9 Logarithms

POINTS TO REMEMBER

1. If $a^b = x$ then $\log_a x = b$
and it is read as
b is the logarithms of x to the base a. Logarithms of only positive real numbers are defined.
Note. (*i*) a^b is called the exponential form and $\log_a x$ is called the logarithmic form.
(*ii*) $\log_a 1 = 0$ as $a^0 = 1$.
(*iii*) $\log_a a = 1$ as $a^1 = a$.

2. **Common Logarithms.** The logarithms to the base 10 is called the common logarithms and is denoted as $\log_{10} a$ or simply $\log a$.
Note. (*i*) $\log 10 = 1$ (*ii*) $\log 100 = 2$
(*iii*) $\log 1000 = 3$ (*iv*) $\log 10000 = 4$ etc.
(*v*) $\log \frac{1}{10}$ or $\log 10^{-1}$ or $\log 0{\cdot}1 = -1$ as $10^{-1} = \frac{1}{10}$
(*vi*) $\log \frac{1}{100}$ or $\log 0{\cdot}001$ or $\log (10)^{-2} = -2$ etc.

3. **Laws of logarithms :**
(*i*) $\log_a mn = \log_a m + \log_a n$ (Product law) (*ii*) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$ (Quotient law)
(*iii*) $\log_a (m)^n = n \log_a m$ (Power law) (*iv*) $\log_a a = 1$ (*v*) $\log_a 1 = 0$.

4. **Characteristic and Mantissa of a logarithm :**
A logarithm of a number has two parts :
(*i*) The integral part which is called characteristic and
(*ii*) The decimal part which is called mantissa.
Note. (*i*) The mantissa is always taken as positive while the characteristic may be positive or negative.
(*ii*) When the characteristic is negative then we denote it by putting a bar on the digit as -2 is written as $\overline{2}$
(*iii*) $\overline{3}{\cdot}5241$ means $(-3 + {\cdot}5241)$

5. **How to find the characteristic and Mantissa of a logarithm :**
Rule 1. The characteristic of logarithm of a number greater than or equal to 1 is one less than the number of digits to the left of the decimal point in the number.

2. The characteristic of logarithm of a number less than 1, is a negative number whose numerical value is one more than the number of zeros immediately following the decimal part.

3. We find mantissa from the log tables.

4. The position of decimal point in a number is immaterial for finding mantissa.

6. **Antilog :** If $\log m = n$, then antilog $n = m$.

7. **How to find antilog of a number.**

(*i*) We use the decimal part for finding the antilog from the antilog table.

(*ii*) After finding the corresponding number from the antilog table, we insert the decimal point as under :

(*a*) If characteristic is n, decimal point is put after $(n + 1)$th digit

If characteristic is $\bar{n}$ *i.e.* $-n$, then decimal point is put in such a way that first significant figure is at the nth place.

EXERCISE 9 (A)

Q. 1. Convert each of the following to logarithmic form :

(*i*) $5^2 = 25$ (*ii*) $3^{-3} = \frac{1}{27}$

(*iii*) $(64)^{\frac{1}{3}} = 4$ (*iv*) $6^0 = 1$

(*v*) $10^{-2} = 0{\cdot}01$ (*vi*) $4^{-1} = \frac{1}{4}$

Sol. (*i*) $5^2 = 25$

$\therefore \log_5 25 = 2$

(*ii*) $3^{-3} = \frac{1}{27}$

$\therefore \log_3\left(\frac{1}{27}\right) = -3$

(*iii*) $(64)^{\frac{1}{3}} = 4$

$\therefore \log_{64} 4 = \frac{1}{3}$

(*iv*) $6^0 = 1$

$\therefore \log_6 1 = 0$

(*v*) $10^{-2} = 0{\cdot}01$

$\therefore \log_{10}(0{\cdot}01) = -2$

(*vi*) $4^{-1} = \frac{1}{4}$

$\therefore \log_4\left(\frac{1}{4}\right) = -1$ **Ans.**

Q. 2. Convert each of the following to exponential form :

(*i*) $\log_3 81 = 4$ (*ii*) $\log_8 4 = \frac{2}{3}$

(*iii*) $\log_2 \frac{1}{8} = -3$ (*iv*) $\log_{10}(0{\cdot}01) = -2$

(*v*) $\log_5\left(\frac{1}{5}\right) = -1$ (*vi*) $\log_a 1 = 0$

Sol. (*i*) $\log_3 81 = 4$ $\therefore 3^4 = 81$

(*ii*) $\log_8 4 = \frac{2}{3}$ $\therefore (8)^{\frac{2}{3}} = 4$

(*iii*) $\log_2 \frac{1}{8} = -3$ $\therefore 2^{-3} = \frac{1}{8}$

(*iv*) $\log_{10}(0{\cdot}01) = -2$ $\therefore 10^{-2} = 0{\cdot}01$

(*v*) $\log_5\left(\frac{1}{5}\right) = -1$ $\therefore 5^{-1} = \frac{1}{5}$

(*vi*) $\log_a 1 = 0$ $\therefore a^0 = 1$

Q. 3. By converting to exponential form, find the value of each of the following :

(*i*) $\log_2 64$ (*ii*) $\log_8 32$

(*iii*) $\log_3 \frac{1}{9}$ (*iv*) $\log_{0\cdot5}(16)$

(*v*) $\log_2(0{\cdot}125)$ (*vi*) $\log_7 7$

Sol. (*i*) Let $\log_2 64 = x$, then

$2^x = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^x = 2^6$

$\therefore \quad x = 6$

Hence $\log_2 64 = 6$ **Ans.**

(*ii*) Let $\log_8 32 = x$, then

$8^x = 32 \Rightarrow 2^{3x} = 2^5$

$\Rightarrow \quad 2^{3x} = 2^5$

$\therefore \quad 3x = 5 \Rightarrow x = \frac{5}{3}$

Hence $\log_8 32 \quad \frac{5}{3}$ **Ans.**

(*iii*) Let $\log_3 \frac{1}{9} = x$, then

$3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$\therefore \quad x = -2$

Hence $\log_3\left(\frac{1}{9}\right) = -2$ **Ans.**

(*iv*) Let $\log_{0\cdot5}(16) = x$, then

$(0\cdot5)^x = 16 \Rightarrow \left(\frac{1}{2}\right)^x = 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^{-x} = 2^4$

$\therefore -x = 4 \Rightarrow x = -4$

Hence $\log_{0\cdot5}(16) = -4$ **Ans.**

(*v*) Let $\log_2(0\cdot125) = x$, then

$2^x = 0\cdot125 = \frac{0\cdot125}{1000} = \frac{1}{8} = \frac{1}{2^3}$

$\Rightarrow 2^x = 2^{-3} \quad \therefore \quad x = -3$

Hence $\log_2(0\cdot125) = -3$ **Ans.**

(*vi*) Let $\log_7 7 = x$, then

$7^x = 7 = 7^1 \quad \therefore \quad x = 1$

Hence $\log_7 7 = 1$ **Ans.**

Q. 4. Find the value of x, when :

(*i*) $\log_2 x = -2$ (*ii*) $\log_x 9 = 1$

(*iii*) $\log_9 243 = x$ (*iv*) $\log_3 x = 0$

(*v*) $\log_{\sqrt{3}}(x-1) = 2$

(*vi*) $\log_5(x^2 - 19) = 3$

(*vii*) $\log_x 64 = \frac{3}{2}$

(*viii*) $\log_2(x^2 - 9) = 4$

(*ix*) $\log_x(0\cdot008) = -3$

Sol. (*i*) $\log_2 x = -2$

$\therefore 2^{-2} = x \Rightarrow x = \frac{1}{2^2} = \frac{1}{2 \times 2} = \frac{1}{4}$

Hence $x = \frac{1}{4}$ **Ans.**

(*ii*) $\log_x 9 = 1$

$\therefore x^1 = 9 \quad \Rightarrow x = 9$

Hence $x = 9$ **Ans.**

(*iii*) $\log_9 243 = x$

$\therefore \quad 9^x = 243$

$\Rightarrow (3^2)^x = 3 \times 3 \times 3 \times 3 \times 3$

$\Rightarrow \quad 3^{2x} = 3^5$

$\therefore \quad 2x = 5 \quad \Rightarrow \quad x = \frac{5}{2} = 2\cdot5$

(*iv*) $\log_3 x = 0$

$\Rightarrow 3^0 = x \quad \Rightarrow \quad x = 1 \quad (\because 3^0 = 1)$

Hence $x = 1$

(*v*) $\log_{\sqrt{3}}(x-1) = 2$

$\therefore \quad (\sqrt{3})^2 = x - 1$

$\Rightarrow \quad (3^{\frac{1}{2}})^2 = x - \quad \Rightarrow 3 = x - 1$

$\therefore \quad x = 3 + 1 = 4$ **Ans.**

(*vi*) $\log_5(x^2 - 19) = 3$

$\Rightarrow \quad x^2 - 19 = 5^3 = 125$

$\Rightarrow x^2 = 125 + 19 = 144$

$\Rightarrow \quad x = \pm\sqrt{144} = \pm 12$ **Ans.**

(*vii*) $\log_x 64 = \frac{3}{2}$

$\Rightarrow x^{3/2} = 64$

$\Rightarrow x = (64)^{2/3} = (4^3)^{2/3}$

$= 4^{3\times\frac{2}{3}} = 4^2 = 16$

$\therefore \quad x = 16$ **Ans.**

(*viii*) $\log_2 (x^2 - 9) = 4$

$\Rightarrow x^2 - 9 = 2^4 = 16$

$\Rightarrow x^2 = 16 + 9 = 25$

$\therefore \quad x = \pm\sqrt{25} = \pm 5$ **Ans.**

(*ix*) $\log_x (0{\cdot}008) = -3$

$\Rightarrow \quad x^{-3} = 0{\cdot}008$

$\Rightarrow \quad \frac{1}{x^3} = \frac{8}{1000}$

$\Rightarrow \quad x^3 = \frac{1000}{8}$

$\Rightarrow \quad x^3 = 125 = (5)^3$

$\therefore \quad x = 5$ **Ans.**

Q. 5. If $\log_{10} x = p$ and $\log_{10} y = q$, show that $xy = (10)^{p+q}$.

Sol. $\because \log_{10} x = p$

$\therefore \quad 10^p = x$

Similarly $\log_{10} y = q$

$\therefore \quad 10^q = y$

Now $xy = 10^p \times 10^q = (10)^{p+q}$

Hence proved.

Q. 6. Given $\log_{10} x = a$, $\log_{10} y = b$,

(*i*) Write down 10^{a+1} in terms of x.

(*ii*) Write down 10^{2b} in terms of y.

(*iii*) If $\log_{10} P = 2a - b$, express P in terms of x and y.

Sol. $\log_{10} x = a$, $\log_{10} y = b$

$\therefore \quad 10^a = x \quad$ and $\quad 10^b = y$

(*i*) $10^{a+1} = 10^a \times 10^1$

$= x \times 10 = 10x$

$(\because 10^a = x)$

(*ii*) $10^{2b} = 10^b \times 10^b$

$= y \times y = y^2 \qquad (\because 10^b = y)$

(*iii*) $\log_{10} P = 2a - b \quad \therefore 10^{2a-b} = P$

$\Rightarrow \quad P = 10^{2a} \div 10^b$

$= (10^a)^2 \div 10^b$

$= x^2 \div y \; (10^a = x$ and $10^b = y)$

$= \frac{x^2}{y}$ **Ans.**

EXERCISE 9 (B)

Q. 1. Evaluate the following without using log tables :

(*i*) $2 \log 5 + \log 8 - \frac{1}{2}\log 4$

(*ii*) $\log 8 + \log 25 + 2 \log 3 - \log 18$

(*iii*) $5 \log 2 + \frac{3}{2}\log 25 + \frac{1}{2}\log 49 - \log 28$

(*iv*) $2 \log 2 + \log 5 - \frac{1}{2}\log 36 - \log \frac{1}{30}$

(*v*) $\log (1{\cdot}2) + 2 \log (0{\cdot}75) - \log (6{\cdot}75)$

Sol. (*i*) $2 \log 5 + \log 8 - \frac{1}{2}\log 4$

$= \log (5)^2 + \log 8 - \log (4)^{\frac{1}{2}}$

$(\log m^n = n \log m)$

$= \log 25 + \log 8 - \log 2 = \log \frac{25 \times 8}{2}$

$\left(\because \log m + \log n = \log mn \text{ and } \log m - \log n = \log \frac{m}{n}\right)$

$= \log 100 = 2$ **Ans.**

(*ii*) $\log 8 + \log 25 + 2 \log 3 - \log 18$

$= \log 8 + \log 25 + \log (3)^2 - \log 18$

$\{\because \log (m)^n = n \log m\}$

$= \log 8 + \log 25 + \log 9 - \log 18$

$= \log \frac{8 \times 25 \times 9}{18}$

$\left\{\because \log m + \log n = \log mn \text{ and } \log m - \log n \quad \log \frac{m}{n}\right\}$

$= \log 100 = 2$ **ans.**

(*iii*) $5\log 2 + \frac{3}{2}\log 25 + \frac{1}{2}\log 49 - \log 28$

$= \log 2^5 + \log (25)^{\frac{3}{2}} + \log (49)^{\frac{1}{2}} - \log 28$

$= \log 32 + \log (5)^3 + \log 7 - \log 28$

$= \log 32 + \log 125 + \log 7 - \log 28$

$= \log \frac{32 \times 125 \times 7}{28} = \log 1000 = 3$ **Ans.**

(*iv*) $2\log 2 + \log 5 - \frac{1}{2}\log 36 - \log \frac{1}{30}$

$= \log 2^2 + \log 5 - \log (36)^{\frac{1}{2}} - \log \frac{1}{30}$

$= \log 4 + \log 5 - \log 6 - [\log 1 - \log 30]$

$= \log 4 + \log 5 - \log 6 - \log 1 + \log 30$

$= \log \frac{4 \times 5 \times 30}{6 \times 1} = \log 100 = 2$ **Ans.**

(*v*) $\log (1{\cdot}2) + 2\log (0{\cdot}75) - \log (6{\cdot}75)$

$= \log (1{\cdot}2) + \log (0{\cdot}75)^2 - \log (6{\cdot}75)$

$= \log (1{\cdot}2) + \log ({\cdot}5625) - \log 6{\cdot}75$

$= \log \frac{1{\cdot}2 \times 0{\cdot}5625}{6{\cdot}25} = \log \frac{12 \times 5625 \times 100}{10 \times 10000 \times 675}$

$= \log \frac{1}{10} = \log 0{\cdot}1 = -1$ **Ans.**

Q. 2. Evaluate :

$\log 5 + 16\log\left(\frac{625}{6}\right) + 12\log\left(\frac{4}{375}\right) + 7\log\left(\frac{81}{1250}\right)$

Sol.

$\log 5 + 16\log\left(\frac{625}{6}\right) + 12\log\left(\frac{4}{375}\right) + 7\log\left(\frac{81}{1250}\right)$

$= \log 5 + \log\left(\frac{625}{6}\right)^{16} + \log\left(\frac{4}{375}\right)^{12} + \log\left(\frac{81}{1250}\right)^{7}$

$(\because n \log m = \log m^n)$

$= \log\left[5 \times \left(\frac{625}{6}\right)^{16} \times \left(\frac{4}{375}\right)^{12} \times \left(\frac{81}{1250}\right)^{7}\right]$

$= \log\left[5 \times \left(\frac{5^4}{2 \times 3}\right)^{16} \times \left(\frac{2^2}{5^3 \times 3}\right)^{12} \times \left(\frac{3^4}{2 \times 5^4}\right)^{7}\right]$

$= \log\left[\frac{5 \times 5^{64} \times 2^{24} \times 3^{28}}{2^{16} \times 3^{16} \times 5^{36} \times 3^{12} \times 2^7 \times 5^{28}}\right]$

$= \log [5^{1+64-36-28} \times 3^{28-16-12} \times 2^{24-16-7}]$

$= \log [5^1 \times 3^0 \times 2^1] = \log (5 \times 1 \times 2)$

$= \log 10 = 1$ **Ans.**

Q. 3. Express $\log_{10}\left(\frac{x^2 y^3}{z}\right)$ in terms of $\log 10^x$, $\log 10^y$ and $\log 10^z$.

Sol. $\log_{10}\left(\frac{x^2 y^3}{z}\right)$

$$\left\{\begin{array}{l} \because \log mn = \log m + \log n \\ \log \frac{m}{n} = \log m - \log n \\ \log m^n = n \log m \end{array}\right.$$

$= \log_{10} x^2 + \log_{10} y^3 - \log 10^z$

$= 2\log_{10} x + 3\log_{10} y - \log_{10}{}^z$ Ans.

Q. 4. Express $\log_{10} \frac{\sqrt{pq^3}}{r^2 s}$ in terms of $\log 10^p$, $\log 10^q$, $\log 10^r$ and $\log 10^s$.

Sol. $\log_{10} \frac{\sqrt{pq^3}}{r^2 s} = \frac{(pq^3)^{\frac{1}{2}}}{r^2 s}$

$= \log_{10}(pq^3)^{\frac{1}{2}} - \log_{10} r^2 s = \frac{1}{2}\log_{10} pq^3 - \log_{10} r_2 s$

$= \frac{1}{2}[\log_{10} p + \log_{10} q^3] - [\log_{10} r^2 + \log_{10} s]$

$= \frac{1}{2}[\log_{10} p + 3\log_{10} q] - [2\log_{10} r + \log_{10} s]$

$= \frac{1}{2}[\log_{10} p + 3\log_{10} q] - 2\log_{10} r - \log_{10} s$ Ans.

Q. 5. Express each of the following as a single logarithm :

(*i*) $2\log_{10} 8 + \log_{10} 36 - \log_{10} (1{\cdot}5) - 3\log_{10} 2$

(*ii*) $2\log_{10} 5 + 2\log_{10} 3 - \log_{10} 2 + 1$

(*iii*) $2 + \frac{1}{2}\log_{10} 9 - 2\log_{10} 5$

(*iv*) $\frac{1}{2}\log_{10} 9 + \frac{1}{4}\log_{10} 81 + 2\log_{10} 6 - \log_{10} 12$

(v) $2\log_{10}\left(\frac{11}{13}\right)+\log_{10}\left(\frac{130}{77}\right)-\log_{10}\left(\frac{55}{91}\right)$

(vi) $1-\frac{1}{3}\log_{10}64$

Sol. (i) $2\log_{10}8+\log_{10}36-\log_{10}(1\cdot5)-3\log_{10}2$

$=\log_{10}(8)^2+\log_{10}36-\log(1\cdot5)-\log_{10}2^3$

$=\log_{10}64+\log_{10}36-\log_{10}1\cdot5-\log_{10}8$

$=\log_{10}\frac{64\times36}{1\cdot5\times8}=\log_{10}\left(\frac{64\times36\times10}{15\times8}\right)$

$=\log_{10}192$

(ii) $2\log_{10}5+2\log_{10}3-\log_{10}2+1$

$=\log_{10}(5)^2+\log_{10}(3)^2-\log_{10}2+\log_{10}10$ $\{\because \log_{10}10=1\}$

$=\log_{10}25+\log_{10}9-\log_{10}2+\log_{10}10$

$=\log_{10}\frac{25\times9\times10}{2}=\log_{10}1125$ **Ans.**

(iii) $2+\frac{1}{2}\log_{10}9-2\log_{10}5$

$=\log_{10}100+\log_{10}(9)^{\frac{1}{2}}-\log_{10}(5)^2$ $\{\because \log_{10}100=2\}$

$=\log_{10}100+\log_{10}3-\log_{10}25$

$=\log_{10}\frac{100\times3}{25}=\log_{10}12$ **Ans.**

(iv) $\frac{1}{2}\log_{10}9+\frac{1}{4}\log_{10}81+2\log_{10}6-\log_{10}12$

$=\log_{10}(9)^{\frac{1}{2}}+\log_{10}(81)^{\frac{1}{4}}+\log_{10}(6)^2-\log_{10}12$

$=\log_{10}3+\log_{10}3+\log_{10}36-\log_{10}12$

$=\log_{10}\frac{3\times3\times36}{12}=\log_{10}27$ **Ans.**

(v) $2\log_{10}\left(\frac{11}{13}\right)+\log_{10}\left(\frac{130}{77}\right)-\log_{10}\left(\frac{55}{91}\right)$

$=\log_{10}\left(\frac{11}{13}\right)^2+\log_{10}\left(\frac{130}{77}\right)-\log_{10}\left(\frac{55}{91}\right)$

$=\log_{10}\frac{121}{169}+\log_{10}\left(\frac{130}{77}\right)-\log_{10}\left(\frac{55}{91}\right)$

$=\log_{10}\left(\frac{121}{169}\times\frac{130}{77}\times\frac{91}{55}\right)=\log_{10}2$ **Ans.**

(vi) $1-\frac{1}{3}\log_{10}64=\log_{10}10-\log_{10}(64)^{\frac{1}{3}}$ $\{\because \log_{10}10=1\}$

$=\log_{10}10-\log_{10}4=\log_{10}\frac{10}{4}=\log_{10}\frac{5}{2}$

$=\log_{10}(2\cdot5)$ **Ans.**

Q. 6. Evaluate the following without using log tables :

(i) $\frac{\log 81}{\log 27}$ (ii) $\frac{\log 128}{\log 32}$

(iii) $\frac{\log 27}{\log\sqrt{3}}$ (iv) $\frac{\log 9-\log 3}{\log 27}$

Sol. (i) $\frac{\log 81}{\log 27}=\frac{\log 3^4}{\log 3^3}=\frac{4\log 3}{3\log 3}=\frac{4}{3}$ **Ans.**

(ii) $\frac{\log 128}{\log 32}=\frac{\log 2^7}{\log 2^5}=\frac{7\log 2}{5\log 2}=\frac{7}{5}$ **Ans.**

(iii) $\frac{\log 27}{\log\sqrt{3}}=\frac{\log 3^3}{\log 3^{\frac{1}{2}}}=\frac{3\log 3}{\frac{1}{2}\log 3}$

$=\frac{3}{\frac{1}{2}}=\frac{3\times2}{1}=6$ **Ans.**

(iv) $\frac{\log 9-\log 3}{\log 27}=\frac{\log 3^2-\log 3}{\log 3^3}$

$=\frac{2\log 3-\log 3}{3\log 3}=\frac{\log 3}{3\log 3}=\frac{1}{3}$ **Ans.**

Q.7 . Given : $\log 2=0\cdot3010$ and $\log 3=0\cdot4771$, find the value of :

(i) $\log 12$ (ii) $\log 25$ (iii) $\log\sqrt{18}$ (iv) $\log\left(\frac{9}{4}\right)$

Sol. (i) $\log 12=\log 4\times3=\log(2^2\times3)$

$=\log 2^2+\log 3=2\log 2+\log 3=2(0\cdot3010)+0\cdot4771$

$=0\cdot6020+0\cdot4771=1\cdot0791$ **Ans.**

(ii) $\log 25=\log\frac{5}{2}=\log\frac{5\times2}{2\times2}=\log\frac{10}{2^2}$

$=\log 10-\log 2^2=\log 10-2\log 2$

$=1-2(0\cdot3010)=1-0\cdot6020=0\cdot3980$ **Ans.**

(*iii*) $\log\sqrt{18} = \log(18)^{\frac{1}{2}}$

$= \log(3 \times 3 \times 2)^{\frac{1}{2}}$

$= \log(3^2 \times 2)^{\frac{1}{2}} = \frac{1}{2}\log(3^2 \times 2)$

$= \frac{1}{2}[\log 3^2 + \log 2]$

$= \frac{1}{2}[2\log 3 + \log 2]$

$= \frac{1}{2} \times 2\log 3 + \frac{1}{2}\log 2$

$= \log 3 + \frac{1}{2}\log 2$

$= 0{\cdot}4771 + \frac{1}{2} \times 0{\cdot}3010$

$= 0{\cdot}4771 + 0{\cdot}1505 = 0{\cdot}6276$ **Ans.**

(*iv*) $\log\left(\frac{9}{4}\right) = \log\left(\frac{3 \times 3}{2 \times 2}\right) = \log\left(\frac{3}{2}\right)^2$

$= 2\log\frac{3}{2} = 2[\log 3 - \log 2]$

$= 2\ [0{\cdot}4771 - 0{\cdot}3010]$

$= 2 \times 0{\cdot}1761 = 0{\cdot}3522$ **Ans.**

Q.8. If $\log 2 = 0{\cdot}3010$, find the value of

$\left(\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}\right)$.

Sol. $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$

$= \log\frac{75}{16} - \log\left(\frac{5}{9}\right)^2 + \log\frac{32}{243}$

$= \log\frac{75}{16} - \log\frac{25}{81} + \log\frac{32}{243}$

$= \log\left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25}\right)$

$= \log 2 = 0{\cdot}3010$ **Ans.**

Q.9. If $\log 8 = 0{\cdot}9030$, find the value of :

(*i*) $\log 4$ (*ii*) $\log\sqrt{32}$

(*iii*) $\log(0{\cdot}125)$

Sol. $\because \log 8 = 0{\cdot}9030$

$\Rightarrow \log(2^3) = 0{\cdot}9030$

$\Rightarrow 3\log 2 = 0{\cdot}9030$

$\therefore \log 2 = \frac{0{\cdot}9030}{3} = 0{\cdot}3010$

Now (*i*) $\log 4 = \log 2^2 = 2\log 2$

$= 2\ (0{\cdot}3010) = 0{\cdot}6020$ **Ans.**

(*ii*) $\log\sqrt{32} = \log(32)^{\frac{1}{2}} = \frac{1}{2}\log 2^5$

$\frac{5}{2}\log 2 = \frac{5}{2}(0{\cdot}3010)$

$= 5 \times 0{\cdot}1505 = 0{\cdot}7525$ **Ans.**

(*iii*) $\log(0{\cdot}125) = \log\frac{125}{1000} = \log\left(\frac{1}{8}\right)$

$= \log\left(\frac{1}{2}\right)^3 = \log(2)^{-3}$

$= -3\log 2 = -3\ (0{\cdot}3010)$

$= -0{\cdot}9030$ **Ans.**

Q.10. If $\log 27 = 1{\cdot}4313$, find the value of :

(*i*) $\log 9$ (*ii*) $\log 30$

Sol. $\log 27 = 1{\cdot}4313$

$\Rightarrow \log 3^3 = 1{\cdot}4313$

$\Rightarrow 3\log 3 = 1{\cdot}4313$

$\therefore \log 3 = \frac{1{\cdot}4313}{3} = 0{\cdot}477$

Now (*i*) $\log 9 = \log 3^2 = 2\log 3$

$= 2\ (0{\cdot}4771) = 0{\cdot}9542$ **Ans.**

(*ii*) $\log 30 = \log(3 \times 10)$

$= \log 3 + \log 10$

$= 0{\cdot}4771 + 1 = 1{\cdot}4771$ **Ans.**

Q.11. Show that $\log(1 + 2 + 3) = \log 1 + \log 2 + \log 3$

Sol. $\log(1 + 2 + 3) = \log 6$

$= \log(1 \times 2 \times 3)$

$= \log 1 + \log 2 + \log 3$

Hence proved.

$[\because \log(mnp) = \log_m + \log_n + \log_p]$

Q.12. (*i*) If $\log(m+n) = \log m + \log n$, show that $m = \frac{n}{n-1}$.

(*ii*) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$, show that $\frac{1}{2}(a+b) = \sqrt{ab}$.

Sol. (*i*) $\log(m+n) = \log m + \log n$

$\Rightarrow \log(m+n) = \log(m \times n)$

$\therefore \quad m + n = mn$

$\Rightarrow \quad m - mn = -n$

$\Rightarrow \quad m(1-n) = -n$

$$\Rightarrow \quad m = \frac{-n}{1-n} = \frac{-n}{-(n-1)} = \frac{n}{n-1}$$

Hence, $\quad m = \frac{n}{n-1}$.

(*ii*) $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$$\Rightarrow \log\left(\frac{a+b}{2}\right) = \frac{1}{2}\log(ab) = \log(ab)^{\frac{1}{2}}$$

$$\Rightarrow \log\left(\frac{a+b}{2}\right) = \log\sqrt{ab}$$

Comparing we get

$$\frac{a+b}{2} = \sqrt{ab} \quad \text{or} \quad \frac{1}{2}(a+b) = \sqrt{ab}$$

Hence proved.

Q.13. Solve for x :

(*i*) $\log(x+2) + \log(x-2) = \log 5$

(*ii*) $\log(x+4) - \log(x-4) = \log 2$

(*iii*) $\log(x+3) - \log(x-3) = 1$

(*iv*) $\log(x^2 - 21) = 2$

(*v*) $2\log x + 1 = \log 250$

(*vi*) $\frac{\log x}{\log 5} = \frac{\log 9}{\log\left(\frac{1}{3}\right)}$

Sol. (*i*) $\log(x+2) + \log(x-2) = \log 5$

$\Rightarrow \quad \log(x+2)(x-2) = \log 5$

($\because \log m + \log n = \log mn$)

$\Rightarrow \quad \log(x^2 - 4) = \log 5$

$\therefore \quad x^2 - 4 = 5 \Rightarrow x^2 = 5 + 4 = 9$

$\Rightarrow \quad x = \pm\sqrt{9} = \pm 3$ **Ans.**

(*ii*) $\log(x+4) - \log(x-4) = \log 2$

$$\Rightarrow \quad \log\frac{x+4}{x-4} = \log 2$$

$$\therefore \quad \frac{x+4}{x-4} = \frac{2}{1} \Rightarrow 2x - 8 = x + 4$$

(By cross multiplication)

$2x - x = 4 + 8 \quad \Rightarrow \quad x = 12$

$\therefore \quad x = 12$ **Ans.**

(*iii*) $\log(x+3) - \log(x-3) = 1$

$$\Rightarrow \log\frac{x+3}{x-3} = \log 10 \quad (\because \log 10 = 1)$$

$$\therefore \quad \frac{x+3}{x-3} = 10 \Rightarrow 10x - 30 = x + 3$$

(By cross multiplication)

$10x - x = 3 + 30 \Rightarrow 9x = 33$

$$\Rightarrow \quad x = \frac{33}{9} = \frac{11}{3}$$

Hence $\quad x = \frac{11}{3}$ **Ans.**

(*iv*) $\log(x^2 - 21) = 2$

$\Rightarrow \log(x^2 - 21) = \log 100 \ (\because \log 100 = 2)$

$\therefore \quad x^2 - 21 = 100$

$\Rightarrow x^2 = 100 + 21 = 121$

$\therefore \quad x = \pm\sqrt{121} = \pm 11$ **Ans.**

(*v*) $2\log x + 1 = \log 250$

$\Rightarrow 2\log x + \log 10 = \log 250$

($\because \log_{10 = 1}$)

$\Rightarrow \log x^2 + \log 10 = \log 250$

$\Rightarrow \quad \log(x^2 \times 10) = \log 250$

$\therefore\ 10x^2 = 250 \Rightarrow x^2 = \frac{250}{10} = 25$

$\therefore\quad x = \pm\sqrt{25} = \pm 5$

(*vi*) $\frac{\log x}{\log 5} = \frac{\log 9}{\log\left(\frac{1}{3}\right)}$

$\Rightarrow \frac{\log x}{\log 5} = \frac{\log 3^2}{\log(3^{-1})} = \frac{2\log 3}{-1\log 3} = -2$

$\Rightarrow \log x = -2 \times \log 5 = \log (5)^{-2}$

$\Rightarrow \quad \log x = \log\frac{1}{25}$

$\therefore \quad x = \frac{1}{25}$ **Ans.**

Q. 14. If

$[\log 7 - \log 2 + \log 16 - 2\log 3 - \log\frac{7}{45}]$

$= 1 + \log n$, find the value of n.

Sol. $\log 7 - \log 2 + \log 16 - 2\log 3 - \log\frac{7}{45}$

$= 1 + \log n$

$\Rightarrow \log 7 - \log 2 + \log 16 - \log 3^2$

$- \log\frac{7}{45} = \log 10 + \log n$

$(\because \log_{10} = 1)$

$\Rightarrow \log 7 - \log 2 + \log 16 - \log 9$

$- \log\frac{7}{45} = \log 10 + \log n$

$\Rightarrow \log\left(\frac{7 \times 16 \times 45}{2 \times 9 \times 7}\right) = \log (10 \times n)$

$= \log (10\, n)$

$\therefore \quad 10n = \frac{7\times16\times45}{2\times9\times7}$

$\Rightarrow \quad n = \frac{7\times16\times45}{10\times2\times9\times7} = 4$

Hence $\quad n = 4$ **Ans.**

Q. 15. Write the logarithmic equation for :

(*i*) $R = \sqrt{\frac{3V}{\pi h}}$ (*ii*) $x = ab\sqrt{\frac{a-b}{a+b}}$

Sol. (*i*) $R = \sqrt{\frac{3V}{\pi h}}$

Taking log both side,

$\log R = \log\sqrt{\frac{3V}{\pi h}} = \log\left(\sqrt{\frac{3V}{\pi h}}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\ [\log 3V - \log\pi h]$

$= \frac{1}{2}\ [\log 3 + \log v - \log\pi - \log h]$

(*ii*) $x = ab\sqrt{\frac{a-b}{a+b}}$

Taking log both sides

$\log x = \log ab\sqrt{\frac{a-b}{a+b}}$

$\Rightarrow \log x = \log a + \log b + \log\left(\frac{a-b}{a+b}\right)^{\frac{1}{2}}$

$= \log a + \log b + \frac{1}{2}\ \log\frac{a-b}{a+b}$

$= \log a + \log b + \frac{1}{2}\ [\log (a-b) - \log(a+b)]$ Ans.