22 Trigonometrical Ratios

UNIT 7 : Trigonometry

[Sine, Cosine, Tangent of an Angle and Their Reciprocals]

22.1 INTRODUCTION

The word **'Trigonometry'** means *measurement of triangles.*

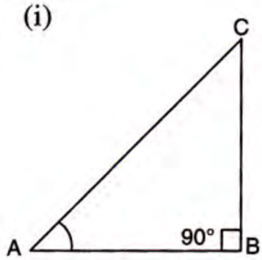
In this unit, we shall be dealing with the relations of sides and angles of right-angled triangles only.

22.2 CONCEPT OF PERPENDICULAR, BASE AND HYPOTENUSE IN A RIGHT TRIANGLE.

For any acute angle (which is also known as the *angle of reference*) in a right-angled triangle; *the side opposite to the acute angle* is called the ***perpendicular****; the side adjacent to it* is called the ***base*** and *the side opposite to the right angle* is called the ***hypotenuse***.

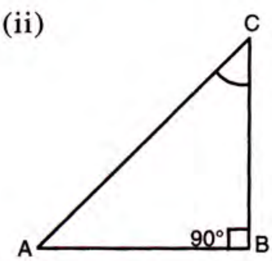
Examples :

(i)



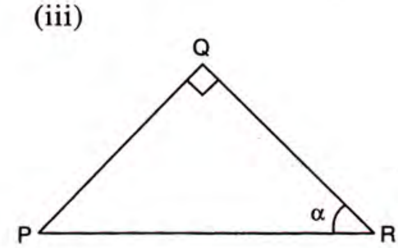
Angle of reference = ∠A
Perpendicular = BC
Base = AB
Hypotenuse = AC

(ii)



Angle of reference = ∠C
Perpendicular = AB
Base = BC
Hypotenuse = AC

(iii)



Angle of reference = α
Perpendicular = PQ
Base = QR
Hypotenuse = PR

22.3 NOTATION OF ANGLES

To indicate an angle, any letter of the English alphabet can be used, but in trigonometry, in general, the following Greek letters are used :

(i) θ (theta) (ii) ϕ (phi) (iii) α (alpha) (iv) β (beta) (v) γ (gamma), etc.

22.4 TRIGONOMETRICAL RATIOS

The *ratio* between the lengths of a pair of two sides of a right-angled triangle is called a ***trigonometrical ratio***.

The three sides of a right-angled triangle give **six** trigonometrical ratios; namely : **sine, cosine, tangent, cotangent, secant** and **cosecant**. In short, these ratios are written as : **sin, cos, tan, cot, sec** and **cosec** respectively.

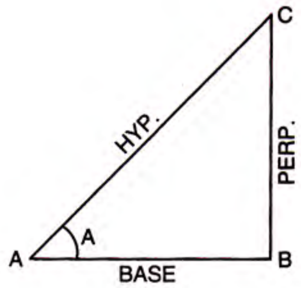
In a right-angled triangle ABC, for acute angle A :

(1) **sine** (sin) is defined as the *ratio* between the lengths of *perpendicular* and *hypotenuse*.

$$\therefore \sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC}$$

[Refer to figure on the next page]

(2) **cosine** (cos) is defined as the *ratio* between the lengths of *base* and *hypotenuse*.



$$\therefore \cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC}$$

(3) **tangent** (tan) is defined as the *ratio* between the lengths of *perpendicular* and *base*.

$$\therefore \tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB}$$

(4) **cotangent** (cot) is defined as the *ratio* between the lengths of *base* and *perpendicular*.

$$\therefore \cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC}$$

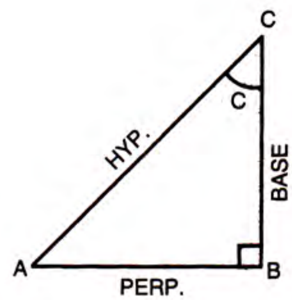
(5) **secant** (sec) is defined as the *ratio* between the lengths of *hypotenuse* and *base*.

$$\therefore \sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB}$$

(6) **cosecant** (cosec) is defined as the *ratio* between the lengths of *hypotenuse* and *perpendicular*.

$$\therefore \text{cosec } A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC}$$

Similarly, for acute angle C in the given right triangle,



(i) $\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC}$

(ii) $\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AC}$

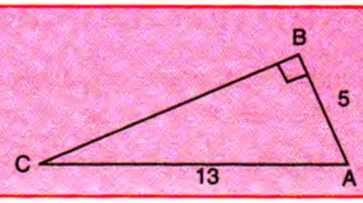
(iii) $\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC}$ and so on.

Each *trigonometrical ratio* is a ***real number*** and has ***no unit***.

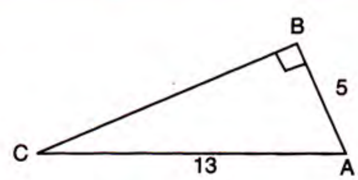
1 From the given figure, find :

(i) sin A (ii) cos C (iii) tan A

(iv) cosec C (v) $\sec^2 A - \tan^2 A$.



Solution :



Given, angle ABC = 90°.

$\Rightarrow$ $AC^2 = AB^2 + BC^2$ (AC is hyp.)

$\Rightarrow$ $13^2 = 5^2 + BC^2$

$\therefore$ $BC^2 = 169 - 25 = 144$ and BC = 12

(i) **sin A** $= \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \mathbf{\frac{12}{13}}$ **Ans.**

(ii) **cos C** $= \frac{\text{base}}{\text{hypotenuse}} = \text{Cos } C = \frac{BC}{AC} = \mathbf{\frac{12}{13}}$ **Ans.**

(iii) $\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \frac{12}{5}$ **Ans.**

(iv) $\text{cosec } C = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{AB} = \frac{13}{5}$ **Ans.**

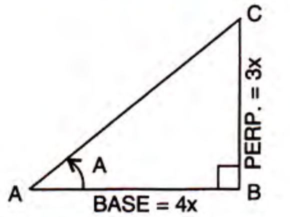
(v) Since, $\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{13}{5}$ and $\tan A = \frac{12}{5}$

$\Rightarrow \sec^2 A - \tan^2 A = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2 = \frac{169}{25} - \frac{144}{25} = 1$ **Ans.**

$\sec^2 A$ means : $(\sec A)^2$; $\tan^2 A$ means : $(\tan A)^2$ and so on.

2 In a right-angled triangle, if angle A is acute and $\cot A = \frac{4}{3}$; find the remaining trigonometrical ratios.

Solution :



Given : $\cot A = \frac{4}{3}$ *i.e.* $\frac{\text{base}}{\text{perpendicular}} = \frac{4}{3}$

$\Rightarrow \frac{AB}{BC} = \frac{4}{3}$

$\therefore$ If length of AB = $4x$ unit, length of BC = $3x$ unit.

Since, $AC^2 = AB^2 + BC^2$ [Using Pythagoras Theorem]

$\Rightarrow AC^2 = (4x)^2 + (3x)^2 = 25x^2$

$\therefore AC = 5x$ unit (hyp.)

$\therefore$ (i) $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3x}{5x} = \frac{3}{5}$ (ii) $\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

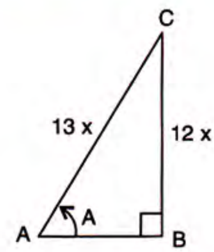
(iii) $\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3x}{4x} = \frac{3}{4}$ (iv) $\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{5x}{4x} = \frac{5}{4}$

and, (v) $\text{cosec } A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{5x}{3x} = \frac{5}{3}$ **Ans.**

3 Given 13 sin A = 12, find :

(i) sec A – tan A (ii) $\frac{1}{\cos^2 A} - \tan^2 A$

Solution :



$13 \sin A = 12 \Rightarrow \sin A = \frac{12}{13}$

i.e. $\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{12}{13} \Rightarrow \frac{BC}{AC} = \frac{12}{13}$

$\therefore$ If length of BC = $12x$, length of AC = $13x$

Since, $AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$\Rightarrow AB^2 + (12x)^2 = (13x)^2$

$\Rightarrow AB^2 = 169x^2 - 144x^2 = 25x^2$

$\therefore AB = 5x$ (base)

$\therefore \sec A = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{13x}{5x} = \dfrac{13}{5}$

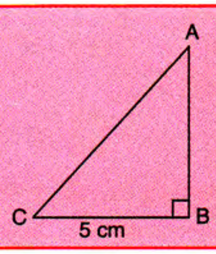
and, $\tan A = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{12x}{5x} = \dfrac{12}{5}$

$\therefore$ (i) **$\sec A - \tan A$** $= \dfrac{13}{5} - \dfrac{12}{5} = \mathbf{\dfrac{1}{5}}$ **Ans.**

(ii) $\dfrac{1}{\cos^2 A} - \tan^2 A = \dfrac{1}{\left(\dfrac{5}{13}\right)^2} - \left(\dfrac{12}{5}\right)^2$ $\left[\cos A = \dfrac{\text{Base}}{\text{Hypot.}} = \dfrac{5x}{13x}\right]$

$= \dfrac{169}{25} - \dfrac{144}{25} = \dfrac{25}{25} = \mathbf{1}$ **Ans.**

4 In the given figure, ABC in a right-angled triangle, right-angled at B. If BC = 5 cm and AC – AB = 1 cm, find the value of cosec A and cos A.



Solution :

$\because$ BC = 5 cm and AC – AB = 1 cm

$\Rightarrow AC = 1 + AB$

In right triangle ABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow (1 + AB)^2 = AB^2 + 5^2$

i.e. $1 + AB^2 + 2AB = AB^2 + 25$

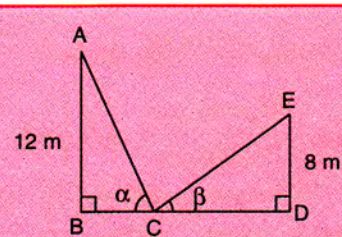
$\Rightarrow 2AB = 24$ and $AB = 12$ cm

$\because AC - AB = 1$

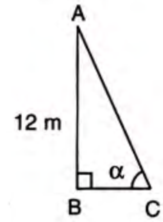
$\Rightarrow AC = 1 + AB = 1 + 12$ cm $= 13$ cm

$\therefore$ **cosec A** $= \dfrac{AC}{BC} = \dfrac{13}{5} = 2\dfrac{3}{5}$ and **cos A** $= \dfrac{AB}{AC} = \dfrac{12}{13}$ **Ans.**

5 For the given figure, if $\cos \alpha = \dfrac{5}{13}$ and $\cos \beta = \dfrac{3}{5}$, find the length of BD.



Solution :



In Δ ABC, $\cos\alpha = \frac{5}{13} \Rightarrow \frac{BC}{AC} = \frac{5}{13}$

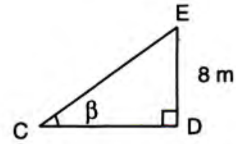
i.e. if BC $= 5k$, AC $= 13k$

$AC^2 = AB^2 + BC^2 \Rightarrow (13k)^2 = (12)^2 + (5k)^2$

i.e. $169k^2 = 144 + 25k^2 \Rightarrow 144k^2 = 144$

i.e. $k^2 = 1$ and $k = 1$

∴ BC $= 5k = 5 \times 1$ m $= 5$ m



In Δ CDE, $\cos\beta = \frac{3}{5} \Rightarrow \frac{CD}{CE} = \frac{3}{5}$

i.e. if CD $= 3p$, CE $= 5p$

$CE^2 = CD^2 + DE^2 \Rightarrow (5p)^2 = (3p)^2 + (8)^2$

i.e. $25p^2 - 9p^2 = 64 \Rightarrow 16p^2 = 64$

i.e. $p^2 = 4$ and $p = 2$

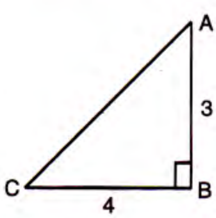
∴ CD $= 3p = 3 \times 2$ m $= 6$ m

Clearly, **BD** = BC + CD

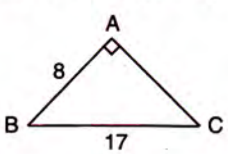
= 5 m + 6 m = **11 m** **Ans.**

EXERCISE 22(A)

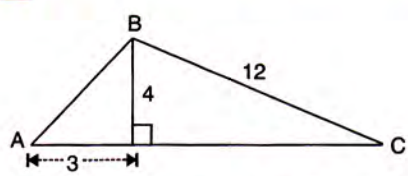
1. From the following figure, find the values of :
   (i) sin A
   (ii) cos A
   (iii) cot A
   (iv) sec C
   (v) cosec C
   (vi) tan C.



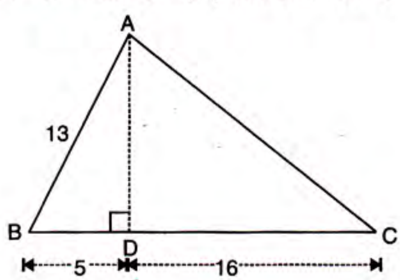
2. From the following figure, find the values of :
   (i) cos B (ii) tan C
   (iii) $\sin^2 B + \cos^2 B$
   (iv) sin B.cos C + cos B.sin C



3. From the following figure, find the values of :
   (i) cos A (ii) cosec A
   (iii) $\tan^2 A - \sec^2 A$ (iv) sin C
   (v) sec C (vi) $\cot^2 C - \frac{1}{\sin^2 C}$

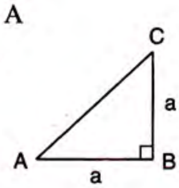


4. From the following figure, find the values of :
   (i) sin B (ii) tan C
   (iii) $\sec^2 B - \tan^2 B$ (iv) $\sin^2 C + \cos^2 C$



5. Given : $\sin A = \frac{3}{5}$, find :
   (i) tan A (ii) cos A

6. From the following figure, find the values of :
   (i) sin A (ii) sec A
   (iii) $\cos^2 A + \sin^2 A$



7. Given : $\cos A = \frac{5}{13}$

evaluate : (i) $\frac{\sin A - \cot A}{2 \tan A}$ (ii) $\cot A + \frac{1}{\cos A}$

8. Given : $\sec A = \frac{29}{21}$, evaluate : $\sin A - \frac{1}{\tan A}$

9. Given : $\tan A = \frac{4}{3}$, find : $\frac{\text{cosec } A}{\cot A - \sec A}$

10. Given : 4 cot A = 3, find :

(i) sin A (ii) sec A

(iii) $\text{cosec}^2 A - \cot^2 A$.

11. Given : cos A = 0.6; find all other trigonometrical ratios for angle A.

12. In a right-angled triangle, it is given that A is an acute angle and $\tan A = \frac{5}{12}$.

Find the values of :

(i) cos A (ii) sin A (iii) $\frac{\cos A + \sin A}{\cos A - \sin A}$

13. Given : $\sin \theta = \frac{p}{q}$,

find cos θ + sin θ in terms of p and q.

14. If $\cos A = \frac{1}{2}$ and $\sin B = \frac{1}{\sqrt{2}}$, find the value of : $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

15. If 5 cot θ = 12, find the value of :

cosec θ + sec θ

16. If $\tan x = 1\frac{1}{3}$, find the value of :

$4 \sin^2 x - 3 \cos^2 x + 2$

17. If $\text{cosec } \theta = \sqrt{5}$, find the value of :

(i) $2 - \sin^2 \theta - \cos^2 \theta$

(ii) $2 + \frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta}$

18. If $\sec A = \sqrt{2}$, find the value of :

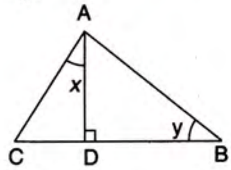
$\frac{3 \cos^2 A + 5 \tan^2 A}{4 \tan^2 A - \sin^2 A}$

19. If cot θ = 1; find the value of :

$5 \tan^2 \theta + 2 \sin^2 \theta - 3$

20. In the following figure :

AD ⊥ BC, AC = 26, CD = 10, BC = 42, ∠DAC = x and ∠B = y.



Find the value of :

(i) cot x

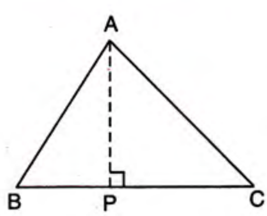
(ii) $\frac{1}{\sin^2 y} - \frac{1}{\tan^2 y}$

(iii) $\frac{6}{\cos x} - \frac{5}{\cos y} + 8 \tan y$.

6 In a triangle ABC, AP is perpendicular to BC. If BC = 112 cm, $\cot B = \frac{4}{3}$ and $\cot C = \frac{12}{5}$, calculate the length of AP.

Solution :

According to the given statement, the figure will be as shown alongside :



Given : $\cot B = \frac{4}{3} \Rightarrow \frac{BP}{AP} = \frac{4}{3}$

$\Rightarrow$ if BP = 4k, AP = 3k

Also, $\cot C = \frac{12}{5} \Rightarrow \frac{CP}{AP} = \frac{12}{5}$

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i.e. $\frac{CP}{3k} = \frac{12}{5} \Rightarrow CP = \frac{36k}{5}$

$\because$ BC = 112 cm $\Rightarrow$ BP + CP = 112

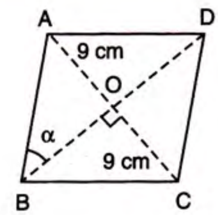
i.e. $4k + \frac{36k}{5} = 112 \Rightarrow \frac{56k}{5} = 112$ and $k = 112 \times \frac{5}{56} = 10$

$\therefore$ **AP** $= 3k = 3 \times 10$ cm = **30 cm** **Ans.**

7 In rhombus ABCD, diagonal AC = 18 cm, the angle between diagonal BD and side AB is α such that cos α = 0·8. Find the diagonal BD and the perimeter of rhombus ABCD.

Solution :

According to the given statement, the figure will be as shown alongside.



We know that the diagonals of a rhombus bisect each other at right angle.

$\therefore$ OA = OC = $\frac{1}{2}$ AC = $\frac{1}{2} \times 18$ cm = 9 cm.

Also, BD = 2 × BO

In right triangle AOB,

$\cos\alpha = 0{\cdot}8 \Rightarrow \frac{BO}{AB} = \frac{4}{5}$ $\left[\because 0{\cdot}8 = \frac{8}{10} = \frac{4}{5}\right]$

i.e. if BO = $4k$, AB = $5k$

$AB^2 = OA^2 + BO^2 \Rightarrow (5k)^2 = 9^2 + (4k)^2$

i.e. $25k^2 = 81 + 16k^2 \Rightarrow 9k^2 = 81$

i.e. $k^2 = 9$ and $k = 3$

Clearly, BO = $4k = 4 \times 3$ cm = 12 cm and AB = $5k = 5 \times 3$ cm = 15 cm

Hence, **the diagonal BD** = 2 × BO

= 2 × 12 cm = **24 cm** **Ans.**

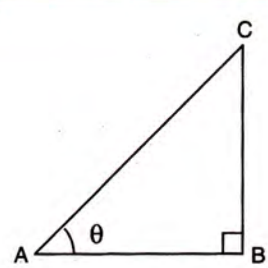
And, **the perimeter** = 4 × AB

= 4 × 15 cm = **60 cm** **Ans.**

8 If $\cos\theta = \frac{2x}{1+x^2}$, find the values of sin θ and cot θ. [Given $x^2 < 1$]

Solution :

In Δ ABC, let ∠A = θ and ∠B = 90°



$\cos\theta = \frac{2x}{1+x^2} \Rightarrow \frac{AB}{AC} = \frac{2x}{1+x^2}$

i.e. if AB = $2x$, AC = $1 + x^2$

$AB^2 + BC^2 = AC^2 \Rightarrow (2x)^2 + BC^2 = (1 + x^2)^2$

i.e. $BC^2 = 1 + x^4 + 2x^2 - 4x^2$

$= 1 + x^4 - 2x^2 = (1 - x^2)^2$

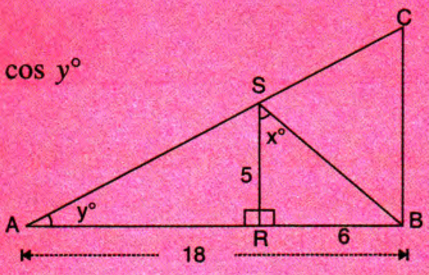
and, $BC = 1 - x^2$

$\therefore \quad \sin\theta = \frac{BC}{AC} = \frac{1-x^2}{1+x^2}$ **Ans.**

And, $\cot\theta = \frac{AB}{BC} = \frac{2x}{1-x^2}$ **Ans.**

9 From the adjoining figure, find :

(i) $\tan x°$ (ii) $\sin y°$ (iii) $\cos y°$



Solution :

(i) In right-angled triangle SRB.

$\tan x° = \frac{\text{perpendicular}}{\text{base}} = \frac{RB}{RS} = \frac{6}{5}$ **Ans.**

(ii) In right-angled triangle ARS,

$AR = 18 - 6 = 12$

$AS^2 = AR^2 + SR^2 = 12^2 + 5^2 = 144 + 25 = 169$

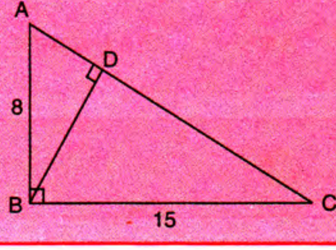
$\therefore AS = 13.$

$\therefore \sin y° = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{SR}{AS} = \frac{5}{13}$ **Ans.**

(iii) $\cos y° = \frac{\text{base}}{\text{hypotenuse}} = \frac{AR}{AS} = \frac{12}{13}$ **Ans.**

10 In the adjoining figure, triangle ABC is right-angled at B and BD is perpendicular to AC. Find :

(i) $\cos \angle ABD$ (ii) $\tan \angle DBC$.



Solution :

In right-angled triangle ABC :

$AC^2 = AB^2 + BC^2$ [Pythagoras Theorem]

$= 8^2 + 15^2$

$= 64 + 225 = 289$

$\therefore AC = 17$

(i) **cos ∠ABD** = cos C

$= \frac{BC}{AC}$

$= \mathbf{\frac{15}{17}}$

Since, ∠ABD +∠A=90° and ∠A+∠C = 90°,
∴ ∠ABD + ∠A = ∠A + ∠C ⇒ ∠ABD = ∠C

Ans.

(ii) **tan ∠DBC** = tan A

$= \frac{BC}{AB}$

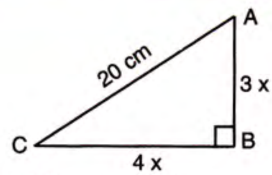
$= \mathbf{\frac{15}{8}}$

Since, ∠DBC +∠C = 90° and ∠C +∠A = 90°
∴ ∠DBC + ∠C = ∠C + ∠A ⇒ ∠DBC = ∠A

Ans.

11 In Δ ABC, right-angled at B, AC = 20 cm and tan ∠ACB = $\frac{3}{4}$; calculate the measures of AB and BC.

Solution :



$\tan \angle ACB = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$

Let AB = 3x cm, ∴ BC = 4x cm.

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$\Rightarrow (3x)^2 + (4x)^2 = (20)^2$

$25x^2 = 400$

$x = 4$

∴ **AB** = $3x = 3 \times 4 =$ **12 cm** and **BC** = $4x = 4 \times 4 =$ **16 cm** **Ans.**

22.5 RECIPROCAL RELATIONS

1. Since, $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$ and $\text{cosec } A = \frac{\text{hypotenuse}}{\text{perpendicular}}$

⇒ sin A and cosec A are reciprocal of each other.

∴ $\sin A = \frac{1}{\text{cosec } A}$ and $\text{cosec } A = \frac{1}{\sin A}$

Similarly, $\cos A = \frac{1}{\sec A}$ and $\sec A = \frac{1}{\cos A}$

$\tan A = \frac{1}{\cot A}$ and $\cot A = \frac{1}{\tan A}$

2. Since, $\frac{\sin A}{\cos A} = \frac{\frac{\text{perpendicular}}{\text{hypotenuse}}}{\frac{\text{base}}{\text{hypotenuse}}} = \frac{\text{perpendicular}}{\text{base}} = \tan A$

∴ $\tan A = \frac{\sin A}{\cos A}$ and $\cot A = \frac{\cos A}{\sin A}$

12 If tan θ + cot θ = 2; find the value of $\tan^2 \theta + \cot^2 \theta$.

Solution :

Given : $\tan \theta + \cot \theta = 2$

⇒ $(\tan \theta + \cot \theta)^2 = (2)^2$

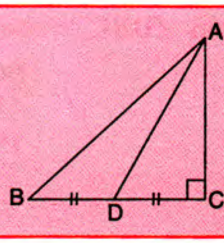
$\Rightarrow \quad \tan^2\theta + \cot^2\theta + 2\tan\theta \cdot \cot\theta = 4$

$\Rightarrow \quad \tan^2\theta + \cot^2\theta + 2\tan\theta \cdot \dfrac{1}{\tan\theta} = 4$ $\quad \left[\text{Since, } \cot\theta = \dfrac{1}{\tan\theta}\right]$

$\Rightarrow \quad \tan^2\theta + \cot^2\theta = 4 - 2 = 2$ **Ans.**

13 In the given figure; $\angle C = 90°$ and BD = DC. Find :

(i) $\dfrac{\cot \angle ABC}{\cot \angle ADC}$ (ii) $\dfrac{\tan \angle DAC}{\tan \angle BAC}$



Solution :

Let BD = DC = $x \Rightarrow$ BC = BD + DC = $x + x = 2x$

$\Rightarrow \cot \angle ABC = \dfrac{BC}{AC} = \dfrac{2x}{AC}, \quad \cot \angle ADC = \dfrac{DC}{AC} = \dfrac{x}{AC}$

$\tan \angle DAC = \dfrac{DC}{AC} = \dfrac{x}{AC}$ and $\tan \angle BAC = \dfrac{BC}{AC} = \dfrac{2x}{AC}$

(i) $\dfrac{\cot \angle ABC}{\cot \angle ADC} = \dfrac{\frac{2x}{AC}}{\frac{x}{AC}} = \dfrac{2x}{AC} \times \dfrac{AC}{x} = 2$ **Ans.**

(ii) $\dfrac{\tan \angle DAC}{\tan \angle BAC} = \dfrac{\frac{x}{AC}}{\frac{2x}{AC}} = \dfrac{x}{AC} \times \dfrac{AC}{2x} = \dfrac{1}{2}$ **Ans.**

14 If 3 sin A = 4 cos A; find the value of :

(i) sin A (ii) cos A (iii) $\tan^2 A - \sec^2 A$

Solution :

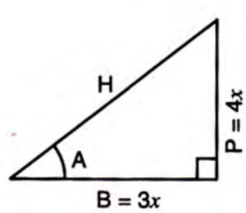
$3\sin A = 4\cos A \Rightarrow \dfrac{\sin A}{\cos A} = \dfrac{4}{3} \Rightarrow \tan A = \dfrac{4}{3}$ $\quad \left[\because \tan A = \dfrac{\sin A}{\cos A}\right]$

$\tan A = \dfrac{4}{3} \quad \Rightarrow \dfrac{P}{B} = \dfrac{4}{3}$

i.e. if $P = 4x$, $B = 3x$

$H^2 = P^2 + B^2 \Rightarrow H^2 = (4x)^2 + (3x)^2 = 16x^2 + 9x^2 = 25x^2$

$\Rightarrow H = 5x$



(i) $\sin A = \dfrac{P}{H} = \dfrac{4x}{5x} = \dfrac{4}{5}$ **Ans.**

(ii) $\cos A = \dfrac{B}{H} = \dfrac{3x}{5x} = \dfrac{3}{5}$ **Ans.**

(iii) $\tan A = \frac{4}{3}$ and $\sec A = \frac{H}{B} = \frac{5x}{3x} = \frac{5}{3}$

$\Rightarrow$ $\tan^2 A - \sec^2 A = \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$

$= \frac{16}{9} - \frac{25}{9} = \frac{16-25}{9} = \frac{-9}{9} = -1$ **Ans.**

15 If $5 \tan \theta = 4$, find the value of : $\frac{8\sin\theta - 3\cos\theta}{8\sin\theta + 2\cos\theta}$.

Solution :

$5 \tan \theta = 4 \Rightarrow \tan \theta = \frac{4}{5}$

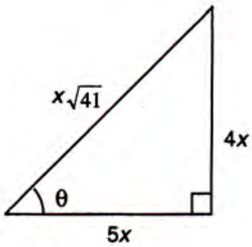
$\therefore \quad \frac{8\sin\theta - 3\cos\theta}{8\sin\theta + 2\cos\theta} = \frac{\frac{8\sin\theta}{\cos\theta} - 3}{\frac{8\sin\theta}{\cos\theta} + 2}$ [Dividing each terms by cos θ]

$= \frac{8\tan\theta - 3}{8\tan\theta + 2}$ $\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

$= \frac{8\times\frac{4}{5} - 3}{8\times\frac{4}{5} + 2} = \frac{\frac{17}{5}}{42} = \frac{17}{42}$ **Ans.**

Alternative method :

$5 \tan \theta = 4 \Rightarrow \tan \theta = \frac{4}{5}$

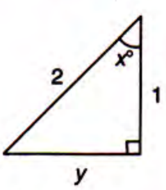


$\Rightarrow \sin\theta = \frac{4x}{x\sqrt{41}} = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5x}{x\sqrt{41}} = \frac{5}{\sqrt{41}}$

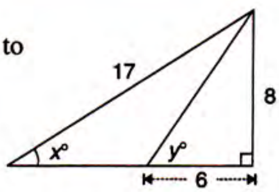
$\therefore \quad \frac{8\sin\theta - 3\cos\theta}{8\sin\theta + 2\cos\theta} = \frac{8\times\frac{4}{\sqrt{41}} - 3\times\frac{5}{\sqrt{41}}}{8\times\frac{4}{\sqrt{41}} + 2\times\frac{5}{\sqrt{41}}} = \frac{32-15}{32+10} = \frac{17}{42}$ **Ans.**

EXERCISE 22(B)

1. From the following figure, find :
   (i) y
   (ii) $\sin x°$
   (iii) $(\sec x° - \tan x°)(\sec x° + \tan x°)$

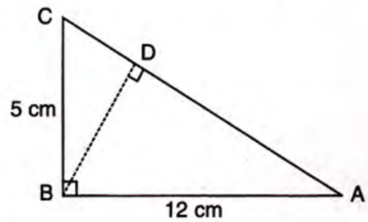


2. Use the given figure to find :
   (i) $\sin x°$
   (ii) $\cos y°$
   (iii) $3 \tan x° - 2 \sin y° + 4 \cos y°$.



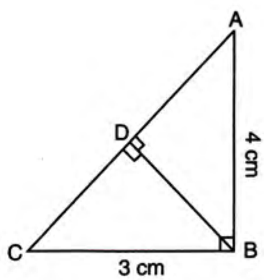
3. In the diagram, given below, triangle ABC is right-angled at B and BD is perpendicular to AC. Find :

   (i) cos ∠DBC (ii) cot ∠DBA



4. In the given figure, triangle ABC is right-angled at B. D is the foot of the perpendicular from B to AC. Given that BC = 3 cm and AB = 4 cm. Find :

   (i) tan ∠DBC (ii) sin ∠DBA

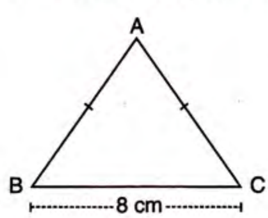


5. In triangle ABC, AB = AC = 15 cm and BC = 18 cm, find cos ∠ABC.

6. In the figure, given below, ABC is an isosceles triangle with BC = 8 cm and AB = AC = 5 cm. Find :

   (i) sin B (ii) tan C

   (iii) $\sin^2 B + \cos^2 B$ (iv) tan C – cot B

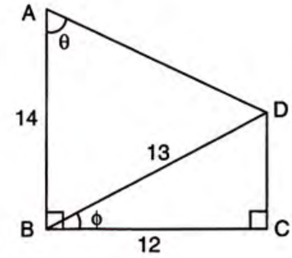


7. In triangle ABC; ∠ABC = 90°, ∠CAB = x°, $\tan x° = \frac{3}{4}$ and BC = 15 cm. Find the measures of AB and AC.

8. Using the measurements given in the following figure :

   (i) Find the value of sin φ and tan θ.

   (ii) Write an expression for AD in terms of θ.

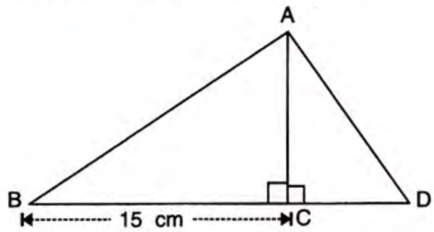


9. In the given figure;

   BC=15 cm and $\sin B = \frac{4}{5}$.

   (i) Calculate the measures of AB and AC.

   (ii) Now, if tan ∠ADC = 1; calculate the measures of CD and AD.



   Also, show that : $\tan^2 B - \frac{1}{\cos^2 B} = -1$.

10. If sin A + cosec A = 2;

    find the value of $\sin^2 A + \text{cosec}^2 A$.

11. If tan A + cot A = 5;

    find the value of $\tan^2 A + \cot^2 A$.

12. Given : 4 sin θ = 3 cos θ; find the value of :

    (i) sin θ (ii) cos θ

    (iii) $\cot^2 \theta - \text{cosec}^2 \theta$.

    (iv) $4 \cos^2 \theta - 3 \sin^2 \theta + 2$

13. Given : 17 cos θ = 15;

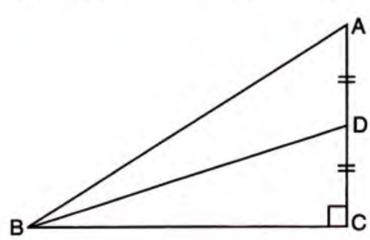
    find the value of tan θ + 2 sec θ.

14. Given: 5 cos A – 12 sin A = 0; evaluate :

    $$\frac{\sin A + \cos A}{2 \cos A - \sin A}$$

15. In the given figure; ∠C = 90° and D is mid-point of AC. Find :

    (i) $\frac{\tan \angle CAB}{\tan \angle CDB}$ (ii) $\frac{\tan \angle ABC}{\tan \angle DBC}$



16. If 3 cos A = 4 sin A, find the value of :
(i) cos A (ii) 3 – $\cot^2$ A + cosec^2 A

17. In triangle ABC, ∠B = 90° and tan A = 0·75. If AC = 30 cm, find the lengths of AB and BC.

18. In rhombus ABCD, diagonals AC and BD intersect each other at point O.
If cosine of angle CAB is 0·6 and OB = 8 cm, find the lengths of the side and the diagonals of the rhombus.

19. In triangle ABC, AB = AC = 15 cm and BC = 18 cm. Find :
(i) cos B
(ii) sin C
(iii) $\tan^2$ B – $\sec^2$ B + 2

20. In triangle ABC, AD is perpendicular to BC. sin B = 0·8, BD = 9 cm and tan C = 1. Find the length of AB, AD, AC and DC.

21. Given : q tan A = p, find the value of :
$$\frac{p\sin A - q\cos A}{p\sin A + q\cos A}.$$

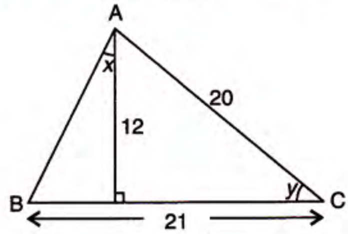
22. If sin A = cos A, find the value of
2 $\tan^2$ A – 2 $\sec^2$ A + 5.

23. In rectangle ABCD, diagonal BD = 26 cm and cotangent of angle ABD = 1·5. Find the area and the perimeter of the rectangle ABCD.

24. If 2 sin x = $\sqrt{3}$, evaluate.
(i) 4 $\sin^3 x$ – 3 sin x.
(ii) 3 cos x – 4 $\cos^3 x$.

25. If sin A = $\frac{\sqrt{3}}{2}$ and cos B = $\frac{\sqrt{3}}{2}$, find the value of : $\frac{\tan A - \tan B}{1 + \tan A \tan B}$.

26. Use the informations given in the following figure to evaluate : $\frac{10}{\sin x} + \frac{6}{\sin y} - 6 \cot y$.



27. If sec A = $\sqrt{2}$, find : $\frac{3\cot^2 A + 2\sin^2 A}{\tan^2 A - \cos^2 A}$.

28. If 5 cos θ = 3, evaluate : $\frac{\text{cosec}\,\theta - \cot\theta}{\text{cosec}\,\theta + \cot\theta}$.

29. If cosec A + sin A = $5\frac{1}{5}$, find the value of cosec^2 A + $\sin^2$ A.

30. If 5 cos θ = 6 sin θ; evaluate :
(i) tan θ (ii) $\frac{12\sin\theta - 3\cos\theta}{12\sin\theta + 3\cos\theta}$