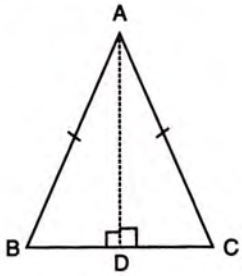
10 Isosceles Triangles

10.1 INTRODUCTION

1. A triangle, with *atleast* two sides equal to each other, is called an ***isosceles triangle***.
2. If all the sides of a triangle are equal to each other, it is called an ***equilateral triangle***.
3. An equilateral triangle satisfies all the properties of an isosceles triangle, whereas it is not necessary for an isosceles triangle to satisfy all the properties of an equilateral triangle.

Theorem 1

If two sides of a triangle are equal, the angles opposite to them are also equal.



Given : A triangle ABC in which AB = AC.

To Prove : ∠B = ∠C.

Construction : Draw AD perpendicular to BC.

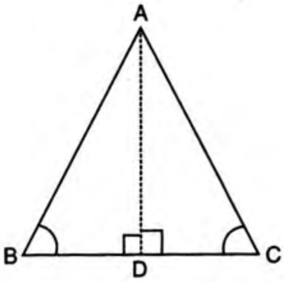
Proof :

Statement :	Reason :
In triangles ABD and ACD :	
1. AB = AC	Given
2. AD = AD	Common
3. ∠ADB = ∠ADC	Each 90°, since AD ⊥ BC
∴ Δ ABD ≅ Δ ACD	R.H.S.
⇒ **∠B = ∠C**	Corresponding parts of congruent triangles are congruent.

Hence Proved.

Theorem 2

If two angles of a triangle are equal, the sides opposite to them are also equal.



Given : A triangle ABC in which ∠B = ∠C.

To Prove : AB = AC.

Construction : Draw AD perpendicular to BC.

Proof :

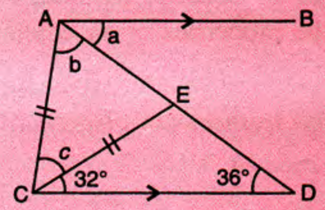
Statement :	Reason :
In triangles ABD and ACD :	
1. ∠B = ∠C	Given
2. ∠ADB = ∠ADC = 90°	AD ⊥ BC
3. AD = AD	Common
∴ Δ ABD ≅ Δ ACD	A.A.S.
⇒ **AB = AC**	Corresponding parts of congruent triangles are congruent.

Hence Proved.

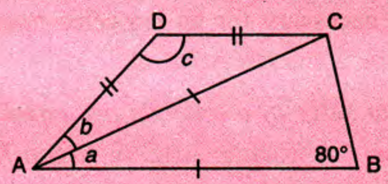
Prove the following :

1. The bisector of the angle at the vertex of an isosceles triangle bisects the base at right angles.
2. If equal sides of an isosceles triangle are produced, the exterior angles so formed are equal.
3. The perpendicular bisector of the base of an isosceles triangle passes through the vertex of the triangle.
4. The line, joining the mid-point of the base of an isosceles triangle and the opposite vertex, is perpendicular to the base and bisects the angle at the vertex.

1 Use the information given in following figures to find the values of a, b and c :

(i) 

[AB is parallel to CD]

(ii) 

[AC bisects angle A]

Solution :

(i) Since, AB is parallel to CD and AD is transversal

$a = 36°$ [Alternate angles] **Ans.**

In ΔCDE,

Ext. ∠CEA = the sum of two interior opposite angles

$= 32° + 36° = 68°$

Now, in ΔCEA, CE = CA

$\Rightarrow b = \angle CEA = 68°$ **Ans.**

In ΔACE, $b + c + \angle CEA = 180°$

$\Rightarrow 68° + c + 68° = 180°$

$\Rightarrow c = 180° - 136° = 44°$ **Ans.**

(ii) In ΔABC, AB = AC $\Rightarrow \angle ACB = \angle ABC = 80°$

And, $a + \angle ACB + \angle ABC = 180°$

$\Rightarrow a + 80° + 80° = 180° \Rightarrow a = 20°$ **Ans.**

$\because$ AC bisects angle A; $b = a = 20°$ **Ans.**

In ΔADC, AD = DC

$\Rightarrow \angle ACD = b = 20°$

And, $b + c + \angle ACD = 180°$

$\Rightarrow 20° + c + 20° = 180°$

$\Rightarrow c = 140°$ **Ans.**

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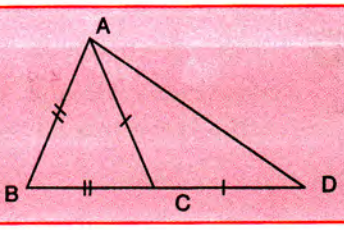
2 In the adjoining figure, AB = BC and AC = CD. Prove that : ∠BAD : ∠ADB = 3 : 1.



Solution :

Let ∠ADB = x

In ΔACD, AC = CD

⇒ ∠CAD = ∠CDA = x

and, ext. ∠ACB = ∠CAD + ∠CDA

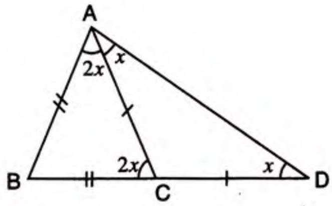
= $x + x = 2x$

In ΔABC, AB = BC

⇒ ∠BAC = ∠ACB = $2x$.

∴ ∠BAD = ∠BAC + ∠CAD

= $2x + x = 3x$



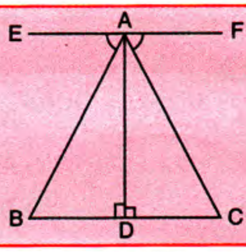
And, $\frac{\angle BAD}{\angle ADB} = \frac{3x}{x} = \frac{3}{1}$

i.e. **∠BAD : ∠ADB = 3 : 1.**

Hence Proved.

3 In the given figure, AD is perpendicular to BC and EF both. If ∠EAB = ∠FAC, show that triangles ABD and ACD are congruent.

Also, find the values of x and y if AB = $2x + 3$, AC = $3y + 1$, BD = x and DC = $y + 1$.



Solution :

AD is perpendicular to EF

⇒ ∠EAD = ∠FAD = 90°

Given : ∠EAB = ∠FAC

⇒ ∠EAD – ∠EAB = ∠FAD – ∠FAC

⇒ ∠BAD = ∠CAD

In Δ ABD and Δ ACD, ∠BAD = ∠CAD [Proved above]

∠ADB = ∠ADC = 90° [Given AD ⊥ BC]

and AD = AD

∴ **Δ ABD ≅ Δ ACD** [By ASA]

Hence Proved.

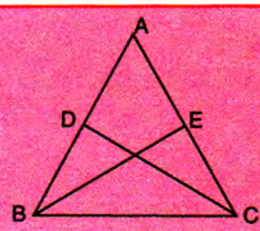
Δ ABD ≅ Δ ACD ⇒ AB = AC and BD = CD [By C.P.C.T.C.]

⇒ $2x + 3 = 3y + 1$ and $x = y + 1$

⇒ $2x - 3y = -2$ and $x - y = 1$

On solving, we get : $x = 5$ and $y = 4$ **Ans.**

4 In the given figure, D and E are mid-points of equal sides AB and AC respectively of triangle ABC. Prove that BE = CD.

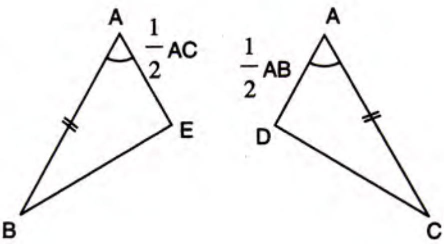


Solution :

In Δ ABE and Δ ACD,

	AB = AC	[Given]
⇒	$\frac{AB}{2} = \frac{AC}{2}$	
⇒	AD = AE	
and,	∠A = ∠A	[Common]
∴	Δ ABE ≅ Δ ACD	[By S.A.S.]
⇒	**BE = CD**	[By C.P.C.T.C.]

Hence proved.

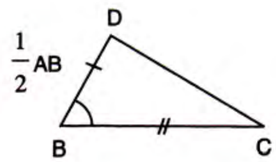


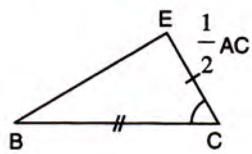
Alternative method :

In Δ BCD and Δ CBE,

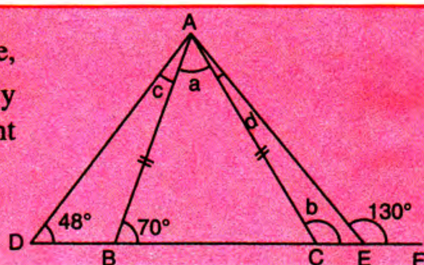
	BD = $\frac{1}{2}$AB and CE = $\frac{1}{2}$AC	
But	AB = AC	[Given]
⇒	BD = CE	.....(i)
	AB = AC	
⇒	∠ABC = ∠ACB	
i.e.	∠DBC = ∠ECB	....(ii)
	BC = BC	[Common] ...(iii)
(i), (ii) and (iii)		
⇒	Δ BCD ≅ Δ CBE	[By S.A.S.]
⇒	**BE = CD**	[By C.P.C.T.C.]

Hence Proved.





5 Use the information; given in the adjoining figure, to find the measures of angles represented by letters *a*, *b*, *c* and *d*. Given DBCEF is a straight line.



Solution :

In Δ ABC,

AB = AC ⇒ ∠ABC = ∠ACB = 70° [Angles opposite to equal sides]

Also, $\angle BAC + \angle ABC + \angle ACB = 180°$

$a + 70° + 70° = 180°$

$\Rightarrow \quad a = 40°$

$\angle ACB + b = 180°$ [$\because$ BCE is a straight line]

$\Rightarrow \quad 70° + b = 180°$ and $b = 180° - 70° = 110°$

In Δ ABD,

$70° = 48° + c$ [Ext. angle of a Δ = sum of int. opp. $\angle s$]

$\Rightarrow \quad c = 70° - 48° = 22°$

In Δ ACE,

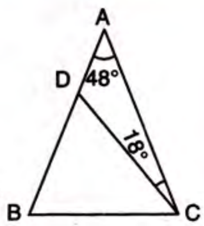
$b + d = 130°$ [Ext. angle of a Δ = sum of int. opp. $\angle s$]

$\Rightarrow \quad 110° + d = 130°$ and $d = 20°$

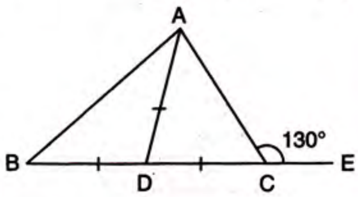
$\therefore \quad a = 40°,\ b = 110°,\ c = 22°$ and $d = 20°$ **Ans.**

EXERCISE 10(A)

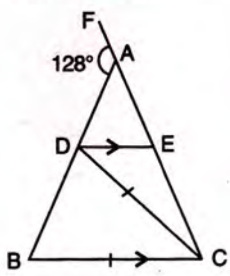
1. In the figure alongside,
   AB = AC
   $\angle A = 48°$ and
   $\angle ACD = 18°$.
   Show that : BC = CD.



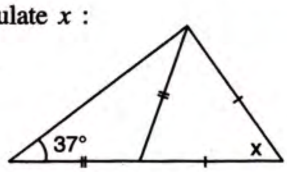
2. Calculate :
   (i) $\angle ADC$
   (ii) $\angle ABC$
   (iii) $\angle BAC$



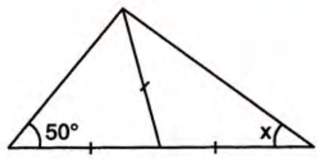
3. In the following figure, AB = AC; BC = CD and DE is parallel to BC. Calculate :
   (i) $\angle CDE$
   (ii) $\angle DCE$



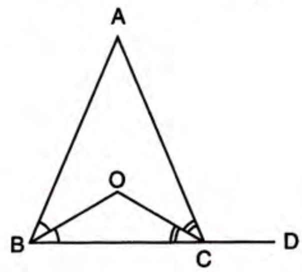
4. Calculate x :
   (i)



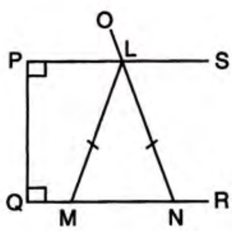
(ii)



5. In the figure, given below, AB = AC.
   Prove that : $\angle BOC = \angle ACD$.

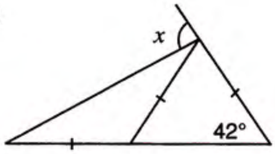


6. In the figure given below, LM = LN; angle PLN = 110°. Calculate :
   (i) $\angle LMN$
   (ii) $\angle MLN$

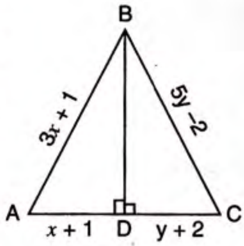


7. An isosceles triangle ABC has AC = BC. CD bisects AB at D and $\angle CAB = 55°$.
   Find : (i) $\angle DCB$ (ii) $\angle CBD$.

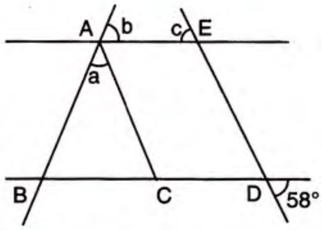
8. Find x :



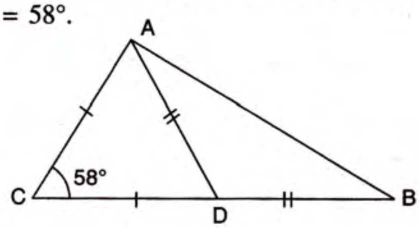
9. In the triangle ABC, BD bisects angle B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.



10. In the given figure ; AE // BD, AC // ED and AB = AC. Find $\angle a$, $\angle b$ and $\angle c$.



11. In the following figure; AC = CD, AD = BD and $\angle$C = 58°.



Find angle CAB.

12. In the figure of Q. no. 11, given above, if AC = AD = CD = BD; find angle ABC.

13. In triangle ABC; AB = AC and $\angle A : \angle B = 8 : 5$; find angle A.

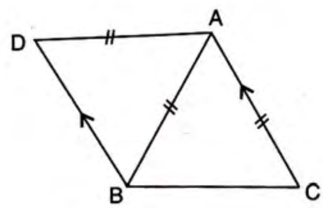
14. In triangle ABC; $\angle A = 60°$, $\angle C = 40°$ and bisector of angle ABC meets side AC at point P. Show that BP = CP.

15. In triangle ABC; angle ABC = 90° and P is a point on AC such that $\angle PBC = \angle PCB$. Show that : PA = PB.

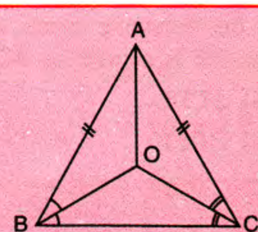
16. ABC is an equilateral triangle. Its side BC is produced upto point E such that C is mid-point of BE. Calculate the measure of angles ACE and AEC.

17. In triangle ABC, D is a point in AB such that AC = CD = DB. If $\angle B = 28°$, find the angle ACD.

18. In the given figure, AD = AB = AC, BD is parallel to CA and angle ACB = 65°. Find angle DAC.



6 In the given figure, AB = AC, BO bisects angle ABC and CO bisects angle ACB, prove that AO bisects angle BAC.



Solution :

Since, BO bisects angle ABC

$\Rightarrow \quad \angle OBC = \angle OBA = \frac{1}{2}\angle ABC$ I

Since, CO bisects angle ACB

$\Rightarrow \quad \angle OCB = \angle OCA = \frac{1}{2}\angle ACB$ II

AB = AC $\Rightarrow$ $\angle ABC = \angle ACB$ ..III [Angles opp. to equal sides are equal]

Combining I, II and III, we get :

$\angle OBC = \angle OBA = \angle OCB = \angle OCA$ ...IV

In Δ BOC,

$\angle OBC = \angle OCB$ [From equation IV]

$\Rightarrow$ $OB = OC$ [Sides opp. to equal angles are equal]

$\angle OBA = \angle OCA$ [From eq. (IV)]

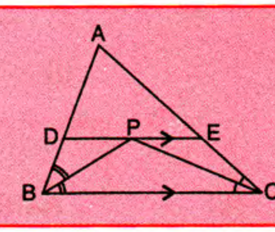
and, $AB = AC$ [Given]

$\Rightarrow$ $\Delta AOB \cong \Delta AOC$ [By S.A.S.]

$\Rightarrow$ $\angle OAB = \angle OAC$ [By C.P.C.T.C.]

$\Rightarrow$ **AO bisects angle BAC** **Hence Proved**

7 In the figure, given alongside, DE is parallel to BC. Prove that : DE = BD + CE.



Solution :

Since, BP bisects angle B

$\therefore$ $\angle DBP = \angle PBC$ I

DE is parallel to BC and BP is transversal,

$\therefore$ $\angle DPB = \angle PBC$ .....II [Alternate angles]

$\Rightarrow$ $\angle DBP = \angle DPB$ [From I and II]

$\Rightarrow$ $DP = BD$ III

In the same way, $\angle PCB = \angle PCE$ and $\angle EPC = \angle PCB$

$\Rightarrow$ $\angle PCE = \angle EPC$ and so $PE = CE$ IV

Adding III and IV, we get :

$DP + PE = BD + CE \Rightarrow$ **DE = BD + CE.**

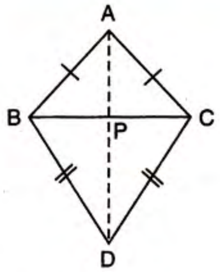
Hence Proved.

8 ABC and DBC are two isosceles triangles on opposite sides of BC. Prove that :

(i) DA bisects BC at right angle (ii) $\angle ABD = \angle ACD$

Solution :

According to the given statement, the figure will be as shown alongside :



(i) Join AD which meets BC at point P.

To Prove : DA bisects BC at right angles

i.e., $BP = CP$ and $\angle APB = 90°$

Proof : In ΔABD and ΔACD

$AB = AC$ and $DB = DC$ [Given]

and, AD = AD [Common]

∴ $\Delta ABD \cong \Delta ACD$ [By S.S.S.]

⇒ $\angle BAD = \angle CAD$ [C.P.C.T.C]

i.e. $\angle BAP = \angle CAP$

Now, in ΔABP and ΔACP

AB = AC and AP = AP [Common]

and, $\angle BAP = \angle CAP$ [Proved above]

∴ $\Delta ABP \cong \Delta ACP$ [By S.A.S.]

⇒ BP = CP and $\angle APB = \angle APC$ [C.P.C.T.C.]

Since, $\angle APB = \angle APC$ and $\angle APB + \angle APC = 180°$

⇒ $\angle APB = \angle APC = \frac{180°}{2} = 90°$

∴ BP = CP and $\angle APB = 90°$

⇒ **DA bisects BC at right angles.**

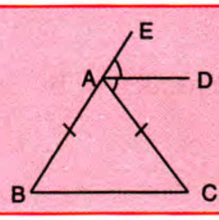
Hence Proved.

(ii) $\Delta ABD \cong \Delta ACD$

⇒ $\angle ABD = \angle ACD$ [C.P.C.T.C]

Hence Proved.

9 In the figure, given alongside, AB = AC and AD bisects exterior angle CAE.
Prove that : AD is parallel to BC.



Solution :

In ΔABC, AB = AC

⇒ $\angle ABC = \angle ACB$

and, ext. $\angle CAE = \angle ABC + \angle ACB$

$= \angle ABC + \angle ABC$

$= 2\angle ABC$ I

∵ AD bisects $\angle CAE$,

∴ $\angle EAD = \angle CAD = \frac{1}{2}\angle CAE$

i.e. $\angle CAE = 2\angle EAD$ II

From I and II, we get :

$2\angle ABC = 2\angle EAD$

⇒ $\angle ABC = \angle EAD$

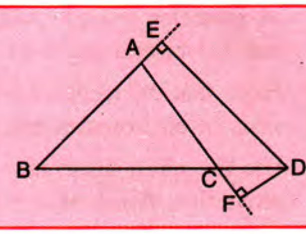
But these are corresponding angles and whenever the corresponding angles are equal, the lines are parallel.

∴ **AD is parallel to BC.**

Hence Proved.

10 In triangle ABC, AB = AC and D is a point on BC produced, DE is perpendicular to BA produced and DF is perpendicular to AC produced.

Prove that : BD bisects angle EDF.



Solution :

In ΔABC, AB = AC

$\Rightarrow$ $\angle ABC = \angle ACB$ [Why ?]

Also, $\angle ACB = \angle DCF$ [Why ?]

$\therefore$ $\angle ABC = \angle DCF$ I

Now, in ΔBDE and ΔCDF,

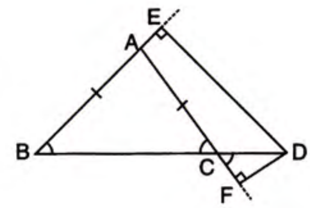
$\angle ABC = \angle DCF$ From I

and, $\angle E = \angle F = 90°$

$\Rightarrow$ $\angle EDB = \angle FDC$

$\therefore$ **BD bisects angle EDF.**

Hence Proved.



EXERCISE 10(B)

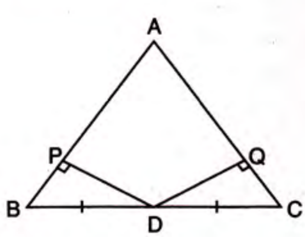
1. If the equal sides of an isosceles triangle are produced, prove that the exterior angles so formed are obtuse and equal.

2. In the given figure, AB = AC. Prove that:

   (i) DP = DQ

   (ii) AP = AQ

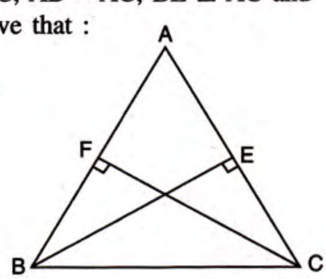
   (iii) AD bisects angle A



3. In triangle ABC, AB = AC; BE ⊥ AC and CF ⊥ AB. Prove that :

   (i) BE = CF

   (ii) AF = AE



4. In isosceles triangle ABC, AB = AC. The side BA is produced to D such that BA=AD. Prove that : $\angle BCD = 90°$.

5. (i) In a triangle ABC, AB = AC and $\angle A = 36°$. If the internal bisector of $\angle C$ meets AB at point D, prove that AD = BC.

   (ii) If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

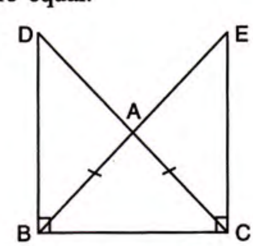
6. Prove that the bisectors of the base angles of an isosceles triangles are equal.

7. In the given figure, AB = AC and $\angle DBC = \angle ECB = 90°$.

   Prove that :

   (i) BD = CE

   (ii) AD = AE



8. ABC and DBC are two isosceles triangles on the same side of BC. Prove that :

   (i) DA (or AD) produced bisects BC at right angle.

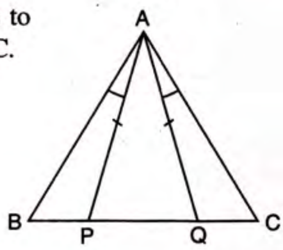
   (ii) $\angle BDA = \angle CDA$.

9. The bisectors of the equal angles B and C of

an isosceles triangle ABC meet at O. Prove that AO bisects angle A.

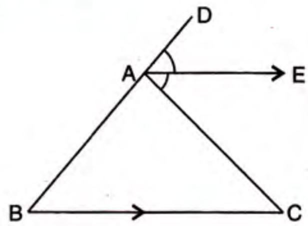
10. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.

11. Use the given figure to prove that, AB = AC.



12. In the given figure; AE bisects exterior angle CAD and AE is parallel to BC.

Prove that : AB = AC.



13. In an equilateral triangle ABC; points P, Q and R are taken on the sides AB, BC and CA respectively such that AP = BQ = CR. Prove that triangle PQR is equilateral.

14. In triangle ABC, altitudes BE and CF are equal. Prove that the triangle is isosceles.

15. Through any point in the bisector of an angle, a straight line is drawn parallel to either arm of the angle. Prove that the triangle so formed is isosceles.

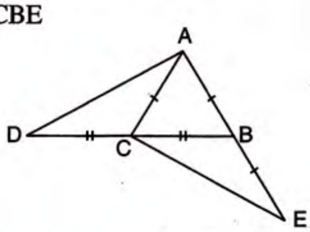
16. In triangle ABC; AB = AC. P, Q and R are mid-points of sides AB, AC and BC respectively. Prove that :

(i) PR = QR (ii) BQ = CP

17. From the following figure, prove that :

(i) ∠ACD = ∠CBE

(ii) AD = CE

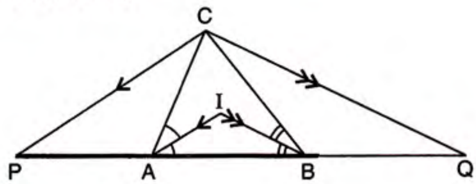


18. Equal sides AB and AC of an isosceles triangle ABC are produced. The bisectors of the exterior angles so formed meet at D. Prove that AD bisects angle A.

19. ABC is a triangle. The bisector of the angle BCA meets AB in X. A point Y lies on CX such that AX = AY.

Prove that : ∠CAY = ∠ABC.

20. In the following figure; IA and IB are bisectors of angles CAB and CBA respectively. CP is parallel to IA and CQ is parallel to IB.



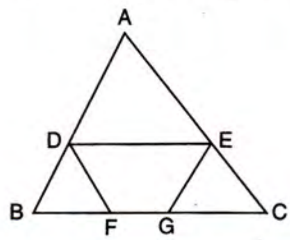
Prove that :

PQ = The perimeter of the Δ ABC.

21. Sides AB and AC of a triangle ABC are equal. BC is produced through C upto point D such that AC = CD. D and A are joined and produced (through vertex A) upto point E. If angle BAE = 108°; find angle ADB.

22. The given figure shows an equilateral triangle ABC with each side 15 cm. Also DE//BC, DF//AC and EG//AB.

If DE + DF + EG = 20 cm, find FG.



Each of triangles ADE, BDF and EGC is an equilateral triangle.

∴ DE + DF = AD + DB = AB = 15 cm

DE + DF + EG = 20 cm *i.e.* AB + EG = 20 cm

⇒ EG = 5 cm = GC = BF

Now, FG = BC – BF – GC

23. If all the three altitudes of a triangle are equal, the triangle is equilateral. Prove it.

24. In a Δ ABC, the internal bisector of angle A meets opposite side BC at point D. Through vertex C, line CE is drawn parallel to DA which meets BA produced at point E. Show that Δ ACE is isosceles.

25. In triangle ABC, bisector of angle BAC meets opposite side BC at point D. If BD = CD, prove that Δ ABC is isosceles.

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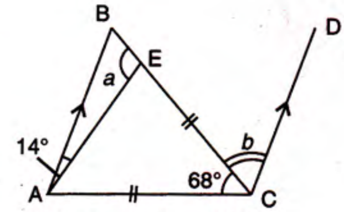
Produce AD upto point E so that AD = DE. Now show that Δ ABD ≅ Δ EDC by SAS
⇒ AB = EC and ∠E = ∠BAD.
But ∠BAD = ∠CAD; therefore ∠E = ∠CAD
⇒ AC = EC. Now AB = EC and AC = EC
⇒ AB = AC *i.e.* triangle is isosceles

26. In Δ ABC, D is a point on BC such that AB = AD = BD = DC. Show that : ∠ADC : ∠C = 4 : 1.

27. Using the information, given in each of the following figures, find the values of *a* and *b*.

(i)



[Given : CE = AC]

(ii)