5 Factorisation

5.1 INTRODUCTION

When a polynomial (an algebraic expression) is expressed as the product of two or more expressions, each of these expressions is called a **factor** of the polynomial.

The polynomial $x^2 + 5x + 6$ can be expressed as the product of the expressions $(x + 3)$ and $(x + 2)$.

That is $x^2 + 5x + 6 = (x + 3)(x + 2) \Rightarrow (x + 3)$ and $(x + 2)$ are factors of $x^2 + 5x + 6$.

The process of writing an expression in the form of terms or brackets multiplied together, is called **factorisation**. Each term and each bracket is called a factor of the expression.

e.g. (i) $5x^2 + 15 = 5(x^2 + 3)$

$\Rightarrow$ 5 and $x^2 + 3$ are factors of $5x^2 + 15$.

(ii) $ax^2 + 5ax + 6a = a(x + 3)(x + 2)$

$\Rightarrow$ a, $(x + 3)$ and $(x + 2)$ are factors of $ax^2 + 5ax + 6a$.

Factorisation is the reverse of multiplication.

5.2 METHODS OF FACTORISATION

Type 1 : Taking out the common factors

When each term of a given expression contains a common factor, divide each term by this factor and enclose the quotient within brackets, keeping the common factor outside the bracket.

Procedure :

Find the H.C.F. of all the terms of the given expression.

For expression $6a^2 - 3ax$, its terms are $6a^2$ and $-3ax$. And, the H.C.F. of these terms is $3a$.

Therefore, $6a^2 - 3ax = 3a\left(\frac{6a^2}{3a} - \frac{3ax}{3a}\right) = 3a(2a - x)$.

1 Factorise :

(i) $8ab^2 + 12a^2b$ (ii) $4(x + y)^2 - 3(x + y)$ (iii) $x(a - 5) + y(5 - a)$

Solution :

(i) $8ab^2 + 12a^2b = 4ab\left[\frac{8ab^2}{4ab} + \frac{12a^2b}{4ab}\right]$ [H.C.F. of $8ab^2$ and $12a^2b$ is $4ab$]

$= 4ab(2b + 3a)$ **Ans.**

Direct method :

It can easily be seen that $4ab$ is the largest expression which divides both the terms $8ab^2$ and $12a^2b$ of the given expression $8ab^2 + 12a^2b$ completely.

$\therefore \quad 8ab^2 + 12a^2b = 4ab(2b + 3a)$ **Ans.**

(ii) $4(x + y)^2 - 3(x + y) = (x + y)[4(x + y) - 3]$

$= (x + y)(4x + 4y - 3)$ **Ans.**

(iii) $x(a - 5) + y(5 - a) = x(a - 5) - y(a - 5)$

$= (a - 5)(x - y)$ **Ans.**

Type 2 : Grouping

An expression of an even number of terms, may be resolved into factors, if the terms are arranged in groups such that each group has a common factor.

Procedure :

1. Group the terms of the given expression in such a way that each group has a common factor.
2. Factorise each group formed.
3. From each group, obtained in step 2, take out the common factor.

2 Factorise : (i) $ab + bc + ax + cx$ (ii) $ab^2 - (a - 1)b - 1$.

Solution :

(i) $ab + bc + ax + cx = (ab + bc) + (ax + cx)$ [Forming groups]

$= b(a + c) + x(a + c)$ [Taking out common factors from each group]

$= (a + c)(b + x)$ **Ans.** [Taking $(a + c)$ common]

(ii) $ab^2 - (a - 1)b - 1 = ab^2 - ab + b - 1$

$= ab(b - 1) + 1(b - 1)$

$= (b - 1)(ab + 1)$ **Ans.**

3 Factorise : $a^2 + \frac{1}{a^2} + 2 - 5a - \frac{5}{a}$.

Solution :

$a^2 + \frac{1}{a^2} + 2 - 5a - \frac{5}{a} = (a^2 + \frac{1}{a^2} + 2) - 5(a + \frac{1}{a})$

$= (a + \frac{1}{a})^2 - 5(a + \frac{1}{a}) = (a + \frac{1}{a})(a + \frac{1}{a} - 5)$ **Ans.**

EXERCISE 5 (A)

Factorise by taking out the common factors :

1. $3a^2 - 9ab$
2. $2(x + y)^3 - 6(x + y)$
3. $x^3(2x - 3y) - x^2(2x - 3y)^2$
4. $2(2x - 5y)(3x + 4y) - 6(2x - 5y)(x - y)$

Factorise by grouping method :

5. $a^3 + a - 3a^2 - 3$
6. $16 (a + b)^2 - 4a - 4b$
7. $a^4 - 2a^3 - 4a + 8$
8. $ab - 2b + a^2 - 2a$
9. $ab (x^2 + 1) + x (a^2 + b^2)$
10. $a^2 + b - ab - a$
11. $(ax + by)^2 + (bx - ay)^2$
12. $a^2x^2 + (ax^2 + 1) x + a$
13. $(2a - b)^2 - 10a + 5b$
14. $a (a - 4) - a + 4$
15. $y^2 - (a + b) y + ab$
16. $a^2 + \frac{1}{a^2} - 2 - 3a + \frac{3}{a}$
17. $x^2 + y^2 + x + y + 2xy$
18. $a^2 + 4b^2 - 3a + 6b - 4ab$
19. $m (x - 3y)^2 + n (3y - x) + 5x - 15y$
20. $x (6x - 5y) - 4 (6x - 5y)^2$

Type 3 : Trinomial of the form $ax^2 \pm bx \pm c$ (By splitting the middle term)

When a trinomial is of the form $ax^2 + bx + c$ (or $a + bx + cx^2$), split b (the coefficient of x in the middle term) into two parts such that the sum of these two parts is equal to b and the product of these two parts is equal to the product of a and c. Then factorize by the grouping method.

4 Factorise : (i) $x^2 + 5x + 6$ (ii) $x^2 - 5x + 6$ (iii) $x^2 - 5x - 6$ (iv) $x^2 + 5x - 6$.

Solution :

(i) $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$ Since, $3 + 2 = 5$
$= x (x + 3) + 2(x + 3)$ and, $3 \times 2 = 6$
$= (x + 3) (x + 2)$ **Ans.**

(ii) $x^2 - 5x + 6 = x^2 - 3x - 2x + 6$ Since, $(-3) + (-2) = -5$
$= x (x - 3) -2 (x - 3)$ and, $(-3) \times (-2) = +6$
$= (x - 3) (x - 2)$ **Ans.**

(iii) $x^2 - 5x - 6 = x^2 - 6x + x - 6$ Since, $-6 + 1 = -5$
$= x (x - 6) + 1 (x - 6)$ and, $(-6) \times 1 = -6$
$= (x - 6) (x + 1)$ **Ans.**

(iv) $x^2 + 5x - 6 = x^2 + 6x - x - 6$ Since, $6 - 1 = 5$
$= x (x + 6) - 1 (x + 6)$ and, $6 \times (-1) = -6$
$= (x + 6) (x - 1)$ **Ans.**

5 Factorise : (i) $2x^2 - 7x + 6$ (ii) $3x^2 - 11x - 4$ (iii) $6 + 11x + 3x^2$ (iv) $7 - 12x - 4x^2$.

Solution :

(i) $2x^2 - 7x + 6 = 2x^2 - 4x - 3x + 6$ Since, $-4 -3 = -7$ and, $(-4) \times (-3) = 12$
$= 2x (x - 2) -3(x - 2) = (x - 2) (2x - 3)$ **Ans.**

(ii) $3x^2 - 11x - 4 = 3x^2 - 12x + x - 4$
$= 3x (x - 4) +1 (x - 4) = (x - 4) (3x + 1)$ **Ans.**

(iii) $6 + 11x + 3x^2 = 6 + 9x + 2x + 3x^2$
$= 3(2 + 3x) + x(2 + 3x) = (2 + 3x)(3 + x)$ **Ans.**

(iv) $7 - 12x - 4x^2 = 7 - 14x + 2x - 4x^2$
$= 7(1 - 2x) + 2x(1 - 2x) = (1 - 2x)(7 + 2x)$ **Ans.**

6 Factorise :

(i) $x^2 + 7x + 6 + px + 6p$ (ii) $12 - (x + x^2)(8 - x - x^2)$.

Solution :

(i) $x^2 + 7x + 6 + px + 6p$

$= x^2 + 6x + x + 6 + px + 6p$ [Factorising $x^2 + 7x + 6$]
$= x(x + 6) + 1(x + 6) + p(x + 6)$
$= (x + 6)(x + 1 + p)$ **Ans.**

(ii) $12 - (x + x^2)(8 - x - x^2) = 12 - (x + x^2)[8 - (x + x^2)]$
$= 12 - a(8 - a)$ [Taking $x + x^2 = a$]
$= 12 - 8a + a^2$
$= 12 - 6a - 2a + a^2$
$= 6(2 - a) - a(2 - a)$
$= (2 - a)(6 - a)$
$= [2 - (x + x^2)][6 - (x + x^2)]$ [$\because a = x + x^2$]
$= (2 - x - x^2)(6 - x - x^2)$
$= (2 - 2x + x - x^2)(6 - 3x + 2x - x^2)$
$= [2(1 - x) + x(1 - x)][3(2 - x) + x(2 - x)]$
$= (1 - x)(2 + x)(2 - x)(3 + x)$ **Ans.**

$ax^2 + bx + c$, where a, b and c are real numbers, is known as a trinomial or a quadratic expression in which a = coefficient of x^2, b = coefficient of x and c = a constant.
If we find the value of $b^2 - 4ac$ and this value is a perfect square, the trinomial $ax^2 + bx + c$ is factorisable, otherwise, not.

7 (i) Is $5x^2 + 17x + 6$ factorisable? If yes, factorise it.
(ii) Is $3x^2 - 8x - 15$ factorisable? If yes, factorise it.

Solution :

(i) Comparing $5x^2 + 17x + 6$ with $ax^2 + bx + c$, we get :
$a = 5$, $b = 17$ and $c = 6$

$\therefore b^2 - 4ac = (17)^2 - 4 \times 5 \times 6$
$= 289 - 120 = 169$, *which is a perfect square.*

$\therefore$ **$5x^2 + 17x + 6$ is factorisable.** **Ans.**

Now, $5x^2 + 17x + 6 = 5x^2 + 15x + 2x + 6$
$= 5x(x + 3) + 2(x + 3) = (x + 3)(5x + 2)$ **Ans.**

Downloaded from https:// www.studiestoday.com

(ii) Comparing $3x^2 - 8x - 15$ with $ax^2 + bx + c$, we get :

$a = 3$, $b = -8$ and $c = -15$

$\therefore \quad b^2 - 4ac = (-8)^2 - 4 \times 3 \times -15$

$= 64 + 180 = 244$, *which is not a perfect square.*

$\therefore$ **$3x^2 - 8x - 15$ is not factorisable.** **Ans.**

EXERCISE 5 (B)

Factorise :

1. $a^2 + 10a + 24$
2. $a^2 - 3a - 40$
3. $1 - 2a - 3a^2$
4. $x^2 - 3ax - 88a^2$
5. $6a^2 - a - 15$
6. $24a^3 + 37a^2 - 5a$
7. $a(3a - 2) - 1$
8. $a^2b^2 + 8ab - 9$
9. $3 - a(4 + 7a)$
10. $(2a + b)^2 - 6a - 3b - 4$
11. $1 - 2a - 2b - 3(a + b)^2$
12. $3a^2 - 1 - 2a$
13. $x^2 + 3x + 2 + ax + 2a$
14. $(3x - 2y)^2 + 3(3x - 2y) - 10$
15. $5 - (3a^2 - 2a)(6 - 3a^2 + 2a)$
16. For each trinomial (quadratic expression), given below, find whether it is factorisable or not. Factorise, if possible.

    (i) $x^2 - 3x - 54$ (ii) $2x^2 - 7x - 15$

    (iii) $2x^2 + 2x - 75$ (iv) $3x^2 + 4x - 10$

    (v) $x(2x - 1) - 1$

Type 4 : Difference of two squares

Since, the product of $(x + y)$ and $(x - y) = (x + y)(x - y) = x^2 - y^2$,

$\therefore$ Factors of $x^2 - y^2$ are $(x + y)$ and $(x - y)$

i.e. $x^2 - y^2 = (x + y)(x - y)$.

8 Factorise : (i) $x^2 - 25$ (ii) $9(x - y)^2 - (x + 2y)^2$

(iii) $48x^3 - 27x$ (iv) $16a^4 - b^4$.

Solution :

(i) $x^2 - 25 = x^2 - 5^2 = (x + 5)(x - 5)$ **Ans.**

(ii) $9(x - y)^2 - (x + 2y)^2 = [3(x - y)]^2 - (x + 2y)^2$

$= (3x - 3y)^2 - (x + 2y)^2$

$= (\overline{3x - 3y} + \overline{x + 2y})(\overline{3x - 3y} - \overline{x + 2y})$

$= (3x - 3y + x + 2y)(3x - 3y - x - 2y)$

$= (4x - y)(2x - 5y)$ **Ans.**

(iii) $48x^3 - 27x = 3x(16x^2 - 9)$

$= 3x[(4x)^2 - (3)^2] = 3x(4x + 3)(4x - 3)$ **Ans.**

(iv) $16a^4 - b^4 = (4a^2)^2 - (b^2)^2$

$= (4a^2 + b^2)(4a^2 - b^2) = (4a^2 + b^2)(2a + b)(2a - b)$ **Ans.**

9 Factorise :

(i) $16x^2 - y^2 + 4yz - 4z^2$ (ii) $(1 - x^2)(1 - y^2) + 4xy$

(iii) $x^4 + x^2y^2 + y^4$ (iv) $(p^2 + q^2 - r^2)^2 - 4p^2q^2$.

Solution :

(i)
$$16x^2 - y^2 + 4yz - 4z^2 = (4x)^2 - (y^2 - 4yz + 4z^2)$$
$$= (4x)^2 - (y - 2z)^2$$
$$= (4x + \overline{y - 2z})(4x - \overline{y - 2z})$$
$$= (4x + y - 2z)(4x - y + 2z) \quad \textbf{Ans.}$$

(ii)
$$(1 - x^2)(1 - y^2) + 4xy = 1 - x^2 - y^2 + x^2y^2 + 4xy$$
$$= 1 + x^2y^2 + 2xy - x^2 - y^2 + 2xy$$
$$= (1 + x^2y^2 + 2xy) - (x^2 + y^2 - 2xy)$$
$$= (1 + xy)^2 - (x - y)^2$$
$$= (\overline{1 + xy} + \overline{x - y})(\overline{1 + xy} - \overline{x - y})$$
$$= (1 + xy + x - y)(1 + xy - x + y) \quad \textbf{Ans.}$$

(iii)
$$x^4 + x^2y^2 + y^4 = x^4 + y^4 + 2x^2y^2 - x^2y^2$$
$$= (x^2 + y^2)^2 - (xy)^2$$
$$= (x^2 + y^2 + xy)(x^2 + y^2 - xy) \quad \textbf{Ans.}$$

(iv)
$$(p^2 + q^2 - r^2)^2 - 4p^2q^2 = (p^2 + q^2 - r^2)^2 - (2pq)^2$$
$$= (p^2 + q^2 - r^2 + 2pq)(p^2 + q^2 - r^2 - 2pq)$$
$$= (p^2 + q^2 + 2pq - r^2)(p^2 + q^2 - 2pq - r^2)$$
$$= [(p + q)^2 - r^2][(p - q)^2 - r^2]$$
$$= (p + q + r)(p + q - r)(p - q + r)(p - q - r) \quad \textbf{Ans.}$$

EXERCISE 5 (C)

Factorise :

1. $25a^2 - 9b^2$
2. $a^2 - (2a + 3b)^2$
3. $a^2 - 81(b - c)^2$
4. $25(2a - b)^2 - 81b^2$
5. $50a^3 - 2a$
6. $4a^2b - 9b^3$
7. $3a^5 - 108a^3$
8. $9(a - 2)^2 - 16(a + 2)^2$
9. $a^4 - 1$
10. $a^3 + 2a^2 - a - 2$
11. $(a + b)^3 - a - b$
12. $a(a - 1) - b(b - 1)$
13. $4a^2 - (4b^2 + 4bc + c^2)$
14. $4a^2 - 49b^2 + 2a - 7b$
15. $9a^2 + 3a - 8b - 64b^2$
16. $4a^2 - 12a + 9 - 49b^2$
17. $4xy - x^2 - 4y^2 + z^2$
18. $a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$
19. $4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$

20. $(a^2 - 1)(b^2 - 1) + 4ab$
21. $x^4 + x^2 + 1$
22. $(a^2 + b^2 - 4c^2)^2 - 4a^2b^2$
23. $(x^2 + 4y^2 - 9z^2)^2 - 16x^2y^2$
24. $(a + b)^2 - a^2 + b^2$
25. $a^2 - b^2 - (a + b)^2$
26. $9a^2 - (a^2 - 4)^2$
27. $x^2 + \frac{1}{x^2} - 11$
28. $4x^2 + \frac{1}{4x^2} + 1$
29. $4x^4 - x^2 - 12x - 36$
30. $a^2(b + c) - (b + c)^3$

Type 5 : The sum or difference of two cubes

We know, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ [Expansion of $a^3 + b^3$]

and, $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ [Expansion of $a^3 - b^3$]

Further
$$\mathbf{a^3 + b^3} = (a + b)^3 - 3ab(a + b)$$
$$= (a + b)[(a + b)^2 - 3ab]$$
$$= (a + b)(a^2 + 2ab + b^2 - 3ab) = \mathbf{(a + b)(a^2 - ab + b^2)}$$

Similarly, $\mathbf{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$

Clearly, factors of $a^3 + b^3$ are $(a + b)$ and $(a^2 - ab + b^2)$

and, factors of $a^3 - b^3$ are $(a - b)$ and $(a^2 + ab + b^2)$

10 Factorise : (i) $a^3 + 27b^3$ (ii) $16a^4 + 54a$ (iii) $125a^3 + \frac{1}{8}$ (iv) $a^3 + b^3 + a + b$.

Solution :

(i)
$$\mathbf{a^3 + 27b^3} = (a)^3 + (3b)^3$$
$$= (a + 3b)[(a)^2 - a \times 3b + (3b)^2]$$
$$= \mathbf{(a + 3b)(a^2 - 3ab + 9b^2)}$$ **Ans.**

(ii)
$$\mathbf{16a^4 + 54a} = 2a(8a^3 + 27)$$
$$= 2a[(2a)^3 + (3)^3]$$
$$= \mathbf{2a(2a + 3)(4a^2 - 6a + 9)}$$ **Ans.**

(iii)
$$\mathbf{125a^3 + \frac{1}{8}} = (5a)^3 + (\frac{1}{2})^3$$
$$= (5a + \frac{1}{2})[(5a)^2 - 5a \times \frac{1}{2} + (\frac{1}{2})^2]$$
$$= \mathbf{(5a + \frac{1}{2})(25a^2 - \frac{5}{2}a + \frac{1}{4})}$$ **Ans.**

(iv)
$$\mathbf{a^3 + b^3 + a + b} = (a + b)(a^2 - ab + b^2) + (a + b)$$
$$= \mathbf{(a + b)(a^2 - ab + b^2 + 1)}$$ **Ans.**

11 Factorise : (i) $8a^3 - 27b^3$ (ii) $2a^7 - 128a$.

Solution :

(i) $8a^3 - 27b^3 = (2a)^3 - (3b)^3$

$= (2a - 3b)(4a^2 + 6ab + 9b^2)$ **Ans.**

(ii) $2a^7 - 128a = 2a(a^6 - 64)$

$= 2a[(a^3)^2 - (8)^2]$

$= 2a(a^3 + 8)(a^3 - 8)$

$= 2a(a^3 + 2^3)(a^3 - 2^3)$

$= 2a(a + 2)(a^2 - 2a + 4)(a - 2)(a^2 + 2a + 4)$

$= 2a(a + 2)(a - 2)(a^2 - 2a + 4)(a^2 + 2a + 4)$ **Ans.**

12 Show that :

(i) $15^3 - 8^3$ is divisible by 7 (ii) $15^3 + 8^3$ is divisible by 23.

Solution :

(i) $\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$\therefore 15^3 - 8^3 = (15 - 8)(15^2 + 15 \times 8 + 8^2)$

$= 7(225 + 120 + 64) = 7 \times 409$

Which is divisible by 7.

(ii) $\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$\therefore 15^3 + 8^3 = (15 + 8)(15^2 - 15 \times 8 + 8^2)$

$= 23(225 - 120 + 64) = 23 \times 169$

Which is divisible by 23.

EXERCISE 5 (D)

Factorise :

1. $a^3 - 27$
2. $1 - 8a^3$
3. $64 - a^3b^3$
4. $a^6 + 27b^3$
5. $3x^7y - 81x^4y^4$
6. $a^3 - \frac{27}{a^3}$
7. $a^3 + 0{\cdot}064$
8. $a^4 - 343a$
9. $(x - y)^3 - 8x^3$
10. $\frac{8a^3}{27} - \frac{b^3}{8}$
11. $a^6 - b^6$
12. $a^6 - 7a^3 - 8$
13. $a^3 - 27b^3 + 2a^2b - 6ab^2$
14. $8a^3 - b^3 - 4ax + 2bx$
15. $a - b - a^3 + b^3$
16. $2x^3 + 54y^3 - 4x - 12y$
17. Show that :
    (i) $13^3 - 5^3$ is divisible by 8.
    (ii) $35^3 + 27^3$ is divisible by 62.

13 Factorise :

(i) $x^2 + \frac{a^2 - 1}{a}x - 1$ (ii) $x^4 + y^4 - 23x^2y^2$.

Solution :

(i) $x^2 + \frac{a^2-1}{a}x - 1 = x^2 + \left(\frac{a^2}{a} - \frac{1}{a}\right)x - 1$

$= x^2 + ax - \frac{x}{a} - 1$

$= (x^2 - \frac{x}{a}) + (ax - 1) = \left(x^2 - \frac{x}{a}\right) + \left(ax - \frac{a}{a}\right)$

$= x\,(x - \frac{1}{a}) + a\,(x - \frac{1}{a}) = (x - \frac{1}{a})\,(x + a)$ **Ans.**

(ii) $x^4 + y^4 - 23x^2y^2 = x^4 + y^4 + 2x^2y^2 - 2x^2y^2 - 23x^2y^2$

$= (x^2 + y^2)^2 - 25x^2y^2$

$= (x^2 + y^2)^2 - (5xy)^2$

$= (x^2 + y^2 + 5xy)\,(x^2 + y^2 - 5xy)$ **Ans.**

14 Factorise :

(i) $a^2 + \frac{1}{a^2} - 18$ (ii) $a^4 - 7a^2 + 1$ (iii) $a^3 + 3a^2b + 3ab^2 + 2b^3$.

Solution :

(i). $a^2 + \frac{1}{a^2} - 18 = (a^2 + \frac{1}{a^2} - 2) - 16$

$= (a - \frac{1}{a})^2 - (4)^2$

$= (a - \frac{1}{a} + 4)\,(a - \frac{1}{a} - 4)$ **Ans.**

(ii) $a^4 - 7a^2 + 1 = a^4 - 7a^2 + 1 + 2a^2 - 2a^2$

$= (a^4 + 1 + 2a^2) - 9a^2$

$= (a^2 + 1)^2 - (3a)^2$

$= (a^2 + 1 + 3a)\,(a^2 + 1 - 3a)$ **Ans.**

(iii) $a^3 + 3a^2b + 3ab^2 + 2b^3 = a^3 + 3a^2b + 3ab^2 + b^3 + b^3$

$= (a + b)^3 + b^3$

$= x^3 + b^3$ [Taking $a + b = x$]

$= (x + b)\,(x^2 - bx + b^2)$

$= (a + b + b)\,[(a + b)^2 - b\,(a + b) + b^2]$

$= (a + 2b)\,(a^2 + 2ab + b^2 - ab - b^2 + b^2)$

$= (a + 2b)\,(a^2 + ab + b^2)$ **Ans.**

15 Factorise : $a^6 - 26a^3 - 27$

Solution :

$$\text{Let } a^3 = x \Rightarrow a^6 = x^2$$

$$\begin{aligned}
\therefore \quad a^6 - 26a^3 - 27 &= x^2 - 26x - 27 \\
&= x^2 - 27x + x - 27 \\
&= x(x - 27) + 1(x - 27) \\
&= (x - 27)(x + 1) \\
&= (a^3 - 27)(a^3 + 1) \\
&= (a^3 - 3^3)(a^3 + 1^3) \\
&= (a - 3)(a^2 + 3a + 9)(a + 1)(a^2 - a + 1) \quad \textbf{Ans.}
\end{aligned}$$

EXERCISE 5 (E)

Factorise :

1. $x^2 + \dfrac{1}{4x^2} + 1 - 7x - \dfrac{7}{2x}$
2. $9a^2 + \dfrac{1}{9a^2} - 2 - 12a + \dfrac{4}{3a}$
3. $x^2 + \dfrac{a^2+1}{a}x + 1$
4. $x^4 + y^4 - 27x^2y^2$
5. $4x^4 + 9y^4 + 11x^2y^2$
6. $x^2 + \dfrac{1}{x^2} - 3$
7. $a - b - 4a^2 + 4b^2$
8. $(2a - 3)^2 - 2(2a - 3)(a - 1) + (a - 1)^2$
9. $(a^2 - 3a)(a^2 - 3a + 7) + 10$
10. $(a^2 - a)(4a^2 - 4a - 5) - 6$
11. $x^4 + y^4 - 3x^2y^2$

$x^4 + y^4 - 3x^2y^2 = x^4 + y^4 - 2x^2y^2 - x^2y^2$
$= (x^2 - y^2)^2 - (xy)^2$ and so on.

12. $5a^2 - b^2 - 4ab + 7a - 7b$

$5a^2 - b^2 - 4ab + 7a - 7b$
$= 4a^2 + a^2 - b^2 - 4ab + 7a - 7b$
$= (a^2 - b^2) + (4a^2 - 4ab) + (7a - 7b)$
$= (a - b)(a + b) + 4a(a - b) + 7(a - b)$
$= (a - b)[a + b + 4a + 7]$
$= (a - b)(5a + b + 7)$

Alternative method :

$5a^2 - 4ab - b^2 + 7a - 7b$
$= 5a^2 - 5ab + ab - b^2 + 7a - 7b$
$= 5a(a - b) + b(a - b) + 7(a - b)$
$= \mathbf{(a - b)(5a + b + 7)}$

13. $12(3x - 2y)^2 - 3x + 2y - 1$

Let $3x - 2y = a$
$\therefore$ Given expression
$= 12(3x - 2y)^2 - (3x - 2y) - 1$
$= 12a^2 - a - 1$ and so on.

14. $4(2x - 3y)^2 - 8x + 12y - 3$
15. $3 - 5x + 5y - 12(x - y)^2$
16. $9x^2 + 3x - 8y - 64y^2$
17. $2\sqrt{3}x^2 + x - 5\sqrt{3}$
18. $\dfrac{1}{4}(a + b)^2 - \dfrac{9}{16}(2a - b)^2$
19. $2(ab + cd) - a^2 - b^2 + c^2 + d^2$
20. Find the value of :
   (i) $(987)^2 - (13)^2$
   (ii) $(67{\cdot}8)^2 - (32{\cdot}2)^2$
   (iii) $\dfrac{(6{\cdot}7)^2 - (3{\cdot}3)^2}{6{\cdot}7 - 3{\cdot}3}$
   (iv) $\dfrac{(18{\cdot}5)^2 - (6{\cdot}5)^2}{18{\cdot}5 + 6{\cdot}5}$