21

Volume and Surface Area of Solids

POINTS TO REMEMBER

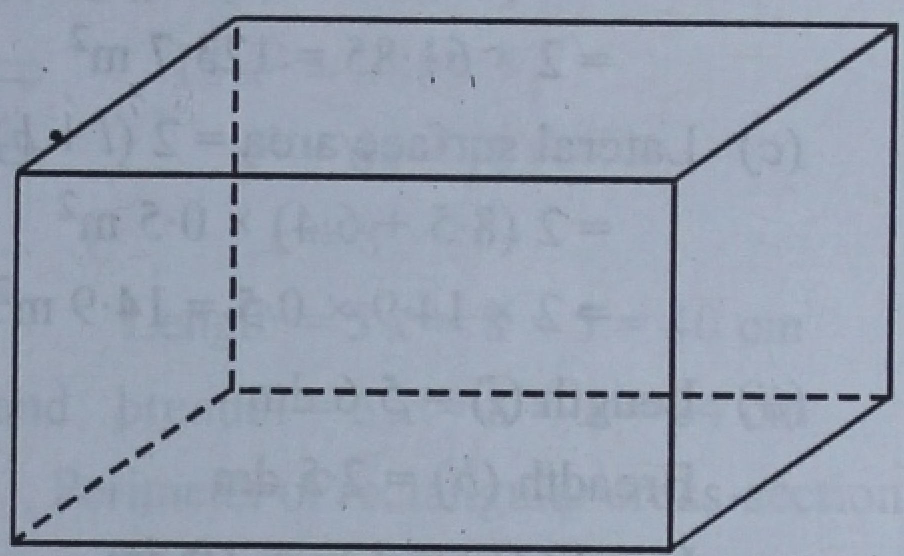
1. **Solids.** *The bodies occupying space are called solids.*

The solid bodies occur in various shapes, such as *a cuboid, a cube, a cylinder, a cone* and *a sphere*, etc.

2. **Volume of a Solid.** *The space occupied by a solid body is called its volume.*

The units of volume are ***cubic cm*** **(i.e., cm^3)** or ***cubic metres*** **(i.e., cm^3)**, etc.

3. **Cuboid.** *A rectangular solid bounded by six rectangular plane faces is called a cuboid. A cuboid has 6 rectangular faces, 12 edges and 8 vertices.*



Cuboid

4. **Cube.** *A cuboid whose length, breadth and height are all equal is called a cube. Each edge of a cube is called its side.* It has 6 square faces, 12 edges and 8 vertices.

5. **Formulae**

1. **Cuboid.** Let length = l units, breadth = b units and height = h units. Then,

(*i*) Volume of the cuboid = $(l \times b \times h)$ cubic units.

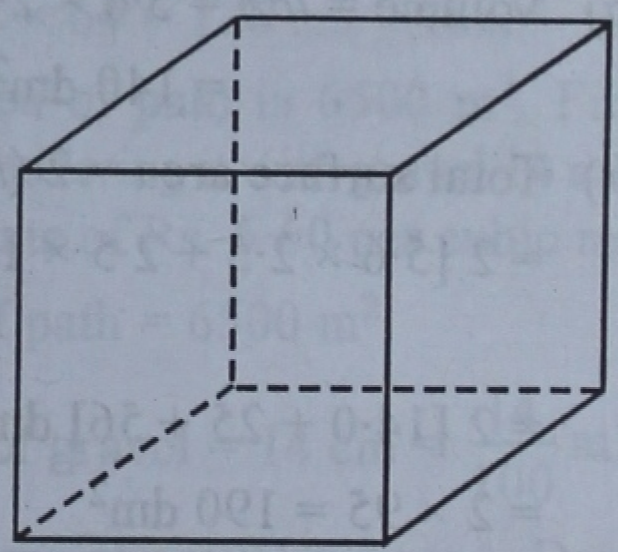
(*ii*) Diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.

(*iii*) Total Surface Area of the cuboid = $2\,(lb + bh + lh)$ sq. units.

(*iv*) Lateral Surface Area of the cuboid = $[2\,(l + b) \times h]$ sq. units.

(*v*) Area of 4 walls of a room = $[2\,(l + b) \times h]$ sq. units.

2. **Cube.** Let edge of a cube = a units. Then,



Cube

(*i*) Volume of the cube = a^3 cubic units.

(*ii*) Diagonal of the cube = $(a\sqrt{3})$ units.

(*iii*) Total Surface Area of the cube = $(6\,a^2)$ sq. units.

(*iv*) Lateral Surface Area of the cube = $(4\,a^2)$ sq. units.

EXERCISE 21 (A)

Q. 1. Find the volume ; total surface area amd the lateral surface area of a rectangular solid having :

(*i*) length = 8·5 m, breadth = 6·4 m and height = 50 cm.

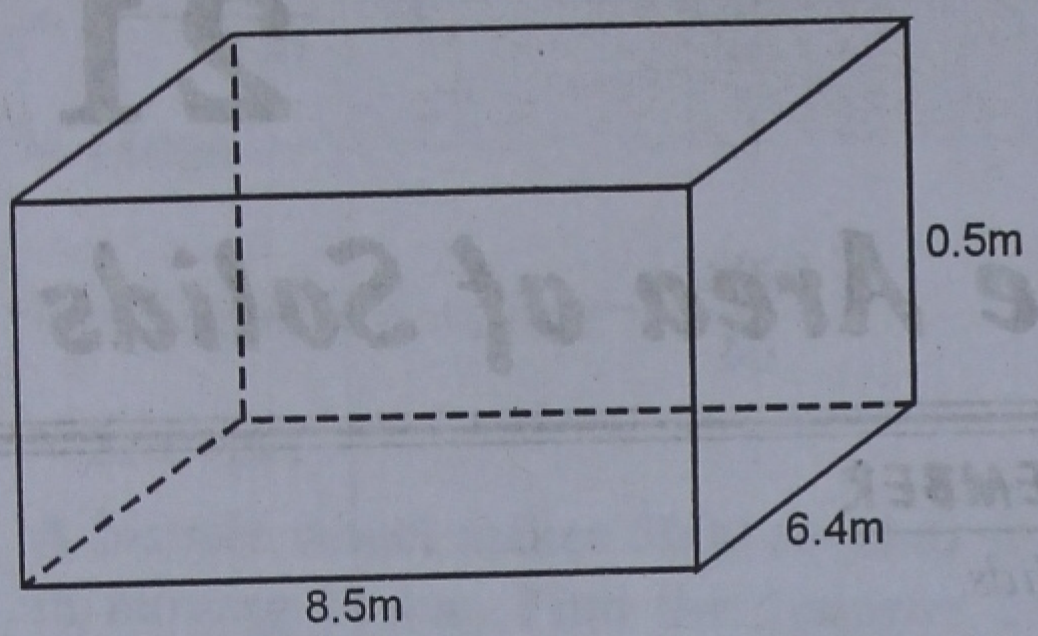
(*ii*) length = 5·6 dm, breadth = 22·5 dm and height = 1 m.

Sol. (*i*) Length = 8·5 m, Breadth = 6·4 m and Height = 50 cm = 0·5 m

(*a*) Volume = Length × Breadth × Height

$= 8{\cdot}5 \text{ m} \times 6{\cdot}4 \text{ m} \times 0{\cdot}5 \text{ m}$

$= 27{\cdot}2 \text{ m}^3$



(*b*) Total surface area $= 2\ (lb \times bh + hl)$

$= 2\ [8{\cdot}5 \times 6{\cdot}4 + 6{\cdot}4 \times 0{\cdot}5 + 0{\cdot}5 \times 8{\cdot}5]\ m^2$

$= 2\ [54{\cdot}4 + 3{\cdot}2 + 4{\cdot}25]\ m^2$

$= 2 \times 61{\cdot}85 = 123{\cdot}7\ m^2$

(*c*) Lateral surface area $= 2\ (l + b)\ h$

$= 2\ (8{\cdot}5 + 6{\cdot}4) \times 0{\cdot}5\ m^2$

$= 2 \times 14{\cdot}9 \times 0{\cdot}5 = 14{\cdot}9\ m^2$ **Ans.**

(*ii*) Length $(l) = 5{\cdot}6$ dm

Breadth $(b) = 2{\cdot}5$ dm

Height $(h) = 1$ m $= 10$ dm

(*a*) Volume $= lbh = 5{\cdot}6 \times 2{\cdot}5 \times 10$

$= 140\ dm^2$

(*b*) Total surface area $= 2\ (lb + bh + hl)$

$= 2\ [5{\cdot}6 \times 2{\cdot}5 + 2{\cdot}5 \times 10 + 10 \times 5{\cdot}6]\ dm^2$

$= 2\ [14{\cdot}0 + 25 + 56]\ dm^2$

$= 2 \times 95 = 190\ dm^2$

(*c*) Lateral surface area $= 2\ (l + b) \times h$

$= 2\ (5{\cdot}6 + 2{\cdot}5) \times 10\ dm^2$

$= 2 \times 8{\cdot}1 \times 10 = 162\ dm^2$ **Ans.**

Q. 2. The volume of a rectangular wall is $33\ m^3$. If its length is 16·5 m and height 8 m, find the width of the wall.

Sol. Volume of rectangular wall $= 33\ m^2$

Length of wall $(l) = 16{\cdot}5$ m

Height of wall $(h) = 8$ m

Let width of wall $= b$

then $lbh =$ volume

$\Rightarrow 16{\cdot}5 \times 8 \times b = 33$

$\Rightarrow b = \dfrac{33}{16{\cdot}5 \times 8} = \dfrac{1}{4}\,m = 0{\cdot}25\,m$ **Ans.**

Q. 3. Find the number of bricks, each measuring 25 cm × 12·5 cm × 7·5 cm, required to construct a wall 6 m long, 5 m high and 50 cm thick, while the cement and the sand mixture occupies $\dfrac{1}{20}$th of the volume of the wall.

Sol. Volume of one brick

$= 25\ cm \times 12{\cdot}5\ cm \times 7{\cdot}5\ cm$

$= \dfrac{25}{100} \times \dfrac{12{\cdot}5}{100} \times \dfrac{7{\cdot}5}{100}\,m^3$

$= \dfrac{1}{4} \times \dfrac{1}{8} \times \dfrac{3}{40}\,m^3$

$= \dfrac{3}{1280}\,m^3$

Length of wall $(l) = 6$ m

Height of wall $(h) = 5$ m

and Thickness $(b) = \dfrac{50}{100}\,m = \dfrac{1}{2}\,m$

$\therefore$ Volume $= lbh = 6 \times \dfrac{1}{2} \times 5 = 15\,m^3$

Volume of cement and sand

$= \dfrac{1}{20}$ of $15\ m^3 = \dfrac{3}{4}\,m^3$

$\therefore$ Volume of bricks $= 15 - \dfrac{3}{4}$

$= \dfrac{60-3}{4} = \dfrac{57}{4}\,m^3$

$\therefore$ No. of bricks

$= \dfrac{\text{Volume of total bricks}}{\text{Volume of one brick}}$

$= \dfrac{\frac{57}{4}}{\frac{3}{1280}} = \dfrac{57}{4} \times \dfrac{1280}{3}$

$= 19 \times 320$

$= 6080$ **Ans.**

Q. 4. A class room is 12·5 m long, 6·4 m broad and 5 m high. How many students can accommodate if each student needs 1·6 m^2 of floor area ? How many cubic metres of air would each student get ?

Sol. Length of room (l) = 12·5 m

Width of room (b) = 6·4 m

and Height (h) = 5 m

$\therefore$ Volume of air inside the room

$= lbh = 12{\cdot}5 \times 6{\cdot}4 \times 5 \text{ m}^3$

$= 400 \text{ m}^3$

Area of floor of the room = $l \times b$

$= 12{\cdot}5 \times 6{\cdot}4 \text{ m}^2$

$= 80 \text{ m}^2$

For each student area required = 1·6 m^2

$\therefore$ No. of students $= \dfrac{80}{1{\cdot}6} = \dfrac{80 \times 10}{16} = 50$

and each student required the air

$$= \frac{\text{Volume of air}}{\text{No. of students}}$$

$$= \frac{400}{50} = 8 \text{ m}^3 \quad \textbf{Ans.}$$

Q. 5. Find the length of the longest rod that can be placed in a room measuring 12 m × 9 m × 8 m.

Sol. Length of room (l) = 12 m

Breadth (b) = 9 m

and height (h) = 8 m

$\therefore$ The longest rod required to place in the room

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(12)^2 + (9)^2 + (8)^2}$$

$$= \sqrt{144 + 81 + 64} \text{ m}$$

$$= \sqrt{289} = 17 \text{ m} \quad \textbf{Ans.}$$

Q. 6. The volume of a cuboid is 14400 cm^3 and its height is 15 cm. The cross-section of the cuboid is a rectangle having its sides in the ratio 5 : 3. Find the perimeter of the cross-section.

Sol. Volume of cuboid = 14400 cm^3

Height (h) = 15 cm

$\therefore$ Length × Breadth

$$= \frac{\text{Volume}}{h} = \frac{14400}{15} \text{cm}^2$$

$= 960 \text{ cm}^2$

Ratio in remaining sides = 5 : 3

Let length = $5x$

and breadth = $3x$

$\therefore \quad 5x \times 3x = 960$

$\Rightarrow \quad 15x^2 = 960$

$\Rightarrow \quad x^2 = 64 = (8)^2$

$\therefore \quad x = 8$

Length = $5x = 8 \times 5 = 40$ cm

and breadth = $3x = 8 \times 3 = 24$ cm

$\therefore$ Perimeter of rectangular cross-section

$= 2(l + b) = 2(40 + 24)$ cm

$= 2 \times 64 = 128$ cm **Ans.**

Q. 7. The area of path is 6500 m^2. Find the cost of covering it with gravel 14 cm deep at the rate of Rs. 5·60 per cubic metre.

Sol. Area of path = 6500 m^2

Depth of gravel = 14 cm $= \dfrac{14}{100}$ m

$\therefore$ Volume of gravel = Area × Depth

$$= 6500 \times \frac{14}{100} = 910 \text{ m}^3$$

Rate of covering the gravel

= Rs. 5·60 per m^3

$\therefore$ Total cost = Rs. 10 × 5·60

$$= \text{Rs. } \frac{910 \times 560}{100} = \text{Rs. } 5096 \quad \textbf{Ans.}$$

Q. 8. The cost of papering the four walls of a room 12 m long at Rs. 6·50 per square metre is Rs. 1638 and the cost of matting the floor at Rs. 3·50 per square metre is Rs. 378. Find the height of the room.

Sol. Rate of papering the walls

$= \text{Rs. } 6{\cdot}50 \text{ per m}^2$

Total cost = Rs. 1638

$\therefore$ Area of four walls $= \dfrac{1638}{6{\cdot}50}$

$= \dfrac{1638 \times 100}{650} \text{ m}^2 = 252 \text{ m}^2$

Rate of matting the floor

$= \text{Rs. } 3{\cdot}50 \text{ per m}^2$

Total cost = Rs. 378

$\therefore$ Area of floor $= \dfrac{378}{3{\cdot}50}$

$= \dfrac{378 \times 100}{350} \text{ m}^2 = 108 \text{ m}^2$

Length of room = 12 m

$\therefore$ Breadth of room $= \dfrac{\text{Area of floor}}{\text{Length}}$

$= \dfrac{108}{12} = 9 \text{ m}$

But Area of 4-walls $= 2\,(l + b)\,h$

$\therefore \quad 2\,(l + b)\,h = 252$

$\Rightarrow \quad 2\,(12 + 9)\,h = 252$

$\Rightarrow \quad 2 \times 21\,h = 252$

$\Rightarrow \quad h = \dfrac{252}{2 \times 21}$

$\Rightarrow \quad h = 6$

Hence height of the room = 6 m **Ans.**

Q. 9. The dimensions of a field are 15 m × 12 m. A pit 7·5 m × 6 m × 1·5 m is dug in one corner of the field and the earth removed from it, is evenly spread over the remaining area of the field, calculate, by how much the level of the field is raised ?

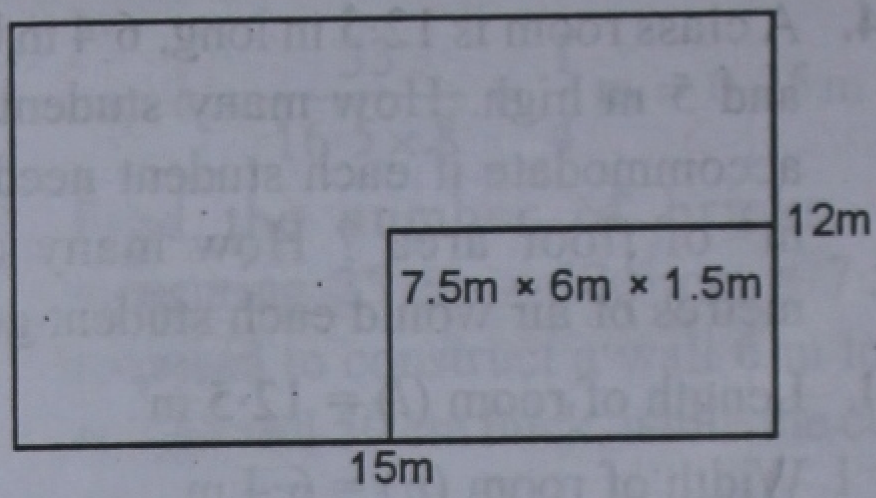
Sol. Length of field (l) = 15 m

and breadth = 12 m

$\therefore$ Area of total field $= l \times b$

$= 15 \times 12 = 180 \text{ m}^2$

Length of pit = 7·5 m



Breadth = 6 m

and depth = 1·5 m

$\therefore$ Area of the pit $= l \times b$

$= 7{\cdot}5 \times 6 = 45 \text{ m}^2$

and volume of earth removed $= l.b.h$

$= 7{\cdot}5 \times 6 \times 1{\cdot}5 \text{ m}^3$

$= 67{\cdot}5 \text{ m}^3$

Area of field leaving pit = 180 – 45

$= 135 \text{ m}^2$

and volume of earth removed $= l.b.h$

$= 7{\cdot}5 \times 6 \times 1{\cdot}5 \text{ m}^3$

$= 67{\cdot}5 \text{ m}^3$

Area of field leaving pit = 180 – 45

$= 135 \text{ m}^2$

$\therefore$ Level (height) of the earth in the field

$= \dfrac{\text{Volume of earth}}{\text{Area of remaining part}}$

$= \dfrac{67{\cdot}5}{135} = \dfrac{675}{10 \times 135} \text{ m}$

$= 0{\cdot}5 \text{ m}$

= 50 cm **Ans.**

Q. 10. The sum of length, breadth and depth of a cuboid is 19 cm, and the length of its diagonal is 11 cm. Find the surface area of the cuboid.

Sol. Let l, b and h be the length, breadth and depth of the cuboid, then

$l + b + h = 19 \text{ cm}$

and $\sqrt{l^2 + b^2 + h^2} = 11 \text{cm}$

$\Rightarrow l^2 + b^2 + h^2 = (11)^2 = 121$

Now we have to find the surface area of the cuboid *i.e.* $2\,(lb + bh + hl)$

We know that $(l+b+h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$

$\Rightarrow (19)^2 = 121 + 2(lb + bh + hl)$

$\Rightarrow 2[lb + bh + hl] = (19)^2 - 121$

$= 361 - 121 = 240 \text{ cm}^2$

$\therefore$ Surface area of the cuboid

$= 240 \text{ cm}^2$ **Ans.**

Q. 11. Three cubes, each of side 6 cm are joined end to end. Find the surface area of the resulting cuboid.

Sol. Each side of a cube = 6 cm

$\therefore$ Length of 3 cubes joined end to end $(l) = 6 \times 3 = 18$ cm

Breadth $(b) = 6$ cm

and height $(h) = 6$ cm

$\therefore$ Surface area of cuboid so formed

$= 2(lb + bh + hl)$

$= 2(18 \times 6 + 6 \times 6 + 6 \times 18) \text{ cm}^2$

$= 2(108 + 36 + 108) = 2 \times 252 \text{ cm}^2$

$= 504 \text{ cm}^2$ **Ans.**

Q. 12. Find *(i)* the volume *(ii)* the total surface area *(iii)* the lateral surface area *(iv)* the length of the diagonal of a cube of side 10 cm.

Sol. Side of cube $(a) = 10$ cm

(i) Volume $= a^3 = (10)^3 = 1000 \text{ cm}^3$

(ii) Total surface area $= 6a^2$

$= 6 \times (10)^2 = 6 \quad 100 = 1000 \text{ cm}^2$

(iii) Lateral surface area $= 4a^2$

$= 4 \times (10)^2 \text{ cm}^2$

$= 4 \times 100 = 400 \text{ cm}^2$

(iv) Length of its diagonal $= \sqrt{3a^2} = \sqrt{3}.a$

$= 1{\cdot}732 \times 10 = 17{\cdot}32$ cm **Ans.**

Q. 13. The surface area of a cube is 1536 cm^2. Find :

(i) The length of its edge. *(ii)* Its volume.

(iii) The volume of its material whose thickness is 5 mm.

Sol. Surface area of a cube = 1536 cm^2

Let each edge of the cube = a

$\therefore 6a^2 = 1536$

$\Rightarrow a^2 = \frac{1536}{6} = 256 = (16)^2$

$\therefore a = 16$

(i) Length of its edge = 16 cm.

(ii) Volume $= a^3 = (16)^3 = 4096 \text{ cm}^2$.

(iii) Thickness of its material = 5 mm

$= \frac{5}{10} = 0{\cdot}5$ cm.

$\therefore$ Inner side $= 16 - 0{\cdot}5 \times 2 = 16 - 1$

$= 15$ cm

$\therefore$ Volume of the material used

$= (16)^3 - (15{\cdot}0)^3$

$= (4096 - 3775) \text{ cm}^3$

$= 721 \text{ cm}^3$ **Ans.**

Q. 14. Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol. Volume of a cube = 512 cm^2

Let side of cube = a

then $a^3 = 512 = (8)^3$

$\therefore a = 8$

$\therefore$ By joining end to end two such cubes, length of cuboid so formed $(l) = 8 + 8$

$= 16$ cm

breadth $(h) = 8$ cm

and height $(h) = 8$ cm

$\therefore$ Surface area $= 2(lb + bh + hl)$

$= 2(16 \times 8 + 8 \times 8 + 8 \times 16) \text{ cm}^2$

$= 2(128 + 64 + 128) \text{ cm}^2$

$= 2 \times 320$

$= 640 \text{ cm}^2$ **Ans.**

Q. 15. *(i)* How many cubic cm of iron are there is an open box whose external dimensions are 36 cm, 25 cm and 16·5 cm, the iron being 1·5 cm thick throughout.

(*ii*) If 1 cm^3 of iron weighs 15 *g*, find the weight of the empty box in kg.

Sol. (*i*) Outer length of open box $(l_1) = 36$ cm

breadth $(b_1) = 25$ cm

and height $(h_1) = 16{\cdot}5$ cm

Thickness of iron = $1{\cdot}5$ cm

$\therefore$ Inner length $(l_2) = 36 - (2 \times 1{\cdot}5)$

$= 36 - 3 = 33$ cm

inner breadth $(b_2) = 25 - (2 \times 1{\cdot}5)$

$= 25 - 3 = 22$ cm

and inner height $(h_2) = 16{\cdot}5 - 1{\cdot}5$

$= 15$ cm

$\therefore$ Volume of iron = Volume of outer box − Volume of inner box

$= [(36 \times 25 \times 16{\cdot}5) - 33 \times 22 \times 15)]$ cm^3

$= 14850 - 10890 = 3960$ cm^3

(*ii*) Weight of 1 $cm^3 = 15$ *g*

$\therefore$ Total weight of the box $= 3960 \times 15$ *g*

$= \frac{3960 \times 15}{1000}$ kg $= 59{\cdot}4$ kg **Ans.**

Q. 16. A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of two smaller cubes are 6 cm and 8 cm, find the edge of third smaller cube.

Sol. Edge of metal cube = 12 cm

$\therefore$ Volume $= (12)^3 = 1728$ cm^3

Edge of first smaller cube = 6 cm

$\therefore$ Volume $= (6)^3 = 216$ cm^3

Edge of second smaller cube = 8 cm

$\therefore$ Volume $= (8)^3 = 512$ cm^3

$\therefore$ Volume of third smaller volume

$= 1728 - (216 + 512)$

$= 1728 - 728 = 1000$ cm^3

$\therefore$ Edge of third cube $= (1000)^{1/3}$ cm

$= [(10)^3]^{1/3}$ cm

$= 10$ cm **Ans.**

Q. 17. The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find (*i*) the edge of the cube (*ii*) the surface area of the cube.

Sol. Length of the metallic cuboid (*l*)

$= 100$ cm

Breadth (*b*) = 80 cm

and height (*h*) = 64 cm

$\therefore$ Volume $= lbh = 100 \times 80 \times 64$

$= 512000$ cm^3

$\therefore$ Volume of cube so formed

$= 512000$ cm^3

(*i*) Edge of the cube $(a) = (512000)^{1/3}$

$= [(80)^3]^{1/3} = 80$ cm **Ans.**

(*ii*) Surface area $= 6\,a^2 = 6\,(80)^2$

$= 6 \times 80 \times 80$ cm^2

$= 38400$ cm^2 **Ans.**

Q. 18. The square on the diagonal of a cube has an area of 1875 cm^2, calculate (*i*) the side of the cube (*ii*) the total surface area of the cube.

Sol. $(\text{Diagonal})^2$ of cube = 1875 cm^2

(*i*) Let side of cube = *a*

$\therefore (\sqrt{3}a)^2 = 1875$

$\Rightarrow 3\,a^2 = 1875 \Rightarrow a^2 = \frac{1875}{3}$

$\Rightarrow a^2 = 625 = (25)^2$

$\therefore a = 25$

Hence side of the cube = 25 cm.

(*ii*) Total surface area of the cube

$= 6\,a^2 = 6\,(25)^2 = 6 \times 625$ cm^2

$= 3750$ cm^2 **Ans.**

Q. 19. The areas of three adjacent faces of a cuboid are *x*, *y* and *z* units. If the volume is V cubic units, prove that $V^2 = xyz$.

Sol. Let length of cuboid = *l*

breadth = *b*

and height = h

$\therefore\ x = lb, y = bh$ and $z = hl$

and $v = lbh$

Now $x.y.z = lb.bh.hl = l^2.b^2.h^2$

$= (lbh)^2 = V^2$

Hence $V^2 = xyz$.

Hence proved.

Q. 20. The diagonal of a cube is $16\sqrt{3}$ cm. Find its surface area and volume.

Sol. Diagonal of a cube = $16\sqrt{3}$ cm

Let a be the length of the edge of cube

then $\sqrt{3}\,a = 16\sqrt{3} \Rightarrow a = 16$

(*i*) Surface area = $6a^2 = 6(16)^2$ cm²

$= 6 \times 256 = 1536$ cm² and

(*ii*) Volume = $a^3 = (16)^3 = 16 \times 16 \times 16$ cm³

$= 4096$ cm³ **Ans.**

Q. 21. Water flows into a tank 150 m long and 100 m broad through a rectangular pipe whose cross-section is 5 dm × 3·5 dm ; at the speed of 15 km/hr. In what time, will the water be 7 m deep ?

Sol. Length of tank = 150 m

and breadth = 100 m

and height of water in the tank (h) = 7 m

$\therefore$ Volume of water in the tank = lbh

$= 150 \times 100 \times 7$ m³

$= 105000$ m³

Cross-section of rectangular pipe

= 5 dm × 3·5 dm

$= \frac{5}{10}\text{ m} \times \frac{3{\cdot}5}{10}\text{ m}$

$\therefore$ Length of water flow

$= \frac{\text{Volume of water}}{\text{Area of cross-section}}$

$= \frac{105000 \times 10 \times 10}{5 \times 3{\cdot}5}\text{ m}$

$= \frac{105000 \times 100 \times 10}{5 \times 35}\text{ m}$

$= 600000$ m

$= \frac{600000}{1000} = 600$ km

Speed of water flow = 15 km/hr

$\therefore$ Time taken $= \frac{600}{15} = 40$ hr **Ans.**

Q. 22. A rectangular tank is 25 m long and 7·5 m deep. If 540 m³ of water be drawn off the tank, the level of water in the tank goes down by 1·8 m.

Calculate : (*i*) the width of the tank,

(*ii*) the capacity of the tank.

Sol. Length of water tank (l) = 25 m

Depth (h) = 7·5 m

Volume of water taken = 540 m³

and height of water level = 1·8 m

(*i*) Let width of the tank = b

$\therefore lbh$ = Volume of water

$25 \times b \times 1{\cdot}8 = 540$

$b = \frac{540}{25 \times 1{\cdot}8} = \frac{540 \times 10}{25 \times 1{\cdot}8}$

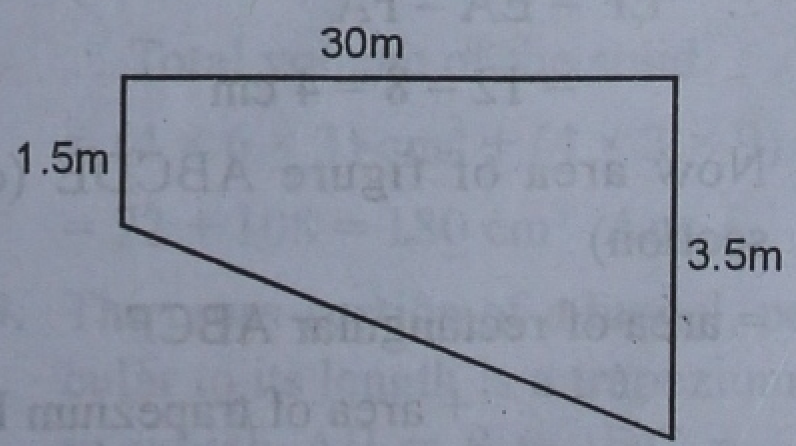
$= 12$ m **Ans.**

(*ii*) Capacity of the tank = lbh

$= 25 \times 12 \times 7{\cdot}5$ m³

$= 2250$ m³ **Ans.**

Q. 23. A swimming pool is 30 m long and 12 m broad. Its shallow and deep ends are 1·5 m and 3·5 m deep respectively. If the bottom of the pool slopes uniformly, how many litres of water will fill the pool ?



Sol. Length of swimming pool (l) = 30 m

breadth (b) = 12 m

Depth of its ends = 1·5 m and 3·5 m

∴ Volume of water

$$= \left[\frac{1}{2}(1\cdot5 + 3\cdot5) \times 30\right] \times 12\,m^3$$

$$= \frac{1}{2} \times 5 \times 30 \times 12\,m^3$$

$$= 900\,m^3$$

∴ Capacity of water in litres

= 900 × 1000 litres

($1\,m^3$ = 1000 litres)

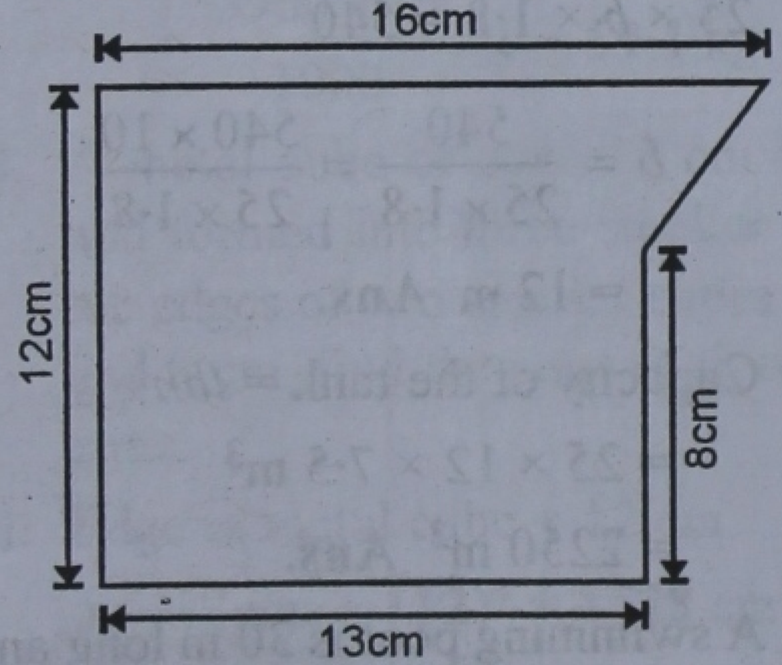
= 900000 litres **Ans.**

Q. 24. The cross-section of a piece of metal 2 m in length is shown in the adjoining figure. Calculate :

(*i*) the area of its cross-section ;

(*ii*) the volume of piece of metal ;

(*iii*) the weight of piece of metal to the nearest kg, if 1 cm^3 of the metal weighs 6·5 g.



Sol. From C, draw CF ∥ AB

then CF = AB = 13 cm

AF = CB = 8 cm

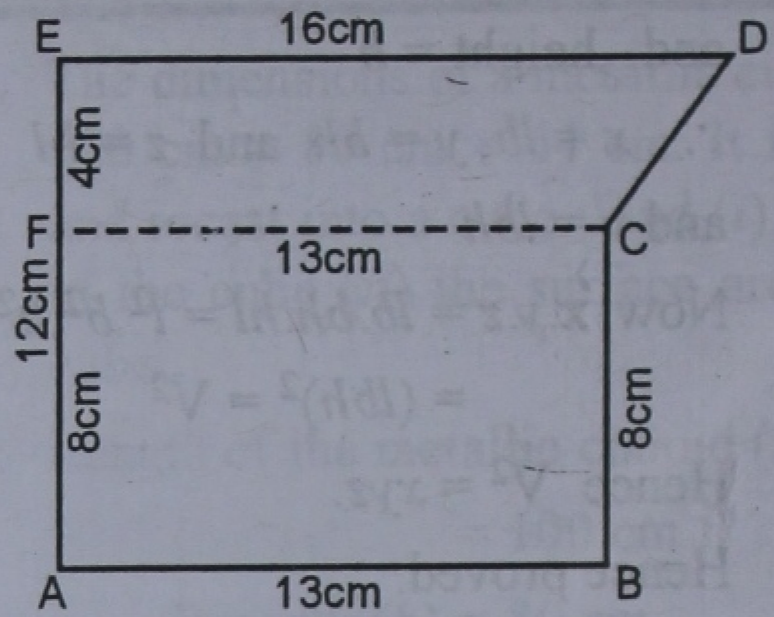
∴ EF = EA – FA

= 12 – 8 = 4 cm

(*i*) Now area of figure ABCDE (cross-section)

= area of rectangular ABCF

+ area of trapezium FCDE



$$= 13\text{ cm} \times 8\text{ cm} + \frac{1}{2}(13 + 16) \times 4\text{ cm}^2$$

= 104 + 29 × 2 = 104 + 58

= 162 cm^2 **Ans.**

(*ii*) Length of the piece = 2 m = 200 cm

∴ Volume of the metal piece

= Area × Length

= 162 × 200 cm^3 = 32400 cm^3

$$= \frac{32400}{100 \times 100} = 3{\cdot}24\,m^3$$ **Ans.**

(*iii*) Weight of 1 cm^3 metal = 6·5 g

∴ Total weigh of the metal piece

= 32400 × 6·5 g

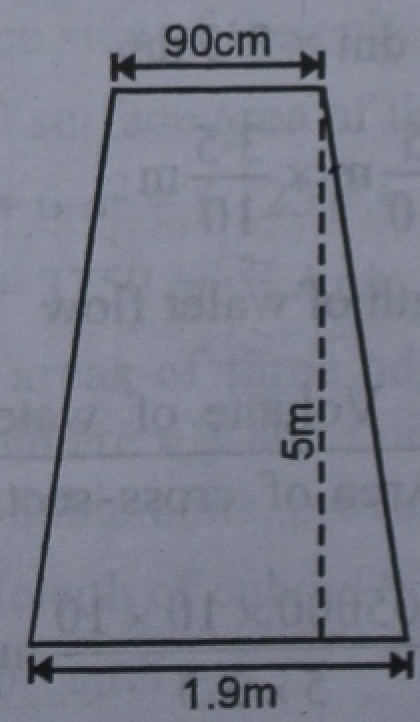
= 210600 g

= 211 kg (approx) **Ans.**

Q. 25. The adjoining figure shows the cross-section of a concrete wall to be constructed. It is 1·9 m wide at the bottom, 90 cm wide at the top and 5 m high. If its length is 20 m, find :

(*i*) the cross-sectional area ;

(*ii*) the volume of the concrete in the wall.



Sol. (*i*) Area of the cross-section which is in form of trapezium

$$= \frac{1}{2}(1\cdot9\text{ m} + 0\cdot9\text{ m}) \times 5\text{ m}$$

$$= \frac{1}{2} \times 2\cdot8 \times 5\text{ m} = 7\text{ m}^2$$

(*ii*) Length of the cross-section = 20 m

∴ Volume = Area × Length

$= 7 \times 20 = 140\text{ m}^3$ **Ans.**

Q. 26. A square brass plate of side x cm is 1 mm thick and weighs 5·44 kg. If 1 cm³ of brass weighs 8·5 g, find the value of x.

Sol. Side of square plate = x cm

Thickness = 1 mm

Total weight = 5·44 kg

Weight of 1 cm³ = 8·5 g

∴ Total volume of the plate

$$= \frac{5\cdot44 \times 1000}{8\cdot5}\text{ cm}^3$$

$= 640\text{ cm}^3$

∴ According to the condition

$$x \times x \times \frac{1}{10} = 640 \Rightarrow x^2 = 6400$$

$\Rightarrow x = \sqrt{6400} = 80$ cm **Ans.**

Q. 27. The area of cross-section of a rectangular pipe is 5·4 cm² and water is pumped out of it at the rate of 27 kmph. Find, in litres, the volume of water which flows out of the pipe in 1 minute.

Sol. Area of cross-section of rectangular pipe = 5·4 cm²

Speed of water pumped out = 27 kmph

Time = 1 minute

∴ Length of water flow $= \frac{27}{60} \times 1000\text{ m}$

$= 450$ m

∴ Volume of water = Area × Length

$$= 450 \times \frac{5\cdot4}{100 \times 100}\text{ m}^3$$

$$= \frac{450 \times 54}{100 \times 100 \times 10}\text{ m}^3$$

$$= \frac{24300}{100000}\text{ m}^3$$

$$= \frac{243}{1000}\text{ m}^3$$

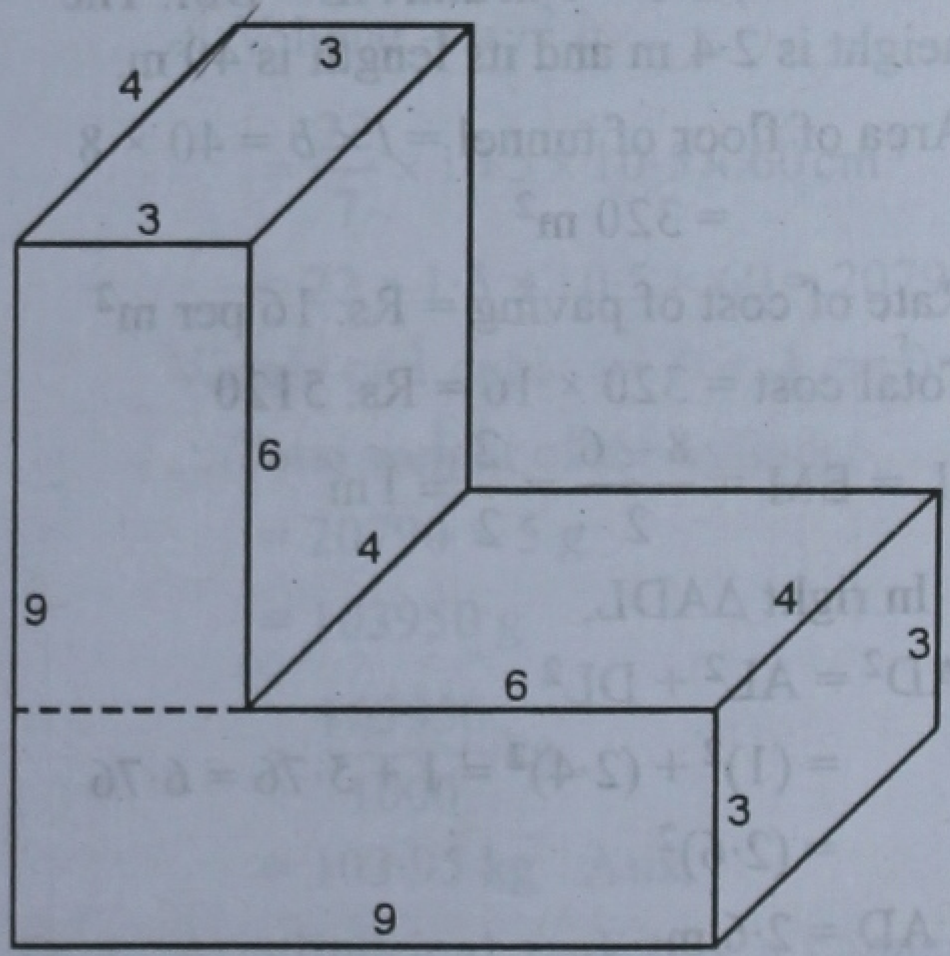
∴ Volume of water in litres

$$= \frac{243}{1000} \times 1000$$

(1 m³ = 1000 litres)

= 243 litres **Ans.**

Q. 28. The adjoining figure shows a solid of uniform cross section. Find the volume of the solid. It is being given that all the measurements are in cm and each angle in the figure is a right angle.



Sol. In the figure, two cuboid are joined together where dimensions are (4 × 6 × 3) cm and (4 × 3 × 9) cm.

∴ Total volume of the solid

= (4 × 6 × 3) cm³ + (4 × 3 × 9) cm³

= 72 + 108 = 180 cm³ **Ans.**

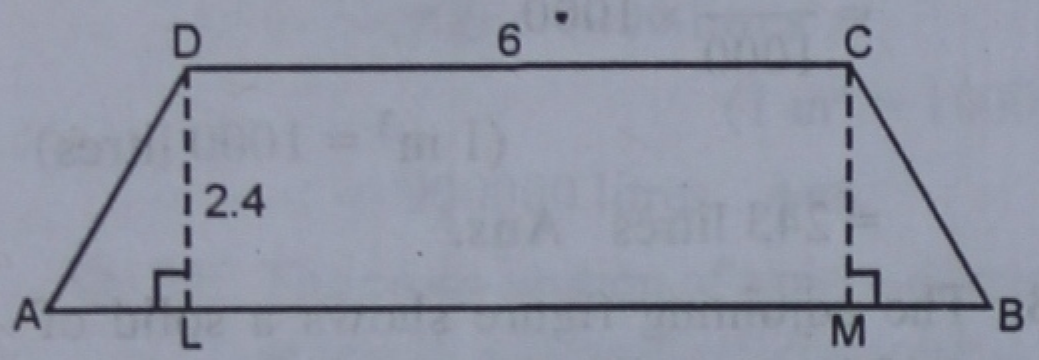
Q. 29. The cross-section of a tunnel, perpendicular to its length is a trapezium ABCD in which AB = 8 m, DC = 6 m and

AL = BM. The height of the tunnel is 2·4 m and its length is 40 m.

Find :

(*i*) the cost of paving the floor of the tunnel at Rs. 16 per m^2.

(*ii*) the cost of painting the internal surface of the tunnel, excluding the floor at the rate of Rs. 5 per m^2.



Sol. The cross-section of a tunnel is of the trapezium shaped ABCD in which AB = 8 m, DC = 6 m and AL = BM. The height is 2·4 m and its length is 40 m.

(*i*) Area of floor of tunnel = $l \times b = 40 \times 8$

$= 320 \text{ m}^2$

Rate of cost of paving = Rs. 16 per m^2

Total cost = 320 × 16 = Rs. 5120

(*ii*) $AL = BM = \frac{8-6}{2} = \frac{2}{2} = 1 \text{ m}$

∴ In right ΔADL,

$AD^2 = AL^2 + DL^2$

$= (1)^2 + (2{\cdot}4)^2 = 1 + 5{\cdot}76 = 6{\cdot}76$

$= (2{\cdot}6)^2$

∴ AD = 2·6 m

Now perimeter of the cross-section of the tunnel

$= 8 + 2{\cdot}6 + 2{\cdot}6 + 6$

$= 19{\cdot}2 \text{ m}$

Length = 40 m

∴ Internal surface area of the tunnel (except floor)

$= 19{\cdot}2 \times 40 - 40 \times 8$

$= 768 - 320 = 448 \text{ m}^2$

Rate of painting = Rs. 5 per m^2

∴ Total cost of painting = Rs. 5 × 448

= Rs. 2240 **Ans.**

EXERCISE 21(B)

Q. 1. Find the curved surface area and the total surface area of the cylinder for which :

(*i*) $h = 16$ cm, $r = 10{\cdot}5$ cm

(*ii*) $h = 5$ cm, $r = 21$ cm

(*iii*) $h = 20$ cm, $r = 14$ cm

(*iv*) $h = 1$ *m*, $r = 1{\cdot}4$ cm

Sol. (*i*) Height (*h*) of cylinder = 16 cm

Radius (*r*) = 10·5 cm

(*a*) ∴ Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 10{\cdot}5 \times 16 \text{ cm}^2$

$= 44 \times 1{\cdot}5 \times 16 \text{ cm}^2 = 1056 \text{ cm}^2$

(*b*) Total surface area = $2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 10{\cdot}5 \times (16 + 10{\cdot}5) \text{ cm}^2$

$= 44 \times 1{\cdot}5 \times 26{\cdot}5 \text{ cm}^2$

$= 1749 \text{ cm}^2$

(*ii*) Height of the cylinder (*h*) = 5 cm and radius (*r*) = 21 cm (*a*) $\therefore$ Curved surface area $= 2\,\pi\, rh$

$= 2 \times \frac{22}{7} \times 21 \times 5\,\text{cm}^2$

$= 44 \times 15 = 660\ \text{cm}^2$

(*b*) and total surface area $= 2\,\pi\, r\,(h + r)$

$= 2 \times \frac{22}{7} \times 21 \times (5 + 21)\,\text{cm}^2$

$= 44 \times 3 \times 26\ \text{cm}^2$

$= 3432\ \text{cm}^2$

(*iii*) Height of the cylinder (h) = 20 cm

Radius (r) = 14 cm

(*a*) $\therefore$ Curved surface area $= 2\,\pi\, rh$

$= 2 \times \frac{22}{7} \times 14 \times 20\,\text{cm}^2$

$= 44 \times 40 = 1760\ \text{cm}^2$

(*b*) and total surface area $= 2\,\pi\, r\,(h + r)$

$= 2 \times \frac{22}{7} \times 14\,(20 + 14)\,\text{cm}^2$

$= 44 \times 2 \times 34 = 2992\ \text{cm}^2$

(*iv*) Height of the cylinder (h) = 1 m

= 100 cm

and radius (r) = 1·4 cm

(*a*) Curved surface area $= 2\,\pi\, rh$

$= 2 \times \frac{22}{7} \times 144 \times 100\,\text{cm}^2$

$= 44 \times 0{\cdot}2 \times 100\ \text{cm}^2$

$= 880\ \text{cm}^2$

(*b*) Total surface area $= 2\,\pi\, r\,(h + r)$

$= 2 \times \frac{22}{7} \times 1{\cdot}4\,(100 + 1{\cdot}4)\,\text{cm}^2$

$= 44 \times 0{\cdot}2 \times 10{\cdot}14\ \text{cm}^2$

$= 892{\cdot}32\ \text{cm}^2$ **Ans.**

Q. 2. Find the volume of the cylinder in which :

(*i*) Height = 21 cm and Base Radius = 5 cm

(*ii*) Diameter = 28 cm and Height = 40 cm.

Sol. (*i*) Height of cylinder (h) = 21 cm

and radius of the base (r) = 5 cm

$\therefore$ Volume $= \pi\, r^2\, h$

$= \frac{22}{7} \times 5 \times 5 \times 21\,\text{cm}^3$

$= 1650\ \text{cm}^3$

(*ii*) Radius of the base of cylinder

$= \frac{28}{2} = 14\,\text{cm}$

Height (h) = 40 cm

$\therefore$ Volume $= \pi\, r^2\, h$

$= \frac{22}{7} \times 14 \times 14 \times 40\,\text{cm}^3$

$= 22 \times 28 \times 40 = 24640\ \text{cm}^3$ **Ans.**

Q. 3. Find the weight of the solid cylinder of radius 10.5 cm and height 60 cm, if the material of the cylinder weighs, 5 grams per cu. cm.

Sol. Radius of solid cylinder (r) = 10·5 cm

Height (h) = 60 cm

$\therefore$ Volume $= \pi\, r^2\, h$

$= \frac{22}{7} \times 10{\cdot}5 \times 10{\cdot}5 \times 60\,\text{cm}^3$

$= 22 \times 1{\cdot}5 \times 10{\cdot}5 \times 60 = 20790\ \text{cm}^3$

Weight of 1 cubic cm (*i.e.* 1 cm^3) = 5 *g*

$\therefore$ Total weight of the cylinder

$= 20790 \times 5\ g$

$= 103950\ g$

$= \frac{103950}{1000}\ \text{kg}$

$= 103{\cdot}95$ kg **Ans.**

Q. 4. A cylindrical tank has a capacity of 6160 m^3. Find its depth if its radius is 14 m. Also find the cost of painting its curved surface at Rs. 3 per m^2.

Sol. Volume of cylinder = 6160 m^3

Radius (r) = 14 m

Let depth = h m

$\therefore \pi\, r^2\, h = 6160$

$\Rightarrow \frac{22}{7} \times 14 \times 14 \times h = 6160$

$\Rightarrow \quad h = \frac{6160 \times 7}{22 \times 14 \times 14} = 10\,\text{m}$

Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 10\,\text{m}^2$

$= 880\ \text{m}^2$

Rate of painting its curved surface

= Rs. 3 per m^2

$\therefore$ Total cost = 880×3 = Rs. 2640 **Ans.**

Q. 5. The curved surface area of a cylinder is 4400 cm^2 and the circumference of its base is 110 cm. Find the height and the volume of the cylinder.

Sol. Curved surface area = 4400 cm^2

Circumference of its base = 110 cm

Let radius be r and height h

$\therefore \quad 2\pi r = 110 \Rightarrow 2 \times \frac{22}{7} r = 110$

$\Rightarrow \quad r = \frac{110 \times 7}{22 \times 2} = \frac{35}{2}\,\text{cm}$

and $2\pi r h = 4400$

$\Rightarrow 2 \times \frac{22}{7} \times \frac{35}{2} \times h = 4400$

$\Rightarrow h = \frac{4400 \times 7 \times 2}{2 \times 22 \times 35} = 40$ cm

Now volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40\,\text{cm}^3$

$= 3850\ \text{cm}^3$ **Ans.**

Q. 6. The total surface area of a solid cylinder is 462 cm^2 and its curved surface area is one-third of its total surface area. Find the volume of the cylinder.

Sol. Total surface area of cylinder = 462 cm^2

and curved surface area

$= \frac{1}{3}$ of total surface area

$= \frac{1}{3} \times 462 = 154\,\text{cm}^2$

Let r be the radius and h be its height, then

$2\pi rh = 154 \Rightarrow 2 \times \frac{22}{7} rh = 154$

$\Rightarrow rh = 154 \times \frac{7}{44} = \frac{49}{2}$ ...(*i*)

and $2\pi r^2 = 462 - 154 = 308$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 308$

$\Rightarrow r^2 = \frac{308 \times 7}{2 \times 22}$

$\Rightarrow r^2 = 49 = (7)^2$

$\therefore \quad r = 7$ cm

Now substituting the value of r in (*i*)

$7 \times h = \frac{49}{2} \Rightarrow h = \frac{49}{2 \times 7} = \frac{7}{2}$ cm

$\therefore$ Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times \frac{7}{2}\,\text{cm}^3$

$= 539\ \text{cm}^3$ **Ans.**

Q. 7. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 m^2, find its volume.

Sol. Let r be the radius and h be the height of the cylinder. Then $r + h = 37$ m

and total surface area = 1628 m^2

$\Rightarrow \quad 2\pi r(r + h) = 1628$

$\Rightarrow 2 \times \frac{22}{7} r \times 37 = 1628$

$\Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37}$

$\therefore \quad r = 7$

but $\quad r + h = 37 \Rightarrow 7 + h = 37$

$\therefore \quad h = 37 - 7 = 30$ m

Now volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 30\,\text{m}^3$

$= 4620\ \text{m}^3$ **Ans.**

Q. 8. Find the height of the solid circular cylinder having total surface area of 660 cm^2 and radius 5 cm.

Sol. Let h be the height of the cylinder

Radius = 5 cm

and total surface area = 660 cm^2

$$\Rightarrow \quad 2\pi r(h+r) = 660$$

$$\Rightarrow 2 \times \frac{22}{7} \times 5(h+5) = 660$$

$$\Rightarrow h + 5 = \frac{660 \times 7}{2 \times 22 \times 5} = 21$$

$\therefore \quad h = 21 - 5 = 16$ cm **Ans.**

Q. 9. Find the total surface area of a hollow cylinder open at both ends, if its length is 12 cm, external diameter is 8 cm and the thickness is 2 cm.

Sol. Length of hollow cylinder (h) = 12 cm

External diameter = 8 cm

$\therefore$ External radius (r) = $\frac{8}{2}$ cm = 4 cm

Thickness of the cylinder = 2 cm

$\therefore$ Inner radius = 4 – 2 = 2 cm

Now total surface area

$$= [2\pi rh + 2\pi rh + 2(\pi R^2 - \pi r^2)] \text{ cm}^2$$

$$= \left[\left(2 \times \frac{22}{7} \times 4 \times 12 + 2 \times \frac{22}{7} \times 2 \times 12\right) + 2\left(\frac{22}{7} \times 4 \times 4 - \frac{22}{7} \times 2 \times 2\right)\right] \text{cm}^2$$

$$= \left[\left(\frac{2112}{7} + \frac{1056}{7}\right) + 2\left(\frac{352}{7} - \frac{88}{7}\right)\right] \text{cm}^2$$

$$= \left[\frac{3168}{7} + 2 \times \frac{264}{7}\right] \text{cm}^2$$

$$= \frac{3168}{7} + \frac{528}{7} = \frac{3696}{7} = 528 \text{ cm}^2 \text{ **Ans.**}$$

Q. 10. Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2 m. In how much time will the cistern be filled ?

Sol. Diameter of cistern = 10 m

$\therefore$ Radius (r) = 5 m

and depth (h) = 2 m

$\therefore$ Volume of water filled in cistern

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 5 \times 5 \times 2 \text{ m}^3$$

$$= \frac{1100}{7} \text{ m}^3$$

Internal diameter of the circular pipe

= 20 cm

$\therefore$ Radius of pipe = 10 cm

and let length of the pipe = h

$$\therefore \quad \pi r^2 h = \frac{1100}{7}$$

$$\Rightarrow \frac{22}{7} \times 10 \times 10 \times h$$

$$= \frac{1100}{7} \times 100 \times 100$$

$$h = \frac{1100}{7} \times \frac{7 \times 100 \times 100}{22 \times 100} = 5000 \text{ m}$$

Speed of water = 3 km/hr

$$\therefore \text{Time taken} = \frac{5000}{3 \times 1000} \text{ hrs} = \frac{5}{3} \text{ hours}$$

= 1 hour 40 minutes **Ans.**

Q. 11. A swimming pool 70 m long, 44 m wide and 3 m deep is filled by water issuing from a pipe of diameter 35 cm at 6 m per second. How many hours does it take to fill the pool ?

Sol. Length of pool (l) = 70 m

Breadth (b) = 44 m

and depth (h) = 3 m

$\therefore$ Volume of water = lbh

$$= 70 \times 44 \times 3 \text{ m}^3 = 9240 \text{ m}^3$$

Diameter of cylinderical pipe = 35 cm

$$\therefore \text{Radius } (r) = \frac{35}{2} \text{ cm} = \frac{35}{200} \text{ m}$$

Speed of water = 6 m/sec

Let h be the length of the flow then

$\pi r^2 h = 9240$

and $\frac{22}{7} \times \frac{35}{200} \times \frac{35}{200} \times h = 9240$

$\therefore h = \frac{9240 \times 7 \times 200 \times 200}{22 \times 35 \times 35} = 96000$ m

$\therefore$ Time taken to fill the pool

$= \frac{96000}{6}$ sec

$= 16000$ sec

$= \frac{16000}{60 \times 60} = \frac{40}{9}$ hr

$= 4\frac{4}{9}$ hours **Ans.**

Q. 12. Water is flowing at the rate of 8 m per second through a circular pipe whose internal diameter is 2 cm, into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the water level in 30 minutes.

Sol. Diameter of pipe = 2 cm

$\therefore$ Radius of pipe = 1 cm

Rate of water flow = 8 m/sec

Time taken = 30 minutes

$\therefore$ Length of water flow into the pipe (h)

$= 30 \times 60 \times 8 = 14400$ m

$\therefore$ Volume of water $= \pi r^2 h$

$= \frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 14400 \text{ m}^3$

$= \frac{144 \times 22}{700} \text{ m}^3$

Now radius of the base of the cylindrical tank

$= 40 \text{ cm} = \frac{40}{100} \text{ m} = \frac{2}{5} \text{ m}$

Let h be the water increase in the tank

$\therefore \pi r^2 h = \frac{144 \times 22}{700}$

$\Rightarrow \frac{22}{7} \times \frac{2}{5} \times \frac{2}{5} h = \frac{144 \times 22}{700}$

$h = \frac{144 \times 22 \times 7 \times 5 \times 5}{700 \times 27 \times 2 \times 2} = 9 \text{ m}$

$\therefore$ Height of water = 9 m **Ans.**

Q. 13. A 20 m deep well with diameter 7 m is dug up and the earth from digging is spread evenly to form a platform 22 m × 14 m. Determine the height of the platform.

Sol. Diameter of well = 7 m

$\therefore$ Radius (r) $= \frac{7}{2}$ m

and depth (h) = 20 m

$\therefore$ Volume of earth dug out $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 \text{ m}^3$

$= 770 \text{ m}^3$

Let h_1 be the height of the platform

then $22 \times 14 \times h_1 = 770$

$\Rightarrow h_1 = \frac{770}{22 \times 14} = \frac{5}{2} = 2{\cdot}5 \text{ m}$ **Ans.**

Q. 14. Find the mass of a metallic hollow cylindrical pipe 24 cm long with internal diameter 10 cm and made of 5 mm thick metal, if 1 cm^3 of the metal weighs 7·5 grams.

Sol. Internal diameter of hollow cylindrical pipe = 10 cm

$\therefore$ Internal radius (r) $= \frac{10}{2} = 5$ cm

Width of metal = 5 mm = 0·5 cm

$\therefore$ Outer radius (R) = 5 + 0·5 = 5·5 cm

Length of pipe (h) = 24 cm

$\therefore$ Volume of metal $= \pi h (R^2 - r^2)$

$= \frac{22}{7} \times 24 \, [(5{\cdot}5)^2 - (5)^2] \text{ cm}^3$

$= \frac{22}{7} \times 24 \times (5{\cdot}5 + 5)(5{\cdot}5 - 5) \text{ cm}^3$

$= \frac{22}{7} \times 24 \times 10{\cdot}5 + 0{\cdot}5\,\text{cm}^3$

$= 396\ \text{cm}^3$

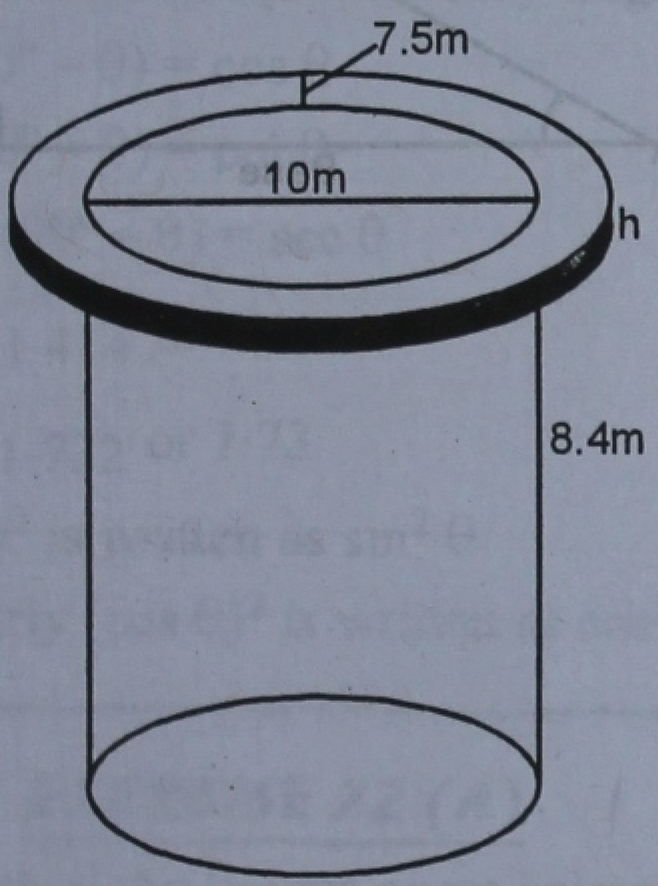
Weight of 1 cm³ metal = 7·5 *g*

∴ Total weight = 396 × 7·5 *g*

= 2970 *g* = 2·97 kg **Ans.**

Q. 15. A well with 10 m inside diameter is dug 8·4 m deep. Earth taken out of it is spread all around it to a width of 7·5 m to form an embankment. Find the height of the embankment.

Sol. Internal diameter of well = 10 m



∴ Radius $(r) = \frac{10}{2} = 5\,\text{m}$

Depth of well $(h) = 8{\cdot}4$ m

Volume of earth dug out

$= \pi r^2 h = \frac{22}{7} \times 5 \times 5 \times 8{\cdot}4\ \text{m}^3$

$= 22 \times 25 \times 1{\cdot}2$

$= 660\ \text{m}^3$

Let h_1 be height of embankment and width = 7·5 m

Outer radius (R) = 5 + 7·5

= 12·5 m

∴ Volume of embankment

= Volume of earth

$\Rightarrow \pi h_1 (R^2 - r^2) = 660$

$\Rightarrow \frac{22}{7} h_1 [(12{\cdot}5)^2 - (5)^2] = 660$

$\Rightarrow \frac{22}{7} h_1 (12{\cdot}5 + 5)(12{\cdot}5 - 5) = 660$

$\Rightarrow \frac{22}{7} h_1 \times 17{\cdot}5 \times 7{\cdot}5 = 660$

$\Rightarrow h_1 = \frac{660 \times 7}{22 \times 17{\cdot}5 \times 7{\cdot}5} = 1{\cdot}6\,\text{m}$

Hence height of embankment

= 1·6 m **Ans.**