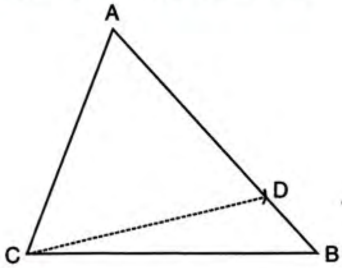
11 Inequalities

11.1 INTRODUCTION

1. The sign > means, "*is greater than*" *i.e.* if '*a*' is greater than '*b*', we write : $a > b$.
2. The sign < means, "*is less than*" *i.e.* if '*a*' is less than '*b*', we write : $a < b$.

Theorem 3

If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.



Given : A triangle ABC in which AB > AC.

To Prove : ∠ACB > ∠B.

Construction : From AB, cut AD = AC. Join C and D.

Proof :

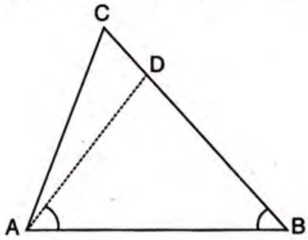
Statement :	Reason :
In Δ ACD :	
1. AC = AD	By construction
2. ∠ACD = ∠ADC	Angles opposite to equal sides
In Δ BDC :	
3. Ext. ∠ADC > ∠B	Ext. angle of a Δ is always greater than each of its interior opposite angles.
∴ ∠ACD > ∠B	From 2 and 3.
∴ **∠ACB > ∠B**	Since, ∠ACD is a part of ∠ACB, ∴ ∠ACB > ∠ACD > ∠B.

Hence Proved.

Theorem 4

(Converse of theorem 3)

If two angles of a triangle are unequal, the greater angle has the greater side opposite to it.



Given : A triangle ABC in which ∠CAB > ∠B.

To Prove : BC > AC.

Construction : Draw ∠BAD = ∠B.

Proof :

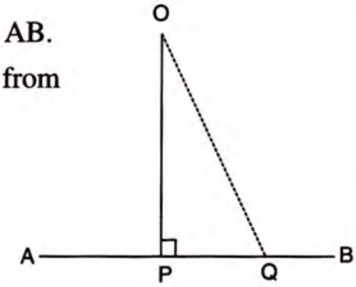
Statement :	Reason :
In Δ ABD :	
1. AD = BD	Sides opposite to equal angles.
In Δ ADC :	
2. AD + DC > AC	Sum of any two sides of a Δ is always greater than the third side.
∴ BD + DC > AC	Since, AD = BD
∴ **BC > AC**	BD + DC = BC

Hence Proved.

Theorem 5

Of all the lines, that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

Given : A point O outside the line AB and OP perpendicular to AB.



To Prove : OP is the shortest of all the lines that can be drawn from O to AB.

Construction : Join O with any point Q in AB.

Proof :

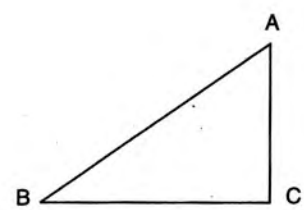
Statement :	Reason :
In right angled ΔOPQ :	
1. $\angle OPQ > \angle OQP$	Right angle is the greatest angle in a right angled Δ.
2. $OQ > OP$	Side opp. to greater angle is greater.

Similarly, it can be shown that OP is smaller than any other line that can be drawn from O to AB.

$\therefore$ **OP is the shortest line drawn from O to AB.**

Hence Proved.

Corollary 1 : The sum of the lengths of any two sides of a triangle is always greater than the third side.



For example :

In triangle ABC.

(i) $AB + AC > BC$,

(ii) $AB + BC > AC$ and (iii) $BC + AC > AB$.

Corollary 2 : The difference between the lengths of any two sides of a triangle is always less than the third side.

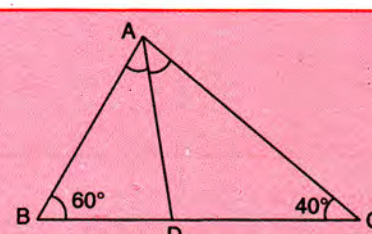
For example :

In triangle ABC, given for corollary 1, if AB is the largest side and AC is the smallest side; then:

(i) $AB - AC < BC$

(ii) $AB - BC < AC$ and (iii) $BC - AC < AB$.

1 In the adjoining figure, AD bisects $\angle A$. Arrange AB, BD and DC in the descending order of their lengths.



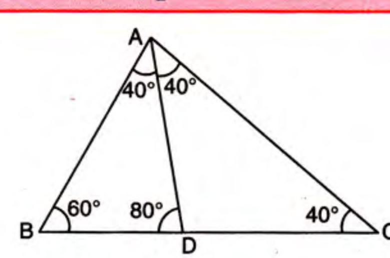
Solution :

$\angle BAC = 180° - (60° + 40°) = 80°$

Since, AD bisects $\angle A$

$\therefore \quad \angle BAD = \angle CAD = \frac{80°}{2} = 40°$

$\angle ADB = 180° - (60° + 40°) = 80°$

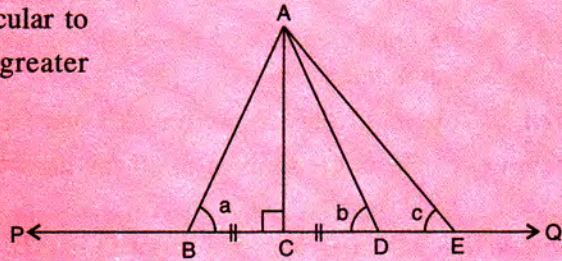


In Δ ABD, $AB > AD > BD$ I   [Side opposite to greater angle is greater]

In Δ ADC, $AD = DC$ II   [Sides opposite to equal angles are equal]

$\therefore$ **AB > DC > BD**   **Ans.**   [From I and II]

2 In the given figure, AC is perpendicular to line PQ and BC = CD. Show that AE is greater than AB.



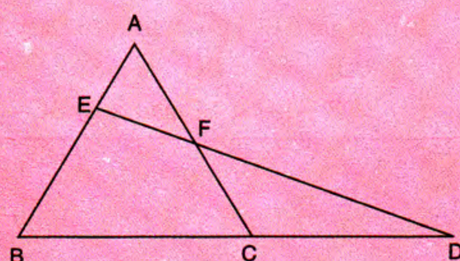
Solution :

In Δ ABC and Δ ADC

	BC = CD	[Given]
	$\angle$ACB = $\angle$ACD	[Each 90°]
and,	AC = AC	[Common]
$\therefore$	Δ ABC $\cong$ Δ ADC	[By S.A.S.]
$\Rightarrow$	AB = AD	[By C.P.C.T.C.]
and so,	$\angle a = \angle b$	[Angles opp. to equal sides]
In Δ ADE,	ext. $\angle b = \angle c + \angle DAE$	[Ext. $\angle$ = sum of int. opp. $\angle s$]
$\Rightarrow$	$\angle b > \angle c$	
$\Rightarrow$	$\angle a > \angle c$	[$\because$ $\angle a = \angle b$]
In Δ ABE,	$\angle a > \angle c$	
$\Rightarrow$	**AE > AB**	[In a Δ, side opp. to greater angle is greater]

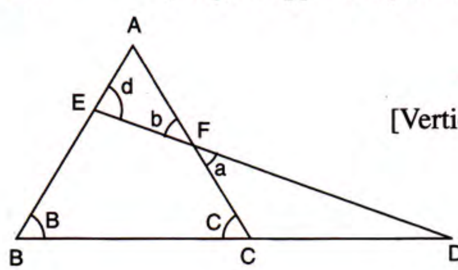
Hence Proved.

3 In the given figure, AB = AC. Prove that AF is greater than AE.



Solution :

Since, AB = AC $\Rightarrow$ $\angle B = \angle C$ [Angles opp. to equal sides are equal]



In Δ FCD,	ext. $\angle C = \angle D + \angle a$	
$\Rightarrow$	$\angle C > \angle a$	
But,	$\angle a = \angle b$	[Vertically opp. $\angle s$]
$\therefore$	$\angle C > \angle b$	I
In Δ EBD,	ext. $\angle d = \angle B + \angle D$	
$\Rightarrow$	$\angle d > \angle B$	
$\Rightarrow$	$\angle d > \angle C$	....II [$\because$ $\angle B = \angle C$]

$\therefore$ $\angle d > \angle C$ and $\angle C > \angle b \Rightarrow \angle d > \angle b$

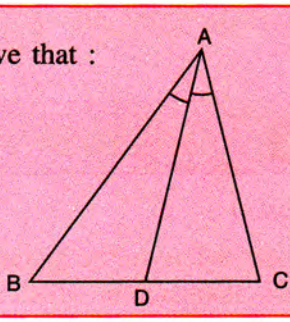
In Δ AEF, $\angle d > \angle b \Rightarrow$ **AF > AE** **Hence Proved.**

4 In the figure, given alongside, AD bisects angle BAC. Prove that :

(i) AB > BD

(ii) AC > CD

(iii) AB + AC > BC



Solution :

Given, AD bisects angle BAC $\Rightarrow \angle BAD = \angle CAD$

(i) In triangle ADC,

ext. $\angle ADB = \angle CAD + \angle C$

$\Rightarrow \angle ADB = \angle BAD + \angle C$ [$\because \angle CAD = \angle BAD$]

$\Rightarrow \angle ADB > \angle BAD$

In triangle ABD, $\angle ADB > \angle BAD$

$\Rightarrow$ **AB > BD** [Greater angle has greater side opposite to it.]

Hence Proved.

(ii) In Δ ABD,

ext. $\angle ADC = \angle BAD + \angle B$

$\Rightarrow \angle ADC = \angle CAD + \angle B$ [$\because \angle BAD = \angle CAD$]

$\Rightarrow \angle ADC > \angle CAD$

In Δ ADC, $\angle ADC > \angle CAD \Rightarrow$ **AC > CD** **Hence Proved.**

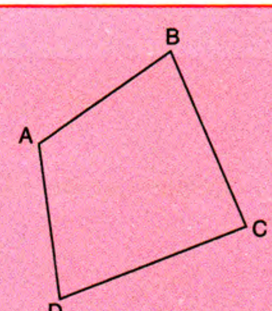
(iii) Since, AB > BD and AC > CD

$\therefore$ AB + AC > BD + CD $\Rightarrow$ **AB + AC > BC** **Hence Proved.**

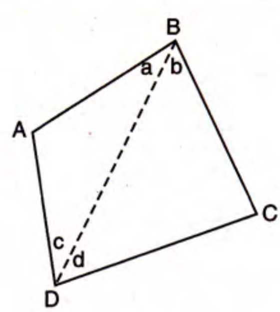
5 In quadrilateral ABCD; AB is the shortest side and DC is the longest side. Prove that :

(i) $\angle B > \angle D$

(ii) $\angle A > \angle C$



Solution :



Join B and D

In Δ ABD, AD > AB [Given, AB is the shortest]

$\therefore$ $\angle a > \angle c$ .... I [Angle opposite to the greater side is greater]

In Δ BCD, CD > BC [Given, CD is the longest side]

$\therefore$ $\angle b > \angle d$ ....II [Angle opposite to greater side is greater]

Downloaded from https:// www.studiestoday.com

$\therefore \quad \angle a + \angle b > \angle c + \angle d$ [Adding I and II]

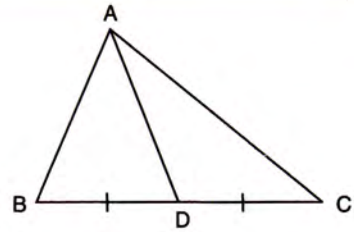
$\Rightarrow \quad \angle B > \angle D$ **Hence Proved.**

Similarly, by joining A and C, it can be proved that $\angle A > \angle C$.

Hence Proved.

6 AD is a median of triangle ABC. Prove that : AB + AC > 2AD

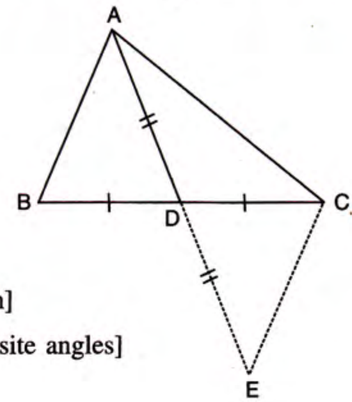
Solution :



According to the given statement, the figure will be as drawn alongside in which AD is a median *i.e.* BD = CD.

Produce AD upto a point E such that AD = DE *i.e.* AE = 2AD.

Also join C and E.



Since, the sum of any two sides of a triangle is greater than the third side, therefore in triangle ACE

$$AC + CE > AE$$

i.e. $AC + CE > 2AD$ I

In Δ ADB and Δ CDE

$BD = CD$ [Given]

$AD = DE$ [By construction]

and, $\angle ADB = \angle CDE$ [Vertically opposite angles]

$\therefore \quad \Delta ADB \cong \Delta EDC$ [By S.A.S.]

$\Rightarrow \quad AB = CE$ [C.P.C.T.C.]

Substituting $CE = AB$ in equation I, we get :

$$AC + AB > 2AD$$

i.e. $AB + AC > 2AD$ **Hence Proved.**

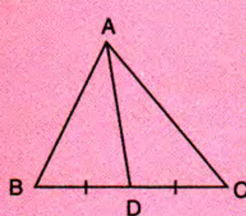
As proved above, if AD is a median of the triangle ABC;

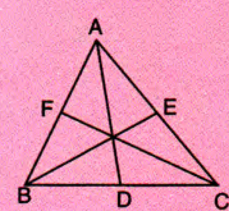
$$AB + AC > 2AD$$

Similarly, if BE and CF are also the medians :

$$AB + BC > 2BE$$

and, $BC + AC > 2CF$





Adding these results, we get :

$$2AB + 2BC + 2AC > 2AD + 2BE + 2CF$$

$\Rightarrow \quad AB + BC + AC > AD + BE + CF$

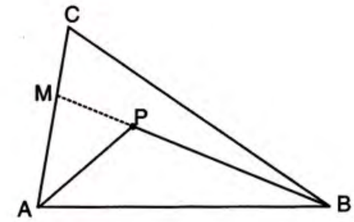
i.e. the perimeter of a triangle is greater than the sum of the lengths of its medians.

7 P is any point in the interior of a triangle ABC.

Prove that : PA + PB < AC + BC

Solution :

According to the given statement, the figure will be as shown alongside.



Produce BP to meet AC at point M.

Since, the sum of any two sides of a triangle is greater than its third side.

∴ In Δ BCM, BC + CM > BM I

and, in Δ APM, AM + PM > AP II

Adding I and II, we get :

BC + CM + AM + PM > BM + AP

⇒ BC + (CM + AM) > BM – PM + AP

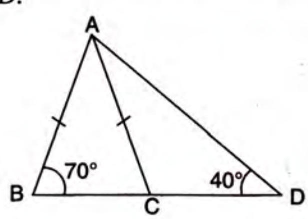
⇒ BC + AC > PB + PA

i.e. **PB + PA < BC + AC**

Hence Proved.

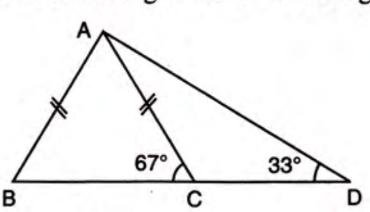
EXERCISE 11

1. From the following figure, prove that : AB > CD.

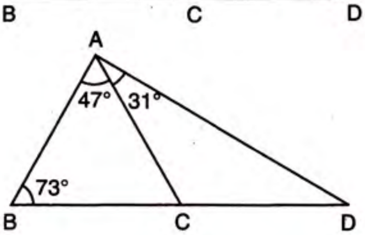


2. In a triangle PQR; QR = PR and ∠P = 36°. Which is the largest side of the triangle ?
3. If two sides of a triangle are 8 cm and 13 cm, then the length of the third side is between *a* cm and *b* cm. Find the values of *a* and *b* such that *a* is less than *b*.
4. In each of following figures, write BC, AC and CD in ascending order of their lengths.

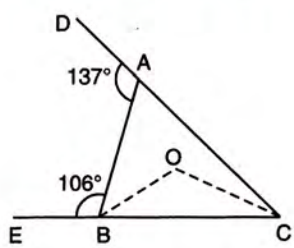
(i)



(ii)



5. Arrange the sides of Δ BOC in descending order of their lengths. BO and CO are bisectors of angles ABC and ACB respectively.



6. D is a point in side BC of triangle ABC. If AD > AC, show that AB > AC.
7. O is a point in the interior of a triangle ABC. Show that : OB + OC < AB + AC.

$x = a + b \Rightarrow a < x$

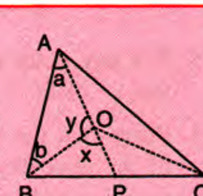
$\Rightarrow a < y$

⇒ OB < AB I

Similarly OC < AC II

Adding I and II, we get :

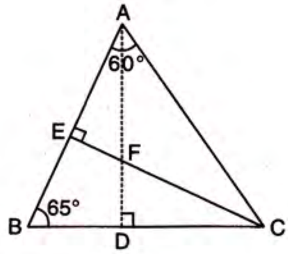
OB + OC < AB + AC



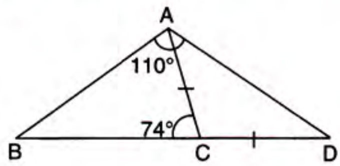
8. In the following figure, ∠BAC = 60° and ∠ABC = 65°.

Prove that :

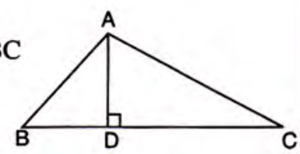
(i) CF > AF (ii) DC > DF



9. In the following figure;
AC = CD; ∠BAD = 110° and ∠ACB = 74°.
Prove that : BC > CD.

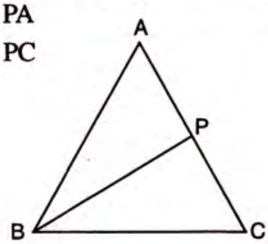


10. From the following figure; prove that :
   (i) AB > BD
   (ii) AC > CD
   (iii) AB + AC > BC



11. In a quadrilateral ABCD; prove that :
   (i) AB + BC + CD > DA
   (ii) AB + BC + CD + DA > 2AC
   (iii) AB + BC + CD + DA > 2BD

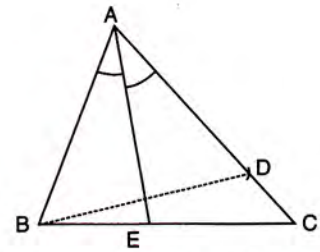
12. In the following figure, ABC is an equilateral triangle and P is any point in AC; prove that :
   (i) BP > PA
   (ii) BP > PC



13. P is any point inside the triangle ABC. Prove that : ∠BPC > ∠BAC.

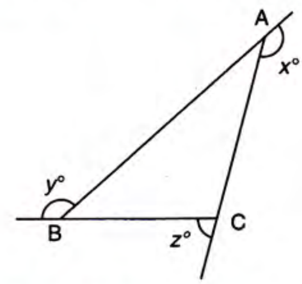
14. Prove that the straight line joining the vertex of an isosceles triangle to any point in the base is smaller than either of the equal sides of the triangle.

15. In the following diagram; AD = AB and AE bisects angle A. Prove that :
   (i) BE = DE   (ii) ∠ABD > ∠C



16. The sides AB and AC of a triangle ABC are produced; and the bisectors of the external angles at B and C meet at P. Prove that if AB > AC, then PC > PB.

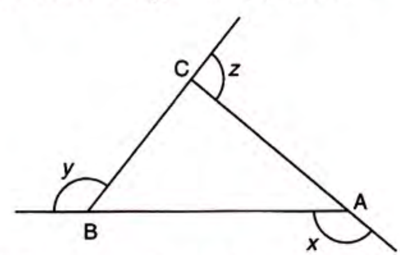
17. In the following figure; AB is the largest side and BC is the smallest side of triangle ABC.



Write the angles $x°$, $y°$ and z° in ascending order of their values.

18. In quadrilateral ABCD, side AB is the longest and side DC is the shortest. Prove that :
   (i) ∠C > ∠A   (ii) ∠D > ∠B

19. The following figure shows a triangle ABC with exterior angles as x, y and z.



   (i) If AB > AC > BC; arrange the angles x, y and z in ascending order of their values.
   (ii) In the same figure, if $y > x >$ z; arrange sides AB, BC and AC in descending order of their lengths.

20. (i) In a right angled triangle prove that hypotenuse is the greatest.
   (ii) In a triangle ABC, ∠ACB = 108°, show that AB is the largest side of the triangle.

21. In triangle ABC, D is any point in side BC. Show that : AB + BC + AC > 2AD

22. In triangle ABC, side AC is greater than side AB. If the internal bisector of angle A meets the opposite side at point D, prove that : ∠ADC is greater than ∠ADB.

23. In isosceles triangle ABC, sides AB and AC are equal. If point D lies in base BC and point E lies on BC produced (BC being produced through vertex C), prove that :

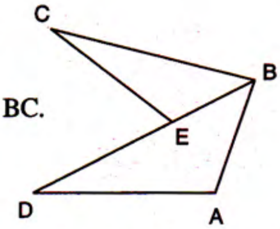
(i) AC > AD

(ii) AE > AC

(iii) AE > AD

24. Given : ED = EC

Prove : AB + AD > BC.



25. In triangle ABC, AB > AC and D is *a* point in side BC. Show that : AB > AD.