24 Solution Of Right Triangles

[Simple 2-D Problems Involving One Right-angled Triangle]

24.1 INTRODUCTION :

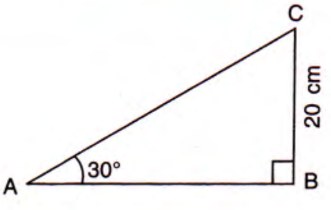
To solve a right-angled triangle means, to find the values of remaining angles and remaining sides; when :

(i) one side and one acute angle are given.

(ii) two sides of the triangle are given.

1 In a triangle ABC, right-angled at B, side BC = 20 cm and angle A = 30°. Find the length of AB.

Solution :



As shown alongside :

$$\frac{AB}{BC} = \cot 30^\circ \qquad [\because \frac{\text{Base}}{\text{Perp.}} = \cot]$$

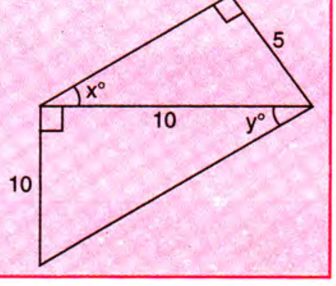
$$\Rightarrow \frac{AB}{20} = \sqrt{3}$$

and, **AB** $= 20 \times \sqrt{3} = 20 \times 1{\cdot}732$ cm = **34.64 cm** **Ans.**

In general, when one side and one acute angle of a right-angled triangle are given; we take

$$\frac{\text{unknown side}}{\text{known side}} = \text{corresponding trigonometrical ratio of the given angle.}$$

2 Without calculating other lengths, use tables to find the angles $x°$ and $y°$.



Solution :

$$\sin x^\circ = \frac{\text{Perp.}}{\text{Hypt.}} = \frac{5}{10} = \frac{1}{2} = \sin 30^\circ$$

$\therefore$ $x^\circ = 30^\circ$ **Ans.**

$$\text{Similarly, } \tan y^\circ = \frac{\text{Perp.}}{\text{Base}} = \frac{10}{10} = 1 = \tan 45^\circ$$

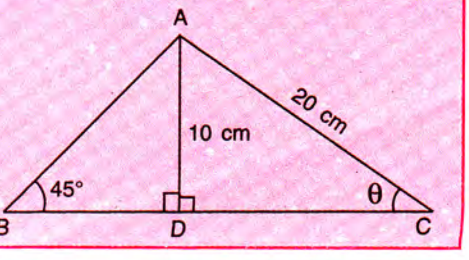
$\Rightarrow$ $y^\circ = 45^\circ$ **Ans.**

3 Use the information, given in the adjoining figure, to find :

(i) length of BD

(ii) angle C *i.e.* θ

(iii) length of BC.



Solution :

(i) In right-angled triangle ABD,

$\tan 45° = \frac{AD}{BD} \Rightarrow 1 = \frac{10}{BD} \Rightarrow$ **BD = 10 cm Ans.**

(ii) In right-triangle ADC,

$\sin \theta = \frac{AD}{AC} = \frac{10}{20} = \frac{1}{2} = \sin 30° \quad \therefore \boldsymbol{\theta = 30°}$ **Ans.**

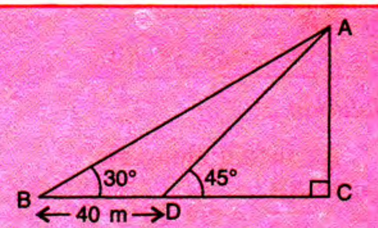
(iii) In right-triangle ADC,

$\tan \theta = \frac{AD}{DC} \Rightarrow \tan 30° = \frac{10}{DC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{DC} \Rightarrow DC = 10 \times 1{\cdot}732 \text{ cm} = 17{\cdot}32 \text{ cm}$

$\therefore$ **BC** = BD + DC = (10 + 17·32) cm = **27.32 cm** **Ans.**

4 From the adjoining figure; find the length of AC.



Solution :

Let DC = x m

$\Rightarrow$ BC = BD + DC = $(40 + x)$ m

In Δ ABC, $\tan 30° = \frac{AC}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AC}{40+x} \Rightarrow 40 + x = AC\sqrt{3}$ --------- I

In Δ ADC, $\tan 45° = \frac{AC}{DC} \Rightarrow 1 = \frac{AC}{x}$ and $x = AC$ ---------II

Substituting x = AC in equation I, we get :

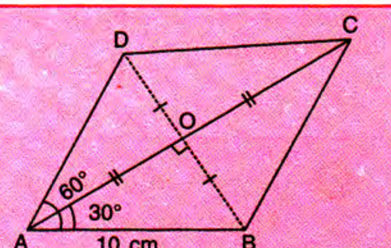
$40 + AC = AC\sqrt{3} \Rightarrow AC\sqrt{3} - AC = 40$

$\Rightarrow AC(\sqrt{3} - 1) = 40$

And, **AC** $= \frac{40}{\sqrt{3}-1} = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{40(1{\cdot}732+1)}{3-1} = 20 \times 2{\cdot}732$ m = **54.64 m** **Ans.**

5 In a rhombus ABCD, length of each side is 10 cm and $\angle A = 60°$. Find the lengths of its diagonals AC and BD.



Solution :

We know that the diagonals of a rhombus bisect each other at right angles and also bisect the angle of the vertex.

$\Rightarrow OA = OC = \frac{1}{2}AC, \; OB = OD = \frac{1}{2}BD; \; \angle AOB = 90°$

and $\angle OAB = \dfrac{60^\circ}{2} = 30^\circ$. Also, given : AB = 10 cm.

In right triangle AOB :

$\sin 30^\circ = \dfrac{OB}{AB} \Rightarrow \dfrac{1}{2} = \dfrac{OB}{10} \Rightarrow OB = 5$ cm

$\cos 30^\circ = \dfrac{OA}{AB} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{OA}{10} \Rightarrow OA = 5\sqrt{3}$ cm $= 5 \times 1{\cdot}732 = 8{\cdot}66$ cm

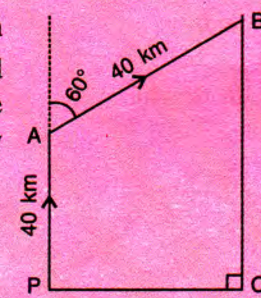
∴ **Length of diagonal AC** = 2 × OA = 2 × 8·66 = **17·32 cm** **Ans.**

And, **length of diagonal BD** = 2 × OB = 2 × 5 cm = **10 cm** **Ans.**

6 In the given figure, a rocket is fired vertically upwards from its launching pad P. It first rises 40 km vertically upwards and then 40 km at 60° to the vertical. PA represents the first stage of the journey and AB the second. C is a point vertically below B on the horizontal level as P, calculate :

(i) the height of the rocket when it is at point B.

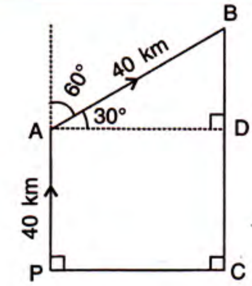
(ii) the horizontal distance of point C from P.



Solution :

Draw AD ⊥ BC.

Clearly $\angle BAD = 90^\circ - 60^\circ = 30^\circ$



(i) In Δ ABD, $\sin 30^\circ = \dfrac{BD}{AB}$

$\Rightarrow \dfrac{1}{2} = \dfrac{BD}{40\,\text{km}}$ *i.e.* BD = 20 km

∴ **The height of rocket when it is at point B**

= BC = BD + DC = 20 km + 40 km = **60 km** **Ans.**

(ii) In Δ ABD, $\cos 30^\circ = \dfrac{AD}{AB}$

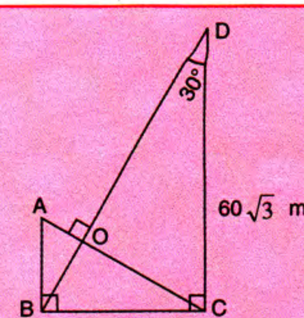
$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{AD}{40\,\text{km}} \Rightarrow AD = 20\sqrt{3}$ km

∴ **The horizontal distance of point C from P**

= PC = AD

$= 20\sqrt{3}$ km = 20 × 1·732 km = **34·64 km** **Ans.**

7 *In the given figure, AB ⊥ BC, DC ⊥ BC,* BD ⊥ AC, ∠D = 30° and DC = $60\sqrt{3}$ m. Find the length of AB.



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Solution :

In right triangle BCD,

$\tan 30° = \frac{BC}{DC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{60\sqrt{3}}$ *i.e.* BC = 60 m

In Δ BCD, $\angle BCD = 90°$

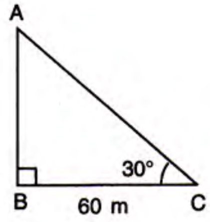
Also, $\angle DBC + 30° + 90° = 180°$

$\Rightarrow \angle DBC = 60°$

In Δ BOC, $\angle BOC + \angle DBC + \angle BCO = 180°$

$90° + 60° + \angle BCO = 180°$

and $\angle BOC = 30°$



In Δ ABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{60}$

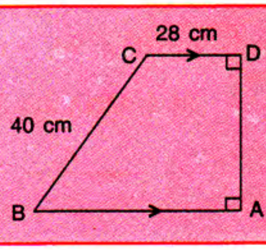
$\therefore$ **AB** $= \frac{60}{\sqrt{3}}$ m $= \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ m

$= \frac{60\sqrt{3}}{3}$ m = **$20\sqrt{3}$ m** **Ans.**

8 In the given figure, ABCD is a trapezium with angle C = 120°, DC = 28 cm and BC = 40 cm. Find :

(i) AB (ii) AD

(iii) the area of the trapezium.



Solution :

Since, CD//BA and BC is transversal

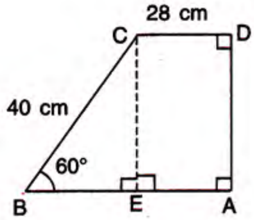
$\therefore \angle B + \angle C = 180°$ [Co-interior angles]

$\Rightarrow \angle B = 180° - \angle C$

$= 180° - 120° = 60°$

Draw CE ⊥ BA

In right triangle CBE,



$\sin 60° = \frac{CE}{40\text{ cm}}$

$\Rightarrow CE = 40 \times \frac{\sqrt{3}}{2}$ cm $= 20\sqrt{3}$ cm

And, $\cos 60° = \frac{BE}{40\text{ cm}}$

$\Rightarrow BE = 40 \times \frac{1}{2} = 20$ cm

(i) **AB** = BE + AE

= BE + CD = 20 cm + 28 cm = **48 cm** **Ans.**

(ii) $AD = CE = 20\sqrt{3}$ cm **Ans.**

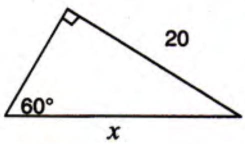
(iii) **Area of trapezium** $= \frac{1}{2}(AB + CD) \times AD$

$$= \frac{1}{2}(48 + 28) \times 20\sqrt{3} \text{ cm}^2 = 760\sqrt{3} \text{ cm}^2$$ **Ans.**

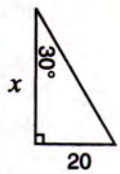
EXERCISE 24

1. Find 'x', if :

(i)



(ii)



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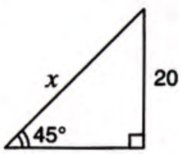
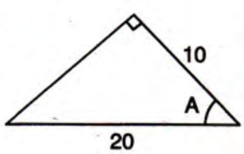
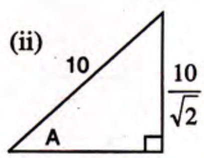


2. Find angle 'A' if :

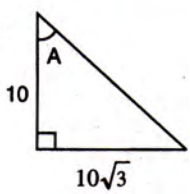
(i)



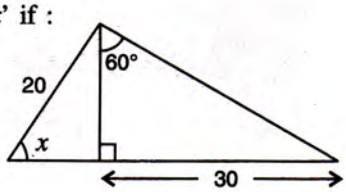
(ii)



(iii)

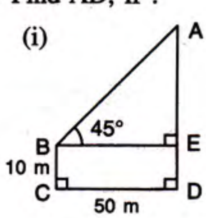


3. Find angle 'x' if :

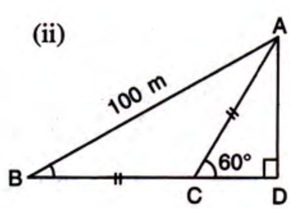


4. Find AD, if :

(i)



(ii)

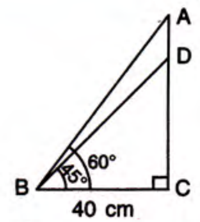


5. Find the length of AD.

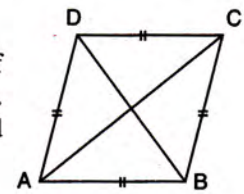
Given : $\angle ABC = 60°$,

$\angle DBC = 45°$

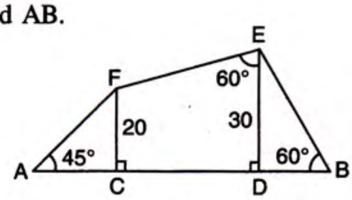
and $BC = 40$ cm.



6. Find lengths of diagonals AC and BD. Given AB = 60 cm and $\angle BAD = 60°$.



7. Find AB.

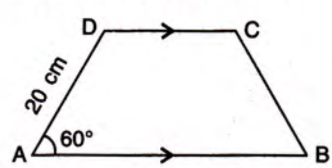


8. In trapezium ABCD, as shown, AB // DC, AD = DC = BC = 20 cm and $\angle A = 60°$.

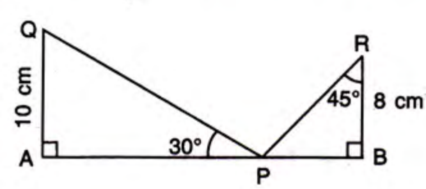
Find :

(i) length of AB

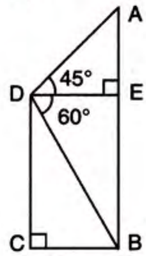
(ii) distance between AB and DC.



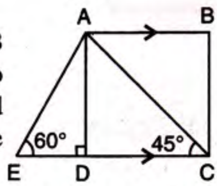
9. Use the information given to find the length of AB.



10. Find the length of AB.



11. In the given figure, AB and EC are parallel to each other. Sides AD and BC are 2 cm each and are perpendicular to AB.

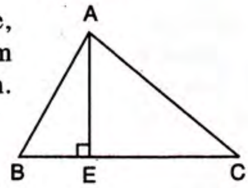


Given that ∠AED = 60° and ∠ACD = 45°; calculate :

(i) AB (ii) AC (iii) AE

12. In the given figure, ∠B = 60°, AB = 8 cm and BC = 25 cm. Calculate :

(i) BE (ii) AC.

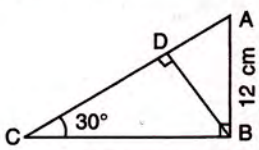


13. Find :

(i) BC

(ii) AD

(iii) AC



14. In right-angled triangle ABC; ∠B = 90°. Find the magnitude of angle A, if :

(i) AB is $\sqrt{3}$ times of BC.

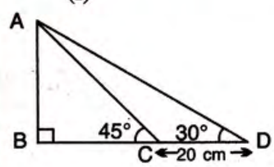
(ii) BC is $\sqrt{3}$ times of AB.

15. A ladder is placed against a vertical tower. If the ladder makes an angle of 30° with the ground and reaches upto a height of 15 m of the tower; find the length of the ladder.

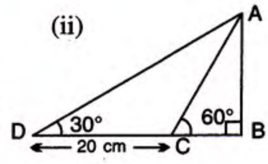
16. A kite is attached to a 100 m long string. Find the greatest height reached by the kite when its string makes an angle of 60° with the level ground.

17. Find AB and BC, if :

(i)



(ii)



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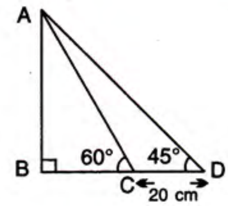
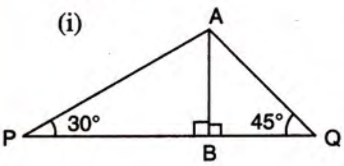
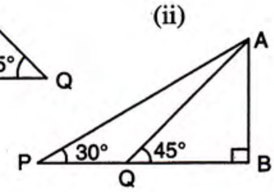


18. Find PQ, if AB = 150 m, ∠P = 30° and ∠Q = 45°

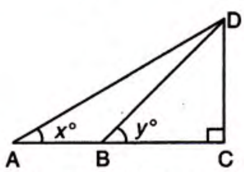
(i)



(ii)



19. If $\tan x° = \frac{5}{12}$, $\tan y° = \frac{3}{4}$ and AB = 48 m; find the length of CD.



20. The perimeter of a rhombus is 96 cm and obtuse angle of it is 120°. Find the lengths of its diagonals.