

## 19

**Mean and Median**

[For Ungrouped Data Only]

**19.1 MEAN OF UNGROUPED DATA**

If  $x_1, x_2, x_3, x_4, \dots, x_n$  are  $n$  observations in an ungrouped data, then their mean, in general denoted by  $\bar{x}$ , is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^n x_i$$

1. The symbol  $\sum_{i=1}^n x_i = x_1 + x_2 + x_3 + x_4 + \dots + x_n$
2. Mean, arithmetic mean and average are the same.
3. To find the mean of any set of observations, divide their sum by the total number of observations.

- 1** The heights of 6 boys are 146 cm, 154 cm, 153 cm, 160 cm, 157 cm and 160 cm. Find their mean height.

**Solution :**

$$\text{Mean height} = \frac{\text{Sum of heights of all the boys}}{\text{Number of boys}}$$

$$\text{i.e. } \bar{x} = \frac{146 + 154 + 153 + 160 + 157 + 160}{6} \text{ cm} = \mathbf{155 \text{ cm}} \quad \text{Ans.}$$

**Alternative method :**

$$\because \sum_{i=1}^6 x_i = 146 \text{ cm} + 154 \text{ cm} + 153 \text{ cm} + 160 \text{ cm} + 157 \text{ cm} + 160 \text{ cm} = 930 \text{ cm}$$

$$\text{and, } n = 6$$

$$\therefore \text{Mean } (\bar{x}) = \frac{1}{n} \sum_{i=1}^6 x_i = \frac{1}{6} \times 930 \text{ cm} = \mathbf{155 \text{ cm}} \quad \text{Ans.}$$

$$\sum_{i=1}^n x_i = x_1 + x_2 + x_3 + \dots + x_n$$

$$\sum_{i=1}^6 x_i = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \quad \text{and so on.}$$

- 2** Find the mean of all prime numbers between 20 and 50.

**Solution :**

Prime numbers between 20 and 50 are 23, 29, 31, 37, 41, 43 and 47.

$$\therefore \text{Mean} = \frac{23+29+31+37+41+43+47}{7} = \frac{251}{7} = 35\frac{6}{7} \quad \text{Ans.}$$

**Alternative method :**

$$\therefore \Sigma x = 23 + 29 + 31 + 37 + 41 + 43 + 47 = 251$$

and  $n = 7$

$$\therefore \text{Mean } (\bar{x}) = \frac{251}{7} = 35\frac{6}{7} \text{ cm} \quad \text{Ans.}$$

For  $\sum_{i=1}^n x_i$ , we can simply write  $\Sigma x$ .

**19.2 PROPERTIES OF MEAN**

**Property 1 :**

If  $\bar{x}$  is the mean of  $n$  number of observations  $x_1, x_2, x_3, x_4, \dots, x_n$ ; then the sum of deviations of  $\bar{x}$  from the observations  $x_1, x_2, x_3, x_4, \dots, x_n$ , is zero i.e.,  $\Sigma(x - \bar{x}) = 0$ .

- 3** (i) Find the mean ( $\bar{x}$ ) of first 5 even natural numbers.  
 (ii) If  $x$  denotes the different even natural numbers, taken in part (i) above, show that  $\Sigma(x - \bar{x}) = 0$ .

**Solution :**

(i) Different values of  $x$  are : 2, 4, 6, 8 and 10

$$\therefore \Sigma x = 2 + 4 + 6 + 8 + 10 = 30 \text{ and } n = 5$$

$$\therefore \text{Mean } (\bar{x}) = \frac{\Sigma x}{n} = \frac{30}{5} = 6 \quad \text{Ans.}$$

(ii)

$x$	$x - \bar{x}$
2	$2 - 6 = -4$
4	$4 - 6 = -2$
6	$6 - 6 = 0$
8	$8 - 6 = 2$
10	$10 - 6 = 4$

$$\begin{aligned} \therefore \Sigma(x - \bar{x}) &= -4 - 2 + 0 + 2 + 4 \\ &= 0 \end{aligned}$$

- 4** Find the sum of the deviations of the data 4, 5, 7, 9 and 15 from their mean.

**Solution :**

$$\text{Mean of given data} = \frac{4+5+7+9+15}{5} = 8$$

$$\begin{aligned} \therefore \text{Sum of deviations of the given data from their mean} &= (4 - 8) + (5 - 8) + (7 - 8) + (9 - 8) + (15 - 8) \\ &= -4 - 3 - 1 + 1 + 7 = 0 \quad \text{Ans.} \end{aligned}$$

**Property 2 :**

If  $\bar{x}$  is the mean of  $n$  number of observations  $x_1, x_2, x_3, \dots, x_n$ , then the mean of observations  $x_1 + a, x_2 + a, x_3 + a, \dots, x_n + a$  is  $\bar{x} + a$ .

*i.e., if each observation under consideration is increased by quantity  $a$ , then their mean is also increased by the same quantity  $a$ .*

**Property 3 :**

If  $\bar{x}$  is the mean of  $n$  number of observations  $x_1, x_2, x_3, \dots, x_n$ , then the mean of observations  $x_1 - a, x_2 - a, x_3 - a, \dots, x_n - a$  is  $\bar{x} - a$ .

*i.e., if each observation under consideration is decreased by quantity  $a$ , then their mean is also decreased by the same quantity  $a$ .*

**Property 4 :**

If  $\bar{x}$  is the mean of  $n$  number of observations  $x_1, x_2, x_3, \dots, x_n$ , then mean of  $ax_1, ax_2, ax_3, \dots, ax_n$  is  $a\bar{x}$ .

*i.e., if each observation under consideration is multiplied by quantity  $a$ , then mean is also multiplied by the same quantity  $a$ .*

**Property 5 :**

If  $\bar{x}$  is the mean of  $n$  number of observations  $x_1, x_2, x_3, \dots, x_n$ , then mean of

$\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \dots, \frac{x_n}{a}$  is  $\frac{\bar{x}}{a}$ .

*i.e., if each observation under consideration is divided by quantity  $a$ , the mean is also divided by the same quantity  $a$ .*

- 5** (a) Find the mean of 5, 6, 17, 8, 9, 15, 23, 18, 10 and 24.  
 (b) Find the resulting mean, if each observation, given above, is :  
 (i) increased by 3. (ii) decreased by 2.  
 (iii) multiplied by 4. (iv) divided by 5.

**Solution :**

$$(a) \quad \text{Mean} = \frac{5+6+17+8+9+15+23+18+10+24}{10} \\ = \frac{135}{10} = 13.5$$

$$(b) \quad (i) \quad \text{According to the property 2, resulting mean} = 13.5 + 3 \\ = 16.5 \quad \text{Ans.}$$

$$(ii) \quad \text{According to the property 3, resulting mean} = 13.5 - 2 \\ = 11.5 \quad \text{Ans.}$$

$$(iii) \quad \text{According to the property 4, resulting mean} = 13.5 \times 4 \\ = 54 \quad \text{Ans.}$$

$$(iv) \quad \text{According to the property 5, resulting mean} = \frac{13.5}{5} \\ = 2.7 \quad \text{Ans.}$$

- 6 The mean of 7, 5, 8,  $p$  and 11 is 8. Find the value of  $p$ .

**Solution :**

$$\text{Given : } \frac{7+5+8+p+11}{5} = 8$$

$$\Rightarrow 31 + p = 40$$

$$\Rightarrow p = 40 - 31 = 9$$

**Ans.**

- 7 The mean of 40 observations was 160. It was detected that the value of 165 was wrongly copied as 125. Find the correct mean.

**Solution :**

$$\frac{\Sigma x}{n} = \bar{x} \Rightarrow \frac{\Sigma x}{40} = 160 \Rightarrow \Sigma x = 40 \times 160 = 6400$$

$$\Rightarrow \text{Incorrect value of } \Sigma x = 6400$$

$$\begin{aligned} \therefore \text{Correct value of } \Sigma x &= \text{Incorrect value of } \Sigma x - \text{Incorrect observation} \\ &\quad + \text{correct observation} \\ &= 6400 - 125 + 165 = 6440 \end{aligned}$$

$$\begin{aligned} \therefore \text{Correct mean} &= \frac{\text{Correct value of } \Sigma x}{n} \\ &= \frac{6440}{40} = 161 \end{aligned}$$

**Ans.**

**Directly,**

Mean (average) of 40 observations = 160

Total sum (incorrect) of 40 observations =  $40 \times 160 = 6400$

Since, 165 was wrongly copied as 125

$\therefore$  Correct sum of 40 observations =  $6400 - 125 + 165 = 6440$

$$\Rightarrow \text{Correct mean} = \frac{6440}{40} = 161$$

**Ans.**

- 8 The mean of 100 observations was found to be 30. If two observations were wrongly taken as 32 and 12 instead of 23 and 11, find the correct mean.

**Solution :**

$$\text{Mean} = \frac{\Sigma x}{n} \Rightarrow 30 = \frac{\Sigma x}{100} \Rightarrow \Sigma x = 3000$$

i.e., incorrect value of  $\Sigma x = 3000$

$$\Rightarrow \text{Correct value of } \Sigma x = 3000 - (32 + 12) + (23 + 11) = 2990$$

$$\begin{aligned} \therefore \text{Correct mean} &= \frac{\text{Correct value of } \Sigma x}{n} \\ &= \frac{2990}{100} = 29.9 \end{aligned}$$

**Ans.**

- 9** The mean monthly salary of 10 people is ₹ 2,890. One more person whose monthly salary is ₹ 3,000 has also been taken into consideration. Find the mean monthly salary of all the 11 people.

**Solution :**

Since, mean monthly salary of 10 people = ₹ 2,890

⇒ Total monthly salary of 10 people =  $10 \times ₹ 2,890 = ₹ 28,900$

On adding the salary of the 11th person,

total monthly salary of 11 people = ₹ 28,900 + ₹ 3,000 = ₹ 31,900

∴ **Average monthly salary** =  $\frac{₹ 31,900}{11} = ₹ 2,900$  **Ans.**

- 10** Mean of 15 observations is 200. If one observation is excluded, the mean of remaining observations is 198. Find the value of the excluded observation.

**Solution :**

Mean of 15 observations = 200

⇒ Total sum of 15 observations =  $15 \times 200 = 3,000$

On excluding an observation, the mean of the remaining 14 observations is 198

∴ Total sum of remaining 14 observations =  $14 \times 198 = 2,772$

⇒ **Excluded observation** = Total of 15 observations – total of remaining 14 observations  
=  $3,000 - 2,772 = 228$  **Ans.**

- 11** Out of 25 numbers, the mean of 15 numbers is 36 and the mean of the remaining numbers is 26; find the mean of all the 25 numbers.

**Solution :**

Mean of 15 numbers = 36

⇒ Total of 15 numbers =  $36 \times 15 = 540$

Mean of remaining (25 – 15) 10 numbers = 26

⇒ Total of remaining 10 numbers =  $26 \times 10 = 260$

∴ Total of all the 25 numbers =  $540 + 260 = 800$

⇒ **Mean of all the 25 numbers** =  $\frac{800}{25} = 32$  **Ans.**

### EXERCISE 19(A)

- Find the mean of 43, 51, 50, 57 and 54.
- Find the mean of first six natural numbers.
- Find the mean of first ten odd natural numbers.
- Find the mean of all factors of 10.
- Find the mean of  $x + 3$ ,  $x + 5$ ,  $x + 7$ ,  $x + 9$  and  $x + 11$ .
- If different values of variable  $x$  are 9·8, 5·4, 3·7, 1·7, 1·8, 2·6, 2·8, 8·6, 10·5 and 11·1; find
  - the mean  $\bar{x}$
  - the value of  $\Sigma(x - \bar{x})$
- The mean of 15 observations is 32. Find the resulting mean, if each observation is :
  - increased by 3
  - decreased by 7

- (iii) multiplied by 2 (iv) divided by 0.5  
 (v) increased by 60% (vi) decreased by 20%
8. The mean of 5 numbers is 18. If one number is excluded, the mean of remaining numbers becomes 16. Find the excluded number.
9. If the mean of observations  $x$ ,  $x + 2$ ,  $x + 4$ ,  $x + 6$  and  $x + 8$  is 11, find :  
 (i) the value of  $x$ ;  
 (ii) the mean of the first three observations.
10. The mean of 100 observations is 40. It is found that an observation 53 was misread as 83. Find the correct mean.
11. The mean of 200 items was 50. Later on, it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.
12. Find the mean of 75 numbers, if the mean of 45 of them is 18 and the mean of the remaining ones is 13.
13. The mean weight of 120 students of a school is 52.75 kg. If the mean weight of 50 of them is 51 kg, find the mean weight of the remaining students.
14. The mean marks (out of 100) of boys and girls in an examination are 70 and 73 respectively. If the mean marks of all the students in that examination is 71, find the ratio of the number of boys to the number of girls.

### 19.3 MEDIAN

For any given set of data, *the median is the value of its middle term.*

For finding the median of a given ungrouped data, adopt the following steps :

**Step 1 :** Arrange the given data in ascending or descending order of their magnitude (value).

**Step 2 :** Count the total number ( $n$ ) of observations in the given data.

**Step 3 :**

(i) If  $n$  is odd, median =  $\left(\frac{n+1}{2}\right)^{\text{th}}$  term.

(ii) If  $n$  is even, median =  $\frac{1}{2}$  [value of  $\left(\frac{n}{2}\right)^{\text{th}}$  term + value of  $\left(\frac{n}{2}+1\right)^{\text{th}}$  term]

**12** Find the median of : 18, 30, 39, 36, 28, 27, 31, 40, 33, 25 and 37.

**Solution :**

On arranging the given set of data in ascending order of magnitude, we get :

18, 25, 27, 28, 30, 31, 33, 36, 37, 39 and 40.

$\therefore$  Number of observations,  $n = 11$  (odd)

$$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{11+1}{2}\right)^{\text{th}} \text{ term} \\ = 6^{\text{th}} \text{ term} = 31$$

**Ans.**

**13** Find the median of : 34, 47, 41, 52, 53, 56, 35, 49, 55 and 42.

**Solution :**

On arranging the given set of data in ascending order, we get :

34, 35, 41, 42, 47, 49, 52, 53, 55 and 56.

∴ Number of observations,  $n = 10$  (even)

$$\begin{aligned}\therefore \text{Median} &= \frac{1}{2} \left[ \text{value of } \left(\frac{10}{2}\right)^{\text{th}} \text{ term} + \text{value of } \left(\frac{10}{2} + 1\right)^{\text{th}} \text{ term} \right] \\ &= \frac{1}{2} [\text{value of } 5^{\text{th}} \text{ term} + \text{value of } 6^{\text{th}} \text{ term}] \\ &= \frac{1}{2} [47 + 49] = 48\end{aligned}$$

**Ans.**

**14** The weight of 12 students (in kg) are :

40, 61, 54, 50, 59, 37, 51, 41, 48, 62, 46 and 34.

Find the median weight.

If the weight of 62 kg is replaced by 35 kg, find the new median weight.

**Solution :**

After writing the given weights in ascending order, we get :

34, 37, 40, 41, 46, 48, 50, 51, 54, 59, 61 and 62.

∴  $n = 12$

$$\begin{aligned}\therefore \text{Median weight} &= \frac{1}{2} \left[ \left(\frac{12}{2}\right)^{\text{th}} \text{ term} + \left(\frac{12}{2} + 1\right)^{\text{th}} \text{ term} \right] \\ &= \frac{1}{2} [6^{\text{th}} \text{ term} + 7^{\text{th}} \text{ term}] \\ &= \frac{1}{2} [48 \text{ kg} + 50 \text{ kg}] = \frac{1}{2} \times 98 \text{ kg} = 49 \text{ kg}\end{aligned}$$

**Ans.**

On replacing 62 kg by 35 kg and on arranging the weights in ascending order, we get :

34, 35, 37, 40, 41, 46, 48, 50, 51, 54, 59 and 61.

$$\begin{aligned}\text{And, median} &= \frac{1}{2} \left[ \left(\frac{12}{2}\right)^{\text{th}} \text{ term} + \left(\frac{12}{2} + 1\right)^{\text{th}} \text{ term} \right] \\ &= \frac{1}{2} [6^{\text{th}} \text{ term} + 7^{\text{th}} \text{ term}] = \frac{1}{2} [46 + 48] \text{ kg} = 47 \text{ kg}\end{aligned}$$

**Ans.**

**15** The median of the following observations, arranged in ascending order, is 20. Find  $x$ .  
3, 4, 7, 8, 10, 18,  $x + 2$ ,  $x + 4$ , 26, 28, 31, 36, 38 and 40.

**Solution :**

∴  $n = 14$

$$\therefore \text{Median} = \frac{1}{2} \left[ \left(\frac{14}{2}\right)^{\text{th}} \text{ term} + \left(\frac{14}{2} + 1\right)^{\text{th}} \text{ term} \right]$$

$$\Rightarrow 20 = \frac{1}{2} [7^{\text{th}} \text{ term} + 8^{\text{th}} \text{ term}]$$

$$\Rightarrow 40 = x + 2 + x + 4 \Rightarrow 2x = 34 \text{ and } x = 17$$

**Ans.**

**EXERCISE 19(B)**

- Find the median of :
  - 25, 16, 26, 16, 32, 31, 19, 28 and 35
  - 241, 243, 347, 350, 327, 299, 261, 292, 271, 258 and 257
  - 63, 17, 50, 9, 25, 43, 21, 50, 14 and 34
  - 233, 173, 189, 208, 194, 204, 194, 185, 200 and 220
- The following data have been arranged in ascending order. If their median is 63, find the value of  $x$ .  
34, 37, 53, 55,  $x$ ,  $x + 2$ , 77, 83, 89 and 100.
- In 10 numbers, arranged in increasing order, the 7<sup>th</sup> number is increased by 8, how much will the median be changed ?
- Out of 10 students, who appeared in a test, three secured less than 30 marks and 3 secured more than 75 marks. The marks secured by the remaining 4 students are 35, 48, 66 and 40. Find the median score of the whole group.
- The median of observations 10, 11, 13, 17,  $x + 5$ , 20, 22, 24 and 53 (arranged in ascending order) is 18; find the value of  $x$ .

**EXERCISE 19(C)**

- Find the mean of 8, 12, 16, 22, 10 and 4. Find the resulting mean, if each of the observations, given above, be :
  - multiplied by 3.
  - divided by 2.
  - multiplied by 3 and then divided by 2.
  - increased by 25%.
  - decreased by 40%.
- The mean of 18, 24, 15,  $2x + 1$  and 12 is 21. Find the value of  $x$ .
- The mean of 6 numbers is 42. If one number is excluded, the mean of remaining numbers is 45. Find the excluded number.
- The mean of 10 numbers is 24. If one more number is included, the new mean is 25. Find the included number.
- The following observations have been arranged in ascending order. If the median of the data is 78, find the value of  $x$ .  
44, 47, 63, 65,  $x+13$ , 87, 93, 99, 110.
- The following observations have been arranged in ascending order. If the median of these observations is 58, find the value of  $x$ .  
24, 27, 43, 48,  $x-1$ ,  $x+3$ , 68, 73, 80, 90.
- Find the mean of the following data :  
30, 32, 24, 34, 26, 28, 30, 35, 33, 25
  - Show that the sum of the deviations of all the given observations from the mean is zero.
  - Find the median of the given data.
- Find the mean and median of the data :  
35, 48, 92, 76, 64, 52, 51, 63 and 71.  
If 51 is replaced by 66, what will be the new median ?
- The mean of  $x$ ,  $x+2$ ,  $x+4$ ,  $x+6$  and  $x+8$  is 11, find the mean of the first three observations.
- Find the mean and median of all the positive factors of 72.