8 Logarithms

8.1 INTRODUCTION

Logarithms are used to make the long and complicated calculations easy.

Consider $3^4 = 81$, this is the exponential form of representing relation between three numbers 3, 4 and 81. Now the same relation between 3, 4 and 81 can be written as

$\log_3 81 = 4$ (read as : logarithm of 81 at base 3 is 4).

Thus : $3^4 = 81 \Leftrightarrow \log_3 81 = 4$

Definition : If a, b and c are three real numbers such that $a \neq 1$ and $a^b = c$ then b is called logarithm of c at the base a and is written as $\log_a c = b$; read as log of c at the base a is b.

$$a^b = c \Leftrightarrow \log_a c = b$$

8.2 INTERCHANGING

(Logarithmic form vis-à-vis exponential form)

$a^b = c$ is called the ***exponential form***

and, $\log_a c = b$ is called the ***logarithmic form.***

i.e., (i) $2^{-3} = 0{\cdot}125$ [Exponential form]

$\Rightarrow$ log of 0·125 to the base 2 $= -3$

i.e., $\log_2 0{\cdot}125 = -3$ [Logarithmic form]

(ii) $\log_{64} 8 = \frac{1}{2}$ [Logarithmic form]

$\Rightarrow$ log of 8 to the base 64 $= \frac{1}{2}$

i.e. $(64)^{\frac{1}{2}} = 8$ [Exponential form] and so on.

Similarly :

If x is positive;

(iii) $x^0 = 1 \Rightarrow \log_x 1 = 0$ *i.e.* log of 1 to the base $x = 0$

In general; *the logarithm of 1 to any base is zero.*

i.e. $\log_5 1 = 0$; $\log_{10} 1 = 0$; $\log_a 1 = 0$ and so on.

(iv) $x^1 = x \Rightarrow \log_x x = 1$ *i.e.* log x to the base $x = 1$

In general, *the logarithm of any number to the same base is always one.*

i.e. $\log_5 5 = 1$; $\log_{10} 10 = 1$; $\log_a a = 1$ and so on.

1 Find : (i) the logarithm of 1000 to the base 10.

(ii) the logarithm of $\frac{1}{9}$ to the base 3.

Solution :

(i) Let $\log_{10}1000 = x \Rightarrow 10^x = 1000$

$\Rightarrow 10^x = 10^3 \Rightarrow x = 3$

$\therefore \log_{10}1000 = 3$ **Ans.**

(ii) Let $\log_3 \frac{1}{9} = x \Rightarrow 3^x = \frac{1}{9}$

$\Rightarrow 3^x = 3^{-2} \Rightarrow x = -2$

$\therefore \log_3 \frac{1}{9} = -2$ **Ans.**

2 Find x, if : (i) $\log_2 x = -2$ (ii) $\log_4(x + 3) = 2$ (iii) $\log_x 64 = \frac{3}{2}$

Solution :

(i) $\log_2 x = -2 \Rightarrow 2^{-2} = x$

$\Rightarrow x = \frac{1}{4}$ **Ans.**

(ii) $\log_4 (x + 3) = 2 \Rightarrow 4^2 = x + 3$

$\Rightarrow x = 16 - 3 = 13$ **Ans.**

(iii) $\log_x 64 = \frac{3}{2} \Rightarrow x^{\frac{3}{2}} = 64$

$\Rightarrow x = (64)^{\frac{2}{3}} = (2^6)^{\frac{2}{3}} = 2^4 = 16$ **Ans.**

EXERCISE 8(A)

1. Express each of the following in *logarithmic form* :

   (i) $5^3 = 125$ (ii) $3^{-2} = \frac{1}{9}$

   (iii) $10^{-3} = 0{\cdot}001$ (iv) $(81)^{\frac{3}{4}} = 27$

2. Express each of the following in *exponential form* :

   (i) $\log_8 0{\cdot}125 = -1$ (ii) $\log_{10}0{\cdot}01 = -2$

   (iii) $\log_a A = x$ (iv) $\log_{10}1 = 0$

3. Solve for x : $\log_{10} x = -2$.

4. Find the logarithm of :

   (i) 100 to the base 10

   (ii) 0·1 to the base 10

   (iii) 0·001 to the base 10

   (iv) 32 to the base 4

   (v) 0·125 to the base 2

   (vi) $\frac{1}{16}$ to the base 4

   (vii) 27 to the base 9

   (viii) $\frac{1}{81}$ to the base 27

5. State, true or false :

   (i) If $\log_{10} x = a$, then $10^x = a$.

   (ii) If $x^y = z$, then $y = \log_z x$.

   (iii) $\log_2 8 = 3$ and $\log_8 2 = \frac{1}{3}$.

6. Find x, if :

   (i) $\log_3 x = 0$ (ii) $\log_x 2 = -1$

   (iii) $\log_9 243 = x$ (iv) $\log_5 (x - 7) = 1$

   (v) $\log_4 32 = x - 4$ (vi) $\log_7 (2x^2 - 1) = 2$

7. Evaluate :

   (i) $\log_{10} 0{\cdot}01$ (ii) $\log_2 (1 \div 8)$

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(iii) $\log_5 1$ (iv) $\log_5 125$
(v) $\log_{16} 8$ (vi) $\log_{0.5} 16$

8. If $\log_a m = n$, express a^{n-1} in terms of a and m.

$\log_a m = n \Rightarrow a^n = m$

$\Rightarrow a^{n-1} = \frac{a^n}{a} = \frac{m}{a}$

9. Given $\log_2 x = m$ and $\log_5 y = n$.
 (i) Express 2^{m-3} in terms of x.
 (ii) Express 5^{3n+2} in terms of y.

10. If $\log_2 x = a$ and $\log_3 y = a$, write 72^a in terms of x and y.

11. Solve for x : $\log(x-1) + \log(x+1) = \log_2 1$.

12. If $\log(x^2 - 21) = 2$, show that $x = \pm 11$.

8.3 LAWS OF LOGARITHM WITH USE

First Law (Product Law) :

The *logarithm of a product at any non-zero base* is equal to the *sum of the logarithms* of its factors at the same base.

i.e. $\mathbf{\log_a (m \times n) = \log_a m + \log_a n}$

$\mathbf{\log_x (m \times n \times p) = \log_x m + \log_x n + \log_x p}$ and so on.

Remember : $\log_a (m + n) \neq \log_a m + \log_a n$

Second Law (Quotient Law) :

The *logarithm of a fraction* at any non-zero base is equal to the *difference between the logarithm of the numerator minus the logarithm of the denominator, both at the same base.*

i.e. $\mathbf{\log_a \frac{m}{n} = \log_a m - \log_a n}$

Remember : $\frac{\log_a m}{\log_a n} \neq \log_a m - \log_a n$. Also, $\log_a (m - n) \neq \log_a m - \log_a n$

Third Law (Power Law) :

The *logarithm of a power of a number at any non-zero base* is equal to the *logarithm of the number (at the same base) multiplied by the power.*

i.e. $\mathbf{\log_a (m)^n = n \log_a m}$

Corollary : Since $\sqrt[n]{m} = m^{\frac{1}{n}}$

$\therefore$ $\log_a \sqrt[n]{m} = \log_a m^{\frac{1}{n}} = \frac{1}{n} \log_a m$

1. Logarithms to the **base 10** are known as ***common logarithms.***
2. If **no base is given,** the **base is** always taken as **10,**
 i.e. $\log 8 = \log_{10} 8$; $\log a = \log_{10} a$; $\log 10 = \log_{10} 10$ and so on.
3. $\log_{10} 1 = 0$; $\log_{10} 10 = 1$;
 $\log_{10} 100 = 2$ [$\log_{10} 100 = \log_{10} 10^2 = 2\log_{10} 10 = 2 \times 1 = 2$]
 Similarly, $\log_{10} 1000 = 3$; $\log_{10} 10000 = 4$ and so on.

8.4 EXPANSION OF EXPRESSIONS WITH THE HELP OF LAWS OF LOGARITHM

Let $y = \frac{a^4 \times b^2}{c^3} \Rightarrow \log y = \log \frac{a^4 \times b^2}{c^3}$

i.e. $\log y = \log(a^4 \times b^2) - \log c^3$ [$\because \log \frac{m}{n} = \log m - \log n$]

$= \log a^4 + \log b^2 - \log c^3$ $\quad [\because \log m \times n = \log m + \log n]$

$= 4 \log a + 2 \log b - 3 \log c$ $\quad [\because \log m^n = n \log m]$

$\therefore$ $\log y = 4 \log a + 2 \log b - 3 \log c$ is the logarithmic expansion of the given expression $y = \frac{a^4 \times b^2}{c^3}$

Similarly,

$$m = \frac{3^x}{5^y \times 8^z} \Rightarrow \log m = \log 3^x - \log (5^y \times 8^z)$$
$$= x \log 3 - [\log 5^y + \log 8^z]$$
$$= x \log 3 - y \log 5 - z \log 8$$
$$\Rightarrow \log m = x \log 3 - y \log 5 - z \log 8$$

Conversely :

$$\log V = \log \pi + 2 \log r + \log h - \log 3$$
$$\Rightarrow \log V = \log \pi + \log r^2 + \log h - \log 3$$
$$= \log \frac{\pi r^2 h}{3} \Rightarrow V = \frac{\pi r^2 h}{3}$$

3 Express $\log_{10} \sqrt[5]{108}$ in terms of $\log_{10}2$ and $\log_{10}3$.

Solution :

$\log_{10} \sqrt[5]{108} = \log_{10}(108)^{\frac{1}{5}}$ $\quad [\sqrt[n]{m} = m^{\frac{1}{n}}]$

$= \frac{1}{5} \log_{10}108$ $\quad [\log_{10}n^m = m \log_{10}n]$

$= \frac{1}{5} \log_{10}(2^2 \times 3^3)$ $\quad [108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3]$

$= \frac{1}{5} [\log_{10}2^2 + \log_{10}3^3]$ $\quad [\log_{10}m \times n = \log_{10}m + \log_{10}n]$

$= \frac{1}{5} [2\log_{10}2 + 3\log_{10}3]$ **Ans.**

4 Express as a single logarithm : $2 + \frac{1}{2}\log_{10}9 - 2\log_{10}5$

Solution :

$= \log_{10}100 + \log_{10}9^{\frac{1}{2}} - \log_{10}5^2$ $\quad [\log_{10}100 = 2;\ \log 9 = \log 3^2$ and $2\log 5 = \log 5^2]$

$= \log_{10}100 + \log_{10}3 - \log_{10}25$

$= \log_{10}\frac{100 \times 3}{25} = \log_{10}12$ $\quad [\log a + \log b - \log c = \log \frac{a \times b}{c}]$ **Ans.**

5 Find x, if : (i) $\log_{10}(x + 5) = 1$.

(ii) $\log_{10}(x + 1) + \log_{10}(x - 1) = \log_{10}11 + 2\log_{10}3$

Solution :

(i) $\Rightarrow \log_{10}(x + 5) = \log_{10}10$

$\Rightarrow x + 5 = 10 \Rightarrow x = 5$ **Ans.**

(ii) $\Rightarrow \log_{10}(x + 1)(x - 1) = \log_{10}11 + \log_{10}3^2$

$\Rightarrow \log (x^2 - 1) = \log (11 \times 9)$

$\Rightarrow x^2 - 1 = 99$

$\therefore x^2 = 100$ and, $x = 10$ **Ans.**

6 If $\log 2 = 0{\cdot}3010$ and $\log 3 = 0{\cdot}4771$, find the value of :

(i) $\log 6$ (ii) $\log 5$ (iii) $\log \sqrt{24}$

Solution :

(i) $\log 6 = \log 2 \times 3 = \log 2 + \log 3 = 0{\cdot}3010 + 0{\cdot}4771 = 0{\cdot}7781$ **Ans.**

(ii) $\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0{\cdot}3010 = 0{\cdot}6990$ **Ans.** [$\because \log 10 = 1$]

(iii) $\log\sqrt{24} = \log (24)^{\frac{1}{2}} = \frac{1}{2}\log (2^3 \times 3)$ [$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$]

$= \frac{1}{2}[3 \times \log 2 + \log 3]$

$= \frac{1}{2}[3 \times 0{\cdot}3010 + 0{\cdot}4771] = 0{\cdot}69005$ **Ans.**

EXERCISE 8(B)

1. Express in terms of log 2 and log 3:
   (i) $\log 36$ (ii) $\log 144$
   (iii) $\log 4{\cdot}5$ (iv) $\log \frac{26}{51} - \log \frac{91}{119}$
   (v) $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243}$
2. Express each of the following in a form free from logarithm :
   (i) $2 \log x - \log y = 1$
   (ii) $2 \log x + 3 \log y = \log a$
   (iii) $a \log x - b \log y = 2 \log 3$
3. Evaluate each of the following without using tables :
   (i) $\log 5 + \log 8 - 2 \log 2$
   (ii) $\log_{10} 8 + \log_{10} 25 + 2 \log_{10} 3 - \log_{10} 18$
   (iii) $\log 4 + \frac{1}{3} \log 125 - \frac{1}{5} \log 32$
4. Prove that :
   $2 \log \frac{15}{18} - \log \frac{25}{162} + \log \frac{4}{9} = \log 2.$
5. Find x, if :
   $x - \log 48 + 3 \log 2 = \frac{1}{3} \log 125 - \log 3.$
6. Express $\log_{10}2 + 1$ in the form of $\log_{10}x$.
7. Solve for x :
   (i) $\log_{10}(x - 10) = 1$
   (ii) $\log (x^2 - 21) = 2$
   (iii) $\log (x - 2) + \log (x + 2) = \log 5$
   (iv) $\log (x + 5) + \log (x - 5) = 4 \log 2 + 2 \log 3$
8. Solve for x :
   (i) $\frac{\log 81}{\log 27} = x$ (ii) $\frac{\log 128}{\log 32} = x$
   (iii) $\frac{\log 64}{\log 8} = \log x$ (iv) $\frac{\log 225}{\log 15} = \log x$

(i) $\dfrac{\log 81}{\log 27} = x$

$\Rightarrow x = \dfrac{\log 3^4}{\log 3^3} = \dfrac{4\log 3}{3\log 3} = \dfrac{4}{3}$ **Ans.**

9. Given $\log x = m + n$ and $\log y = m - n$, express the value of $\log \dfrac{10x}{y^2}$ in terms of m and n.

10. State, true or false :
   (i) $\log 1 \times \log 1000 = 0$
   (ii) $\dfrac{\log x}{\log y} = \log x - \log y$
   (iii) If $\dfrac{\log 25}{\log 5} = \log x$, then $x = 2$
   (iv) $\log x \times \log y = \log x + \log y$

11. If $\log_{10}2 = a$ and $\log_{10}3 = b$; express each of the following in terms of 'a' and 'b' :
   (i) $\log 12$ (ii) $\log 2{\cdot}25$ (iii) $\log 2\frac{1}{4}$
   (iv) $\log 5{\cdot}4$ (v) $\log 60$ (vi) $\log 3\frac{1}{8}$

12. If $\log 2 = 0{\cdot}3010$ and $\log 3 = 0{\cdot}4771$; find the value of :
   (i) $\log 12$ (ii) $\log 1{\cdot}2$
   (iii) $\log 3{\cdot}6$ (iv) $\log 15$
   (v) $\log 25$ (vi) $\frac{2}{3}\log 8$

13. Given $2\log_{10}x + 1 = \log_{10}250$, find :
   (i) x (ii) $\log_{10}2x$

14. Given $3\log x + \frac{1}{2}\log y = 2$, express y in term of x.

15. If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, find $\log \dfrac{10\sqrt{y}}{x^2z^3}$ in terms of a, b and c.

16. If $3(\log 5 - \log 3) - (\log 5 - 2\log 6) = 2 - \log x$, find x.

7 If $\log_{10}4 = 0{\cdot}6020$; find the value of :
(i) $\log_{10}8$ (ii) $\log_{10}2{\cdot}5$

Solution :

If $\log_{10}4 = 0{\cdot}6020 \Rightarrow 2\log 2 = 0{\cdot}6020$ [$\because \log 4 = \log 2^2 = 2\log 2$]

$\Rightarrow \log 2 = \dfrac{0{\cdot}6020}{2} = 0{\cdot}3010$

(i) $\log_{10}8 = \log_{10}2^3$
$= 3\log 2 = 3 \times 0{\cdot}3010 = \mathbf{0{\cdot}9030}$ **Ans.**

(ii) $\log_{10}2{\cdot}5 = \log\dfrac{25}{10} = \log\dfrac{5}{2} = \log\dfrac{10}{2\times 2}$
$= \log 10 - 2\log 2 = 1 - 2 \times 0{\cdot}3010 = \mathbf{0{\cdot}3980}$ **Ans.**

8 Given $\log_{10}x = a$ and $\log_{10}y = b$.
(i) Write down 10^{a-1} in terms of x.
(ii) Write down 10^{2b} in terms of y.
(iii) If $\log_{10}P = 2a - b$; express P in terms of x and y.

Solution :

(i) $\log_{10}x = a \Rightarrow 10^a = x$

$\therefore 10^{a-1} = \dfrac{10^a}{10^1} = \dfrac{x}{10}$ **Ans.**

(ii) $\log_{10}^{y} = b \quad \Rightarrow \quad 10^b = y$

$\therefore \quad 10^{2b} = (10^b)^2 = y^2$ **Ans.**

(iii) $\log_{10} P = 2a - b$

$\Rightarrow \quad \log_{10} P = 2\log_{10} x - \log_{10} y$

$\Rightarrow \quad \log P = \log x^2 - \log y \Rightarrow \log P = \log \frac{x^2}{y} \quad \therefore P = \frac{x^2}{y}$ **Ans.**

EXERCISE 8(C)

1. If $\log_{10} 8 = 0{\cdot}90$; find the value of :
   (i) $\log_{10} 4$ (ii) $\log \sqrt{32}$
   (iii) $\log 0{\cdot}125$
2. If $\log 27 = 1{\cdot}431$, find the value of :
   (i) $\log 9$ (ii) $\log 300$
3. If $\log_{10} a = b$, find 10^{3b-2} in terms of a.
4. If $\log_5 x = y$, find 5^{2y+3} in terms of x.
5. Given: $\log_3 m = x$ and $\log_3 n = y$.
   (i) Express 3^{2x-3} in terms of m.
   (ii) Write down $3^{1-2y+3x}$ in terms of m and n.
   (iii) If $2 \log_3 A = 5x - 3y$; find A in terms of m and n.
6. Simplify :
   (i) $\log (a)^3 - \log a$ (ii) $\log (a)^3 \div \log a$
7. If $\log (a + b) = \log a + \log b$, find a in terms of b.
8. Prove that :
   (i) $(\log a)^2 - (\log b)^2 = \log \left(\frac{a}{b}\right) . \log (ab)$
   (ii) If $a \log b + b \log a - 1 = 0$, then $b^a . a^b = 10$
9. (i) If $\log (a + 1) = \log (4a - 3) - \log 3$; find a.
   (ii) If $2 \log y - \log x - 3 = 0$, express x in terms of y.
   (iii) Prove that : $\log_{10}125 = 3(1 - \log_{10} 2)$.
10. Given $\log x = 2m - n$, $\log y = n - 2m$ and $\log z = 3m - 2n$, find in terms of m and n, the value of $\log \frac{x^2 y^3}{z^4}$.
11. Given $\log_x 25 - \log_x 5 = 2 - \log_x \frac{1}{125}$; find x.

8.5 MORE ABOUT LOGARITHMS

1. Since, $\quad 2^3 = 8 \Rightarrow \log_2 8 = 3$

Also, $\quad 2^3 = 8 \Rightarrow 8^{\frac{1}{3}} = 2 \Rightarrow \log_8 2 = \frac{1}{3}$

Thus, $\quad \log_2 8 = 3$ and $\log_8 2 = \frac{1}{3} \Rightarrow$ $\boxed{\log_2 8 = \frac{1}{\log_8 2}}$

In the same way,

$5^4 = 625 \Rightarrow \log_5 625 = 4$

and, $\quad 5^4 = 625 \Rightarrow 625^{\frac{1}{4}} = 5 \Rightarrow \log_{625} 5 = \frac{1}{4}$

$\therefore \quad \log_5 625 = 4$ and $\log_{625} 5 = \frac{1}{4} \Rightarrow$ $\boxed{\log_5 625 = \frac{1}{\log_{625} 5}}$

Thus, if a and b are two positive numbers

$$\log_b a = \frac{1}{\log_a b} \text{ and } \log_a b = \frac{1}{\log_b a}.$$

2. Since, $\log_b a = \frac{1}{\log_a b} \Rightarrow \boxed{\log_b a \times \log_a b = 1}$

$\Rightarrow$ (i) $\log_5 3 \times \log_3 5 = 1$ (ii) $\log_8 12 \times \log_{12} 8 = 1$

(iii) $\log_{18} 35 \times \log_{35} 18 = 1$ and so on.

3. Since, log of a number at the same base is 1 (one)

$\log_a a = 1 \Rightarrow x \log_a a = x$ [On multiplying both the sides by x]

$\Rightarrow \boxed{\log_a a^x = x}$

$\Rightarrow$ (i) $\log_2 2^5 = 5$ (ii) $\log_5 5^8 = 8$

(iii) $\log_8 8^4 = 4$ and so on.

4. $\log_b a = \frac{\log_x a}{\log_x b}$, where a, b and x all are positive.

For example :

$$\log_{100} 1000 = \frac{\log_{10} 1000}{\log_{10} 100}$$

$$= \frac{\log_{10} 10^3}{\log_{10} 10^2} = \frac{3\log_{10} 10}{2\log_{10} 10} = \frac{3\times1}{2\times1} = \frac{3}{2}$$

9 Evaluate :

(i) $\log_{125} 625 - \log_{16} 64$ (ii) $\log_{16} 32 - \log_{25} 125 + \log_9 27$.

Solution :

(i) $\log_{125} 625 - \log_{16} 64 = \frac{\log_{10} 625}{\log_{10} 125} - \frac{\log_{10} 64}{\log_{10} 16}$ $\left[\because \log_n m = \frac{\log_a m}{\log_a n}\right]$

$= \frac{\log 5^4}{\log 5^3} - \frac{\log 2^6}{\log 2^4}$ $[\because \log_{10} x = \log x]$

$= \frac{4\log 5}{3\log 5} - \frac{6\log 2}{4\log 2} = \frac{4}{3} - \frac{3}{2} = -\frac{1}{6}$ **Ans.**

(ii) $\log_{16} 32 - \log_{25} 125 + \log_9 27$

$$= \frac{\log 32}{\log 16} - \frac{\log 125}{\log 25} + \frac{\log 27}{\log 9}$$

$$= \frac{\log 2^5}{\log 2^4} - \frac{\log 5^3}{\log 5^2} + \frac{\log 3^3}{\log 3^2}$$

$$= \frac{5 \log 2}{4 \log 2} - \frac{3 \log 5}{2 \log 5} + \frac{3 \log 3}{2 \log 3}$$

$$= \frac{5}{4} - \frac{3}{2} + \frac{3}{2} = \frac{5}{4} = 1\frac{1}{4}$$ **Ans.**

10 If $\frac{1}{\log_a x} + \frac{1}{\log_b x} = \frac{2}{\log_c x}$, prove that : $c^2 = ab$.

Solution :

Since, $\log_b a = \frac{1}{\log_a b} \Rightarrow \frac{1}{\log_a x} = \log_x a$, $\frac{1}{\log_b x} = \log_x b$ and $\frac{1}{\log_c x} = \log_x c$

$$\therefore \frac{1}{\log_a x} + \frac{1}{\log_b x} = \frac{2}{\log_c x} \Rightarrow \log_x a + \log_x b = 2 \log_x c$$

$$\Rightarrow \log_x ab = \log_x c^2 \Rightarrow ab = c^2$$

Hence Proved.

EXERCISE 8(D)

1. If $\frac{3}{2}\log a + \frac{2}{3}\log b - 1 = 0$, find the value of $a^9.b^4$.
2. If $x = 1 + \log 2 - \log 5$, $y = 2 \log 3$ and $z = \log a - \log 5$; find the value of a, if $x + y = 2z$.
3. If $x = \log 0{\cdot}6$; $y = \log 1{\cdot}25$ and $z = \log 3 - 2 \log 2$, find the values of :
   (i) $x + y - z$ (ii) $5^{x + y - z}$
4. If $a^2 = \log x$, $b^3 = \log y$ and $3a^2 - 2b^3 = 6 \log z$, express y in terms of x and z.
5. If $\log \frac{a-b}{2} = \frac{1}{2}(\log a + \log b)$, show that: $a^2 + b^2 = 6ab$.
6. If $a^2 + b^2 = 23ab$, show that : $\log \frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$.
7. If $m = \log 20$ and $n = \log 25$, find the value of x, so that : $2 \log (x - 4) = 2m - n$.
8. Solve for x and y; if $x > 0$ and $y > 0$: $\log xy = \log \frac{x}{y} + 2 \log 2 = 2$.
9. Find x, if :
   (i) $\log_x 625 = -4$ (ii) $\log_x (5x - 6) = 2$.
10. If $p = \log 20$ and $q = \log 25$, find the value of x, if $2 \log (x + 1) = 2p - q$.
11. If $\log_2 (x + y) = \log_3 (x - y) = \frac{\log 25}{\log 0{\cdot}2}$, find the values of x and y.
12. Given : $\frac{\log x}{\log y} = \frac{3}{2}$ and $\log (xy) = 5$; find the values of x and y.
13. Given $\log_{10}x = 2a$ and $\log_{10}y = \frac{b}{2}$.
   (i) Write 10^a in terms of x.
   (ii) Write 10^{2b+1} in terms of y.
   (iii) If $\log_{10}P = 3a - 2b$, express P in terms of x and y.
14. Solve :
   $\log_5(x + 1) - 1 = 1 + \log_5(x - 1)$.
15. Solve for x, if :
   $\log_x 49 - \log_x 7 + \log_x \frac{1}{343} + 2 = 0$.

16. If $a^2 = \log x$, $b^3 = \log y$ and $\frac{a^2}{2} - \frac{b^3}{3} = \log c$, find c in terms of x and y.

17. Given $x = \log_{10}12$, $y = \log_4 2 \times \log_{10}9$ and $z = \log_{10}0{\cdot}4$, find :

    (i) $x - y - z$ (ii) 13^{x-y-z}

18. Solve for x, $\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$.

19. Evaluate :

    (i) $\log_b a \times \log_c b \times \log_a c$

    (ii) $\log_3 8 \div \log_9 16$

    (iii) $\frac{\log_5 8}{\log_{25} 16 \times \log_{100} 10}$

20. Show that :

    $\log_a m \div \log_{ab} m = 1 + \log_a b$

$$\log_a m \div \log_{ab} m = \frac{\log_a m}{\log_{ab} m}$$

$$= \frac{\log_m ab}{\log_m a} \quad \left[\because \log_b a = \frac{1}{\log_a b}\right]$$

$$= \log_a ab \quad \left[\because \frac{\log_x a}{\log_x b} = \log_b a\right]$$

$$= \log_a a + \log_a b$$

$$= \mathbf{1 + \log_a b}$$