3 Compound Interest

[Using Formula]

3.1 INTRODUCTION

In the previous chapter, we have learnt to calculate the amount and the compound interest on a given sum (principal) at a given rate and for a given period.

In the above process, we found compound interest as a repeated simple interest computation with a growing principal.

The computation of compound interest and amount, as found above, becomes quite tedious as the number of conversion periods (no. of years, no. of half-years, etc.) increase.

3.2 USING FORMULA

In order to make the above said calculation easy and fast, we use certain formulae.

First formula :

1. When the interest is compounded yearly, the formula for finding the amount is :

$$A = P\left(1+\frac{r}{100}\right)^n$$

where A = amount; P = principal;
r = rate of interest compounded yearly;
and n = number of years.

1 Calculate the amount on ₹ 7,500 in 2 years and at 6% compounded annually.

Solution :

Given : P = ₹ 7,500; n = 2 years and r = 6%

$\Rightarrow$ $A = ₹\ 7{,}500\left(1+\frac{6}{100}\right)^2$

$\because A = P\left(1+\frac{r}{100}\right)^n$

$= ₹\ 7{,}500 \times \left(\frac{106}{100}\right)^2 = ₹\ 8{,}427$

$\therefore$ **Required amount = ₹ 8,427** **Ans.**

And, C.I. = ₹ 8,427 − ₹ 7,500 = ₹ 927 **Ans.**

2 Calculate the compound interest on ₹ 18,000 in 2 years at 15% per annum.

Solution :

$A = P\left(1+\frac{r}{100}\right)^n \Rightarrow A = ₹\ 18{,}000\left(1+\frac{15}{100}\right)^2$

$= ₹\ 23{,}805$

$\therefore$ **Compound Interest** = A – P

= ₹ 23,805 – ₹ 18,000 = **₹ 5,805** **Ans.**

Direct method :

Compound Interest $= ₹\ 18{,}000\left[\left(1+\frac{15}{100}\right)^2-1\right]$

$= ₹\ 18{,}000\ (1.3225 - 1)$

$= ₹\ 18{,}000 \times 0.3225$

$= ₹\ 5{,}805$ **Ans.**

C.I. = A – P

$= P\left(1+\frac{r}{100}\right)^n - P$

$= P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

2. When the rates for successive years are different then:

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right) \text{ ... and so on}$$

where $r_1\%$, $r_2\%$, $r_3\%$ ... and so on are the rates for successive years.

3 Calculate the amount and the compound interest on ₹ 12,000 in 3 years when the rates of interest for successive years are 8%, 10% and 15% respectively.

Solution :

Required amount, $A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$

$\Rightarrow$ $A = ₹\ 12{,}000\left(1+\frac{8}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{15}{100}\right)$

$= ₹\ 16{,}394.40$ **Ans.**

And, **C.I.** = ₹ 16,394.40 – ₹ 12,000 = **₹ 4,394.40** **Ans.**

3.3 INVERSE PROBLEMS:

1. To find the principal :

4 What sum of money will amount to ₹ 3,630/- in 2 years at 10% per annum compound interest ?

$$₹\ 3{,}630 = P\left(1+\frac{10}{100}\right)^2$$

$\because A = P\left(1+\frac{r}{100}\right)^n$

$\Rightarrow$ $₹\ 3{,}630 = P \times \frac{11}{10} \times \frac{11}{10}$

$\Rightarrow$ **The required sum of money,** $P = ₹\ 3{,}630 \times \frac{10}{11} \times \frac{10}{11}$

$= ₹\ 3{,}000$ **Ans.**

5 On what sum of money will compound interest for 2 years at 5 percent per year amount to ₹ 164 ?

Solution :

Since, $\text{C.I.} = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

$\Rightarrow$ ₹ 164 $= P\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$

$\Rightarrow$ ₹ 164 $= P\left[\frac{21}{20}\times\frac{21}{20} - 1\right]$

On further simplification, we get : **P = ₹ 1,600** **Ans.**

2. To find the rate percent :

6 At what rate percent per annum C.I. will ₹ 2,000 amount to ₹ 2,315.25 in 3 years ?

Solution:

$$₹\ 2{,}315.25 = ₹\ 2{,}000\left(1+\frac{r}{100}\right)^3$$

$\because A = P\left(1+\frac{r}{100}\right)^n$

$$\Rightarrow \frac{2{,}315.25}{2{,}000} = \left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \left(\frac{21}{20}\right)^3 = \left(1+\frac{r}{100}\right)^3$$

$$\frac{2{,}315.25}{2{,}000} = \frac{231525}{2{,}000\times100} = \frac{9{,}261}{8{,}000} = \frac{21\times21\times21}{20\times20\times20}$$

$$\Rightarrow \frac{21}{20} = 1+\frac{r}{100}$$

On further simplification, we get : **r = 5%** **Ans.**

7 A person invests ₹ 10,000 for two years at a certain rate of interest compounded annually. At the end of one year this sum amounts to ₹ 11,200. Calculate :

(i) the rate of interest per annum.

(ii) the amount at the end of the second year.

Solution :

(i) $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow 11{,}200 = 10{,}000\left(1+\frac{r}{100}\right)^1$

$$\Rightarrow \frac{11{,}200}{10{,}000} = 1+\frac{r}{100}$$

$$\Rightarrow r = \frac{112}{100} - 1 = \frac{12}{100}$$

$$\Rightarrow r\% = \frac{12}{100}\times 100\% = 12\%$$

$\therefore$ **Rate of interest p.a. = 12%** **Ans.**

(ii) $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow A = ₹\ 11{,}200\left(1+\frac{12}{100}\right)^1$ [For 2nd year, P = ₹ 11,200]

$= ₹\ 11{,}200 \times \frac{112}{100} = ₹\ 12{,}544$ **Ans.**

3. To find the number of years (*i.e.*, time):

8 In how many years will ₹ 2,000 amount to ₹ 2,662 at 10 percent C.I.?

Solution :

$₹\ 2{,}662 = ₹\ 2{,}000\left(1+\frac{10}{100}\right)^n$ $\because A = P\left(1+\frac{r}{100}\right)^n$

$\Rightarrow \frac{2{,}662}{2{,}000} = \left(\frac{11}{10}\right)^n$ $\frac{2{,}662}{2{,}000} = \frac{1{,}331}{1{,}000} = \frac{11\times11\times11}{10\times10\times10}$

$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n \Rightarrow n = 3$ years **Ans.**

9 ₹ 16,820 is divided between Govind and Geeta, both aged 27 and 25 years respectively. Their money is invested at 5% per annum compound interest in such a way that both receive equal money at the age of 40 years. Find the share of each out of ₹ 16,820.

Solution :

Let the share of Govind be ₹ x ⇒ share of Geeta = ₹ $(16{,}820 - x)$

For Govind : P = ₹ x, r = 5% and n = (40 – 27) years = 13 years

$\therefore A = P\left(1+\frac{r}{100}\right)^n = ₹\ x\left(1+\frac{5}{100}\right)^{13} = ₹\ x\left(\frac{21}{20}\right)^{13}$

For Geeta : P = ₹ $(16{,}820 - x)$, r = 5% and n = (40 – 25) years = 15 years.

$\therefore A = P\left(1+\frac{r}{100}\right)^n = ₹\ (16,\ 820 - x)\left(1+\frac{5}{100}\right)^{15}$

$= ₹\ (16{,}820 - x)\left(\frac{21}{20}\right)^{15}$

Given; both receive equal sums on reaching the age of 40 years.

$\therefore x\left(\frac{21}{20}\right)^{13} = (16{,}820 - x)\left(\frac{21}{20}\right)^{15}$

$\Rightarrow x = (16{,}820 - x) \times \left(\frac{21}{20}\right)^2$

$\Rightarrow x = 8{,}820$ [On solving]

and $16{,}820 - x = 16{,}820 - 8{,}820 = 8{,}000$

$\therefore$ **The share of Govind = ₹ 8,820 and the share of Geeta = ₹ 8,000** **Ans.**

EXERCISE 3 (A)

1. Find the amount and the compound interest on ₹ 12,000 in 3 years at 5%; interest being compounded annually.
2. Calculate the amount, if ₹ 15,000 is lent at compound interest for 2 years and the rates for the successive years are 8% p.a. and 10% p.a. respectively.
3. Calculate the compound interest accrued on ₹ 6,000 in 3 years, compounded yearly, if the rates for the successive years are 5%, 8% and 10% respectively.
4. What sum of money will amount to ₹ 5,445 in 2 years at 10% per annum compound interest?
5. On what sum of money will the compound interest for 2 years at 5 per cent per annum amount to ₹ 768·75 ?
6. Find the sum on which the compound interest for 3 years at 10% per annum amounts to ₹ 1,655.
7. What principal will amount to ₹ 9,856 in two years, if the rates of interest for successive years are 10% and 12% respectively ?
8. On a certain sum, the compound interest in 2 years amounts to ₹ 4,240. If the rates of interest for successive years are 10% and 15% respectively, find the sum.
9. At what rate per cent per annum will ₹ 6,000 amount to ₹ 6,615 in 2 years when interest is compounded annually ?
10. At what rate per cent compound interest, does a sum of money become 1·44 times of itself in 2 years ?
11. At what rate per cent will a sum of ₹ 4,000 yield ₹ 1,324 as compound interest in 3 years ?
12. A person invests ₹ 5,000 for three years at a certain rate of interest compounded annually. At the end of two years this sum amounts to ₹ 6,272. Calculate :
    (i) the rate of interest per annum.
    (ii) the amount at the end of the third year.
13. In how many years will ₹ 7,000 amount to ₹ 9,317 at 10 per cent per annum compound interest ?
14. Find the time, in years, in which ₹ 4,000 will produce ₹ 630·50 as compound interest at 5 per cent p.a. interest being compounded annually.
15. Divide ₹ 28,730 between A and B so that when their shares are lent out at 10 per cent compound interest compounded per year, the amount that A receives in 3 years is the same as what B receives in 5 years.
16. A sum of ₹ 44,200 is divided between John and Smith, 12 years and 14 years old respectively, in such a way that if their portions be invested at 10 percent per annum compound interest, they will receive equal amounts on reaching 16 years of age.
    (i) What is the share of each out of ₹ 44,200 ?
    (ii) What will each receive, when 16 years old ?
17. The simple interest on a certain sum of money at 10% per annum is ₹ 6,000 in 2 years. Find :
    (i) the sum.
    (ii) the amount due at the end of 3 years and at the same rate of interest compounded annually.
    (iii) the compound interest earned in 3 years.
18. Find the difference between compound interest and simple interest on ₹ 8,000 in 2 years and at 5% per annum.

3.4 MISCELLANEOUS PROBLEMS :

To find the principal, when difference between C.I. and S.I. is given :

10 On what sum of money will the difference between simple interest and compound interest for 2 years at 5% per annum be equal to ₹ 25 ?

Solution :

$$\text{C.I.} = P\left(1+\frac{5}{100}\right)^2 - P = \frac{41P}{400} \quad \text{and} \quad \text{S.I.} = \frac{P \times 5 \times 2}{100} = \frac{P}{10}$$

Given : C.I. – S.I. = ₹ 25

$\Rightarrow \quad \frac{41P}{400} - \frac{P}{10} = ₹\ 25 \quad \Rightarrow \quad P = ₹\ 10{,}000$ **Ans.**

11 A certain sum of money at compound interest amounts to ₹ 6,600 in 1 year and to ₹ 7,986 in 3 years. Find the sum and the rate percent.

Solution :

Amount in 1 year = ₹ 6,600 $\Rightarrow \quad P\left(1+\frac{r}{100}\right)^1 = 6{,}600$ I

Amount in 3 years = ₹ 7,986 $\Rightarrow \quad P\left(1+\frac{r}{100}\right)^3 = 7{,}986$ II

On dividing II by I, we get :

$$\frac{P\left(1+\frac{r}{100}\right)^3}{P\left(1+\frac{r}{100}\right)^1} = \frac{7{,}986}{6{,}600} \quad \Rightarrow \quad \left(1+\frac{r}{100}\right)^2 = \frac{121}{100}$$

$$\Rightarrow \quad 1+\frac{r}{100} = \frac{11}{10}$$

$\Rightarrow$ **rate r = 10%** **Ans.**

Now, $P\left(1+\frac{r}{100}\right)^1 = 6{,}600 \quad \Rightarrow \quad P\left(1+\frac{10}{100}\right)^1 = 6{,}600$

$\Rightarrow$ **P = ₹ 6,000** **Ans.**

Alternative method :

Since, amount of first year = ₹ 6,600

$\therefore$ For the 2nd year and the 3rd year,

P = ₹ 6,600; A = ₹ 7,986 and n = 2 years

$\therefore \quad A = P\left(1+\frac{r}{100}\right)^n \quad \Rightarrow \quad ₹\ 7{,}986 = ₹\ 6{,}600\left(1+\frac{r}{100}\right)^2$

i.e., $\frac{7{,}986}{6{,}600} = \left(1+\frac{r}{100}\right)^2 \quad \Rightarrow \quad \frac{121}{100} = \left(1+\frac{r}{100}\right)^2$

i.e., $\frac{11}{10} = 1 + \frac{r}{100} \quad \Rightarrow \quad r = 10\%$

Now for the first year :

A = ₹ 6,600, r = 10% and n = 1 year

$\therefore \quad A = P\left(1+\frac{r}{100}\right)^n \quad \Rightarrow \quad 6{,}600 = P\left(1+\frac{10}{100}\right)^1$

$\Rightarrow \quad P = ₹\ 6{,}000$

$\therefore$ **Sum = ₹ 6,000** and **rate = 10%** **Ans.**

12 The difference between the C.I. and the S.I. on ₹ 8,400 for two years is ₹ 21 at the same rate of interest per year. Find the rate of interest.

Solution :

Let the rate of interest per year = $r\%$

$$\text{S.I. in 2 years} = \frac{₹\,8{,}400 \times r \times 2}{100} = ₹\ 168\ r$$

$$\text{And, C.I. in 2 years} = A - P = ₹\ 8{,}400\left(1 + \frac{r}{100}\right)^2 - ₹\ 8{,}400$$

$$\text{Given C.I.} - \text{S.I.} = ₹\ 21$$

$$\Rightarrow \quad 8{,}400\left(1 + \frac{r}{100}\right)^2 - 8{,}400 - 168\ r = 21$$

$$\Rightarrow \quad 8{,}400\left(1 + \frac{r^2}{10{,}000} + \frac{2r}{100}\right) - 8{,}400 - 168\ r = 21$$

$$\Rightarrow \quad 8{,}400 + \frac{84\,r^2}{100} + 168\ r - 8{,}400 - 168\ r = 21$$

$$\Rightarrow \quad \frac{84\,r^2}{100} = 21$$

$$\Rightarrow \quad r^2 = 21 \times \frac{100}{84} = 25 \text{ and } r = 5$$

$\therefore$ **Rate of interest = 5%** **Ans.**

EXERCISE 3 (B)

1. The difference between simple interest and compound interest on a certain sum is ₹ 54.40 for 2 years at 8 percent per annum. Find the sum.
2. A sum of money, invested at compound interest, amounts to ₹ 19,360 in 2 years and to ₹ 23,425.60 in 4 years. Find the rate percent and the original sum of money.
3. A sum of money lent out at C.I. at a certain rate per annum becomes three times of itself in 8 years. Find in how many years will the money become twenty-seven times of itself at the same rate of interest p.a.
4. On what sum of money will compound interest (payable annually) for 2 years be the same as simple interest on ₹ 9,430 for 10 years, both at the rate of 5 percent per annum ?
5. Kamal and Anand each lent the same sum of money for 2 years at 5% at simple interest and compound interest respectively. Anand received ₹ 15 more than Kamal. Find the amount of money lent by each and the interest received.
6. Simple interest on a sum of money for 2 years at 4% is ₹ 450. Find compound interest on the same sum and at the same rate for 2 years.
7. Simple interest on a certain sum of money for 4 years at 4% per annum exceeds the compound interest on the same sum for 3 years at 5 percent per annum by ₹ 228. Find the sum.
8. Compound interest on a certain sum of money at 5% per annum for two years is ₹ 246. Calculate simple interest on the same sum for 3 years at 6% per annum.
9. A certain sum of money amounts to ₹ 23,400 in 3 years at 10% per annum simple interest. Find the amount of the same sum in 2 years and at 10% p.a. compound interest.
10. Mohit borrowed a certain sum at 5% per annum compound interest and cleared this loan by paying ₹ 12,600 at the end of the first year and ₹ 17,640 at the end of the second year. Find the sum borrowed.

For ₹ 12,600 : A = ₹ 12,600; r = 5% and n = 1 year. Find the sum (P).

For ₹ 17,640 : A = ₹ 17,640; r = 5% and n = 2 years. Find the sum (P).

Required answer is the addition of the two sums obtained above

3.5 WHEN THE INTEREST IS COMPOUNDED HALF-YEARLY (TWO TIMES IN A YEAR)

$$A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$$

i.e. the rate percent is divided by 2 and the number of years is multiplied by 2.

Although it does not belong to the I.C.S.E. Syllabus but whenever the interest is compounded quarterly (four times in a year) :

$$A = P\left(1+\frac{r}{4\times 100}\right)^{n\times 4}$$

i.e. the rate percent is divided by 4 and the number of years is multiplied by 4.

13 Calculate the compound interest on ₹ 4,000 in $1\frac{1}{2}$ years at 10% per annum compounded half-yearly.

Solution :

$$A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2} \Rightarrow A = ₹\,4{,}000\left(1+\frac{10}{2\times 100}\right)^{\frac{3}{2}\times 2} = ₹\,4{,}630.50$$

$\therefore$ **C.I.** = A – P

= ₹ 4,630.50 – ₹ 4,000 = **₹ 630.50** **Ans.**

3.6 WHEN THE TIME IS NOT AN EXACT NUMBER OF YEARS AND THE INTEREST IS COMPOUNDED YEARLY

14 Find the amount when ₹ 10,000 is invested for $2\frac{1}{2}$ years at 10% interest compounded yearly.

Solution :

First of all find the amount in 2 years.

Amount in 2 years = ₹ 10,000 $\left(1+\frac{10}{100}\right)^2$ = ₹ 12,100

After two years, ₹ 12,100 is the principal for the remaining half year and so :

$$A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$$

$$= ₹\,12{,}100\left(1+\frac{10}{2\times 100}\right)^{\frac{1}{2}\times 2} = ₹\,12{,}705$$

$\Rightarrow$ **Amount in $2\frac{1}{2}$ years = ₹ 12,705** **Ans.**

Note : For the example given above, we can directly use the formula :

$$A = P\left(1+\frac{r}{100}\right)^2\left(1+\frac{r}{2\times 100}\right)^{\frac{1}{2}\times 2}$$

$\Rightarrow$ **Amount in $2\frac{1}{2}$ years** = ₹ 10,000 $\left(1+\frac{10}{100}\right)^2\left(1+\frac{10}{2\times 100}\right)$ **= ₹ 12,705** **Ans.**

Study the following table carefully :

No. of years :	Compounded yearly :	Compounded half-yearly :
(i) n = 1 year	$A = P\left(1+\frac{r}{100}\right)^{1}$	$A = P\left(1+\frac{r}{2\times100}\right)^{1\times2}$
(ii) $n = 1\frac{1}{2}$ years	$A = P\left(1+\frac{r}{100}\right)^{1}\cdot\left(1+\frac{r}{2\times100}\right)^{\frac{1}{2}\times2}$	$A = P\left(1+\frac{r}{2\times100}\right)^{\frac{3}{2}\times2}$
(iii) n = 2 years	$A = P\left(1+\frac{r}{100}\right)^{2}$	$A = P\left(1+\frac{r}{2\times100}\right)^{2\times2}$
(iv) $n = 2\frac{1}{2}$ years	$A = P\left(1+\frac{r}{100}\right)^{2}\cdot\left(1+\frac{r}{2\times100}\right)^{\frac{1}{2}\times2}$	$A = P\left(1+\frac{r}{2\times100}\right)^{\frac{5}{2}\times2}$

15 John borrowed ₹ 20,000 for 4 years under the following conditions :

10% simple interest for the first $2\frac{1}{2}$ years.

10% C.I. for the remaining one and a half years on the amount due after $2\frac{1}{2}$ years, the interest being compounded half-yearly.

Find the total amount to be paid at the end of fourth year.

Solution :

For first $2\frac{1}{2}$ years : P = ₹ 20,000; R = 10% and T = $\frac{5}{2}$ years.

$\therefore$ Interest = ₹ $\frac{20{,}000\times10\times5}{100\times2}$

= ₹ 5,000

As, interest is simple

$\Rightarrow$ Amount due after $2\frac{1}{2}$ years. = ₹ 20,000 + ₹ 5,000 = ₹ 25,000

For remaining $1\frac{1}{2}$ years : P = ₹ 25,000; $n = \frac{3}{2}$ years and

r = 10% per annum compounded half-yearly.

$\therefore$ **A** $= P\left(1+\frac{r}{2\times100}\right)^{n\times2}$ = ₹ 25,000 $\left(1+\frac{10}{2\times100}\right)^{\frac{3}{2}\times2}$ = **₹ 28,940.63**

$\Rightarrow$ **The total amount to be paid by John at the end of 4 years = ₹ 28,940·63** **Ans.**

16 A sum of money is lent out at compound interest for two years at 20% per annum, compound interest being reckoned yearly. If the same sum of money was lent out at compound interest at the same rate percent per annum, compound interest being reckoned half-yearly, it would have fetched ₹ 482 more by way of interest in two years. Calculate the sum of money lent out.

Solution :

Let the sum of money lent out be ₹ x

In the 1st case:

$$A_1 = ₹\ x\left(1+\frac{20}{100}\right)^2 = ₹\ \frac{36x}{25}$$

$$\because A = P\left(1+\frac{r}{100}\right)^n$$

In the 2nd case:

$$A_2 = ₹\ x\left(1+\frac{20}{2\times 100}\right)^{2\times 2} = ₹\ \frac{14{,}641\,x}{10{,}000}$$

$$\because A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$$

Given, C.I. in the 2nd case is ₹ 482 more than the C.I. in the 1st case.

$\Rightarrow$ For the same principal, amount in the 2nd case is ₹ 482 more than the amount in the 1st case.

$$\Rightarrow \frac{14{,}641\,x}{10{,}000} - \frac{36\,x}{25} = 482 \Rightarrow x = 20{,}000$$

$\therefore$ **The sum of money lent out = ₹ 20,000** **Ans.**

17 A sum of ₹ 6,400 earns a compound interest of ₹ 1,008.80 in 18 months, when the interest is reckoned half-yearly. Find the rate of interest.

Solution :

Given; P = ₹ 6,400, C.I. = ₹ 1,008.80 and time = 18 months = $\frac{3}{2}$ years.

$\therefore$ Amount (A) = P + C.I. = ₹ 6,400 + ₹ 1,008.80 = ₹ 7,408.80

$$7{,}408.80 = 6{,}400\left(1+\frac{r}{2\times 100}\right)^{\frac{3}{2}\times 2}$$

$$\because A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$$

$$\Rightarrow \left(1+\frac{r}{200}\right)^3 = \frac{7{,}408.80}{6{,}400} = \left(\frac{21}{20}\right)^3$$

On simplifying, we get : $r = 10\%$ **Ans.**

18 The simple interest on a sum of money for 2 years at 4% per annum is ₹ 340. Find :

(i) the sum of money and

(ii) the compound interest on this sum for one year payable half-hearly at the same rate.

Solution :

(i) Given : I = ₹ 340, T = 2 years and R = 4%

$$\therefore \quad P = \frac{I\times 100}{R\times T} = ₹\ \frac{340\times 100}{4\times 2} = ₹\ 4{,}250$$ **Ans.**

(ii) **C.I.** $= P\left(1+\frac{r}{2\times100}\right)^{n\times2} - P$

$= ₹\ 4{,}250\left(1+\frac{4}{2\times100}\right)^{1\times2} - ₹\ 4{,}250$

$= ₹\ 4{,}421.70 - ₹\ 4{,}250 =$ **₹ 171.70** **Ans.**

EXERCISE 3 (C)

1. If the interest is compounded half-yearly, calculate the amount when principal is ₹ 7,400; the rate of interest is 5% per annum and the duration is one year.
2. Find the difference between the compound interest compounded yearly and half-yearly on ₹ 10,000 for 18 months at 10% per annum.
3. A man borrowed ₹ 16,000 for 3 years under the following terms :

   20% simple interest for the first 2 years.

   20% C.I. for the remaining one year on the amount due after 2 years, the interest being compounded half-yearly.

   Find the total amount to be paid at the end of three years.
4. What sum of money will amount to ₹ 27,783 in one and a half years at 10% per annum compounded half yearly ?
5. Ashok invests a certain sum of money at 20% per annum, compounded yearly. Geeta invests an equal amount of money at the same rate of interest per annum compounded half-yearly. If Geeta gets ₹ 33 more than Ashok in 18 months, calculate the money invested.
6. At what rate of interrest per annum will a sum of ₹ 62,500 earn a compound interest of ₹ 5,100 in one year ? The interest is to be compounded half-yearly.
7. In what time will ₹ 1,500 yield ₹ 496.50 as compound interest at 20% per year compounded half-yearly ?
8. Calcualte the C.I. on ₹ 3,500 at 6% per annum for 3 years, the interest being compounded half-yearly.

   Do not use mathematical tables. Use the necessary information from the following :

   $(1{\cdot}06)^3 = 1{\cdot}191016$; $(1{\cdot}03)^3 = 1{\cdot}092727$

   $(1{\cdot}06)^6 = 1{\cdot}418519$; $(1{\cdot}03)^6 = 1{\cdot}194052$
9. Find the difference between compound interest and simple interest on ₹ 12,000 and in $1\frac{1}{2}$ years at 10% p.a. compounded yearly.
10. Find the difference between compound interest and simple interest on ₹ 12,000 and in $1\frac{1}{2}$ years at 10% compounded half-yealry.

3.7 OTHER APPLICATIONS OF THE FORMULA

1. Growth :

The word growth can be used in several ways, such as :

(i) the growth of industries in the country

(ii) the rapid growth of plants, inflation, etc.

When the growth of industries (or, production in any particular industry) is **taken into consideration** :

The formula $A = P\left(1 + \frac{r}{100}\right)^n$ can be used as :

1. Production after n years = Initial (original) production $\times \left(1 + \frac{r}{100}\right)^n$
2. Present production = Production n years ago $\times \left(1 + \frac{r}{100}\right)^n$

In each case, rate of growth in production is $r\%$.

In a similar manner, the formula $A = P\left(1 + \frac{r}{100}\right)^n$ can be used for the growth of plants, growth of inflation, etc.

19 The total number of industries in a particular portion of the country is approximately 1,600. If the government has decided to increase the number of industries in the area by 20% every year; find the approximate number of industries after 2 years.

Solution :

Number of industries after 2 years

$$= \text{Original number of industries} \times \left(1 + \frac{r}{100}\right)^n$$

$$= 1{,}600\left(1 + \frac{20}{100}\right)^2 = \mathbf{2{,}304} \qquad \textbf{Ans.}$$

2. Depreciation :

If the cost of a machine depreciates by $r\%$ every year, then its value after n years can be obtained by the formula :

Value after n years = Present value $\times \left(1 - \frac{r}{100}\right)^n$

Also, the present value of the machine = Its value n years ago $\times \left(1 - \frac{r}{100}\right)^n$.

20 A machine depreciates every year at the rate of 20% of its value at the beginning of the year. The machine was purchased for ₹ 2,50,000 when new, and the scrap value realised when sold was ₹ 1,28,000. Find the number of years that the machine was used.

Solution:

Let the required number of years = n years

$\therefore$ Value of machine after n years = Its value when new $\times \left(1 - \frac{r}{100}\right)^n$

$$\Rightarrow \quad ₹\,1{,}28{,}000 = ₹\,2{,}50{,}000\left(1 - \frac{20}{100}\right)^n$$

$$\Rightarrow \quad \frac{₹\,1{,}28{,}000}{₹\,2{,}50{,}000} = \left(\frac{80}{100}\right)^n = \left(\frac{4}{5}\right)^n$$

On simplifying, we get :

$$\left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^n \Rightarrow \mathbf{n = 3 \text{ years}} \qquad \textbf{Ans.}$$

3. Population Problems :

When the population of a town, city, village, etc., increases at a certain rate per year, then the formula $A = P\left(1 + \frac{r}{100}\right)^n$ can be used as :

(i) Population after n years = Present population $\times \left(1 + \frac{r}{100}\right)^n$

(ii) Present population = Population n years ago $\times \left(1 + \frac{r}{100}\right)^n$

i.e. for the population at two different times, the former population will come (in the formula) at the place of principal (P) and the later population will come at the place of amount (A).

21 The population of a town in China increases by 20% every year. If its present population is 2,16,000, find :

(i) its population after 2 years, (ii) its population 2 years ago.

Solution:

(i) Population after n years = Present population $\times \left(1 + \frac{r}{100}\right)^n$

$\Rightarrow$ **Population after 2 years** = $2{,}16{,}000\left(1 + \frac{20}{100}\right)^2$ = **3,11,040** **Ans.**

(ii) Present population = Population n years ago $\times \left(1 + \frac{r}{100}\right)^n$

$\Rightarrow$ $2{,}16{,}000$ = Population 2 years ago $\times \left(1 + \frac{20}{100}\right)^2$

$\Rightarrow$ **Population 2 years ago = 1,50,000** **Ans.**

22 A sum of money lent out at C.I. at a certain rate per annum doubles itself in 5 years. Find in how many years will the money become eight times of itself at the same rate of interest p.a.

***Solution* :**

Let Principal = ₹ x $\Rightarrow$ Amount in 5 years = ₹ $2x$

$A = P\left(1 + \frac{r}{100}\right)^n \Rightarrow 2x = x\left(1 + \frac{r}{100}\right)^5$

$\Rightarrow 2 = \left(1 + \frac{r}{100}\right)^5$ I

For the second part : P = ₹ x and A = ₹ $8x$

$\therefore$ $A = P\left(1 + \frac{r}{100}\right)^n \Rightarrow 8x = x\left(1 + \frac{r}{100}\right)^n$

$$\Rightarrow \quad 8 = \left(1+\frac{r}{100}\right)^n$$

i.e. $(2)^3 = \left(1+\frac{r}{100}\right)^n \Rightarrow \left[\left(1+\frac{r}{100}\right)^5\right]^3 = \left(1+\frac{r}{100}\right)^n \quad \left[\because 2=\left(1+\frac{r}{100}\right)^5\right]$

$$\Rightarrow \left(1+\frac{r}{100}\right)^{15} = \left(1+\frac{r}{100}\right)^n \Rightarrow n = 15$$

i.e. **Time required = 15 years** **Ans.**

23 A man borrowed a sum of money and agrees to pay it off by paying ₹ 43,200 at the end of the first year and ₹ 34,992 at the end of the second year. If the rate of compound interest is 8% per annum, find the sum borrowed.

Solution :

For the payment of ₹ 43,200 at the end of the first year :

A = ₹ 43,200 ; n = 1 year and r = 8%. **To find P.**

$$A = P\left(1+\frac{r}{100}\right)^n \Rightarrow ₹\ 43{,}200 = P\left(1+\frac{8}{100}\right)^1$$

$$\Rightarrow \quad P = ₹\ 43{,}200 \times \frac{100}{108} = ₹\ 40{,}000$$

For the payment of ₹ 34,992 at the end of the second year :

$$₹\ 34{,}992 = P\left(1+\frac{8}{100}\right)^2 \Rightarrow P \Rightarrow ₹\ 34{,}992 \times \left(\frac{100}{108}\right)^2 = ₹\ 30{,}000$$

$\therefore$ **Sum borrowed = ₹ 40,000 + ₹ 30,000 = ₹ 70,000** **Ans.**

EXERCISE 3 (D)

1. The cost of a machine is supposed to depreciate each year by 12% of its value at the beginning of the year. If the machine is valued at ₹ 44,000 at the beginning of 2008, find its value :
   (i) at the end of 2009.
   (ii) at the beginning of 2007.
2. The value of an article decreased for two years at the rate of 10% per year and then in the third year it increased by 10%. Find the original value of the article, if its value at the end of 3 years is ₹ 40,095.
3. According to a census taken towards the end of the year 2009, the population of a rural town was found to be 64,000. The census authority also found that the population of this particular town had a growth of 5% per annum. In how many years after 2009 did the population of this town reach 74,088 ?
4. The population of a town decreased by 12% during 1998 and then increased by 8% during 1999. Find the population of the town, at the beginning of 1998, if at the end of 1999 its population was 2,85,120.
5. A sum of money, invested at compound interest, amounts to ₹ 16,500 in 1 year and to ₹ 19,965 in 3 years. Find the rate per cent and the original sum of money invested.
6. The difference between C.I. and S.I. on ₹ 7,500 for two years is ₹ 12 at the same rate of interest per annum. Find the rate of interest.
7. A sum of money lent out at C.I. at a certain rate per annum becomes three times of itself in 10 years. Find in how many years will the

money become twenty-seven times of itself at the same rate of interest p.a.

8. Mr. Sharma borrowed a certain sum of money at 10% per annum compounded annually. If by paying ₹ 19,360 at the end of the second year and ₹ 31,944 at the end of the third year he clears the debt; find the sum borrowed by him.
9. The difference between compound interest for a year payable half-yearly and simple interest on a certain sum of money lent out at 10% for a year is ₹ 15. Find the sum of money lent out. **[1998]**
10. The ages of Pramod and Rohit are 16 years and 18 years respectively. In what ratio must they invest money at 5% p.a. compounded yearly so that both get the same sum on attaining the age of 25 years ?

EXERCISE 3 (E)

1. Simple interest on a sum of money for 2 years at 4% is ₹ 450. Find compound interest on the same sum and at the same rate for 1 year, if the interest is reckoned half-yearly.
2. Find the compound interest to the nearest rupee on ₹ 10,800 for $2\frac{1}{2}$ years at 10% per annum.
3. The value of a machine, purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97,200, find :
   (i) its value after 2 years.
   (ii) its value when it was purchased.
4. Anuj and Rajesh each lent the same sum of money for 2 years at 8% simple interest and compound interest respectively. Rajesh received ₹ 64 more than Anuj. Find the money lent by each and interest received.
5. Calculate the sum of money on which the compound interest (payable annually) for 2 years be four times the simple interest on ₹ 4,715 for 5 years, both at the rate of 5 per cent per annum.
6. A sum of money was invested for 3 years, interest being compounded annually. The rates for successive years were 10%, 15% and 18% respectively. If the compound interest for the second year amounted to ₹ 4,950, find the sum invested.
7. A sum of money is invested at 10% per annum compounded half-yearly. If the difference of amounts at the end of 6 months and 12 months is ₹ 189, find the sum of money invested.
8. Rohit borrows ₹ 86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of two years.
9. The simple interest on a certain sum of money for 3 years at 5% per annum is ₹ 1,200. Find the amount due and the compound interest on this sum of money at the same rate and after 2 years, interest is reckoned annually.
10. Nikita invests ₹ 6,000 for two years at a certain rate of interest compounded annually. At the end of first year it amounts to ₹ 6,720. Calculate :
    (a) the rate of interest.
    (b) the amount at the end of the second year.