

$$\therefore \quad E = \frac{L}{5}$$

$$\text{and M.A.} = \frac{L}{E} = \frac{5T}{T} = 5$$

Similarly in Fig. 3.26, the tension in the *four* segments of string supports the load L. Therefore,

$$L = 4T \text{ and } E = T.$$

$$\therefore \quad E = \frac{L}{4}$$

$$\text{and M.A.} = \frac{L}{E} = \frac{4T}{T} = 4$$

In general, if the total number of pulleys used in both the blocks is n and *the effort is applied in the downward direction*, then the tension in the n segments of string supports the load, therefore,

$$L = nT \quad \text{and} \quad E = T \text{ or } E = \frac{L}{n}$$

Thus, the effort required to balance the load is :

$$E = \frac{L \text{ (load)}}{n \text{ (number of pulleys)}} \qquad \text{...(3.22)}$$

Thus in a block and tackle system, the effort gets multiplied n times, where n is the total number of pulleys in the system. It therefore acts as a force multiplier.

$$\text{Hence} \quad \text{M.A.} = \frac{\text{load } L}{\text{effort } E} = \frac{nT}{T} = n$$

$$= \text{Total number of pulleys in both the blocks}$$

...(3.23)

Velocity ratio : In a system of n pulleys if the load moves up through a distance d, each segment of string supporting the load is loosened by a length d, so the effort end moves through a distance nd, *i.e.*, if $d_L = d$, then $d_E = nd$.

$$\therefore \quad \text{Velocity ratio} = \frac{d_E}{d_L} = \frac{nd}{d} = n \qquad \text{......(3.24)}$$

Thus, *the velocity ratio is always equal to the number of strands of tackle* (or number of sections the string) *supporting the load.*

$$\text{Efficiency } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{n}{n} = 1 \text{ or } 100\% \qquad \text{..(3.25)}$$

The efficiency is 100% only in the ideal situation. But in actual practice there is always some friction in the bearings of the pulleys and the weight of the string alongwith the weight of the lower block with pulleys is not negligible, so the mechanical advantage decreases and hence the efficiency decreases.

> ***Note :*** The pulley system is only a *force multiplier* and just like other machines, there is no gain in energy. This can be seen as follows:
>
> The work done by the effort (or the input energy)
>
> $= \text{effort} \times \text{distance moved by the effort}$
>
> $= E \times nd = nEd$
>
> The work done on the load (or the output energy)
>
> $= \text{load} \times \text{distance moved by the load}$
>
> $= L \times d$
>
> $= nE \times d = nEd \qquad (\text{since M.A.} = \frac{L}{E} = n)$
>
> Thus, there is no gain in energy.

Effect of weight of pulleys on M.A., V.R. and η

Consider a system of n pulleys. Let w be the total weight of the lower block alongwith the pulleys in it. In the balanced position,

$$E = T \text{ and } L + w = nT$$

or $$L = nT - w = nE - w$$

$$\text{M.A.} = \frac{L}{E} = \frac{nE - w}{E} = n - \frac{w}{E} \quad .....(3.26)$$

Thus the mechanical advantage is less than the ideal value n.

The velocity ratio does not change, it remains n, *i.e.*,

$$\text{V. R.} = n \quad .....(3.27)$$

Hence, efficiency $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{n - \frac{w}{E}}{n}$

or $$\eta = 1 - \frac{w}{nE} \quad .....(3.28)$$

Thus, the efficiency is reduced due to the weight of the lower block of pulleys. More the weight of the lower block, less is the efficiency.

For greater efficiency, the pulleys in the lower block should be as light as possible and the friction in bearings of pulleys should be minimised by the use of lubricants.

EXAMPLES

1. The adjacent Fig. 3.27 shows a fixed pulley used by a boy to lift a load of 400 N through a vertical height of 5 m in 10 s. The effort applied by the boy on the other end of the rope is 480 N.

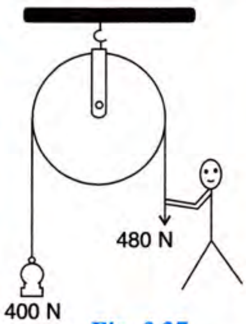


Fig. 3.27

(a) What is the velocity ratio of the pulley ?

(b) What is the mechanical advantage ?

(c) Calculate the efficiency of the pulley.

(d) Why is the efficiency of the pulley not 100% ?

(e) What is the energy gained by the load in 10 s ?

(f) How much power was developed by the boy in raising the load ?

(g) The boy has to apply an effort which is greater than the load he is lifting. What is the justification for using the pulley ?

(a) When the effort moves a distance d downwards, the load moves a distance d upwards.

$$\text{Velocity ratio} = \frac{\text{displacement of effort}}{\text{displacement of load}} = \frac{d}{d} = \mathbf{1}$$

(b) Mechanical advantage $= \frac{\text{load}}{\text{effort}}$

or $\text{M.A.} = \frac{400}{480} = \frac{5}{6} = \mathbf{0{\cdot}833}$

(c) Efficiency $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{0{\cdot}833}{1} = \mathbf{0{\cdot}833 \text{ or } 83{\cdot}3\%}$

(d) The efficiency is less than 100% because **some energy is wasted in overcoming the friction of the pulley bearings.**

(e) Energy gained by the load

$= \text{load} \times \text{displacement of load}$

$= 400 \times 5 = \mathbf{2000\ J}$

(f) Power developed by the boy

$= \frac{\text{effort} \times \text{displacement}}{\text{time}}$

$= \frac{480 \times 5}{10} = \mathbf{240\ W}$

(g) Use of pulley helps **in changing the direction of the applied force to a convenient direction** (instead of upward direction to the downward direction). So one may also use his own weight as effort.

2. The diagram in Fig. 3.28 shows the combination of two pulleys P_1 and P_2 used to lift up a load *W*.

(a) State the kind of pulleys P_1 and P_2.

(b) State the function of the pulley P_2.

(c) If the free end C of the string moves through a distance *x*, by what distance is the load *W* raised ?

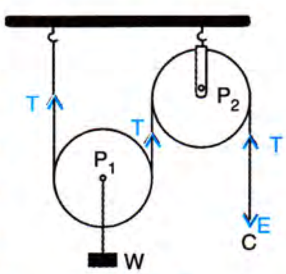


Fig. 3.28

(d) What effort E has to be applied at C to just raise the load W = 20 kgf ? Neglect both the weight of the pulley P_1 and the friction.

(a) P_1 is the movable pulley while P_2 is the fixed pulley.

(b) The fixed pulley P_2 is used **to change the direction of effort to be applied from upward direction to downward direction.**

(c) If the free end C of the string moves through a distance x, the load W will rise by a distance $x/2$.

(d) Given W = 20 kgf, E = ?

In equilibrium, $W = 2T$ and $E = T$

$\therefore$ Effort needed $E = \frac{W}{2} = \frac{20 \text{ kgf}}{2}$ = **10 kgf.**

3. A pulley system with a velocity ratio of 4 is used to lift a load of 150 kgf through a vertical height of 20 m. The effort required is 50 kgf in the downward direction. Calculate :

(a) the distance moved by the effort,

(b) the work done by the effort,

(c) the mechanical advantage,

(d) the efficiency of the pulley system, and

(e) the total number of pulleys and the number of pulleys in each block.

(g = 10 N kg^{-1})

Given : V.R. = 4, L = 150 kgf, d_L = 20 m,

E = 50 kgf = 50 × 10 N = 500 N, d_E = ?

(a) $\text{V.R.} = \frac{\text{distance moved by effort } d_E}{\text{distance moved by load } d_L}$ or $4 = \frac{d_E}{20}$

$\therefore$ Distance moved by the effort $d_E = 20 \times 4$ = **80 m**

(b) Work done by the effort = effort E × distance d_E

= 500 N × 80 m

= **40000 J**

(c) Mechanical advantage $= \frac{L}{E} = \frac{150}{50}$ = **3**

(d) Efficiency $= \frac{\text{M.A.}}{\text{V.R.}} \times 100\% = \frac{3}{4} \times 100\%$ = **75%**

(e) The total number of pulleys = **4**

The number of pulleys in each block = **2**

(Note that tackle will be tied to the hook of the upper block so as to apply the effort downward as shown in Fig. 3.26).

4. A block and tackle has two pulleys in each block, with the tackle tied to the hook of the lower block and the effort being applied upwards.

(a) Draw a neat diagram to show this arrangement and calculate its mechanical advantage.

(b) If the load moves up a distance x, by what distance will the free end of the string move up ?

(a) The arrangement is shown in Fig. 3.29. Here the load is being supported by *five* segments of the string. Therefore,

$L = 5T$ and $E = T$

$\therefore \text{M.A.} = \frac{L}{E} = \frac{5T}{T}$ = **5**

(b) If the load moves up a distance x, the free end of the string will move up by a distance **$5x$**.

Note : Here effort is applied upwards, so its mechanical advantage and velocity ratio is more than the total number of pulleys used. It is equal to the number of strands supporting the load.

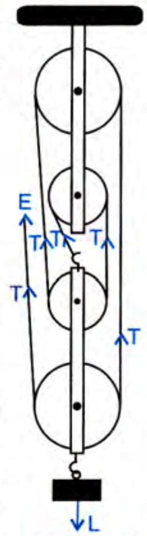


Fig. 3.29

EXERCISE–3(B)

1. What is a fixed pulley ? State its *one* use.

2. What is the ideal mechanical advantage of a single fixed pulley ? Can it be used as a force multiplier ? **Ans.** 1, No

3. Name the pulley which has no gain in mechanical advantage. Explain, why is such a pulley then used ?

4. What is the velocity ratio of a single fixed pulley ? **Ans.** 1

5. In a single fixed pulley, if the effort moves by a distance x downwards, by what height is the load raised upwards ? **Ans.** x

6. What is a single movable pulley ? What is its mechanical advantage in the ideal case ?

7. Name the type of a single pulley that has an ideal mechanical advantage equal to 2. Draw a labelled diagram of the pulley mentioned by you.

8. Give *two* reasons why the efficiency of a single movable pulley is not 100%.

9. In which direction does the force need to be applied,

when a single pulley is used with a mechanical advantage greater than 1 ? How can you change the direction of force applied without altering its mechanical advantage ? Draw a labelled diagram of the system.

10. What is the velocity ratio of a single movable pulley ? How does the friction in the pulley bearing affect it ? **Ans.** 2, no effect

11. In a single movable pulley, if the effort moves by a distance x upwards, by what height is the load raised ? **Ans.** $x/2$

12. Draw a labelled diagram of an arrangement of two pulleys, one fixed and other movable. In the diagram, mark the directions of all forces acting on it. What is the ideal mechanical advantage of the system ? How can it be achieved ?

13. The diagram alongside shows a pulley arrangement.

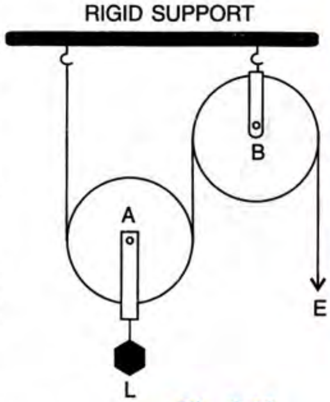


Fig. 3.30

(a) Name the pulleys A and B.

(b) In the diagram, mark the direction of tension on each strand of string.

(c) What is the purpose of the pulley B ?

(d) If the tension is T, deduce the relation between (i) T and E, and (ii) E and L.

(e) What is the velocity ratio of the arrangement ?

(f) Assuming that the efficiency of the system is 100%, what is the mechanical advantage ?

Ans. (a) A – movable, B – fixed (d) (i) $T = E$, (ii) $E = L/2$ (e) 2, (f) 2

14. State *four* differences between a single fixed pulley and a single movable pulley.

15. The diagram alongside shows an arrangement of three pulleys A, B and C. The load is marked as L and the effort as E.

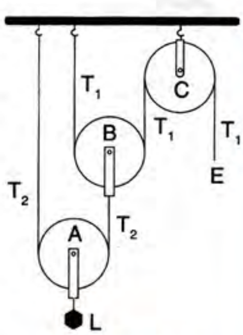


Fig. 3.31

(a) Name the pulleys A, B and C.

(b) Mark in the diagram the directions of load (L), effort (E) and tension T_1 and T_2 in the two strings.

(c) How are the magnitudes of L and E related to the tension T_1 ?

(d) Calculate the mechanical advantage and velocity ratio of the arrangement.

(e) What assumptions have you made in parts (c) and (d) ?

Ans. (a) A and B – movable pulleys, C – fixed pulley. (c) $L = 4T_1$, $E = T_1$, (d) M.A. = 4, V.R. = 4 (e) The pulleys A and B are weightless and there is no friction

16. Draw a diagram of combination of three movable pulleys and one fixed pulley to lift up a load. In the diagram, show the directions of load, effort and tension in each strand. Find : (i) the mechanical advantage, (ii) the velocity ratio, and (iii) the efficiency of the combination, in the ideal situation.

Ans. (i) 2^3 (ii) 2^3 (iii) 1

17. Draw a diagram of a block and tackle system of pulleys having a velocity ratio of 5. In your diagram indicate clearly the points of application and the directions of the load and effort. Also mark the tension in each strand.

18. Give reasons for the following :

(a) In a single fixed pulley, the velocity ratio is always more than the mechanical advantage.

(b) The efficiency of a movable pulley is always less than 100%.

(c) In case of a block and tackle system, the mechanical advantage increases with the increase in the number of pulleys.

(d) The lower block of a block and tackle pulley system must be of negligible weight.

19. Name a machine which is used to :

(a) multiply force,

(b) multiply speed, and

(c) change the direction of force applied.

Ans. (a) a movable pulley (b) class III lever (c) single fixed pulley

20. State whether the following statements are *true* or *false* by writing T or F.

(a) The velocity ratio of a single fixed pulley is always more than 1.

(b) The velocity ratio of a single movable pulley is always 2.

(c) The velocity ratio of a combination of n movable pulleys with a fixed pulley is always 2^n.

(d) The velocity ratio of a block and tackle system is always equal to the number of strands of the tackle supporting the load.

Ans. (a) F (b) T (c) T (d) T

MULTIPLE CHOICE TYPE

1. A single fixed pulley is used because it :
   (a) has the mechanical advantage greater than 1
   (b) has the velocity ratio less than 1
   (c) gives 100% efficiency
   (d) helps to apply the effort in a convenient direction.

   Ans. (d) helps to apply the effort in a convenient direction.

2. The mechanical advantage of an ideal single movable pulley is :
   (a) 1 (b) 2
   (c) less than 2 (d) less than 1. **Ans.** (b) 2

3. A movable pulley is used as :
   (a) a force multiplier
   (b) a speed multiplier
   (c) a device to change the direction of effort
   (d) an energy multiplier

   Ans. (a) a force multiplier

NUMERICALS

1. A woman draws water from a well using a fixed pulley. The mass of bucket and water together is 6 kg. The force applied by the woman is 70 N. Calculate the mechanical advantage. (Take $g = 10 \text{ m s}^{-2}$) **Ans.** 0·857

2. A fixed pulley is driven by a 100 kg mass falling at a rate of 8·0 m in 4·0 s. It lifts a load of 75·0 kgf. Calculate :
   (a) the power input to the pulley taking the force of gravity on 1 kg as 10 N.
   (b) the efficiency of the pulley, and
   (c) the height to which the load is raised in 4·0 s.

   Ans. (a) 2000 W, (b) 0·75, (c) 8·0 m

3. In a block and tackle system consisting of 3 pulleys, a load of 75 kgf is raised with an effort of 25 kgf. Find : (i) the mechanical advantage, (ii) the velocity ratio, and (iii) the efficiency.

   Ans. (i) 3, (ii) 3, (iii) 100%

4. A block and tackle system has 5 pulleys. If an effort of 1000 N is needed in the downward direction to raise a load of 4500 N, calculate :
   (a) the mechanical advantage,
   (b) the velocity ratio, and
   (c) the efficiency of the system.

   Ans. (a) 4·5 (b) 5 (c) 90%

5. In Fig. 3.32, draw a tackle to lift the load by applying the force in the downward direction.
   (a) Mark in the diagram the direction of load and effort.
   (b) If the load is raised by 1 m, through what distance will the effort move ?
   (c) State the number of strands of tackle supporting the load.
   (d) What is the mechanical advantage of the system ?

   Ans. (b) 5 m (c) 5 (d) 5

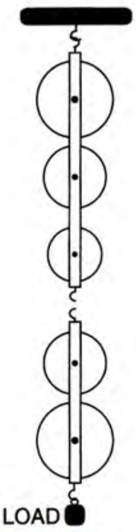


Fig. 3.32

6. A pulley system has a velocity ratio 3 and an efficiency of 80%. Draw a labelled diagram of this pulley system. Calculate :
   (a) the mechanical advantage of the system, and
   (b) the effort required to raise a load of 300 N.

   Ans. (a) 2·4, (b) 125 N

7. Fig. 3.33 shows a system of four pulleys. The upper two pulleys are fixed and the lower two are movable.
   (a) Draw a string around the pulleys. Also show the point of application and direction in which the effort is applied.
   (b) What is the velocity ratio of the system ?
   (c) How are load and effort of the pulley system related ?
   (d) What assumption do you make in arriving at your answer in part (c)?

   Ans. (b) 4 (c) load = 4 × effort

   Assumption: (1) There is no friction in the pulley bearings, (2) Weight of lower block of pulleys is negligible, (3) The effort is applied downwards.

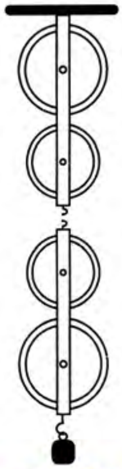


Fig. 3.33

8. Fig. 3.34 shows a block and tackle system of pulleys used to lift a load.
   (a) How many strands of tackle are supporting the load ?
   (b) Draw arrows to represent tension in each strand.
   (c) What is the mechanical advantage of the system ?

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(d) When load is pulled up by a distance 1 m, how far does the effort end move ?

(e) How much effort is needed to lift a load of 100 N ?

Ans. (a) 4 (c) 4 (d) 4 m (e) 25 N

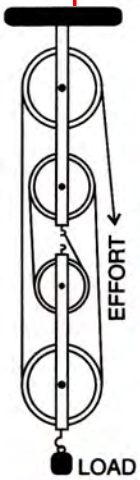

Fig. 3.34

9. A block and tackle system has the velocity ratio 3. Draw a labelled diagram of the system indicating the points of application and the directions of load and effort. A man can exert a pull of 200 kgf. (a) What is the maximum load he can raise with this pulley system if its efficiency is 60% ? (b) If the effort end moves a distance 60 cm, what distance does the load move ? **Ans.** (a) 360 kgf (b) 20 cm.

10. You are given four pulleys and three strings. Draw a neat and labelled diagram to use them so as to obtain a maximum mechanical advantage equal to 8. In your diagram mark the directions of load, effort and tension in each strand.

What assumptions have you made to obtain the required mechanical advantage ?

[**Hint :** Three movable pulleys with one fixed pulley]