28 Distance Formula

28.1 INTRODUCTION

For any two known (given) points in a co-ordinate (Cartesian) plane, the knowledge of *co-ordinate geometry* may be used to find :

(i) the distance between the given points,

(ii) the co-ordinates of a point which divides the line joining the given points in a given ratio,

(iii) the co-ordinates of the mid-point of the line segment joining the two given points,

(iv) equation of the straight line through the given points,

(v) equation of the perpendicular bisector of a line segment, etc.

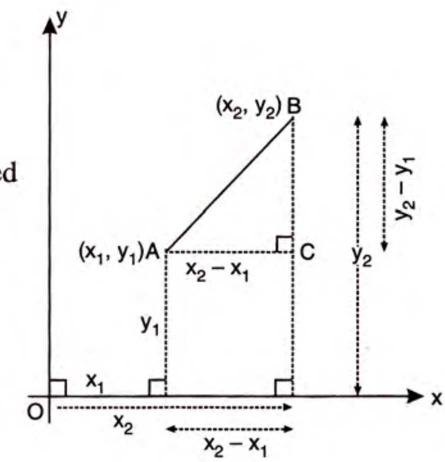
28.2 THE DISTANCE FORMULA

To find the distance between two given points :

Let the two given points be

A (x_1, y_1) and B (x_2, y_2).

It is clear from the adjoining figure that in right-angled triangle ABC,



AC $= x_2 - x_1$ = difference between abscissae of A and B

BC $= y_2 - y_1$ = difference between ordinates of A and B.

Using Pythagoras' Theorem, we get :

$$AB^2 = AC^2 + BC^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\therefore$ The distance between two given points (x_1, y_1) and (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

As, $(a - b)^2 = (b - a)^2$

$$= \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Note :

In co-ordinate geometry, all the formulae are obtained by taking the points in the first quadrant (as in the case of the distance formula, obtained above), but every formula will always remain true in whatever quadrant the points may lie. Only the proper signs must be used with the co-ordinates.

1 Find the distance between the points (3, 6) and (0, 2).

Solution :

Let (3, 6) = (x_1, y_1) and (0, 2) = (x_2, y_2)

$\therefore$ **Distance between the given points** $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-3)^2 + (2-6)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = \mathbf{5}$ **Ans.**

The distance of any point (x, y) from the origin (0, 0)

$$= \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

2 Find the distance between the origin and the point :
(i) (–12, 5) (ii) (15, –8).

Solution :

(i) Since, distance between origin and $(x, y) = \sqrt{x^2 + y^2}$

$\therefore$ Distance between origin and the point (–12, 5)

$= \sqrt{(-12)^2 + (5)^2} = \sqrt{144 + 25} = \mathbf{13}$ **Ans.**

(ii) Distance between origin and the point (15, –8)

$= \sqrt{(15)^2 + (-8)^2} = \sqrt{225 + 64} = \mathbf{17}$ **Ans.**

Remember

If a point lies on the x-axis; its ordinate is zero, therefore a point on the x-axis is taken as $(x, 0)$, such as (3, 0), (–5, 0), etc.
In the same way, if a point lies on the y-axis, its abscissa is zero, therefore a point on the y-axis is taken as $(0, y)$, such as (0, 4), (0, –7), etc.

3 Find the co-ordinates of points on the x-axis which are at a distance of 5 units from the point (6, –3).

Solution :

Let the co-ordinates of the point on the x-axis be $(x, 0)$

Since, distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore \quad 5 = \sqrt{(x-6)^2 + (0+3)^2}$ Taking (6, –3) = (x_1, y_1) and $(x, 0) = (x_2, y_2)$

$\Rightarrow \quad 25 = x^2 - 12x + 36 + 9$ On squaring

$\Rightarrow \quad x^2 - 12x + 20 = 0$

$\Rightarrow \quad x = 2$, or $x = 10$ On solving the quadratic equation

$\therefore$ **Required points on the x-axis are (2, 0) and (10, 0)** **Ans.**

4 KM is a straight line of 13 units. If K has the co-ordinates (2, 5) and M has the co-ordinates $(x, -7)$, find the values of x.

Solution :

Let $K(2, 5) = (x_1, y_1)$ and $M(x, -7) = (x_2, y_2)$

$$KM = 13 \text{ units} \Rightarrow \sqrt{(x-2)^2 + (-7-5)^2} = 13$$

$$\Rightarrow x^2 - 4x + 4 + 144 = 169$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow (x-7)(x+3) = 0 \Rightarrow x = 7 \text{ or } x = -3$$ **Ans.**

5 Which point on the y-axis is equidistant from the points (12, 3) and (–5, 10).

Solution :

Let the required point on the y-axis be $(0, y)$.

Given $(0, y)$ is equidistant from (12, 3) and (–5, 10)

i.e. distance between (0, y) and (12, 3)

= distance between $(0, y)$ and (–5, 10)

$$\Rightarrow \sqrt{(12-0)^2 + (3-y)^2} = \sqrt{(-5-0)^2 + (10-y)^2}$$

$$\Rightarrow 144 + 9 + y^2 - 6y = 25 + 100 + y^2 - 20y$$

$$\Rightarrow 14y = -28 \text{ and } y = -2$$

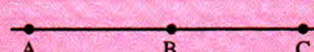
∴ **Required point on the y-axis = (0, –2)** **Ans.**

6 Use the distance formula to show that the points A(1, –1), B(6, 4) and C(4, 2) are collinear.

Solution :

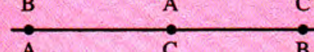
Three points A, B and C are collinear

if and only if : (i) AB + BC = AC *i.e.*



or, (ii) AB + AC = BC *i.e.*

or, (iii) AC + BC = AB *i.e.*



Since, $AB = \sqrt{(6-1)^2 + (4+1)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

$BC = \sqrt{(4-6)^2 + (2-4)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

and, $AC = \sqrt{(4-1)^2 + (2+1)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$\Rightarrow$ **BC + AC** $= 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} =$ **AB**

∴ **Given points A, B and C are collinear.** **Ans.**

7 Show that the points A (8, 3), B (0, 9) and C (14, 11) are the vertices of an isosceles right-angled triangle.

Solution :

$$AB = \sqrt{(0-8)^2 + (9-3)^2} = \sqrt{64+36} = \sqrt{100} = \mathbf{10}$$

$$BC = \sqrt{(14-0)^2 + (11-9)^2} = \sqrt{196+4} = \sqrt{200} = \mathbf{10\sqrt{2}}$$

$$CA = \sqrt{(8-14)^2 + (3-11)^2} = \sqrt{36+64} = \sqrt{100} = \mathbf{10}$$

$AB^2 + CA^2 = 100 + 100 = 200 = BC^2$

$BC^2 = AB^2 + CA^2 \Rightarrow$ the triangle is right-angled triangle.

and, $AB = CA \Rightarrow$ the triangle is isosceles.

Hence, the triangle ABC is an isosceles right-angled triangle.

Remember

A given quadrilateral will be a :

(i) *parallelogram*; if both the pairs of opposite sides are equal.

(ii) *rectangle*; if both the pairs of opposite sides are equal and diagonals are also equal.

(iii) *rhombus*; if all the sides are equal.

(iv) *square*; if all the sides are equal and diagonals are also equal.

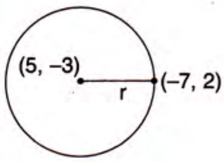
8 Find the area of a circle, whose centre is (5, –3) and which passes through the point (–7, 2). (Take π = 3·14)

Solution :

The radius (r) of the circle = distance between the points (5, –3) and (–7, 2)

$$= \sqrt{(-7-5)^2 + (2+3)^2}$$

$$= \sqrt{144+25} = \sqrt{169} = 13$$



∴ **Area of the circle** $= \pi r^2$

$= 3.14 \times 13^2 =$ **530·66 sq. units** **Ans.**

9 Find the points on the x-axis whose distances from the points A(7, 6) and B(–3, 4) are in the ratio 1 : 2.

Solution :

Let the required point P on x-axis = $(x, 0)$

| P = $(x, 0)$
| A = (7, 6) and
| B = (–3, 4)

Given : $\frac{PA}{PB} = \frac{1}{2} \Rightarrow 2\,PA = PB$

i.e. $2\sqrt{(x-7)^2 + (0-6)^2} = \sqrt{(x+3)^2 + (0-4)^2}$

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$$\Rightarrow \quad 4(x^2 - 14x + 49 + 36) = x^2 + 6x + 9 + 16$$

$$\Rightarrow \quad 4x^2 - 56x + 196 + 144 = x^2 + 6x + 25$$

$$\Rightarrow \quad 3x^2 - 62x + 315 = 0$$

$$\Rightarrow \quad 3x^2 - 27x - 35x + 315 = 0$$

$$\Rightarrow \quad (x - 9)(3x - 35) = 0$$

$$\Rightarrow \quad x = 9 \text{ or } x = \frac{35}{3}$$

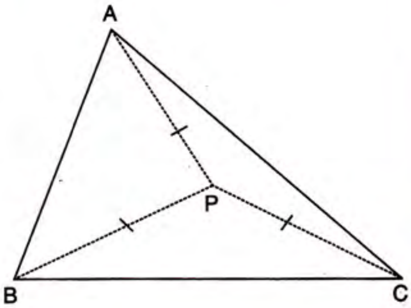
∴ **Required points on x-axis are : (9, 0) and $\left(\frac{35}{3}, 0\right)$** **Ans.**

28.3 CIRCUMCENTRE OF A TRIANGLE

It is the point which is equidistant from the vertices of the triangle *i.e.* if P is the circumcentre of the triangle ABC, then :

PA = PB = PC = Circumradius.

(If a circle is drawn with P as centre and PA or PB or PC as radius, the circle will pass through all the three vertices of the triangle).



10 Find the co-ordinates of the circumcentre of the triangle ABC; whose vertices A, B and C are (4, 6), (0, 4) and (6, 2) respectively.

Solution :

Let the circumcentre be P (x, y).

then, PA = PB

$$\Rightarrow \quad \sqrt{(x-4)^2 + (y-6)^2} = \sqrt{(x-0)^2 + (y-4)^2}$$

$$\Rightarrow \quad x^2 - 8x + 16 + y^2 - 12y + 36 = x^2 + y^2 - 8y + 16$$

$$\Rightarrow \quad -8x - 4y = -36$$

$$\Rightarrow \quad 2x + y = 9 \qquad .....\text{I}$$

and PA = PC

$$\Rightarrow \quad \sqrt{(x-4)^2 + (y-6)^2} = \sqrt{(x-6)^2 + (y-2)^2}$$

$$\Rightarrow \quad x^2 - 8x + 16 + y^2 - 12y + 36 = x^2 - 12x + 36 + y^2 - 4y + 4$$

$$\Rightarrow \quad 4x - 8y = -12$$

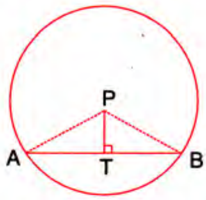
$$\Rightarrow \quad x - 2y = -3 \qquad .....\text{II}$$

On solving I and II, we get : $x = 3$ and $y = 3$

∴ **The circumcentre of the given triangle = (3, 3)** **Ans.**

EXERCISE 28

1. Find the distance between the following pairs of points :
   (i) (–3, 6) and (2, –6)
   (ii) (–a, –b) and (a, b)
   (iii) $\left(\frac{3}{5}, 2\right)$ and $\left(-\frac{1}{5}, 1\frac{2}{5}\right)$
   (iv) $\left(\sqrt{3}+1, 1\right)$ and $\left(0, \sqrt{3}\right)$
2. Find the distance between the origin and the point :
   (i) (–8, 6) (ii) (–5, –12)
   (iii) (8, –15)
3. The distance between the points (3, 1) and (0, x) is 5. Find x.
4. Find the co-ordinates of points on the x-axis which are at a distance of 17 units from the point (11, –8)
5. Find the co-ordinates of the points on the y-axis, which are at a distance of 10 units from the point (–8, 4)
6. A point A is at a distance of $\sqrt{10}$ unit from the point (4, 3). Find the co-ordinates of point A, if its ordinate is twice its abscissa.
7. A point P (2, –1) is equidistant from the points (a, 7) and (–3, a). Find a.
8. What point on the x-axis is equidistant from the points (7, 6) and (–3, 4) ?
9. Find a point on the y-axis which is equidistant from the points (5, 2) and (–4, 3).
10. A point P lies on the x-axis and another point Q lies on the y-axis.
    (i) Write the ordinate of point P.
    (ii) Write the abscissa of point Q.
    (iii) If the abscissa of point P is –12 and the ordinate of point Q is –16; calculate the length of line segment PQ.
11. Show that the points P (0, 5), Q (5, 10) and R (6, 3) are the vertices of an isosceles triangle.
12. Prove that the points P (0, –4), Q (6, 2), R (3, 5) and S (–3, –1) are the vertices of a rectangle PQRS.
13. Prove that the points A (1, –3), B (–3, 0) and C (4, 1) are the vertices of an isosceles right-angled triangle. Find the area of the triangle.
14. Show that the points A (5, 6), B (1, 5), C (2, 1) and D (6, 2) are the vertices of a square ABCD.
15. Show that (–3, 2), (–5, –5), (2, –3) and (4, 4) are the vertices of a rhombus.
16. Points A (–3, –2), B (–6, a), C (–3, –4) and D(0, –1) are the vertices of quadrilateral ABCD; find a if 'a' is negative and AB = CD.
17. The vertices of a triangle are (5, 1), (11, 1) and (11, 9). Find the co-ordinates of the circumcentre of the triangle.
18. Given A = (3, 1) and B = (0, y – 1). Find y if AB = 5.
19. Given A = (x + 2, –2) and B = (11, 6). Find x if AB = 17.
20. The centre of a circle is (2x – 1, 3x + 1). Find x if the circle passes through (–3, –1) and the length of its diameter is 20 unit.
21. The length of line PQ is 10 units and the co-ordinates of P are (2, –3); calculate the co-ordinates of point Q, if its abscissa is 10.
22. Point P (2, –7) is the centre of a circle with radius 13 unit, PT is perpendicular to chord AB and T = (–2, –4); Calculate the length of :
    (i) AT (ii) AB.



23. Calculate the distance between the points P(2, 2) and Q(5, 4) correct to three significant figures.
24. Calculate the distance between A(7, 3) and B on the x-axis whose abscissa is 11.
25. Calculate the distance between A(5, –3) and B on the y-axis whose ordinate is 9.
26. Find the point on y-axis whose distances from the points A(6, 7) and B(4, –3) are in the ratio 1 : 2.
27. The distances of point P(x, y) from the points A(1, –3) and B(–2, 2) are in the ratio 2 : 3. Show that :
    $$5x^2 + 5y^2 - 34x + 70y + 58 = 0.$$
28. The points A(3, 0), B(a, –2) and C(4, –1) are the vertices of triangle ABC right angled at vertex A. Find the value of a.