# 14 Rectilinear Figures

## [Quadrilaterals : Parallelogram, Rectangle, Rhombus, Square and Trapezium]

## 14.1 INTRODUCTION

1. ***Rectilinear*** means along a straight line or in a straight line or forming a straight line.
2. A plane figure bounded by straight lines is called a *rectilinear figure*.
3. A closed plane figure, bounded by at least three line segments, is called a *polygon*.

## 14.2 NAMES OF POLYGONS

A polygon is named by the number of sides in it, as given below :

| No. of sides | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| Name | Triangle | Quadrilateral | Pentagon | Hexagon | Heptagon | Octagon |

,etc.

**1. *Convex Polygon :***

If each angle of a polygon is *less* than 180°; the polygon is called a *convex polygon*.

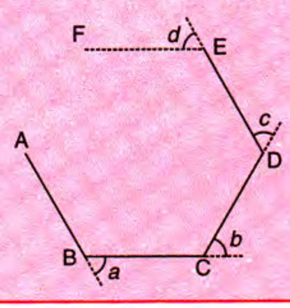
**2. *Concave Polygon :***

If *at least* one angle of a polygon is *greater* than 180°; it is called a *concave polygon*.

Unless otherwise stated, a polygon means a *convex polygon*.

1. *In a polygon of n sides, the sum of the interior angles is equal to (2n – 4) right angles.*
2. If the sides of a polygon are produced in order (*i.e.* all the sides are produced either in clockwise direction or in anti-clockwise direction); the sum of exterior angles so formed is 4 right angles. *i.e.*

   $\angle a + \angle b + \angle c + \angle d + \ldots\ldots\ldots = 4$ rt. angles $= 360°$.




**1** The sum of the interior angles of a polygon is five times the sum of its exterior angles. Find the number of sides in the polygon.

***Solution :***

Let the number of sides be $n$.

Given : The sum of the interior angles of the polygon

$= 5 \times$ the sum of its exterior angles.

$\therefore \quad (2n - 4) \times 90° = 5 \times 360°$

On solving, we get : $2n = 24$ and $n = 12$

$\therefore$ **The required no. of sides in the polygon = 12** **Ans.**

**2** One angle of an eight-sided polygon is 100° and the other angles are equal. Find the measure of each equal angle.

***Solution :***

The sum of the interior angles of an eight-sided polygon

$$= (2n - 4) \times 90° = (2 \times 8 - 4) \times 90° = 1080°$$

Since, one angle of the polygon $= 100°$

$\therefore$ The sum of the remaining seven angles

$$= 1080° - 100° = 980°$$

Since, these angles are equal

$\therefore$ **The measure of each equal angle** $= \dfrac{980°}{7} = \mathbf{140°}$ **Ans.**

**Alternative method :**

Let each of the remaining seven equal angles $= x°$.

$\therefore$ The sum of these seven angles $= 7x°$

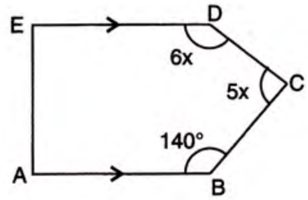
And, $7x° + 100° = (2 \times 8 - 4) \times 90° \Rightarrow 7x° = 1080° - 100° = 980°$

$\Rightarrow \quad x° = \dfrac{980°}{7} = \mathbf{140°}$ **Ans.**

**3** In a pentagon ABCDE, AB is parallel to ED and angle B = 140°. Find the angles C and D, if ∠C : ∠D = 5 : 6.

***Solution :***

The rough sketch of the given pentagon will be as shown alongside.




Since, AB//ED $\Rightarrow \angle A + \angle E = 180°$.

Given : $\angle C : \angle D = 5 : 6 \Rightarrow$ if $\angle C = 5x$, $\angle D = 6x$.

Now, $\angle A + \angle B + \angle C + \angle D + \angle E = (2 \times 5 - 4) \times 90°$

$\Rightarrow (\angle A + \angle E) + 140° + 5x + 6x = 540°$

$\Rightarrow 180° + 140° + 5x + 6x = 540°$

*i.e.* $11x = 540° - 320° \Rightarrow 11x = 220°$ and $x = \dfrac{220°}{11} = 20°$

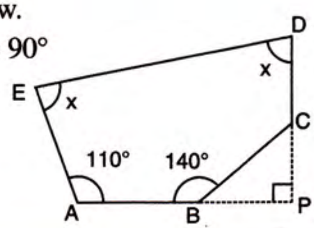
$\therefore$ $\angle C = 5x = 5 \times 20° = \mathbf{100°}$ and $\angle D = 6x = 6 \times 20° = \mathbf{120°}$ **Ans.**

**4** In the pentagon ABCDE, angle A = 110°, angle B = 140° and angle D = angle E. The sides AB and DC, when produced, meet at right angle. Calculate angles BCD and E.

***Solution :***

According to the given statement, the figure will be as shown below.

In the figure, AB and DC produced meet at point P, therefore $\angle P = 90°$




$\angle B = 140° \Rightarrow \angle PBC = 180° - 140° = 40°$

and $\angle BCP = 90° - 40° = 50°$

$\therefore$ $\angle BCD = 180° - 50° = \mathbf{130°}$ **Ans.**

Let angle D = angle E = $x$

Since, $\angle A + \angle B + \angle BCD + \angle D + \angle E = (2 \times 5 - 4) \times 90°$

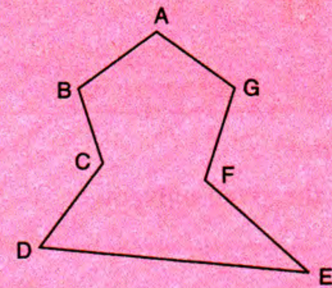
$\Rightarrow$ $110° + 140° + 130° + x + x = 540°$

*i.e.* $2x = 540° - 380° = 160°$ $\Rightarrow$ $x = 80°$

$\therefore$ **Angle E = $x$ = 80°** **Ans.**

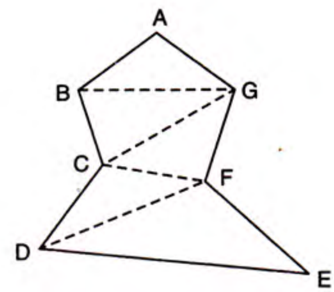
**5** By dividing into triangles, find the sum of the angles of the doubly re-entrant heptagon ABCDEFG as shown alongside.

Does the general value of $(2n - 4)$ right-angles hold for re-entrant polygon?




***Solution :***

On dividing the given figure into triangles, we get 5 triangles.




**Sum of the angles of given heptagon**

= Sum of the angles of 5 triangles

= $5 \times 180°$ = **900°** **Ans.**

$\because$ $n = 7$

$\therefore$ **$(2n - 4)$ right angles** $= (2 \times 7 - 4) \times 90°$

$= 10 \times 90° =$ **900°** **Ans.**

$\therefore$ **General value $(2n - 4)$ right angles holds for re-entrant polygons.** **Ans.**

## 14.3 REGULAR POLYGON

If *all the sides and all the angles of a polygon are equal,* it is called a ***regular polygon***.

A *regular polygon* means; a polygon with :

(i) *all its sides equal to each other,*

(ii) *all its interior angles equal to each other*

and, (iii) *all its exterior angles also equal to each other.*

1. Sum of interior angles of an '$n$' sided polygon (whether it is regular or not)

$= (2n - 4)$ rt. angles

and sum of its exterior angles = 4 right angles = 360°.

2. At each vertex of every polygon, Exterior angle + Interior angle = 180°.

3. Each interior angle of a regular polygon $= \dfrac{(2n-4) \text{ rt. angles}}{n} = \dfrac{(2n-4)\times 90°}{n}$.

4. Each exterior angle of a regular polygon $= \dfrac{4 \text{rt. angles}}{n} = \dfrac{360°}{n}$.

5. If each exterior angle of a regular polygon is $x°$, the number of sides in it = $\frac{360}{x}$.
6. Greater the number of sides in a regular polygon; *greater is* the value of its each *interior angle* and *smaller* is the value of its each *exterior angle*.

**6** Each interior angle of a regular polygon is 160°. Find the interior angle of another regular polygon whose number of sides is two-thirds the number of sides of the given polygon.

***Solution :***

**For the given polygon :**

$$\text{Each interior angle} = 160°$$

$$\Rightarrow \quad \text{each exterior angle} = (180 - 160)° = 20°$$

$$\therefore \quad \text{No. of sides in it} = \frac{360°}{20°} = 18$$

**For the other polygon :**

$$\text{No. of sides} = \frac{2}{3} \times 18 = 12$$

$$\Rightarrow \quad \text{Each exterior angle} = \frac{360°}{12} = 30°$$

and, **each interior angle** $= 180° - 30°$

$= \mathbf{150°}$ **Ans.**

**7** If the difference between an exterior angle of a regular polygon of '$n$' sides and an exterior angle of another regular polygon of '$(n + 1)$' sides is equal to 5°; find the value of '$n$'.

***Solution :***

Each exterior angle of n-sided regular polygon = $\frac{360°}{n}$.

Each exterior angle of $(n + 1)$ sided regular polygon = $\frac{360°}{n+1}$

*Given* : $\frac{360°}{n} - \frac{360°}{n+1} = 5$ [Greater the no. of sides, smaller is the exterior angle]

On solving, we get : $\mathbf{n = 8}$ **Ans.**

## EXERCISE 14(A)

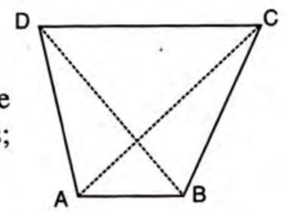
1. The sum of the interior angles of a polygon is four times the sum of its exterior angles. Find the number of sides in the polygon.
2. The angles of a pentagon are in the ratio 4 : 8 : 6 : 4 : 5. Find each angle of the pentagon.
3. One angle of a six-sided polygon is 140° and the other angles are equal. Find the measure of each equal angle.
4. In a polygon, there are 5 right angles and the remaining angles are equal to 195° each. Find the number of sides in the polygon.

5. Three angles of a seven sided polygon are 132° each and the remaining four angles are equal. Find the value of each equal angle.
6. Two angles of an eight sided polygon are 142° and 176°. If the remaining angles are equal to each other; find the magnitude of each of the equal angles.
7. In a pentagon ABCDE, AB is parallel to DC and ∠A : ∠E : ∠D = 3 : 4 : 5. Find angle E.
8. AB, BC and CD are the three consecutive sides of a regular polygon. If ∠BAC = 15°; find,
   (i) each interior angle of the polygon.
   (ii) each exterior angle of the polygon.
   (iii) number of sides of the polygon.
9. The ratio between an exterior angle and an interior angle of a regular polygon is 2 : 3. Find the number of sides in the polygon.
10. The difference between an exterior angle of $(n - 1)$ sided regular polygon and an exterior angle of $(n + 2)$ sided regular polygon is 6°. Find the value of $n$.
11. Two alternate sides of a regular polygon, when produced, meet at right angle. Find :
    (i) the value of each exterior angle of the polygon;
    (ii) the number of sides in the polygon.

## 14.4 QUADRILATERALS

A closed plane figure bounded by four line segments is called a ***quadrilateral***.

The adjoining figure shows a *quadrilateral*. It is bounded by four line segments AB, BC, CD and DA, called the *sides* of the quadrilateral.




The points A, B, C and D are its four ***vertices***.

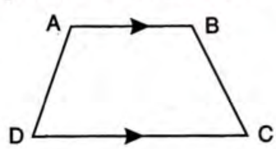
The line segments joining the opposite vertices of a quadrilateral are called its ***diagonals***. Thus, a quadrilateral ABCD has two diagonals; namely, AC and BD.

**Sum of the angles of a quadrilateral is 360°.**

## 14.5 SPECIAL KINDS OF QUADRILATERALS

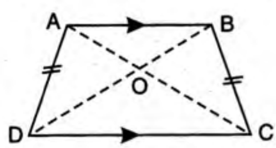
### 1. Trapezium :

A ***trapezium*** is a quadrilateral in which *one pair of opposite sides is parallel* but the other pair of opposite sides is *non-parallel.*




The given figure shows a quadrilateral in which the sides AB and DC are *parallel* whereas the sides AD and BC are *non-parallel.*

Therefore, quadrilateral ABCD is a ***trapezium***.



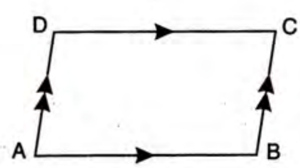

If the *non-parallel* sides **AD** and **BC** of the trapezium ABCD are **equal**, it is called an ***isosceles trapezium***. In this case:

(i) ∠D = ∠C and ∠A = ∠B. (ii) AC = BD

(iii) If diagonals intersect at point O, OA = OB and OC = OD.

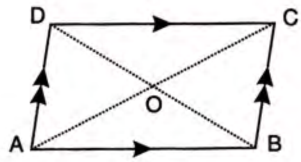
### 2. Parallelogram :

A *parallelogram* is a quadrilateral in which *opposite sides* are *parallel*. It is denoted by  .




The adjoining figure shows a **parallelogram** ABCD, since, AB // DC and AD // BC.

*In a parallelogram ABCD :*




(i) **opposite sides are parallel** *i.e.* AB // DC and AD // BC,

(ii) **opposite sides are equal** *i.e.* AB = DC and AD = BC,

(iii) **opposite angles are equal** *i.e.* $\angle A = \angle C$ and $\angle B = \angle D$,

(iv) **consecutive angles** (conjoined angles) **are supplementary**,

*i.e.* $\angle A + \angle B = 180°$, $\angle B + \angle C = 180°$

$\angle C + \angle D = 180°$ and $\angle D + \angle A = 180°$,

(v) diagonals bisect each other,

*i.e.* $OA = OC = \frac{1}{2} AC$ and $OB = OD = \frac{1}{2} BD$,

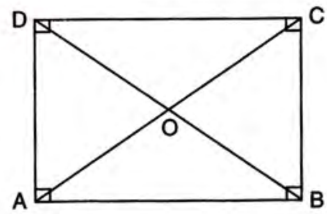
(vi) each diagonal divides the parallelogram into two congruent triangles,

*i.e.* $\Delta ABC \cong \Delta CDA$ and $\Delta ABD \cong \Delta CDB$

(vii) diagonals divide the parallelogram into four triangles of equal area,

*i.e.* $\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA = \frac{1}{4}$ (parallelogram ABCD)

**3. Rectangle :**




A *rectangle* is a parallelogram in which :

(i) diagonals are equal, *i.e.* AC = BD;

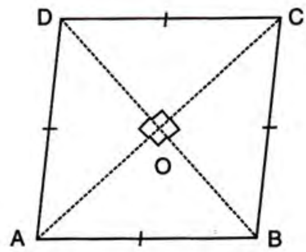
(ii) diagonals bisect each other,

*i.e.* OA = OC and OB = OD;

(iii) each angle is a right angle,

*i.e.* $\angle A = \angle B = \angle C = \angle D = 90°$.

**4. Rhombus :**




A *rhombus* is a parallelogram in which :

(i) all the sides are equal, *i.e.* AB = BC = CD = DA,

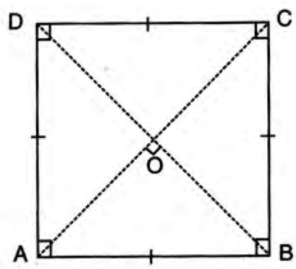
(ii) diagonals bisect each other at 90°, *i.e.*

OA = OC; OB = OD and

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$.

(iii) each diagonal bisects angles at the vertices *i.e.* diagonal AC bisects angles A and C; and diagonal BD bisects angles B and D.

**5. Square :**




A *square* is a parallelogram in which :

(i) all the sides are equal;

(ii) each angle is 90°;

(iii) diagonals are equal;

(iv) diagonals bisect each other at right angle and

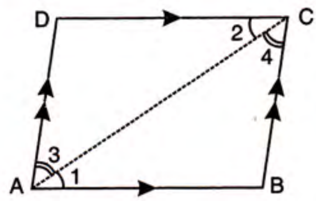
(v) each diagonal bisects angles at the vertices.

## DIAGONAL PROPERTIES OF DIFFERENT KINDS OF PARALLELOGRAMS

| Properties | Parallelogram | Rectangle | Rhombus | Square |
|---|---|---|---|---|
| Diagonals bisect each other | √ | √ | √ | √ |
| Diagonals are equal | — | √ | — | √ |
| Diagonals bisect vertex angles | — | — | √ | √ |
| Diagonals are perpendicular to each other | — | — | √ | √ |
| Each diagonal forms 2 congruent triangles | √ | √ | √ | √ |
| Diagonals form 4 equal triangles | √ | √ | √ | √ |
| [Equal triangles means, triangles equal in area] | | | | |
| Diagonals form 4 congruent triangles | — | — | √ | √ |

### Theorem 11

*In a parallelogram, both the pairs of opposite sides are equal.*




**Given :** A parallelogram ABCD.

**To Prove :** AB = DC and AD = BC

**Construction :** Join A and C.

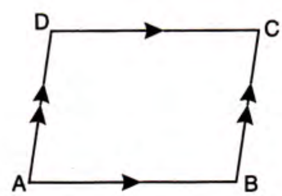
**Proof :**

| Statement : | Reason : |
|---|---|
| In triangles ABC and CDA : | |
| 1. $\angle 1 = \angle 2$ | [Alternate angles as AC cuts parallel sides AB and DC] |
| 2. $\angle 3 = \angle 4$ | [Alternate angles as AC cuts parallel sides AD and BC]. |
| 3. AC = AC | [Common]. |
| $\Rightarrow$ $\Delta ABC \cong \Delta CDA$ | [A.S.A.] |
| $\Rightarrow$ **AB = DC and AD = BC** | [Corresponding parts of congruent triangles are congruent]. |

**Hence Proved.**

### Theorem 12

*In a parallelogram, both the pairs of opposite angles are equal.*

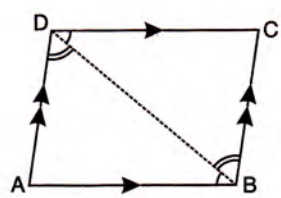



**Given :** A parallelogram ABCD in which AB // DC and AD // BC.

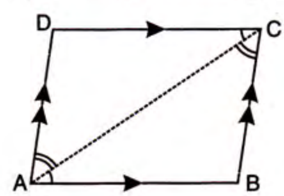
**To Prove :** $\angle A = \angle C$ and $\angle B = \angle D$

**Construction :** Draw diagonal BD.

**Proof :**




| Statement : | Reason : |
|---|---|
| Since, $\Delta ABD \cong \Delta CDB$ | [By A.S.A.] |
| $\Rightarrow$ $\angle A = \angle C$ | ......I [By CPCTC] |
| Now, draw diagonal AC and | |
| show that $\Delta ABC \cong \Delta CDA$ | [By A.S.A.] |
| $\Rightarrow$ $\angle B = \angle D$ | ....II [By CPCTC] |
| Equations I and II give : | |
| $\angle A = \angle C$ and $\angle B = \angle D$ | |




**Hence Proved.**

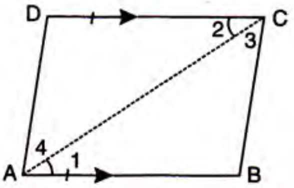
## Theorem 13

*If one pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.*

**Given :** A quadrilateral ABCD in which AB = DC and AB // DC.

**To Prove :** ABCD is a parallelogram.

**Construction :** Join A and C.




**Proof :**

| Statement : | Reason : |
|---|---|
| In triangles ABC and CDA : | |
| 1. $\angle 1 = \angle 2$ | [Alternate angles, since AC cuts parallel sides AB and DC] |
| 2. AB = DC | [Given] |
| 3. AC = AC | [Common] |
| $\Rightarrow$ $\Delta ABC \cong \Delta CDA$ | [S.A.S.] |
| $\Rightarrow$ $\angle 3 = \angle 4$ | [Corresponding parts of congruent $\Delta$s are congruent.] |
| But, $\angle 3$ and $\angle 4$ are alternate angles | |
| $\Rightarrow$ AD // BC | [If alternate angles are equal, lines are parallel.] |
| $\therefore$ **ABCD is a parallelogram** | [Opposite sides are parallel] |

**Hence Proved.**

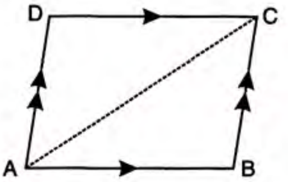
## Theorem 14

*Each diagonal of a parallelogram bisects the parallelogram.*

**Given :** A parallelogram ABCD

**To Prove :** Diagonal AC bisects the parallelogram ABCD.

**Proof :** Proceed as in theorem 11 to prove that $\Delta ABC \cong \Delta CDA$.




$\therefore$ $\Delta ABC = \Delta CDA = \frac{1}{2}$ (parallelogram ABCD)

$\therefore$ **AC bisects the parallelogram ABCD.**

Similarly, prove that diagonal **BD also bisects the parallelogram ABCD.**

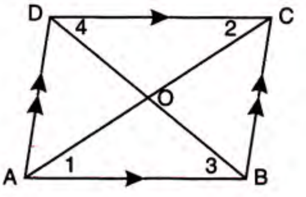
**Hence Proved.**

## Theorem 15

*The diagonals of a parallelogram bisect each other.*

**Given :** A parallelogram ABCD whose diagonals AC and BD intersect at O.

**To Prove :** OA = OC and OB = OD



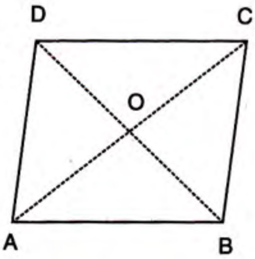

**Proof :**

| Statement : | Reason : |
|---|---|
| In triangles AOB and COD : | |
| 1. AB = DC | [Opposite sides of a parallelogram are equal]. |
| 2. $\angle 1 = \angle 2$ | [Alternate angles] |
| 3. $\angle 3 = \angle 4$ | [Alternate angles] |

| | | |
|---|---|---|
| ∴ | Δ AOB ≅ Δ COD | [A.S.A.] |
| ⇒ | **OA = OC** and **OB = OD** | [Corresponding parts of congruent triangles are congruent]. |

**Hence Proved.**

## Theorem 16




*Rhombus is a special parallelogram whose diagonals meet at right angles.*

***Given :*** A rhombus ABCD whose diagonals AC and BD meet at point O.

***To Prove :*** Diagonals AC and BD meet at right angle.

**Proof :**

Since, ABCD is a rhombus and sides of a rhombus are equal.

⇒ AB = BC ..........I

Since, rhombus ABCD is a parallelogram also and diagonals of a parallelogram bisect each other

⇒ OA = OC .........II

Also OB = OB .........III [Common]

From equations I, II and III, we get :

Δ AOB ≅ Δ COB [By S.S.S.]

⇒ ∠AOB = ∠BOC [By CPCTC]

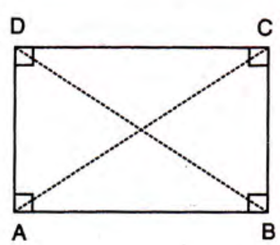
But ∠AOB + ∠BOC = 180° [AOC is a straight line]

⇒ $\angle AOB = \angle BOC = \frac{180^\circ}{2} = 90^\circ$

⇒ **Diagonals meet at right angles.**

**Hence Proved.**

## Theorem 17




*In a rectangle, diagonals are equal.*

***Given :*** Rectangle ABCD with diagonals AC and BD.

***To Prove :*** Diagonals are equal *i.e.* AC = BD

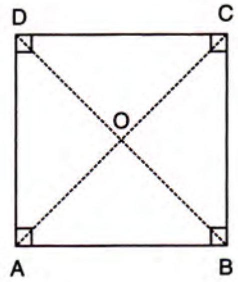
**Proof :**

In Δ DAB and Δ CBA

| | | |
|---|---|---|
| 1. | AD = BC | [Opp. sides of a rectangle] |
| 2. | AB = AB | [Common] |
| 3. | ∠DAB = ∠CBA | [Each 90°] |
| ∴ | Δ DAB ≅ Δ CBA | [By SAS] |
| ⇒ | **AC = BD** | |

**Hence Proved.**

### Theorem 18




*In a square, diagonals are equal and meet at right angles.*

***Given :*** A square ABCD with diagonals AC and BD intersecting each other at point O.

***To Prove :*** AC = BD and ∠AOB = 90°.

**Proof :**

In Δ DAB and Δ CBA

| | | |
|---|---|---|
| 1. | AD = BC | [Sides of a square] |
| 2. | ∠DAB = ∠CBA | [Each 90°] |
| 3. | AB = AB | [Common] |
| ⇒ | Δ DAB ≅ Δ CBA | [By S.A.S.] |
| ⇒ | **AC = BD** | [By CPCTC] |
| Now, prove that | Δ AOB ≅ Δ BOC | [By S.S.S.] |
| ⇒ | ∠AOB = ∠BOC | |

But, ∠AOB + ∠BOC = 180°

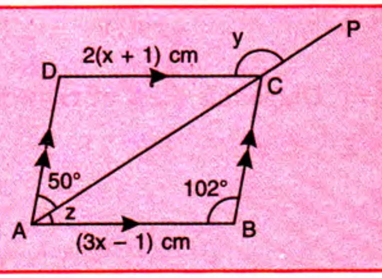
∴ $\angle AOB = \angle BOC = \dfrac{180^\circ}{2} = 90^\circ$

∴ **AC = BD ⇒ diagonals are equal**

and, **∠AOB = 90° ⇒ diagonals meet at right angles**

**Hence Proved.**

**8** In the given figure, ABCD is a parallelogram. Find the values of $x$, $y$ and $z$.




***Solution :***

Since, opposite sides of a parallelogram are equal, therefore AB = DC

⇒ $3x - 1 = 2(x + 1)$ *i.e.* $3x - 1 = 2x + 2$

⇒ $x = 3$

Since, AD//BC and AC is transversal

⇒ ∠BCA = ∠DAC [Alternate angles]

= 50°

In Δ ABC,

$z$ + ∠ABC + ∠BCA = 180°

⇒ $z + 102^\circ + 50^\circ = 180^\circ \Rightarrow z = 180^\circ - 152^\circ = 28^\circ$

**Alternative method :**

∠DAB + ∠ABC = 180° [Co-interior angles]

$(50^\circ + z) + 102^\circ = 180^\circ$ *i.e.* $z = 180^\circ - 152^\circ = 28^\circ$

Since, AB//DC and AC is transversal

$\Rightarrow$ $\angle ACD = z = 28°$ [Alternate angles]

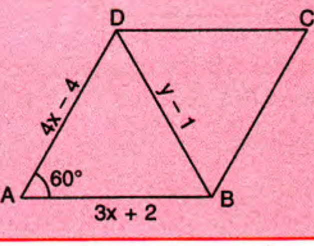
ACP is a straight line

$\Rightarrow$ $\angle ACD + y = 180°$ *i.e.* $28° + y = 180°$

$\Rightarrow$ $y = 180° - 28° = 152°$

$\therefore$ $x = 3, y = 152°$ and $z = 28°$ **Ans.**

**9** The given figure shows a rhombus ABCD. Find $x$ and $y$.




***Solution :***

Since, a rhombus has all its sides equal

$\therefore$ AB = AD

$\Rightarrow$ $3x + 2 = 4x - 4 \Rightarrow x = 6$

$\because$ $\angle DAB + \angle ABC = 180°$ [Co-interior angles]

$\Rightarrow$ $60° + \angle ABC = 180°$ *i.e.* $\angle ABC = 120°$

Each diagonal of a rhombus bisects angle at vertex

$\Rightarrow$ BD bisects $\angle ABC$, so $\angle ABD = \frac{1}{2}\angle ABC = \frac{1}{2} \times 120° = 60°$

Triangle ABD has two of its angles 60° each, so its third angle is also 60°

$\Rightarrow$ $\Delta$ ABD is an equilateral triangle

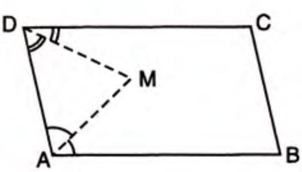
$\Rightarrow$ BD = AB *i.e.* $y - 1 = 3x + 2$

$\Rightarrow$ $y = 3 \times 6 + 2 + 1 = 21$ $\therefore$ $x = 6$ and $y = 21$ **Ans.**

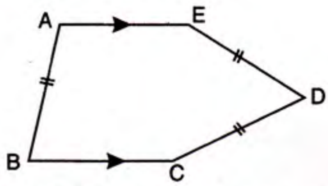
## EXERCISE 14(B)

1. State, 'true' or 'false'
   (i) The diagonals of a rectangle bisect each other.
   (ii) The diagonals of a quadrilateral bisect each other.
   (iii) The diagonals of a parallelogram bisect each other at right angle.
   (iv) Each diagonal of a rhombus bisects it.
   (v) The quadrilateral, whose four sides are equal, is a square.
   (vi) Every rhombus is a parallelogram.
   (vii) Every parallelogram is a rhombus.
   (viii) Diagonals of a rhombus are equal.
   (ix) If two adjacent sides of a parallelogram are equal, it is a rhombus.
   (x) If the diagonals of a quadrilateral bisect each other at right angle, the quadrilateral is a square.

2. In the figure, given below, AM bisects angle A and DM bisects angle D of parallelogram ABCD. Prove that : $\angle AMD = 90°$.



3. In the following figure, AE and BC are equal and parallel and the three sides AB, CD and DE are equal to one another. If angle A is 102°. Find angles AEC and BCD.




4. In a square ABCD, diagonals meet at O. P is a point on BC, such that OB = BP.

Show that :

(i) $\angle POC = \left(22\frac{1}{2}\right)^\circ$

(ii) $\angle BDC = 2\ \angle POC$

(iii) $\angle BOP = 3\ \angle COP$

5. The given figure shows a square ABCD and an equilateral triangle ABP.

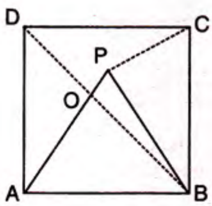
Calculate :

(i) $\angle AOB$

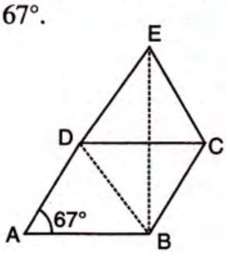
(ii) $\angle BPC$

(iii) $\angle PCD$

(iv) reflex $\angle APC$




6. In the given figure; ABCD is a rhombus with angle A = 67°.

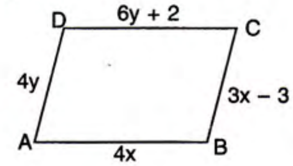



If DEC is an equilateral triangle, calculate :

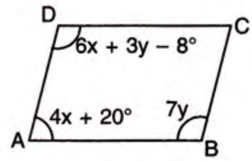
(i) $\angle CBE$ (ii) $\angle DBE$

7. In each of the following figures, ABCD is a parallelogram.

(i)




(ii)

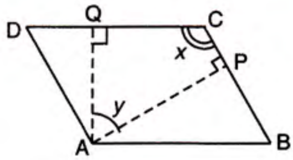



In each case, given above, find the values of $x$ and $y$.

8. The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Show that the quadrilateral is a trapezium.

9. In a parallelogram ABCD, AB = 20 cm and AD = 12 cm. The bisector of angle A meets DC at E and BC produced at F. Find the length of CF.

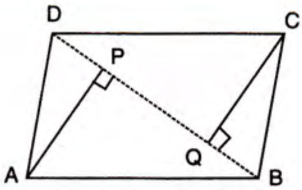
10. In parallelogram ABCD, AP and AQ are perpendiculars from vertex of obtuse angle A as shown. If $\angle x : \angle y = 2 : 1$; find the angles of the parallelogram.




**10** ABCD is a parallelogram in which AP and CQ are perpendiculars from vertices A and C respectively on diagonal BD. Prove that :

(i) $\Delta$ APB $\cong$ $\Delta$ CQD (ii) AP = CQ

***Solution :***




According to the given statement, the figure will be as shown alongside.

(i) In $\Delta$ APB and $\Delta$ CQD :

1. $\angle APB = \angle CQD$ [Each 90°]

2. ∠ABP = ∠CDQ [Alternate angles as AB // CD]

3. ∠PAB = ∠QCD [When two angles of a triangle are equal to two angles of the other triangle, their third angles are also equal]

4. AB = CD [Opp. sides of the parallelogram]

∴ **Δ APB ≅ Δ CQD** [By A.S.A.]

**Hence Proved.**

(ii) ∵ Δ APB ≅ Δ CQD :

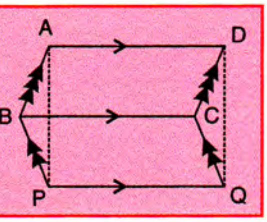
⇒ **AP = CQ** [By CPCTC]

**Hence Proved.**

**11** In the adjoining figure, ABCD and PBCQ are parallelograms. Prove that :

(i) APQD is a parallelogram.

(ii) AP = DQ (iii) Δ ABP ≅ Δ DCQ




***Solution :***

(i) Since, opposite sides of a parallelogram are parallel and equal, therefore

in parallelogram ABCD, AD // BC and AD = BC ......I

and, in parallelogram PBCQ, PQ // BC and PQ = BC .....II

Combining I and II, we get : AD // PQ and AD = PQ

⇒ **APQD is a parallelogram**

[If a pair of opposite sides of a quadrilateral are parallel and equal; the quadrilateral is parallelogram.] **Hence Proved.**

(ii) Since, opposite sides of a parallelogram are equal.

∴ In parallelogram APQD, **AP = DQ** **Hence Proved.**

(iii) In Δ ABP and Δ DCQ,

AB = DC [Opp. sides of //gm. ABCD]

BP = CQ [Opp. sides of //gm PBCQ]

and, AP = DQ [Proved above]

⇒ **Δ ABP ≅ Δ DCQ** [By S.S.S.]

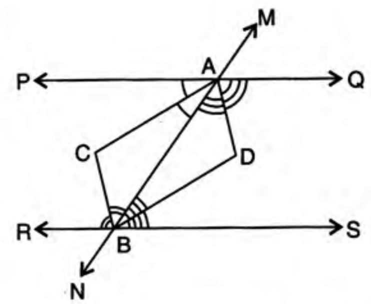
**Hence Proved.**

**12** A transversal cuts two parallel lines PQ and RS at points A and B respectively. The two interior angles at A are bisected and so are the two interior angles at B; the four bisectors form a quadrilateral ACBD. Prove that :

(i) ACBD is a rectangle.

(ii) CD is parallel to given parallel lines PQ and RS.

**Solution :**




According to the given statement, the figure will be as shown alongside.

In the figure, the transversal MN cuts parallel lines PQ and RS at points A and B respectively.

Bisectors of interior angles at A and bisectors of interior angles at B meet at points C and D to form a quadrilateral ACBD.

(i) **To prove :** ACBD is a rectangle *i.e.* each angle of the quadrilateral ACBD is 90°.

**Proof :**

AC bisects $\angle PAB \Rightarrow \angle CAB = \frac{1}{2}\angle PAB$ .......I

AD bisects $\angle QAB \Rightarrow \angle DAB = \frac{1}{2}\angle QAB$ .......II

BC bisects $\angle RBA \Rightarrow \angle CBA = \frac{1}{2}\angle RBA$ .......III

and, BD bisects $\angle SBA \Rightarrow \angle DBA = \frac{1}{2}\angle SBA$ .......IV

$\angle PAB + \angle RBA = 180°$ [Co-interior angles]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle RBA = 90°$

$\Rightarrow \angle CAB + \angle CBA = 90°$ [From I and III]

In $\Delta$ ACB, $\angle ACB + \angle CAB + \angle CBA = 180°$

$\Rightarrow \angle ACB + 90° = 180°$ and $\angle ACB = 90°$

Similarly, $\angle BDA = 90°$

$\angle PAB + \angle QAB = 180°$ [Straight line angle]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QAB = 90°$

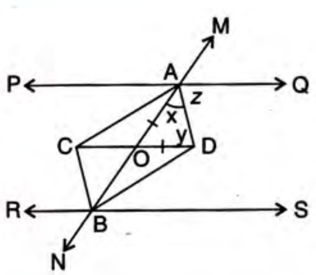
$\Rightarrow \angle CAB + \angle DAB = 90° \Rightarrow \angle CAD = 90°$

Similarly, $\angle CBD = 90°$

Since, each angle of quadrilateral ACBD is 90°, **ACBD is a rectangle.**

**Hence Proved.**

(ii) **To Prove :** CD is parallel to given parallel lines *i.e.* CD // PQ // RS.




**Proof :** Since, diagonals of a rectangle are equal and bisect each other, therefore AB = CD and let AB and CD bisect each other at point O.

Clearly, OA = OD $\Rightarrow \angle x = \angle y$

Given, AD bisects QAB $\Rightarrow \angle x = \angle z$

$\therefore \angle y = \angle z$

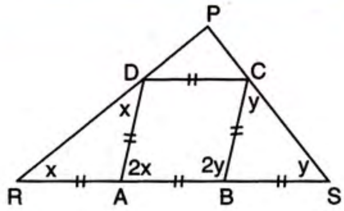
But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

$\therefore$ PQ // CD *i.e.* **CD // PQ // RS**

**Hence Proved.**

**13** ABCD is a rhombus. RABS is a straight line such that RA = AB = BS. Prove that : RD and SC when produced meet at right angle.

***Solution :***




According to the given statement, the figure will be as shown alongside, in which RD and SC produced meet at point P.

Also, RA = AB = BS = BC = CD = DA.

**To Prove :** $\angle P = 90°$

**Proof :** In $\Delta$ RAD, RA = AD $\Rightarrow \angle R = \angle ADR = x$ (let)

and, exterior angle DAB $= \angle R + \angle ADR = x + x = 2x$.

In $\Delta$ SBC, BS = BC $\Rightarrow \angle S = \angle BCS = y$ (let)

and, exterior angle ABC $= \angle S + \angle BCS = y + y = 2y$.

Since, the adjacent angles of a rhombus are supplementary

*i.e.* $\angle DAB + \angle ABC = 180° \Rightarrow 2x + 2y = 180°$ and $x + y = 90°$

In $\Delta$ RPS, $x + y = 90°$

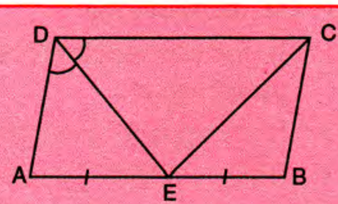
*i.e.* $\angle R + \angle S = 90° \Rightarrow \angle P = 180° - 90° = 90°$

**Hence Proved.**

**14** ABCD is a parallelogram. E is mid-point of AB and DE bisects angle D. Prove that :

(i) BC = BE (ii) CE bisects angle C

(iii) $\angle DEC = 90°$.




***Solution :***

(i) **To Prove :** BC = BE.

| | **Statement :** | **Reason :** |
|---|---|---|
| 1. | $\angle CDE = \angle DEA$ | [Alternate angles] |
| 2. | $\angle CDE = \angle EDA$ | [Given DE bisects $\angle D$] |
| $\therefore$ | $\angle DEA = \angle EDA$ | [From 1 and 2]. |
| 3. | AD = AE | [In a $\Delta$, sides opposite to equal angles are equal]. |
| 4. | AD = BC | [Opposite sides of parallelogram ABCD] |
| 5. | AE = BE | [Given E is mid-point of AB] |
| $\therefore$ | **BC = BE** | [From 3, 4 and 5] |

**Hence Proved.**

(ii) **To Prove :** CE bisects angle C.

**Proof :**

| | **Statement :** | **Reason :** |
|---|---|---|
| 1. | BC = BE | [From (i)] |
| $\Rightarrow$ | $\angle BEC = \angle BCE$ | [In a $\Delta$, angles opposite to equal sides are equal]. |

2. $\angle BEC = \angle ECD$ [Alternate angles]

$\therefore$ $\angle BCE = \angle ECD$ [From 1 and 2]

$\therefore$ **CE bisects angle C.**

**Hence Proved.**

(iii) **To Prove :** $\angle DEC = 90°$

**Proof :**

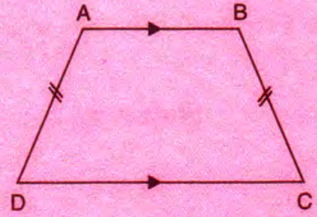
| **Statement :** | **Reason :** |
|---|---|
| 1. $\angle DCE = \frac{1}{2} \angle C$ | [CE bisects angle C]. |
| 2. $\angle CDE = \frac{1}{2} \angle D$ | [DE bisects angle D] |
| $\therefore$ $\angle DCE + \angle CDE = \frac{1}{2} (\angle C + \angle D)$ | [Adding 1 and 2] |
| $= \frac{1}{2} \times 180°$ | [$\angle C + \angle D = 180°$] |
| $= 90°$ | |
| $\therefore$ In $\Delta$ DEC, $\angle DEC = 180° - 90°$ | [Sum of the angles of a $\Delta$ is 180°]. |
| $= 90°$ | |

**Hence Proved.**

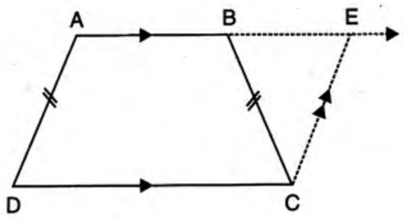
**Prove the following :**

1. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.
2. If the opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram.
3. If the opposite angles of a quadrilateral are equal, the quadrilateral is a parallelogram.
4. If the diagonals of a parallelogram are equal, the parallelogram is a rectangle.
5. If the diagonals of a parallelogram are equal and intersect each other at right angle, the parallelogram is a square.
6. If the diagonals of a rectangle intersect each other at right angles, the rectangle is a square.
7. A diagonal of a square makes an angle of 45° with the sides of the square.
8. A diagonal of a rhombus bisects the angles at the vertices.
9. The diagonals of a rhombus intersect each other at right angles.
10. If the diagonals of a parallelogram intersect each other at right angles, the parallelogram is a rhombus.
11. If the diagonals of a rhombus are equal, the rhombus is a square.

**15** The figure, given alongside, shows a trapezium ABCD in which AB // DC and AD = BC.
Prove that :
(i) $\angle A = \angle B$ (ii) $\angle C = \angle D$
(iii) $\Delta$ ABC $\cong$ $\Delta$ BAD
(iv) Diagonal AC = diagonal BD



**Solution :**




Draw a line through C and parallel to DA which intersects AB produced at point E

(i) Since, AE // DC and CE // DA

∴ AECD is a parallelogram.

⇒ AD = CE [Opp. sides of the parallelogram]

But, AD = BC [Given]

∴ CE = BC

⇒ ∠CBE = ∠E [Angles opp. to equal sides of ΔBCE]

Since, ABE is a straight line,

∠ABC + ∠CBE = 180°

⇒ ∠ABC + ∠E = 180° ........I [∵ ∠CBE = ∠E]

Since, AD // EC and AE is transversal

∠A + ∠E = 180° ........II [Co-interior angles]

∴ ∠ABC + ∠E = ∠A + ∠E [From I and II]

⇒ ∠ABC = ∠A *i.e.* **∠B = ∠A**

**Hence Proved.**

(ii) AB is parallel to DC

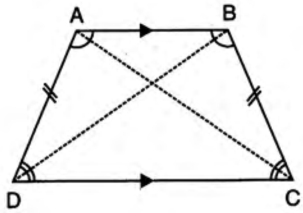
⇒ ∠A + ∠D = 180° and ∠B + ∠C = 180° [Co-interior angles]

⇒ ∠A + ∠D = ∠B + ∠C

⇒ **∠D = ∠C** [As, ∠A = ∠B]

**Hence Proved.**

(iii) In Δ ABC and Δ BAD




1. AB = AB [Common]
2. AD = BC [Given]
3. ∠BAD = ∠ABC [Proved above ∠A = ∠B]

∴ **Δ ABC ≅ Δ BAD** [By S.A.S.]

**Hence Proved.**

(iv) Since, Δ ABC ≅ Δ BAD [Proved above]

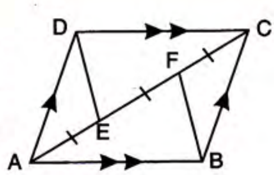
⇒ AC = BD

*i.e.* **diagonal AC = diagonal BD**

**Hence Proved.**

## EXERCISE 14(C)

1. E is the mid-point of side AB and F is the mid point of side DC of parallelogram ABCD. Prove that AEFD is a parallelogram.
2. The diagonal BD of a parallelogram ABCD bisects angles B and D. Prove that ABCD is a rhombus.
3. The alongside figure shows a parallelogram ABCD in which AE = EF = FC.

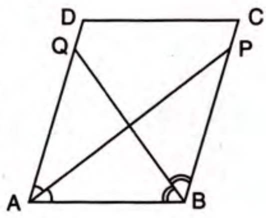


Prove that :

(i) DE is parallel to FB (ii) DE = FB

(iii) DEBF is a parallelogram.

4. In the alongside figure, ABCD is a parallelogram in which AP bisects angle A and BQ bisects angle B. Prove that :

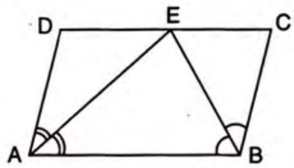



(i) AQ = BP

(ii) PQ = CD.

(iii) ABPQ is a parallelogram

5. In the given figure, ABCD is a parallelogram. Prove that : AB = 2 BC.




6. Prove that the bisectors of opposite angles of a parallelogram are parallel.

7. Prove that the bisectors of interior angles of a parallelogram form a rectangle.

8. Prove that the bisectors of the interior angles of a rectangle form a square.

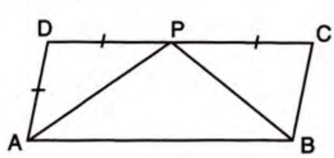
9. In parallelogram ABCD, the bisector of angle A meets DC at P and AB = 2AD.

Prove that :

(i) BP bisects angle B. (ii) Angle APB = 90°.

10. Points M and N are taken on the diagonal AC of a parallelogram ABCD such that AM = CN. Prove that BMDN is a parallelogram.

11. In the following figure, ABCD is a parallelogram. Prove that :




(i) AP bisects angle A

(ii) BP bisects angle B

(iii) $\angle$DAP + $\angle$CBP = $\angle$APB

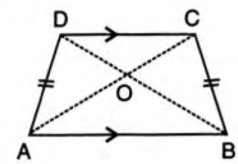
12. ABCD is a square. A is joined to a point P on BC and D is joined to a point Q on AB. If AP = DQ; prove that AP and DQ are perpendicular to each other.

13. In a quadrilateral ABCD, AB = AD and CB = CD. Prove that :

(i) AC bisects angle BAD.

(ii) AC is perpendicular bisector of BD.

14. The following figure shows a trapezium ABCD in which AB is parallel to DC and AD = BC.




Prove that :

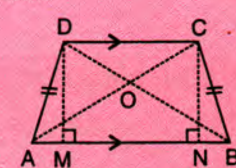
(i) $\angle$DAB = $\angle$CBA

(ii) $\angle$ADC = $\angle$BCD

(iii) AC = BD

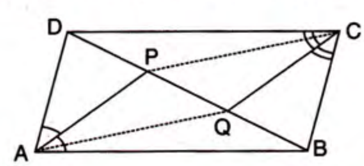
(iv) OA = OB and OC = OD

Draw DM $\perp$ AB and CN $\perp$ AB.
$\Delta$ DAM $\equiv$ $\Delta$ CBN by R.H.S.
$\therefore$ $\angle$DAB = $\angle$CBA
$\Rightarrow$ $\angle$ADC = $\angle$BCD




(Supplements of equal angles DAB and CBA).
Now by SAS, $\Delta$ ADB $\equiv$ $\Delta$ BCA
$\Rightarrow$ AC = BD and $\angle$ACB = $\angle$ADB
Further, $\Delta$ OAD $\equiv$ $\Delta$ OBC by ASA
$\Rightarrow$ OA = OB and OC = OD

15. In the given figure, AP is bisector of $\angle$A and CQ is bisector of $\angle$C of parallelogram ABCD. Prove that APCQ is a parallelogram




Join AC and show that diagonals AC and PQ bisect each other.

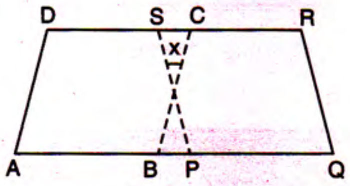
16. In case of a parallelogram prove that :

(i) the bisectors of any two adjacent angles intersect at 90°.

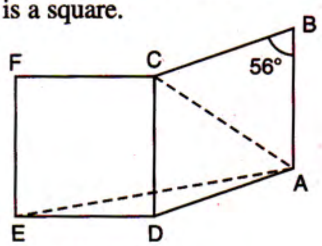
(ii) the bisectors of opposite angles are parallel to each other.

17. The diagonals of a rectangle intersect each other at right angles. prove that the rectangle is a square.

18. In the following figure, ABCD and PQRS are two parallelograms such that $\angle D = 120°$ and $\angle Q = 70°$. Find the value of $x$.




19. In the following figure, ABCD is a rhombus and DCFE is a square.




If $\angle ABC = 56°$, find :

(i) $\angle DAE$ (ii) $\angle FEA$

(iii) $\angle EAC$ (iv) $\angle AEC$