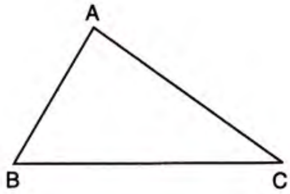
UNIT 4 : Geometry

9 Triangles

[Congruency in Triangles]

9.1 INTRODUCTION

A plane figure bounded by three straight line segments, is called a ***triangle***. Every triangle has three vertices and three sides.

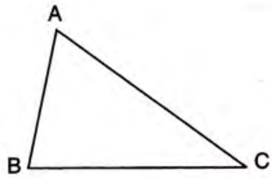


The adjoining figure shows a triangle ABC (ΔABC), whose three

(i) vertices are A, B and C. (ii) sides are AB, BC and CA.

9.2 RELATION BETWEEN SIDES AND ANGLES OF TRIANGLES

1. *If all the sides of a triangle are of different lengths, its angles are also of different measures in such a way that, the greater side has greater angle opposite to it.*



In the given triangle ABC, sides AB, BC and AC are all of different lengths. Therefore, its angles *i.e.*, ∠A, ∠B and ∠C are also of different measures.

Thus, in Δ ABC,

$AB \neq BC \neq AC$

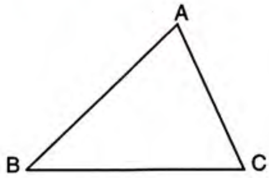
$\Rightarrow \quad \angle A \neq \angle B \neq \angle C$

Also, according to the given figure, AC > BC > AB

⇒ Angle opposite to AC > angle opposite to BC > angle opposite to AB

$\Rightarrow \quad \angle B > \angle A > \angle C$

2. *Conversely, if all the angles of a triangle have different measures, its sides are also of different lengths in such a way that, the greater angle has greater side opposite to it.*



In the given figure,

$\angle A \neq \angle B \neq \angle C$

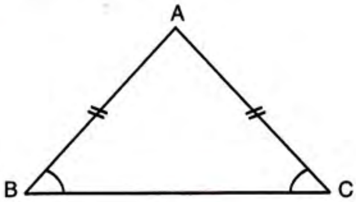
$\Rightarrow \quad AB \neq BC \neq AC$

Also, according to the given figure, ∠C > ∠A > ∠B

⇒ Side opposite to ∠C > side opposite to ∠A > side opposite to ∠B

$\Rightarrow \quad AB > BC > AC$

3. *If any two sides of a triangle are equal, the angles opposite to them are also equal. Conversely, if any two angles of a triangle are equal, the sides opposite to them are also equal.*

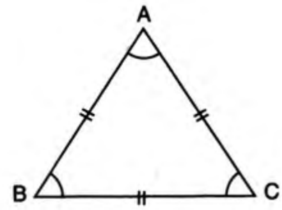


Thus, in triangle ABC,

(i) $AB = AC \Rightarrow \angle B = \angle C$

and, (ii) $\angle B = \angle C \Rightarrow AB = AC$

4. *If all the sides of a triangle are equal, all its angles are also equal. Conversely, if all the angles of a triangle are equal, all its sides are also equal.*



Thus, in triangle ABC,

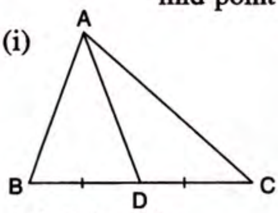
(i) AB = BC = AC $\Rightarrow \angle A = \angle B = \angle C$

and, (ii) $\angle A = \angle B = \angle C \Rightarrow$ AB = BC = AC

9.3 SOME IMPORTANT TERMS

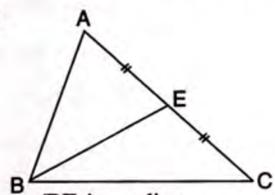
1. Median : The ***median*** of a triangle, corresponding to any side, is the line joining the mid-point of that side with the opposite vertex.

e.g. (i)



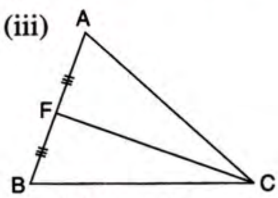
(AD is median corresponding to side BC)

(ii)



(BE is median corresponding to side AC)

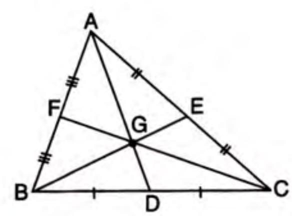
(iii)



(CF is median corresponding to side AB)

1. A triangle has three medians and all the three medians are always concurrent *i.e.,* they intersect each other at one point only.
2. The point of intersection of the medians is called the *centroid* of the triangle.

   In the figure, G is the centroid of Δ ABC.



3. Also, the centroid of a triangle divides each median in the ratio 2 : 1.

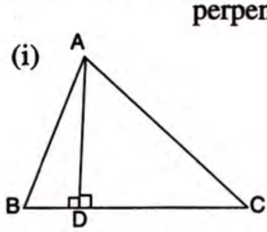
   i.e., in the given figure :

   AG : GD = 2 : 1; BG : GE = 2 : 1

   and CG : GF = 2 : 1.

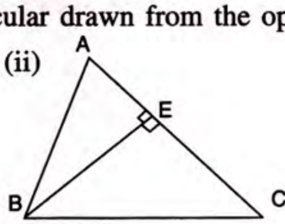
2. Altitude : An ***altitude*** of a triangle, corresponding to any side, is the length of the perpendicular drawn from the opposite vertex to that side.

e.g. (i)



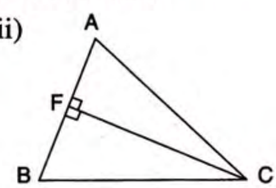
(AD is altitude corresponding to side BC)

(ii)



(BE is altitude corresponding to side AC)

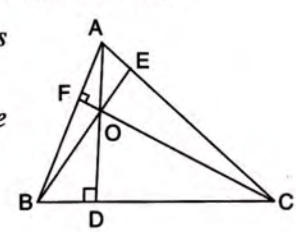
(iii)



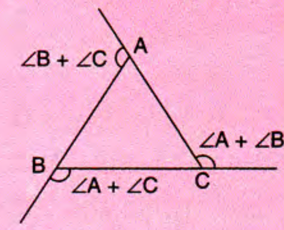
(CF is altitude corresponding to side AB)

1. A triangle has three altitudes and all the three altitudes are *always concurrent i.e.,* they intersect each other at one point only.
2. The point of intersection of the altitudes of a triangle is called *the* ***orthocentre***.

In the given figure, O is the orthocentre of the triangle ABC.



1. *The sum of the angles of a triangle is equal to two right angles i.e. 180°.*
2. *If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.*
3. *As shown, in triangle ABC,*
   (i) $\angle A + \angle B + \angle C = 2$ right angles $= 180°$
   (ii) Exterior angle at A $= \angle B + \angle C$.
   (iii) Exterior angle at B $= \angle A + \angle C$.
   (iv) Exterior angle at C $= \angle A + \angle B$.



Prove the following corollaries :

Corollary 1 : If one side of a triangle is produced, the exterior angle so formed is greater than each of the interior opposite angles.

Corollary 2 : A triangle cannot have more than one right angle.

Corollary 3 : A triangle cannot have more than one obtuse angle.

Corollary 4 : In a right angled triangle, the sum of the other two angles (acute angles) is 90°.

Corollary 5 : In every triangle, at least two angles are acute.

Corollary 6 : If two angles of a triangle are equal to two angles of any other triangle, each to each, then the third angles of both the triangles are also equal.

9.4 CONGRUENT TRIANGLES

Two triangles are said to be **congruent** to each other, if on placing one over the other, they exactly coincide.

In fact, two triangles are congruent, if they have exactly the *same shape* and the *same size. i.e.*, all the angles and all the sides of one triangle are equal to the corresponding angles and the corresponding sides of the other triangle each to each.

Triangles with same shape means : Angles of one triangle are equal to angles of other triangle each to each.

Triangles with same size means : Sides of one triangle are equal to sides of other triangle each to each.

The given figure shows two triangles ABC and DEF such that :

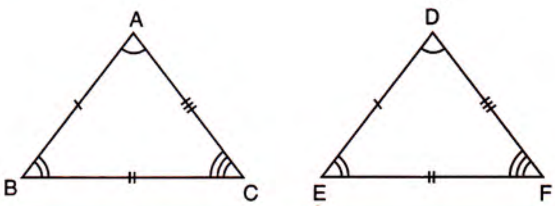
(i) $\angle A = \angle D$; $\angle B = \angle E$ and $\angle C = \angle F$.

(ii) AB = DE; BC = EF and AC = DF.

$\therefore$ Δ ABC is congruent to Δ DEF

and we write : $\mathbf{\Delta\ ABC \cong \Delta\ DEF}$.

The symbol $\cong$ is read as *"is congruent to"*.



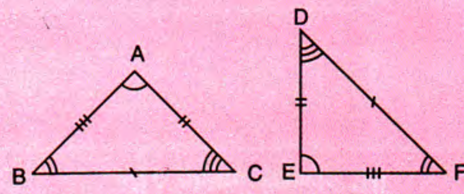
1. Congruent figures (triangles) always coincide by *superposition i.e.* by placing one figure over the other.
2. In congruent triangles, the sides and the angles that *coincide* by *superposition* are called *corresponding sides* and *corresponding angles.*
3. The corresponding sides lie *opposite* to the *equal angles* and corresponding angles lie *opposite* to the *equal sides.*

In the figure alongside, Δ ABC ≅ Δ EFD.

Since, ∠A = ∠E, therefore the side opposite to ∠A and the side opposite to ∠E are corresponding sides *i.e.,* BC and DF are corresponding sides.

Similarly, AB and EF are corresponding sides as ∠C = ∠D.

Also, AC and DE are corresponding sides.



Conversely, as side AB = side EF, therefore, angles opposite to these sides *i.e.* ∠C and ∠D are the corresponding angles and so on.

4. Corresponding Parts of Congruent Triangles are also Congruent.

Abbreviated as : **C.P.C.T.C.**

9.5 CONDITIONS FOR CONGRUENCY OF TRIANGLES

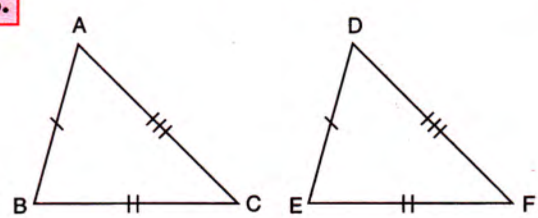
1. *If three sides of one triangle are equal to three sides of the other triangle, each to each, the triangles are congruent.* *Abbreviated* as : **S.S.S.**

In the figure alongside, AB = DE,

BC = EF

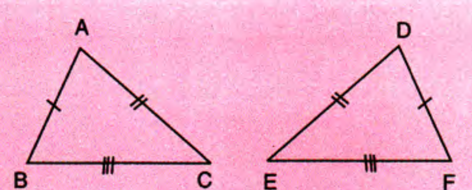
and, AC = DF

∴ Δ ABC ≅ Δ DEF. [By S.S.S.].



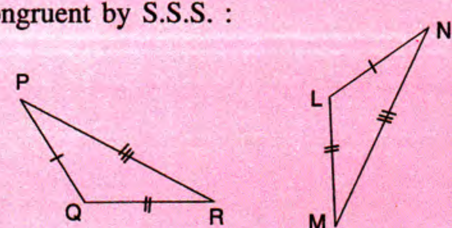
In each of the following figures, triangles are congruent by S.S.S. :

(i)



[Δ ABC ≅ Δ DFE]

(ii)



[Δ PQR ≅ Δ NLM]

2. *If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, the triangles are congruent.*

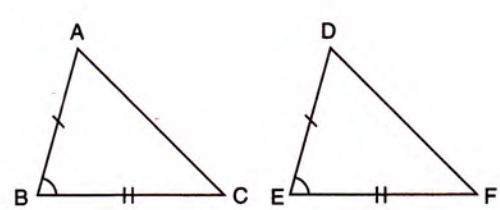
Abbreviated as : **S.A.S.**

In the figure alongside,

AB = DE;

BC = EF and ∠B = ∠E.

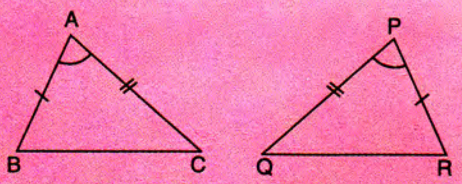
∴ Δ ABC ≅ Δ DEF [By S.A.S.]



Triangles will be congruent only when the equal angles are the included angles.

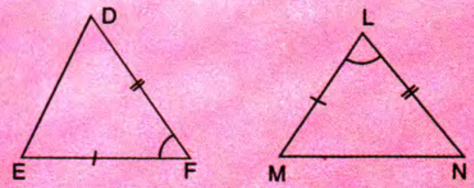
In each of the following figures, triangles are congruent by S.A.S. :

(i)



[Δ ABC ≅ Δ PRQ]

(ii)



[Δ DEF ≅ Δ NML]

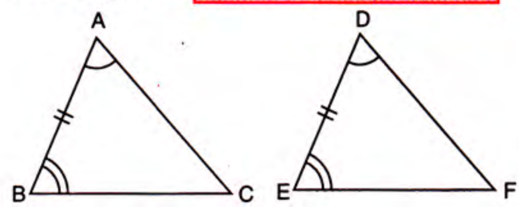
3. *If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, the triangles are congruent.* *Abbreviated* as : **A.S.A.**

In the figure alongside, ∠A = ∠D,

∠B = ∠E

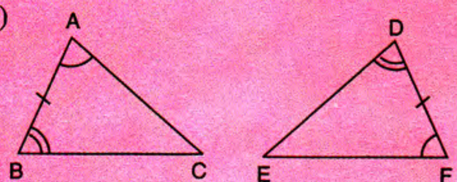
and, AB = DE

∴ Δ ABC ≅ Δ DEF. [By A.S.A.]



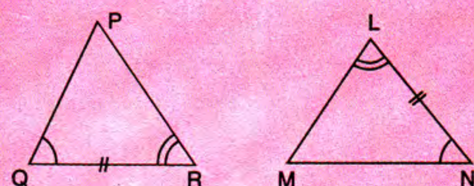
In each of the following figures, triangles are congruent by A.S.A. :

(i)



[Δ ABC ≅ Δ FDE]

(ii)



[Δ PQR ≅ Δ MNL]

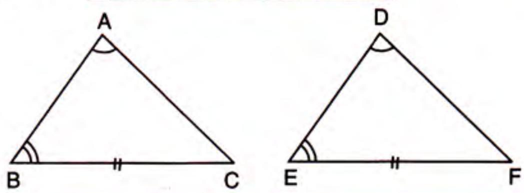
4. *If two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle, the triangles are congruent.* *Abbreviated* as : **A.A.S.**

In the figure alongside, ∠A = ∠D,

∠B = ∠E

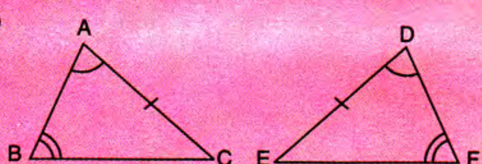
and, BC = EF

∴ Δ ABC ≅ Δ DEF. [By A.A.S.].



In each of the following figures, triangles are congruent by A.A.S. :

(i)



[Δ ABC ≅ Δ DFE]

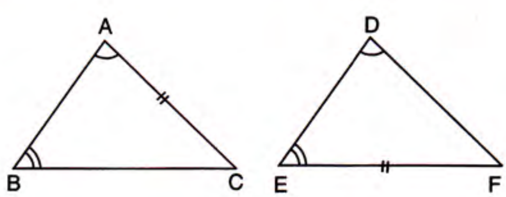
(ii)

[Δ PQR ≅ Δ LNM]

Precaution :

While using the axiom A.A.S., the two equal sides must be the corresponding sides, that is, the sides must be opposite to equal angles of the two triangles under consideration.

In the figure, given alongside, the equal sides are AC and EF. And these two sides (*i.e.*, AC



and EF) are not opposite to equal angles.

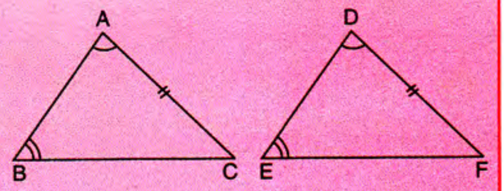
∴ Δ ABC and Δ DEF are not congruent by A.A.S.

The A.A.S. axiom is another form of axiom A.S.A.

In the figure, given alongside :

∠A = ∠D, ∠B = ∠E

and, AC = DF ⇒ ΔABC ≡ ΔDEF by A.A.S.



Since, the sum of the angles of every triangle is 180°, therefore, if any two angles of a triangle are equal to two angles of another triangle, each to each, then their third angles will also be equal.

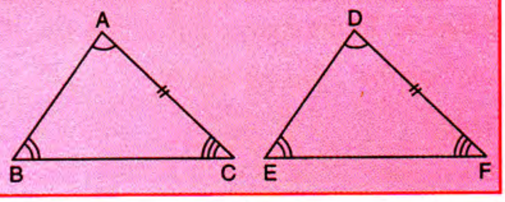
∴ In triangles ABC and DEF;

∠A = ∠D and ∠B = ∠E ⇒ ∠C = ∠F.

Reconsidering the given figure, now we have :

∠A = ∠D, ∠C = ∠F and AC = DF

⇒ **Δ ABC ≡ Δ DEF** by A.S.A.



5. *Two right-angled triangles are congruent, if the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of the other triangle.* *Abbreviated as :* ***R.H.S.***

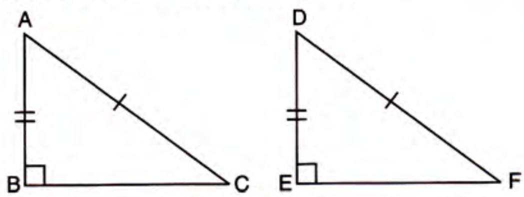
The given figure shows two right-angled triangles ABC and DEF such that :

∠B = ∠E = 90°;

AC = DF

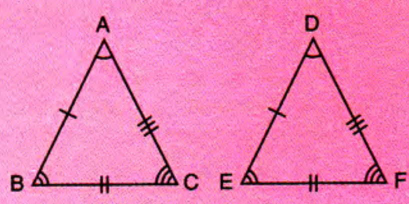
and, AB = DE

∴ Δ ABC ≅ Δ DEF. [By R.H.S.]



Important :

1. The given figure shows two congruent triangles ABC and DEF such that vertex A corresponds to vertex D (as, ∠A = ∠D); vertex B corresponds to vertex E (as, ∠B = ∠E) and, vertex C corresponds to vertex F (as, ∠C = ∠F).



We write : Δ ABC ≅ Δ DEF and not Δ ABC ≅ Δ DFE or Δ BAC ≅ Δ DEF, etc.

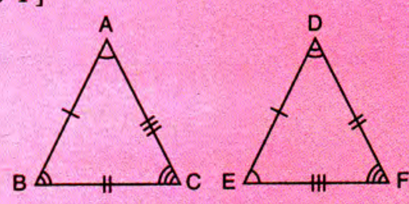
In fact, the order of vertices of two congruent triangles must be writen in such a way that the corresponding vertices occupy the same position.

Thus, triangle ABC is congruent to triangle DEF

⇒ Δ ABC ≅ Δ DEF [A ↔ D, B ↔ E and C ↔ F]

2. The adjoining figure shows two congruent triangles such that, the corresponding vertices are: A ↔ E; B ↔ D and C ↔ F.

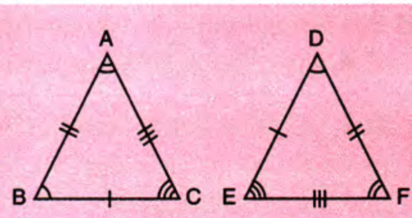
∴ Δ ABC ≅ Δ EDF



3. In the adjoining figure, the corresponding vertices are : A and F, B and D and C and E.

$\therefore$ Δ ABC $\cong$ Δ FDE

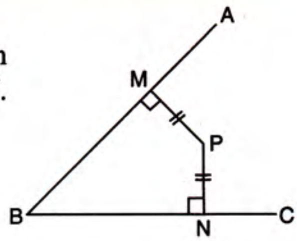
4. The same procedure is used for **Similar Triangles.**



1 P is any point in the angle ABC such that the perpendiculars drawn from P on AB and BC are equal. Prove that BP bisects angle ABC.

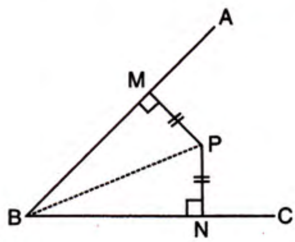
Solution :

According to the given statement, the figure will be as shown alongside. In the figure, PM ⊥ AB and PN ⊥ BC such that PM = PN.



Construction : Join P and B.

Proof :

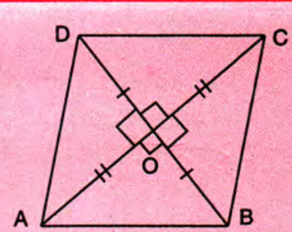


Statement :	**Reason :**
In Δ PMB and Δ PNB :	
(i) PM = PN	[Given]
(ii) $\angle$PMB = $\angle$PNB	[Each 90°]
(iii) PB = PB	[Common]
$\therefore$ Δ PMB $\cong$ Δ PNB	[R.H.S.]
$\Rightarrow$ $\angle$PBM = $\angle$PBN	[C.P.C.T.C.]

$\Rightarrow$ **BP bisects angle ABC.** **Hence proved.**

2 If the diagonals of a quadrilateral bisect each other at right angle; prove that the quadrilateral is a rhombus.

A rhombus has all its four sides equal.



Solution :

Given : A quadrilateral ABCD, in which diagonals AC and BD bisect each other at 90°.

To Prove : ABCD is a rhombus *i.e.,* AB = BC = CD = DA.

Proof :

Statement :	**Reason :**
In triangles AOB and COB :	
1. OA = OC	[Given, diagonals bisect each other]
2. OB = OB	[Common]
3. $\angle$AOB = $\angle$BOC	[Given, diagonals bisect each other at 90°]
$\therefore$ Δ AOB $\cong$ Δ COB	[S.A.S.]
$\therefore$ AB = BC ----------- I	[Corresponding parts of congruent Δs]

Similarly, by proving that Δ BOC ≅ Δ DOC and Δ COD ≅ Δ AOD; we get :

BC = CD and CD = DA -------------- II

∴ AB = BC = CD = DA [Combining I and II]

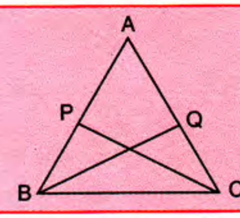
i.e. **ABCD is a rhombus.**

Hence Proved.

3 In triangle ABC, the sides AB and AC are equal. If P is a point on AB and Q is a point on AC such that AP = AQ; prove that :

(i) triangle APC and AQB are congruent.

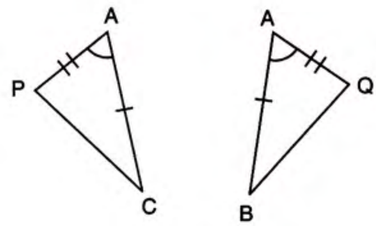
(ii) triangle BPC and CQB are congruent.



Solution :

Given : A triangle ABC, in which AB = AC and AP = AQ.

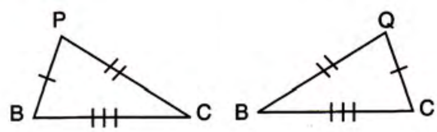
(i) ***Proof :***



Statement :	**Reason :**
In triangles APC and AQB :	
1. AC = AB	[Given]
2. AP = AQ	[Given]
3. ∠CAP = ∠BAQ	[Common]
∴ **Δ APC ≅ Δ AQB.**	[S.A.S.]

Hence Proved.

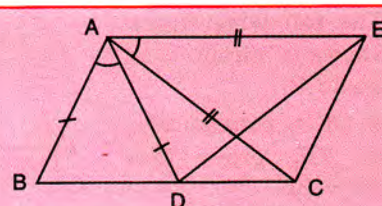
(ii) ***Proof :***



Statement :	**Reason :**
Since; AB = AC and AP = AQ	[Given]
∴ AB − AP = AC − AQ	
i.e. BP = CQ.	
In triangles BPC and CQB :	
1. BP = CQ	[From above]
2. CP = BQ	[Corresponding sides of ≅ Δs APC and AQB]
3. BC = BC	[Common]
∴ **Δ BPC ≅ Δ CQB**	[S.S.S.]

Hence Proved.

4 In the given figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that : BC = DE.



Solution :

∠BAD = ∠EAC [Given]

⇒ ∠BAD + ∠DAC = ∠EAC + ∠DAC [Adding ∠DAC on both the sides]

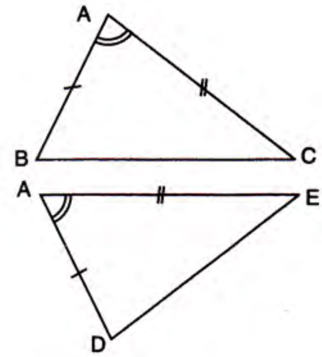
$\Rightarrow$ $\angle BAC = \angle DAE$

$AB = AD$ [Given]

and, $AC = AE$ [Given]

$\Rightarrow$ $\Delta\ BAC = \Delta\ DAE$ [By S.A.S.]

$\Rightarrow$ **BC = DE** [C.P.C.T.C.]

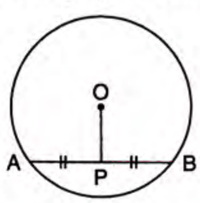


EXERCISE 9(A)

1. Which of the following pairs of triangles are congruent ? In each case, state the condition of congruency :
   (a) In Δ ABC and Δ DEF, AB = DE, BC = EF and $\angle B = \angle E$.
   (b) In Δ ABC and Δ DEF, $\angle B = \angle E = 90°$; AC = DF and BC = EF.
   (c) In Δ ABC and Δ QRP, AB = QR, $\angle B = \angle R$ and $\angle C = \angle P$.
   (d) In Δ ABC and Δ PQR, AB = PQ, AC = PR and BC = QR.
   (e) In Δ ABC and Δ PQR, BC = QR, $\angle A = 90°$, $\angle C = \angle R = 40°$ and $\angle Q = 50°$.

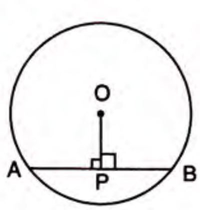
2. The given figure shows a circle with centre O. P is mid-point of chord AB.

   Show that OP is perpendicular to AB.



3. The following figure shows a circle with centre O.

   If OP is perpendicular to AB, prove that AP = BP.



4. In a triangle ABC, D is mid-point of BC; AD is produced upto E so that DE = AD.

   Prove that :
   (i) Δ ABD and Δ ECD are congruent.
   (ii) AB = EC.
   (iii) AB is parallel to EC.

5. A triangle ABC has $\angle B = \angle C$.

   Prove that :
   (i) the perpendiculars from the mid-point of BC to AB and AC are equal.
   (ii) the perpendiculars from B and C to the opposite sides are equal.

6. The perpendicular bisectors of the sides of a triangle ABC meet at I.

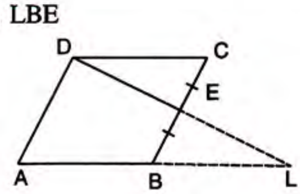
   Prove that : IA = IB = IC.

7. A line segment AB is bisected at point P and through point P another line segment PQ, which is perpendicular to AB, is drawn. Show that : QA = QB.

8. If AP bisects angle BAC and M is any point on AP, prove that the perpendiculars drawn from M to AB and AC are equal.

9. From the given diagram, in which ABCD is a parallelogram, ABL is a line segment and E is mid point of BC.

   Prove that :
   (i) $\Delta\ DCE \cong \Delta\ LBE$
   (ii) AB = BL.
   (iii) AL = 2DC



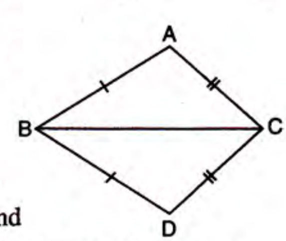
10. In the given figure, AB = DB and AC = DC.

    If $\angle ABD = 58°$,

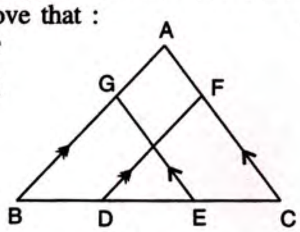
    $\angle DBC = (2x - 4)°$,

    $\angle ACB = y + 15°$ and

    $\angle DCB = 63°$; find the values of x and y.

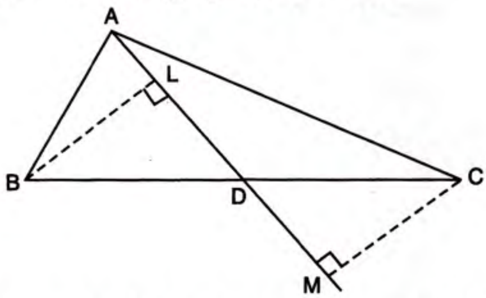


11. In the given figure : AB//FD, AC//GE and BD = CE; prove that :
 (i) BG = DF
 (ii) CF = EG



12. In a triangle ABC, AB = AC. Show that the altitude AD is median also.

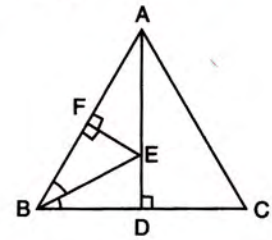
13. In the following figure, BL = CM.



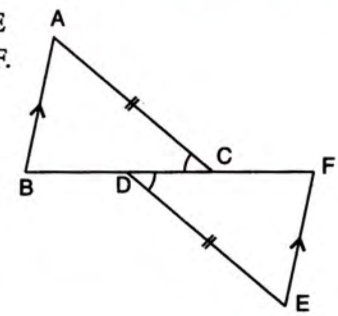
Prove that AD is a median of triangle ABC.

14. In the following figure, AB = AC and AD is perpendicular to BC. BE bisects angle B and EF is perpendicular to AB.

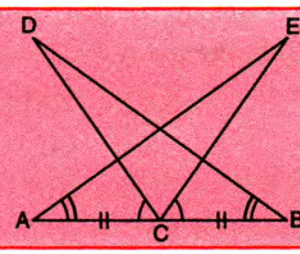
Prove that :
 (i) BD = CD
 (ii) ED = EF



15. Use the information in the given figure to prove :
 (i) AB = FE
 (ii) BD = CF.



5 Use the information given in the adjoining figure, to prove :
 (i) Δ DBC ≅ Δ EAC.
 (ii) DC = EC.



Solution :

(i) Let $\angle ACD = \angle BCE = x$

∴ $\angle ACE = \angle DCE + \angle ACD = \angle DCE + x$ .... (i)

and, $\angle BCD = \angle DCE + \angle BCE = \angle DCE + x$ .... (ii)

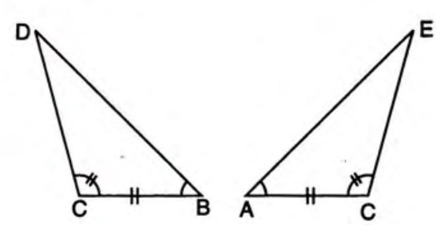
From (i) and (ii), we get : $\angle ACE = \angle BCD$

Now, in Δ DBC and Δ EAC,

$\angle ACE = \angle BCD$ [Proved above]

BC = AC [Given]

$\angle CBD = \angle EAC$ [Given]



∴ **Δ DBC ≅ Δ EAC** [By A.S.A.] **Hence proved.**

(ii) Since, Δ DBC ≅ Δ EAC

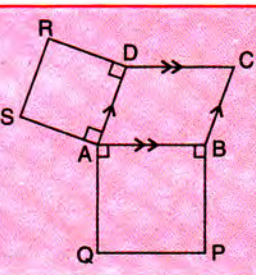
⇒ **DC = EC.** **Hence proved.**

6 The given figure shows a parallelogram ABCD. Squares ABPQ and ADRS are drawn on sides AB and AD of the parallelogram ABCD.

Prove that :

(i) ∠SAQ = ∠ABC.

(ii) SQ = AC.



Solution :

(i) Let ∠ABC = x

Since, adjacent angles of a parallelogram are supplementary, ∠DAB + ∠ABC = 180°

i.e. ∠DAB + x = 180° ⇒ ∠DAB = 180° − x

Consider the angles around the point A :

∠SAQ + ∠SAD + ∠QAB + ∠DAB = 360°

⇒ ∠SAQ + 90° + 90° + 180° − x = 360°

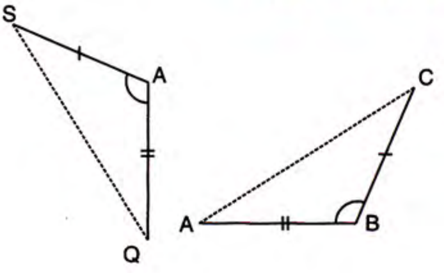
⇒ **∠SAQ = x = ∠ABC.** **Hence proved.**

(ii) Since, sides of a square are equal, therefore in square ADRS, SA = AD.

Since, opposite sides of a parallelogram are equal, therefore in parallelogram ABCD, AD = BC.

Combining the two results, we get : SA = BC.

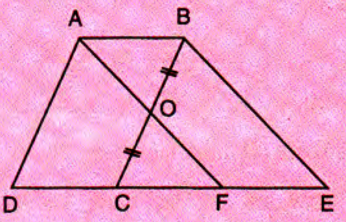
Consider the Δ SAQ and Δ ABC :



	∠SAQ = ∠ABC	[Proved in part (i)]
	SA = BC	[Proved above]
	AQ = AB	[Sides of the square ABPQ]
∴	Δ SAQ ≅ Δ CBA	[By S.A.S.]
⇒	**SQ = AC.**	[By C.P.C.T.C.]

Hence proved.

7 In the given figure, ABCD and ABEF are parallalograms. If O is the mid-point of BC, show that : DC = CF = FE.



Solution :

In Δ ABO and Δ FCO,

	OB = OC	[Given]
	∠AOB = ∠FOC	[Vertically opposite angles]
	∠OBA = ∠OCF	[Alternate angles as AB//CF and BC is transversal]
⇒	Δ ABO ≅ Δ FCO	[BY A.S.A.]
⇒	AB = CF	[By C.P.C.T.C.]

Also,	AB = DC	[Opposite sides of parallelogram ABCD]
and,	AB = FE	[Opp. sides of parallelogram ABEF]
∴	**DC = CF = FE**	

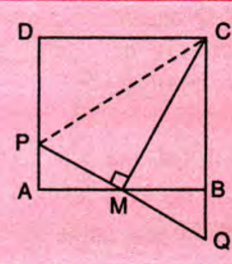
Hence Proved

8 In the given figure, ABCD is a square. M is the mid-point of AB and PQ is perpendicular to CM. CB produced meets PQ at point Q.

Prove that :

(i) PA = BQ

(ii) CP = AB + PA.



Solution :

(i) In Δ PAM and Δ QBM,

	∠A = ∠ABC	[Each angle of a square is 90°]
	= ∠MBQ	[∠MBQ = 180° – ∠ABC = 180° – 90° = 90°]
	∠AMP = ∠BMQ	[Vertically opposite angles]
	AM = BM	[M is mid-point of AB]
⇒	Δ PAM ≅ Δ QBM	[BY A.S.A.]
⇒	**PA = BQ**	[By C.P.C.T.C.]

Hence Proved

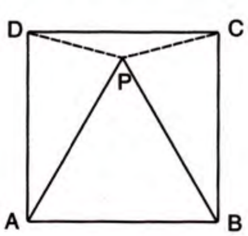
(ii)	Δ PAM ≅ Δ QBM	[Proved above]
⇒	PM = QM	[By C.P.C.T.C.]
	∠CMP = ∠CMQ = 90°	
and,	CM = CM	[Common]
⇒	Δ PCM ≅ Δ QCM	[By S.A.S.]
⇒	**CP** = CQ	[By C.P.C.T.C.]
	= CB + BQ	
	= **AB + PA**	[CB = AB = side of square ABCD and BQ = PA (proved above)]

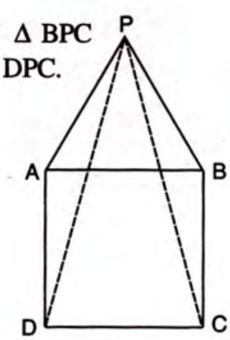
Hence Proved

EXERCISE 9(B)

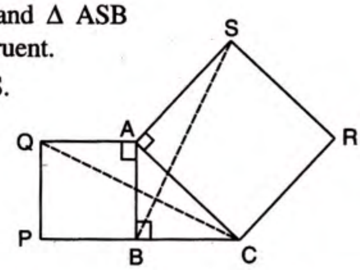
1. On the sides AB and AC of triangle ABC, equilateral triangles ABD and ACE are drawn. Prove that: (i) ∠CAD =∠BAE (ii) CD = BE.

2. In the following diagrams, ABCD is a square and APB is an equilateral triangle. In each case,
   (i) Prove that : Δ APD Δ BPC
   (ii) Find the angles of Δ DPC.



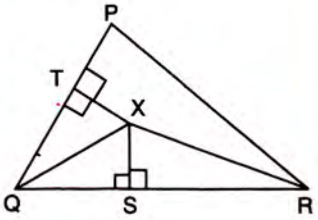


3. In the figure, given below, triangle ABC is right-angled at B. ABPQ and ACRS are squares. Prove that :
   (i) Δ ACQ and Δ ASB are congruent.
   (ii) CQ = BS.



4. In a Δ ABC, BD is the median to the side AC, BD is produced to E such that BD = DE. Prove that : AE is parallel to BC.

5. In the adjoining figure, QX and RX are the bisectors of the angles Q and R respectively of the triangle PQR.

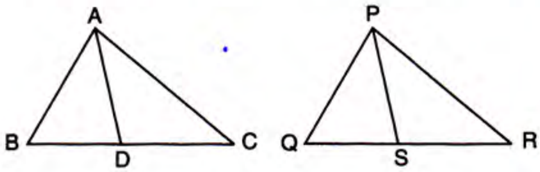


If XS ⊥ QR and XT ⊥ PQ; prove that :
(i) Δ XTQ ≅ Δ XSQ
(ii) PX bisects angle P.

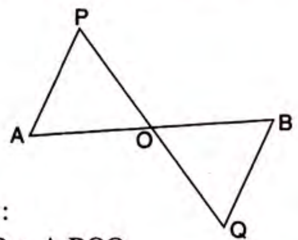
6. In the parallelogram ABCD, the angles A and C are obtuse. Points X and Y are taken on the diagonal BD such that the angles XAD and YCB are right angles.
Prove that : XA = YC.

7. ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively, such that AB = BE and AD = DF.
Prove that : Δ BEC ≅ Δ DCF.

8. In the following figures, the sides AB and BC and the median AD of triangle ABC are respectively equal to the sides PQ and QR and median PS of the triangle PQR. Prove that Δ ABC and Δ PQR are congruent.

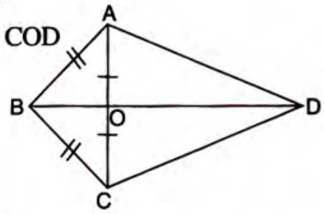


9. In the following diagram, AP and BQ are equal and parallel to each other.

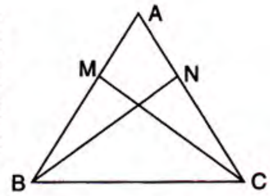


Prove that :
(i) Δ AOP ≅ Δ BOQ.
(ii) AB and PQ bisect each other.

10. In the following figure, OA = OC and AB = BC.
Prove that :
(i) ∠AOB = 90°
(ii) Δ AOD ≅ Δ COD
(iii) AD = CD



11. The following figure shows a triangle ABC in which AB = AC. M is a point on AB and N is a point on AC such that BM = CN.

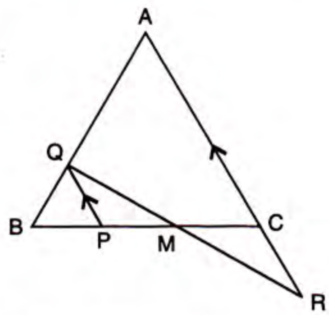


Prove that :
(i) AM = AN
(ii) Δ AMC ≅ Δ ANB
(iii) BN = CM
(iv) Δ BMC ≅ Δ CNB

12. In a triangle ABC, AB = BC, AD is perpendicular to side BC and CE is perpendicular to side AB. Prove that : AD = CE.

13. PQRS is a parallelogram. L and M are points on PQ and SR respectively such that PL = MR. Show that LM and QS bisect each other.

14. In the following figure, ABC is an equilateral triangle in which QP is parallel to AC. Side AC is produced upto point R so that CR = BP.



Prove that QR bisects PC.

Show that Δ QBP is equilateral
⇒ BP = PQ, but BP = CR
⇒ PQ = CR ⇒ Δ QPM ≅ Δ RCM