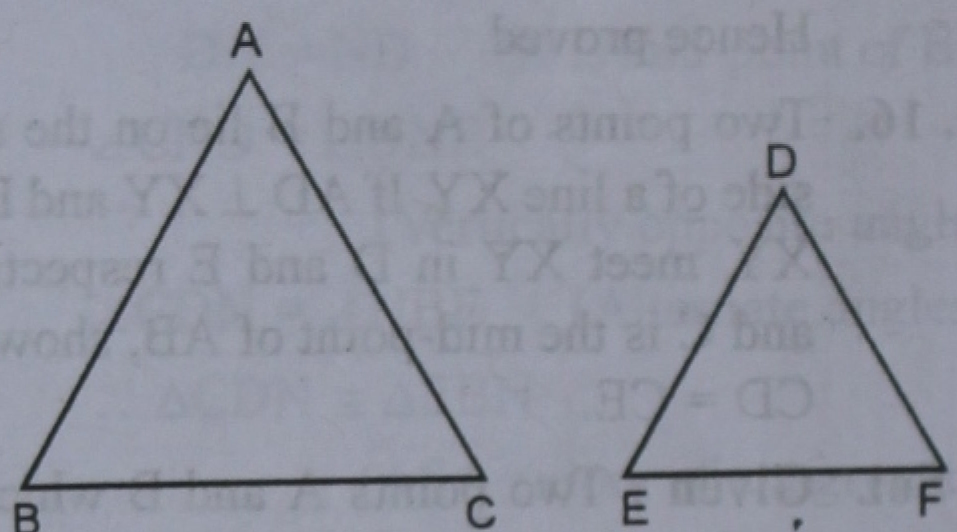
12 Similarity

POINTS TO REMEMBER

1. **Similar figures :** Two figures are similar if they have the same shape. Similar figures may differ in size. The sign '~' is used for similarity.

2. **Similar Triangles :** *Two Δs ABC and DEF are said to be similar, if their corresponding sides are proportional and we write,*



$\Delta ABC \sim \Delta DEF$.

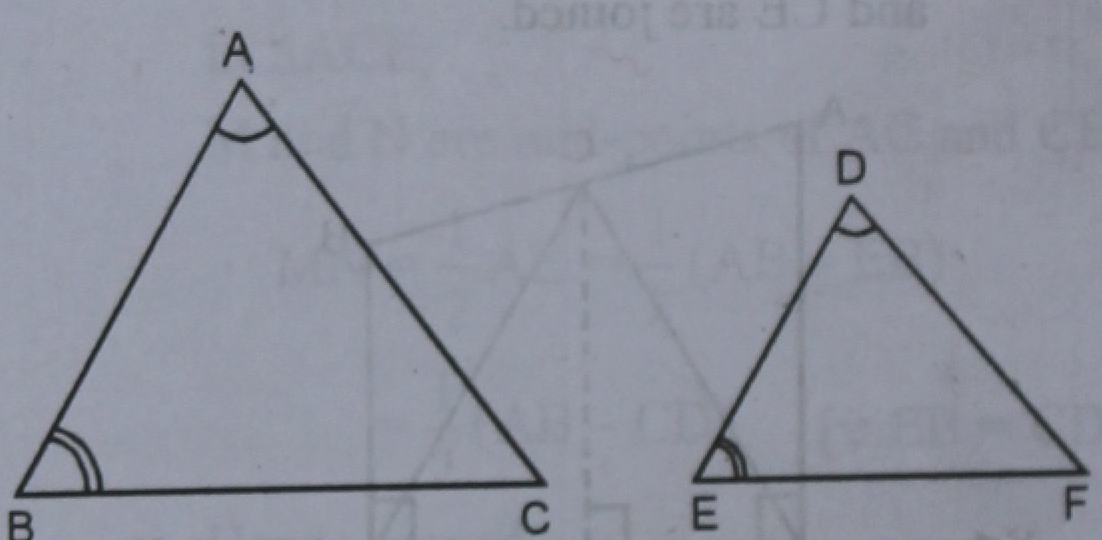
Thus, $\Delta ABC \sim \Delta DEF$

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.

3. **Axioms of Similarity of Triangles**

(i) (AA – axiom of Similarity) :

If two triangles have two pairs of corresponding angles equal, then the triangles are similar.



In the given figure, ΔABC and ΔDEF are such that

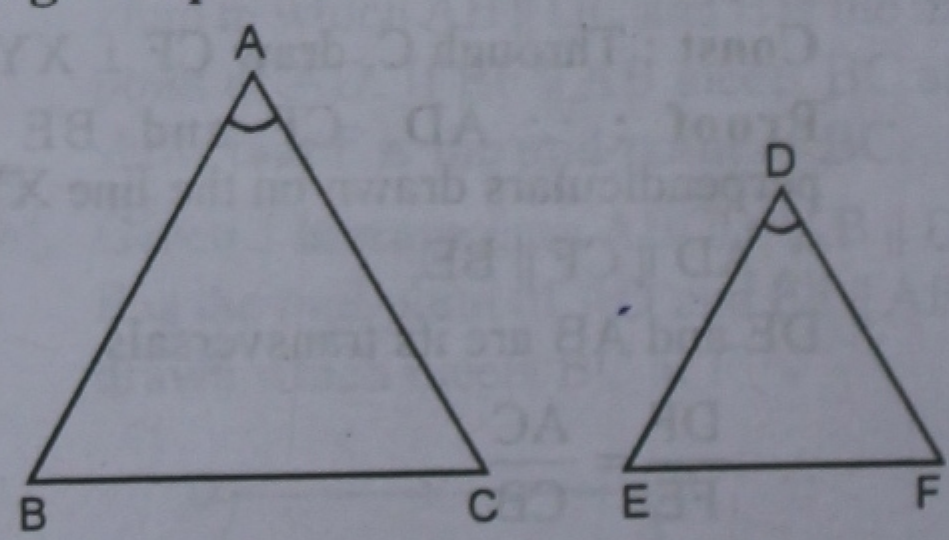
$\angle A = \angle D$

and $\angle B = \angle E$.

$\therefore \quad \Delta ABC \sim \angle DEF$.

(ii) (SAS – axiom of Similarity) :

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.



In the given figure, ΔABC and ΔDEF are such that

$\angle A = \angle D$

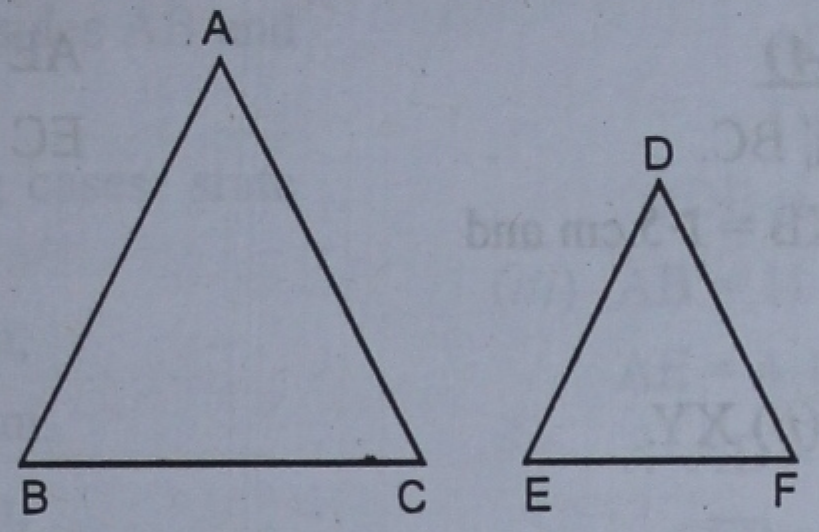
and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$.

$\therefore \quad \Delta ABC \sim \Delta DEF$.

(iii) (SSS – axiom of Similarity) :

If two triangles have three pairs of corresponding sides proportional then the triangles are similar.

In the given figure, ΔABC and ΔDEF are such that $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.



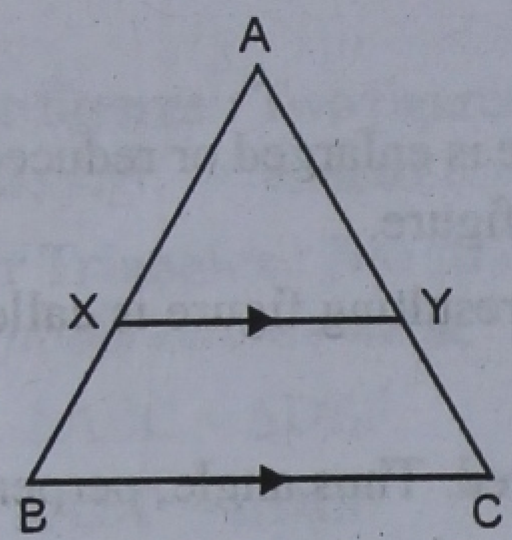
$\therefore \quad \Delta ABC \sim \Delta DEF.$

4. **Size Transformation :** It is the process in which a given figure is enlarged or reduced by the scale factor 'k' such that the resulting figure is similar to the given figure.
   The given figure is called an object on the **Preimage** and the resulting figure is called its **image**.
5. **Properties of size-Transformation.**
   (*i*) In the size transformation, the shape of the figure is preserved. Thus angle, perpendicular ship, parallelism etc. are preserved.
   (*ii*) Let k be the scale factor of the given size transformation, then
   (*a*) $k > 1 \Rightarrow$ The transformation is enlargement.
   (*b*) $k < 1 \Rightarrow$ The transformation is reduction.
   (*c*) $k = 1 \Rightarrow$ The transformation is an identity transformation.
   (*iii*) Each side of resulting image = k times the corresponding side of the given object.
   (*iv*) Area of resulting image = $k^2 \times$ (Area of given object).
   (*v*) Volume of resulting image = $k^3 \times$ (Volume of given object).
6. **Model.** The model of a plane figure and the actual figure are similar to one another.
   Let the model of a plane figure be drawn to the scale 1 : p, then
   $$\text{scale factor, } k = \frac{1}{p}$$
   (*a*) Length of model = $k \times$ (Length of actual object).
   (*b*) Area of model = $k^2 \times$ (Area of actual object).
   (*c*) Volume of model = $k^3 \times$ (Volume of actual object).
7. **Map.** Let the map of a plane figure be drawn to the scale 1 : p, then
   $$\text{scale factor, } k = \frac{1}{p}$$
   (*a*) Length in the map = $k \times$ (Actual length).
   (*b*) Area in the map = $k^2 \times$ (Actual area).
8. **Theorems.** (*i*) (Basic Proportionality Theorem)
   A line drawn parallel to one side of a triangle divides the other two sides proportionally.
   (*ii*) (Converse of Theorem) If a line divides any two sides of a triangle proportionally, the line is parallel to the third side.

EXERCISE 12 (A)

Q. 1. In the given figure, XY ∥ BC.
Given that AX = 3 cm, XB = 1·5 cm and BC = 6 cm.

Calculate : (*i*) $\frac{AY}{YC}$ (*ii*) XY.



Sol. In ΔABC, XY ∥ BC

AX = 3 cm, XB = 1·5 cm and BC = 6 cm

(*i*) $\therefore \frac{AX}{XB} = \frac{AY}{YC}$

$\Rightarrow \frac{3}{1 \cdot 5} = \frac{AY}{YC} \Rightarrow \frac{AY}{YC} = 2$ **Ans.**

(*ii*) In ΔAXY and ΔABC

∠A = ∠A (Common)

∠AXY = ∠ABC (Corresponding angles)

∠AYX = ∠ACB (Corresponding angles)

∴ ΔAXY ~ ΔABC (AAA axiom of similarity)

$\therefore \frac{AX}{AB} = \frac{XY}{BC} \Rightarrow \frac{AX}{AX + XB} = \frac{XY}{BC}$

$\Rightarrow \frac{XY}{6} = \frac{3}{3 + 1 \cdot 5} \Rightarrow \frac{XY}{6} = \frac{3}{4 \cdot 5}$

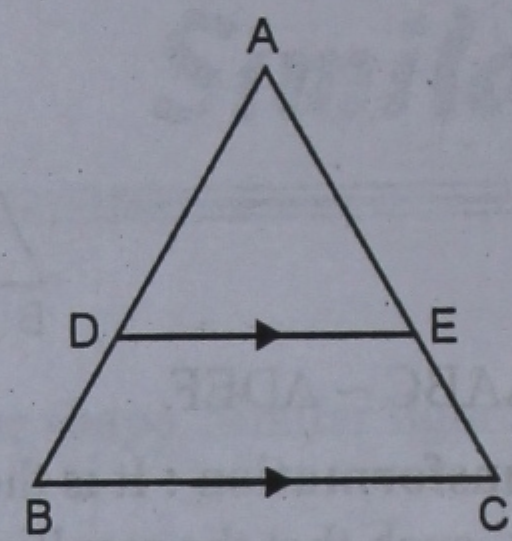
$\Rightarrow XY = \frac{3 \times 6}{4 \cdot 5} = 4$ **Ans.**

Q. 2. In the given figure, DE ∥ BC.

(*i*) If AD = 3·6 cm, AB = 9 cm and AE = 2·4 cm, find EC.

(*ii*) If $\frac{AD}{DB} = \frac{3}{5}$ and AC = 5·6 cm, find AE.

(*iii*) If AD = *x* cm, DB = (*x* – 2) cm, AE = (*x* + 2) cm and EC = (*x* – 1) cm, find the value of *x*.



Sol. (*i*) AD = 3·6 cm, AB = 9 cm,
AE = 2·4 cm
DB = AB – AD = 9·0 – 3·6 = 5·4 cm

Now in ΔABC, DE ∥ BC

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3 \cdot 6}{5 \cdot 4} = \frac{2 \cdot 4}{EC}$

$\Rightarrow EC \times 3 \cdot 6 = 2 \cdot 4 \times 5 \cdot 4$

$\Rightarrow EC = \frac{2 \cdot 4 \times 5 \cdot 4}{3 \cdot 6} = 3 \cdot 6$ cm **Ans.**

(*ii*) Let AE = *x*,
AC = 5·6 cm.

∴ EC = AC – AE = 5·6 – *x*

∵ DE ∥ BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{5} = \frac{x}{5 \cdot 6 - x}$

$\Rightarrow (5 \cdot 6 - x) \times 3 = 5x$

$\Rightarrow 16 \cdot 8 - 3x = 5x \Rightarrow 16 \cdot 8 = 5x + 3x$

$\Rightarrow 8x = 16 \cdot 8 \Rightarrow x = \frac{16 \cdot 8}{8} = 2 \cdot 1$

Hence AE = 2·1 cm **Ans.**

(*iii*) AD = *x* cm DB = (*x* – 2) cm
AE = (*x* + 2) cm and
EC = (*x* – 1) cm

∵ DE ∥ BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x-1) = (x-2)(x+2)$

$\Rightarrow x^2 - x = x^2 - 4 \Rightarrow -x = -4$

$\therefore x = 4$ **Ans.**

Q. 3. D and E are points on the sides AB and AC respectively of ΔABC.

For each of the following cases, state whether $DE \parallel BC$:

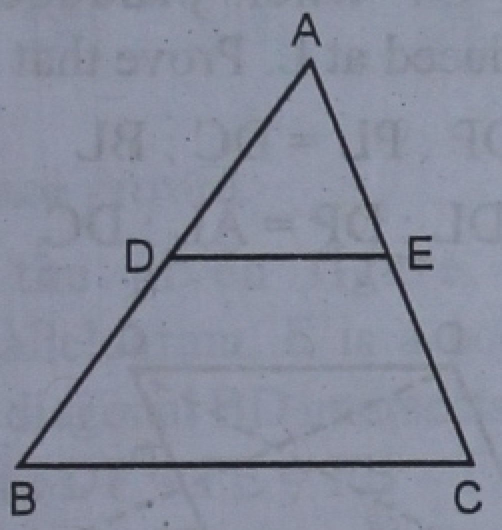
(*i*) AD = 5·7 cm, BD = 9·5 cm,
AE = 3·6 cm and EC = 6 cm.

(*ii*) AB = 5·6 cm, AD = 1·4 cm,
AC = 9·6 cm and EC = 2·4 cm.

(*iii*) AB = 11·7 cm, BD = 5·2 cm,
AE = 4·4 cm and AC = 9·9 cm.

(*iv*) AB = 10·8 cm, BD = 4·5 cm,
AC = 4·8 cm and AE = 2·8 cm.

Sol. In ΔABC, D and E are points on the sides AB and AC respectively. D, E are joined.



(*i*) Here AD = 5·7 cm, BD = 9·5 cm,
AE = 3·6 cm and EC = 6 cm

Now $\dfrac{AD}{DB} = \dfrac{5 \cdot 7}{9 \cdot 5} = \dfrac{3}{5}$

and $\dfrac{AE}{EC} = \dfrac{3 \cdot 6}{6 \cdot 0} = \dfrac{3}{5}$

$\because \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\therefore \quad DE \parallel BC$

(*ii*) Here AB = 5·6 cm, AD = 1·4 cm,
AC = 9·6 cm and EC = 2·4 cm.

DB = AB – AD = 5·6 – 1·4 = 4·2 cm

AE = AC – EC = 9·6 – 2·4 = 7·2 cm

Now $\dfrac{AD}{DB} = \dfrac{1 \cdot 4}{4 \cdot 2} = \dfrac{1}{3}$

and $\dfrac{AE}{EC} = \dfrac{7 \cdot 2}{2 \cdot 4} = \dfrac{3}{1}$

$\because \quad \dfrac{AD}{DB} \neq \dfrac{AE}{EC}$

$\therefore$ DE is not parallel to BC.

(*iii*) AB = 11·7 cm, BD = 5·2 cm,
AE = 4·4 cm, AC = 9·9 cm

$\therefore$ AD = AB – BD = 11·7 – 5·2 = 6·5 cm

EB = AC – AE = 9·9 – 4·4 = 5·5 cm.

Now $\dfrac{AD}{DB} = \dfrac{6 \cdot 5}{5 \cdot 2} = \dfrac{5}{4}$

$\dfrac{AE}{EC} = \dfrac{4 \cdot 4}{5 \cdot 5} = \dfrac{4}{5}$

$\because \quad \dfrac{AD}{DB} \neq \dfrac{AE}{EC}$

$\therefore$ DE is not parallel to BC.

(*iv*) AB = 10·8 cm, BD = 4·5 cm,
AC = 4·8 cm and AE = 2·8 cm

AD = AB – BD = 10·8 – 4·5 = 6·3 cm

EC = AC – AE = 4·8 – 2·8 = 2·0 cm

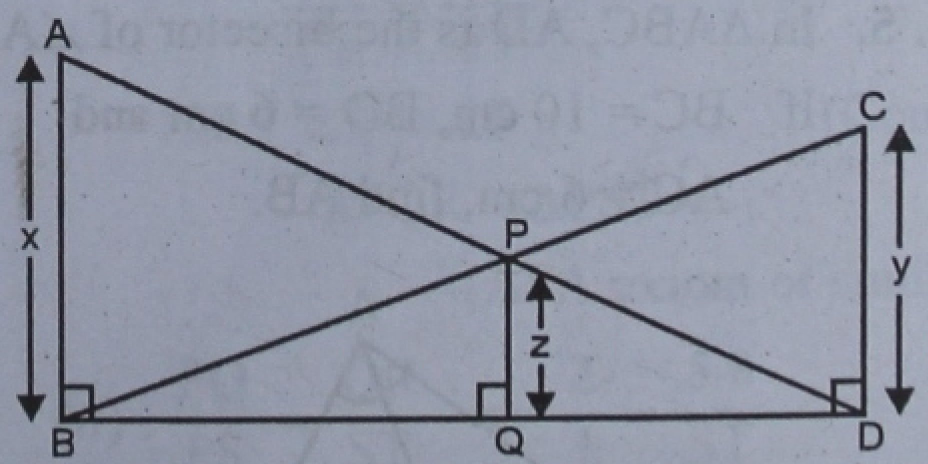
Now $\dfrac{AD}{DB} = \dfrac{6 \cdot 3}{4 \cdot 5} = \dfrac{7}{5}$

and $\dfrac{AE}{EC} = \dfrac{2 \cdot 8}{2 \cdot 0} = \dfrac{7}{5}$

$\because \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\therefore \quad DE \parallel BC$

Q. 4. In the given figure, it is given that $\angle ABD = \angle CDB = \angle PQB = 90^\circ$. If AB = x units, CD = y units and PQ = z units, prove that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$.



Sol. In ΔABD and ΔPQD

$\angle D = \angle D$ (Common)

$\angle B = \angle Q$ (Each 90°)

$\therefore\ \Delta ABD \sim \Delta PQD$ (AA Axiom)

$\therefore\ \frac{BD}{QD} = \frac{AB}{PQ} = \frac{x}{z}$

$\Rightarrow\ \frac{BD}{QD} = \frac{x}{z}$

$\Rightarrow\ \frac{QD}{BD} = \frac{z}{x}$

Similarly, we can prove that

ΔBDC and ΔBQP are similar

$\therefore\ \frac{BD}{BQ} = \frac{CD}{PQ} = \frac{y}{z}$

$\Rightarrow\ \frac{BD}{BQ} = \frac{y}{z}$

$\therefore\ \frac{BQ}{BD} = \frac{z}{y}$

Adding the two results,

$$\frac{QD}{BD} + \frac{BQ}{BD} = \frac{z}{x} + \frac{z}{y}$$

$$\Rightarrow\ \frac{QD + BQ}{BD} = z\left(\frac{1}{x} + \frac{1}{y}\right)$$

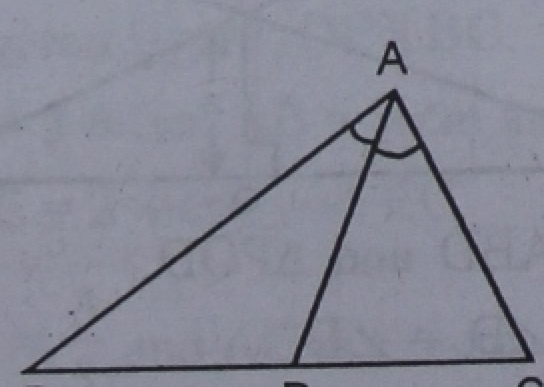
$$\Rightarrow\ \frac{BD}{BD} = z\left(\frac{1}{x} + \frac{1}{y}\right)$$

$$\Rightarrow\ z\left(\frac{1}{x} + \frac{1}{y}\right) = 1$$

$$\Rightarrow\ \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Hence proved.

Q. 5. In ΔABC, AD is the bisector of $\angle A$. If BC = 10 cm, BD = 6 cm and AC = 6 cm, find AB.



Sol. In ΔABC, AD is the bisector of $\angle A$

$\therefore\ \frac{AB}{AC} = \frac{BD}{DC}$

But BC = 10 cm, BD = 6 cm, AC = 6 cm

$\therefore$ DC = BC – BD = 10 – 6 = 4 cm

$\therefore\ \frac{AB}{6} = \frac{6}{4}$

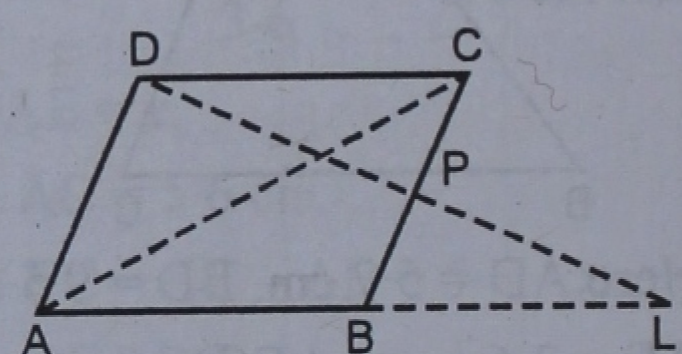
$\Rightarrow\ AB = \frac{6 \times 6}{4} = 9$

Hence AB = 9 cm.

Q. 6. In the adjoining figure, ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that :

(*i*) DP : PL = DC : BL

(*ii*) DL : DP = AL : DC.



Sol. Given : ABCD is a parallelogram. P is a point on side BC. DP is joined and produced to meet AB produced at L.

To prove :

(*i*) DP : PL = DC : BL

(*ii*) DL : DP = AL : DC

Proof : (*i*) In ΔDCP and ΔBPL,

$\angle CPD = \angle BPL$ (Vertically opposite angles)

$\angle CDP = \angle PLB$ (Alternate angles)

$\angle DCP = \angle PBL$ (Alternate angles)

$\therefore\ \Delta DCP \sim \Delta BPL$ (AAA axioms of similarity)

$\therefore\ \frac{DP}{PL} = \frac{DC}{BL}$

$\Rightarrow$ DP : PL = DC : BL.

(*ii*) Now in ΔALD

BP || AD

(opposite sides of a parallelogram)

$$\therefore \quad \frac{LP}{PD} = \frac{LB}{BA}$$

$$\Rightarrow \quad \frac{DP}{PL} = \frac{AB}{BL}$$

$$\Rightarrow \quad \frac{PL}{DP} = \frac{BL}{AB}$$

Adding 1 both sides

$$\frac{PL}{DP} + 1 = \frac{BC}{AB} + 1$$

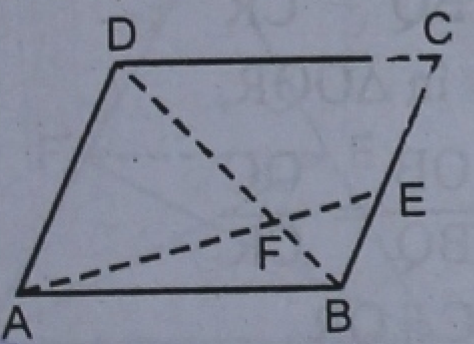
$$\frac{PL + DP}{DP} = \frac{BL + AB}{AB}$$

$$\Rightarrow \frac{DL}{DP} = \frac{AL}{AB} \Rightarrow \frac{DL}{DP} = \frac{AL}{DC}$$

($\because$ AB = DC)

Hence proved.

Q. 7. In the given figure, ABCD is a parallelogram, E is a point on BC and the diagonal BD intersects AE at F. Prove that : DF × FE = FB × FA.



Sol. Given : ABCD is a parallelogram. E is a point on BC and diagonal BD intersects AE at F.

To prove : DF × FE = FB × FA.

Proof : In ΔAFD and ΔBFE

∠AFD = ∠BFE

(Vertically opposite angles)

∠ADF = ∠FBE (Alternate angles)

∴ ΔAFD ~ ΔBFE

(AA axiom of similarity)

$$\therefore \quad \frac{FD}{FB} = \frac{FA}{FE}$$

⇒ DF × FE = FB × FA

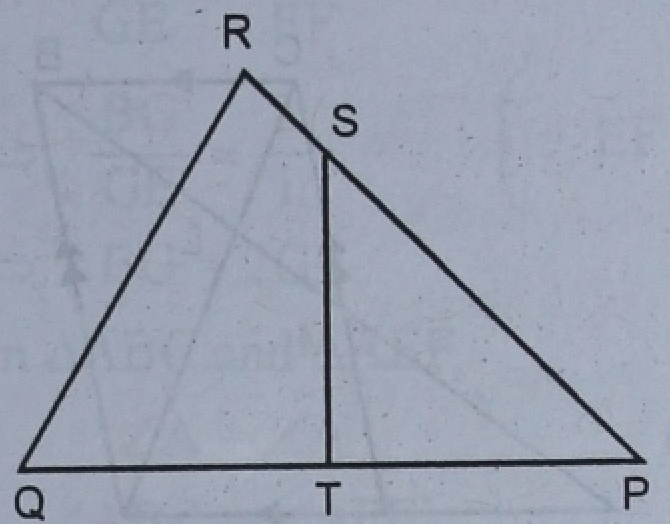
Hence proved.

Q. 8. In the adjoining figure (not drawn to scale),

PS = 4 cm, SR = 2 cm, PT = 3 cm and QT = 5 cm.

(*i*) Show that ΔPQR ~ ΔPST

(*ii*) Calculate ST, if QR = 5·8 cm.



Sol. Given : PS = 4 cm, SR = 2 cm, PT = 3 cm, QT = 5 cm

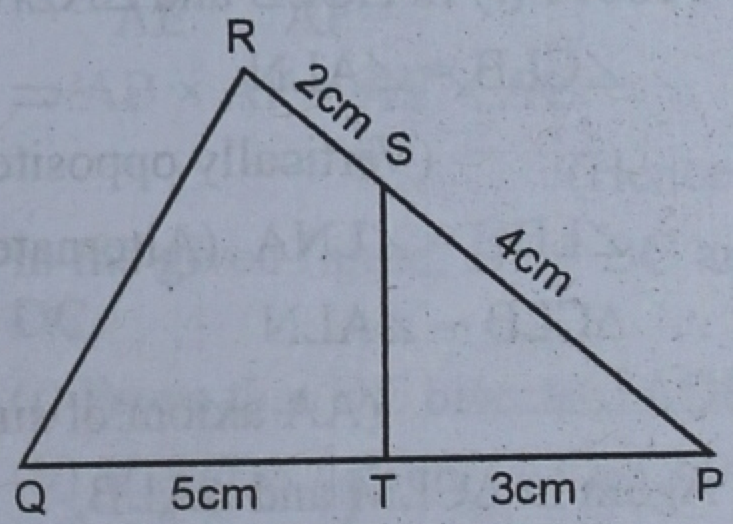
To prove : (*i*) Show that ΔPQR ~ ΔPST

(*ii*) Calculate ST, if QR = 5·8 cm.

Proof : (*i*) In ΔPQR and ΔPST

PQ = 5 + 3 = 8 cm

PR = 4 + 2 = 6 cm



$$\therefore \quad \frac{PQ}{PS} = \frac{8}{4} = \frac{2}{1}$$

$$\text{and} \quad \frac{PR}{PT} = \frac{6}{3} = \frac{2}{1}$$

$$\therefore \quad \frac{PQ}{PS} = \frac{PR}{PT}$$

and ∠P = ∠P (Common)

∴ ΔPQR ~ ΔPST

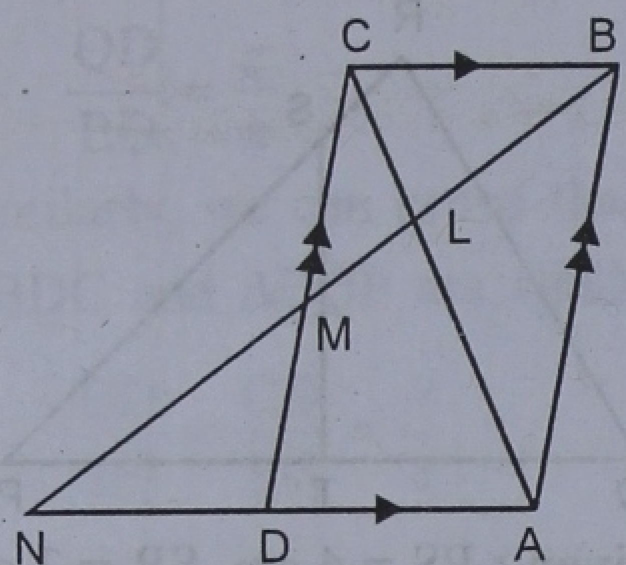
(ASA axiom of similarity)

$$(ii) \quad \frac{PQ}{PS} = \frac{QR}{ST} \Rightarrow \frac{2}{1} = \frac{5{\cdot}8}{ST}$$

⇒ 2 × ST = 5·8 × 1

$$\Rightarrow \quad ST = \frac{5{\cdot}8}{2} = 2{\cdot}9\,\text{cm} \quad \textbf{Ans.}$$

Q. 9. In the adjoining figure, ABCD is a parallelogram in which AB = 16 cm, BC = 10 cm and L is a point on AC such that CL : LA = 2 : 3. If BL produced meets CD at M and AD produced at N, prove that : (*i*) ΔCLB ~ ΔALN (*ii*) ΔCLM ~ ΔALB.



Sol. **Given :** In ||gm ABCD, AB = 16 cm, BC = 10 cm and L is a point on AC such that CL : LA = 2 : 3. BL is produced to meet CD at M and AD produced at N.

To prove : (*i*) ΔCLB ~ ΔALN

(*ii*) ΔCLM ~ ΔALB

Proof : (*i*) In ΔCLB and ΔALN,

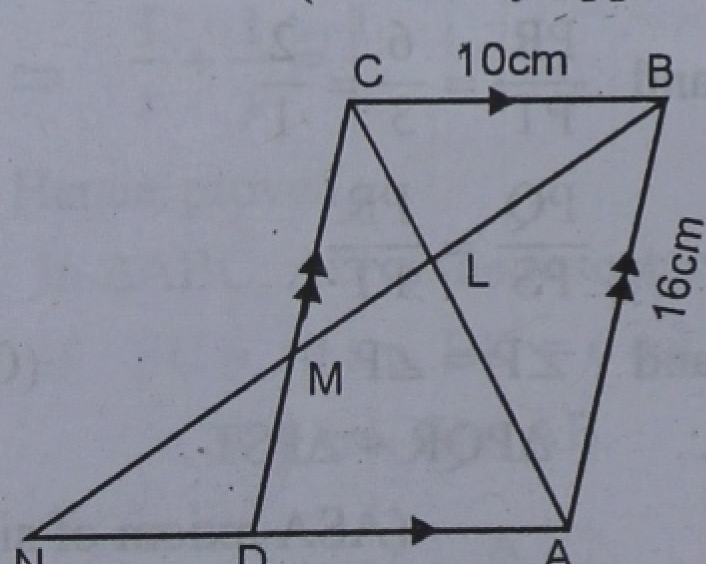
∠CLB = ∠ALN (Vertically opposite angles)

∠LBC = ∠LNA (Alternate angles)

∴ ΔCLB ~ ∠ALN (AA axiom of similarity)

Again in ΔCLM and ΔALB,

∠CLM = ∠ALB (Vertically opposite angles)



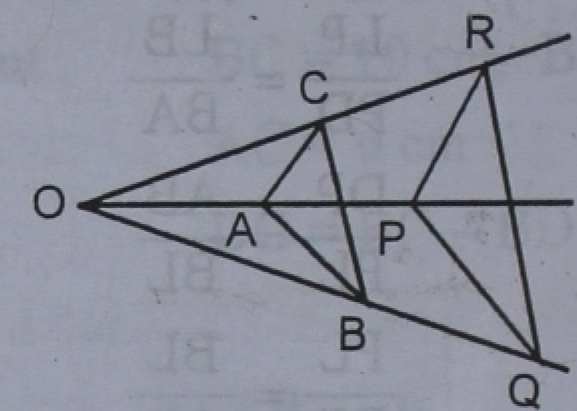
∠LCM = ∠LAB (Alternate angles)

∴ ΔCLM ~ ΔALB (AA axiom of similarity)

Hence proved.

Q. 10. In the given figure, AB || PQ and AC || PR.

Prove that BC || QR.



Sol. **Given :** In figure, AB || PQ and AC || PR

To prove : BC || QR

Proof : In ΔOPQ,

AB || PQ (given)

$$\therefore \frac{OB}{BQ} = \frac{OA}{AP} \quad ...(i)$$

Similarly in ΔOPR,

AC || PR (given)

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad ...(ii)$$

from (*i*) and (*ii*)

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Now in ΔOQR,

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

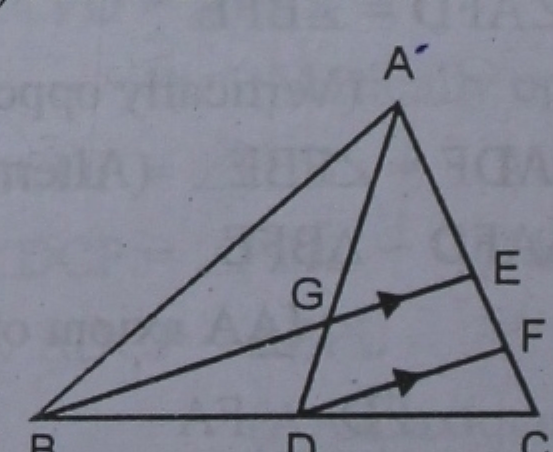
∴ BC || QR

Hence proved.

Q. 11. In the given figure, medians AD and BE, of ΔABC meet at G and DF || BE.

Prove that : (*i*) EF = FC

(*ii*) AG : GD = 2 : 1.



Sol. **Given :** In ΔABC, AD and BE are the medians which intersect each other at G and DF || BE.

To prove : (*i*) EF = FG

(*ii*) AG : GD = 2 : 1

Proof : (*i*) In ΔBCE,

D is mid point of BC and DF ∥ BE

∴ F is mid point of EC

∴ EF = FC

(*ii*) ∵ E is mid point of AC

∴ AE = EC = 2EF

Now in ΔADF,

DF ∥ BGE

$$\therefore \frac{AG}{GD} = \frac{AE}{EF} = \frac{2EF}{EF} = \frac{2}{1}$$

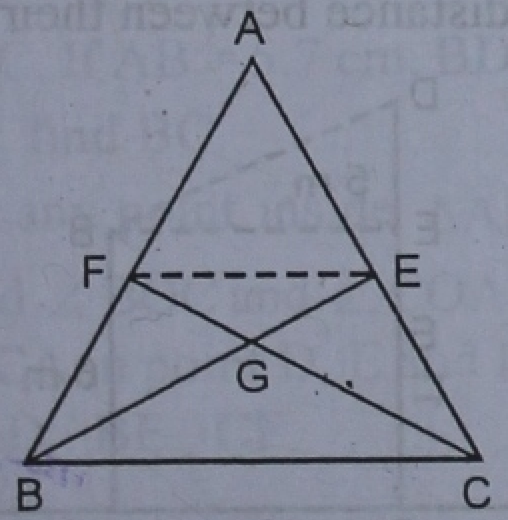
∴ AG : GD = 2 : 1

Hence proved.

Q. 12. In the given figure, the medians BE and CF of ΔABC meet at G.

Prove that : (*i*) ΔGEF ~ ΔGBC and therefore, BG = 2GE

(*ii*) AB × AF = AE × AC.



Sol. Given : In ΔABC, BE and CF are the medians which intersect each other at G.

To prove : (*i*) ΔGEF ~ ΔGBC and therefore, BG = 2GE

(*ii*) AB × AF = AE × AC

Construction. Join EF

Proof : ∵ E and F are the mid points of AC and AB respectively.

$$\therefore EF \parallel BC \text{ and } EF = \frac{1}{2}BC$$

(*i*) In ΔGEF and ΔGBC,

∠GEF = ∠GBC (Alternate angles)

∠EFG = ∠BCF (Alternate angles)

∠EGF = ∠BGC (Vertically opposite angles)

∴ ΔGEF ~ ΔGBC (AAA axiom of similarlity)

$$\therefore \frac{BG}{GE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{BG}{GE} = \frac{2}{1} \qquad \left(\because EF = \frac{1}{2}BC\right)$$

⇒ BG = 2GE

(*ii*) In ΔABC and ΔAEF,

∠A = ∠A (common)

∠ABC = ∠AFE (Corresponding angles)

∠ACB = ∠AEF (Corresponding angles)

∴ ΔABC ~ ΔAEF (AAA axiom of similarity)

$$\therefore \frac{AB}{AE} = \frac{AC}{AF}$$

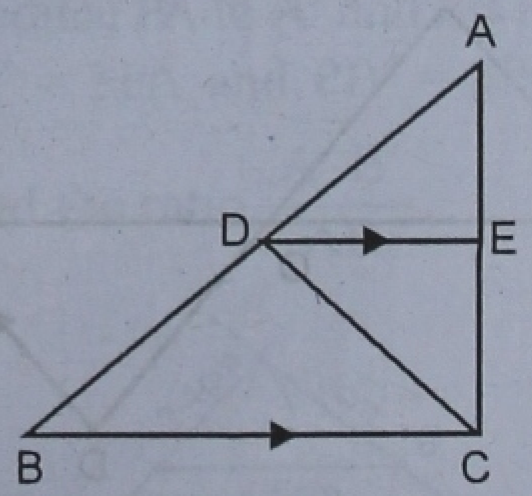
⇒ AB × AF = AE × AC (Hence proved)

Q. 13. In the given figure, DE ∥ BC and BD = DC.

(*i*) Prove that DE bisects ∠ADC

(*ii*) If AD = 4·5 cm, AE = 3·9 cm and DC = 7·5 cm, find CE

(*iii*) Find the ratio AD : DB.



Sol. Given : In ΔABC, DE ∥ BC

BD = DC

To prove : (*i*) DE bisects ∠ADC

(*ii*) If AD = 4·5 cm, AE = 3·9 cm and DC = 7·5 cm, find CE

(*iii*) Find the ratio AD : DB.

Proof : (*i*) In ΔABC,

$\because$ DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

(Basic proportionality Theorem)

$\Rightarrow \frac{AD}{DC} = \frac{AE}{EC}$ [$\because$ DB = DC (given)]

$\therefore$ In ΔADC

DE bisects $\angle ADC$

Hence proved

(*ii*) AD = 4·5 cm, AE = 3·9 cm,

DC = 7·5 cm

Now $\frac{AD}{DC} = \frac{AE}{CE} \Rightarrow \frac{4 \cdot 5}{7 \cdot 5} = \frac{3 \cdot 9}{CE}$

$\Rightarrow 4 \cdot 5 \times CE = 3 \cdot 9 \times 7 \cdot 5$

$\Rightarrow CE = \frac{3 \cdot 9 \times 7 \cdot 5}{4 \cdot 5} = 6 \cdot 5 \text{cm}$

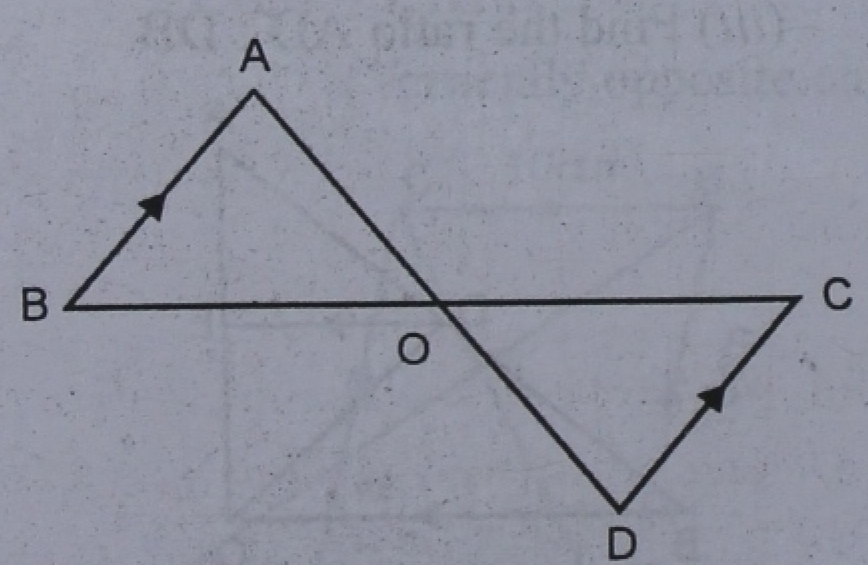
(*iii*) $\because \frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \frac{AD}{DB} = \frac{3 \cdot 9}{6 \cdot 5} = \frac{3}{5}$

$\Rightarrow$ AD : DB = 3 : 5 **Ans.**

Q. 14. In the given figure, BA || DC. Show that $\Delta OAB \sim \Delta ODC$.

If AB = 4 cm, CD = 3 cm, OC = 5·7 cm and OD = 3·6 cm, find OA and OB.



Sol. In ΔOAB and ΔOCD

$\angle AOB = \angle COD$

(Vertically opposite angles)

$\angle BAO = \angle ODC$ (Alternate angles)

$\angle ABO = \angle OCD$ (Alternate angles)

$\therefore \Delta OAB \sim \Delta OCD$

(AAA axiom of similarity)

$\therefore \frac{OA}{OD} = \frac{OB}{OC} = \frac{AB}{CD}$

Now AB = 4 cm, CD = 3 cm,

OC = 5·7 cm and OD = 3·6 cm

$\therefore \frac{OA}{3 \cdot 6} = \frac{OB}{5 \cdot 7} = \frac{4}{3} \Rightarrow \frac{OA}{3 \cdot 6} = \frac{4}{3}$

$\Rightarrow OA = \frac{3 \times 3 \cdot 6}{3} = 4 \cdot 8 \text{cm}$

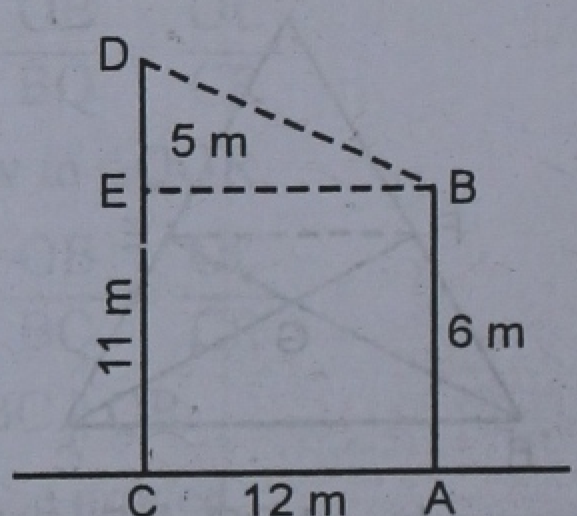
and $\frac{OB}{5 \cdot 7} = \frac{4}{3}$

$\Rightarrow OB = \frac{4 \times 5 \cdot 7}{3} = 7 \cdot 6 \text{cm}$

Hence OA = 4·8 cm and

OB = 7·6 cm **Ans.**

Q. 15. AB and CD are two vertical poles of height 6 m and 11 m respectively. If the distance between their feet is 12 m, find the distance between their tops.



Sol. AB and CD are two vertical poles such that AB = 6 m and CD = 11 m and distance between them *i.e.* CA = 12 m.

Draw BE $\perp$ CD.

$\therefore$ EC = BA = 6 m

and DE = CD – EC

= 11 – 6 = 5 m

and EB = CA = 12 m

Now in right ΔBDE,

$BD^2 = EB^2 + DE^2 = (12)^2 + (5)^2$

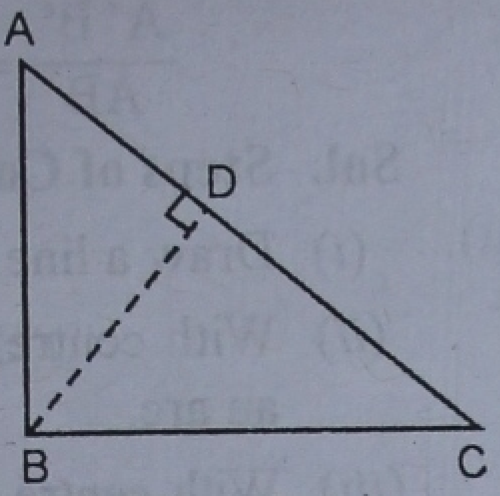
$= 144 + 25 = 169 = (13)^2$

$\therefore$ BD = 13 m.

Hence distance between their tops

= 13 m **Ans.**

Q. 16. In the given figure $\angle ABC = 90^\circ$ and $BD \perp AC$. If $AB = 5{\cdot}7$ cm, $BD = 3{\cdot}8$ cm and $CD = 5{\cdot}4$ cm, find BC.



Sol. In ΔABC, $\angle ABC = 90^\circ$ and $BD \perp AC$.

$AB = 5{\cdot}7$ cm, $BD = 3{\cdot}8$ cm and $CD = 5{\cdot}4$ cm

Now in ΔABC and ΔBDC,

$\angle ABC = \angle BDC$ (each $= 90^\circ$)

$\angle C = \angle C$ (Common)

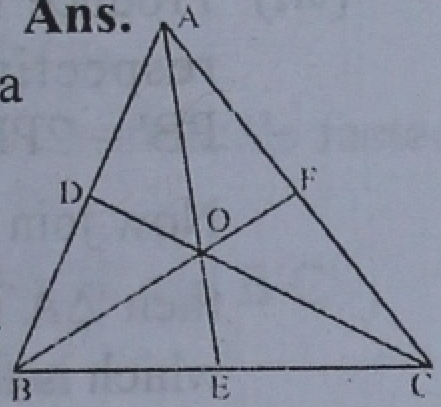
$\therefore \Delta ABC \sim \Delta BDC$

(AA axiom of similarity)

$\therefore \dfrac{AB}{BD} = \dfrac{BC}{DC} \Rightarrow \dfrac{5{\cdot}7}{3{\cdot}8} = \dfrac{BC}{5{\cdot}4} \Rightarrow BC = \dfrac{5{\cdot}7 \times 5{\cdot}4}{3{\cdot}8}$

Hence $BC = 8{\cdot}1$ cm. **Ans.**

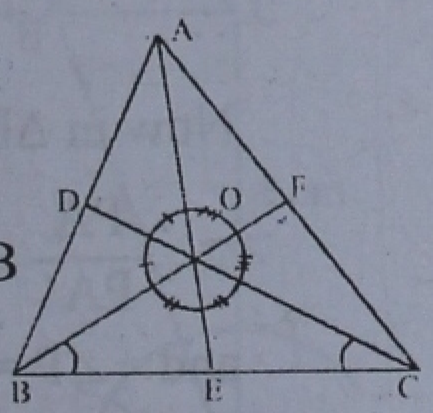
Q. 17. O is any point inside a ΔABC. The bisectors of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in points D, E and F respectively. Prove that AD.BE.CF = DB.EC.FA.



Sol. O is any point inside a ΔABC = 90° and BD $\perp$ AC. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.

Given : O is any point inside ΔABC Bisectors of $\angle AOB$ and $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in point D, E and F respectively.

To prove : AD . BE . CF = DB . EC . FA



Proof : In ΔAOB,

$\because$ OD is the bisector of $\angle AOB$

$\therefore \dfrac{OA}{OB} = \dfrac{AD}{DB}$ ...(*i*)

(Angle bisector Theorem)

Similarly in ΔBOC,

$\dfrac{OB}{OC} = \dfrac{BE}{EC}$ ...(*ii*)

and in ΔCOA,

$\dfrac{OC}{OA} = \dfrac{CF}{FA}$ ...(*iii*)

Multiplying (*i*), (*ii*) and (*iii*), we set

$\dfrac{OA}{OB} \times \dfrac{OB}{OC} \times \dfrac{OC}{OA} = \dfrac{AD}{DB} \times \dfrac{BE}{EC} \times \dfrac{CF}{FA}$

$\Rightarrow 1 = \dfrac{AD.BE.CF}{DB.EC.FA} \Rightarrow AD . BE. CF = DB.EC . FA$

Hence proved.

EXERCISE 12 (B)

Q. 1. A ΔABC has been reduced by scale factor $0{\cdot}8$ to $\Delta A'B'C'$. Calculate :

(*i*) the length $A'B'$, when $AB = 10$ cm.

(*ii*) the length BC, when $B'C' = 8{\cdot}4$ cm.

Sol. Scale factor $= k = 0{\cdot}8 = \dfrac{8}{10} = \dfrac{4}{5}$

(*i*) $AB = 10$ cm.

$\therefore A'B' = k \times AB = \dfrac{4}{5} \times 10 = 8$ cm

(*ii*) $B'C' = 8{\cdot}4$ cm,

then $BC = \dfrac{1}{k} B'C' = \dfrac{1}{4} \times 8{\cdot}4 = \dfrac{8{\cdot}4 \times 5}{4} = 10{\cdot}5$ cm

Q. 2. A ΔPQR has been enlarged by scale factor $2{\cdot}5$ to $\Delta P'Q'R'$. Calculate :

(*i*) the length $Q'R'$, when $QR = 6$ cm.

(*ii*) the length PQ, when $P'Q' = 10$ cm.

Sol. Scale factor $= k = 2{\cdot}5$

(*i*) $QR = 6$ cm.

then $Q'R' = kQR = 2{\cdot}5 \times 6 = 15$ cm.

(*ii*) $P'Q' = 10$ cm

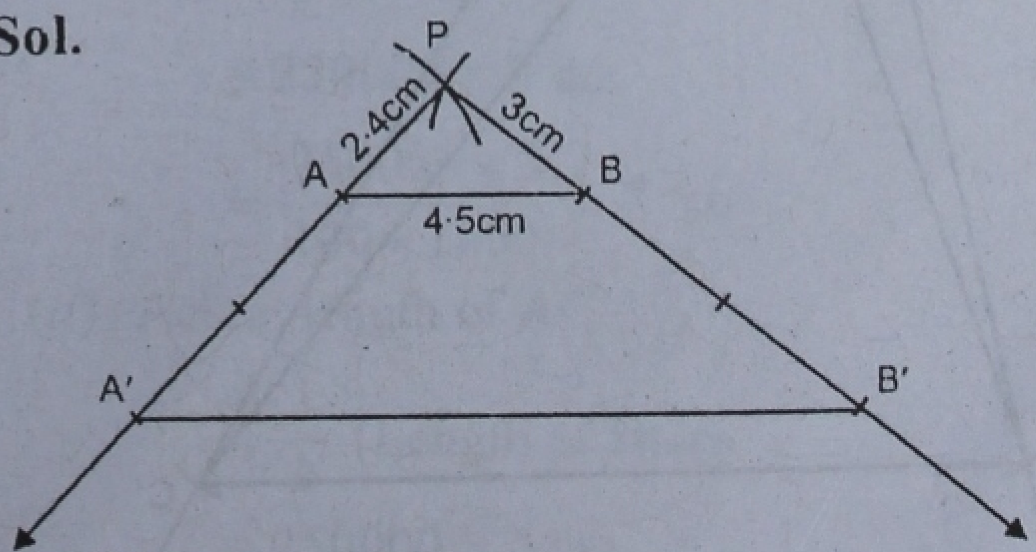
then $PQ = \dfrac{1}{k} \times P'Q' = \dfrac{1}{2{\cdot}5} \times 10$

$= 10 \times \dfrac{10}{25} = 4$ cm. **Ans.**

Q. 3. Draw a line segment $AB = 4{\cdot}5$ cm. Locate a point P outside AB such that $PQ = 2{\cdot}4$ cm and $PB = 3$ cm. Produce PA to A′ and PB to B′ such that $PA' = 3PA$ and $PB' = 3PB$. Join A′B′.

Find the ratio $\dfrac{A'B'}{AB}$.

Sol.



Steps of Construction :

(*i*) Draw a line segment $AB = 4{\cdot}5$ cm.

(*ii*) With centre A and radius 2·4 cm, draw an arc.

(*iii*) With centre B and radius 3 cm, draw another arc intersecting the first arc at P.

(*iv*) Join PA and PB.

(*v*) Now produce PA to A′ and PB to B′ such that PA′ = 3PA and PB′ = 3PB.

(*vi*) Join A′B′.

Now $\frac{PA'}{PA} = \frac{3PA}{PA} = \frac{3}{1}$

and $\frac{PB'}{PB} = \frac{3PB}{PB} = \frac{3}{1}$

$\therefore \frac{PA'}{PA} = \frac{PB'}{PB}$

and $\angle P = \angle P$ (common)

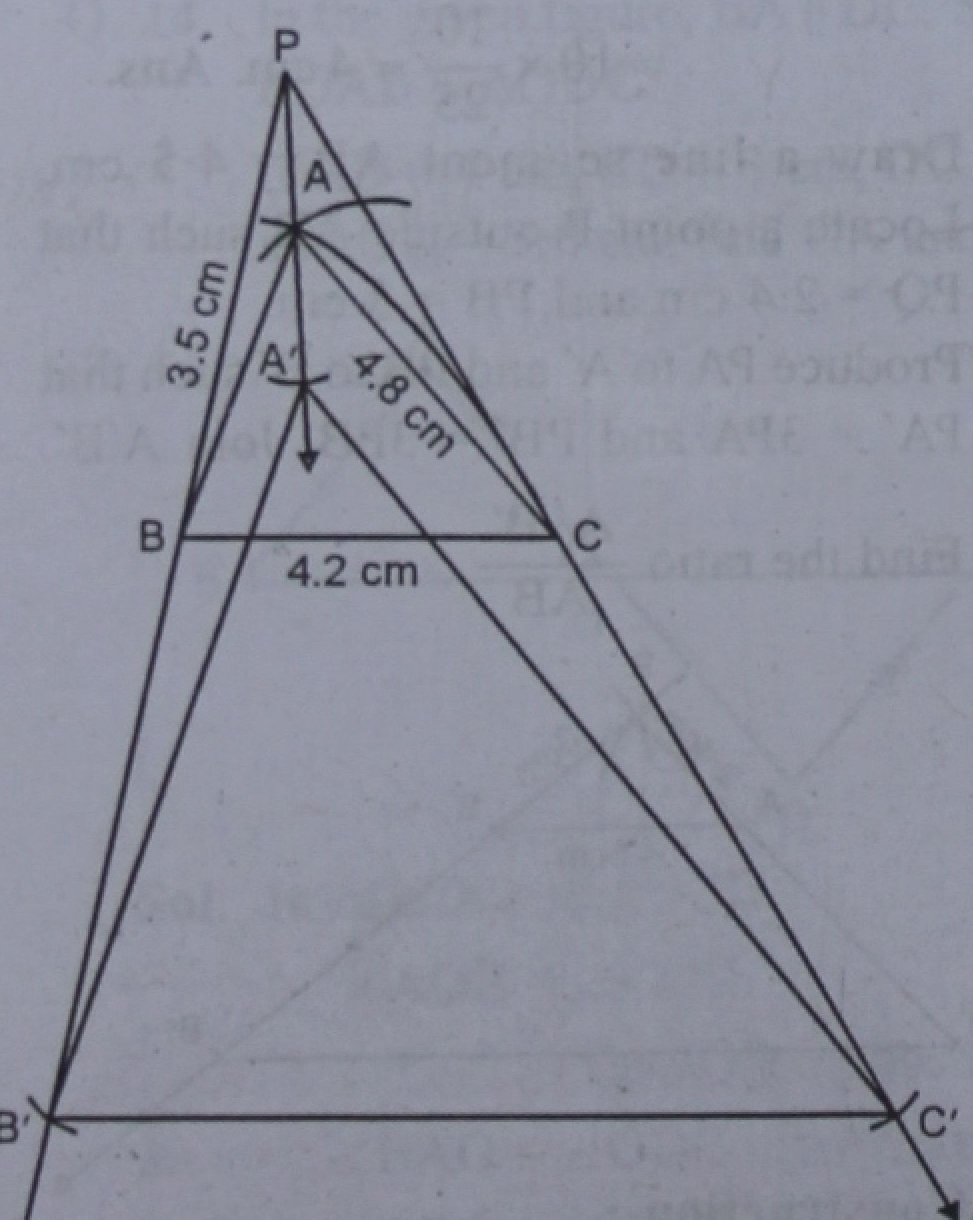
$\therefore \Delta PA'B' \sim \Delta PAB$

(SAS axiom of similarity)

$\therefore \frac{A'B'}{AB} = \frac{PA'}{PA} = \frac{3}{1}$

or A′B′ : AB = 3 : 1

Q. 4. Construct a ΔABC with AB = 3·5 cm, BC = 4·2 cm and AC = 4·8 cm. Enlarge ΔABC with scale factor $k = 2$ to ΔA′B′C′. Show that



$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = 2$$

Sol. Steps of Construction :

(*i*) Draw a line segment BC = 4·2 cm.

(*ii*) With centre B and radius 3·5 cm draw an arc.

(*iii*) With centre C and radius 4·8 cm, draw another arc intersecting the first arc at A.

(*iv*) Join AB and AC.

Then ΔABC is the triangle.

Now, the scale factor $k = 2$.

(*v*) Take a point P outside ΔABC.

(*vi*) Join PA, PB and PC.

(*vii*) Produce PA, PB and PC to A′, B′, C′ respectively such that PA′ = 2PA, PB′ = 2PB and PC′ = 2PC.

Now join A′B′, B′C′ and A′C′ then ΔA′B′C′ is the required triangle which is the image of ΔABC.

Proof. $\because \frac{PB}{PB'} = \frac{PB}{2PB} = \frac{1}{2}$

and $\frac{PA}{PA'} = \frac{PA}{2PA} = \frac{1}{2}$

and $\frac{PC}{PC'} = \frac{PC}{2PC} = \frac{1}{2}$

$\therefore \frac{PA}{PA'} = \frac{PB}{PB'} = \frac{PC}{PC'}$

Now in ΔPAB and ΔP′A′B′

$\because \frac{PA}{PA'} = \frac{PB}{PB'}$ (proved)

and $\angle P = \angle P$ (Common)

$\therefore \Delta PAB \sim \Delta PA'B'$

(SAS axiom of similarity)

$\therefore \frac{AB}{A'B'} = \frac{PA}{PA'} = \frac{1}{2}$

$\Rightarrow \frac{A'B'}{AB} = \frac{2}{1}$ ...(*i*)

Similarly we can prove that

ΔPAC ~ ΔPA′C′

$\therefore \quad \dfrac{AC}{A'C'} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{A'C'}{AC} = \dfrac{2}{1}$ ...(*ii*)

and $\Delta PBC \sim \Delta PB'C'$

$\therefore \quad \dfrac{BC}{B'C'} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{B'C'}{BC} = \dfrac{2}{1}$ ...(*iii*)

$\therefore$ From (*i*), (*ii*) and (*iii*)

$$\dfrac{A'B'}{AB} = \dfrac{A'C'}{AC} = \dfrac{B'C'}{BC} = \dfrac{2}{1}$$

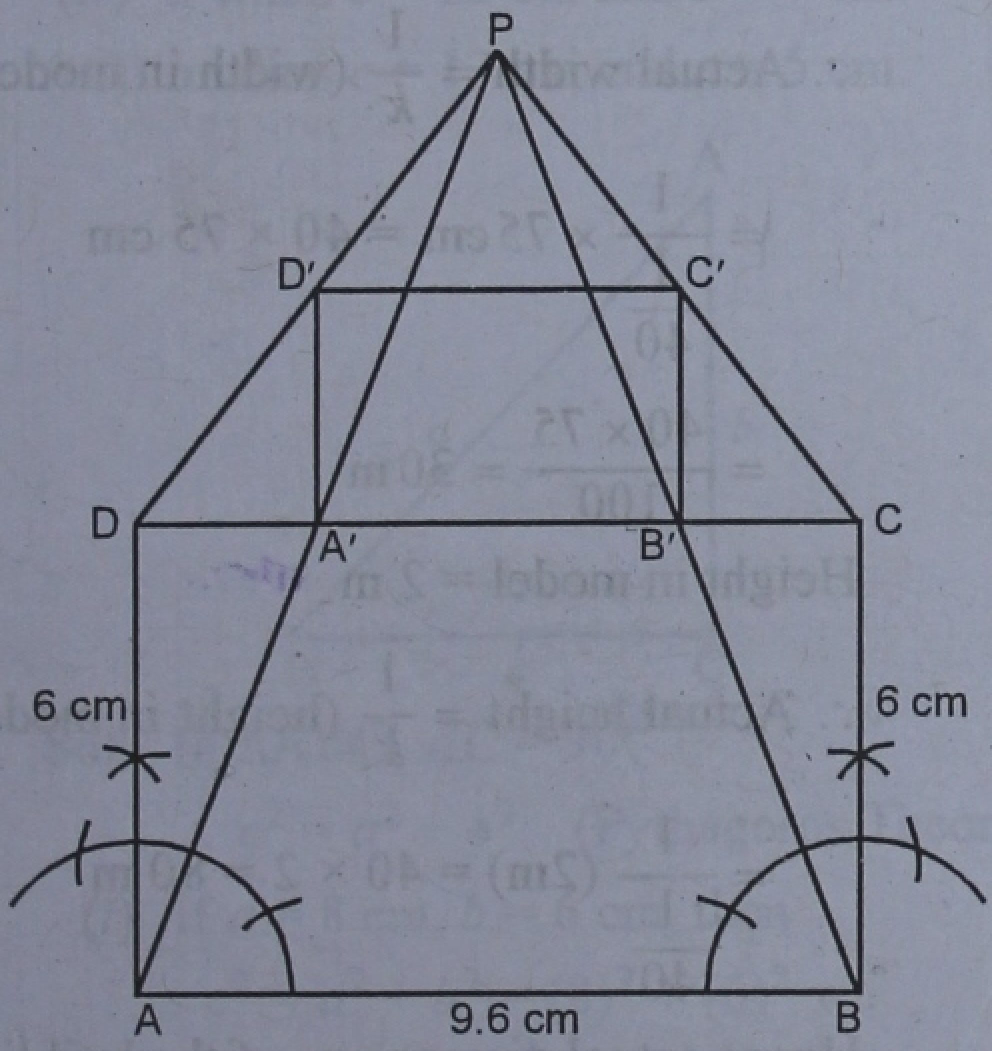
Hence proved.

Q. 5. Construct a rectangle ABCD with AB = 9·6 cm and BC = 6 cm.

(*i*) Reduce it to A′B′C′D′ with scale factor 0·5.

(*ii*) Measure the lengths A′B′ and B′C′.

Steps of Construction :



(*i*) Draw a line segment AB = 9·6 cm.

(*ii*) At A and B, draw perpendiculars and cut off AD = BC = 6 cm.

(*iii*) Join CD.

ABCD is a rectangle.

(*iv*) Take a point P outside the rectangle.

(*v*) Join PA, PB, PC and PD.

(*vi*) As $k = 0{\cdot}5$, then take mid points of line segments PA, PB, PC and PD as A′, B′, C′ and D′.

(*vii*) Join A′B′, B′C′, C′D′, D′A′.

Then A′B′C′D′ is the required rectangle.

Measuring the lengths of A′B′ = 4·8 cm and B′C′ = 3 cm.

Q. 6. On a map drawn to a scale 1 : 250000, a triangular plot of land has the following measurements :

AB = 3 cm, BC = 4 cm, $\angle ABC = 90^\circ$

Calculate : (*i*) the actual length of AB in km.

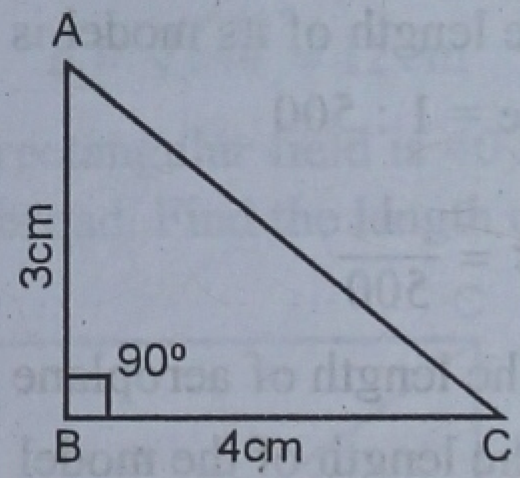
(*ii*) the actual length of AC in km.

(*iii*) the actual area of the plot in km^2.

Sol. Scale = 1 : 250000 $\Rightarrow k = \dfrac{1}{250000}$

On map of a plot, measurements are

AB = 3 cm, BC = 4 cm, $\angle ABC = 90^\circ$



$\therefore$ Lengths of AC $= \sqrt{AB^2 + BC^2}$

$= \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5\,\text{cm}$

(*i*) The actual length AB

$= \dfrac{1}{k}$ (length of AB on the map)

$= 250000 \times 3$ cm

$= \dfrac{250000 \times 3}{100 \times 1000} = 7{\cdot}5\,\text{km}$

(*ii*) Actual length of AC

$= \dfrac{1}{k}$ (Length of map)

$= 250000 \times 5$ cm

$$= \frac{250000 \times 5}{100 \times 1000} = 12{\cdot}5\,\text{km}$$

(*iii*) Actual area of the plot

$$= \frac{1}{k^2} \text{ (Area of plot on the map)}$$

$$= \frac{1}{k^2}\left(\frac{3 \times 4}{2}\right) = \frac{6}{k^2}$$

$$= (250000)^2 \times 6 \text{ cm}^2$$

$$= \frac{250000 \times 250000 \times 6}{100 \times 1000 \times 100 \times 1000}$$

$$= \frac{75}{2} = 37{\cdot}5\,\text{km}^2 \text{ **Ans.**}$$

Q. 7. A model of an aeroplane is made to a scale 1 : 500

Calculate : (*i*) the length in cm of the model, if the length of the aeroplane is 50 m.

(*ii*) the length in meters of the aeroplane, if the length of its model is 18 cm.

Sol. Scale = 1 : 500

$$\therefore\ k = \frac{1}{500}$$

(*i*) The length of aeroplane = 50 m.

∴ The length of the model

$$= \frac{1}{k} \text{ (actual length)}$$

$$= \frac{1}{500} \times 50 = \frac{1}{10}\,\text{m} = 10 \text{ cm}$$

(*ii*) The length of model = 18 cm

∴ The length of actual aeroplane

$$= \frac{1}{k} \text{ (length in model)}$$

$$= \frac{1}{\frac{1}{500}} (18\,\text{cm})$$

$$= 500 \times 18 \text{ cm}$$

$$= \frac{500 \times 18}{100} = 90\,\text{m} \text{ **Ans.**}$$

Q. 8. The dimensions of the model of a multistorey building are 1·2 m × 75 cm × 2 m. If scale factor is 1 : 40, find the actual dimensions of the building.

Sol. Scale factor = 1 : 40

$$\therefore\ k = \frac{1}{40}$$

Length in model = 1·2 m

$$\therefore \text{ Actual length} = \frac{1}{k} \text{ (length in model)}$$

$$= \frac{1}{\frac{1}{40}} (1{\cdot}2 \text{ m}) = 40 \times 1{\cdot}2 \text{ m}$$

$$= 48 \text{ m}$$

Width in model = 75 cm.

$$\therefore \text{ Actual width} = \frac{1}{k} \text{ (width in model)}$$

$$= \frac{1}{\frac{1}{40}} \times 75\,\text{cm} = 40 \times 75 \text{ cm}$$

$$= \frac{40 \times 75}{100} = 30\,\text{m}$$

Height in model = 2 m

$$\therefore \text{ Actual height} = \frac{1}{k} \text{ (height in model)}$$

$$= \frac{1}{\frac{1}{40}} (2\text{m}) = 40 \times 2 = 80 \text{ m}$$

Hence actual dimensions of the building are 48 m × 30 m × 80 m **Ans.**