27 Graphical Solution

[Solution of Simultaneous Linear Equations, Graphically]

27.1 GRAPHS OF LINEAR EQUATIONS IN TWO VARIABLES

An equation of the form $ax + by + c = 0$ is called a linear equation in two variables with x and y as the variables and a, b, c as the constants.

Steps to draw the graph of a linear equation $ax + by + c = 0$:

1. Make x or y, the subject of the equation.
2. Give *at least* three suitable values to the variable on the right-hand side (the subject of equation being on the left-hand side) and find the corresponding values of the variable on the left-hand side.
3. Construct a table for the different pairs of values of x and y.
4. Plot *at least* three ordered pairs (points) from the table on a graph paper.
5. Draw a straight line passing through the points plotted on the graph.

1 Draw the graph of $3x + y - 4 = 0$.

Solution :

$3x + y - 4 = 0$

$\Rightarrow y = -3x + 4$ [Making y, the subject]

Now give *at least* three different values to the variable x and find the corresponding values of y.

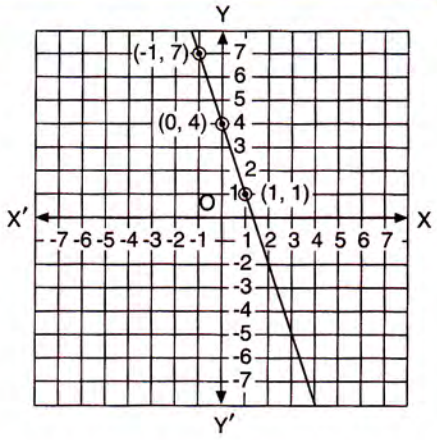
Let $x = 1$; then $y = -3 \times 1 + 4 = 1$

Let $x = -1$; then $y = -3 \times -1 + 4 = 7$

Let $x = 0$; then $y = -3 \times 0 + 4 = 4$

$\therefore$ Table for x and y is :

x	1	–1	0
y	1	7	4



Plot the points (1, 1), (–1, 7) and (0, 4) on a graph paper.
Draw a straight line passing through the points plotted on the graph paper.

2 Use the table given alongside to draw the graph.
Use the graph drawn to find the values of 'a' and 'b'.
State the linear relation between the variables x and y.

x	–2	0	2	1	b
y	–3	1	a	3	–7

Solution :

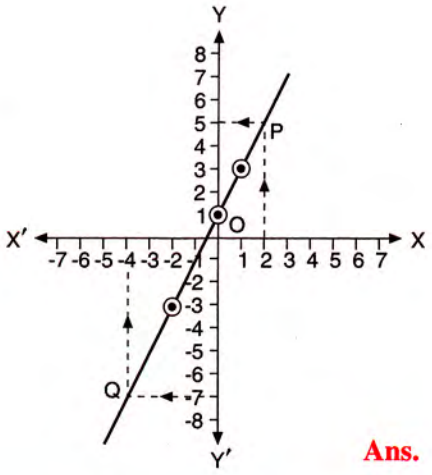
Plot the given points (–2, –3), (0, 1) and (1, 3) on a graph paper.
Draw a straight line passing through these points.

To find the value of 'a' :

Through $x = 2$, draw a vertical line which meets the graph at a point, say P. Through P, draw a horizontal line which meets the y-axis at $y = 5$.

$\therefore$ The value of unknown **'a'** in the table is **5**.

Ans.



Similarly, through $y = -7$, draw a horizontal line which meets the graph at a point, say Q.

Through Q, draw a vertical line which meets the x-axis at -4.

$\therefore$ $b = -4$ **Ans.**

The linear relation between x and y is $y = 2x + 1$. **Ans.**

Reason : Let the linear relation between the variables x and y be $y = mx + c$,

Since, the graph passes through $(-2, -3)$; substitute $x = -2$ and $y = -3$ in $y = mx + c$.

This gives : $-3 = -2m + c$ I

Again, the graph passes through $(0, 1)$; substitute $x = 0$ and $y = 1$ in $y = mx + c$.

This gives : $1 = m \times 0 + c$ *i.e.* $c = 1$

Now, $-3 = -2m + c \Rightarrow -3 = -2m + 1$ *i.e.* $m = 2$

$\therefore$ **Required relation is :** $y = mx + c$ *i.e.* $y = 2x + 1$

3 On a graph paper, draw a straight line represented by the equation $2x - 3y + 12 = 0$. Use the graph drawn to find the values of m and n so that the points $(m, -2)$ and $(3, n)$ lie on the given straight line.

Solution :

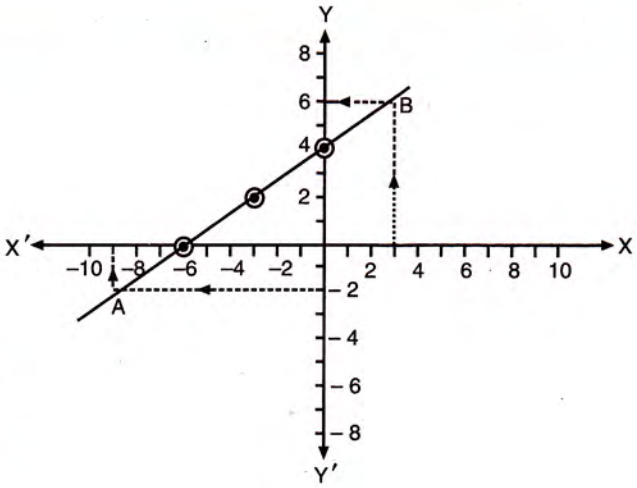
$$2x - 3y + 12 = 0$$

$$\Rightarrow \quad 2x = 3y - 12$$

$$\Rightarrow \quad x = \frac{3y - 12}{2}$$

$\Rightarrow$

x	-6	-3	0
y	0	2	4



Plot the points $(-6, 0)$, $(-3, 2)$ and $(0, 4)$ on a graph paper; then draw a straight line through these points which will represent the given equation $2x - 3y + 12 = 0$.

Since, point $(m, -2)$ lies on the straight line drawn, through $y = -2$ draw a horizontal line which meets the straight line at point A. Through point A, draw a vertical line which meets x-axis at point -9.

$\therefore$ $m = -9$ **Ans.**

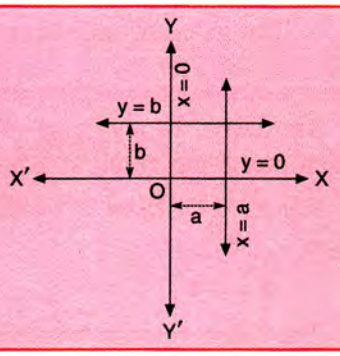
Also, as $(3, n)$ lies on the straight line drawn, through $x = 3$ draw a vertical line which meets the straight line at point B. Through point B, draw a horizontal line which meets y-axis at the point 6.

$\therefore$ $n = 6$ **Ans.**

Remember :

1. The equation of the x-axis is $y = 0$; and equation of the y-axis is $x = 0$.
2. The graph of $x = a$ (where a is constant) is a straight line parallel to the y-axis and at a distance of 'a' units from the y-axis.

   Similarly, the graph of $y = b$ (a constant) is a straight line parallel to the x-axis and at a distance of 'b' units from it.



EXERCISE 27(A)

1. Draw the graph for each equation, given below :

   (i) $x = 5$ (ii) $x + 5 = 0$

   (iii) $y = 7$ (iv) $y + 7 = 0$

   (v) $2x + 3y = 0$ (vi) $3x + 2y = 6$

   (vii) $x - 5y + 4 = 0$ (viii) $5x + y + 5 = 0$

2. Draw the graph for each equation given below; hence find the co-ordinates of the points where the graph drawn meets the co-ordinate axes :

   (i) $\frac{1}{3}x + \frac{1}{5}y = 1$ (ii) $\frac{2x + 15}{3} = y - 1$

3. Draw the graph of the straight line given by the equation $4x - 3y + 36 = 0$

   Calculate the area of the triangle formed by the line drawn and the co-ordinate axes.

4. Draw the graph of the equation

   $$2x - 3y - 5 = 0$$

   From the graph, find :

   (i) x_1, the value of x, when $y = 7$

   (ii) x_2, the value of x, when $y = -5$

5. Draw the graph of the equation

   $$4x + 3y + 6 = 0$$

   From the graph, find :

   (i) y_1, the value of y, when $x = 12$

   (ii) y_2, the value of y, when $x = -6$

6. Use the table given below to draw the graph.

x	-5	-1	3	b	13
y	-2	a	2	5	7

   From your graph, find the values of 'a' and 'b'. State a linear relation between the variables x and y.

7. Draw the graph obtained from the table below :

x	a	3	-5	5	c	-1
y	-1	2	b	3	4	0

   Use the graph to find the values of a, b and c. State a linear relation between the variables x and y.

8. A straight line passes through the points $(2, 4)$ and $(5, -2)$. Taking 1 cm = 1 unit; mark these points on a graph paper and draw the straight line through these points. If points $(m, -4)$ and $(3, n)$ lie on the line drawn; find the values of m and n.

9. Draw the graph (straight line) given by equation $x - 3y = 18$. If the straight line drawn passes through the points $(m, -5)$ and $(6, n)$; find the values of m and n.

10. Use the graphical method to find the value of k, if :

    (i) $(k, -3)$ lies on the straight line

    $$2x + 3y = 1$$

    (ii) $(5, k - 2)$ lies on the straight line

    $$x - 2y + 1 = 0$$

27.2 SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS GRAPHICALLY

In order to solve simultaneous linear equations graphically :

1. Draw a graph (straight line) on the same graph paper for each given equation.
2. Find the coordinates of the point of intersection of the two lines drawn.
3. The coordinates of the point of intersection give the solution of the given equations.

4 Solve the given equations graphically :
$3x - 2y = 4$ and $5x - 2y = 0$

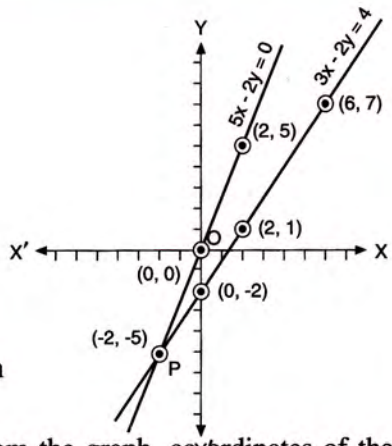
Solution :

Table for $3x - 2y = 4$:

x	0	2	6
y	–2	1	7

Table for $5x - 2y = 0$:

x	–2	0	2
y	–5	0	5



On the same graph paper, draw the graph for each given equation.

Both the straight lines drawn meet at point P. As is clear from the graph, coordinates of the common point P are (–2, –5).

Solution of the given equations is : $x = -2$ and $y = -5$ **Ans.**

5 Use graph paper for this question :

(i) Draw the graphs of $3x - y - 2 = 0$ and $2x + y - 8 = 0$. Take 1 cm = 1 unit on both the axes and plot only three points per line.

(ii) Write down the co-ordinates of the point of intersection and the area of the triangle formed by the lines and the x-axis.

Solution :

(i) $3x - y - 2 = 0$

$\Rightarrow \quad y = 3x - 2$

$\Rightarrow$

x	0	2	4
y	–2	4	10

Plot the points (0, –2), (2, 4) and (4, 10) on a graph paper, taking 1 cm = 1 unit.

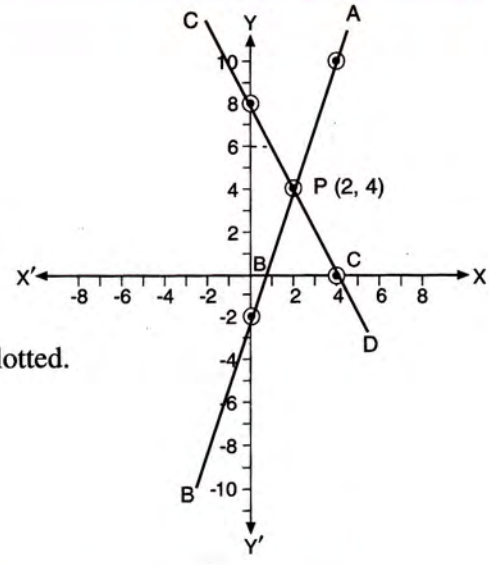
Now, draw a straight line AB through the points plotted.

$2x + y - 8 = 0$

$\Rightarrow \quad y = 8 - 2x$

$\Rightarrow$

x	0	2	4
y	8	4	0



After plotting the points (0, 8), (2, 4) and (4, 0); draw a line CD through these points.

(ii) It is clear from the graph obtained that the point of intersection of the given two lines AB and CD is **P (2, 4)** **Ans.**

Now the area of the triangle formed by the lines and the

x-axis $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times \frac{10}{3} \times 4$ sq. units $\quad [\because$ Base BC $= 4 - \frac{2}{3} = \frac{10}{3}]$

$= 6\frac{2}{3}$ **sq. unit** **Ans.**

6 In a factory, the cost of manufacturing x articles is ₹ $(20 + 2x)$ and the selling price of x articles is ₹ $(2.5x)$. On the same graph paper, with the same axes, draw two graphs, first for the cost of manufacturing against no. of articles and the second for the selling price against no. of articles.
Take 2 cm = 10 articles on one axis and 2 cm = ₹ 20 on the other axis. Provide for x upto 80. Use your graph to determine :
(i) No. of articles to be manufactured and sold to reach breakeven point (no profit and no loss situation).
(ii) The profit made when 60 articles are manufactured and sold.

Solution :

Given : C.P. = ₹ $(20 + 2x)$

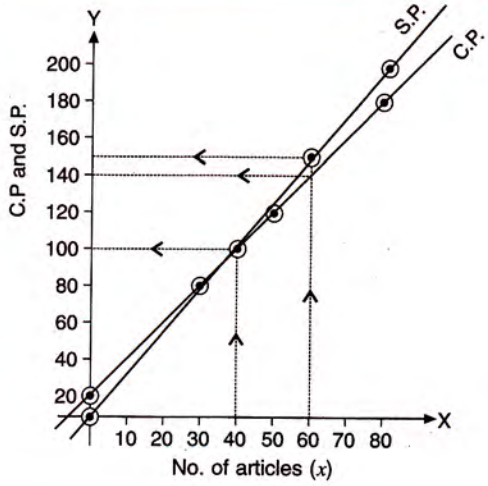
$\Rightarrow$

x	0	30	50	80
C.P.	20	80	120	180

S.P. = ₹ $2.5x$

$\Rightarrow$

x	0	40	60	80
S.P.	0	100	150	200



(i) The figure alongside shows the graphs of C.P. and S.P. Since the two straight lines meet at $x = 40$, it shows that the cost price of 40 articles is the same as their selling price.

∴ No. of articles that must be manufactured and sold to reach breakeven point is 40. **Ans.**

(ii) Draw the vertical line through $x = 60$; which meets graph for C.P. at ₹ 140 and graph for S.P. at ₹ 150.

∴ **Profit made** = ₹ 150 − ₹ 140 = **₹ 10** **Ans.**

EXERCISE 27(B)

1. Solve, graphically, the following pairs of equations :

   (i) $x - 5 = 0$, $y + 4 = 0$

   (ii) $2x + y = 23$, $4x - y = 19$

   (iii) $3x + 7y = 27$, $8 - y = \frac{5}{2}x$

   (iv) $\frac{x+1}{4} = \frac{2}{3}(1 - 2y)$, $\frac{2 + 5y}{3} = \frac{x}{7} - 2$

2. Solve graphically the simultaneous equations given below. Take the scale as 2 cm = 1 unit on both the axes.

   $x - 2y - 4 = 0$
   $2x + y = 3$

3. Use graph paper for this question. Draw the graph of $2x - y - 1 = 0$ and $2x + y = 9$ on the same axes. Use 2 cm = 1 unit on both axes and plot only 3 points per line.

   Write down the co-ordinates of the point of intersection of the two lines.

4. Use graph paper for this question. Take 2 cm = 2 units on x-axis and 2 cm = 1 unit on y-axis.

   Solve graphically the following equations :

   $3x + 5y = 12$; $3x - 5y + 18 = 0$

   (Plot only three points per line)

5. Use graph paper for this question. Take 2 cm = 1 unit on both the axes.

   (i) Draw the graphs of $x + y + 3 = 0$ and $3x - 2y + 4 = 0$. Plot only three points per line.

   (ii) Write down the co-ordinates of the point of intersection of the lines.

   (iii) Measure and record the distance of the point of intersection of the lines from the origin in cm.

6. The sides of a triangle are given by the equations $y - 2 = 0$; $y + 1 = 3(x - 2)$ and $x + 2y = 0$.

   Find, graphically :

   (i) the area of triangle;

   (ii) the co-ordinates of the vertices of the triangle.

7. By drawing a graph for each of the equations $3x + y + 5 = 0$; $3y - x = 5$ and $2x + 5y = 1$ on the same graph paper; show that the lines given by these equations are concurrent (*i.e.* they pass through the same point).

   Take 2 cm = 1 unit on both the axes.

8. Using a scale of 1 cm to 1 unit for both the axes, draw the graphs of the following equations : $6y = 5x + 10$, $y = 5x - 15$.

   From the graph find :

   (i) the co-ordinates of the point where the two lines intersect;

   (ii) the area of the triangle between the lines and the x-axis.

9. The cost of manufacturing x articles is ₹ $(50 + 3x)$. The selling price of x articles is ₹ $4x$.

   On a graph sheet, with the same axes, and taking suitable scales draw two graphs, first for the cost of manufacturing against no. of articles and the second for the selling price against number of articles.

   Use your graph to determine :

   (i) No. of articles to be manufactured and sold to break even (no profit and no loss),

   (ii) The profit or loss made when (a) 30 (b) 60 articles are manufactured and sold.

10. Find graphically, the vertices of the triangle whose sides have the equations $2y - x = 8$; $5y - x = 14$ and $y - 2x = 1$ respectively.
    Take 1 cm = 1 unit on both the axes.

11. Using the same axes of co-ordinates and the same unit, solve graphically :

    $x + y = 0$ and $3x - 2y = 10$.

    (Take at least 3 points for each line drawn).

12. Solve graphically, the following equations.

    $x + 2y = 4$; $3x - 2y = 4$.

    Take 2 cm = 1 unit on each axis.

    Also, find the area of the triangle formed by the lines and the x-axis.

13. Use the graphical method to find the value of 'x' for which the expressions $\frac{3x + 2}{2}$ and $\frac{3}{4}x - 2$ are equal.

14. The course of an enemy submarine, as plotted on rectangular co-ordinate axes, gives the equation $2x + 3y = 4$. On the same axes, a destroyer's course is indicated by the graph $x - y = 7$. Use the graphical method to find the point at which the paths of the submarine and the destroyer intersect ?