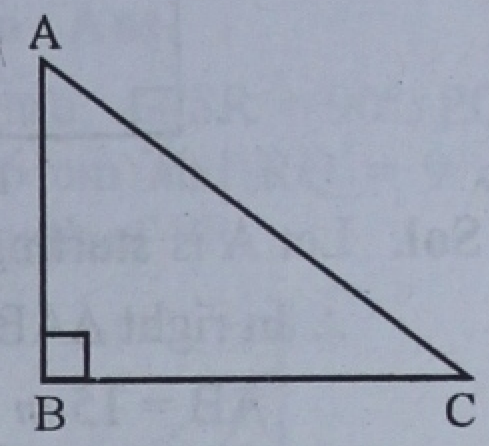
13

Pythagoras Theorem

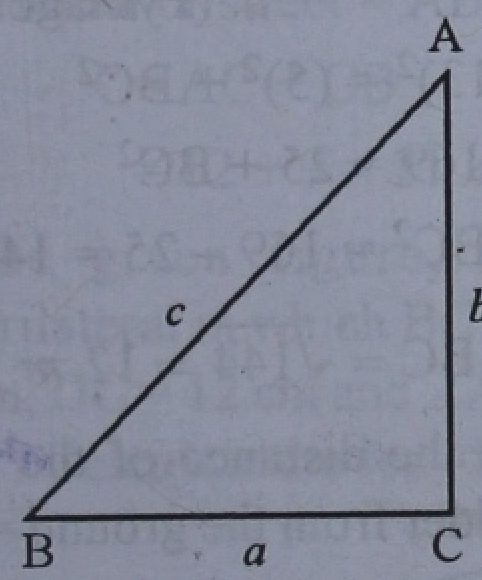
POINTS TO REMEMBER



1. **Pythagoras Theorem :** In a right-angled triangle the square on the hypotenuse is equal to the sum of squares on the other two sides.
   In ΔABC, $\angle B = 90°$
   $\therefore AC^2 = AB^2 + BC^2$.
2. **Converse of Pythagoras Theorem :** In a triangle, if the square of one side is equal to the sum of the square of other two sides, then the triangle is a right-angled.

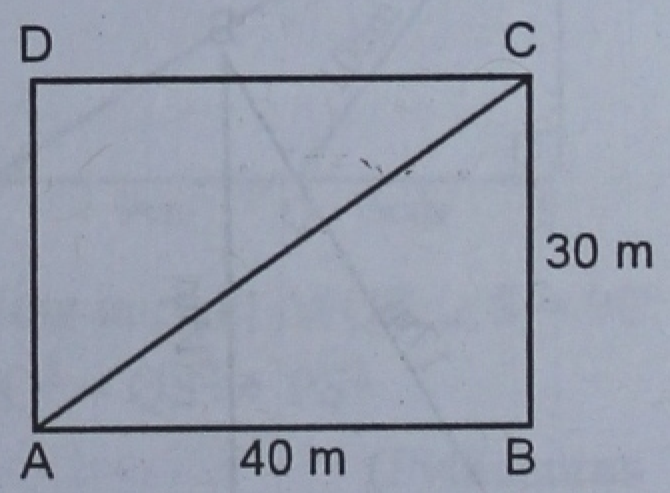
EXERCISE 13

Q. 1. In ΔABC, $\angle C = 90°$
If $BC = a$, $AC = b$ and $AB = c$, find :
(*i*) c when $a = 8$ cm and $b = 6$ cm.
(*ii*) a when $c = 25$ cm and $b = 7$ cm.
(*iii*) b when $c = 13$ cm and $a = 5$ cm.



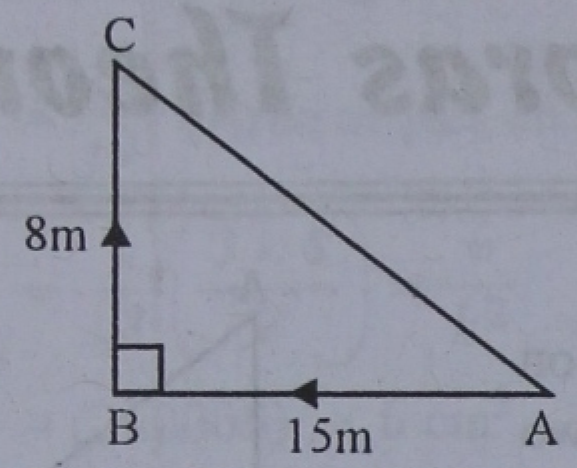
Sol. In ΔABC, $\angle C = 90°$
$\therefore c^2 = a^2 + b^2$ (Pythagoras Theorem)
(*i*) If $a = 8$ cm, $b = 6$ cm, then
$c^2 = a^2 + b^2 = (8)^2 + (6)^2$
$= 64 + 36 = 100$
$\therefore c = \sqrt{100} = 10$ cm.
(*ii*) $c = 25$ cm, $b = 7$ cm
But $c^2 = a^2 + b^2 \Rightarrow (25)^2 = a^2 + (7)^2$
$\Rightarrow 625 = a^2 + 49 \Rightarrow a^2 = 625 - 49$
$\Rightarrow a^2 = 576 \Rightarrow c = \sqrt{576} = 24$ cm.
(*iii*) $c = 13$ cm, $a = 5$ cm
But $c^2 = a^2 + b^2 \Rightarrow (13)^2 = (5)^2 + b^2$
$\Rightarrow 169 = 25 + b^2$
$\Rightarrow b^2 = 169 - 25 = 144$
$\therefore b = \sqrt{144} = 12$ cm.

Q. 2. A rectangular field is 40 *m* long and 30 *m* broad. Find the length of its diagonal.



Sol. In rectangular field ABCD, $AB = 40\ m$ and $BC = 30\ m$
AC is its diagonal.
$\therefore$ In ΔABC, $\angle B = 90°$
$AC^2 = AB^2 + BC^2$
(Pythagoras Theorem)
$= (40)^2 + (30)^2 = 1600 + 900$
$= 2500$
$\therefore AC = \sqrt{2500} = 50\ m$
Hence diagonal $AC = 50\ m$ **Ans.**

Q. 3. A man goes 15 m due west and then 8 m due north. How far is he from the starting point ?



Sol. Let A is starting point.

$\therefore$ In right ΔABC, $\angle B = 90^\circ$

$AB = 15\ m$ and $BC = 8\ m$

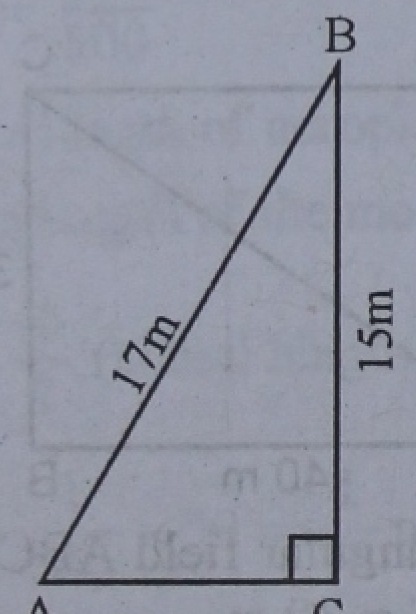
$\therefore AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow AC^2 = (15)^2 + (8)^2 = 225 + 64 = 289$

$\therefore AC = \sqrt{289} = 17\ m$

Hence he is 17 m far from the starting point.

Q. 4. A ladder 17 m long reaches the window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.



Sol. Let AB the ladder, CB be the building and B is window, then

$AB = 17\ m$, $BC = 15\ m$

Now in right ΔACB, $\angle C = 90^\circ$

$AB^2 = AC^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (17)^2 = AC^2 + (15)^2$

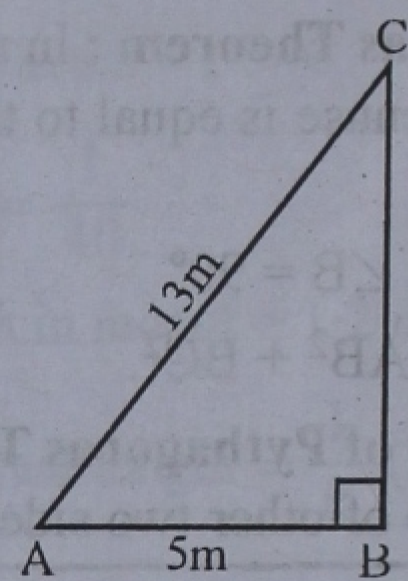
$\Rightarrow 289 = AC^2 - 225$

$\Rightarrow AC^2 = 289 - 225 = 64 = (8)^2$

$\therefore AC = 8$

Hence the distance of the foot of the ladder from the building = 8 m **Ans.**

Q. 5. A ladder 13 m long rests against a vertical wall. If the foot of the ladder is 5 m from the foot of the wall, find the distance of the other end of the ladder from the ground.



Sol. Let AC be the ladder, BC be the building and AB be the distance from the foot of the ladder from the building.

$\therefore AB = 5\ m$, $AC = 13\ m$.

Now in right ΔABC, $\angle B = 90^\circ$

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (13)^2 = (5)^2 + BC^2$

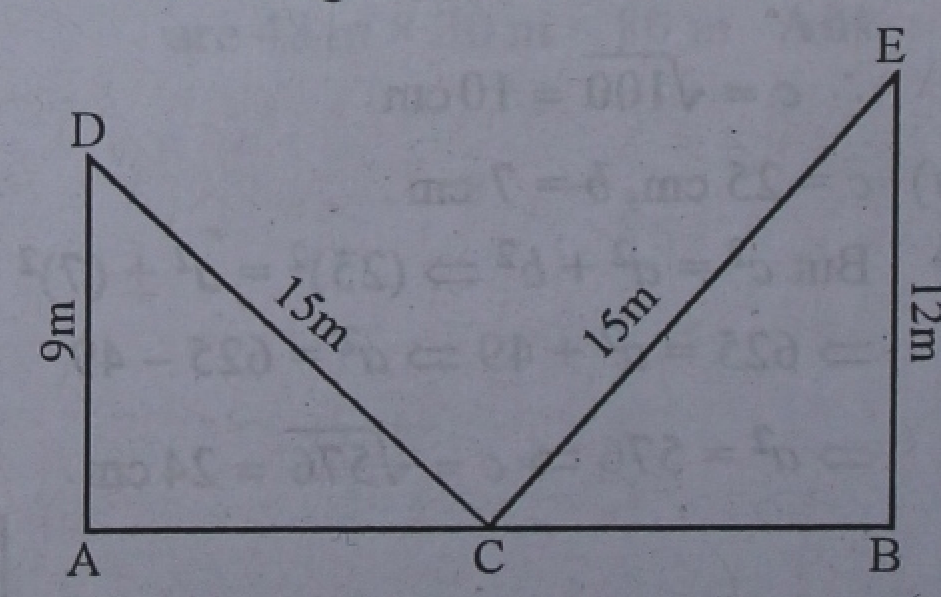
$\Rightarrow 169 = 25 + BC^2$

$\Rightarrow BC^2 = 169 - 25 = 144$

$\therefore BC = \sqrt{144} = 12\ m$

Hence the distance of the other end of the ladder from the ground = 12 m **Ans.**

Q. 6. A ladder 15 m long reaches a window which is 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. Find the width of the street.



Sol. Let CD and CE be two positions of the ladder, D is the window on one side and E is the window on the other side and AB be the width of the street.

$\therefore$ CD = CE = 15 *m*, AD = 9 *m*,

and BE = 12 *m*

Now in right ΔCAD, by Pythagoras Theorem,

$CD^2 = AC^2 + AD^2$

$\Rightarrow (15)^2 = AC^2 + (9)^2$

$\Rightarrow 225 = AC^2 + 81$

$\Rightarrow AC^2 = 225 - 81 = 144 = (12)^2$

$\therefore AC = 12$

Similarly, in right ΔCBE,

$CE^2 = CB^2 + BE^2$

$\Rightarrow (15)^2 = CB^2 + (12)^2$

$\Rightarrow 225 = CB^2 + 144$

$\Rightarrow CB^2 = 225 - 144 = 81 = (9)^2$

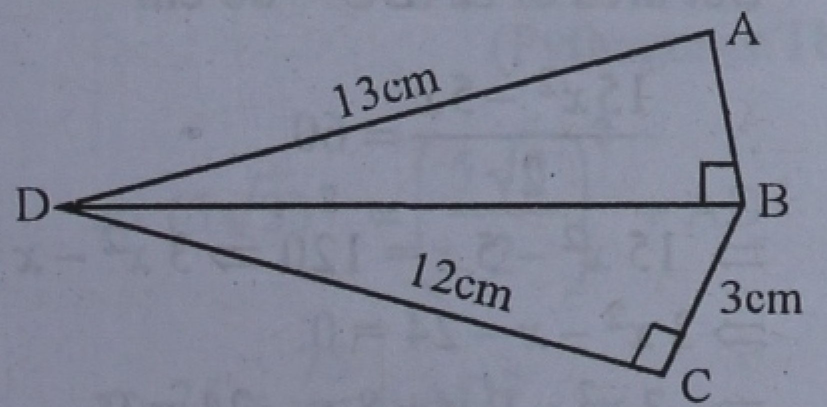
$\therefore CB = 9$

Now width of street = AB

= AC + CB

= 12 + 9 = 21 *m* **Ans.**

Q. 7. In the given figure, ABCD is a quadrilateral in which BC = 3 cm, AD = 13 cm, DC = 12 cm and $\angle ABD = \angle BCD = 90^\circ$. Calculate the length of AB.



Sol. In quadrilateral ABCD,

BC = 3 cm, AD = 13 cm, DC = 12 cm

$\angle ABD = \angle BCD = 90^\circ$

In right ΔBCD,

$BD^2 = CB + CD^2$

(Pythagoras Theorem)

$= (12)^2 + (3)^2 = 144 + 9 = 153$

Similarly, in right ΔABD,

$AD^2 = BD^2 + AB^2$

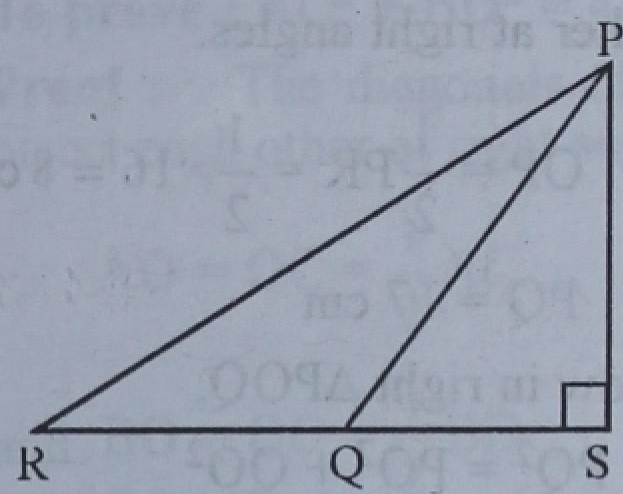
$\Rightarrow (13)^2 = 153 + AB^2$

$\Rightarrow 169 = 153 + AB^2$

$\Rightarrow AB^2 = 169 - 153 = 16 = (4)^2$

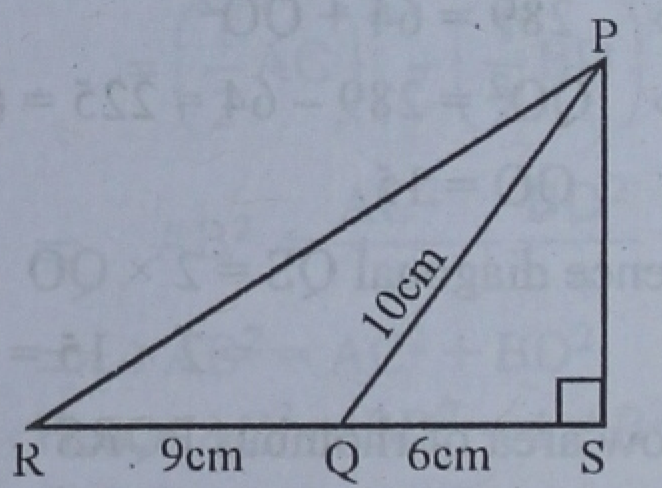
$\therefore AB = 4$ cm **Ans.**

Q. 8. In the given figure, $\angle PSR = 90^\circ$, PQ = 10 cm, QS = 6 cm and RQ = 9 cm, calculate the length of PR.



Sol. In the figure,

QS = 6 cm, RQ = 9 cm, PQ = 10 cm.



Now in right ΔPQS, $\angle S = 90^\circ$

$PQ^2 = QS^2 + PS^2$

(Pythagoras Theorem)

$\Rightarrow (10)^2 = (6)^2 + PS^2 \Rightarrow 100 = 36 + PS^2$

$\Rightarrow PS^2 = 100 - 36 = 64 = (8)^2$

PS = 8 cm

Similarly, in right ΔPRS,

$PR^2 = RS^2 + PS^2 = (RQ + QS)^2 + PS^2$

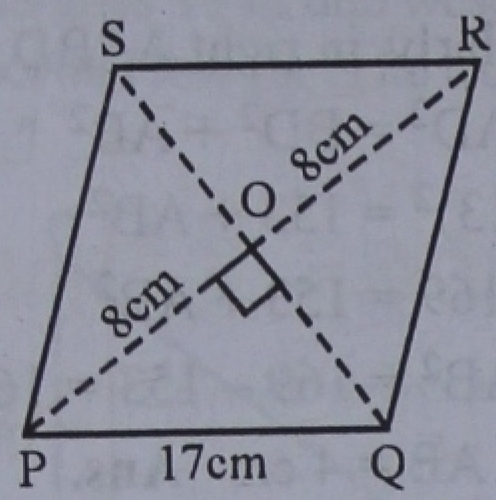
$= (9 + 6)^2 + (8)^2 = (15)^2 + (8)^2$

$= 225 + 64 = 289$

$\therefore PR = \sqrt{289} = 17$

Hence PR = 17 cm **Ans.**

Q. 9. In a rhombus PQRS, side PQ = 17 cm and diagonal PR = 16 cm. Calculate the area of the rhombus.



Sol. In rhombus PQRS, PQ = 17 cm and diagonal PR = 16 cm.

$\because$ The diagonals of a rhombus bisect each other at right angles.

$\therefore$ $OP = \frac{1}{2}PR = \frac{1}{2}\times 16 = 8\text{ cm}$

$PQ = 17$ cm

Now in right ΔPOQ,

$PQ^2 = PO^2 + QO^2$

(Pythagoras Theorem)

$\Rightarrow (17)^2 = (8)^2 + QO^2$

$\Rightarrow 289 = 64 + QO^2$

$\Rightarrow QO^2 = 289 - 64 = 225 = (15)^2$

$QO = 15$

Hence diagonal $QS = 2 \times QO$

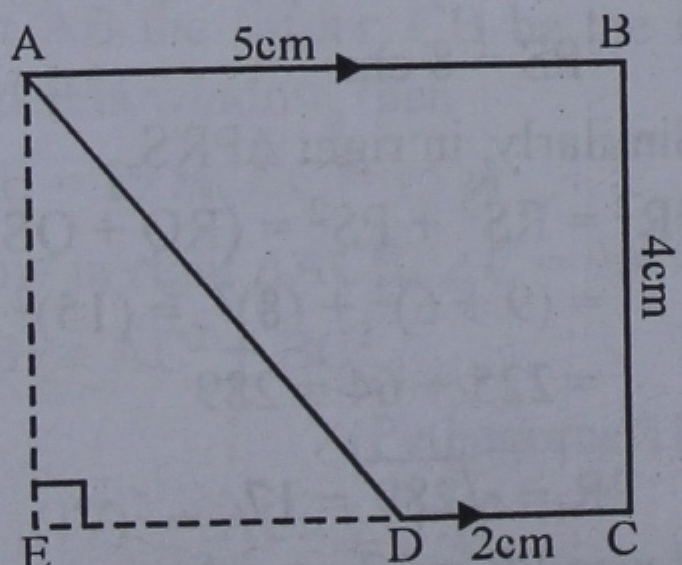
$= 2 \times 15 = 30$ cm

Now area of rhombus PQRS

$= \frac{1}{2}PR \times QS = \frac{1}{2} \times 16 \times 30\text{ cm}^2$

$= 240\text{ cm}^2$ **Ans.**

Q. 10. From the given figure, find the area of trapezium ABCD.



Sol. In trapezium ABCD,

AB || CD

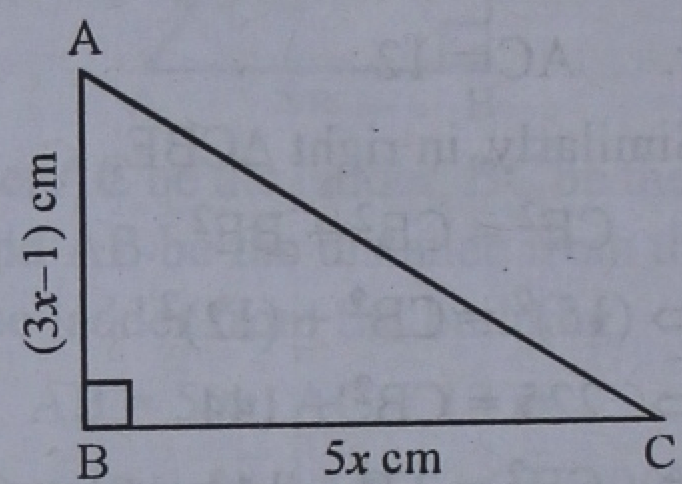
AB = 5 cm, CD = 2 cm, BC = 4 cm

Area of trapezium $= \frac{1}{2}$ (Sum of parallel sides) height

$= \frac{1}{2}(2+5) \times 4$

$= \frac{1}{2} \times 7 \times 4 = 14\text{ cm}^2$ **Ans.**

Q. 11. The sides of a right triangle containing the right angle are $(5x)$ cm and $(3x - 1)$ cm. If the area of the triangle be 60 cm^2, calculate the length of the sides of the triangle.



Sol. In right ΔABC,

$\angle B = 90^\circ$, $AB = 3x - 1$, $BC = 5x$

Now area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times (3x - 1)5x$

$= \frac{15x^2 - 5x}{2}$

But area of $\Delta ABC = 60\text{ cm}^2$

$\therefore \frac{15x^2 - 5x}{2} = 60$

$\Rightarrow 15x^2 - 5x = 120 \Rightarrow 3x^2 - x = 24$

$\Rightarrow 3x^2 - x - 24 = 0$

$\Rightarrow 3x^2 - 9x + 8x - 24 = 0$

$\Rightarrow 3x(x - 3) + 8(x - 3) = 0$

$\Rightarrow (x - 3)(3x + 8) = 0$

Either $x - 3 = 0$, then $x = 3$

or $3x + 8 = 0$, then $3x = -8 \Rightarrow x = \frac{-8}{3}$

But it is not possible.

$\therefore x = 3$

Now $AB = 3x - 1 = 3 \times 3 - 1$

$= 9 - 1 = 8$ cm

$BC = 5x = 5 \times 3 = 15$ cm

But $AC^2 = AB^2 + BC^2$

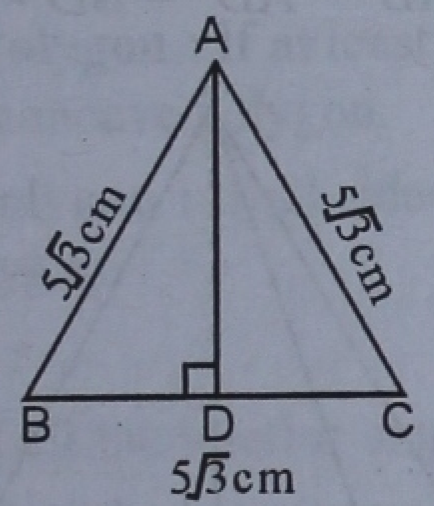
(Pythagoras Theorem)

$= (8)^2 + (15)^2 = 64 + 225$

$= 289 = (17)^2$

$\therefore$ AC = 17 cm **Ans.**

Q. 12. Find the altitude of an equilateral triangle of side $5\sqrt{3}$ cm.



Sol. In ΔABC,

$AB = BC = CA = 5\sqrt{3}$ cm

$\because AD \perp BC$.

$\therefore$ It bisects BC at D.

$\therefore BD = \frac{1}{2} BC = \frac{1}{2} \times 5\sqrt{3} = \frac{5\sqrt{3}}{2}$ cm

Now in right ΔABD, $\angle D = 90^\circ$,

$AB^2 = BD^2 + AD^2$

(Pythagoras Theorem)

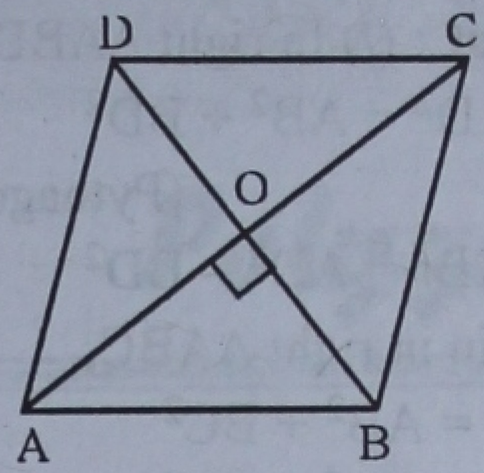
$\Rightarrow (5\sqrt{3})^2 = \left(\frac{5\sqrt{3}}{2}\right)^2 + AD^2$

$\Rightarrow 75 = \frac{75}{4} + AD^2$

$\Rightarrow AD^2 = 75 - \frac{75}{4} = \frac{300 - 75}{4} = \frac{225}{4}$

$\therefore AD = \sqrt{\frac{225}{4}} = \frac{15}{2} = 7{\cdot}5$ cm **Ans.**

Q. 13. In rhombus ABCD, prove that $AC^2 + BD^2 = 4\,AB^2$.



Sol. **Given :** In rhombus ABCD, AC and BD are its diagonals which intersect each other at O.

To prove : $AC^2 + BD^2 = 4\,AB^2$

Proof : $\because$ The diagonals of a rhombus bisect each other at right angles.

$\therefore \quad AO = OC = \frac{1}{2} AC$

and $BO = OD = \frac{1}{2} BD$

Now in right ΔAOB

$AB^2 = AO^2 + BO^2$

$= \left(\frac{1}{2} AC\right)^2 + \left(\frac{1}{2} BD\right)^2$

$\Rightarrow \quad AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$

$\Rightarrow 4\,AB^2 = AC^2 + BD^2$

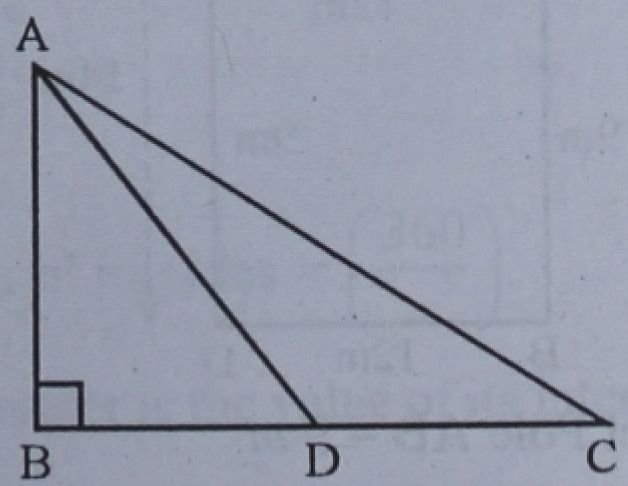
Hence $AC^2 + BD^2 = 4\,AB^2$

Q.E.D.

Q. 14. In ΔABC, $\angle B = 90^\circ$ and D is the mid-point of BC. Prove that :

(*i*) $AC^2 = AD^2 + 3\,CD^2$

(*ii*) $BC^2 = 4\,(AD^2 - AB^2)$.



Sol. **Given :** In ΔABC, $\angle B = 90^\circ$, D is mid-point of BC. AD is joined.

To prove :

(*i*) $AC^2 = AD^2 + 3\,CD^2$

(*ii*) $BC^2 = 4\,(AD^2 - AB^2)$

Proof : (*i*) In right ΔABD, $\angle B = 90^\circ$

$\therefore AD^2 = AB^2 + BD^2$ (Pythagoras Theorem)

$\Rightarrow AB^2 = AD^2 - BD^2$ ...(*i*)

Again in right ΔABC,

$AC^2 = AB^2 + BC^2$

$= AD^2 - BD^2 + (BD + DC)^2$ [From (*i*)]

$= AD^2 - BD^2 + (2\ CD)^2$ ($\because$ D is mid-point of BC)

$= AD^2 - CD^2 + 4\ CD^2$

$= AD^2 + 3\ CD^2$

(*ii*) From (*i*),

$AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = AB^2 + \left(\frac{1}{2}BC\right)^2$

$\Rightarrow AD^2 = AB^2 + \frac{BC^2}{4}$

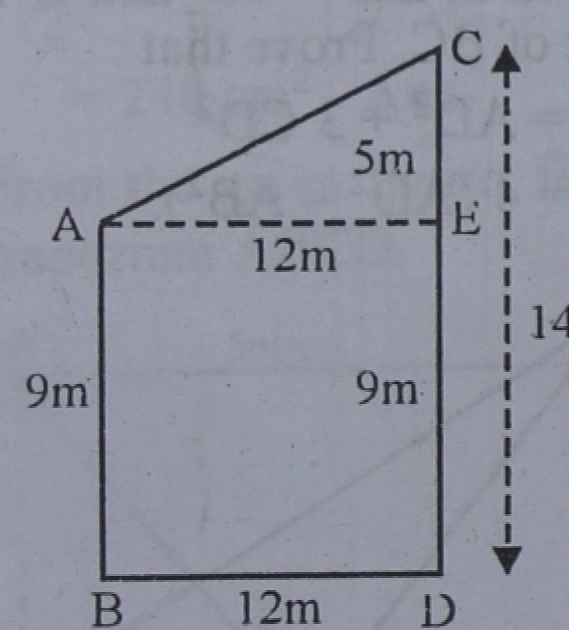
$\Rightarrow 4\ AD^2 = 4\ AB^2 + BC^2$

$\Rightarrow BC^2 = 4\ AD^2 - 4\ AB^2$

$\Rightarrow BC^2 = 4\ (AD^2 - AB^2)$

Hence proved.

Q. 15. Two poles of height 9 *m* and 14 *m* stand vertically on a plane ground. If the distance between their feet is 12 *m*, find the distance between their tops.



Sol. Let Pole AB = 9 *m*

Pole CD = 14 *m*

Distance between them (BD) = 12 *m*

From A, draw AE || BD

then AB = ED = 9 *m*

CE = CD – ED = 14 – 9 = 5 *m*

AE = BD = 12 *m*

Now in right ΔAEC, $\angle E = 90^\circ$

$AC^2 = AE^2 + CE^2$ (Pythagoras Theorem)

$= (12)^2 + (5)^2$

$= 144 + 25 = 169 = (13)^2$

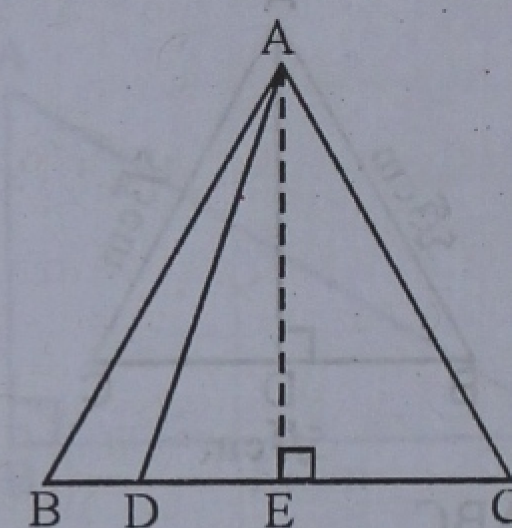
$\therefore AC = 13$

Hence distance between their tops

= 13 *m* **Ans.**

Q. 16. In ΔABC, if AB = AC and D is a point on BC. Prove that

$AB^2 - AD^2 = BD \times CD$.



Sol. Given : In ΔABC, AB = AC

D is a point on BC.

To prove : $AB^2 - AD^2 = BD \times CD$

Const : Draw $AE \perp BC$

Proof : Now in right ΔAEB and ΔAEC

Side AE = AE (Common)

Hyp. AB = AC (Given)

$\therefore \Delta ABE \cong \Delta ACE$ (R.H.S. axiom of congruency)

$\therefore$ BE = EC (C.P.C.T.)

Now in right ΔABE,

$AB^2 = AE^2 + BE^2$ ...(*i*) (Pythagoras Theorem)

Similarly in right ΔADE

$AD^2 = AE^2 + DE^2$ ...(*ii*)

Subtracting (*ii*) from (*i*)

$AB^2 - AD^2 = AE^2 + BE^2 - AE^2 - DE^2$

$= BE^2 - DE^2$

$= (BE + ED)(BE - ED)$

$= (CE + ED)(BE - ED)$ [BE = EC]

$= CD \times BD$

$= BD \times CD$

Hence proved.