1 Rational and Irrational Numbers

1.1 INTRODUCTION

The complete number system is divided into two types of numbers :

1. Imaginary numbers **2.** Real numbers

For example :

1. If $x = 4$, $\sqrt{-x}$ *i.e.* $\sqrt{-4}$ is an imaginary number and $\sqrt{x} = \sqrt{4} = 2$ is a real number.
2. $\sqrt{-5}$ is imaginary but $\sqrt{5}$ is real and so on.

Thus, square root of every negative number is an imaginary number and if the number is not imaginary, it is a real number.

In this chapter, we confine our studies only upto real numbers.

Starting from real numbers, the complete number system is as shown below :

Real Numbers (R)
- Rational Numbers (Q) $\left\{-8, 0, 6, \frac{5}{8}, \frac{2}{9}, \text{etc.}\right\}$
  - Integers (I or Z) {...., –2, –1, 0, 1, 2, ...}
    - Negative Integers {...., –3, –2, –1}
    - Zero {0}
    - Positive Integers (Natural Numbers, N) {1, 2, 3, 4, ...}
    - Zero and Positive Integers → Whole-Numbers (W) {0, 1, 2, 3, 4, 5, etc.}
  - Non-Integral Rationals $\left\{-\frac{5}{8}, \frac{8}{15}, 2\frac{3}{7}, \text{etc.}\right\}$
- Irrational Numbers ($\overline{Q}$) $\left\{\sqrt{5}, \sqrt{3}, -\sqrt{8}, 3\sqrt{2}, 2-\sqrt{7}, \text{etc.}\right\}$

1.2 RATIONAL NUMBERS (Q)

A number which can be expressed as $\frac{a}{b}$, where 'a' and 'b' both are integers and 'b' is not equal to zero, is called a **rational number.**

In general, the set of rational numbers is denoted by the letter Q.

$$\therefore Q = \{\frac{a}{b} : a, b \in Z \text{ and } b \neq 0\}$$

1. $\frac{a}{b}$ is a rational number

$\Rightarrow$ (i) $b \neq 0$

(ii) a and b have no common factor other than 1 (one) *i.e.* a and b are **co-primes**.

(iii) b is usually positive, whereas a may be positive, negative or zero.

2. Every integer (positive, negative or zero) and every decimal number is a rational number.

3. *Corresponding to every rational number $\frac{a}{b}$, its negative rational number is $\frac{-a}{b}$.*

Also, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$ *e.g.* $\frac{-3}{5} = \frac{3}{-5} = -\frac{3}{5}$ and so on.

4. *Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal, if and only if : $a \times d = b \times c$.*

i.e. $\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \times d = b \times c$

Also, $\frac{a}{b} > \frac{c}{d} \Leftrightarrow a \times d > b \times c$ *and* $\frac{a}{b} < \frac{c}{d} \Leftrightarrow a \times d < b \times c$.

5. *For any two rational numbers a and b, $\frac{a+b}{2}$ is also a rational number which lies between a and b. Thus :*

if $a > b \Rightarrow a > \frac{a+b}{2} > b$ *and* *if* $a < b \Rightarrow a < \frac{a+b}{2} < b$.

1 Insert three rational numbers between 3 and 5.

Solution :

Since, $3 < 5 \Rightarrow 3 < \frac{3+5}{2} < 5.$ [Inserting one rational number between 3 and 5]

$\Rightarrow 3 < 4 < 5$

$\Rightarrow 3 < \frac{3+4}{2} < 4 < \frac{4+5}{2} < 5$

$\Rightarrow 3 < \frac{7}{2} < 4 < \frac{9}{2} < 5 \Rightarrow 3 < 3\frac{1}{2} < 4 < 4\frac{1}{2} < 5$

$\therefore$ **$3\frac{1}{2}$, 4 and $4\frac{1}{2}$ are three rational numbers between 3 and 5.** **Ans.**

1. There are infinitely many rational numbers between each pair of rational numbers.

2. For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a+c}{b+d}$ is also a rational number with its value lying between $\frac{a}{b}$ and $\frac{c}{d}$.

For example 1, given above :

Consider the rational numbers 3 and 5

i.e. $\frac{3}{1}$ and $\frac{5}{1}$ where $\frac{3}{1} < \frac{5}{1}$

$$\Rightarrow \quad \frac{3}{1} < \frac{3+5}{1+1} < \frac{5}{1} \quad i.e. \quad \frac{3}{1} < \frac{4}{1} < \frac{5}{1}$$

$$\Rightarrow \quad \frac{3}{1} < \frac{3+4}{1+1} < \frac{4}{1} < \frac{4+5}{1+1} < \frac{5}{1}$$

$$\Rightarrow \quad 3 < \frac{7}{2} < 4 < \frac{9}{2} < 5 \quad i.e. \quad \mathbf{3 < 3\frac{1}{2} < 4 < 4\frac{1}{2} < 5}$$

Also, every terminating and non-terminating recurring decimal number between 3 and 5 is a rational number between 3 and 5.

For example :

(i) $3{\cdot}2 < 3{\cdot}85 < 4{\cdot}3$ (ii) $4{\cdot}\overline{97} > 4{\cdot}2\overline{94} > 3{\cdot}8\dot{7} > 3{\cdot}\bar{2}$

1.3 METHOD FOR FINDING LARGE NUMBER OF RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

Let x and y be two rational numbers such that $x < y$.

In order to find n rational numbers between x and y, first of all find $d = \frac{y-x}{n+1}$.

Then, n rational number between x and y are :
$x + d, x + 2d, x + 3d, \ldots\ldots\ldots\ldots, x + nd.$

In example 1, given above : $3 < 5$

$\Rightarrow$ $x = 3$ and $y = 5$

To insert 3 rational numbers between 3 and 5, $n = 3$

$$\Rightarrow d = \frac{y-x}{n+1} = \frac{5-3}{3+1} = \frac{2}{4} = \frac{1}{2}.$$

$\therefore$ **Required rational numbers are :** $x + d$, $x + 2d$ and $x + 3d$

$= 3 + \frac{1}{2}, 3 + 2 \times \frac{1}{2}$ and $3 + 3 \times \frac{1}{2} = \mathbf{3\frac{1}{2}, 4}$ **and** $\mathbf{4\frac{1}{2}}$ **Ans.**

2 Find four rational numbers between $\frac{2}{3}$ and $\frac{5}{6}$.

Solution :

Since, $\frac{2}{3} < \frac{5}{6}$ [As, $2 \times 6 < 5 \times 3$]

$$\Rightarrow \quad x = \frac{2}{3}, y = \frac{5}{6} \text{ and } n = 4$$

$$\therefore \quad d = \frac{y-x}{n+1} = \frac{\frac{5}{6}-\frac{2}{3}}{4+1} = \frac{\frac{5-2\times2}{6}}{5} = \frac{5-4}{5\times6} = \frac{1}{30}$$

$\Rightarrow$ **Required rational numbers are :**

$$= x + d, x + 2d, x + 3d \text{ and } x + 4d$$

$$= \frac{2}{3} + \frac{1}{30}, \frac{2}{3} + 2 \times \frac{1}{30}, \frac{2}{3} + 3 \times \frac{1}{30} \text{ and } \frac{2}{3} + 4 \times \frac{1}{30}$$

$$= \frac{21}{30}, \frac{22}{30}, \frac{23}{30} \text{ and } \frac{24}{30} = \mathbf{\frac{7}{10}, \frac{11}{15}, \frac{23}{30} \text{ and } \frac{4}{5}}$$ **Ans.**

Alternative method :

For finding 4 rational numbers between $\frac{2}{3}$ and $\frac{5}{6}$.

1. Find L.C.M. of the denominators. L.C.M. of denominators 3 and 6 = 6.
2. Make denominator of each given rational number equal to 6 (the L.C.M.).

   $\therefore \quad \frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ and $\frac{5}{6} = \frac{5}{6}$.
3. Since, 4 rational numbers are required, multiply the numerator and denominator of each rational number (obtained in step 2) by 4 + 1 = 5.

   $\therefore \quad \frac{4}{6} = \frac{4 \times 5}{6 \times 5} = \frac{20}{30}$ and $\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$.

   Now, every rational number with denominator 30 and numerator between 20 and 25 will have its value between the given rational numbers $\frac{2}{3}$ and $\frac{5}{6}$.

   $\Rightarrow$ **Required rational numbers between $\frac{2}{3}$ and $\frac{5}{6}$ are :**

   $$= \frac{21}{30}, \frac{22}{30}, \frac{23}{30} \text{ and } \frac{24}{30} = \mathbf{\frac{7}{10}, \frac{11}{15}, \frac{23}{30} \text{ and } \frac{4}{5}}$$ **Ans.**

3 Insert three rational numbers between 2·6 and 3·1.

Solution :

First method :

$$2{\cdot}6 < 3{\cdot}1 \Rightarrow 2{\cdot}6 < \frac{2{\cdot}6 + 3{\cdot}1}{2} < 3{\cdot}1$$

$$\Rightarrow 2{\cdot}6 < 2{\cdot}85 < 3{\cdot}1$$

$$\Rightarrow 2{\cdot}6 < \frac{2{\cdot}6 + 2{\cdot}85}{2} < 2{\cdot}85 < \frac{2{\cdot}85 + 3{\cdot}1}{2} < 3{\cdot}1$$

$$\Rightarrow 2{\cdot}6 < 2{\cdot}725 < 2{\cdot}85 < 2{\cdot}975 < 3{\cdot}1$$

$\therefore$ **Required rational numbers are : 2·725, 2·85 and 2·975** **Ans.**

Second method :

$$2{\cdot}6 < 3{\cdot}1 \Rightarrow \frac{26}{10} < \frac{31}{10}$$

$$\Rightarrow \frac{26}{10} < \frac{26+31}{10+10} < \frac{31}{10}$$

$$\Rightarrow \frac{26}{10} < \frac{57}{20} < \frac{31}{10}$$

$$\Rightarrow \frac{26}{10} < \frac{26+57}{10+20} < \frac{57}{20} < \frac{57+31}{20+10} < \frac{31}{10}$$

$$\Rightarrow \quad \frac{26}{10} < \frac{83}{30} < \frac{57}{20} < \frac{88}{30} < \frac{31}{10}$$

$$\Rightarrow \quad 2{\cdot}6 < 2{\cdot}77 < 2{\cdot}85 < 2{\cdot}93 < 3{\cdot}1$$

$\therefore$ **Required rational numbers are : 2·77, 2·85 and 2·93** **Ans.**

Third method :

Since, 2·6 < 3·1, therefore let $x = 2{\cdot}6$ and $y = 3{\cdot}1$. Also, $n = 3$

$$\therefore \quad d = \frac{y-x}{n+1} = \frac{3{\cdot}1-2{\cdot}6}{3+1} = \frac{0{\cdot}5}{4} = 0{\cdot}125 = 0{\cdot}13 \qquad \text{(Correct to two decimal places)}$$

$\Rightarrow$ **Required rational numbers are :**

$x + d$, $x + 2d$ and $x + 3d$

$= 2{\cdot}6 + 0{\cdot}13$, $2{\cdot}6 + 2 \times 0{\cdot}13$ and $2{\cdot}6 + 3 \times 0{\cdot}13$ = **2·73, 2·86 and 2·99** **Ans.**

Fourth method :

Since, 2·6 < 3·1, therefore let $x = 2{\cdot}6$ and $y = 3{\cdot}1$

$$2{\cdot}6,\ 3{\cdot}1 = \frac{26}{10}, \frac{31}{10}$$

$$= \frac{26\times4}{10\times4}, \frac{31\times4}{10\times4} \qquad [\because\ n + 1 = 3 + 1 = 4]$$

$$= \frac{104}{40}, \frac{124}{40}$$

Now, every rational number with denominator 40 and numerator between 104 and 124 will lie between given rational numbers 2·6 and 3·1.

$\therefore$ **Required rational numbers can be taken as :**

$\frac{106}{40}, \frac{110}{40}$ and $\frac{120}{40}$ = **2·65, 2·75 and 3** **Ans.**

4 Which of the rational numbers $\frac{3}{5}$ and $\frac{5}{7}$ is greater. Insert three rational numbers between $\frac{3}{5}$ and $\frac{5}{7}$ so that all the five numbers are in ascending order of their values.

Solution :

$$\frac{3}{5} \text{ and } \frac{5}{7} = \frac{3\times7}{5\times7} \text{ and } \frac{5\times5}{7\times5} = \frac{21}{35} \text{ and } \frac{25}{35} \qquad \text{[L.C.M. of 5 and 7 = 35]}$$

Since, $21 < 25 \Rightarrow \frac{21}{35} < \frac{25}{35} \Rightarrow \frac{3}{5} < \frac{5}{7} \Rightarrow \frac{5}{7}$ **is greater.** **Ans.**

Now, $\frac{3}{5} < \frac{5}{7} \Rightarrow \frac{3}{5}, \frac{\frac{3}{5}+\frac{5}{7}}{2} < \frac{5}{7}$

$$\left[\frac{\frac{3}{5}+\frac{5}{7}}{2} = \frac{21+25}{2\times35} = \frac{46}{70} = \frac{23}{35}\right]$$

$$\Rightarrow \frac{3}{5} < \frac{23}{35} < \frac{5}{7}$$

$$\Rightarrow \frac{3}{5} < \frac{\frac{3}{5}+\frac{23}{35}}{2} < \frac{23}{35} < \frac{\frac{23}{35}+\frac{5}{7}}{2} < \frac{5}{7}$$

$$\Rightarrow \quad \frac{3}{5} < \frac{22}{35} < \frac{23}{35} < \frac{24}{35} < \frac{5}{7}$$

Which are in ascending order of their values. **Ans.**

1.4 PROPERTIES OF RATIONAL NUMBERS (Q)

1. *The sum of two or more rational numbers is always a rational number.*
2. *The difference of two rational numbers is always a rational number.*

   If a and b are any two rational numbers, then each of $a - b$ and $b - a$ is also a rational number.
3. *The product of two or more rational numbers is always a rational number.*
4. *The division of a rational number by a non-zero rational number is always a rational number.*

If a and b are any two rational numbers and $b \neq 0$; then $\frac{a}{b}$ is always a rational number.

Since, the sum (addition) of two rational numbers is always a rational number; we say that *the set of rational numbers is closed for addition.*

In the same way, the set of rational numbers is closed for :

(i) subtraction (ii) multiplication and

(iii) division; if divisor $\neq 0$.

EXERCISE 1 (A)

1. Insert **two** rational numbers between :

   (i) $\frac{3}{8}$ and $\frac{7}{12}$ (ii) $\frac{1}{3}$ and $\frac{1}{4}$
2. Insert **three** rational numbers between :

   (i) $\frac{2}{5}$ and $\frac{3}{7}$ (ii) $\frac{4}{11}$ and $\frac{9}{16}$
3. (i) Find **three** rational numbers between 5 and –2.

   (ii) Find **three** rational numbers between $-\frac{3}{4}$ and $\frac{1}{2}$.
4. Insert 4 rational numbers between 5 and 8.
5. Insert 5 rational numbers between $\frac{1}{3}$ and $\frac{5}{9}$.
6. Insert 6 rational numbers between 4·6 and 8·4.
7. Insert 7 rational numbers between 1 and 2.
8. Insert 8 rational numbers between 1·8 and 3·6.
9. Arrange $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ and $\frac{11}{18}$ in the ascending order of their magnitudes.

   Also, find the difference between the largest and the smallest of these rational numbers. Express this difference as a decimal fraction correct to one decimal place.
10. Arrange $\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4}$ and $\frac{17}{32}$ in the descending order of their magnitudes.

    Also, find the sum of the lowest and the largest of these rational numbers. Express the result obtained as a decimal fraction correct to two decimal places.

1.5 DECIMAL REPRESENTATION OF RATIONAL NUMBERS

[Terminating decimals and non-terminating recurring decimals]

Every rational number can be expressed either as a terminating decimal or a non-terminating decimal.

(a) Examine the following rational numbers :

(i) $\frac{1}{8} = 0{\cdot}125$ (ii) $\frac{1}{25} = 0{\cdot}04$ (iii) $3\frac{2}{5} = 3{\cdot}4$

In each example, given above, the division is exact *i.e.* no remainder is left. The quotients of such divisions are called ***terminating decimals.***

(b) Now, examine the following divisions :

(i) $\frac{3}{7} = 0{\cdot}428571428.......$ (ii) $\frac{18}{23} = 0{\cdot}7826086.......$

In each example, given above, the division never ends, no matter how long it continues. The quotients of such divisions are called ***non-terminating decimals.***

(c) Further, examine the following divisions :

(i) $\frac{4}{9} = 0{\cdot}4444.......$ (ii) $\frac{11}{30} = 0{\cdot}36666.......$ (iii) $\frac{4}{7} = 0{\cdot}571428571428.......$

In (i); digit '4' is repeated again and again.

In (ii); digit '3' is not repeated but digit '6' is repeated again and again.

In (iii); the set of digits '571428' is repeated again and again.

Similarly, in $\frac{13}{44} = 0{\cdot}29545454.......$; '29' is not repeated but '54' is repeated.

In such cases; the process of division will never end (terminate) and in the decimal part, either a single digit or a set of digits repeats in some specific order.

Such a non-terminating decimal, in which a digit or a set of digits repeats continually, is called a ***recurring*** or a ***periodic*** or a ***circulating decimal***. The repeating digit or the set of repeating digits is called the ***period of the recurring decimal.***

Therefore, in (i) $\frac{4}{9} = 0{\cdot}4444.......$; **4 is the period,**

in (ii) $\frac{11}{30} = 0{\cdot}36666.......$; **6 is the period and**

in (iii) $\frac{4}{7} = 0{\cdot}571428571428.......$; **571428 is the period.**

Notation : The period of recurring decimals is indicated as follows :

(i) If only one digit repeats, a *dot* or a *bar* is put above it.

Thus, $\frac{4}{9} = 0{\cdot}4444...... = 0{\cdot}\dot{4}$ or $0{\cdot}\overline{4}$;

$\frac{11}{30} = 0{\cdot}36666.... = 0{\cdot}3\dot{6}$ or $0{\cdot}3\overline{6}$.

(ii) If two digits repeat, a *dot* or a *bar* is put above each.

Thus, $\frac{13}{44} = 0{\cdot}29545454.... = 0{\cdot}29\dot{5}\dot{4}$ or $0{\cdot}29\overline{54}$

(iii) If three or more digits repeat, *dots* are put above the first and the last repeating digits or a *bar* is put over all the repeating digits.

Thus, $\frac{4}{7} = 0{\cdot}571428\ 571428.....$

$= 0{\cdot}\dot{5}7142\dot{8}$ or $0{\cdot}\overline{571428}$

5 Convert each of the following recurring decimals into a rational number :

(i) $0{\cdot}\overline{82}$ (ii) $1{\cdot}3\dot{8}$ (iii) $0{\cdot}64\overline{38}$

Solution :

(i) Let $x = 0{\cdot}\overline{82}$

If required, multiply both the sides by 10, 100, 1000, etc. so that decimal point is shifted just before the repeating digit/digits.

Here, decimal point is already just before the repeating digit *i.e.* before 8.

$\therefore \quad x = 0{\cdot}\overline{82}$

$= 0{\cdot}8282.......$ I

Since, the recurring part has two digits, multiply both the sides by 100 to get :

$100x = 082{\cdot}82.......$ II

On subtracting equation I from equation II, we get :

$99x = 82 \Rightarrow x = \frac{82}{99} \quad \therefore \quad 0{\cdot}\overline{82} = \frac{82}{99}$ **Ans.**

(ii) Let $x = 1{\cdot}3\dot{8}$

$\Rightarrow 10x = 13{\cdot}\overline{8}$ [Bringing decimal point just before the repeating digit/digits]

$= 13{\cdot}888.......$ I

$\Rightarrow 10 \times 10x = 138{\cdot}888.......$

i.e. $100x = 138{\cdot}888.......$ II

$10x = 13{\cdot}888.......$ I

$- \quad -$

$90x = 125 \Rightarrow x = \frac{125}{90} = \frac{25}{18}$

$\therefore \quad 1{\cdot}3\dot{8} = \frac{25}{18} = 1\frac{7}{18}$ **Ans.**

(iii) Let $x = 0{\cdot}64\overline{38}$

$\Rightarrow 100x = 064{\cdot}\overline{38}$

$= 64{\cdot}3838.......$

$100 \times 100x = 6438{\cdot}38.......$

$\Rightarrow \quad 10000x = 6438{\cdot}38.......$

$\quad 100x = 64{\cdot}3838.......$

$\quad - \qquad -$

$9900x = 6374 \quad \Rightarrow \quad x = \frac{6374}{9900} = \frac{3187}{4950}$

$\therefore \quad 0{\cdot}64\overline{38} = \frac{3187}{4950}$ **Ans.**

In a recurring decimal, if all the digits in the decimal part are not repeating, it is called a ***mixed recurring decimal***.

e.g. $0{\cdot}7\dot{8}$, $0{\cdot}4\overline{39}$, $3{\cdot}54\dot{7}$, etc.

6 Find the rational number equivalent to recurring decimal : $0{\cdot}54\dot{7}$

Solution :

Let $\quad x = 0{\cdot}54\dot{7}$

$100x = 54{\cdot}\dot{7}$

$= 54{\cdot}7777.......$

$10 \times 100x = 547{\cdot}777.......$

$\Rightarrow \quad 1000x = 547{\cdot}777.......$

$100x = 54{\cdot}777.......$

$- \qquad -$

$900x = 493 \quad \Rightarrow \quad x = \frac{493}{900}$

$\therefore \quad 0{\cdot}54\dot{7} = \frac{493}{900}$ **Ans.**

Alternative method :

Form a fraction with :

numerator = all the digits in the decimal part of the given recurring decimal minus all the non-recurring digits in the decimal part.

$\therefore$ For $0{\cdot}54\dot{7}$, numerator = 547 – 54 [$3{\cdot}54\dot{7} = 3 + 0{\cdot}54\dot{7}$]

denominator = number of nines equal to number of repeating digits followed by number of zeroes equal to number of non-repeating digits in the decimal part.

$\therefore$ For $0{\cdot}54\dot{7}$, denominator = 900 [9 for 7 and 00 for 54]

And, so $0{\cdot}54\dot{7} = \frac{547-54}{900} = \frac{493}{900} = \frac{493}{900}$ **Ans.**

Similarly :

(i) $0.4\dot{8} = \frac{48-4}{90} = \frac{44}{90} = \frac{22}{45}$.

(ii) $2.\overline{573} = 2 + 0.\overline{573} = 2 + \frac{573-5}{990} = 2 + \frac{568}{990} = 2\frac{284}{495}$.

(iii) $0.7\overline{325} = \frac{7325-7}{9990} = \frac{7318}{9990} = \frac{3659}{4995}$ and so on.

7 (a) Find the decimal representation of $\frac{1}{11}$.

(b) Use the above result to find $\frac{3}{11}$ and $\frac{7}{11}$.

Solution :

(a) $\frac{1}{11} = 0.09090909.......$ [Dividing 1 by 11]

$= \mathbf{0.\overline{09}}$ **Ans.**

(b) $\frac{3}{11} = 3 \times \frac{1}{11} = 3 \times 0.\overline{09} = \mathbf{0.\overline{27}}$ **Ans.**

And, $\frac{7}{11} = 7 \times \frac{1}{11} = 7 \times 0.\overline{09} = \mathbf{0.\overline{63}}$ **Ans.**

8 Without doing any actual division, find whether each of the following is a terminating decimal or not; (i) $\frac{17}{50}$ (ii) $\frac{7}{8}$ (iii) $\frac{23}{72}$.

Solution :

If the denominator of a rational number can be expressed as $2^m \times 5^n$; where m and n both are whole numbers, the rational number is convertible into terminating decimal.

(i) Since, $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$ *i.e.* 50 can be expressed as $2^m \times 5^n$

$\therefore$ **Rational number $\frac{17}{50}$ is a terminating decimal.** **Ans.**

(ii) Since, $8 = 2 \times 2 \times 2 = 2^3 = 2^3 \times 5^0$ *i.e.* 8 can be expressed as $2^m \times 5^n$

$\therefore$ **Rational number $\frac{7}{8}$ is a terminating decimal.** **Ans.**

(iii) Since, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ *i.e.* 72 cannot be expressed as $2^m \times 5^n$

$\therefore$ **Rational number $\frac{23}{72}$ is not a terminating decimal.** **Ans.**

EXERCISE 1 (B)

1. State, which of the following decimal numbers are pure recurring decimals and which are mixed recurring decimals :

   (i) $0{\cdot}\overline{083}$ (ii) $0{\cdot}0\overline{83}$

   (iii) $0{\cdot}\overline{227}$ (iv) $3{\cdot}5\dot{4}$

   (v) $2{\cdot}\overline{81}$

2. Represent as a decimal number :

   (i) $\frac{4}{15}$ (ii) $\frac{2}{7}$ (iii) $\frac{4}{9}$

   (iv) $\frac{5}{24}$ (v) $\frac{8}{13}$

3. Express each of the following as a rational number *i.e.* in the form $\frac{a}{b}$; where $a, b \in Z$ and $b \neq 0$.

   (i) $0{\cdot}5\dot{3}$ (ii) $0{\cdot}2\overline{27}$ (iii) $0{\cdot}2\overline{104}$

   (iv) $3{\cdot}5\dot{2}$ (v) $2{\cdot}24\overline{689}$ (vi) $0{\cdot}\overline{572}$

   (vii) $0{\cdot}15\dot{8}$ (viii) $0{\cdot}03\overline{84}$

4. Find the decimal representation of $\frac{1}{7}$ and $\frac{2}{7}$

   Deduce from the decimal representation of $\frac{1}{7}$, without actual calculation, the decimal representation of $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$.

5. Without doing any actual division, find which of the following rational numbers have terminating decimal representation :

   (i) $\frac{7}{16}$ (ii) $\frac{23}{125}$ (iii) $\frac{9}{14}$

   (iv) $\frac{32}{45}$ (v) $\frac{43}{50}$ (vi) $\frac{17}{40}$

   (vii) $\frac{61}{75}$ (viii) $\frac{123}{250}$

1.6 IRRATIONAL NUMBERS ($\overline{Q}$)

(a) *The square roots, cube roots, etc., of natural numbers are irrational numbers; if their exact values cannot be obtained.*

$\sqrt{m}$ is irrational, if exact square root of m does not exist.

Similarly, $\sqrt[3]{m}$ is irrational, if exact cube root of m does not exist.

(b) *A non-terminating and non-recurring decimal is an irrational number.*

e.g. (i) 0·42434445, ---------- (ii) 3·862045 ---------- and so on.

(c) *The number* $\pi = \dfrac{\text{Circumference of a circle}}{\text{Diameter of the circle taken}}$

$= 3{\cdot}14159265358979323846264338327950$----------

$=$ An irrational number

[We often take $\frac{22}{7}$ as an approximate value of π, but $\pi \neq \frac{22}{7}$]

(i) $\sqrt{3} + \sqrt{5} \neq \sqrt{8}$; $\sqrt{7} - \sqrt{5} \neq \sqrt{2}$

(ii) $\sqrt{5} + \sqrt{5} \neq \sqrt{10}$; but $\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ and $\sqrt{5} \times \sqrt{5} = 5$

(iii) $\frac{5}{\sqrt{5}} = \sqrt{5}$, $\frac{2}{\sqrt{2}} = \sqrt{2}$, $\frac{7}{\sqrt{7}} = \sqrt{7}$ and so on.

(iv) $\sqrt{48} = \sqrt{2\times2\times2\times2\times3} = 2 \times 2 \times \sqrt{3} = 4\sqrt{3}$ and so on.

9 Show that $\sqrt{2}$ is an irrational number.

Solution :

Division method :

	1·4 1 4 2 1 3 5 ....
1	2·00 00 00 00 00 00 00 ....
	1
24	100
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
282841	383600
	282841
2828423	10075900
	8485269
28284265	159063100
	141421325
	17641775 .....

Clearly,

$\sqrt{2}$ = 1·4142135 .... ;

which is a non-repeating non-recurring representation.

$\therefore$ **$\sqrt{2}$ is an irrational number.**

Hence Proved.

Alternative method :

Let $\sqrt{2}$ is a rational number.

$\therefore \quad \sqrt{2} = \frac{a}{b}$ [Where $a, b \in Z$ and $b \neq$ o]

$\Rightarrow \quad 2 = \frac{a^2}{b^2}$ [Squaring both the sides]

$\Rightarrow \quad a^2 = 2b^2$

$\Rightarrow$ a^2 is divisible by 2

$\Rightarrow$ a is also divisible by 2 I

Let $a = 2c$

$\Rightarrow \quad a^2 = 4c^2$

[Squaring both the sides]

$\Rightarrow \quad 2b^2 = 4c^2$ [$\because a^2 = 2b^2$]

$\Rightarrow \quad b^2 = 2c^2$

$\Rightarrow$ b^2 is divisible by 2

$\Rightarrow$ b is also divisible by 2 II

From I and II, we get a and b both are divisible by 2

i.e. a and b have a common factor (2).

This contradicts our assumption that $\frac{a}{b}$ is rational *i.e.* a and b do not have any common factor other than unity (1).

$\Rightarrow$ $\frac{a}{b}$ is not rational $\Rightarrow$ $\sqrt{2}$ is not rational *i.e.* **$\sqrt{2}$ is irrational.** **Ans.**

Similarly, each of $\sqrt{3}$, $\sqrt{5}$, $3\sqrt{2}$, etc., can be proved to be an irrational number.

As per classical definition of rational numbers, if a number can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$; it is a rational number. But in cases like $\sqrt{2}$, $\sqrt{3}$, *etc.*, such representation is not possible, so, such numbers are irrational numbers.

Remember that :

1. If p is a number whose square (p^2) is divisible by 2, then necessarily p is also divisible by 2. Similarly, if :
   (i) p^2 is divisible by 3 $\Rightarrow$ p is divisible by 3,
   (ii) p^2 is divisible by 5 $\Rightarrow$ p is divisible by 5 and so on.
2. A number is rational if :
   (i) the number can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
   (ii) p and q do not have any common factor other than unity (1)

Suppose, p and q both have a common factor 2; then $\frac{p}{q}$ is not rational. Similarly, if p and q both have 3 as a common factor, then $\frac{p}{q}$ is not rational and so on.

10 Identify each of the following as rational or irrational number.
(i) $\sqrt{12}$ (ii) $3\sqrt{2} \times \sqrt{8}$

Solution :

(i) $\sqrt{12} = \sqrt{2\times2\times3}$

$= 2\sqrt{3}$; *which is the product of a rational number (2) and an irrational number* ($\sqrt{3}$).

$\therefore$ **$\sqrt{12}$ is an irrational number.** **Ans.**

(ii) $3\sqrt{2} \times \sqrt{8} = 3\sqrt{2} \times \sqrt{2\times2\times2}$

$= 3\sqrt{2} \times 2\sqrt{2}$

$= 3 \times 2 \times 2$ $[\because \sqrt{2} \times \sqrt{2} = 2]$

$= 12$; *which is a rational number.*

$\therefore$ **$3\sqrt{2} \times \sqrt{8}$ is a rational number.** **Ans.**

11 Insert a rational number and an irrational number between 3 and 4.

Solution :

If a and b are two positive numbers such that ab is not a perfect square, then :

(i) a rational number between a and b = $\frac{a+b}{2}$

and, (ii) an irrational number between a and b = $\sqrt{ab}$

Since, 3 and 4 are positive rational numbers and 3 × 4 = 12 is not a perfect square, therefore :

(i) **a rational number between 3 and 4** = $\frac{3+4}{2}$

$= \frac{7}{2} = 3\frac{1}{2}$ **Ans.**

(ii) **an irrational number between 3 and 4** = $\sqrt{3\times 4}$

$= \sqrt{3\times 2\times 2} = 2\sqrt{3}$ **Ans.**

12 Find two irrational numbers between 2 and 3.

Solution :

Since, 2 and 3 are rational numbers and 2 × 3 = 6 is not a perfect square.

∴ One irrational number between 2 and 3 = $\sqrt{2\times 3} = \sqrt{6}$

And, an irrational number between 2 and $\sqrt{6}$ = $\sqrt{2\times\sqrt{6}} = \sqrt{2\sqrt{6}}$

∴ **Required irrational numbers are :** $\sqrt{6}$ **and** $\sqrt{2\sqrt{6}}$ **Ans.**

Alternative method :

Since, 2 = $\sqrt{4}$ and 3 = $\sqrt{9}$

∴ Each of $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ and $\sqrt{8}$ is an irrational number between 2 and 3.

13 Examine each of the following as a rational number or an irrational number :

(i) $(3+\sqrt{2})^2$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$ (iii) $\frac{6}{\sqrt{3}}$

Solution :

(i) $(3+\sqrt{2})^2 = 3^2 + (\sqrt{2})^2 + 2\times 3\times\sqrt{2}$

$= 9 + 2 + 6\sqrt{2} = 11 + 6\sqrt{2}$

Since, 11 is rational, $6\sqrt{2}$ is irrational and we know that the sum of a rational and an irrational number is always irrational.

∴ $11 + 6\sqrt{2}$ is irrational

∴ $(3+\sqrt{2})^2$ **is an irrational number** **Ans.**

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2 - (\sqrt{3})^2$

$= 9 - 3 = 6$; *which is rational.*

∴ $(3+\sqrt{3})(3-\sqrt{3})$ **is a rational number** **Ans.**

(iii) $\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$; *which is irrational.*

$\therefore$ **$\frac{6}{\sqrt{3}}$ is an irrational number** **Ans.**

14 Insert two rational numbers and two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$.

Solution :

Since, square of $\sqrt{3} = 3$ and square of $\sqrt{7} = 7$.

(i) Choose any two rational numbers between 3 and 7 each of which is a perfect square. The square roots of such numbers will be the required rational numbers.

Let the numbers be 4 and 5·76, where $\sqrt{4} = 2$ and $\sqrt{5{\cdot}76} = 2{\cdot}4$.

$\therefore$ **Required rational numbers are 2 and 2·4** **Ans.**

(ii) Now, choose any two rational numbers between 3 and 7 each of which is not a perfect square. The square root of such numbers will be the required irrational numbers.

Let the numbers be 5 and 6

$\therefore$ **$\sqrt{5}$ and $\sqrt{6}$ are the required irrational numbers.** **Ans.**

More about irrational numbers :

1. For any two positive rational numbers x and y

   If $\sqrt{x}$ and $\sqrt{y}$ are irrationals then :

   $\sqrt{x} > \sqrt{y} \Rightarrow x > y$ and $\sqrt{x} < \sqrt{y} \Rightarrow x < y$.

2. (i) $a + b\sqrt{x} = c + d\sqrt{x} \Rightarrow a = c$ and $b = d$.

   (ii) $5 - a\sqrt{3} = b - 2\sqrt{3} \Rightarrow b = 5$ and $a = 2$.

   (iii) $x\sqrt{5} - 3\sqrt{2} = 4\sqrt{5} + y\sqrt{2} \Rightarrow x = 4$ and $y = -3$ and so on.

3. The negative of an irrational number is always irrational.
4. The sum of a rational and an irrational number is always irrational.
5. The product of a non-zero rational number and an irrational number is always irrational.

Note 1 : *The sum of two irrational numbers may or may not be irrational.*

e.g. (i) $(3 + \sqrt{5}) + (6 - \sqrt{5}) = 9$; *which is not an irrational number.*

(ii) $(\sqrt{7} - 3) + (\sqrt{2} + 3) = \sqrt{7} - 3 + \sqrt{2} + 3$

$= \sqrt{7} + \sqrt{2}$; *which is an irrational number.*

Note 2 : *The difference of two irrational numbers may or may not be irrational.*

e.g. (i) $(8 - \sqrt{10}) - (3 - \sqrt{10}) = 5$; *which is not an irrational number.*

(ii) $(3\sqrt{2} + 5) - (-7\sqrt{2} - 12) = 3\sqrt{2} + 5 + 7\sqrt{2} + 12$

$= 10\sqrt{2} + 17$; *which is an irrational number.*

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Note 3 : *The product of two irrational numbers may or may not be irrational.*

e.g. (i) $(3-\sqrt{5})\times(3+\sqrt{5}) = 9-5 = 4$; *which is not an irrational number.*

(ii) $(2+\sqrt{3})\times(3-\sqrt{2}) = 6-2\sqrt{2}+3\sqrt{3}-\sqrt{6}$; *which is an irrational number.*

15 Which of the following numbers is greater :

(i) $3\sqrt{2}$ and $2\sqrt{3}$ (ii) $6\sqrt[3]{3}$ and $5\sqrt[3]{4}$

Solution :

(i) Since, $3\sqrt{2} = \sqrt{3^2\times 2} = \sqrt{18}$ and $2\sqrt{3} = \sqrt{2^2\times 3} = \sqrt{12}$

$\therefore$ **$3\sqrt{2}$ is greater.** **Ans.**

(ii) Since, $6\sqrt[3]{3} = \sqrt[3]{6^3\times 3} = \sqrt[3]{648}$, $5\sqrt[3]{4} = \sqrt[3]{5^3\times 4} = \sqrt[3]{500}$

$\therefore$ **$6\sqrt[3]{3}$ is greater.** **Ans.**

16 Compare : (i) $\sqrt[3]{4}$ and $\sqrt{3}$ (ii) $\sqrt[4]{8}$ and $\sqrt[6]{22}$

Solution :

Make the index (power) of each pair of numbers to be compared same and then compare.

(i) Since, $\sqrt[3]{4} = 4^{\frac{1}{3}}$ has power $\frac{1}{3}$ and $\sqrt{3} = 3^{\frac{1}{2}}$ has power $\frac{1}{2}$.

Convert both the powers *i.e.* $\frac{1}{3}$ and $\frac{1}{2}$ into like fractions (fractions with same denominator).

As, the L.C.M. of 3 and 2 is 6

$\therefore \frac{1}{3} = \frac{1\times 2}{3\times 2} = \frac{2}{6}$ and $\frac{1}{2} = \frac{1\times 3}{2\times 3} = \frac{3}{6}$. [$\frac{2}{6}$ and $\frac{3}{6}$ are like fractions]

Now, $\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\frac{2}{6}} = (4^2)^{\frac{1}{6}} = (16)^{\frac{1}{6}}$

and, $\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{3}{6}} = (3^3)^{\frac{1}{6}} = (27)^{\frac{1}{6}}$

Clearly, $27 > 16 \Rightarrow 27^{\frac{1}{6}} > 16^{\frac{1}{6}} \Rightarrow$ **$\sqrt{3} > \sqrt[3]{4}$** **Ans.**

(ii) Since, $\sqrt[4]{8} = 8^{\frac{1}{4}}$, $\sqrt[6]{22} = (22)^{\frac{1}{6}}$ and L.C.M. of 4 and 6 = 12

$\therefore \sqrt[4]{8} = 8^{\frac{1}{4}} = 8^{\frac{3}{12}} = (8^3)^{\frac{1}{12}} = (512)^{\frac{1}{12}}$

$\sqrt[6]{22} = 22^{\frac{1}{6}} = 22^{\frac{2}{12}} = (22^2)^{\frac{1}{12}} = (484)^{\frac{1}{12}}$

Clearly, $512 > 484 \Rightarrow (512)^{\frac{1}{12}} > (484)^{\frac{1}{12}} \Rightarrow$ **$\sqrt[4]{8} > \sqrt[6]{22}$** **Ans.**

1. Since, $5 = \sqrt{25}$ and $6 = \sqrt{36}$; therefore $\sqrt{26}$, $\sqrt{27}$, $\sqrt{28}$, $\sqrt{29}$, $\sqrt{30}$, $\sqrt{31}$, $\sqrt{32}$, $\sqrt{33}$, $\sqrt{34}$ and $\sqrt{35}$ are irrational numbers between 5 and 6.

2. Since, $3\sqrt{2} = \sqrt{3^2 \times 2} = \sqrt{18}$ and $2\sqrt{3} = \sqrt{2^2 \times 3} = \sqrt{12}$; therefore $\sqrt{17}$, $\sqrt{15}$, $\sqrt{14}$ and $\sqrt{13}$ are irrational numbers between $3\sqrt{2}$ and $2\sqrt{3}$.

1.7 REAL NUMBERS (R)

The *union* of the set of *rational numbers* and the set of *irrational numbers* is called the *set* of *real numbers, i.e.* $R = Q \cup \overline{Q}$.

Rational number (Q) :

= Set of all terminating or recurring decimals.

Irrational numbers ($\overline{Q}$) :

= Set of all non-terminating and non recurring decimals.

EXERCISE 1 (C)

1. State, whether the following numbers are rational or not :

   (i) $(2 + \sqrt{2})^2$ (ii) $(3 - \sqrt{3})^2$

   (iii) $(5 + \sqrt{5})(5 - \sqrt{5})$ (iv) $(\sqrt{3} - \sqrt{2})^2$

   (v) $\left(\frac{3}{2\sqrt{2}}\right)^2$ (vi) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2$

2. Find the square of :

   (i) $\frac{3\sqrt{5}}{5}$ (ii) $\sqrt{3} + \sqrt{2}$

   (iii) $\sqrt{5} - 2$ (iv) $3 + 2\sqrt{5}$

3. State, in each case, whether *true* or *false* :

   (i) $\sqrt{2} + \sqrt{3} = \sqrt{5}$

   (ii) $2\sqrt{4} + 2 = 6$

   (iii) $3\sqrt{7} - 2\sqrt{7} = \sqrt{7}$

   (iv) $\frac{2}{7}$ is an irrational number.

   (v) $\frac{5}{11}$ is a rational number.

   (vi) All rational numbers are real numbers.

   (vii) All real numbers are rational numbers.

   (viii) Some real numbers are rational numbers.

4. Given universal set = $\{-6, -5\frac{3}{4}, -\sqrt{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, \sqrt{8}, 3.01, \pi, 8.47\}$

   From the given set, find :

   (i) set of rational numbers

   (ii) set of irrational numbers

   (iii) set of integers

   (iv) set of non-negative integers

5. Use division method to show that $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers.

6. Use method of contradiction to show that $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers.

7. Write a pair of irrational numbers whose sum is irrational.

8. Write a pair of irrational numbers whose sum is rational.

9. Write a pair of irrational numbers whose difference is irrational.

10. Write a pair of irrational numbers whose difference is rational.

11. Write a pair of irrational numbers whose product is irrational.

12. Write a pair of irrational numbers whose product is rational.

13. Write in ascending order :

(i) $3\sqrt{5}$ and $4\sqrt{3}$

(ii) $2\sqrt[3]{5}$ and $3\sqrt[3]{2}$

(iii) $6\sqrt{5}$, $7\sqrt{3}$ and $8\sqrt{2}$

14. Write in descending order :

(i) $2\sqrt[4]{6}$ and $3\sqrt[4]{2}$

(ii) $7\sqrt{3}$ and $3\sqrt{7}$

15. Compare :

(i) $\sqrt[6]{15}$ and $\sqrt[4]{12}$ (ii) $\sqrt{24}$ and $\sqrt[3]{35}$

16. Insert two irrational numbers between 5 and 6.

17. Insert five irrational numbers between $2\sqrt{5}$ and $3\sqrt{3}$.

18. Write two rational numbers between $\sqrt{2}$ and $\sqrt{3}$.

Take any two rational numbers between 2 and 3 which are perfect squares; such as : 2·25, 2·56, 2·89, etc.

$\therefore \sqrt{2} < \sqrt{3} \Rightarrow \sqrt{2} < \sqrt{2 \cdot 25} < \sqrt{2 \cdot 56} < \sqrt{3}$

$\Rightarrow \sqrt{2} < 1 \cdot 5 < 1 \cdot 6 < \sqrt{3}$

19. Write three rational numbers between $\sqrt{3}$ and $\sqrt{5}$.

1.8 SURDS (Radicals)

If x is a positive rational number and n is a positive integer such that $x^{\frac{1}{n}}$ *i.e.* $\sqrt[n]{x}$ is irrational; then $x^{\frac{1}{n}}$ is called a **surd** or a **radical**.

$\therefore$ $\sqrt[3]{6}$ is a surd, $\because$ (i) 6 is a positive rational number, (ii) $\sqrt[3]{6}$ is an irrational number.

(i) Similarly, $\sqrt{5}$, $\sqrt[4]{8}$, $\sqrt[3]{20}$, etc. are surds as 5, 8, 20, etc. are positive rational numbers.

(ii) But $\sqrt{4}$, $\sqrt[3]{27}$ and $\sqrt[4]{625}$ are not surds because $\sqrt{4} = 2$, $\sqrt[3]{27} = 3$ and $\sqrt[4]{625} = 5$ *i.e.* $\sqrt{4}$, $\sqrt[3]{27}$ and $\sqrt[4]{625}$ are not irrational numbers.

1. Every surd is an irrational number, but every irrational number is not a surd.
For example, 'π' is an irrational number but not a surd.

2. Let a be a rational number and n be a positive number greater than 1,
then $\sqrt[n]{a}$ *i.e.* $a^{\frac{1}{n}}$ is called a surd of order n.

$\therefore$ (a) $\sqrt{5}$ is a surd of order 2.

(b) $\sqrt[3]{10}$ is a surd of order 3.

(c) $\sqrt[5]{7}$ is a surd of order 5 and so on.

17 State, with reasons, which of the following are surds and which are not :

(i) $\sqrt{27}$ (ii) $\sqrt{225} \times \sqrt{4}$

Solution :

(i) $\sqrt{27} = \sqrt{3\times3\times3} = 3\sqrt{3}$; **which is irrational.**

$\therefore$ $\sqrt{27}$ is an irrational number.

Since, 27 is a positive rational number and $\sqrt{27}$ is irrational.

$\therefore$ **$\sqrt{27}$ is a surd.** **Ans.**

(ii) $\sqrt{225} \times \sqrt{4} = \sqrt{15\times15} \times \sqrt{2\times2} = 15 \times 2 = 30$; **which is a rational number.**

$\therefore$ **$\sqrt{225} \times \sqrt{4}$ is not a surd.** **Ans.**

1.9 RATIONALISATION (For surds of order 2)

When two surds are multiplied together such that their product is a rational number, the two surds are called *rationalising factors of each other.*

The process of rationalising a surd by multiplying it with its rationalising factor is called ***rationalisation***.

Examples :

(i) Since, $5\sqrt{2} \times 3\sqrt{2} = 15 \times 2 = 30$; which is a rational number, therefore $5\sqrt{2}$ and $3\sqrt{2}$ are rationalising factors of each other.

(ii) $3\sqrt{7}$ and $4\sqrt{7}$ are rationalising factors of each other,

as $3\sqrt{7} \times 4\sqrt{7} = 12 \times 7 = 84$; *which is a rational number.*

Since : (i) $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30 \Rightarrow$ $2\sqrt{5}$ and $3\sqrt{5}$ are rationalising factors of each other.

(ii) $2\sqrt{5} \times \sqrt{5} = 2 \times 5 = 10 \Rightarrow$ $2\sqrt{5}$ and $\sqrt{5}$ are rationalising factors of each other.

(iii) $2\sqrt{5} \times \frac{3}{\sqrt{5}} = 6 \Rightarrow 2\sqrt{5}$ and $\frac{3}{\sqrt{5}}$ are rationalising factors of each other.

Therefore, from examples, given above, we can conclude that the *rationalising factor of a surd is not unique.*

18 Find the least rationalising factor of : (i) $\sqrt{27}$, (ii) $2\sqrt{125}$.

Solution :

(i) $\because$ $\sqrt{27} = \sqrt{3\times3\times3} = 3\sqrt{3}$

And, $3\sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$; *which is a rational number.*

$\therefore$ **The least rationalising factor of $\sqrt{27} = \sqrt{3}$** **Ans.**

(ii) $\because$ $2\sqrt{125} = 2\sqrt{25\times5} = 2 \times 5\sqrt{5} = 10\sqrt{5}$

And, $10\sqrt{5} \times \sqrt{5} = 10 \times 5 = 50$; *which is a rational number.*

$\therefore$ **The least rationalising factor of $2\sqrt{125} = \sqrt{5}$** **Ans.**

(i) $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right) = \left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2 = 3 - 2 = 1$

$\Rightarrow$ $\left(\sqrt{3}-\sqrt{2}\right)$ and $\left(\sqrt{3}+\sqrt{2}\right)$ are rationalising factors of each other.

(ii) $\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right) = (3)^2 - (\sqrt{5})^2 = 9 - 5 = 4$

$\Rightarrow$ $\left(3+\sqrt{5}\right)$ and $\left(3-\sqrt{5}\right)$ are rationalising factors of each other.

1.10 SIMPLIFYING AN EXPRESSION BY RATIONALISING ITS DENOMINATOR

19 Rationalise the denominator of : (i) $\frac{1}{\sqrt{2}}$ (ii) $\frac{5}{2\sqrt{2}}$.

Solution :

Method : Multiply and divide the given expression by the least rationalising factor of its denominator. Simplify, if necessary.

(i) $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ [Rationalising factor of denominator $\sqrt{2}$ is $\sqrt{2}$]

$= \frac{\sqrt{2}}{2}$ **Ans.**

(ii) As the least rationalising factor of the denominator $2\sqrt{2}$ is $\sqrt{2}$

$\therefore \frac{5}{2\sqrt{2}} = \frac{5}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2\times 2} = \frac{5\sqrt{2}}{4}$ **Ans.**

20 Simplify each of the following by rationalising the denominator :

(i) $\frac{1}{3-\sqrt{7}}$ (ii) $\frac{3}{\sqrt{5}+\sqrt{3}}$ (iii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(iv) $\frac{7}{\sqrt{15}+2\sqrt{2}}$ (v) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

Solution :

(i) Since, the denominator = $3 - \sqrt{7}$ and its rationalising factor = $3 + \sqrt{7}$

$\therefore \frac{1}{3-\sqrt{7}} = \frac{1}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$

$= \frac{3+\sqrt{7}}{9-7}$ [$\because \left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right) = (3)^2 - (\sqrt{7})^2 = 9 - 7 = 2$]

$= \frac{3+\sqrt{7}}{2}$ **Ans.**

(ii) $\because$ R.F. (Rationalising factor) of denominator $\sqrt{5}+\sqrt{3}$ is $\sqrt{5}-\sqrt{3}$

$\therefore \frac{3}{\sqrt{5}+\sqrt{3}} = \frac{3}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{3\left(\sqrt{5}-\sqrt{3}\right)}{5-3} = \frac{3}{2}\left(\sqrt{5}-\sqrt{3}\right)$ **Ans.**

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(iii) $\because$ R.F. of denominator $= \sqrt{3} - \sqrt{2}$

$$\therefore \quad \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{3+2-2\times\sqrt{3}\times\sqrt{2}}{3-2} = \frac{5-2\sqrt{6}}{1} = 5 - 2\sqrt{6}$$ **Ans.**

(iv) $\because$ R.F. of denominator $= \sqrt{15} - 2\sqrt{2}$

And, $(\sqrt{15}+2\sqrt{2})(\sqrt{15}-2\sqrt{2}) = (\sqrt{15})^2 - (2\sqrt{2})^2 = 15 - 8 = 7$

$$\therefore \quad \frac{7}{\sqrt{15}+2\sqrt{2}} = \frac{7}{\sqrt{15}+2\sqrt{2}} \times \frac{\sqrt{15}-2\sqrt{2}}{\sqrt{15}-2\sqrt{2}}$$

$$= \frac{7(\sqrt{15}-2\sqrt{2})}{7} = \sqrt{15}-2\sqrt{2}$$ **Ans.**

(v) $\because$ R.F. of denominator $= 5\sqrt{3} + 3\sqrt{5}$

And, $(5\sqrt{3} - 3\sqrt{5})(5\sqrt{3} + 3\sqrt{5}) = (5\sqrt{3})^2 - (3\sqrt{5})^2 = 25 \times 3 - 9 \times 5 = 30$

$$\therefore \quad \frac{30}{5\sqrt{3}-3\sqrt{5}} = \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}$$

$$= \frac{30(5\sqrt{3}+3\sqrt{5})}{30} = 5\sqrt{3}+3\sqrt{5}$$ **Ans.**

21 Find the values of 'a' and 'b', if : $\dfrac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} = a + b\sqrt{6}$.

Solution :

Since, $$\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} = \frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} \times \frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}$$

$$= \frac{4\times3+9\times2+2\times2\sqrt{3}\times3\sqrt{2}}{(2\sqrt{3})^2-(3\sqrt{2})^2}$$

$$= \frac{12+18+12\sqrt{6}}{12-18} = \frac{30+12\sqrt{6}}{-6} = -5-2\sqrt{6}$$

Given : $\dfrac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} = a + b\sqrt{6} \Rightarrow -5 - 2\sqrt{6} \Rightarrow a = -5$ and $b = -2$ **Ans.**

22 Prove that : $\dfrac{1}{\sqrt{2}+1} + \dfrac{1}{\sqrt{3}+\sqrt{2}} + \dfrac{1}{2+\sqrt{3}} = 1.$

Solution :

L.H.S. $$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{2-\sqrt{3}}{4-3}$$

$$= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + 2 - \sqrt{3}$$

$= 1 =$ **R.H.S.** **Hence Proved**

23 Rationalise the denominator of : $\frac{1}{\sqrt{3}+\sqrt{2}-1}$.

Solution :

$$\frac{1}{\sqrt{3}+\sqrt{2}-1} = \frac{1}{(\sqrt{3}+\sqrt{2})-(1)} \times \frac{(\sqrt{3}+\sqrt{2})+(1)}{(\sqrt{3}+\sqrt{2})+(1)}$$

$$= \frac{\sqrt{3}+\sqrt{2}+1}{3+2+2\sqrt{6}-1} \qquad \left[\text{Denominator} = (\sqrt{3}+\sqrt{2})^2 - (1)^2\right]$$

$$= \frac{\sqrt{3}+\sqrt{2}+1}{4+2\sqrt{6}}$$

$$= \frac{\sqrt{3}+\sqrt{2}+1}{2(2+\sqrt{6})} \times \frac{2-\sqrt{6}}{2-\sqrt{6}}$$

$$= \frac{2\sqrt{3}-\sqrt{18}+2\sqrt{2}-\sqrt{12}+2-\sqrt{6}}{2(4-6)}$$

$$= \frac{2\sqrt{3}-3\sqrt{2}+2\sqrt{2}-2\sqrt{3}+2-\sqrt{6}}{-4} \qquad \left[\because \sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2} \text{ similarly, } \sqrt{12} = 2\sqrt{3}\right]$$

$$= -\frac{1}{4}(-\sqrt{2}+2-\sqrt{6}) = \frac{1}{4}(\sqrt{2}-2+\sqrt{6})$$ **Ans.**

EXERCISE 1 (D)

1. State, with reason, which of the following are surds and which are not :
   (i) $\sqrt{180}$ (ii) $\sqrt[4]{27}$
   (iii) $\sqrt[5]{128}$ (iv) $\sqrt[3]{64}$
   (v) $\sqrt[3]{25} \cdot \sqrt[3]{40}$ (vi) $\sqrt[3]{-125}$
   (vii) $\sqrt{\pi}$ (viii) $\sqrt{3+\sqrt{2}}$

2. Write the lowest rationalising factor of :
   (i) $5\sqrt{2}$ (ii) $\sqrt{24}$
   (iii) $\sqrt{5}-3$ (iv) $7-\sqrt{7}$
   (v) $\sqrt{18}-\sqrt{50}$ (vi) $\sqrt{5}-\sqrt{2}$
   (vii) $\sqrt{13}+3$ (viii) $15-3\sqrt{2}$
   (ix) $3\sqrt{2}+2\sqrt{3}$

3. Rationalise the denominators of :
   (i) $\frac{3}{\sqrt{5}}$ (ii) $\frac{2\sqrt{3}}{\sqrt{5}}$
   (iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (iv) $\frac{3}{\sqrt{5}+\sqrt{2}}$
   (v) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$ (vi) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(vii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (viii) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

(ix) $\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

4. Find the values of '*a*' and '*b*' in each of the following :

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

5. Simplify :

(i) $\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1}$

(ii) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

6. If $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y = \frac{\sqrt{5}+2}{\sqrt{5}-2}$; find :

(i) x^2 (ii) y^2

(iii) xy (iv) $x^2 + y^2 + xy$.

7. If $m = \frac{1}{3-2\sqrt{2}}$ and $n = \frac{1}{3+2\sqrt{2}}$, find :

(i) m^2 (ii) n^2 (iii) mn

8. If $x = 2\sqrt{3} + 2\sqrt{2}$, find :

(i) $\frac{1}{x}$ (ii) $x + \frac{1}{x}$ (iii) $\left(x + \frac{1}{x}\right)^2$

9. If $x = 1 - \sqrt{2}$, find the value of $\left(x - \frac{1}{x}\right)^3$.

10. If $x = 5 - 2\sqrt{6}$, find : $x^2 + \frac{1}{x^2}$

$$x^2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} - 2 + 2 = \left(x - \frac{1}{x}\right)^2 + 2$$

11. Show that : $\frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5.$

12. Rationalise the denominator of :

$\frac{1}{\sqrt{3}-\sqrt{2}+1}$

13. If $\sqrt{2} = 1{\cdot}4$ and $\sqrt{3} = 1{\cdot}7$, find the value of :

(i) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (ii) $\frac{1}{3+2\sqrt{2}}$