6 Simultaneous (Linear) Equations [Including Problems]

6.1 INTRODUCTION :

An equation of the form $ax + by + c = 0$ is called a **linear equation** in which a, b and c are constants (real numbers) and x and y are variables each with degree 1 (one).

Consider the two linear equations: $3x + 4y = 6$ and $8x + 5y = 3$. These two equations contain same two variables (x and y in this case). Together such equations are called **simultaneous** (linear) **equations**.

Consider the simultaneous linear equations $2x - y = 1$ and $3x + y = 14$.

If $x = 3$ and $y = 5$

$2x - y = 1 \Rightarrow 2 \times 3 - 5 = 1 \Rightarrow 1 = 1$

$3x + y = 14 \Rightarrow 3 \times 3 + 5 = 14 \Rightarrow 14 = 14$

Since, $x = 3$ and $y = 5$ satisfy both the equations $2x - y = 1$ and $3x + y = 14$.

Therefore, $x = 3$ and $y = 5$ is the solution of simultaneous linear equations under consideration.

6.2 METHODS OF SOLVING SIMULTANEOUS EQUATIONS

To solve two simultaneous linear equations means, to find the values of variables used in the given equations.

Out of the different algebraic methods for solving simultaneous equations, we shall be discussing the following three methods only :

1. Method of **elimination by substitution**.
2. Method of **elimination by equating coefficients**.
3. Method of **cross-multiplication**.

6.3 METHOD OF ELIMINATION BY SUBSTITUTION :

Steps: 1. From any of the given two equations, find the value of one variable in terms of the other.

2. Substitute the value of the variable, obtained in step (1), in the other equation and solve it.

3. Substitute the value of the variable obtained in step (2), in the result of step (1) and get the value of the remaining unknown variable.

1 Solve the following system of equations using the method of elimination by substitution.
$x + y = 7$ and $3x - 2y = 11$.

Step 1: $x + y = 7 \Rightarrow y = 7 - x$

Step 2: $3x - 2y = 11 \Rightarrow 3x - 2(7 - x) = 11$

$\Rightarrow 3x - 14 + 2x = 11$

$\Rightarrow 5x = 25$ and $x = 5$

Step 3: $y = 7 - x \Rightarrow y = 7 - 5 = 2$

$\therefore$ **Solution is : $x = 5$ and $y = 2$** **Ans.**

Alternative method :

Instead of finding the value of y in terms of x; if we find the value of x in terms of y and proceed as above; the result will remain the same, For this :

Step 1: $x + y = 7 \Rightarrow x = 7 - y$

Step 2: $3x - 2y = 11 \Rightarrow 3(7 - y) - 2y = 11$

$\Rightarrow 21 - 3y - 2y = 11$

$\Rightarrow -5y = -10$ and $y = 2$

Step 3: $x = 7 - y \Rightarrow x = 7 - 2 = 5$

$\therefore$ **Solution is : $x = 5$ and $y = 2$** **Ans.**

2 Solve using elimination by substitution :

$$\frac{x+7}{5} - \frac{2x-y}{4} = 3y - 5 \text{ and } \frac{4x-3}{6} + \frac{5y-7}{2} = 18 - 5x.$$

Solution :

$$\frac{x+7}{5} - \frac{2x-y}{4} = 3y - 5 \quad \Rightarrow \quad \frac{4(x+7)-5(2x-y)}{20} = 3y - 5$$

i.e. $4x + 28 - 10x + 5y = 60y - 100$

$\Rightarrow$ $-6x - 55y = -128$

i.e. $6x = 128 - 55y$ and $x = \frac{128-55y}{6}$

$$\frac{4x-3}{6} + \frac{5y-7}{2} = 18 - 5x \quad \Rightarrow \quad \frac{4x-3+15y-21}{6} = 18 - 5x$$

i.e. $4x + 15y - 24 = 108 - 30x \Rightarrow 34x + 15y = 132$

$\Rightarrow$ $34\left(\frac{128-55y}{6}\right) + 15y = 132$ $\left[\because x = \frac{128-55y}{6}\right]$

$\Rightarrow 34 \times 128 - 34 \times 55y + 90y = 132 \times 6$

$\Rightarrow$ $4352 - 1870y + 90y = 792$

$\Rightarrow$ $1780y = 3560$ *i.e.* $y = \frac{3560}{1780} = 2$

$\therefore$ $x = \frac{128-55y}{6} = \frac{128-55\times2}{6} = \frac{18}{6} = 3$

Solution is : $x = 3$ and $y = 2$ **Ans.**

EXERCISE 6 (A)

Solve the following pairs of linear (simultaneous) equations using method of elimination by substitution:

1. $8x + 5y = 9$
   $3x + 2y = 4$
2. $2x - 3y = 7$
   $5x + y = 9$
3. $2x + 3y = 8$
   $2x = 2 + 3y$
4. $0{\cdot}2x + 0{\cdot}1y = 25$
   $2(x - 2) - 1{\cdot}6y = 116$
5. $6x = 7y + 7$
   $7y - x = 8$
6. $y = 4x - 7$
   $16x - 5y = 25$
7. $2x + 7y = 39$
   $3x + 5y = 31$
8. $1{\cdot}5x + 0{\cdot}1y = 6{\cdot}2$
   $3x - 0{\cdot}4y = 11{\cdot}2$
9. $2(x - 3) + 3(y - 5) = 0$
   $5(x - 1) + 4(y - 4) = 0$
10. $\frac{2x+1}{7} + \frac{5y-3}{3} = 12$
    $\frac{3x+2}{2} - \frac{4y+3}{9} = 13$

6.4 METHOD OF ELIMINATION BY EQUATING COEFFICIENTS

Steps :

1. Multiply one or both of the equations by a suitable number or numbers so that either the coefficients of x or the coefficients of y in both the equations become numerically equal.
2. Add both the equations, as obtained in step 1, or subtract one equation from the other, so that the terms with equal numerical coefficients cancel mutually.
3. Solve the resulting equation to find the value of one of the unknowns.
4. Substitute this value in any of the two given equations and find the value of the other unknown.

3 Solve, using the method of elimination by equating coefficients :
$3x - 4y = 10$ and $5x - 3y = 24$

Solution :

$3x - 4y = 10$ ....(i)

$5x - 3y = 24$ ....(ii)

Step 1 : Multiply equation (i) by 5 and equation (ii) by 3.

The resulting equations are :

$15x - 20y = 50$

$15x - 9y = 72$

Step 2 : $-\quad +\quad -$ [Subtracting]

$-11y = -22$

Step 3 : $\therefore \quad y = 2$

Step 4 : Substituting $y = 2$, in eq. (i), we get :

$$3x - 4 \times 2 = 10 \qquad \Rightarrow \quad x = \frac{18}{3} = 6$$

Hence, **$x = 6$ and $y = 2$** **Ans.**

4 Solve, using the method of elimination by equating coefficients :
$x + y = 3{\cdot}3$ and $\frac{0{\cdot}6}{3x-2y} = -1$. Given $3x - 2y \neq 0$.

Solution :

$$x + y = 3{\cdot}3 \qquad \text{.....(i)}$$

$$\frac{0{\cdot}6}{3x-2y} = -1 \quad \Rightarrow \quad 0{\cdot}6 = -3x + 2y$$

i.e. $3x - 2y = -0{\cdot}6$ ....(ii)

Eq, (i) × 2 ⇒ $2x + 2y = 6{\cdot}6$

$5x = 6$ (On adding)

$$\Rightarrow \quad x = \frac{6}{5} = 1{\cdot}2$$

$x + y = 3{\cdot}3 \;\Rightarrow\; 1{\cdot}2 + y = 3{\cdot}3$ *i.e.* $y = 2{\cdot}1$

∴ **Solution is : $x = 1{\cdot}2$ and $y = 2{\cdot}1$** **Ans.**

5 Solve : $65x - 33y = 97$ and $33x - 65y = 1$

Solution :

In this example, the coefficient of x in the first equation is numerically equal to the coefficient of y in the second equation and the coefficient of y in the first equation is numerically equal to the coefficient of x in the second equation.

Such equations are solved by the method, given below:

$65x - 33y = 97$ .... (i)

and, $33x - 65y = 1$ .... (ii)

On adding (i) and (ii), we get :

$$98x - 98y = 98$$

⇒ $x - y = 1$ .... (iii) [Dividing each term by 98]

On subtracting (ii) from (i), we get :

$$32x + 32y = 96$$

⇒ $x + y = 3$ .... (iv) [Dividing each term by 32]

Now, on solving equations (iii) and (iv), we get :

$x = 2$ and $y = 1$ **Ans.**

EXERCISE 6 (B)

For solving each pair of equations, in this exercise, use the method of elimination by equating coefficients :

1. $13 + 2y = 9x$
   $3y = 7x$
2. $3x - y = 23$
   $\frac{x}{3} + \frac{y}{4} = 4$
3. $\frac{5y}{2} - \frac{x}{3} = 8$
   $\frac{y}{2} + \frac{5x}{3} = 12$
4. $\frac{1}{5}(x - 2) = \frac{1}{4}(1 - y)$
   $26x + 3y + 4 = 0$
5. $y = 2x - 6$
   $y = 0$
6. $\frac{x-y}{6} = 2(4 - x)$
   $2x + y = 3(x - 4)$
7. $3 - (x - 5) = y + 2$
   $2(x + y) = 4 - 3y$
8. $2x - 3y - 3 = 0$
   $\frac{2x}{3} + 4y + \frac{1}{2} = 0$
9. $13x + 11y = 70$
   $11x + 13y = 74$
10. $41x + 53y = 135$
    $53x + 41y = 147$
11. If $2x + y = 23$ and $4x - y = 19$; find the values of $x - 3y$ and $5y - 2x$.
12. If $10y = 7x - 4$ and $12x + 18y = 1$; find the values of $4x + 6y$ and $8y - x$.
13. Solve for x and y :
    (i) $\frac{y+7}{5} = \frac{2y-x}{4} + 3x - 5$
    $\frac{7-5x}{2} + \frac{3-4y}{6} = 5y - 18$
    (ii) $4x = 17 - \frac{x-y}{8}$
    $2y + x = 2 + \frac{5y+2}{3}$
14. Find the value of m, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = m$.
15. 10% of x + 20% of y = 24
    $3x - y = 20$
16. The value of expression $mx - ny$ is 3 when $x = 5$ and $y = 6$. And its value is 8 when $x = 6$ and $y = 5$. Find the values of m and n.
17. Solve : $11(x - 5) + 10(y - 2) + 54 = 0$
    $7(2x - 1) + 9(3y - 1) = 25$
18. Solve :
    $\frac{7+x}{5} - \frac{2x-y}{4} = 3y - 5$
    $\frac{5y-7}{2} + \frac{4x-3}{6} = 18 - 5x$
19. Solve :
    $4x + \frac{x-y}{8} = 17$
    $2y + x - \frac{5y+2}{3} = 2$

6.5 METHOD OF CROSS-MULTIPLICATION

When the given simultaneous equations are expressed as :

$$a_1x + b_1y + c_1 = 0$$

and, $$a_2x + b_2y + c_2 = 0$$

Then, $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Steps :

1. Express the two given simultaneous equations as :

$$a_1x + b_1y + c_1 = 0$$

and, $$a_2x + b_2y + c_2 = 0$$

2. Write the coefficients of x and y and also the constant terms as :

$$\begin{array}{cccc} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{array}$$

3. Mark arrows as shown below :

$$\begin{array}{ccccccc} b_1 & \searrow\!\!\!\nearrow & c_1 & \searrow\!\!\!\nearrow & a_1 & \searrow\!\!\!\nearrow & b_1 \\ b_2 & & c_2 & & a_2 & & b_2 \end{array}$$

(i) The two numbers connected by an arrow are to be multiplied together.

(ii) Numbers with downward arrow are multiplied first and from this product, the product of numbers with upward arrow is subtracted.

Thus, for $\begin{array}{ccc} b_1 & \searrow\!\!\!\nearrow & c_1 \\ b_2 & & c_2 \end{array}$, we get : $b_1c_2 - b_2c_1$

for $\begin{array}{ccc} c_1 & \searrow\!\!\!\nearrow & a_1 \\ c_2 & & a_2 \end{array}$, we get : $c_1a_2 - c_2a_1$ and

for $\begin{array}{ccc} a_1 & \searrow\!\!\!\nearrow & b_1 \\ a_2 & & b_2 \end{array}$, we get : $a_1b_2 - a_2b_1$

Now, $$\frac{x}{\begin{array}{ccc} b_1 & \searrow\!\!\!\nearrow & c_1 \\ b_2 & & c_2 \end{array}} = \frac{y}{\begin{array}{ccc} c_1 & \searrow\!\!\!\nearrow & a_1 \\ c_2 & & a_2 \end{array}} = \frac{1}{\begin{array}{ccc} a_1 & \searrow\!\!\!\nearrow & b_1 \\ a_2 & & b_2 \end{array}}$$

i.e. $$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

6 Solve, using cross-multiplication :
$4x - 7y + 28 = 0$ and $7x - 5y - 9 = 0$.

Solution :

Comparing $a_1x + b_1y + c_1 = 0$ with $4x - 7y + 28 = 0$

and, $a_2x + b_2y + c_2 = 0$ with $7x - 5y - 9 = 0$, we get :

$$\begin{matrix} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{matrix} = \begin{matrix} -7 & 28 & 4 & -7 \\ -5 & -9 & 7 & -5 \end{matrix}$$

$$\Rightarrow \frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\Rightarrow \frac{x}{\begin{matrix} -7 & 28 \\ -5 & -9 \end{matrix}} = \frac{y}{\begin{matrix} 28 & 4 \\ -9 & 7 \end{matrix}} = \frac{1}{\begin{matrix} 4 & -7 \\ 7 & -5 \end{matrix}}$$

$$\Rightarrow \frac{x}{-7\times-9-28\times-5} = \frac{y}{28\times7-(-9)\times4} = \frac{1}{4\times(-5)-7\times-7}$$

$$\Rightarrow \frac{x}{63+140} = \frac{y}{196+36} = \frac{1}{-20+49}$$

$$\Rightarrow \frac{x}{203} = \frac{y}{232} = \frac{1}{29}$$

$\Rightarrow x = \frac{203}{29}$ and $y = \frac{232}{29}$ *i.e.* $x = 7$ **and** $y = 8$ **Ans.**

7 Solve, using cross-multiplication :
$2x + 3y - 17 = 0$ and $3x - 2y - 6 = 0$.

Solution :

Comparing $a_1x + b_1y + c_1 = 0$ with $2x + 3y - 17 = 0$

and, $a_2x + b_2y + c_2 = 0$ with $3x - 2y - 6 = 0$, we get :

Since, $\begin{matrix} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{matrix} = \begin{matrix} 3 & -17 & 2 & 3 \\ -2 & -6 & 3 & -2 \end{matrix}$

$$\Rightarrow \frac{x}{\begin{matrix} 3 & -17 \\ -2 & -6 \end{matrix}} = \frac{y}{\begin{matrix} -17 & 2 \\ -6 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 2 & 3 \\ 3 & -2 \end{matrix}}$$

$$\Rightarrow \frac{x}{-18-34} = \frac{y}{-51-(-12)} = \frac{1}{-4-9}$$

$$\Rightarrow \frac{x}{-52} = \frac{y}{-39} = \frac{1}{-13}$$

$\Rightarrow x = \frac{-52}{-13}$ and $y = \frac{-39}{-13}$ *i.e.* $x = 4$ **and** $y = 3$ **Ans.**

8 Solve by cross-multiplication :
$2x + 3y = 6$ and $6x - 5y = 4$.

Solution :

$$2x + 3y = 6 \Rightarrow 2x + 3y - 6 = 0 \equiv a_1x + b_1y + c_1 = 0$$

and, $$6x - 5y = 4 \Rightarrow 6x - 5y - 4 = 0 \equiv a_2x + b_2y + c_2 = 0$$

$$\Rightarrow \begin{matrix} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{matrix} = \begin{matrix} 3 & -6 & 2 & 3 \\ -5 & -4 & 6 & -5 \end{matrix}$$

$$\therefore \quad \frac{x}{\begin{matrix} 3 & -6 \\ -5 & -4 \end{matrix}} = \frac{y}{\begin{matrix} -6 & 2 \\ -4 & 6 \end{matrix}} = \frac{1}{\begin{matrix} 2 & 3 \\ 6 & -5 \end{matrix}}$$

$$\Rightarrow \frac{x}{(-12)-(30)} = \frac{y}{(-36)-(-8)} = \frac{1}{(-10)-(18)}$$

$$\Rightarrow \frac{x}{-12-30} = \frac{y}{-36+8} = \frac{1}{-10-18}$$

$$\Rightarrow \frac{x}{-42} = \frac{y}{-28} = \frac{1}{-28}$$

$$\Rightarrow \frac{x}{-42} = \frac{1}{-28} \text{ and } \frac{y}{-28} = \frac{1}{-28}$$

$$\Rightarrow x = \frac{-42}{-28} \text{ and } y = \frac{-28}{-28} \quad i.e., \quad x = 1\frac{1}{2} \text{ and } y = 1$$ **Ans.**

9 Solve by cross-multiplication :
$3x + y = 13$ and $x - 3y + 9 = 0$.

Solution :

Given equations are : $3x + y - 13 = 0$ and $x - 3y + 9 = 0$

Comparing $a_1x + b_1y + c_1 = 0$ with $3x + y - 13 = 0$

and, $a_2x + b_2y + c_2 = 0$ with $x - 3y + 9 = 0$, we get :

$$\begin{matrix} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{matrix} = \begin{matrix} 1 & -13 & 3 & 1 \\ -3 & 9 & 1 & -3 \end{matrix}$$

$$\therefore \quad \frac{x}{\begin{matrix} 1 & -13 \\ -3 & 9 \end{matrix}} = \frac{y}{\begin{matrix} -13 & 3 \\ 9 & 1 \end{matrix}} = \frac{1}{\begin{matrix} 3 & 1 \\ 1 & -3 \end{matrix}}$$

$$\Rightarrow \frac{x}{(9)-(39)} = \frac{y}{(-13)-(27)} = \frac{1}{(-9)-(1)}$$

$$\Rightarrow \frac{x}{-30} = \frac{y}{-40} = \frac{1}{-10}$$

$$\Rightarrow x = 3 \text{ and } y = 4$$ **Ans.**

EXERCISE 6 (C)

Solve, using cross-multiplication :

1. $4x + 3y = 17$
   $3x - 4y + 6 = 0$
2. $3x + 4y = 11$
   $2x + 3y = 8$
3. $6x + 7y - 11 = 0$
   $5x + 2y = 13$
4. $5x + 4y + 14 = 0$
   $3x = -10 - 4y$
5. $x - y + 2 = 0$
   $7x + 9y = 130$
6. $4x - y = 5$
   $5y - 4x = 7$
7. $4x - 3y = 0$
   $2x + 3y = 18$
8. $8x + 5y = 9$
   $3x + 2y = 4$
9. $4x - 3y - 11 = 0$
   $6x + 7y - 5 = 0$
10. $4x + 6y = 15$
    $3x - 4y = 7$

6.6 EQUATIONS REDUCIBLE TO LINEAR EQUATIONS

10 Solve : $\frac{7}{x} + \frac{8}{y} = 2$ and $\frac{2}{x} + \frac{13}{y} = 22$

Solution :

Given : $\frac{7}{x} + \frac{8}{y} = 2$ I

and, $\frac{2}{x} + \frac{13}{y} = 22$ II

Multiplying equation (I) by 2 and equation (II) by 7, we get :

$$\frac{14}{x} + \frac{16}{y} = 4$$

$$\frac{14}{x} + \frac{91}{y} = 154$$

$$- \quad - \quad -$$ [Subtracting]

$$-\frac{75}{y} = -150$$

$\Rightarrow$ $-150y = -75$ or $y = \frac{1}{2}$

> When the given system of linear simultaneous equations has its variables in denominator only, solve the given equations without taking L.C.M. of denominators.

Substituting $y = \frac{1}{2}$ in eq. (I), we get :

$$\frac{7}{x} + \frac{8\times2}{1} = 2 \quad \Rightarrow \quad \frac{7}{x} = 2 - 16 = -14 \quad \Rightarrow \quad x = \frac{7}{-14} = -\frac{1}{2}$$

$\therefore$ $\mathbf{x = -\frac{1}{2}}$ **and** $\mathbf{y = \frac{1}{2}}$ **Ans.**

Alternative method : Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$\therefore$ $\frac{7}{x} + \frac{8}{y} = 2$ becomes : $7a + 8b = 2$ ....(i)

And, $\frac{2}{x} + \frac{13}{y} = 22$ becomes : $2a + 13b = 22$ ....(ii)

Multiplying equation (i) by 2 and equation (ii) by 7; we get :

$$14a + 16b = 4$$
$$14a + 91b = 154$$
$$- \quad - \quad -$$
$$-75b = -150 \Rightarrow b = \frac{-150}{-75} = 2$$

Now, $2a + 13b = 22 \Rightarrow 2a + 13 \times 2 = 22$

$\Rightarrow 2a = -4$ and $a = -2$

$a = -2 \Rightarrow \frac{1}{x} = -2$ and $x = -\frac{1}{2}$

$b = 2 \Rightarrow \frac{1}{y} = 2$ and $y = \frac{1}{2}$

$\therefore$ **Solution is : $x = -\frac{1}{2}$ and $y = \frac{1}{2}$** **Ans.**

11 Solve : $\frac{30}{x-y} + \frac{44}{x+y} = 10$ and $\frac{40}{x-y} + \frac{55}{x+y} = 13$

Solution :

Since, $\frac{30}{x-y} + \frac{44}{x+y} = 10$ .... (i)

and, $\frac{40}{x-y} + \frac{55}{x+y} = 13$ .... (ii)

Multiplying equation (i) by 4 and equation (ii) by 3; we get :

$$\frac{120}{x-y} + \frac{176}{x+y} = 40$$
$$\frac{120}{x-y} + \frac{165}{x+y} = 39$$
$$- \quad - \quad -$$
$$\frac{11}{x+y} = 1 \Rightarrow x + y = 11 \qquad \text{.......... (iii)}$$

Now, $\frac{30}{x-y} + \frac{44}{x+y} = 10 \Rightarrow \frac{30}{x-y} + \frac{44}{11} = 10$

$\Rightarrow \frac{30}{x-y} = 10 - 4 = 6$

$\Rightarrow 6(x - y) = 30$ and $x - y = 5$ .... (iv)

On solving equations (iii) and (iv), we get : $x = 8$ and $y = 3$

$\therefore$ **Solution is : $x = 8$ and $y = 3$** **Ans.**

Alternative method :

Let $x - y = a$ and $x + y = b$

$\therefore \frac{30}{x-y} + \frac{44}{x+y} = 10 \Rightarrow \frac{30}{a} + \frac{44}{b} = 10$ .... (i)

and, $\frac{40}{x-y} + \frac{55}{x+y} = 13 \Rightarrow \frac{40}{a} + \frac{55}{b} = 13$ .... (ii)

Multiplying equation (i) by 4 and equation (ii) by 3; we get :

$$\frac{120}{a} + \frac{176}{b} = 40$$

$$\frac{120}{a} + \frac{165}{b} = 39$$

$$- \quad - \quad -$$

$$\frac{11}{b} = 1 \Rightarrow b = 11 \text{ i.e. } x + y = 11 \quad \text{......... (iii)}$$

$$\frac{30}{a} + \frac{44}{b} = 10 \Rightarrow \frac{30}{a} + \frac{44}{11} = 10 \Rightarrow \frac{30}{a} = 10 - 4$$

$$\Rightarrow a = 5 \text{ i.e. } x - y = 5 \quad \text{......... (iv)}$$

On solving equations (iii) and (iv), we get :

$x = 8$ and $y = 3$ **Ans.**

Third method :

$$\text{Let } \frac{1}{x-y} = a \quad \text{and} \quad \frac{1}{x+y} = b$$

$$\therefore \frac{30}{x-y} + \frac{44}{x+y} = 10 \Rightarrow 30a + 44b = 10 \quad \text{........... (i)}$$

$$\text{and, } \frac{40}{x-y} + \frac{55}{x+y} = 13 \Rightarrow 40a + 55b = 13 \quad \text{.......... (ii)}$$

On solving equations (i) and (ii); we get :

$$a = \frac{1}{5} \quad \text{and} \quad b = \frac{1}{11}$$

$$a = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5 \quad \text{.......... (iii)}$$

$$b = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11 \quad \text{.......... (iv)}$$

On solving equations (iii) and (iv), we get :

$x = 8$ and $y = 3$ **Ans.**

EXERCISE 6 (D)

1. $\frac{9}{x} - \frac{4}{y} = 8$

   $\frac{13}{x} + \frac{7}{y} = 101$

2. $\frac{3}{x} + \frac{2}{y} = 10$

   $\frac{9}{x} - \frac{7}{y} = 10{\cdot}5$

3. $5x + \frac{8}{y} = 19$

   $3x - \frac{4}{y} = 7$

4. Solve : $4x + \frac{6}{y} = 15$ and $3x - \frac{4}{y} = 7$.

   Hence, find 'a' if $y = ax - 2$.

5. Solve : $\frac{3}{x} - \frac{2}{y} = 0$ and $\frac{2}{x} + \frac{5}{y} = 19$.
Hence, find 'a' if $y = ax + 3$.

6. Solve :

(i) $\frac{20}{x+y} + \frac{3}{x-y} = 7$

$\frac{8}{x-y} - \frac{15}{x+y} = 5$

(ii) $\frac{34}{3x+4y} + \frac{15}{3x-2y} = 5$

$\frac{25}{3x-2y} - \frac{8 \cdot 50}{3x+4y} = 4 \cdot 5$

7. Solve :

(i) $x + y = 2xy$

$x - y = 6xy$

(ii) $x + y = 7xy$

$2x - 3y = -xy$

(i) Dividing both the sides of each equation by xy, we get :

$$\frac{x+y}{xy} = \frac{2xy}{xy} \Rightarrow \frac{x}{xy} + \frac{y}{xy} = 2$$

$$\Rightarrow \frac{1}{y} + \frac{1}{x} = 2 \quad ..... \text{I}$$

and, $$\frac{x-y}{xy} = \frac{6xy}{xy} \Rightarrow \frac{x}{xy} - \frac{y}{xy} = 6$$

$$\Rightarrow \frac{1}{y} - \frac{1}{x} = 6 \; ... \text{II}$$

Now, solve equations I and II.

8. Solve :

$\frac{a}{x} - \frac{b}{y} = 0$

$\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$.

9. Solve : $\frac{2xy}{x+y} = \frac{3}{2}$

$\frac{xy}{2x-y} = -\frac{3}{10}$;

$x + y \neq 0$ and $2x - y \neq 0$

10. Solve : $\frac{3}{2x} + \frac{2}{3y} = -\frac{1}{3}$

$\frac{3}{4x} + \frac{1}{2y} = -\frac{1}{8}$

6.7 PROBLEMS BASED ON SIMULTANEOUS EQUATIONS

A. Based on numbers

12 The sum of two numbers is 12 and their difference is 2. Find the numbers.

Solution :

Let the numbers be x and y.

$\therefore$ $x + y = 12$ and $x - y = 2$

On solving, we get : $x = 7$ and $y = 5$

$\therefore$ **Required numbers are 7 and 5** **Ans.**

Therefore, to solve a problem based on simultaneous equations, adopt the following steps:

Steps : 1. Assume the two variables (unknowns) as x and y.

2. According to the given statement, set up two equations in terms of x and y.

3. Solve the equations.

B. Based on fractions

13 If the numerator of a fraction is decreased by 1 its value becomes $\frac{2}{3}$, but if the denominator is increased by 5 its value becomes $\frac{1}{2}$. What is the fraction ?

Solution :

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-1}{y} = \frac{2}{3}$ and $\frac{x}{y+5} = \frac{1}{2}$

$\Rightarrow \quad 3x - 3 = 2y$ and $2x = y + 5$

$\Rightarrow \quad 3x - 2y = 3$ and $2x - y = 5$

On solving, we get : $x = 7$ and $y = 9$.

$\therefore$ **Required fraction** $= \frac{7}{9}$ **Ans.**

C. Based on two-digit numbers

14 The sum of the digits of a two-digit number is 7. If the digits are reversed, the new number increased by 3, equals 4 times the original number. Find the original number.

Solution :

Let x be the digit at ten's place and y be the digit at unit place.

$\therefore$ The number $= 10x + y$ and the sum of its digits $= x + y$.

On reversing the digits, the number becomes $10y + x$.

According to the given statement :

$x + y = 7$ ...... (i)

and, $10y + x + 3 = 4(10x + y)$ ..... (ii)

On solving, we get : $x = 1$ and $y = 6$

$\therefore$ **Required number** $= 10x + y = 10 \times 1 + 6 =$ **16** **Ans.**

15 The ratio between a two digit number and the number obtained on reversing its digits is 4 : 7. If the difference between the digits of the number is 3, find the number.

Solution :

Let the required number of two digits be $10x + y$

$\therefore$ The number obtained on reversing its digits

$= 10y + x$

Given : $\frac{10x+y}{10y+x} = \frac{4}{7}$ *i.e.* $70x + 7y = 40y + 4x$

$\Rightarrow \quad 70x - 4x = 40y - 7y$ *i.e.* $66x = 33y$

$\Rightarrow \quad 2x - y = 0$ I

Given, difference between the digits = 3

i.e. $x - y = 3$ or $y - x = 3$

For a two digit number **$10x + y$**, if on reversing the digits the number increases, the unit digit is bigger than the tens digit and so the difference between the digits must be taken as $y - x$.

For example : On reversing the digits of number 27.

We get 72, which is bigger than the original number 27.

Clearly, the unit digit of the original number is greater than its tens digit.

Similarly, for **$10x + y$**, if on reversing the digits the number decreases, the tens digit is bigger and so the difference between the digits must be taken as $x - y$.

For example : On reversing the digits of number 62.
We get 26, which is smaller than the original number 62.
Clearly, the tens digit of the original number is greater than its unit digit.

In this example, the ratio between the required number and the number obtained on reversing its digits = 4 : 7.

∴ On reversing the digits, the number increases

and so $y - x = 3$ II

On solving I and II, we get :

$x = 3$ and $y = 6$

∴ **Required number** $= 10x + y$

$= 10 \times 3 + 6 =$ **36** **Ans.**

EXERCISE 6 (E)

1. The ratio of two numbers is $\frac{2}{3}$. If 2 is subtracted from the first and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.
2. Two numbers are in the ratio 4 : 7. If thrice the larger be added to twice the smaller, the sum is 59. Find the numbers.
3. When the greater of the two numbers increased by 1 divides the sum of the numbers, the result is $\frac{3}{2}$. When the difference of these numbers is divided by the smaller, the result is $\frac{1}{2}$. Find the numbers.
4. Two numbers are in the ratio 4 : 5. If 30 is subtracted from each of the numbers, the ratio becomes 1 : 2. Find the numbers.
5. If the numerator of a fraction is increased by 2 and denominator is decreased by 1, it becomes $\frac{2}{3}$. If the numerator is increased by 1 and denominator is increased by 2, it becomes $\frac{1}{3}$. Find the fraction.
6. The sum of the numerator and the denominator of a fraction is equal to 7. Four times the numerator is 8 less than 5 times the denominator. Find the fraction.
7. If the numerator of a fraction is multiplied by 2 and its denominator is increased by 1, it becomes 1. However, if the numerator is increased by 4 and denominator is multiplied by 2, the fraction becomes $\frac{1}{2}$. Find the fraction.
8. A fraction becomes $\frac{1}{2}$ if 5 is subtracted from its numerator and 3 is subtracted from its denominator. If the denominator of this fraction is 5 more than its numerator, find the fraction.
9. The sum of the digits of a two digit number is 5. If the digits are reversed, the number is reduced by 27. Find the number.
10. The sum of the digits of a two digit number is 7. If the digits are reversed, the new number decreased by 2, equals twice the original number. Find the number.
11. The ten's digit of a two digit number is three times the unit digit. The sum of the number and the unit digit is 32. Find the number.
12. A two-digit number is such that the ten's digit exceeds twice the unit's digit by 2 and the number obtained by inter-changing the digits is 5 more than three times the sum of the digits. Find the two digit number.
13. Four times a certain two digit number is seven times the number obtained on interchanging its digits. If the difference between the digits is 4; find the number.
14. The sum of a two digit number and the number obtained by interchanging the digits of the number is 121. If the digits of the number differ by 3, find the number.
15. A two digit number is obtained by multiplying the sum of the digits by 8. Also, it is obtained by multiplying the difference of the digits by 14 and adding 2. Find the number.

D. Based on ages

16 The present ages of A and B are in the ratio 9 : 4. Seven years hence, the ratio of their ages will be 5 : 3. Find their present ages.

Solution :

Let the present age of A = x years and that of B = y years

$\therefore \frac{x}{y} = \frac{9}{4} \quad \Rightarrow 4x - 9y = 0$ ...... (i)

7 years hence : A's age will be $(x + 7)$ years and B's age will be $(y + 7)$ years.

$\therefore \frac{x+7}{y+7} = \frac{5}{3} \quad \Rightarrow \quad 3x + 21 = 5y + 35$

$\Rightarrow \quad 3x - 5y = 14$ ..... (ii)

On solving, we get : $x = 18$ and $y = 8$

$\therefore$ **A's present age = 18 years and B's present age = 8 years** **Ans.**

E. Based on C.P. and S.P.

17 A farmer sold a calf and a cow for ₹ 7,600, thereby making a profit of 25% on the calf and 10% on the cow. By selling them for ₹ 7,675, he would have realised a profit of 10% on the calf and 25% on the cow. Find the cost price of each.

Solution :

Let the C.P. of the calf = ₹ x and the C.P. of the cow = ₹ y

In first case :

S.P. of the calf = ₹ x + 25% of ₹ x = ₹ $\left(x + \frac{25x}{100}\right)$ = ₹ $\frac{5x}{4}$

S.P. of the cow = ₹ y + 10% of ₹ y = ₹ $\left(y + \frac{10y}{100}\right)$ = ₹ $\frac{11y}{10}$

$\therefore \quad \frac{5x}{4} + \frac{11y}{10} = 7{,}600 \Rightarrow 25x + 22y = 1{,}52{,}000$ ...... (i)

In second case :

S.P. of the calf = ₹ x + 10% of ₹ x = ₹ $\frac{11x}{10}$

S.P. of the cow = ₹ y + 25% of ₹ y = ₹ $\frac{5y}{4}$

$\therefore \quad \frac{11x}{10} + \frac{5y}{4} = 7{,}675 \Rightarrow 22x + 25y = 1{,}53{,}500$ .... (ii)

On solving equations (i) and (ii), we get :

$x = 3{,}000$ and $y = 3{,}500$

$\therefore$ **C.P. of a calf = ₹ 3,000** and **C.P. of the cow = ₹ 3,500** **Ans.**

F. Based on time and work

18 A and B together can do a piece of work in 15 days. If one day's work of A be $1\frac{1}{2}$ times one day's work of B, find how many days will each take to finish the work alone ?

Solution :

Let A alone will do the work in x days and B alone will do the same work in y days.

$\therefore \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{15}$ .... (i) and $\frac{1}{x} = \frac{3}{2} \times \frac{1}{y} \Rightarrow \frac{1}{x} - \frac{3}{2y} = 0$ .... (ii)

On solving equations (i) and (ii), we get :

$$x = 25 \quad \text{and} \quad y = 37{\cdot}5$$

$\therefore$ **A alone will do the work in 25 days** and

B alone will do the same work in 37·5 days. **Ans.**

EXERCISE 6 (F)

1. Five years ago, A's age was four times the age of B. Five years hence, A's age will be twice the age of B. Find their present ages.
2. A is 20 years older than B. 5 years ago, A was 3 times as old as B. Find their present ages.
3. Four years ago, a mother was four times as old as her daughter. Six years later, the mother will be two and a half times as old as her daughter at that time. Find the present ages of mother and her daughter.
4. The age of a man is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children at that time. Find the present age of the man.

If at present :

Man's age = x years and sum of the ages of two children = y years

Then $x = 2y$

In 20 years :

Age of each child increases by 20 years

$\Rightarrow$ in 20 years, ages of both the children will increase by (20 + 20) years.

$\therefore \; x + 20 = y + 20 + 20$

5. The annual incomes of A and B are in the ratio 3 : 4 and their annual expenditures are in the ratio 5 : 7. If each saves ₹ 5,000; find their annual incomes.
6. In an examination, the ratio of passes to failures was 4 : 1. Had 30 less appeared and 20 less passed, the ratio of passes to failures would have been 5 : 1. Find the number of students who appeared for the examination.

Suppose x candidates passed and y failed;

therefore, $\frac{x}{y} = \frac{4}{1}$ .... I.

In the second case; no. of students appeared = $x + y - 30$ and no. of those who passed = $x - 20$;

$\therefore$ No of failed = $x + y - 30 - (x - 20)$

$= y - 10$

Given; $\frac{x - 20}{y - 10} = \frac{5}{1}$ II

7. A and B both have some pencils. If A gives 10 pencils to B, then B will have twice as many as A. And if B gives 10 pencils to A, then they will have the same number of pencils. How many pencils does each have?
8. 1250 persons went to see a circus-show. Each adult paid ₹ 75 and each child paid ₹ 25 for the admission ticket. Find the number of adults and number of children, if the total collection from them amounts to ₹ 61,250.
9. Two articles A and B are sold for ₹ 1,167 making 5% profit on A and 7% profit on B. If the two articles are sold for ₹ 1,165, a profit of 7% is made on A and a profit of 5% is made on B. Find the cost price of each article.
10. Pooja and Ritu can do a piece of work in $17\frac{1}{7}$ days. If one day work of Pooja be three fourth of one day work of Ritu; find in how many days each will do the work alone.

G. Miscellaneous Problems

19 A and B each have certain number of oranges. A says to B, "if you give me 10 of your oranges, I will have twice the number of oranges left with you." B replies, "if you give me 10 of your oranges, I will have the same number of oranges as left with you." Find the number of oranges with A and B separately.

Solution :

Let A has x number of oranges and B has y number of oranges.

In 1st case (if B gives 10 oranges to A) :

$$x + 10 = 2(y - 10) \quad \Rightarrow \quad x - 2y = -30 \qquad ...(I)$$

In 2nd case (if A gives 10 oranges to B) :

$$y + 10 = x - 10 \quad \Rightarrow \quad x - y = 20 \qquad ...(II)$$

On solving equations (I) and (II), we get :

$$x = 70 \text{ and } y = 50$$

∴ **A has 70 oranges and B has 50 oranges** **Ans.**

20 The sum of a two digit number and the number obtained on reversing the digits is 165. If the digits differ by 3, find the number.

Solution :

Let the digit at tens place be x and the digit at unit place be y.

∴ Number = $10x + y$

Number on reversing the digits = $10y + x$

And, the difference between the digits = $x - y$ or $y - x$

Given : $(10x + y) + (10y + x) = 165$

$\Rightarrow$ $x + y = 15$ ...(I)

And, $x - y = 3$ ...(II)

or $y - x = 3$ ...(III)

On solving equations (I) and (II), we get :

$$x = 9 \text{ and } y = 6$$

∴ Number = $10x + y = 10 \times 9 + 6 = 96$

On solving equations (I) and (III), we get :

$$x = 6 \text{ and } y = 9$$

∴ Number = $10x + y = 10 \times 6 + 9 = 69$

∴ **Required number = 96 or 69** **Ans.**

21 The total railway fare for 5 members in 3-tier and 3 members in 2-tier is ₹ 2,050 whereas, the total railway fare for 8 members in 3-tier and 5 members in 2-tier is ₹ 3,350. Find the fare to be paid by a couple travelling through 2-tier.

Solution :

Let fare for one person in 3-tier be ₹ x

and, fare for one person in 2-tier be ₹ y

∴ **In 1st case :** $5x + 3y = 2{,}050$ I

And, **in 2nd case :** $8x + 5y = 3{,}350$ II

On solving equations I and II, we get : $x = 200$ and $y = 350$

∴ **Fare to be paid by a couple travelling through 2-tier** = 2 × ₹ y = 2 × ₹ 350 = ₹ 700 **Ans.**

EXERCISE 6 (G)

1. Rohit says to Ajay, "Give me a hundred, I shall then become twice as rich as you." Ajay replies, "if you give me ten, I shall be six times as rich as you." How much does each have originally ?
2. The sum of a two digit number and the number obtained by reversing the order of the digits is 99. Find the number, if the digits differ by 3.
3. Seven times a two digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3, find the number.
4. From Delhi station, if we buy 2 tickets for station A and 3 tickets for station B, the total cost is ₹ 77. But if we buy 3 tickets for station A and 5 tickets for station B, the total cost is ₹ 124. What are the fares from Delhi to station A and to station B ?
5. The sum of digits of a two digit number is 11. If the digit at ten's place is increased by 5 and the digit at unit's place is decreased by 5, the digits of the number are found to be reversed. Find the original number.
6. 90% acid solution (90% pure acid and 10% water) and 97% acid solution are mixed to obtain 21 litres of 95% acid solution. How many litres of each solution are mixed.

> Let x litres of 90% and y litres of 97% be mixed; then $x + y = 21$
> and 90% of x + 97% of y = 95% of 21.

7. Class XI students of a school wanted to give a farewell party to the outgoing students of class XII. They decided to purchase two kinds of sweets, one costing ₹ 250 per kg and the other costing ₹ 350 per kg. They estimated that 40 kg of sweets were needed. If the total budget for the sweets was ₹ 11,800; find how much sweets of each kind were bought ?
8. Mr. and Mrs. Ahuja weigh x kg and y kg respectively. They both take a dieting course, at the end of which Mr. Ahuja loses 5 kg and weighs as much as his wife weighed before the course. Mrs. Ahuja loses 4 kg and weighs $\frac{7}{8}$th of what her husband weighed before the course. Form two equations in x and y to find their weights before taking the dieting course.
9. A part of monthly expenses of a family is constant and the remaining vary with the number of members in the family. For a family of 4 persons, the total monthly expenses are ₹ 10,400; whereas for a family of 7 persons, the total monthly expenses are ₹ 15,800. Find the constant expenses per month and the monthly expenses on each member of a family.

> Let the constant monthly expenses be ₹ x and for each member of the family the monthly expenses be ₹ y.
> $\therefore \quad x + 4y = 10{,}400$ and $x + 7y = 15{,}800$.

10. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 315 and for a journey of 15 km, the charge paid is ₹ 465.. What are the fixed charges and the charge per kilometer ? How much does a person have to pay for travelling a distance of 32 km ?
11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Geeta paid ₹ 27 for a book kept for seven days, while Mohit paid ₹ 21 for the book he kept for five days. Find the fixed charges and the charge for each extra day.
12. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. However, if the length of this rectangle increases by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

> Let the length of the rectangle = x units and its breadth = y units
> According to first condition :
> $$(x - 5)(y + 3) = xy - 9 \text{ and}$$
> According to second condition :
> $$(x + 3)(y + 2) = xy + 67$$

13. It takes 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter is used for 9 hours, only half of the pool is filled. How long would each pipe take to fill the swimming pool ?