15 Construction of Polygons

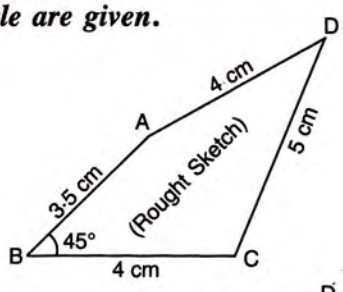
[Using Ruler and Compass Only]

15.1 CONSTRUCTION OF QUADRILATERALS

1. To construct a quadrilateral means; to find (locate) its four vertices.
2. Always, draw a rough free-hand sketch, before starting the actual construction.

1. ***To construct a quadrilateral, whose four sides*** **and** ***one angle are given.***

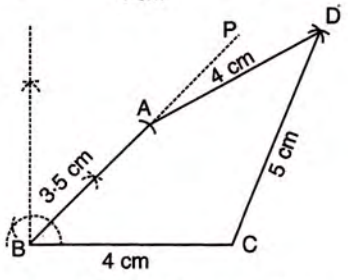
Let quadrilateral ABCD has AB = 3·5 cm, BC = 4·0 cm, CD = 5·0 cm, DA = 4·0 cm and ∠B = 45°.



Steps :

1. Draw BC = 4·0 cm.
2. Through B, draw BP such that ∠B = 45°.
3. From BP, cut BA = 3·5 cm.
4. With A and C as centres and radii 4 cm and 5 cm respectively, draw arcs cutting each other at D.
5. Join AD and CD.

ABCD is the required quadrilateral.



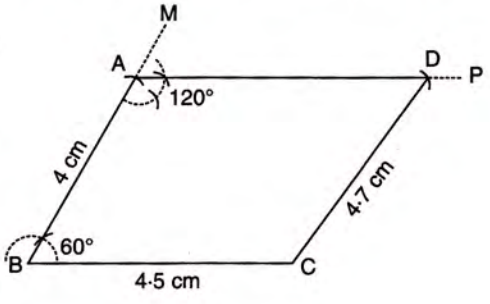
2. ***To construct a quadrilateral, whose three sides and two consecutive angles are given.***

Let quadrilateral ABCD has AB = 4·0 cm, BC = 4·5 cm, CD = 4·7 cm, ∠B = 60° and ∠A = 120°.

Steps :

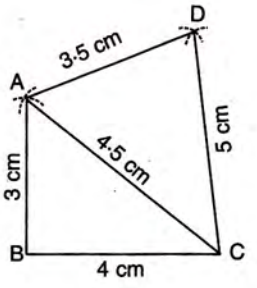
1. Draw BC = 4·5 cm
2. Construct angle MBC = 60° and then from BM cut BA = 4·0 cm.
3. Draw AP such that ∠A = 120°.
4. With C as centre and radius = 4·7 cm draw an arc cutting AP at D.
5. Join C and D.

ABCD is the required quadrilateral.



3. ***To construct a quadrilateral, whose four sides and one diagonal are given.***

Let quadrilateral ABCD has AB = 3·0 cm, BC = 4·0 cm, CD = 5·0 cm, DA = 3·5 cm and diagonal AC = 4·5 cm.



Steps :

Using given dimensions; first of all construct triangle ABC and then triangle ADC.

4. ***To construct a quadrilateral, whose three sides*** **and** ***two diagonals*** **are given.**

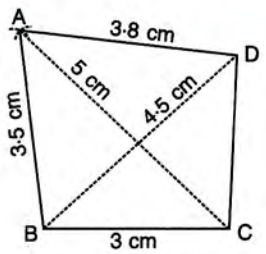
Let quadrilateral ABCD has AB = 3·5 cm, BC = 3·0 cm, AD = 3·8 cm, diagonal AC = 5·0 cm and diagonal BD = 4·5 cm

Steps :

(i) First of all, construct triangle ABC and then triangle ABD.

(ii) Join C and D.

ABCD is the required quadrilateral.



15.2 CONSTRUCTION OF PARALLELOGRAMS

1. ***To construct a parallelogram, whose two consecutive sides and the*** **included angle** ***are given.***

Let parallelogram ABCD has AB = 3·0 cm, BC = 4·0 cm and ∠B = 60°.

Since, opposite sides of a parallelogram are equal,

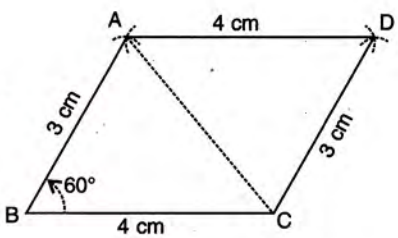
∴ AB = DC = 3·0 cm and BC = AD = 4·0 cm.

Steps :

(i) Taking AB = 3 cm, BC = 4 cm and ∠B = 60°, construct triangle ABC.

(ii) Now, construct triangle ADC.

ABCD is the required parallelogram.



2. ***To construct a parallelogram, whose one side and both the diagonals are given.***

Let parallelogram ABCD has side BC = 4·5 cm, diagonal AC = 5·6 cm and diagonal BD = 5·0 cm.

Steps :

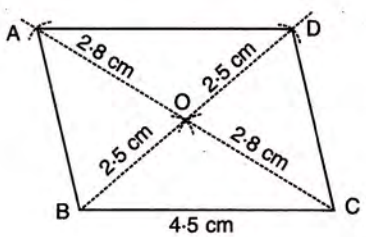
(i) Since diagonals of a parallelogram bisect each other;

construct triangle OBC, such that :

$OB = \frac{1}{2} BD = \frac{1}{2} \times 5{\cdot}0 \text{ cm} = 2{\cdot}5 \text{ cm}$

$OC = \frac{1}{2} AC = \frac{1}{2} \times 5{\cdot}6 \text{ cm} = 2{\cdot}8 \text{ cm}$

and, BC = 4·5 cm



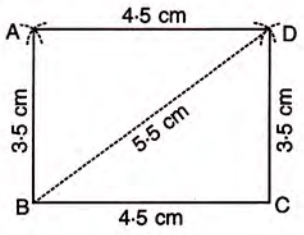
(ii) Produce BO upto D, such that OD = OB = 2·5 cm and produce CO upto A, such that OA = OC = 2·8 cm.

(iii) Join AB, AD and CD.

ABCD is the required parallelogram.

3. ***To construct a parallelogram, whose two consecutive sides*** **and** ***one diagonal are given.***

Let parallelogram ABCD has AB = 3·5 cm, BC = 4·5 cm and diagonal BD = 5·5 cm.



Steps :

1. Draw triangle BCD.
2. Draw triangle BAD.

4. ***To construct a parallelogram, whose two diagonals*** **and** ***included angle are given.***

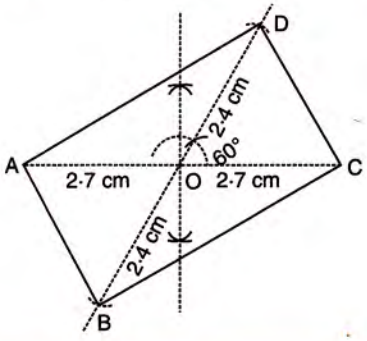
Let parallelogram ABCD has diagonal AC = 5·4 cm, diagonal BD = 4·8 cm and the acute angle between the diagonals = 60°.

Steps :

1. Draw AC = 5·4 cm and locate its mid point O.
2. Draw line BOD such that ∠DOC = 60° and

   $OB = OD = \frac{1}{2} BD = \frac{1}{2} \times 4{\cdot}8 \text{ cm} = 2{\cdot}4 \text{ cm}$.
3. Join AB, BC, CD and DA.

   ABCD is the required parallelogram.



In this construction, we are given diagonal AC = 5·4 cm and diagonal BD = 4·8 cm. The lengths of both the diagonals (5·4 cm and 4·8 cm) are divisible by 2, correct to one place of decimal to get 2·7 cm and 2·4 cm respectively. Lengths 2·7 cm and 2·4 cm can easily be measured with the help of a scale in your geometry box.

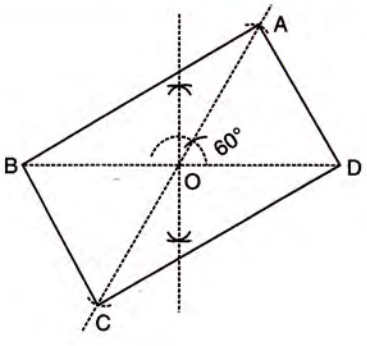
But, if we are given AC = 5·4 cm and BD = 4·5 cm, now $\frac{1}{2}BD = \frac{1}{2} \times 4{\cdot}5 \text{ cm} = 2{\cdot}25$ cm, which can not correctly be measured with the help of the scale you use.

To overcome this situation, instead of starting with AC = 5·4 cm, you start the construction of the parallelogram with BD = 4·5 cm and continue accordingly.

Steps :

1. Draw BD = 4·5 cm.
2. Draw perpendicular bisector of BD to get its mid-point O.
3. Draw line AOC, such that ∠AOD = 60°.
4. From AOC, cut $OA = OC = \frac{1}{2}AC = \frac{1}{2} \times 5{\cdot}4$ cm $= 2{\cdot}7$ cm.
5. Join AB, BC, CD and DA.

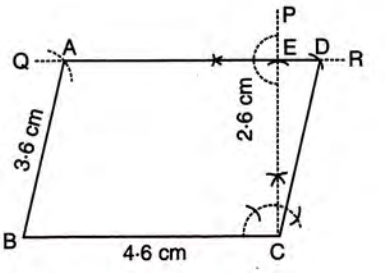
   ABCD is the required parallelogram.



5. ***To construct a parallelogram, whose two adjacent sides and height are given.***

Let parallelogram ABCD has adjacent sides AB = 3·6 cm, BC = 4·6 cm and height corresponding to side BC = 2·6 cm.

Steps :



1. Draw BC = 4·6 cm.
2. At C, draw CP ⊥ BC.
3. From CP, cut CE = 2·6 cm = height of parallelogram.
4. Through E, draw perpendicular to CP to get QR parallel to BC.
5. With B as centre and radius = AB = 3·6 cm, draw an arc which cuts QR at A.
6. With C as centre and radius = 3·6 cm, draw one more arc which cuts QR at D.

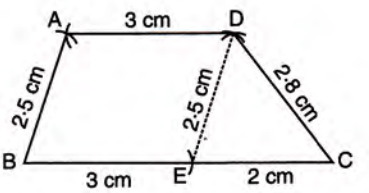
ABCD is the required parallelogram.

15.3 CONSTRUCTION OF TRAPEZIUM

To construct a trapezium ABCD, whose four sides are given.

Let AD // BC, AD = 3·0 cm, AB = 2·5 cm, BC = 5·0 cm and CD = 2·8 cm.

Steps :



1. Draw BC = 5·0 cm.
2. From BC, cut BE = AD = 3·0 cm.
3. Draw triangle DEC, such that
   DE = AB = 2·5 cm and CD = 2·8 cm.
4. Taking B and D as centres and radii 2·5 cm and 3·0 cm respectively, draw arcs cutting each other at A.
5. Join AB and AD.

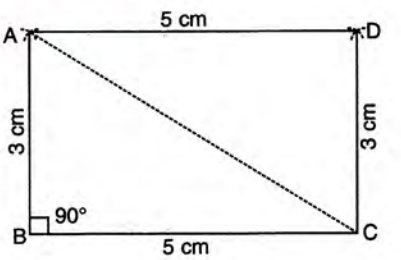
ABCD is the required trapezium.

15.4 CONSTRUCTION OF RECTANGLES

1. ***To construct a rectangle whose adjacent sides are given.***

Let rectangle ABCD has adjacent sides AB = 3·0 cm and BC = 5·0 cm.

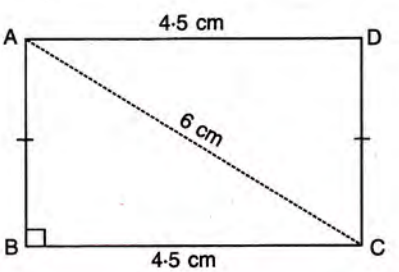
Since each angle of a rectangle is 90° and opposite sides are equal, therefore, first of all construct Δ ABC and then Δ ADC.



ABCD is the required rectangle.

2. ***To construct a rectangle, whose one side and one diagonal are given.***

Let for rectangle ABCD, side BC = 4·5 cm and diagonal AC = 6·0 cm.



Steps :

1. Draw a right-angled triangle ABC.
2. Draw another right-angled triangle ADC.

ABCD is the required rectangle.

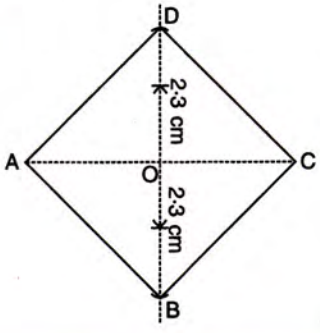
15.5 CONSTRUCTION OF RHOMBUS

To construct a rhombus, whose diagonals are given.

Let rhombus ABCD has diagonal AC = 6·0 cm and diagonal BD = 4·6 cm.

Note : Diagonals of a rhombus bisect each other at right angle.

Steps :



1. Draw AC = 6·0 cm.
2. Draw perpendicular bisector to AC which cuts AC at O.
3. From this perpendicular, cut OD and OB such that,

   $OD = OB = \frac{1}{2} BD = \frac{1}{2} \times 4{\cdot}6 \text{ cm} = 2{\cdot}3 \text{ cm}.$
4. Join AB, BC, CD and DA.

   ABCD is the required rhombus.

What will be your steps of construction, if :

1. diagonal AC = 6·3 cm and diagonal BD = 4·6 cm ?
2. diagonal AC = 6·2 cm and diagonal BD = 4·7 ?

15.6 CONSTRUCTION OF SQUARE :

To construct a square whose diagonal is given.

The diagonals of a square are equal and bisect each other at right angle.

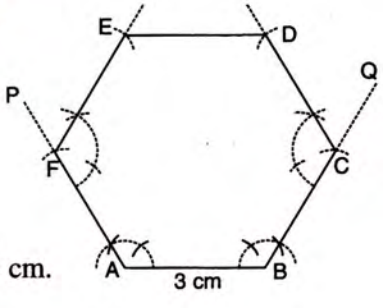
∴ Similar method is used as for the construction of a rhombus.

15.7 TO CONSTRUCT A REGULAR HEXAGON

Construct a regular hexagon of side 3·0 cm.

Method 1 : (Each interior angle of a regular hexagon is 120° and its opposite sides are parallel)

Steps :

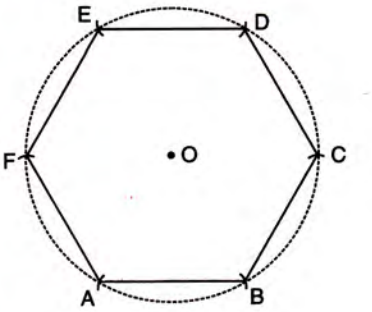


1. Draw a line segment AB = 3·0 cm.
2. Through A, draw AP so that ∠PAB = 120° and through B, draw BQ so that ∠ABQ = 120°.
3. From AP, cut AF = 3·0 cm and from BQ, cut BC = 3·0 cm.
4. At C, construct CD = 3·0 cm, so that ∠BCD = 120°.
5. At F, construct FE = 3·0 cm so that ∠AFE = 120°.
6. Join E and D.

   ABCDEF is the required regular hexagon.

Method II : (The length of the side of a regular hexagon is equal to the radius of its circumcircle)

Steps :



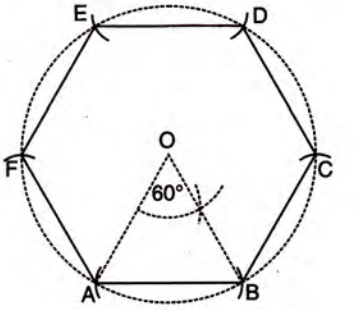
1. Draw a circle of radius 3·0 cm.
2. Taking any point A on the circumference of the circle as centre, draw arcs of same radii (*i.e.* 3·0 cm) which cut the circumference at B and F.
3. With B and F as centres, again draw two arcs of same radii (3 cm) which cut the circumference at C and E respectively.
4. With C or E as centre, draw one more arc of the same radius (3·0 cm) which cuts the circumference at point D.

   In this way, the circumference of the circle is divided into six equal parts.
5. Join AB, BC, CD, DE, EF and FA.

ABCDEF is the required regular hexagon.

Method III : (The angle subtended by each side of a regular hexagon at the centre of its circumcircle is $\frac{360^\circ}{6} = 60^\circ$)

Steps :



1. Construct an isosceles triangle AOB so that base AB = 3 cm and $\angle AOB = 60^\circ$.
2. With O as centre and OA or OB as radius, draw a circle.
3. With B as centre and BA as radius, draw an arc which cuts the circle at point C.
4. Again with C as centre and radius equal to AB, draw one more arc which cuts the circle at point D.
5. Continue in the similar manner to get the points E and F.
6. Join BC, CD, DE, EF and FA.

ABCDEF is the required regular hexagon.

Note : In this method, for steps 1 and 2, students can first draw a circle with centre O and radius = 3 cm; and then construct angle AOB = 60°, whose arms cut the circle at points A and B.

EXERCISE 15

Construct a quadrilateral ABCD, when :

1. AB = 3·2 cm, BC = 5·2 cm, CD = 6·2 cm, DA = 4·2 cm and BD = 5·2 cm.
2. AB = 7·2 cm, BC = 5·8 cm, CD = 6·3 cm, AD = 4·3 cm and angle A = 75°.
3. Angle A = 90°, AB = 4·6 cm, BD = 6·4 cm, AC = 6·0 cm and CD = 4·2 cm.
4. AB = 3·8 cm, AC = 4·8 cm, AD = 2·8 cm, angle A = 105° and angle B = 60°.
5. BC = 7·5 cm, AC = 5·8 cm, AD = 3·6 cm, CD = 4·2 cm and angle A = 120°.
6. AD = AB = 4 cm, BC = 2·8 cm, CD = 2·5 cm and angle BAD = 45°.
7. AB = 6·3 cm, BC = CD = 4·2 cm and ∠ABC = ∠BCD = 90°.

Construct a parallelogram ABCD, when :

8. AB = 4·4 cm, AD = 6·2 cm and AC = 4·8 cm.
9. Diagonal AC = 6·4 cm, diagonal BD = 8·2 cm and angle between the diagonals = 60°.
10. AB = 5·8 cm, diagonal AC = 8·2 cm and diagonal BD = 6·2 cm.
11. AB = 6·0 cm, AD = 5·0 cm and ∠A = 45°
12. Base AB = 6·5 cm, BC = 4 cm and the altitude corresponding to AB = 3·1 cm.
13. AB = 4·5 cm, ∠B = 120° and the distance between AB and DC = 3·0 cm.
14. Base BC = 5·6 cm, diagonal BD = 6·5 cm and altitude = 3·2 cm.

Construct a rectangle ABCD, when :

15. Its sides are 6·0 cm and 7·2 cm.
16. One side = 4 cm and one diagonal is 5 cm. Measure the length of other side.
17. One diagonal = 6·0 cm and the acute angle between the diagonals = 45°.
18. Area = 24 cm^2 and base = 4·8 cm.
19. Area = 36 cm^2 and height = 4·5 cm.

Construct a trapezium ABCD, when :

20. AB = 4·8 cm, BC = 6·8 cm, CD = 5·4 cm, angle B = 60° and AD // BC.
21. AB = CD = 3·2 cm, BC = 6·0 cm, AD = 4·4 cm and AD // BC.

Construct a rhombus ABCD, when :

22. Its one side = 6 cm and ∠A = 60°.
23. One side = 5·4 cm and one diagonal is 7·0 cm.
24. Diagonal AC = 6·3 cm and diagonal BD = 5·8 cm.
25. One side = 5·0 cm and height = 2·6 cm.
26. ∠A = 60° and height = 3·0 cm.
27. Diagonal AC = 6·0 cm and height = 3·5 cm.

Construct a square ABCD, when :

28. One side = 4·5 cm.
29. One diagonal = 5·4 cm.
30. Perimeter = 24 cm.
31. Construct a rhombus, having given one side = 4·8 cm and one angle = 75°.
32. Construct a regular hexagon of side
    (i) 2·5 cm (ii) 3·2 cm.
33. Using ruler and compasses only, construct the quadrilateral ABCD, having given AB = 5 cm, BC = 2·5 cm, CD = 6 cm, angle BAD = 90° and the diagonal AC = 5·5 cm.
34. Using ruler and compasses only, construct a trapezium ABCD, in which the parallel sides AB and DC are 3·3 cm apart; AB = 4·5 cm, angle A = 120°, BC = 3·6 cm and angle B is obtuse.
35. Using ruler and compasses only, construct the quadrilateral ABCD, having given AB = 5 cm, BC = 2·5cm, CD = 6 cm, ∠BAD = 90° and diagonal BD = 5·5 cm.
36. Using ruler and compasses only, construct a parallelogram ABCD using the following data:
    AB = 6 cm, AD = 3 cm and ∠DAB = 45°. If the bisector of ∠DAB meets DC at P, prove that ∠APB is a right angle.
37. The perpendicular distances between the pair of opposite sides of a parallelogram are 3 cm and 4 cm, and one of its angles measures 60°. Using ruler and compasses only, construct the parallelogram.
38. Draw parallelogram ABCD with the following data :
    AB = 6 cm, AD = 5 cm and ∠DAB = 45°.
    Let AC and DB meet in O and let E be the mid-point of BC. Join OE. Prove that :
    (i) OE // AB (ii) $OE = \frac{1}{2}AB$.
39. Using ruler and compasses only, construct a rectangle each of whose diagonals measure 6 cm and the diagonals intersect at an angle of 45°.