

2

Compound Interest [Without Using Formula]

UNIT 2 : Commercial Mathematics

2.1 INTRODUCTION

Sometimes, in need, we borrow money from a bank or some other agency doing financial business. In general, the money is borrowed for a specified period and has to be returned at the end of that period. At the end of the period, we pay the money borrowed **plus** some extra money for utilising the money of the lender.

The money borrowed is called the **principal**, the extra money paid for using lender's money is called **interest** and the total money, paid to the lender at the end of the specified period is called the **amount**.

$$\text{Amount} = \text{Principal} + \text{Interest } i.e. A = P + I$$

2.2 INTEREST (Simple Interest)

Interest is said to be simple, if it is calculated on the original principal throughout the loan period, irrespective of the length of the period for which it is borrowed.

1. We know, S. I. = $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$ *i.e.* $I = \frac{P \times R \times T}{100}$
2. When we say, **interest**, it always means **simple interest**.

2.3 COMPOUND INTEREST (C.I.)

Money is said to be lent at **compound interest**, when the interest, which has become due at the end of a certain fixed period (one year, half year, etc., as given), is not paid to the money lender, but is added to the sum lent. The amount thus obtained becomes the principal for the next period. This process is repeated until the amount for the last period is found.

The difference between the final amount and the original principal is the required **compound interest**.

$$\therefore \text{Compound Interest} = \text{Final Amount} - \text{Original Principal } i.e. C.I. = A - P$$

The difference between **simple interest** (S.I.) and **compound interest** (C.I.), is made clear by the table given on next page.

[Here, in the table, we have taken sum borrowed (principal)] = ₹ 1,000 at 10% per annum and for 3 years.]

At simple interest	At compound interest
<p><u>For 1st year</u></p> $P = ₹ 1,000$ $I = \frac{₹ 1,000 \times 10 \times 1}{100}$ $= ₹ 100 \text{ (S.I.)}$ <p>\therefore Amount = ₹ 1,000 + ₹ 100</p> $= ₹ 1,100$	<p>$P = ₹ 1,000$</p> $I = \frac{₹ 1,000 \times 10 \times 1}{100}$ $= ₹ 100 \text{ (C.I.)}$ <p>\therefore Amount = ₹ 1,000 + ₹ 100</p> $= ₹ 1,100$
For 1st year : C.I. = S.I.	
<p><u>For 2nd year</u></p> $P = ₹ 1,000$ $I = \frac{₹ 1,000 \times 10 \times 1}{100}$ $= ₹ 100 \text{ (S.I.)}$ <p>\therefore Amount = ₹ 1,100 + ₹ 100</p> $= ₹ 1,200$	<p>$P = ₹ 1,100$</p> $I = \frac{₹ 1,100 \times 10 \times 1}{100}$ $= ₹ 110 \text{ (C.I.)}$ <p>\therefore Amount = ₹ 1,100 + ₹ 110</p> $= ₹ 1,210$
For 2nd year : C.I. is more than the S.I.	
<p><u>For 3rd year</u></p> $P = ₹ 1,000$ $I = \frac{₹ 1,000 \times 10 \times 1}{100}$ $= ₹ 100 \text{ (S.I.)}$ <p>\therefore Amount = ₹ 1,200 + ₹ 100</p> $= ₹ 1,300$	<p>$P = ₹ 1,210$</p> $I = \frac{₹ 1,210 \times 10 \times 1}{100}$ $= ₹ 121 \text{ (C.I.)}$ <p>\therefore Amount = ₹ 1,210 + ₹ 121</p> $= ₹ 1,331$
Every year, C.I. increases but the S.I. remains the same.	

2.4 COMPOUND INTEREST AS A REPEATED SIMPLE INTEREST COMPUTATION WITH A GROWING PRINCIPAL

As shown in the table, given above, the principal for 1st year is ₹ 1,000 and interest (C.I.) on it is ₹ 100. The principal for 2nd year is ₹ 1,100 and interest (C.I.) on it is ₹ 110; whereas, the principal for 3rd year is ₹ 1,210 and the interest (C.I.) on it is ₹ 121.

It is observed that the compound interest is growing (increasing) every year which increases the principal for next year.

As shown in the table, given above, **compound interest in 3 years**

$$= \text{C.I. of 1st year} + \text{C.I. of 2nd year} + \text{C.I. of 3rd year}$$

$$= ₹ 100 + ₹ 110 + ₹ 121 = ₹ 331$$

Also, compound interest in 3 years.

$$= \text{Amount at the end of 3 years} - \text{Original sum (Principal for 1st year)}$$

$$= ₹ 1,331 - ₹ 1,000 = ₹ 331$$

- 1** ₹ 8,000 is lent at 5 percent compound interest per year for 2 years. Find the amount and the compound interest.

Solution :

For the first year :

$$\text{Principal (P)} = ₹ 8,000; \text{Rate (R)} = 5\%$$

and, $\text{Time (T)} = 1 \text{ year}$

$$\begin{aligned} \therefore \text{Interest} &= \frac{P \times R \times T}{100} \\ &= \frac{₹ 8,000 \times 5 \times 1}{100} = ₹ 400 \end{aligned}$$

$$\begin{aligned} \text{Amount} &= \text{Principal} + \text{Interest} \\ &= ₹ 8,000 + ₹ 400 = ₹ 8,400 \end{aligned}$$

According to the definition of the compound interest, the amount of the first year will work as principal for the next (second) year.

\therefore **For the second year :**

$$\text{Principal (P)} = ₹ 8,400; R = 5\% \text{ and } T = 1 \text{ year}$$

$$\text{Interest} = ₹ \frac{8,400 \times 5 \times 1}{100} = ₹ 420$$

$$\therefore I = \frac{P \times R \times T}{100}$$

$$\text{Amount at the end of 2nd year} = ₹ 8,400 + ₹ 420 = ₹ 8,820 \quad \text{Ans.}$$

$$\begin{aligned} \text{Compound Interest} &= \text{Final Amount} - \text{Initial Principal} \\ &= ₹ 8,820 - ₹ 8,000 = ₹ 820 \quad \text{Ans.} \end{aligned}$$

Also, **C.I. of 2 years** = C.I. of 1st year + C.I. of 2nd years
= ₹ 400 + ₹ 420 = ₹ 820 Ans.

- 2** Find the amount and the compound interest on ₹ 10,000 at 8 per cent per annum and in 1 year; interest being compounded half-yearly.

Solution :

For 1st $\frac{1}{2}$ year : $P = ₹ 10,000; R = 8\% \text{ and } T = \frac{1}{2} \text{ year}$

$$\therefore \text{Interest, } I = ₹ \frac{10,000 \times 8 \times 1}{100 \times 2} = ₹ 400$$

$$\therefore I = \frac{P \times R \times T}{100}$$

And, $A = P + I$
 $= ₹ 10,000 + ₹ 400 = ₹ 10,400$

For 2nd $\frac{1}{2}$ year : $P = ₹ 10,400; R = 8\% \text{ and } T = \frac{1}{2} \text{ year.}$

$$\therefore I = ₹ \frac{10,400 \times 8 \times 1}{100 \times 2} = ₹ 416$$

$$\begin{aligned} \text{And,} \quad A &= P + I \\ &= ₹ 10,400 + ₹ 416 = ₹ 10,816 \end{aligned}$$

$$\therefore \text{Required amount} = ₹ 10,816 \quad \text{Ans.}$$

$$\begin{aligned} \text{And,} \quad \text{compound interest} &= A - P \\ &= ₹ 10,816 - ₹ 10,000 = ₹ 816 \quad \text{Ans.} \end{aligned}$$

It is clear from examples, given above, that :

1. When the interest is compounded yearly, the principal changes (increases) every year.
2. When the interest is compounded half-yearly, the principal increases every six months.
3. The period (time), after which the principal changes, is called the **conversion period**.

In example 1, given above, the **conversion period** is **one year**.

And, in example 2, given above, the **conversion period** is **half-year**.

- 3** Calculate the compound interest accrued on ₹ 16,000 in 3 years, when the rates of interest for successive years are 10%, 12% and 15% respectively.

Solution :

$$\text{For 1st year : } P = ₹ 16,000; R = 10\% \text{ and } T = 1 \text{ year}$$

$$\therefore \text{Interest} = ₹ \frac{16,000 \times 10 \times 1}{100} = ₹ 1,600$$

$$\text{And,} \quad \text{amount} = ₹ 16,000 + ₹ 1,600 = ₹ 17,600$$

$$\text{For 2nd year : } P = ₹ 17,600; R = 12\% \text{ and } T = 1 \text{ year}$$

$$\therefore \text{Interest} = ₹ \frac{17,600 \times 12 \times 1}{100} = ₹ 2,112$$

$$\text{And,} \quad \text{amount} = ₹ 17,600 + ₹ 2,112 = ₹ 19,712$$

$$\text{For 3rd year : } P = ₹ 19,712; R = 15\% \text{ and } T = 1 \text{ year}$$

$$\therefore \text{Interest} = ₹ \frac{19,712 \times 15 \times 1}{100} = ₹ 2,956.80$$

$$\text{And,} \quad \text{amount} = ₹ 19,712 + ₹ 2,956.80 = ₹ 22,668.80$$

$$\begin{aligned} \therefore \text{C.I. accrued} &= \text{Final amount} - \text{Initial principal} \\ &= ₹ 22,668.80 - ₹ 16,000 = ₹ 6,668.80 \end{aligned} \quad \text{Ans.}$$

- 4** Calculate the compound interest due in $2\frac{1}{2}$ years on ₹ 6,000 at 10 percent compounded annually.

Solution :

$$\text{For the 1st year : } P = ₹ 6,000, R = 10\% \text{ and } T = 1 \text{ year}$$

$$\therefore I = ₹ \frac{6,000 \times 10 \times 1}{100} = ₹ 600$$

$$\therefore I = \frac{P \times R \times T}{100}$$

and amount, $A = P + I = ₹ (6,000 + 600) = ₹ 6,600$ **For the 2nd year :** $P = ₹ 6,600, R = 10\%$ and $T = 1$ year

$$\therefore I = ₹ \frac{6,600 \times 10 \times 1}{100} = ₹ 660$$

and amount, $A = ₹ 6,600 + ₹ 660 = ₹ 7,260$ **For the last $\frac{1}{2}$ year :** $P = ₹ 7,260, R = 10\%$ and $T = \frac{1}{2}$ year

$$\therefore I = ₹ \frac{7,260 \times 10 \times 1}{100 \times 2} = ₹ 363$$

and amount, $A = ₹ 7,260 + ₹ 363 = ₹ 7,623$ Since, the amount in $2\frac{1}{2}$ years is ₹ 7,623 and the original principal is ₹ 6,000

$$\therefore \text{Compound Interest} = ₹ 7,623 - ₹ 6,000 = ₹ 1,623$$

Ans.**EXERCISE 2 (A)**

- Calculate the amount and the compound interest on :
 - ₹ 3,500 at 10% per annum in 2 years
 - ₹ 6,000 in 3 years at 5% per year.
- Calculate the amount and the compound interest on :
 - ₹ 8,000 in $2\frac{1}{2}$ years at 15% per annum.
 - ₹ 20,000 in $2\frac{1}{4}$ years at 10% per annum.
- Calculate the amount and the compound interest on :
 - ₹ 4,600 in 2 years when the rates of interest of successive years are 10% and 12% respectively.
 - ₹ 16,000 in 3 years, when the rates of the interest for successive years are 10%, 14% and 15% respectively.
- Find the compound interest, correct to the nearest rupee, on ₹ 2,400 for $2\frac{1}{2}$ years at 5 percent per annum.
- Calculate the compound interest for the second year on ₹ 8,000/- invested for 3 years at 10% per annum.
- A borrowed ₹ 2,500 from B at 12% per annum compound interest. After 2 years, A gave ₹ 2,936 and a watch to B to clear the account. Find the cost of the watch.
- How much will ₹ 50,000 amount to in 3 years, compounded yearly, if the rates for the successive years are 6%, 8% and 10% respectively.
- Meenal lends ₹ 75,000 at C.I. for 3 years. If the rate of interest for the first two years is 15% per year and for the third year it is 16%, calculate the sum Meenal will get at the end of the third year.
- Govind borrows ₹ 18,000 at 10% simple interest. He immediately invests the money borrowed at 10% compound interest compounded half-yearly. How much money does Govind gain in one year ?
- Find the compound interest on ₹ 4,000 accrued in three years, when the rate of interest is 8% for the first year and 10% per year for the second and the third years.

5 Calculate the difference between the compound interest and the simple interest on ₹ 4,000 at 8 per cent per annum and in 2 years.

Solution :

For S.I. : P = ₹ 4,000; R = 8% and T = 2 years

$$\therefore \text{Simple interest} = ₹ \frac{4,000 \times 8 \times 2}{100} = ₹ 640$$

For C.I. : Principal for 1st year = ₹ 4,000

$$\Rightarrow \text{Interest on it} = ₹ \frac{4,000 \times 8 \times 1}{100} = ₹ 320$$

$$\text{Amount} = ₹ 4,000 + ₹ 320 = ₹ 4,320$$

\therefore Principal for 2nd year = ₹ 4,320

$$\Rightarrow \text{Interest on it} = ₹ \frac{4,320 \times 8 \times 1}{100} = ₹ 345.60$$

$$\therefore \text{C.I. of 2 years} = ₹ 320 + ₹ 345.60 \\ = ₹ 665.60$$

$$\Rightarrow \text{Required difference between C.I. and S.I.} = \text{C.I.} - \text{S.I.} \\ = ₹ 665.60 - ₹ 640 \\ = ₹ 25.60$$

Ans.

6 Mrs. Kapoor invested ₹ 6,000 every year at the beginning of the year, at 10% per annum compound interest. Calculate the amount of her total savings :

(i) upto the end of the second year. (ii) at the beginning of the third year.

Solution :

(i) **For 1st year :**

Since, money invested at the beginning of the year = ₹ 6,000

$$\Rightarrow \text{Principal for 1st year} = ₹ 6,000$$

$$\therefore \text{Interest} = ₹ \frac{6,000 \times 10 \times 1}{100} = ₹ 600$$

$$\text{And, amount} = ₹ 6,000 + ₹ 600 = ₹ 6,600$$

For 2nd year :

Since, ₹ 6,000 is invested again at the beginning of the second year, therefore, for the second year, principal = ₹ 6,600 + ₹ 6,000 = ₹ 12,600

$$\text{Interest} = ₹ \frac{12,600 \times 10 \times 1}{100} = ₹ 1,260$$

$$\text{And, amount} = ₹ 12,600 + ₹ 1,260 = ₹ 13,860$$

$$\therefore \text{Amount of her total savings upto the end of the second year} \\ = ₹ 13,860$$

Ans.

(ii) Since, ₹ 6,000 is invested again at the beginning of the third year,

$$\therefore \text{Amount of her total savings at the beginning of the third year} \\ = ₹ 13,860 + ₹ 6,000 = ₹ 19,860$$

Ans.

7 Ranbir borrows ₹ 20,000 at 12 per cent C.I. If he repays ₹ 8,400 at the end of first year and ₹ 9,680 at the end of second year, find the amount of loan outstanding at the beginning of the third year.

Solution :

For 1st year : $P = ₹ 20,000$; $R = 12\%$ and $T = 1$ year

$$\Rightarrow \text{Interest} = ₹ \frac{20,000 \times 12 \times 1}{100} = ₹ 2,400$$

And, amount = ₹ 20,000 + ₹ 2,400 = ₹ 22,400

Since, the man pays ₹ 8,400 at the end of 1st year

$$\therefore \text{Principal for 2nd year} = ₹ 22,400 - ₹ 8,400 = ₹ 14,000$$

For 2nd year : $P = ₹ 14,000$; $R = 12\%$ and $T = 1$ year

$$\therefore \text{Interest} = ₹ \frac{14,000 \times 12 \times 1}{100} = ₹ 1,680$$

And, amount = ₹ 14,000 + ₹ 1,680 = ₹ 15,680

Since, the man pays ₹ 9,680 at the end of 2nd year

$$\therefore \text{Principal for 3rd year} = ₹ 15,680 - ₹ 9,680 = ₹ 6,000$$

\Rightarrow **The amount of loan outstanding at the beginning of the 3rd year**
= ₹ 6,000

Ans.

8 A man borrows ₹ 8,000 at 10% compound interest payable every six months. He repays ₹ 2,500 at the end of every six months. Calculate the third payment he has to make at the end of 18 months in order to clear the entire loan.

Solution :

For 1st six months : $P = ₹ 8,000$; $R = 10\%$ and $T = \frac{1}{2}$ year

$$\therefore \text{Interest} = ₹ \frac{8,000 \times 10 \times 1}{100 \times 2} = ₹ 400$$

And, amount = ₹ 8,000 + ₹ 400 = ₹ 8,400

\therefore Money repaid = ₹ 2,500

\therefore Balance = ₹ 8,400 - ₹ 2,500 = ₹ 5,900

For the 2nd six months : $P = ₹ 5,900$; $R = 10\%$ and $T = \frac{1}{2}$ year

$$\therefore \text{Interest} = ₹ \frac{5,900 \times 10 \times 1}{100 \times 2} = ₹ 295$$

And, amount = ₹ 5,900 + ₹ 295 = ₹ 6,195

Again, money repaid = ₹ 2,500

\Rightarrow Balance = ₹ 6,195 - ₹ 2,500 = ₹ 3,695

For the 3rd six months : $P = ₹ 3,695$; $R = 10\%$ and $T = \frac{1}{2}$ year

$$\therefore \text{Interest} = ₹ \frac{3,695 \times 10 \times 1}{100 \times 2} = ₹ 184.75$$

And, amount = ₹ 3,695 + ₹ 184.75 = ₹ 3,879.75

\therefore The 3rd instalment to be made to clear the entire loan = ₹ 3,879.75 Ans.

9 On a certain sum of money, invested at the rate of 5% per annum compounded annually, the difference between the interest of the first year and the interest of the third year is ₹ 61.50. Find the sum.

Solution :

Let the sum (principal) = ₹ 100

$$\text{C.I. of 1st year} = ₹ \frac{100 \times 5 \times 1}{100} = ₹ 5$$

And, amount of 1st year = ₹ 100 + ₹ 5 = ₹ 105

\Rightarrow The principal for 2nd year = ₹ 105

$$\text{C.I. of 2nd year} = ₹ \frac{105 \times 5 \times 1}{100} = ₹ 5.25$$

And, amount of 2nd year = ₹ 105 + ₹ 5.25 = ₹ 110.25

\Rightarrow The principal for 3rd year = ₹ 110.25

$$\text{C.I. of 3rd year} = ₹ \frac{110.25 \times 5 \times 1}{100} = ₹ 5.5125$$

Difference between C.I. of 1st year and C.I. of 3rd year

$$= ₹ 5.5125 - ₹ 5 = ₹ 0.5125$$

Now, when the difference of interest = ₹ 0.5125, sum = ₹ 100

And, when the difference of interest = ₹ 61.50, **sum** = ₹ $\frac{100}{0.5125} \times 61.50$
= ₹ 12,000 Ans.

10 During every financial year, the value of a machine depreciates by 10%. Find the original value (cost) of a machine which depreciates by ₹ 2,250 during the second year.

Solution :

Let the original cost of the machine = ₹ 100

\therefore Depreciation during 1st year = 10% of ₹ 100 = ₹ 10

Value of the machine at the beginning of 2nd year

$$= ₹ 100 - ₹ 10 = ₹ 90$$

\therefore Depreciation during 2nd year = 10% of ₹ 90 = ₹ 9

Now, when depreciation during 2nd year = ₹ 9, original cost = ₹ 100

\Rightarrow when depreciation during 2nd year = ₹ 2,250,

$$\text{original cost} = ₹ \frac{100}{9} \times 2,250 = ₹ 25,000 \quad \text{Ans.}$$

11 A man invests ₹ 46,875 at 4% per annum compound interest for 3 years. Calculate :

- (i) the interest for the 1st year;
 (ii) the amount standing to his credit at the end of the 2nd year;
 (iii) the interest for the 3rd year.

Solution :

(i) **For 1st year :** $P = ₹ 46,875$, $R = 4\%$ and $T = 1$ year

$$\Rightarrow \text{Interest (I)} = ₹ \frac{46,875 \times 4 \times 1}{100} = ₹ 1,875$$

Ans.

(ii) **For 2nd year :** $P = ₹ 46,875 + ₹ 1,875 = ₹ 48,750$

$$\Rightarrow I = ₹ \frac{48,750 \times 4 \times 1}{100} = ₹ 1,950$$

∴ Amount standing to his credit at the end of the second year

$$= ₹ 48,750 + ₹ 1,950 = ₹ 50,700$$

Ans.

(iii) **For 3rd year :** $P = ₹ 50,700$, $R = 4\%$ and $T = 1$ year

$$\therefore \text{Interest} = ₹ \frac{50,700 \times 4 \times 1}{100} = ₹ 2,028$$

Ans.

12 Find the sum invested at 10% compounded annually, on which the interest for the first year plus the interest for the third year amount to ₹ 1,768.

Solution :

Let the sum (principal) = ₹ 100

$$\text{C.I. of 1st year} = ₹ \frac{100 \times 10 \times 1}{100} = ₹ 10$$

And, amount of 1st year = ₹ 100 + ₹ 10 = ₹ 110

⇒ The principal for 2nd year = ₹ 110

$$\text{C.I. of 2nd year} = ₹ \frac{110 \times 10 \times 1}{100} = ₹ 11$$

And, amount of 2nd year = ₹ (110 + 11) = ₹ 121

⇒ The principal for 3rd year = ₹ 121

$$\text{C.I. of 3rd year} = ₹ \frac{121 \times 10 \times 1}{100} = ₹ 12.10$$

Sum of the C.I. of 1st year and C.I. of 3rd year

$$= ₹ 10 + ₹ 12.10 = ₹ 22.10$$

Now, when sum of two interests = ₹ 22.10, principal (sum) = ₹ 100

$$\begin{aligned} \text{And, when sum of two interests} &= ₹ 1,768, \text{ sum} = ₹ \frac{100}{22.10} \times 1,768 \\ &= ₹ 8,000 \end{aligned}$$

Ans.

EXERCISE 2 (B)

- Calculate the difference between the simple interest and the compound interest on ₹ 4,000 in 2 years at 8% per annum compounded yearly.
- A man lends ₹ 12,500 at 12% for the first year, at 15% for the second year and at 18% for the third year. If the rates of interest are compounded yearly; find the difference between the C.I. of the first year and the compound interest for the third year.
- A sum of money is lent at 8% per annum compound interest. If the interest for the second year exceeds that for the first year by ₹ 96, find the sum of money.
- A man borrows ₹ 6,000 at 5 percent C.I. per annum. If he repays ₹ 1,200 at the end of each year, find the amount of the loan outstanding at the beginning of the third year.
- A man borrows ₹ 5,000 at 12 percent compound interest payable every six months. He repays ₹ 1,800 at the end of every six months. Calculate the third payment he has to make at the end of 18 months in order to clear the entire loan.
- On a certain sum of money, the difference between the compound interest for a year, payable half-yearly, and the simple interest for a year is ₹ 180/-. Find the sum lent out, if the rate of interest in both the cases is 10% per annum.
- A manufacturer estimates that his machine depreciates by 15% of its value at the beginning of the year. Find the original value (cost) of the machine, if it depreciates by ₹ 5,355 during the second year.
- A man invests ₹ 5,600 at 14% per annum compound interest for 2 years. Calculate :
 - the interest for the first year.
 - the amount at the end of the first year.
 - the interest for the second year, correct to the nearest rupee.
- Find the difference between the compound interest of second year and the compound interest of third year on ₹ 48,000 invested for 5 years at 10% per annum compounded yearly.
 - A sum of ₹ 50,000 is invested for 8 years at compound interest, the rate of interest being 10%, 12%, 14% and 16% respectively for the first 4 consecutive years. Find the total of interests earned during the first and third years.
- A man saves ₹ 3,000 every year and invests it at the end of the year at 10% compound interest. Calculate the total amount of his savings at the end of the third year.
- A man borrows ₹ 10,000 at 5% per annum compound interest. He repays 35% of the sum borrowed at the end of the first year and 42% of the sum borrowed at the end of the second year. How much must he pay at the end of the third year in order to clear the debt ?
- Mr. Mehta invested ₹ 8,000 every year at the beginning of the year, at 10% per annum compound interest. Calculate his total savings at the beginning of the third year.

2.5 MORE ABOUT COMPOUND INTEREST

1. On the same sum and at the same rate of interest compounded yearly :

- C.I. of 2nd year is always more than C.I. of 1st year.
- C.I. of 3rd year is more than C.I. of 2nd year and so on.

In the same way, if the interest is compounded half-yearly :

- C.I. of 2nd half-year is more than C.I. of 1st half-year.
- C.I. of 3rd half-year is more than C.I. of 2nd half-year and so on.

In general for any period, the C.I. is more than the C.I. of the previous period.

2. The difference between the compound interests for any two consecutive conversion periods (year or half-year) is the interest of one period on the C.I. of the preceding conversion period.

For example, if ₹ 700 and ₹ 750 are the C.I. for any two consecutive years; then their difference ₹ 750 – ₹ 700 = ₹ 50 is the interest of one year on ₹ 700. And, if ₹ 700 and ₹ 750 are the C.I. for two consecutive half years, then their difference (₹ 50) is the interest of half year on ₹ 700.

Similarly; the difference between the amounts for any two consecutive conversion periods is also the interest of one period on the amount of the preceding period.

For example, if ₹ 1,500 and ₹ 1,600 are the amounts for any two consecutive conversion periods, then their difference ₹ 1,600 – ₹ 1,500 = ₹ 100 is the interest on ₹ 1,500 for one conversion period.

- 13** A sum of money is invested at C.I. payable annually. The amounts of interest in two successive years are ₹ 2,700 and ₹ 2,880. Find the rate of interest.

Solution :

∴ Difference between the C.I. of two successive years

$$= ₹ 2,880 - ₹ 2,700 = ₹ 180$$

⇒ ₹ 180 is the interest of one year on ₹ 2,700.

$$\therefore \text{Rate of interest} = \frac{100 \times I}{P \times T} \% = \frac{100 \times 180}{2,700 \times 1} \% = 6\frac{2}{3}\% \quad \text{Ans.}$$

Directly :

$$\begin{aligned} \text{Rate of interest} &= \frac{\text{Difference in interest of two consecutive periods} \times 100}{\text{C.I. of preceding year} \times \text{Time}} \% \\ &= \frac{(2,880 - 2,700) \times 100}{2,700 \times 1} \% = 6\frac{2}{3}\% \quad \text{Ans.} \end{aligned}$$

- 14** A certain sum of money, placed out at compound interest, amounts to ₹ 6,272 in 2 years and to ₹ 7,024.64 in 3 years. Find the rate of interest and the sum of money.

Solution :

Difference between the amounts of two consecutive years

$$= ₹ 7,024.64 - ₹ 6,272 = ₹ 752.64$$

⇒ Interest for one year on ₹ 6,272 = ₹ 752.64

$$\therefore \text{Rate of interest} = \frac{752.64}{6,272} \times 100\% = 12\% \quad \text{Ans.}$$

Directly :

$$\begin{aligned} \text{Rate of interest} &= \frac{\text{Difference between the amounts of two consecutive periods} \times 100}{\text{Preceding amount} \times \text{Time}} \% \\ &= \frac{(7,024.64 - 6,272) \times 100}{6,272 \times 1} \% = 12\% \quad \text{Ans.} \end{aligned}$$

Let the sum of money = ₹ 100

∴ Interest on it for 1st year = 12% of ₹ 100 = ₹ 12

⇒ Amount in one year = ₹ 100 + ₹ 12 = ₹ 112

Similarly, amount in two years = ₹ 112 + 12% of ₹ 112 = ₹ 125.44

When amount in two years = ₹ 125.44, sum = ₹ 100

⇒ When amount in two years = ₹ 6,272, **sum** = ₹ $\frac{100}{125.44} \times 6,272$
= ₹ 5,000 **Ans.**

15 A person invests ₹ 10,000 for three years at a certain rate of interest compounded annually. At the end of one year this sum amounts to ₹ 11,200. Calculate :

- (i) the rate of interest per annum.
- (ii) the amount at the end of the second year.
- (iii) the amount at the end of the third year.

Solution :

(i) **For the first year :**

Principal = ₹ 10,000 and amount = ₹ 11,200

∴ $I = A - P = ₹ 11,200 - ₹ 10,000 = ₹ 1,200$

$$\text{Rate} = \frac{I \times 100}{P \times T} \%$$

⇒ **Rate of interest p.a.** = $\frac{1,200 \times 100}{10,000 \times 1} \% = 12\%$ **Ans.**

(ii) **For the second year :**

P = ₹ 11,200; R = 12% and T = 1 year

$$I = \frac{P \times R \times T}{100} = \frac{₹ 11,200 \times 12 \times 1}{100} = ₹ 1,344$$

∴ **Amount** = P + I = ₹ 11,200 + ₹ 1,344 = ₹ 12,544 **Ans.**

(iii) **For the third year :**

Principal = ₹ 12,544; R = 12% and T = 1 year

$$\text{Interest} = \frac{₹ 12,544 \times 12 \times 1}{100} = ₹ 1,505.28$$

Amount = P + I = ₹ 12,544 + ₹ 1,505.28 = ₹ 14,049.28 **Ans.**

2.6 RELATION BETWEEN SIMPLE INTEREST (S.I.) AND COMPOUND INTEREST (C.I.)

1. S.I. on a certain sum and at a certain fixed interest rate is the same every year.

e.g. (i) If S.I. on a certain sum is ₹ 500 for the 1st year; then for every year the S.I. on that sum will be ₹ 500 only, provided the rate of interest is also the same every year.

- (ii) If S.I. on a certain sum is ₹ 1,200 in 4 years; the S.I. on that sum for one year will be ₹ $\frac{1,200}{4} = ₹ 300$ and so on.

2. S.I. and C.I. are same for the first year on the same sum and at the same rate percent.

16 The simple interest on a certain sum computes to ₹ 600 in 3 years and the compound interest on the same sum, at the same rate and for 2 years computes to ₹ 410. Find the rate per cent.

Solution :

Since, S.I. of 3 years = ₹ 600

$$\therefore \text{S.I. of 1 year} = ₹ \frac{600}{3} = ₹ 200$$

S.I. is same every year

\therefore C.I. for first year = ₹ 200

For 1st year : C.I. = S.I.

Given, C.I. for two years = ₹ 410

$$\therefore \text{C.I. for 2nd year} = ₹ 410 - ₹ 200 = ₹ 210.$$

Difference between the C.I. of two successive years = ₹ 210 - ₹ 200 = ₹ 10

\Rightarrow ₹ 10 is the interest for one year on the interest of 1st year *i.e.* on ₹ 200.

$$\therefore \text{Rate \%} = \frac{100 \times I}{P \times T} \% = \frac{100 \times 10}{200 \times 1} \% = 5\%$$

Ans.

Important Results :

For any two consecutive conversion periods (years or half-years as given) :

- If the C.I. of 1st period is ₹ x ; then the C.I. for the next period on the same sum and at the same rate = ₹ $x + \text{Interest for one period on ₹ } x$.
- In the same way*, if the amount at C.I. in a particular period is ₹ x ; then the amount for the next period, on the same sum and at the same rate
= ₹ $x + \text{Interest on ₹ } x \text{ for one period.}$

17 The compound interest calculated yearly at 10% on a certain sum of money amounts to ₹ 665.50 in the fifth year. Calculate :

- C.I. for the sixth year at the same rate and on the same sum.
- C.I. for the fourth year on the same sum and at the same rate.

Solution :

(i) **C.I. for 6th year** = C.I. of 5th year + Interest on it for 1 year
= ₹ 665.50 + 10% of ₹ 665.50 = ₹ 732.05

Ans.

(ii) Let C.I. for 4th year = ₹ x

Since, C.I. for 5th year = C.I. of 4th year + Interest on it for 1 year

$$\Rightarrow ₹ 665.50 = ₹ x + 10\% \text{ of ₹ } x$$

$$\Rightarrow x = 605$$

$$\Rightarrow \text{C.I. for 4th year} = ₹ 605$$

Ans.

18 A sum of money, at compound interest, amounts to ₹ 8,100 in 5 years and to ₹ 8,748 in 6 years. Find :

- (i) the rate per cent (ii) amount in 7 years and (iii) amount in 4 years.

Solution :

(i) \therefore Amount in 5 years = ₹ 8,100 and amount in 6 years = ₹ 8,748

\therefore ₹ 8,748 – ₹ 8,100 = ₹ 648 is the interest of 1 year on ₹ 8,100

$$\therefore \text{Rate \%} = \frac{648 \times 100}{8100 \times 1} \% = 8\% \quad \text{Ans.}$$

(ii) **Amount in 7 years** = Amount in 6 years + Int. on it for 1 year
= ₹ 8,748 + 8% of ₹ 8,748 = **₹ 9,447.84** Ans.

(iii) Let amount in 4 years = ₹ x

\Rightarrow Amount in 5 years = Amount in 4 years + Int. on it for 1 year

\Rightarrow ₹ 8,100 = ₹ x + 8% of ₹ x

On solving, we get $x = 7500 \Rightarrow$ **Amount in 4 years = ₹ 7,500** Ans.

19 A sum of ₹ 9,600 is invested for 3 years at 10% per annum at compound interest.

- (i) What is the sum due at the end of the first year ?
 (ii) What is the sum due at the end of the second year ?
 (iii) Find the difference between the answers in (ii) and (i) and find the interest on this sum (difference) for one year.
 (iv) Hence, write down the compound interest for the third year.

Solution :

(i) Interest for the 1st year = ₹ $\frac{9,600 \times 10 \times 1}{100}$ = ₹ 960

\Rightarrow **The sum due at the end of the 1st year** = ₹ 9,600 + ₹ 960
= **₹ 10,560** Ans.

(ii) Interest for the 2nd year = ₹ $\frac{10,560 \times 10 \times 1}{100}$ = ₹ 1,056

\Rightarrow **The sum due at the end of the 2nd year** = ₹ 10,560 + ₹ 1,056
= **₹ 11,616** Ans.

(iii) **Required difference** = ₹ 11,616 – ₹ 10,560 = **₹ 1,056**
[₹ 1,056 is C.I. for the 2nd year]

\therefore **Interest for one year on this difference** = ₹ $\frac{1,056 \times 10 \times 1}{100}$ = **₹ 105.60** Ans.

(iv) **C.I. for the 3rd year** = C.I. of the 2nd year + Int. on it for 1 yr.
= ₹ 1,056 + ₹ 105.60
= **₹ 1,161.60** Ans.

EXERCISE 2 (C)

- A sum is invested at compound interest compounded yearly. If the interest for two successive years be ₹ 5,700 and ₹ 7,410, calculate the rate of interest.
- A certain sum of money is put at compound interest, compounded half-yearly. If the interest for two successive half-years are ₹ 650 and ₹ 760.50; find the rate of interest.
- A certain sum amounts to ₹ 5,292 in two years and ₹ 5,556.60 in three years, interest being compounded annually. Find:
 - the rate of interest
 - the original sum.
- The compound interest, calculated yearly, on a certain sum of money for the second year is ₹ 1,089 and for the third year it is ₹ 1,197.90. Calculate the rate of interest and the sum of money.
- Mohit invests ₹ 8,000 for 3 years at a certain rate of interest, compounded annually. At the end of one year it amounts to ₹ 9,440. Calculate :
 - the rate of interest per annum.
 - the amount at the end of the second year.
 - the interest accrued in the third year.
- Geeta borrowed ₹ 15,000 for 18 months at a certain rate of interest compounded semi-annually. If at the end of six months it amounted to ₹ 15,600; calculate :
 - the rate of interest per annum.
 - the total amount of money that Geeta must pay at the end of 18 months in order to clear the account.
- Ramesh invests ₹ 12,800 for three years at the rate of 10% per annum compound interest. Find:
 - the sum due to Ramesh at the end of the first year.
 - the interest he earns for the second year.
 - the total amount due to him at the end of the third year.
- The compound interest, calculated yearly, on a certain sum of money for the second year is ₹ 864 and for the third year is ₹ 933.12. Calculate the rate of interest and the compound interest on the same sum and at the same rate, for the fourth year.
- A sum of money placed out at compound interest amounts to ₹ 20,160 in 3 years and to ₹ 24,192 in 4 years. Calculate :
 - the rate of interest.
 - amount in 2 years and
 - amount in 5 years.
- ₹ 8,000 is lent out at 7% compound interest for 2 years. At the end of the first year ₹ 3,560 are returned. Calculate :
 - the interest paid for the second year.
 - the total interest paid in two years
 - the total amount of money paid in two years to clear the debt.
- The cost of a machine depreciated by ₹ 4,000 during the first year and by ₹ 3,600 during the second year. Calculate :
 - the rate of depreciation.
 - the original cost of the machine.
 - its cost at the end of the third year.
- The cost of a machine is ₹ 32,000. Its value depreciates at the rate of 5% every year. Find the total depreciation in its value by the end of 2 years.
- Find the sum, invested at 10% compounded annually, on which the interest for the third year exceeds the interest of the first year by ₹ 252.
- A man borrows ₹ 10,000 at 10% compound interest compounded yearly. At the end of each year, he pays back 30% of the sum borrowed. How much money is left unpaid just after the second year ?
- A man borrows ₹ 10,000 at 10% compound interest compounded yearly. At the end of each year, he pays back 20% of the amount for that year. How much money is left unpaid just after the second year ?

EXERCISE 2 (D)

- What sum will amount to ₹ 6,593.40 in 2 years at C.I., if the rates are 10 percent and 11 percent for the two successive years ?
- The value of a machine depreciated by 10% per year during the first two years and 15% per year during the third year. Express the total depreciation of the machine, as percent, during the three years
- Rachna borrows ₹ 12,000 at 10 per cent per annum interest compounded half-yearly. She

repays ₹ 4,000 at the end of every six months. Calculate the third payment she has to make at the end of 18 months in order to clear the entire loan.

4. On a certain sum of money, invested at the rate of 10 percent per annum compounded annually, the interest for the first year plus the interest for the third year is ₹ 2,652. Find the sum.
5. During every financial year, the value of a machine depreciates by 12%. Find the original cost of a machine which depreciates by ₹ 2,640 during the second financial year of its purchase.
6. Find the sum on which the difference between the simple interest and the compound interest at the rate of 8% per annum compounded annually be ₹ 64 in 2 years.
7. A sum of ₹ 13,500 is invested at 16% per annum compound interest for 5 years. Calculate :
 - (i) the interest for the first year.
 - (ii) the amount at the end of the first year.
 - (iii) the interest for the second year, correct to the nearest rupee.
8. Saurabh invests ₹ 48,000 for 7 years at 10% per annum compound interest. Calculate :
 - (i) the interest for the first year.
 - (ii) the amount at the end of the second year.
 - (iii) the interest for the third year.
9. Ashok borrowed ₹ 12,000 at some rate per cent compound interest. After a year, he paid back ₹ 4,000. If compound interest for the second year be ₹ 920, find :
 - (i) the rate of interest charged
 - (ii) the amount of debt at the end of the second year.
10. On a certain sum of money, lent out at C.I., interests for first, second and third years are ₹ 1,500; ₹ 1,725 and ₹ 2,070 respectively. Find the rate of interest for the (i) second year (ii) third year.