16 Area Theorems

[Proof and Use]

16.1 INTRODUCTION

Area of a plane figure is the region bounded by it.

Students have already used formulae for finding the areas of different geometrical figures. For example :

$$\text{area of a triangle} = \frac{1}{2} \text{ base} \times \text{height}$$

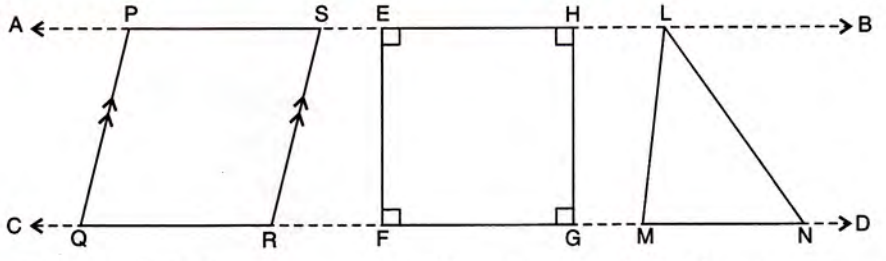
$$\text{area of a rectangle} = \text{length} \times \text{breadth}$$

$$\text{area of a parallelogram} = \text{base} \times \text{height} \qquad \text{and so on.}$$

In the current chapter, we shall be *comparing* the areas of different geometrical figures, such as parallelograms, rectangles and triangles subject to certain conditions.

1. Equal figures mean, the figures equal in area.
2. Congruent figures are always equal in area, but the converse is not always true.

16.2 FIGURES BETWEEN THE SAME PARALLELS

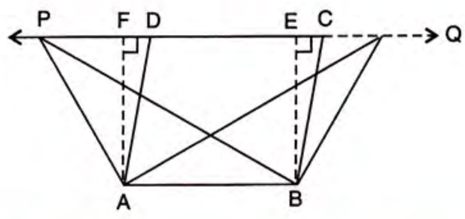


If a parallelogram PQRS, a rectangle EFGH and a triangle LMN are so drawn that their bases lie on the same straight line (say, CD) and their other vertices lie on another straight line (say, AB) parallel to CD, then the parallelogram PQRS, the rectangle EFGH and the triangle LMN are said to be between the same parallels.

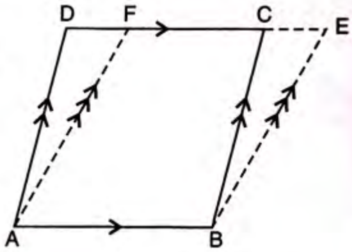
It is obvious that *the parallelogram, the rectangle and the triangle between the same parallels have equal altitudes* (height).

Note : 1. In the figure, given above, if QR = FG = MN, we say that the figures PQRS, EFGH and LMN are on equal bases and between the same parallels.

2. In the figure, given alongside, if PQ is parallel to AB, then the figures PAB, FABE, DABC, QAB, etc. are said to be on the same base and between the same parallels.



Theorem 19



Parallelograms on the same base and between the same parallels are equal in area.

Given : Parallelograms ABCD and ABEF are on the same base AB and between the same parallels AB and DE.

To Prove : Area of (//gm ABCD) = Area of (//gm ABEF).

Proof :

Statement :	Reason :
In Δ ADF and Δ BCE,	
1. AD = BC	[Opposite sides of //gm ABCD]
2. $\angle$ADF = $\angle$BCE	[Corresponding angles]
3. $\angle$AFD = $\angle$BEC	[Corresponding angles]
$\therefore$ $\angle$DAF = $\angle$CBE	[Since, two angles of both the Δs are equal; therefore their third angle will also be equal]
$\Rightarrow$ Δ ADF $\cong$ Δ BCE	[A.S.A.]
$\Rightarrow$ Area (Δ ADF) = Area (Δ BCE)	[Congruent Δs are equal in area]
$\Rightarrow$ Area (Δ ADF) + Area (ABCF) = Area (Δ BCE) + Area (ABCF)	[Adding, area (ABCF) on both the sides]
$\Rightarrow$ **Area (//gm ABCD) = Area (//gm ABEF)**	

Hence proved.

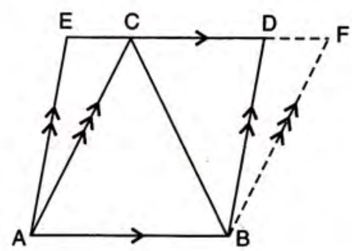
Corollary :

Since, rectangle is a parallelogram also, the above theorem can also be stated as :

"The area of a parallelogram is equal to the area of a rectangle on the same base and between the same parallels."

Theorem 20

The area of a triangle is half that of a parallelogram on the same base and between the same parallels.



Given : Triangle ABC and parallelogram ABDE on the same base AB and between the same parallels AB and ED.

To Prove : Area (Δ ABC) = $\frac{1}{2}$ Area (//gm ABDE)

Construction : Complete the parallelogram ABFC.

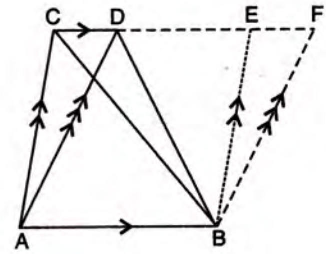
Proof :

Statement :	Reason :
1. Since, BC is the diagonal of //gm ABFC.	
$\therefore$ Area (Δ ABC) = $\frac{1}{2}$ Area (//gm ABFC)	[Diagonal divides a //gm into two equal triangles]
2. Area (//gm ABFC) = Area (//gm ABDE)	[//gms on the same base and between the same parallels are equal in area]
$\therefore$ **Area (Δ ABC) = $\frac{1}{2}$ Area (//gm ABDE)**	[From statements 1 and 2]

Hence Proved.

Theorem 21

Triangles on the same base and between the same parallels are equal in area.



Given : Δ ABC and Δ ABD are on the same base AB and between the same parallels AB and CD.

To Prove : Area (Δ ABC) = Area (Δ ABD).

Construction : Complete the //gms ABEC and ABFD.

Proof :

	Statement :	Reason :
1.	BC is diagonal of //gm ABEC,	
	∴ Area (Δ ABC) = $\frac{1}{2}$ Area (//gm ABEC)	[Diagonal bisects the //gm]
2.	BD is diagonal of //gm ABFD,	
	∴ Area (Δ ABD) = $\frac{1}{2}$ Area (//gm ABFD)	[Diagonal bisects the //gm]
3.	Area (//gm ABEC) = Area (//gm ABFD)	[//gms on the same base and between the same parallels are equal in area]
	∴ **Area (Δ ABC) = Area (Δ ABD)**	[From statements 1, 2 and 3]

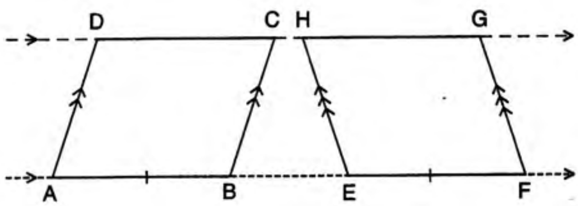
Hence Proved.

Corollaries :

1. *Parallelograms on equal bases and between the same parallels are equal in area.*

From the given figure,

Area (//gm ABCD) = Area (//gm EFGH).



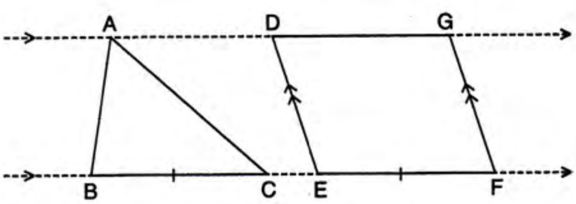
Similarly, if ABCD is a parallelogram and EFGH is a rectangle on equal bases and between the same parallels, then also

Area (//gm ABCD)

= Area (rect. EFGH).

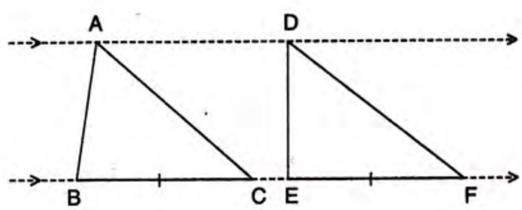
2. *Area of a triangle is half the area of the parallelogram, if both are on equal bases and between the same parallels.*
In the given figure,

Area (Δ ABC) = $\frac{1}{2}$ Area (//gm DEFG)



3. *Two triangles are equal in area if they are on the equal bases and between the same parallels.*
In the given figure,

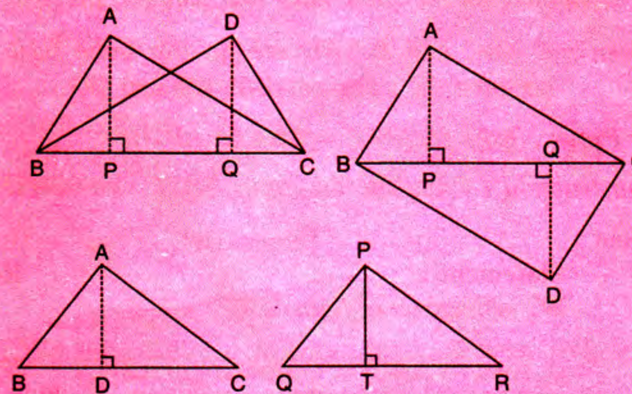
Area (Δ ABC) = Area (Δ DEF)



If two triangles have equal area and stand on the same base (or, equal bases) then their corresponding altitudes are euqal.

In each of the given figures, Δ ABC and Δ DBC are on the same base (BC) and have equal areas, then their corresponding altitudes are equal, *i.e.*, AP = DQ.

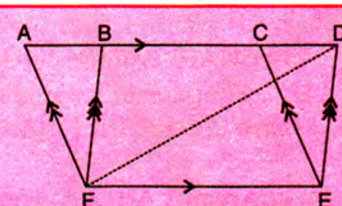
Similarly, if base BC of Δ ABC is equal to base QR of Δ PQR and their areas are also equal, then the corresponding altitudes AD and PT of these two triangles are also equal *i.e.* AD = PT



1 In the adjoining figure, area of parallelogram AFEC is 140 cm². State, giving reason, the area of :

(i) parallelogram BFED.

(ii) triangle BFD.



Solution :

(i) Parallelograms BFED and AFEC are on the same base FE and between the same parallels AD // FE, so they are equal in area.

∴ **Ar. (parallelogram BFED)** = Ar. (parallelogram AFEC)

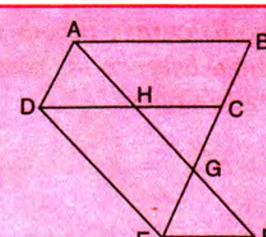
= **140 cm²** **Ans.**

(ii) Δ BFD and parallelogram BFED are on the same base BD and between the same parallels BD // FE, so area of the triangle BFD is half the area of parallelogram BFED.

∴ **Ar. (Δ BFD)** $= \frac{1}{2} \times$ Ar. (parallelogram BFED)

$= \frac{1}{2} \times 140\ \text{cm}^2 =$ **70 cm²** **Ans.**

2 In the given figure, AB // DC // EF, AD // BE and DE // AF. Prove that the area of parallelogram DEFH is equal to the area of parallelogram ABCD.



Solution :

Parallelogram DEFH and parallelogram DEGA are on the same base DE and between the same parallels DE // AF, so they are equal in area.

i.e. Area of DEFH = Area of DEGA I

Parallelogram ABCD and parallelogram DEGA are on the same base AD and between the same parallels AD // BE; so they are equal in area.

i.e. Area of ABCD = Area of DEGA II

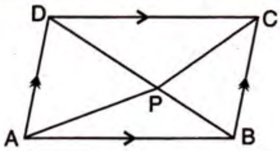
From I and II, Area of DEFH = Area of ABCD **Hence proved.**

3 P is any point inside a parallelogram ABCD. Prove that :

Area (Δ APB) + Area (Δ CPD)

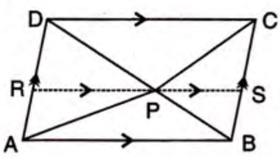
= Area (Δ APD) + Area (Δ BPC)

Solution :



The adjoining figure shows a parallelogram ABCD. Point P is inside ABCD and PA, PB, PC and PD are joined.

Through point P, draw RS parallel to AB which meets AD at R and BC at S.



Since, Δ APB and parallelogram ARSB are on the same base AB and between the same parallels *i.e.* AB // RS.

$\therefore$ Area (Δ APB) = $\frac{1}{2}$ × Area (// gm ARSB) I

Similarly, Δ CPD and parallelogram DRSC are on the same base DC and between the same parallels DC // RS.

$\therefore$ Area (Δ CPD) = $\frac{1}{2}$ × Area (// gm DRSC) II

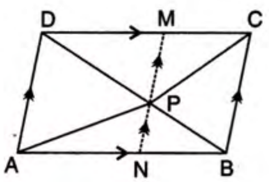
Adding I and II, we get : Area (Δ APB) + Area (Δ CPD)

= $\frac{1}{2}$ × Area (// gm ARSB) + $\frac{1}{2}$ × Area (// gm DRSC)

= $\frac{1}{2}$ [Area (// gm ARSB) + Area (// gm DRSC)]

= $\frac{1}{2}$ × Area (// gm ABCD) III

Now, draw MN through point P such that MN // AD and cuts AB at N and CD at M.



Since Δ APD and parallelogram ANMD are on the same base AD and between the same parallels (AD // MN).

$\therefore$ Area (Δ APD) = $\frac{1}{2}$ × Area (// gm ANMD)

Similarly, Δ BPC and parallelogram BNMC are on the same base BC and between the same parallels (BC // MN).

$\therefore$ Area (Δ BPC) = $\frac{1}{2}$ × Area (// gm BNMC)

Adding, Area (Δ APD) + Area (Δ BPC)

= $\frac{1}{2}$ × Area (// gm ABCD) IV

From equations III and IV, we get :

Area (Δ APB) + Area (Δ CPD)

= Area (Δ APD) + Area (Δ BPC)

Hence proved.

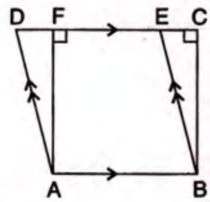
EXERCISE 16(A)

1. In the given figure, if area of triangle ADE is 60 cm^2; state, giving reason, the area of :

   (i) parallelogram ABED;

   (ii) rectangle ABCF;

   (iii) triangle ABE.



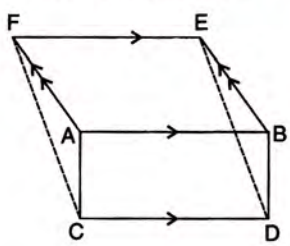
2. The given figure shows a rectangle ABDC and a parallelogram ABEF;drawn on opposite sides of AB. Prove that :

   (i) quadrilateral CDEF is a parallelogram;

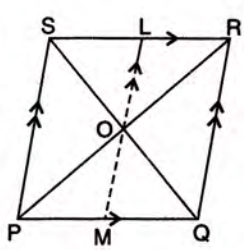
   (ii) Area of quad. CDEF

   = Area of rect. ABDC

   + Area of //gm. ABEF.



3. In the given figure, diagonals PR and QS of the parallelogram PQRS intersect at point O and LM is parallel to PS. Show that :



   (i) 2 Area (Δ POS) = Area (//gm PMLS)

   (ii) Area (Δ POS) + Area (Δ QOR)

   $= \frac{1}{2}$ Area (//gm PQRS)

   (iii) Area (Δ POS) + Area (Δ QOR)

   =Area (Δ POQ) + Area (Δ SOR).

4. In parallelogram ABCD, P is a point on side AB and Q is a point on side BC.

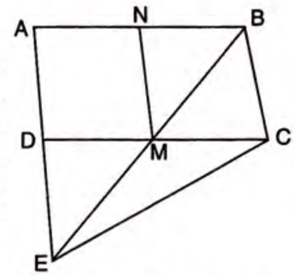
   Prove that :

   (i) Δ CPD and Δ AQD are equal in area.

   (ii) Area (Δ AQD)

   = Area (Δ APD) + Area (Δ CPB)

5. In the given figure, M and N are the mid-points of the sides DC and AB respectively of the parallelogram ABCD.



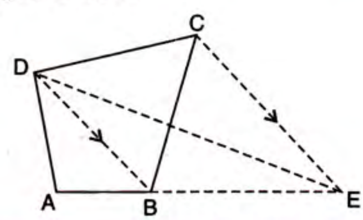
   If the area of parallelogram ABCD is 48 cm^2;

   (i) state the area of the triangle BEC.

   (ii) name the parallelogram which is equal in area to the triangle BEC.

6. In the following figure, CE is drawn parallel to diagonal DB of the quadrilateral ABCD which meets AB produced at point E.

   Prove that Δ ADE and quadrilateral ABCD are equal in area.

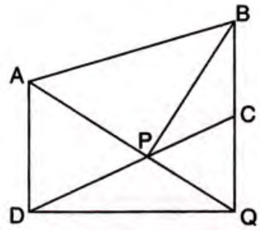


ΔADE = ΔADB + ΔBDE

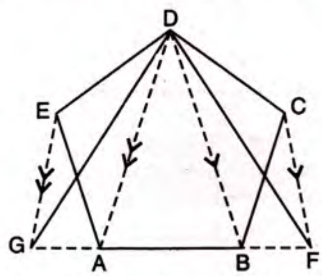
= ΔADB + ΔBDC

= Quad. ABCD

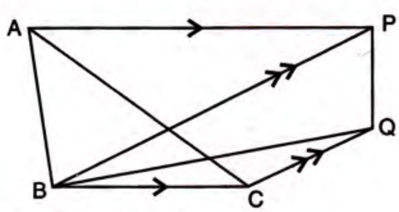
7. ABCD is a parallelogram, a line through A cuts DC at point P and BC produced at Q. Prove that triangle BCP is equal in area to triangle DPQ.



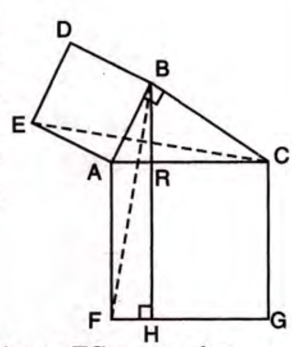
8. The given figure shows a pentagon ABCDE. EG drawn parallel to DA meets BA produced at G and CF drawn parallel to DB meets AB produced at F.
Prove that the area of pentagon ABCDE is equal to the area of triangle GDF.



9. In the given figure, AP is parallel to BC, BP is parallel to CQ. Prove that the areas of triangles ABC and BQP are equal.



10. In the figure given alongside, squares ABDE and AFGC are drawn on the side AB and the hypotenuse AC of the right triangle ABC.



If BH is perpendicular to FG, prove that :

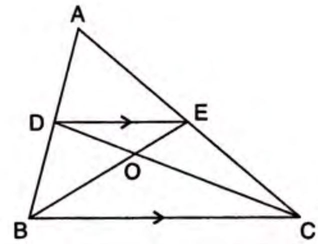
(i) Δ EAC $\cong$ Δ BAF.

(ii) Area of the square ABDE
= Area of the rectangle ARHF.

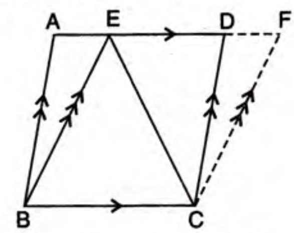
11. In the following figure, DE is parallel to BC. Show that :

(i) Area (Δ ADC) = Area (Δ AEB)

(ii) Area (Δ BOD) = Area (Δ COE).

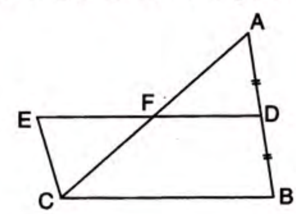


12. ABCD and BCFE are parallelograms. If area of triangle EBC = 480 cm^2, AB = 30 cm and BC = 40 cm; Calculate;



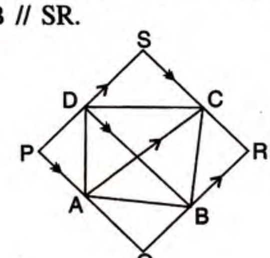
(i) area of parallelogram ABCD;
(ii) area of the parallelogram BCFE;
(iii) length of altitude from A on CD;
(iv) area of triangle ECF.

13. In the given figure, D is mid-point of side AB of ΔABC and BDEC is a parallelogram.



Prove that :
Area of Δ ABC = Area of // gm BDEC.

14. In the following figure, AC // PS // QR and PQ // DB // SR.

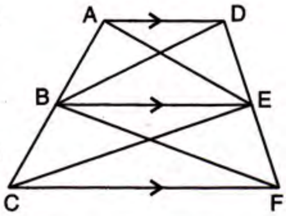


Prove that :
Area of quadrilateral PQRS = 2 × Area of quad. ABCD.

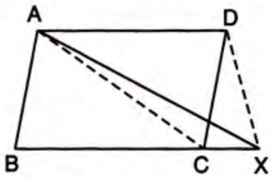
15. ABCD is a trapezium with AB // DC. A line parallel to AC intersects AB at point M and BC at point N. Prove that :
area of Δ ADM = area of Δ ACN.

Join C and M

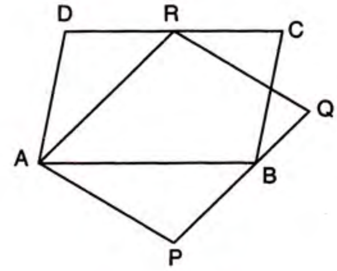
16. In the given figure, AD // BE // CF.
Prove that :
area (ΔAEC)
= area (ΔDBF)



17. In the given figure, ABCD is a parallelogram BC is produced to point X. Prove that :
area (Δ ABX) = area (quad. ACXD)



18. The given figure shows parallelograms ABCD and APQR. Show that these parallelograms are equal in area.
[Join B and R]



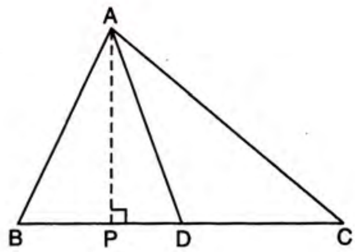
4 Prove that a median divides a triangle into two triangles of equal area.

Solution :

Given : Δ ABC with AD as median.

To prove : Area (Δ ABD) = Area (Δ ADC) = $\frac{1}{2}$ Area (Δ ABC)

Construction : Draw AP ⊥ BC.



Proof : Since, area of a Δ = $\frac{1}{2}$ base × height (altitude)

∴ Area (Δ ABD) = $\frac{1}{2}$ BD × AP

and, Area (Δ ADC) = $\frac{1}{2}$ DC × AP.

= $\frac{1}{2}$ BD × AP [∵ DC = BD]

∴ **Area (Δ ABD) = Area (Δ ADC) = $\frac{1}{2}$ Area (Δ ABC)**

Hence Proved.

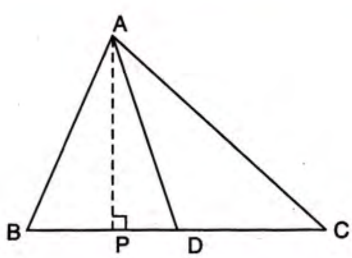
5 In Δ ABC, AD divides BC in the ratio $m : n$. Show that $\frac{\text{Area } (\Delta\, ABD)}{\text{Area } (\Delta\, ADC)} = \frac{m}{n}$.

Solution :

Given : AD divides BC in the ratio $m : n$, therefore, $\frac{BD}{DC} = \frac{m}{n}$

To Prove : $\frac{\text{Area}(\Delta\, ABD)}{\text{Area}(\Delta\, ADC)} = \frac{m}{n}$

Construction : Draw AP ⊥ BC.



Proof : Since, Area (Δ ABD) $= \frac{1}{2}$ BD × AP

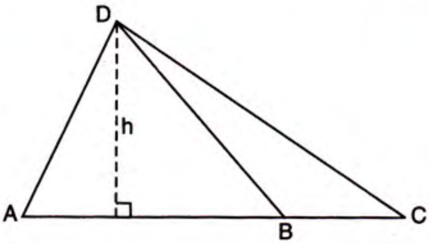
and, Area (Δ ADC) $= \frac{1}{2}$ DC × AP

$$\therefore \quad \frac{\text{Area } (\Delta \text{ABD})}{\text{Area } (\Delta \text{ADC})} = \frac{\frac{1}{2}\text{BD} \times \text{AP}}{\frac{1}{2}\text{DC} \times \text{AP}} = \frac{\text{BD}}{\text{DC}} = \frac{m}{n} \qquad \left(\text{Given : } \frac{\text{BD}}{\text{DC}} = \frac{m}{n}\right)$$

Hence Proved.

16.3 TRIANGLES WITH THE SAME VERTEX AND BASES ALONG THE SAME LINE

The given figure shows Δ ABD, Δ BCD and Δ ACD with the same vertex D and bases along the same straight line AC, so they have the same height. In such a case, the areas of triangles are in the ratio of their bases.



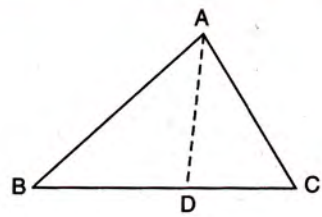
$$\therefore \text{(i)} \quad \frac{\text{Area of } \Delta\text{ABD}}{\text{Area of } \Delta\text{BCD}} = \frac{\text{AB}}{\text{BC}}$$

$$\text{(ii)} \quad \frac{\text{Area of } \Delta\text{ABD}}{\text{Area of } \Delta\text{ACD}} = \frac{\text{AB}}{\text{AC}} \quad \text{and}$$

$$\text{(iii)} \quad \frac{\text{Area of } \Delta\text{BCD}}{\text{Area of } \Delta\text{ACD}} = \frac{\text{BC}}{\text{AC}}$$

6 In triangle ABC, D is a point in side BC such that 2BD = 3DC. Prove that the area of triangle ABD $= \frac{3}{5}$ × Area of Δ ABC.

Solution :



$$2\text{BD} = 3\text{DC}$$

$$\Rightarrow \quad \frac{\text{BD}}{\text{DC}} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{\text{BD}}{\text{BD}+\text{DC}} = \frac{3}{3+2} \Rightarrow \frac{\text{BD}}{\text{BC}} = \frac{3}{5}$$

Since, Δ ABD and Δ ABC have the same vertex A and their bases along the same straight line BC, the areas of the triangles are in the ratio of their bases.

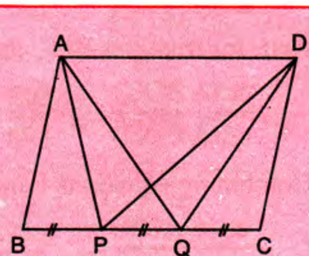
$$\therefore \frac{\text{Ar.}(\Delta \text{ABD})}{\text{Ar.}(\Delta \text{ABC})} = \frac{\text{BD}}{\text{BC}} \Rightarrow \frac{\text{Ar.}(\Delta \text{ABD})}{\text{Ar.}(\Delta \text{ABC})} = \frac{3}{5} \Rightarrow \textbf{Ar. } (\Delta \textbf{ ABD}) = \frac{3}{5} \times \textbf{Ar. } (\Delta \textbf{ ABC})$$

Hence proved.

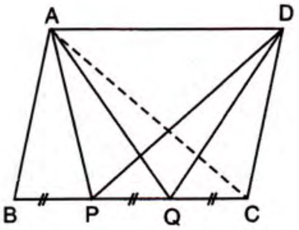
7 In parallelogram ABCD, points P and Q lie on side BC and trisect it. Prove that :

ar.(Δ APQ) = ar.(Δ DPQ)

$= \frac{1}{6}$ × ar. (parallelogram ABCD)



Solution :



Given : BP = PQ=QC

$\Rightarrow$ Bases of triangles ABP and ABC are in the ratio 1 : 3

i.e. BP : BC = 1 : 3

and their vertices are at the same point (point A)

$$\therefore \quad \frac{ar.(\Delta\ ABP)}{ar.(\Delta\ ABC)} = \frac{BP}{BC} = \frac{1}{3}$$

$$\Rightarrow \quad ar(\Delta\ ABP) = \frac{1}{3} \times ar.(\Delta\ ABC) \qquad ....I$$

In parallelogram ABCD, AC is diagonal so it bisects the parallelogram.

$$\Rightarrow \quad ar.(\Delta\ ABC) = \frac{1}{2} \times ar.(//gm\ ABCD) \qquad ...II$$

Equations I and II give :

$$ar.(\Delta\ ABP) = \frac{1}{3} \times \frac{1}{2} \times ar.(//gm\ ABCD)$$

$$\Rightarrow \quad ar.(\Delta\ ABP) = \frac{1}{6} \times ar.(//gm\ ABCD) \qquad ....III$$

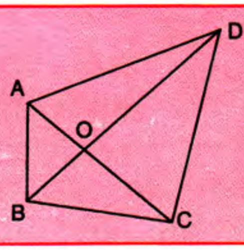
Since, Δ ABP, Δ APQ and Δ DPQ are on equal bases and between the same parallels; therefore:

$$ar.(\Delta\ ABP) = ar.(\Delta\ APQ) = ar.(\Delta\ DPQ) \qquad .....IV$$

$$\mathbf{ar.(\Delta\ APQ) = ar.(\Delta\ DPQ) = \frac{1}{6} \times ar.(//gm\ ABCD)} \qquad \text{(From equations III and IV)}$$

Hence proved.

8 In the given figure, ABCD is a quadrilateral with diagonals AC and BD intersecting at point O.
Prove that :
ar.(Δ AOD) × ar.(Δ BOC) = ar.(Δ AOB) × ar.(Δ COD)



Solution :

Whenever, the triangles have their bases along the same line and vertices at the same point, the ratio between their areas is equal to ratio between their bases.

$\therefore$ For triangles AOB and AOD,

$$\frac{ar.(\Delta\ AOB)}{ar.(\Delta\ AOD)} = \frac{BO}{DO} \qquad ....I$$

And, for triangles BOC and COD,

$$\frac{ar.(\Delta\ BOC)}{ar.(\Delta\ COD)} = \frac{BO}{DO} \qquad ....II$$

Combining equations I and II, we get :

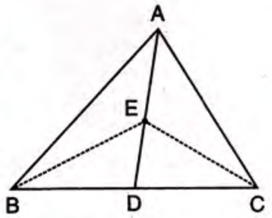
$$\frac{ar.(\Delta\ AOB)}{ar.(\Delta\ AOD)} = \frac{ar.(\Delta\ BOC)}{ar.(\Delta\ COD)}$$

$\Rightarrow$ **ar.(Δ AOD) × ar.(Δ BOC) = ar.(Δ AOB) × ar.(Δ COD)** **Hence proved.**

EXERCISE 16(B)

1. Show that :
   (i) a diagonal divides a parallelogram into two triangles of equal area.
   (ii) the ratio of the areas of two triangles of the same height is equal to the ratio of their bases.
   (iii) the ratio of the areas of two triangles on the same base is equal to the ratio of their heights.

2. In the given figure; AD is median of Δ ABC and E is any point on median AD. Prove that Area (Δ ABE) = Area (Δ ACE).



3. In the figure of question 2, if E is the mid point of median AD, then prove that :
   Area (Δ ABE) = $\frac{1}{4}$ Area (Δ ABC).

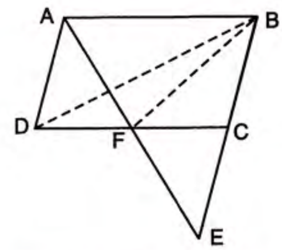
4. ABCD is a parallelogram. P and Q are the mid-points of sides AB and AD respectively.
   Prove that area of triangle APQ = $\frac{1}{8}$ of the area of parallelogram ABCD.

   Join PD and BD.

5. The base BC of triangle ABC is divided at D so that BD = $\frac{1}{2}$ DC.
   Prove that area of Δ ABD = $\frac{1}{3}$ of the area of Δ ABC.

6. In a parallelogram ABCD, point P lies in DC such that DP : PC = 3 : 2. If area of Δ DPB = 30 sq. cm, find the area of the parallelogram ABCD.

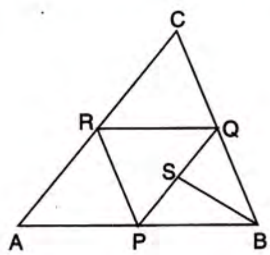
7. ABCD is a parallelogram in which BC is produced to E such that CE = BC and AE intersects CD at F.



   If ar.(Δ DFB) = 30 cm^2; find the area of parallelgoram

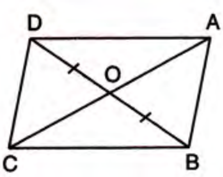
   By A.S.A. Δ ADF $\cong$ Δ ECF
   $\Rightarrow$ DF = CF and so BF is median of Δ BDC.

8. The following figure shows a triangle ABC in which P, Q and R are mid-points of sides AB, BC and CA respectively. S is mid-point of PQ.
   Prove that : ar.(Δ ABC) = 8 $\times$ ar.(Δ QSB)



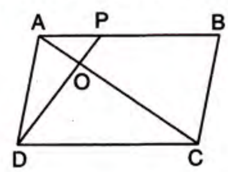
EXERCISE 16(C)

1. In the given figure, the diagonals AC and BD intersect at point O. If OB = OD and AB//DC, prove that :



   (i) Area (Δ DOC) = Area (Δ AOB).
   (ii) Area (Δ DCB) = Area (Δ ACB).
   (iii) ABCD is a parallelogram.

2. The given figure shows a parallelogram ABCD with area 324 sq. cm. P is a point in AB such that AP : PB = 1 : 2. Find :



   (i) the area of Δ APD.
   (ii) the ratio OP : OD.

3. In Δ ABC, E and F are mid-points of sides AB and AC respectively. If BF and CE intersect each other at point O, prove that the Δ OBC and quadrilateral AEOF are equal in area.

> First of all prove that :
> Ar. of Δ BOE = Ar. of Δ COF
> Now, BF is a median
> ⇒ Δ ABF = Δ CBF
> ⇒ Δ ABF – Δ BOE = Δ CBF – Δ COF

4. In parallelogram ABCD, P is mid-point of AB. CP and BD intersect each other at point O. If area of Δ POB = 40 cm^2, find :
   (i) OP : OC
   (ii) Areas of Δ BOC and Δ PBC
   (iii) Areas of Δ ABC and parallelogram ABCD.

5. The medians of a triangle ABC intersect each other at point G. If one of its medians is AD, prove that :
   (i) Area (Δ ABD) = 3 × Area (Δ BGD)
   (ii) Area (Δ ACD) = 3 × Area (CGD)
   (iii) Area (Δ BGC) = $\frac{1}{3}$ × Area (Δ ABC)

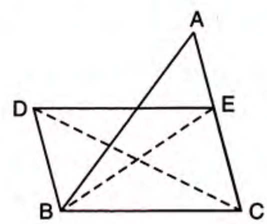
6. The perimeter of a triangle ABC is 37 cm and the ratio between the lengths of its altitudes be 6 : 5 : 4. Find the lengths of its sides.

> Let the sides be x cm, y cm and $(37 - x - y)$ cm.
> Also, let the lengths of altitudes be $6a$ cm, $5a$ cm and $4a$ cm
> ∵ Area of a triangle = $\frac{1}{2}$ × base × altitude
> $\therefore \frac{1}{2} \times x \times 6a = \frac{1}{2} \times y \times 5a = \frac{1}{2}(37 - x - y) \times 4a$
> $\Rightarrow 6x = 5y = 148 - 4x - 4y$
> $\Rightarrow 6x = 5y$ and $6x = 148 - 4x - 4y$
> $\Rightarrow 6x - 5y = 0$ and $10x + 4y = 148$

7. In the given figure, E is mid-point of AB and DE meets diagonal AC at point F. If ABCD is a parallelogram and area of Δ ADF is 60 cm^2; find :
   (i) DF : FE
   (ii) area of Δ ADE
   (iii) area of Δ ADB
   (iv) area of // gm ABCD

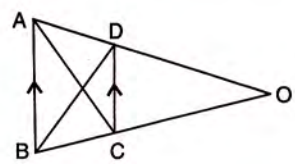
> $AE = \frac{1}{2} AB = \frac{1}{2} DC$
> Δ DFC ~ Δ EFA
> $\Rightarrow \frac{DF}{FE} = \frac{DC}{AE} = \frac{DC}{\frac{1}{2}DC} = \frac{2}{1}$

8. In the following figure, BD is parallel to CA, E is mid-point of CA and BD = $\frac{1}{2}$CA.



Prove that : ar.(Δ ABC) = 2 × ar.(Δ DBC)

9. In the following figure, OAB is a triangle and AB//DC.



If the area of Δ CAD = 140 cm^2 and the area of Δ ODC = 172 cm^2, find
   (i) the area of Δ DBC
   (ii) the area of Δ OAC
   (iii) the area of Δ ODB.