# 11

# Mid-Point and Intercept Theorems

## POINTS TO REMEMBER

**Theorems.**

1. **Mid-point Theorem :** The line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
2. The straight line drawn through the mid-point of one side of a triangle and parallel to the other side, bisects the third side.
3. **Intercept Theorem :** If a transversal makes equal intercepts on three or more parallel lines, then any other line cutting them will also make equal intercepts.

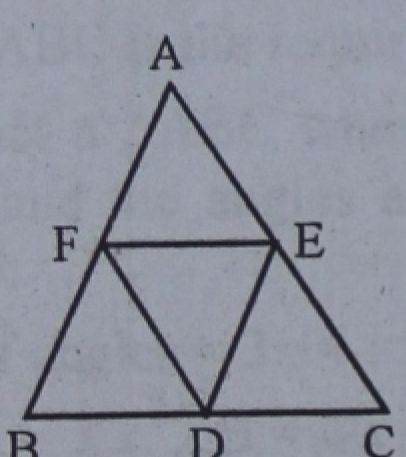
## EXERCISE 11

**Q. 1.** In the given figure, D, E, F are the mid-points of the sides BC, CA and AB respectively.

(*i*) If AB = 6·2 cm, find DE.

(*ii*) If DF = 3·8 cm, find AC.

(*iii*) If perimeter of $\Delta ABC$ is 21 cm, find FE.




**Sol.** D, E and F are the mid-points of the sides BC, CA and AB respectively.

$\therefore$ EF || BC and $EF = \frac{1}{2}BC$

Similarly DE || AB and $DE = \frac{1}{2}AB$

and FD || AC and $FD = \frac{1}{2}AC$

(*i*) Now if AB = 6·2 cm, then

$DE = \frac{1}{2}AB = \frac{1}{2} \times 6{\cdot}2 = 3{\cdot}1\,\text{cm}$.

(*ii*) DF = 3·8 cm,

But $DF = \frac{1}{2}AC \Rightarrow AC = 2\,DF$

$\therefore$ $AC = 2 \times 3{\cdot}8 = 7{\cdot}6$ cm

(*iii*). If perimeter of $\Delta ABC = 21$ cm

$\Rightarrow AB + BC + CA = 21$ cm

$\Rightarrow 6{\cdot}2 + BC + 7{\cdot}6 = 21$ cm

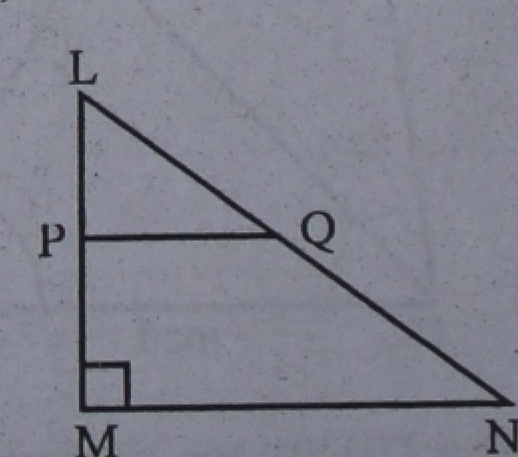
$\Rightarrow BC + 13{\cdot}8 = 21 \Rightarrow BC = 21 - 13{\cdot}8$

$\Rightarrow BC = 7{\cdot}2$ cm

But $FE = \frac{1}{2}BC$

$= \frac{1}{2} \times 7{\cdot}2 = 3{\cdot}6\,\text{cm}$ **Ans.**

**Q. 2.** In the given figure, LMN is a right triangle in which $\angle M = 90°$, P and Q are mid-points of LM and LN respectively. If LM = 9 cm, MN = 12 cm and LN = 15 cm, find :



(*i*) the perimeter of trapezium MNQP,

(*ii*) the area of trapezium MNQP.

**Sol.** In right angled $\Delta LMN$, $\angle M = 90^\circ$

P and Q are the mid-points of sides LM and LN respectively.

$\therefore PQ \parallel MN$ and $PQ = \frac{1}{2} MN$

Now LM = 9 cm, MN = 12 cm

and LN = 15 cm

$\therefore \quad PQ = \frac{1}{2} MN = \frac{1}{2} \times 12 = 6 \text{ cm}$

$PM = \frac{1}{2} LM = \frac{1}{2} \times 9 = 4{\cdot}5 \text{ cm}$

$QN = \frac{1}{2} LN = \frac{1}{2} \times 15 = 7{\cdot}5 \text{ cm}$.

(*i*) Perimeter of trapezium MNQP

$= PQ + QN + MN + PM$

$= 6 + 7{\cdot}5 + 12 + 4{\cdot}5 = 30$ cm.

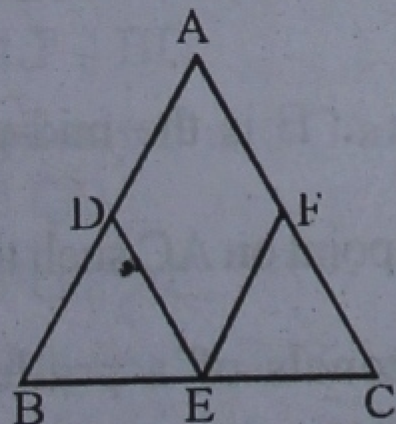
(*ii*) Area of trapezium MNQP

$= \frac{1}{2}(PQ + MN) \times PM \quad (\because \angle M = 90^\circ)$

$= \frac{1}{2}(6 + 12) \times 4{\cdot}5 \text{ cm}^2$

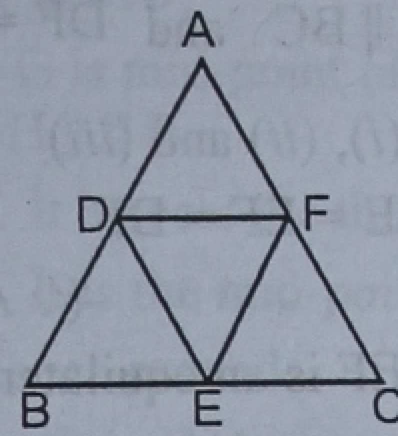
$= \frac{1}{2} \times 18 \times 4{\cdot}5 \text{ cm}^2 = 40{\cdot}5 \text{ cm}^2$. **Ans.**

**Q. 3.** In the given figure, D, E, F are respectively the mid-points of the sides AB, BC and CA of $\Delta ABC$. Prove that ADEF is a parallelogram.




**Sol. Given :** In the $\Delta ABC$, D, E and F are the mid-points of sides AB, BC and CA respectively.

**To prove :** ADEF is a parallelogram.




**Const.** Join DE, EF and FD.

**Proof :** $\because$ E and F are the mid-points of sides BC and CA respectively.

$\therefore \quad EF \parallel AB$ ...(*i*)

Similarly D and E are the mid-points of sides AB and BC respectively.

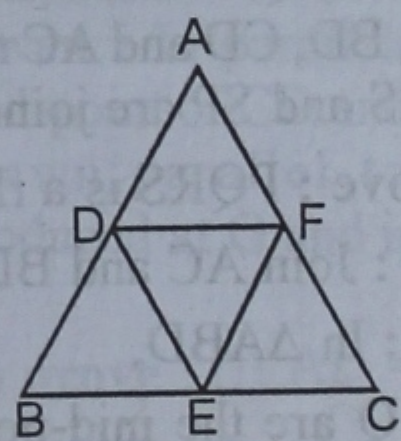
$\therefore \quad DE \parallel AC$ ...(*ii*)

From (*i*) and (*ii*)

ADEF is a parallelogram.

Hence proved.

**Q. 4.** If D, E, F are respectively the mid-points of the sides AB, BC and CA of an equilateral triangle ABC, prove that $\Delta DEF$ is also an equilateral triangle.




**Sol. Given :** $\Delta ABC$ is an equilateral triangle D, E and F are the mid-points of sides AB, BC and CA respectively. DE, EF and FD are joined.

**To prove :** $\Delta DEF$ is also an equilateral triangle.

**Proof :** $\because$ D and E are the mid-point of AB and BC respectively.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2} AC$ ...(*i*)

Similarly E and F are the mid-points of BC and CA respectively.

$\therefore EF \parallel AB$ and $EF = \frac{1}{2} AB$ ...(*ii*)

and F and D are the mid-points of AC and AB respectively.

$\therefore$ DF || BC and $DF = \frac{1}{2}BC$ ...(*iii*)

From (*i*), (*ii*) and (*iii*)

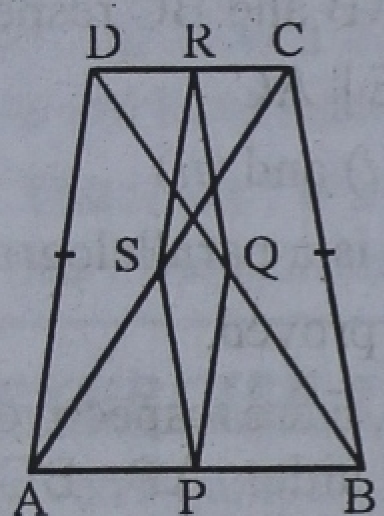
DE = EF = DF

($\because$ AB = BC = CA)

$\therefore$ ΔDEF is an equilateral triangle.

Hence proved.

**Q. 5.** In the adjoining figure, ABCD is a quadrilateral in which AD = BC and P, Q, R, S are the mid-points of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.




**Sol.** **Given :** In quadrilateral ABCD, AD = BC and P, Q, R and S are the mid-points of AB, BD, CD and AC respectively. PQ, QR, RS and SP are joined.

**To prove :** PQRS is a rhombus.

**Const :** Join AC and BD.

**Proof :** In ΔABD,

P and Q are the mid-points of AB and AD

$\therefore$ PQ || AD and $PQ = \frac{1}{2}AD$ ...(*i*)

In ΔACD,

S and R are the mid-points of AC and CD

$\therefore$ SR || AD and $SR = \frac{1}{2}AD$ ...(*ii*)

In ΔBCD,

Q and R are the mid-points of BD and CD

$\therefore$ QR || BC and $QR = \frac{1}{2}BC$ ...(*iii*)

and in ΔABC,

P and S are the mid-points of AB and AC

PS || BC and $PS = \frac{1}{2}BC$ ...(*iv*)

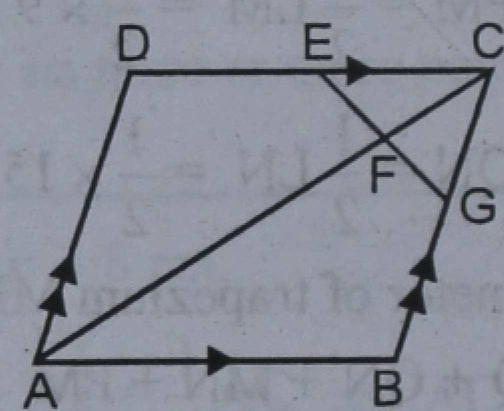
From (*i*), (*ii*) and (*iii*) and (*iv*)

PS = QR = RS = SP ($\because$ AD = BC)

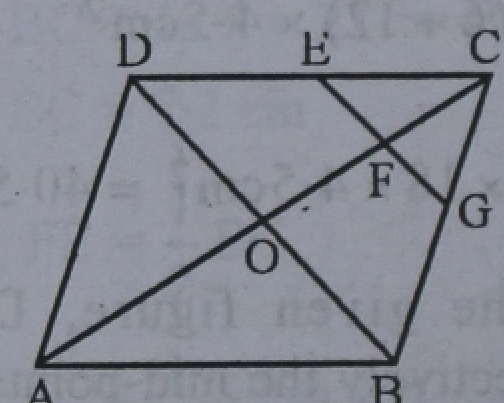
$\therefore$ PQRS is a rhombus.

Hence proved.

**Q. 6.** In the adjoining figure, ABCD is a parallelogram in which E is the mid-point of DC and F is a point on AC such that $CF = \frac{1}{4}AC$. If EF is produced to meet BÇ in G, prove that G is the mid-point of BC.




**Sol.** **Given :** In parallelogram ABCD, in which E is the mid-point of DC and F is a point on AC such that $CF = \frac{1}{4}AC$. EF is produced to meet BC at G.




**To prove :** G is the mid-point of BC.

**Const :** Join BD which intersects AC at O.

**Proof :** $\because$ E is the mid-point of DC and F is the point on AC such that $CF = \frac{1}{4}AC$.

$\because$ Diagonals of a prallelogram bisects each other.

$\therefore$ AO = OC

$CF = \frac{1}{2}OC$

⇒ F is the mid-point of OC.

Now in $\Delta DOC$,

EF || DO or BD

We know that any line drawn from the mid-point of one side of a triangle, parallel to the other side, bisects the third side.

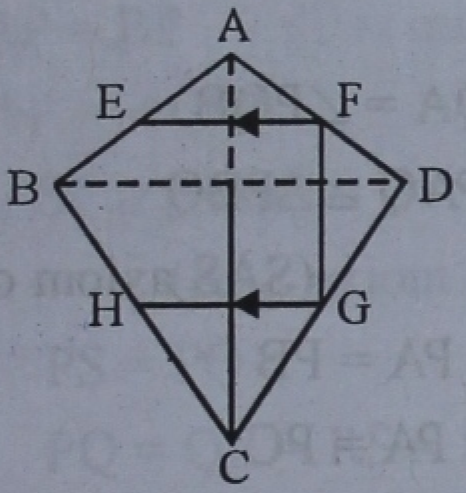
∴ G is mid-point of BC of $\Delta BCD$.

**Q.E.D.**

**Q. 7.** In the adjoining figure, ABCD is a kite in which AB = AD and CB = CD. If E, F, G are respectively the mid-points of AB, AD and CD, prove that :

(*i*) $\angle EFG = 90^\circ$,

(*ii*) If GH || FE, then H bisects CB.




**Sol.** In kite shaped quadrilateral ABCD, AB = AD and CB = CD. E, F and D are the mid-points of AB, AD and CD respectively.

**To prove :** (*i*) $\angle EFG = 90^\circ$

(*ii*) If GH || FE, then H bisects CB.

**Const :** Join AC and BD.

**Proof :** (*i*) In $\Delta ABD$,

E and F are mid-points of AB and AD.

∴ EF || BD.

Similarly F and G are mid-points of AD and CD.

∴ FG || AC.

∵ Diagonals of a kite intersect each other at right angles.

∴ AC and BD are perpendicular to each other.

∴ $\angle EFG = 90^\circ$.

(*ii*) In $\Delta BCD$,

∵ G is mid-point of DC and GH || EF or GH || BD.

∴ It bisects the third side.

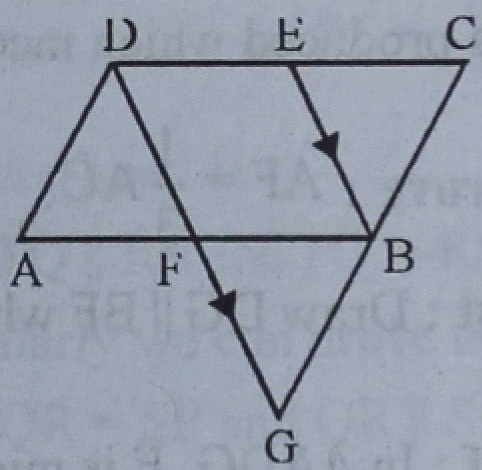
∴ It is the mid-point of BC.

Hence proved.

**Q. 8.** In the adjoining figure, ABCD is a parallelogram, E is the mid-point of CD and through D, a line is drawn parallel to EB to meet CB produced at G and intersecting AB at F.

Prove that : (*i*) $AD = \frac{1}{2} GC$

(*ii*) DG = 2 EB.




**Sol. Given :** In parallelogram ABCD, E is mid-point of CD and through D, a line is drawn parallel to EB to meet CB produced at G and intersects AB at F.

**To prove :** (*i*) $AD = \frac{1}{2} GC$

(*ii*) DG = 2 EB.

**Proof :** (*i*) In $\Delta CDG$, E is mid-point of DC and EB || GD.

∴ B is mid-point of CG.

$$\Rightarrow \quad BC = \frac{1}{2} GC$$

$$\Rightarrow \quad AD = \frac{1}{2} GC$$

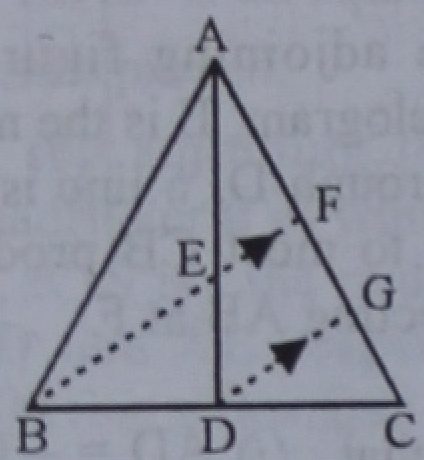
(∵ AD = BC, Opposite sides of the parallelogram)

(*ii*) and $EB = \frac{1}{2} DG$

⇒ 2 EB = DG ⇒ DG = 2 EB

Hence proved.

**Q. 9.** In the adjoining figure, in $\Delta ABC$, AD is the median through A and E is the mid-point of AD. If BE produced meets AC in F, prove that $AF = \frac{1}{3} AC$.




**Sol.** **Given :** In $\Delta ABC$, AD is the median through A and E is the mid-point of AD. BE is produced which meets AC in F.

**To prove :** $AF = \frac{1}{3} AC$.

**Const :** Draw $DG \parallel BF$ which meets AC at G.

**Proof :** In $\Delta ADG$, E is mid-point of AD and $BEF \parallel DG$.

$\therefore$ F is mid-point of AG

or $\quad AF = FG \quad$ ...(*i*)

Again in $\Delta BCF$,

D is mid-point of BC and $DG \parallel BEF$

$\therefore$ G is mid-point of FC

or $\quad FG = GC \quad$ ...(*ii*)

From (*i*) and (*ii*)

$$AF = FG = GC$$

But $AF + FG + GC = AC$

$\Rightarrow AF + AF + AF = AC$

$\Rightarrow 3\,AF = AC$

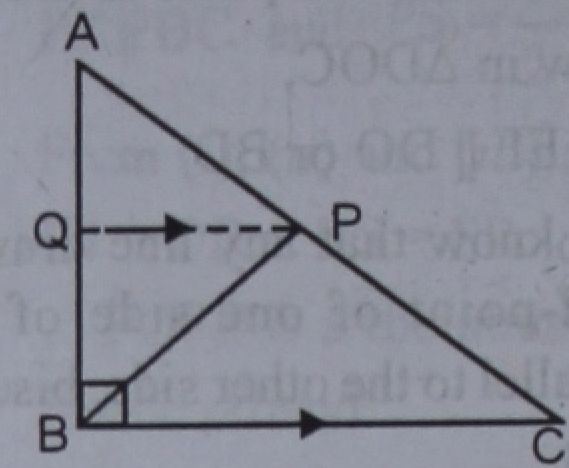
$\therefore \quad AF = \frac{1}{3} AC$

Hence proved.

**Q. 10.** In the adjoining figure, $\Delta ABC$ is a right-angled at B and P is the mid-point of AC. Show that PA = PB = PC.

**Sol.** **Given :** In $\Delta ABC$, $\angle B = 90^o$

P is mid-point of AC. PB is joined.




**To prove :** PA = PB = PC.

**Const :** Through P, draw $PQ \parallel BC$.

**Proof :** $\because$ P is mid-point of AC and $PQ \parallel BC$.

$\therefore$ Q is mid-point of AB.

In $\Delta PAQ$ and $\Delta PBQ$,

PQ = PQ (common)

AQ = BQ ($\because$ Q is mid-point of AB)

$\angle PQA = \angle PQB$ (each $= 90^o$)

$\therefore \Delta PAQ \cong \Delta PBQ$

(SAS axiom of congruency)

$\therefore \quad PA = PB$

But $\quad PA = PC$

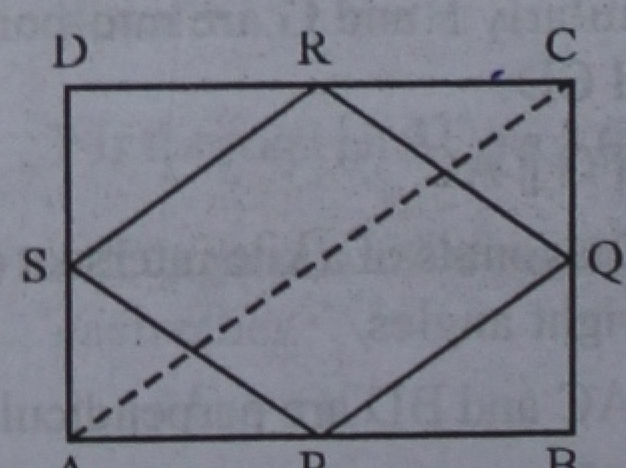
($\because$ P is mid-point of AC)

$\therefore \quad PA = PB = PC$

Hence proved.

**Q. 11.** Show that the quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a rectangle is a rhombus.

**Sol.** **Given :** In rectangle ABCD, P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.




**To prove :** PQRS is a rhombus.

**Const :** Join AC.

**Proof :** In $\Delta ABC$,

P and Q are the mid-points of sides AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(*i*)

Similarly S and R are the mid-points of sides AD and DC respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*)

$PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

Now in $\Delta ASP$ and $\Delta BQP$,

$AS = BQ$ (Half of equal sides)

$AP = BP$ (P is mid-point of AB)

$\angle SAP = \angle QBP$ (Each = 90°)

$\therefore \Delta ASP \cong \Delta BQP$

(SAS axiom of congruency)

$\therefore PS = PQ$ (c.p.c.t.)

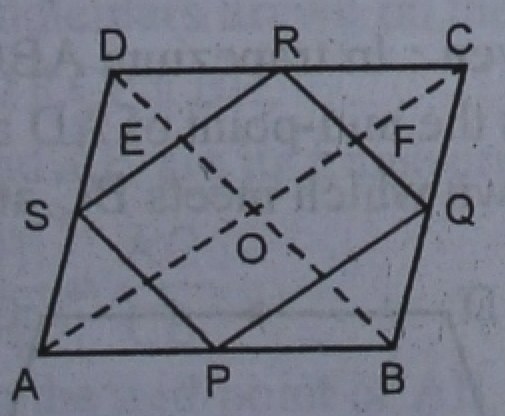
$\therefore PQ = QR = RS = SP$

Hence PQRS is a rhombus.

Hence proved.

**Q. 12.** Show that the quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a rhombus is a rectangle.

**Sol. Given :** In rhombus ABCD, P, Q, R and S are the mid-points of its sides AB, BC, CD and DA respectively and PQ, QR, RS and SP are joined.




**To prove :** PQRS is a rectangle.

**Const :** Join AC and BD which intersect each other at O.

**Proof :** $\because$ The diagonals of a rhombus bisect each other at right angles.

$\therefore AO = OC$, $BO = OD$ and AC and BD bisect each other at right angles.

Now in $\Delta ABC$,

P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(*i*)

Similarly, In $\Delta ADC$,

S and R are the mid-points of CD and DA respectively.

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*)

$PQ \parallel RS$ and $PQ = RS$

Similarly we can prove that

$QR = SP$ and $QR \parallel SP$

$\therefore$ PQRS is a parallelogram.

$\because PQ \parallel AR$ and $QR \parallel BD$

and $AC \perp BD$

$\therefore$ PQRS is a rectangle.

Hence proved.

**Q. 13.** Show that the quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a square is a square.

**Sol. Given :** In square ABCD, P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

**To prove :** PQRS is a square.

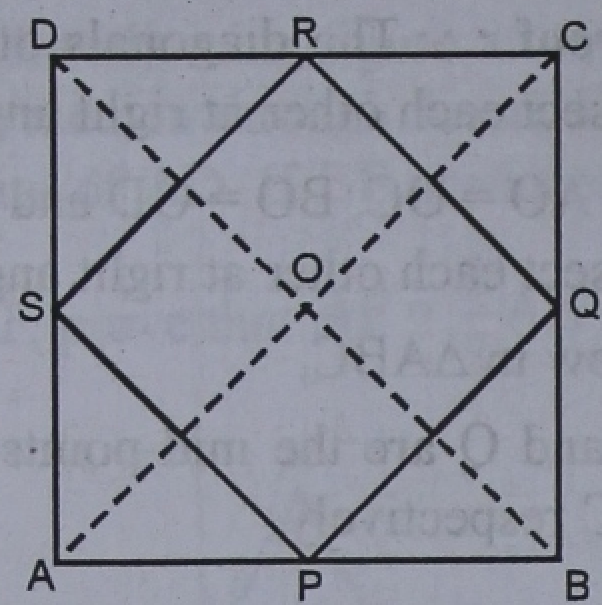
**Construction :** Join AC and BD.

**Proof :** In $\Delta ABC$,

P and Q are the mid-points of sides AB and BC respectively.

$\therefore PQ \parallel AC$

and $PQ = \frac{1}{2}AC$ ...(*i*)




Similarly in $\Delta ADC$,

S and R are the mid-points of AD and CD respectively

$\therefore$ SR $\parallel$ AC and $SR = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*),

PQ = SR and PQ $\parallel$ SR

$\therefore$ PQRS is a parallelogram.

Now, in $\Delta APS$ and $\Delta BPQ$

AP = PB

($\because$ P, Q, S are the mid-points of the sides)

AS = BQ

and $\angle A = \angle B$ (Each = 90°)

$\therefore$ $\Delta APS \cong \Delta BPQ$ (SAS axioms)

$\therefore$ PS = PQ (c.p.c.t.)

$\therefore$ PQ = QR = RS = SP

$\therefore$ PQRS is a rhombus or a square.

$\because$ PQ $\parallel$ AC : QR and PS $\parallel$ BD $\parallel$ QR

(Proved)

But, diagonals AC and BD of a square bisect each other at right angles.

$\therefore$ Each angle of PQRS is a right angle

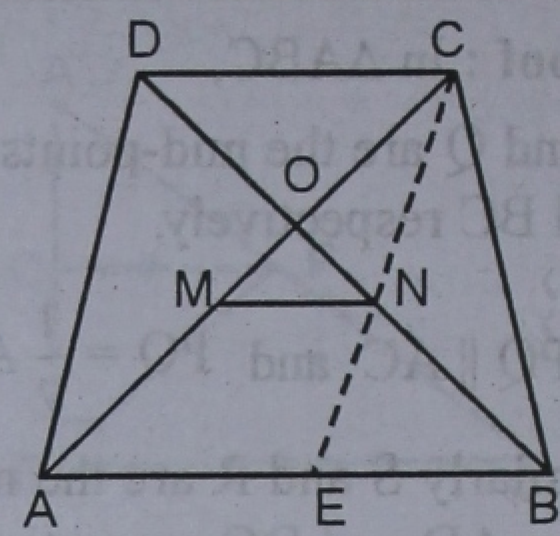
Hence, PQRS is square.

Hence proved.

**Q. 14.** In the adjoining figure, ABCD is a trapezium in which AB $\parallel$ DC. If M and N are the mid-points of AC and BD respectively, prove that

$$MN = \frac{1}{2}(AB - CD).$$

**Sol. Given :** In trapezium ABCD, AB $\parallel$ DC. M and N are the mid-points of AC and BD respectively and MN are joined.




**To prove :** $MN = \frac{1}{2}(AB - CD)$.

**Const :** Join CN and produce it to meet AB at E.

**Proof :** In $\Delta CDN$ and $\Delta EBN$

DN = NB (N is mid-point of BD)

$\angle CND = \angle BNE$

(Vertically opposite angles)

$\angle CDN = \angle NBE$ (Alternate angles)

$\therefore$ $\Delta CDN \cong \Delta EBN$

(ASA axiom of congruency)

$\therefore$ CD = EB (C.P.C.T.)

and CN = NE (C.P.C.T.)

In $\Delta ACE$,

M and N are mid-points of AC and CE

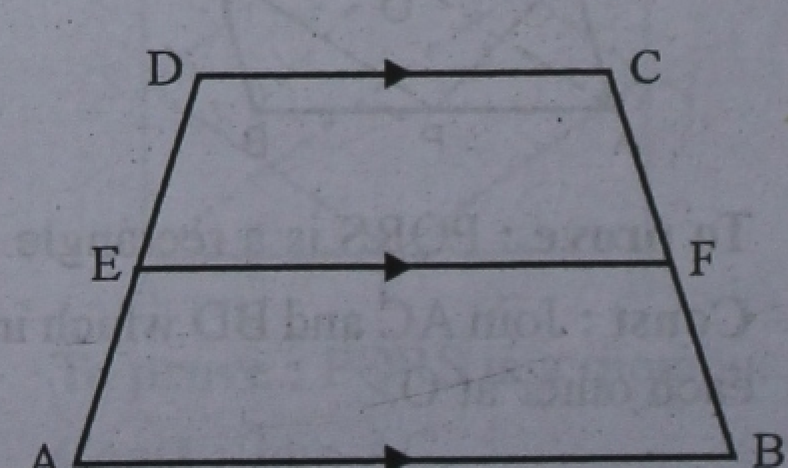
$\therefore$ $MN = \frac{1}{2}AE = \frac{1}{2}(AB - EB)$

$= \frac{1}{2}(AB - CD)$ [$\because$ EB = CD]

Hence proved.

**Q. 15.** In the adjoining figure, ABCD is a trapezium in which AB $\parallel$ DC and E is the mid-point of AD. If EF $\parallel$ AB meets BC at F, show that F is the mid-point of BC.

**Sol. Given :** In trapezium ABCD, AB $\parallel$ DC. E is the mid-point of AD and EF $\parallel$ AB is drawn which meets BC at F.



**To prove :** F is the mid-point of BC.

**Proof :** ∵ AB || EF || DC, CB and AD are its transversals.

$\therefore \quad \frac{DE}{EA} = \frac{CF}{FB}$ (Intercept Theorem)

But DE = EA $\Rightarrow \frac{DE}{EA} = 1$

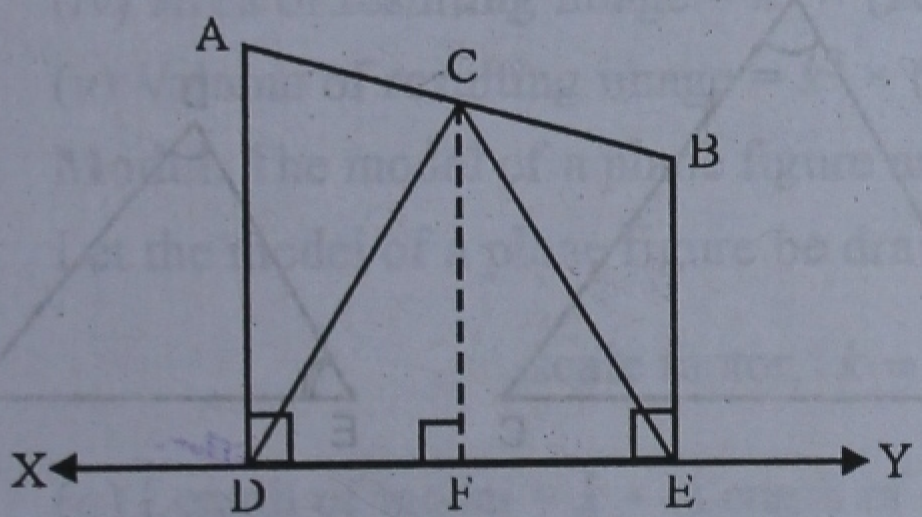
$\therefore \quad \frac{CF}{FB} = 1 \quad \Rightarrow \quad CF = FB$

Hence F is the mid-point of BC.

Hence proved.

**Q. 16.** Two points of A and B lie on the same side of a line XY. If AD ⊥ XY and BE ⊥ XY, meet XY in D and E respectively and C is the mid-point of AB, show that CD = CE.

**Sol. Given :** Two points A and B which lie on the same side of a line XY. AD ⊥ XY and BE ⊥ XY. C is mid-point of AB. CD and CE are joined.




**To prove :** CD = CE.

**Const :** Through C, draw CF ⊥ XY.

**Proof :** ∵ AD, CF and BE are perpendiculars drawn on the line XY.

∴ AD || CF || BE,

DE and AB are its transversals.

$\therefore \quad \frac{DF}{FE} = \frac{AC}{CB}$

∵ C is the mid-point of AB.

$\therefore \quad AC = CB \Rightarrow \frac{AC}{CB} = 1$

$\Rightarrow \frac{DF}{FE} = \quad \Rightarrow \quad DF = FE$

Now in ΔCDF and ΔCEF,

CF = CF (Common)

∠CFD = ∠CFE (Each 90°)

DF = FE (Proved)

∴ ΔCDF ≅ ΔCEF

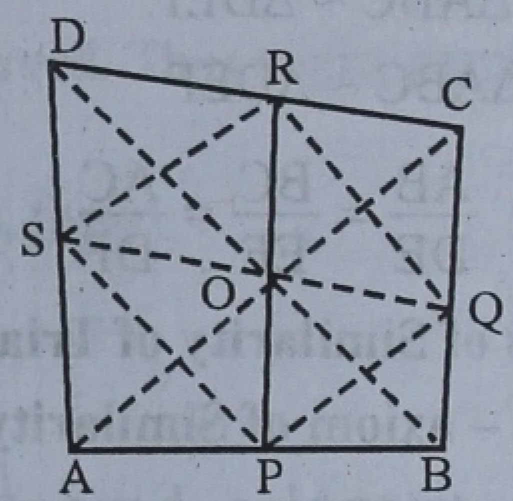
(SAS axiom of congruency)

∴ CD = CE (C.P.C.T.)

Hence proved.

**Q. 17.** Prove that the straight lines joining the mid-points of the opposite sides of a quadrilateral bisect each other.

**Sol. Given :** In quadrilateral ABCD, P, Q, R and S are the mid-points of its sides AB, BC, CD and DA respectively.




PR and QS are joined to intersect each other at O.

**To prove :** QO = OS and PO = OR.

**Const :** Join PQ, QR, RS and SP, AC and BD.

**Proof :** In ΔABC,

P and Q are the mid-points of AB and BC respectively.

$\therefore$ PQ || AC and $PQ = \frac{1}{2}AC$ ...(*i*)

Similarly in ΔADC,

S and R are the mid-points of AD and CD respectively.

$\therefore$ SR || AC and $SR = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*)

PQ || SR and PQ = SR

∴ PQRS is a parallelogram.

But the diagonals of a parallelogram bisect each other.

∴ PR and QS bisect each other at O.

Hence PO = OR and QO = OS.

Hence proved.