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8 Indices

POINTS TO REMEMBER

1. **Indices :** For any real number 'a' and positive integer 'n', we define $a \times a \times a$ ... to n factors $= a^n$.

Here 'a' is called base and n is called index or exponent.

Some Laws of Indices

(*i*) $a^o = 1$, (*ii*) $a^{-n} = \frac{1}{a^n}$ (*iii*) $a^m \times a^n = a^{m+n}$ (*iv*) $a^m \div a^n = a^{m-n}$

(*v*) $(a^m)^n = a^{mn}$ (*vi*) $a^{-m} = \frac{1}{a^m}$ (*vii*) $(ab)^m = a^m \cdot b^m$ (*viii*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(*ix*) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ (*x*) If $a^m = a^n$, then $m = n$ where $a > 0$ and $a \neq 1$.

(*xi*) If $p^m \times q^n \times r^o = p^a q^b r^c$, then $m = a$, $n = b$ and $o = c$ where p, q and r are different primes.

(*xii*) If $a^m = b^m$, then $a = b$ where a and b are positive.

(*xiii*) $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a} = a^{\frac{1}{3}}$, $\sqrt[4]{a} = a^{\frac{1}{4}}$

Some formulae which are used

(*i*) $(a + b)(a - b) = a^2 - b^2$ (*ii*) $(a - b)(a^2 + ab + b^2) = a^3 - b^3$

EXERCISE 8

Evaluate :

Q. 1. (*i*) $(125)^{\frac{1}{3}}$ (*ii*) $(8)^{\frac{2}{3}}$

(*iii*) $\left(\frac{1}{5}\right)^{-2}$ (*iv*) $(16)^{\frac{-3}{4}}$

(*v*) $(32)^{\frac{-4}{5}}$ (*vi*) $\left(\frac{8}{125}\right)^{\frac{-1}{3}}$

(*vii*) $(-27)^{\frac{2}{3}}$ (*viii*) $(0{\cdot}001)^{\frac{-1}{3}}$

(*ix*) $(0{\cdot}027)^{\frac{-2}{3}}$

Sol. (*i*) $(125)^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}}$

$= 5^{3 \times \frac{1}{3}} = 5$ **Ans.**

(*ii*) $(8)^{\frac{2}{3}} = (2 \times 2 \times 2)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$

$= 2^{3 \times \frac{2}{3}} = 2^2 = 2 \times 2 = 4$ **Ans.**

$\{\because (a^m)^n = a^{mn}\}$

(*iii*) $\left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 5 \times 5 = 25$ **Ans.**

(*iv*) $(16)^{\frac{-3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{-3}{4}} = (2^4)^{\frac{-3}{4}}$

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$= 2^{\frac{-3}{4}\times 4} = 2^{-3} = \frac{1}{2^3}$

$\left\{\because (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m}\right\}$

$= \frac{1}{2\times 2\times 2} = \frac{1}{8}$ **Ans.**

(*v*) $(32)^{\frac{-4}{5}} = (2\times 2\times 2\times 2\times 2)^{\frac{-4}{5}}$

$= (2^5)^{\frac{-4}{5}} = 2^{\frac{-4}{5}\times 5} = 2^{-4} = \frac{1}{2^4}$

$\left\{\because (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m}\right\}$

$= \frac{1}{2\times 2\times 2\times 2} = \frac{1}{16}$ **Ans.**

(*vi*) $\left(\frac{8}{125}\right)^{\frac{-1}{3}} = \left(\frac{2\times 2\times 2}{5\times 5\times 5}\right)^{\frac{-1}{3}}$

$= \left[\left(\frac{2}{5}\right)^3\right]^{\frac{-1}{3}}$

$= \left(\frac{2}{5}\right)^{\frac{-1}{3}\times 3} = \left(\frac{2}{5}\right)^{-1} = \frac{5}{2}$ **Ans.**

$\left\{\because (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m}\right\}$

(*vii*) $(-27)^{\frac{2}{3}} = [(-3)\times(-3)\times(-3)]^{\frac{2}{3}}$

$= (-3)^{3\times\frac{2}{3}} = (-3)^2 \quad \{\because (a^m)^n = a^{mn}\}$

$= -3\times -3 = 9$ **Ans.**

(*viii*) $(0\cdot 001)^{\frac{-1}{3}} = (0\cdot 1\times 0\cdot 1\times 0\cdot 1)^{\frac{-1}{3}}$

$= (0\cdot 1)^{3\times\left(\frac{-1}{3}\right)}$

$\{\because (a^m)^n = a^{mn}\}$

$= (0\cdot 1)^{-1} = \frac{1}{0\cdot 1} = \frac{1}{\frac{1}{10}} = \frac{1\times 10}{1}$

$\left\{\because a^{-m} = \frac{1}{a^m}\right\}$

$= 10$ **Ans.**

(*ix*) $(0\cdot 027)^{\frac{-2}{3}} = (0\cdot 3\times 0\cdot 3\times 0\cdot 3)^{\frac{-2}{3}}$

$= (0\cdot 3)^{\frac{-2}{3}\times 3}$

$\{\because (a^m)^n = a^{mn}\}$

$= (0\cdot 3)^{-2} = \frac{1}{(0\cdot 3)^2} = \frac{1}{0\cdot 3\times 0\cdot 3}$

$\left(\because a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{0\cdot 09} = \frac{1}{\frac{9}{100}} = \frac{100}{9}$ **Ans.**

Evaluate the following :

Q. 2. (*i*) $\left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}}\times 5^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$

(*ii*) $\sqrt{\frac{1}{4}} + (0\cdot 01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}\times 3^0$

Sol. (*i*) $\left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}}\times 5^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$

$= \left(\frac{1}{2}\times\frac{1}{2}\right)^{-2} - 3\times(2\times 2\times 2)^{\frac{2}{3}}\times 5^0 + \left(\frac{3}{4}\times\frac{3}{4}\right)^{-\frac{1}{2}}$

$= \left(\frac{1}{2}\right)^{2\times(-2)} - 3\times(2^3)^{\frac{2}{3}}\times 5^0 + \left(\frac{3}{4}\right)^{2\times\left(-\frac{1}{2}\right)}$

$\{\because (a^m)^n = a^{mn}\}$

$= \left(\frac{1}{2}\right)^{-4} - 3\times 2^2\times 1 + \left(\frac{3}{4}\right)^{-1} \quad \{\because a^0 = 1\}$

$= (2)^4 - 3\times 4\times 1 + \frac{4}{3}$

$= (2\times 2\times 2\times 2) - 12 + \frac{4}{3} \quad \left\{\because a^{-m} = \frac{1}{a^m}\right\}$

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$$= 16 - 12 + \frac{4}{3} = 4 + \frac{4}{3}$$

$$= \frac{12+4}{3} = \frac{16}{3} = 5\frac{1}{3} \textbf{ Ans.}$$

(*ii*) $\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} \times 3^0$

$$= \left[\left(\frac{1}{2}\right)^2\right]^{\frac{1}{2}} + (0.1 \times 0.1)^{-\frac{1}{2}} - (3 \times 3 \times 3)^{\frac{2}{3}} \times 3^0$$

$$= \left(\frac{1}{2}\right)^{2 \times \frac{1}{2}} + (0.1)^{2 \times \left(-\frac{1}{2}\right)} - (3^3)^{\frac{2}{3}} \times 3^0$$

$$= \left(\frac{1}{2}\right)^1 + (0.1)^{-1} - (3)^2 \times 3^0$$

$\{\because (a^m)^n = a^{mn}\}$

$$= \frac{1}{2} + \frac{1}{0.1} - 9 \times 1 \quad \left\{\because a^{-m} = \frac{1}{a^m} \text{ and } a^0 = 1\right\}$$

$$= \frac{1}{2} + \frac{10}{1} - 9 = 10\frac{1}{2} - 9 = 1\frac{1}{2} \textbf{ Ans.}$$

Q. 3. (*i*) $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

(*ii*) $\left[(64)^{\frac{2}{3}} \times 2^{-2} \div 7^0\right]^{\frac{-1}{2}}$

Sol. (*i*) $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

$$= \left(\frac{3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2}\right)^{\frac{-3}{4}} \times \left[\left(\frac{5 \times 5}{3 \times 3}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{4 \times \left(\frac{-3}{4}\right)} \times \left[\left(\frac{5}{3}\right)^{2 \times \left(\frac{-3}{2}\right)} \div \left(\frac{2}{5}\right)^3\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{3}\right)^{-3} \div \left(\frac{2}{5}\right)^3\right]$$

$\{\because (a^m)^n = a^{mn}\}$

$$= \left(\frac{2}{3}\right)^3 \times \left[\left(\frac{3}{5}\right)^3 \div \left(\frac{2}{5}\right)^3\right]$$

$\left\{\because a^{-m} = \frac{1}{a^m}\right\}$

$$= \left[\frac{2 \times 2 \times 2}{3 \times 3 \times 3}\right] \times \left[\left(\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}\right) \div \left(\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right)\right]$$

$$= \frac{8}{27} \times \left[\frac{27}{125} \times \frac{125}{8}\right] = \frac{8}{27} \times \frac{27}{8} = 1 \textbf{ Ans.}$$

(*ii*) $\left[(64)^{\frac{2}{3}} \times 2^{-2} \div 7^0\right]^{\frac{-1}{2}}$

$$= \left[(4 \times 4 \times 4)^{\frac{2}{3}} \times \frac{1}{2^2} \div 1\right]^{\frac{-1}{2}}$$

$$= \left[(4^3)^{\frac{2}{3}} \times \frac{1}{2 \times 2} \div 1\right]^{\frac{-1}{2}}$$

$$= \left[4^{\left(3 \times \frac{2}{3}\right)} \times \frac{1}{4} \div 1\right]^{\frac{-1}{2}}$$

$$= \left[4^2 \times \frac{1}{4} \div 1\right]^{\frac{-1}{2}} = \left(16 \times \frac{1}{4} \times 1\right)^{\frac{-1}{2}}$$

$$= (4)^{\frac{-1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{(2 \times 2)^{\frac{1}{2}}} = \frac{1}{2^{2 \times \frac{1}{2}}}$$

$$= \frac{1}{2} \textbf{ Ans.}$$

Q. 4. (*i*) $(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{\frac{-2}{5}} + (8)^{\frac{1}{3}} . \left(\frac{1}{2}\right)^{-1} . (2)^0$

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(*ii*) $\left(\frac{16}{81}\right)^{\frac{-3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$

Sol. (*i*) $(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{\frac{-2}{5}} + (8)^{\frac{1}{3}} . \left(\frac{1}{2}\right)^{-1} . 2^0$

$= (3 \times 3 \times 3 \times 3)^{\frac{3}{4}} - \left(\frac{1}{2 \times 2 \times 2 \times 2 \times 2}\right)^{\frac{-2}{5}} + (2 \times 2 \times 2)^{\frac{1}{3}} \times \left(\frac{1}{2}\right)^{-1} \times 2^0$

$= (3^4)^{\frac{3}{4}} - \left(\frac{1}{2^5}\right)^{\frac{-2}{5}} + (2^3)^{\frac{1}{3}} \times \left(\frac{1}{2}\right)^{-1} \times 2^0$

$= 3^{4 \times \frac{3}{4}} - \left(\frac{2^{-5}}{1}\right)^{\frac{-2}{5}} + 2^{3 \times \frac{1}{3}} \times (2)^1 \times 1$

$= 3^3 - 2^2 + 2^1 \times 2^1 \times 1$

$= 27 - 4 + 2 \times 2 \times 1 = 27 - 4 + 4 = 27$ **Ans.**

(*ii*) $\left(\frac{16}{81}\right)^{\frac{-3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$

$= \left(\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}\right)^{\frac{-3}{4}} \times \left(\frac{7 \times 7}{3 \times 3}\right)^{\frac{3}{2}} \div \left(\frac{7 \times 7 \times 7}{6 \times 6 \times 6}\right)^{\frac{2}{3}}$

$= \left[\left(\frac{2}{3}\right)^4\right]^{\frac{-3}{4}} \times \left[\left(\frac{7}{3}\right)^2\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^3\right]^{\frac{2}{3}}$

$= \left(\frac{2}{3}\right)^{4 \times \left(\frac{-3}{4}\right)} \times \left(\frac{7}{3}\right)^{2 \times \frac{3}{2}} \div \left(\frac{7}{6}\right)^{3 \times \frac{2}{3}}$

$= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^3 \div \left(\frac{7}{6}\right)^2$

$= \left(\frac{3}{2}\right)^3 \times \frac{7 \times 7 \times 7}{3 \times 3 \times 3} \div \frac{7 \times 7}{6 \times 6}$

$= \frac{3 \times 3 \times 3}{2 \times 2 \times 2} \times \frac{343}{27} \div \frac{49}{36}$

$= \frac{27}{8} \times \frac{343}{27} \times \frac{36}{49}$

$= \frac{63}{2} = 31\frac{1}{2}$ **Ans.**

Q. 5. (*i*) $\left(\frac{64}{125}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$

(*ii*) $\frac{(32)^{\frac{2}{5}} \times (4)^{\frac{-1}{2}} \times (8)^{\frac{1}{3}}}{(2)^{-2} \div (64)^{\frac{-1}{3}}}$

Sol. (*i*) $\left(\frac{64}{125}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$

$= \left(\frac{4 \times 4 \times 4}{5 \times 5 \times 5}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{4 \times 4 \times 4 \times 4}{5 \times 5 \times 5 \times 5}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{5^2}}{\sqrt[3]{4^3}}\right)^0$

$= \left[\left(\frac{4}{5}\right)^3\right]^{\frac{-2}{3}} \div \frac{1}{\left[\left(\frac{4}{5}\right)^4\right]^{\frac{1}{4}}} + \left(\frac{5}{4}\right)^0$

$= \left(\frac{4}{5}\right)^{3 \times \left(\frac{-2}{3}\right)} \div \frac{1}{\left(\frac{4}{5}\right)^{4 \times \frac{1}{4}}} + 1 \quad [\because a^o = 1]$

$= \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)^1} + 1 = \left(\frac{5}{4}\right)^2 \div \frac{5}{4} + 1$

$\left[\because a^{-m} = \frac{1}{a^m}\right]$

$= \frac{25}{16} \times \frac{4}{5} + 1 = \frac{5}{4} + \quad = \frac{9}{4} = 2\frac{1}{4}$ **Ans.**

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(*ii*) $\dfrac{(32)^{\frac{2}{5}} \times (4)^{\frac{-1}{2}} \times (8)^{\frac{1}{3}}}{(2)^{-2} \div (64)^{\frac{-1}{3}}}$

$= \dfrac{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}} \times (2 \times 2)^{\frac{-1}{2}} \times (2 \times 2 \times 2)^{\frac{1}{3}}}{(2)^{-2} \div (4 \times 4 \times 4)^{\frac{-1}{3}}}$

$= \dfrac{(2^5)^{\frac{2}{5}} \times (2^2)^{\frac{-1}{2}} \times (2^3)^{\frac{1}{3}}}{(2)^{-2} \div (4^3)^{\frac{-1}{3}}}$

$= \dfrac{(2)^{5\times\frac{2}{5}} \times (2)^{2\times\left(\frac{-1}{2}\right)} \times (2)^{3\times\frac{1}{3}}}{(2)^{-2} \div (4)^{3\times\left(\frac{-1}{3}\right)}}$

$= \dfrac{2^2 \times 2^{-1} \times 2^1}{2^{-2} \div 4^{-1}} = \dfrac{2^{2-1+1}}{2^{-2} \div (2^2)^{-1}}$

$= \dfrac{2^2}{2^{-2} \div 2^{-2}} = \dfrac{2 \times 2}{1} = 4$ **Ans.**

Q. 6. (*i*) $(27)^{\frac{4}{3}} + (32)^{0\cdot 8} + (0\cdot 8)^{-1} + (0\cdot 8)^0$

(*ii*) $\left[\dfrac{(27)^{-3}}{(9)^{-3}}\right]^{\frac{1}{3}}$

Sol. (*i*) $(27)^{\frac{4}{3}} + (32)^{0\cdot 8} + (0\cdot 8)^{-1} + (0\cdot 8)^0$

$= (3 \times 3 \times 3)^{\frac{4}{3}} + (2 \times 2 \times 2 \times 2 \times 2)^{\frac{4}{5}} + \dfrac{1}{0\cdot 8} + 1$

$= 3^{3\times\frac{4}{3}} + 2^{5\times\frac{4}{5}} + \dfrac{5}{4} + \quad = 3^4 + 2^4 + \dfrac{5}{4} + 1$

$= 81 + 16 + \dfrac{5}{4} + \quad = 99\dfrac{1}{4} = 99\cdot 25$ **Ans.**

(*ii*) $\left[\dfrac{(27)^{-3}}{(9)^{-3}}\right]^{\frac{1}{3}} = \left[\left(\dfrac{27}{9}\right)^{-3}\right]^{\frac{1}{3}} = \left(\dfrac{27}{9}\right)^{-3\times\frac{1}{3}}$

$= \left(\dfrac{27}{9}\right)^{-1} = \dfrac{9}{27} = \dfrac{1}{3}$ **Ans.**

Q. 7. (*i*) $(\sqrt{32} - \sqrt{5})^{\frac{1}{3}} \; (\sqrt{32} + \sqrt{5})^{\frac{1}{3}}$

(*ii*) $(9)^{\frac{5}{2}} - 3\cdot(4)^0 - \left(\dfrac{1}{81}\right)^{\frac{-1}{2}}$

Sol. (*i*) $(\sqrt{32} - \sqrt{5})^{\frac{1}{3}} \; (\sqrt{32} + \sqrt{5})^{\frac{1}{3}}$

$[\because a^m b^m = (ab)^m]$

$= [(\sqrt{32} - \sqrt{5}) \; (\sqrt{32} + \sqrt{5})]^{\frac{1}{3}}$

$= [(\sqrt{32})^2 - (\sqrt{5})^2]^{\frac{1}{3}}$

$\{\because (a - b)(a + b) = a^2 - b^2\}$

$= (32 - 5)^{\frac{1}{3}} = (27)^{\frac{1}{3}}$

$= (3 \times 3 \times 3)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3$ **Ans.**

(*ii*) $9^{\frac{5}{2}} - 3\cdot(4)^0 - \left(\dfrac{1}{81}\right)^{\frac{-1}{2}}$

$= (3 \times 3)^{\frac{5}{2}} - 3 \times 1 - \left(\dfrac{1}{9 \times 9}\right)^{\frac{-1}{2}}$

$[\because a^0 = 1]$

$= 3^{2\times\frac{5}{2}} - 3 - \left(\dfrac{1}{9}\right)^{2\times\left(-\frac{1}{2}\right)}$

$= 3^5 - 3 - \left(\dfrac{1}{9}\right)^{-1}$

$= 3 \times 3 \times 3 \times 3 \times 3 - 3 - 9$

$= 243 - 12 = 231$ **Ans.**

Q. 8. Simplify : $\dfrac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Sol. $\dfrac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} = \dfrac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}}$

$= \dfrac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$

$= \dfrac{3^{n+2n+2}}{3^{n-1+2n-2}} = \dfrac{3^{3n+2}}{3^{3n-3}}$

$= 3^{3n+2\,-\,3n+3} = 3^5$ **Ans.**

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Q. 9. $\dfrac{(27)^{\frac{2n}{3}} \times (8)^{-\frac{n}{6}}}{(18)^{-\frac{n}{2}}}$

Sol. $\dfrac{(27)^{\frac{2n}{3}} \times (8)^{-\frac{n}{6}}}{(18)^{-\frac{n}{2}}} = \dfrac{(3^3)^{\frac{2n}{3}} \times (2^3)^{-\frac{n}{6}}}{(2 \times 3^2)^{-\frac{n}{2}}}$

$= \dfrac{3^{3\times\frac{2n}{3}} \times 2^{3\times\left(-\frac{n}{6}\right)}}{2^{-\frac{n}{2}} \times 3^{2\times\left(-\frac{n}{2}\right)}} = \dfrac{3^{2n} \times 2^{-\frac{n}{2}}}{2^{-\frac{n}{2}} \times 3^{-n}}$

$= 3^{2n+n} \times 2^{-\frac{n}{2}+\frac{n}{2}} = 3^{3n} \times 2^0$

$= 3^{3n} \times 1 = 3^{3n}$ **Ans.**

Q. 10. $\dfrac{5^{2(n+6)} \times (25)^{-7+2n}}{(125)^{2n}}$

Sol. $\dfrac{5^{2(n+6)} \times (25)^{-7+2n}}{(125)^{2n}}$

$= \dfrac{5^{2n+12} \times (5^2)^{-7+2n}}{(5^3)^{2n}}$

$= \dfrac{5^{2n+12} \times 5^{-14+4n}}{5^{6n}} = \dfrac{5^{2n+12-14+4n}}{5^{6n}}$

$= \dfrac{5^{6n-2}}{5^{6n}} = 5^{6n-2-6n} = 5^{-2}$

$= \dfrac{1}{5^2} = \dfrac{1}{5 \times 5} = \dfrac{1}{25}$ **Ans.**

Q. 11. $\dfrac{5^{n+3} - 16 \times 5^{n+1}}{12 \times 5^n - 2 \times 5^{n+1}}$

Sol. $\dfrac{5^{n+3} - 16 \times 5^{n+1}}{12 \times 5^n - 2 \times 5^{n+1}}$

$= \dfrac{5^n(5^3 - 16 \times 5^1)}{5^n(12 - 2 \times 5^1)} = \dfrac{125 - 80}{12 - 10}$

$= \dfrac{45}{2} = 22{\cdot}5$ **Ans.**

Q. 12. $\dfrac{3 \times (27)^{n+1} + 9 \times 3^{(3n-1)}}{8 \times 3^{3n} - 5 \times (27)^n}$

Sol. $\dfrac{3 \times (27)^{n+1} + 9 \times 3^{(3n-1)}}{8 \times 3^{3n} - 5 \times (27)^n}$

$= \dfrac{3 \times (3^3)^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times (3^3)^n}$

$= \dfrac{3 \times 3^{3n+3} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 3^{3n}}$

$= \dfrac{3^{3n}(3 \times 3^3 + 9 \times 3^{-1})}{3^{3n}(8 - 5)}$

$= \dfrac{3 \times 27 + 9 \times \frac{1}{3}}{3} = \dfrac{81 + 3}{3} = \dfrac{84}{3}$

$= 28$ **Ans.**

Q. 13. $\dfrac{5 \times (25)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{(2n+3)} - (25)^{n+1}}$

Sol. $\dfrac{5 \times (25)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (25)^{n+1}}$

$= \dfrac{5 \times (5^2)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (5^2)^{n+1}}$

$= \dfrac{5 \times 5^{2n+2} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - 5^{2n+2}}$

$= \dfrac{5^{2n}(5 \times 5^2 - 25)}{5^{2n}(5 \times 5^3 - 5^2)} = \dfrac{5 \times 25 - 25}{5 \times 125 - 25}$

$= \dfrac{125 - 25}{625 - 25} = \dfrac{100}{600} = \dfrac{1}{6}$ **Ans.**

Q. 14. $\dfrac{7^{(2n+3)} - (49)^{n+2}}{[(343)^{n+1}]^{\frac{2}{3}}}$

Sol. $\dfrac{7^{(2n+3)} - (49)^{n+2}}{[(343)^{n+1}]^{\frac{2}{3}}}$

$= \dfrac{7^{2n+3} - (7^2)^{n+2}}{[(7^3)^{n+1}]^{\frac{2}{3}}} = \dfrac{7^{2n+3} - 7^{2n+4}}{7^{3(n+1)\times\frac{2}{3}}}$

$= \dfrac{7^{2n+3} - 7^{2n+4}}{7^{2n+2}} = \dfrac{7^{2n}(7^3 - 7^4)}{7^{2n} \times 7^2}$

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$= \frac{7^3 - 7^4}{7^2} = \frac{7^3(1-7)}{7^2}$

$= 7^{3-2}(1-7) = 7(-6) = -42$ **Ans.**

Q. 15. $(x^{\frac{1}{3}} - x^{-\frac{1}{3}})(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}})$

Sol. $(x^{\frac{1}{3}} - x^{-\frac{1}{3}})(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}})$

Let $x^{\frac{1}{3}} = a$ and $x^{-\frac{1}{3}} = \frac{1}{a}$, then

Given expression

$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2}\right)$

$= a^3 - \frac{1}{a^3} = (x^{\frac{1}{3}})^3 - (x^{-\frac{1}{3}})^3$

$= x^1 - x^{-1} = x - \frac{1}{x}$ **Ans.**

Q. 16. Simplify :

(i) $a^7 \times a^4 \times a^{-6} \times a^0$

(ii) $a^{\frac{4}{3}} \div a^{\frac{-2}{3}}$

(iii) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

(iv) $(a^{-1} + b^{-1}) \div (ab)^{-1}$

(v) $(a^{-1} \times b^{-1}) \div (a^{-1} + b^{-1})$

(vi) $(a+b)^{-1} \times (a^{-1} + b^{-1})$

(vii) $\frac{a+b+c}{(a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1})}$

Sol. (i) $a^7 \times a^4 \times a^{-6} \times a^0$

$= a^{7+4-6+0} = a^{11-6} = a^5$ **Ans.**

(ii) $a^{\frac{4}{3}} \div a^{\frac{-2}{3}} = a^{\frac{4}{3} - \left(-\frac{2}{3}\right)}$

$= a^{\frac{4}{3} + \frac{2}{3}} = a^{\frac{6}{3}} = a^2$ **Ans.**

(iii) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

$= \left(\frac{1}{a} + \frac{1}{b}\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$

$= \frac{b+a}{ab} \div \frac{b^2 - a^2}{a^2b^2}$

$= \frac{(b+a)}{ab} \times \frac{a^2b^2}{b^2 - a^2} = \frac{(b+a)a^2b^2}{ab(b+a)(b-a)}$

$= \frac{ab}{b-a}$ **Ans.**

(iv) $(a^{-1} + b^{-1}) \div (ab)^{-1}$

$= \left(\frac{1}{a} + \frac{1}{b}\right) \div \frac{1}{ab} = \frac{b+a}{ab} \times \frac{ab}{1} = b + a$

$= a + b$ **Ans.**

(v) $(a^{-1} \times b^{-1}) \div (a^{-1} + b^{-1})$

$= \left(\frac{1}{a} \times \frac{1}{b}\right) \div \left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{ab} \div \left(\frac{b+a}{ab}\right)$

$= \frac{1}{ab} \times \frac{ab}{(b+a)} = \frac{1}{(b+a)} = \frac{1}{(a+b)}$ **Ans.**

(vi) $(a+b)^{-1} \times (a^{-1} + b^{-1})$

$= \left(\frac{1}{a+b}\right) \times \left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{a+b} \times \frac{b+a}{ab}$

$= \frac{1}{a+b} \times \frac{a+b}{ab} = \frac{1}{ab}$ **Ans.**

(vii) $\frac{a+b+c}{a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}}$

$= \frac{a+b+c}{\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}} = \frac{a+b+c}{\frac{c+a+b}{abc}}$

$= \frac{(a+b+c)(abc)}{(a+b+c)} = abc$ **Ans.**

Q. 17. Prove that

(i) $\left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)} =$

(ii) $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1.$

(iii) $\left(\frac{x^a}{x^b}\right)^{(a+b-c)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} = 1.$

Sol. (i) L.H.S.

$= \left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)}$

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$$= x^{(a-b)(a+b)} . x^{(b-c)(b+c)} . x^{(c-a)(c+a)}$$

$$= x^{a^2-b^2} . x^{b^2-c^2} . x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^o = 1$$

= R.H.S.

(*ii*) $\text{L.H.S.} = \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} . \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} . \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$$= x^{(a-b)\frac{1}{ab}} . x^{(b-c)\frac{1}{bc}} . x^{(c-a)\frac{1}{ca}}$$

$$= x^{\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ca}}$$

$$= x^{\frac{c(a-b)+a(b-c)+b(c-a)}{abc}}$$

$$= x^{\frac{ca-bc+ab-ca+bc-ab}{abc}} = x^0 = 1$$

= R.H.S.

(*iii*) L.H.S.

$$= \left(\frac{x^a}{x^b}\right)^{(a+b-c)} . \left(\frac{x^b}{x^c}\right)^{(b+c-a)} . \left(\frac{x^c}{x^a}\right)^{(c+a-b)}$$

$$= x^{(a-b)(a+b-c)} . x^{(b-c)(b+c-a)} . x^{(c-a)(c+a-b)}$$

$$= x^{a^2-b^2-ac+bc+b^2-c^2-ab+ca+c^2-a^2-bc+ab}$$

$$= x^0 = 1 = \text{R.H.S.}$$

Q. 18. Prove that :

$$\frac{a^{-1}}{(a^{-1}+b^{-1})} + \frac{a^{-1}}{(a^{-1}-b^{-1})} = \frac{2b^2}{b^2-a^2}.$$

Sol. $\text{L.H.S.} = \frac{a^{-1}}{(a^{-1}+b^{-1})} + \frac{a^{-1}}{(a^{-1}-b^{-1})}$

$$= \frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}} + \frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}$$

$$= \frac{\frac{1}{a}}{\frac{b+a}{ab}} + \frac{\frac{1}{a}}{\frac{b-a}{ab}}$$

$$= \frac{1}{a} \times \frac{ab}{b+a} + \frac{1}{a} \times \frac{ab}{b-a}$$

$$= \frac{b}{b+a} + \frac{b}{b-a}$$

$$= \frac{b^2-ab+b^2+ab}{(b+a)(b-a)} = \frac{2b^2}{b^2-a^2}$$

= R.H.S.

Q. 19. Prove that :

$$\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1.$$

Sol. L.H.S.

$$= \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$$

$$= \frac{1}{x^{a-a}+x^{b-a}+x^{c-a}} + \frac{1}{x^{b-b}+x^{a-b}+x^{c-b}} + \frac{1}{x^{c-c}+x^{b-c}+x^{a-c}}$$

$$= \frac{1}{x^{-a}(x^a+x^b+x^c)} + \frac{1}{x^{-b}(x^b+x^a+x^c)} + \frac{1}{x^{-c}(x^c+x^b+x^a)}$$

$$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = \; = \text{R.H.S.}$$

Q. 20. If $abc = 1$, prove that :

$$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1.$$

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Sol. $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}=1$

$\therefore abc=1,\ a=\frac{1}{bc}$

$a^{-1}=\left(\frac{1}{bc}\right)^{-1}=bc$

$\therefore \text{L.H.S.}=\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}$

Putting values of a and a^{-1}

$\text{L.H.S.}=\frac{1}{1+\frac{1}{bc}+\frac{1}{b}}+\frac{1}{1+b+\frac{1}{c}}+\frac{1}{1+c+bc}$

$=\frac{1}{\left(\frac{bc+1+c}{bc}\right)}+\frac{1}{\left(\frac{c+bc+1}{c}\right)}+\frac{1}{1+c+bc}$

$=\frac{bc}{bc+1+c}+\frac{c}{c+bc+1}+\frac{1}{1+c+bc}$

$=\frac{(bc+c+1)}{(bc+c+1)}=1=\text{R.H.S.}$

Hence the result is proved.

Q. 21. If a, b, c are positive real numbers, show that : $\sqrt{a^{-1}b}\,.\sqrt{b^{-1}c}\,.\sqrt{c^{-1}a}=1$.

Sol. $\text{L.H.S.}=\sqrt{a^{-1}b}\,.\sqrt{b^{-1}c}\,.\sqrt{c^{-1}a}$

$=\sqrt{\frac{b}{a}}\cdot\sqrt{\frac{c}{b}}\cdot\sqrt{\frac{a}{c}}$

$=\sqrt{\frac{b}{a}\times\frac{c}{b}\times\frac{a}{c}}=\sqrt{\frac{abc}{abc}}$

$=\sqrt{1}=1=\text{R.H.S.}$

($\because a, b, c$ are positive real numbers)

Q. 22. If $\frac{9^n\times 3^2\times 3^n-(27)^n}{3^{3m}\times 2^3}=3^{-3}$, prove that $(m-n)=1$.

Sol. $\frac{9^n\times 3^2\times 3^n-(27)^n}{3^{3m}\times 2^3}=3^{-3}$

$\Rightarrow \frac{[(3)^2]^n\times 3^2\times 3^n-(3^3)^n}{3^{3m}\times 2^3}=3^{-3}$

$\Rightarrow \frac{3^{2n}\times 3^2\times 3^n-3^{3n}}{3^{3m}\times 8}=3^{-3}$

$\Rightarrow \frac{3^{2n+2+n}-3^{3n}}{8.3^{3m}}=3^{-3}$

$\Rightarrow \frac{3^{3n+2}-3^{3n}}{8.3^{3m}}=3^{-3}$

$\Rightarrow \frac{3^{3n}(3^2-1)}{8.3^{3m}}=3^{-3}$

$\Rightarrow \frac{3^{3n}\times(9-1)}{8\times 3^{3m}}=3^{-3}$

$\Rightarrow \frac{8.3^{3n}}{8.3^{3m}}=3^{-3}$

$\Rightarrow 3^{3n-3m}=3^{-3}$

$\Rightarrow 3^{3(n-m)}=3^{-3}$

Comparing we get,

$\therefore\ 3(n-m)=-3 \Rightarrow n-m=-1$

$\Rightarrow m-n=1$ Hence proved.

Q. 23. If $21168=x^4\times y^3\times z^2$, find the values of x, y, z.

Sol. $21168=x^4\times y^3\times z^2$

$\Rightarrow 2^4\times 3^3\times 7^2=x^4\times y^3\times z^2$

2	21168
2	10584
2	5292
2	2646
3	1323
3	441
3	147
7	49
7	7
	1

Comparing, we get

$x^4=2^4 \quad\Rightarrow\quad x=2$

$y^3=3^3 \quad\Rightarrow\quad y=3$

and $z^2=7^2 \quad\Rightarrow\quad z=7$

Q. 24. If $1960=2^a\times 5^b\times 7^c$, find the values of a, b, c.

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Hence calculate the value of $(2^{-a} \times 5^{-c} \times 7^{b})$.

Sol. $1960 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$

2	1960
2	980
2	490
5	245
7	49
7	7
	1

Comparing, we get

$a = 3, b = 1, c = 2.$

Now $2^{-a} \times 5^{-c} \times 7^b$

$= 2^{-3} \times 5^{-2} \times 7^1 = \frac{1}{2^3 \times 5^2} \times 7^1$

$= \frac{7}{8 \times 25} = \frac{7}{200}$ **Ans.**

Solve for x :

Q. 25. $3^x = \frac{1}{3}$

Sol. $3^x = \frac{1}{3} = 3^{-1}$ $\left(\because \frac{1}{3} = 3^{-1}\right)$

Comparing, we get,

$\therefore x = -1$ **Ans.**

Q. 26. $3^{2x+1} = 1$

Sol. $3^{2x+1} = 1 = 3^0$ $(\because 3^0 = 1)$

Comparing, we get,

$\therefore 2x + 1 = 0 \Rightarrow 2x = -1$

$x = \frac{-1}{2}$ **Ans.**

Q. 27. $\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-3x}$

Sol. $\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-3x}$

$\left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-(1-3x)} = \left(\frac{a}{b}\right)^{3x-1}$

Comparing, we get

$3x - 1 = \frac{1}{2} \Rightarrow 3x = \frac{1}{2} + 1 = \frac{3}{2}$

$\therefore x = \frac{3}{2} \times 3 = \frac{1}{2}$ **Ans.**

Q. 28. $\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$

Sol. $\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8} = \frac{(3)^3}{(2)^3} = \left(\frac{3}{2}\right)^3$

$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3}$

Comparing, we get

$\frac{x-1}{3} = -3 \Rightarrow x - 1 = -9$

$\Rightarrow x = -9 + 1 = -8$

$\therefore x = -8$ **Ans.**

Q. 29. $\left(\sqrt{\frac{3}{5}}\right)^{x-1} = \left(\frac{27}{125}\right)^{-1}$

Sol. $\left(\sqrt{\frac{3}{5}}\right)^{x-1} = \left(\frac{27}{125}\right)^{-1}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x-1}{2}} = \left(\frac{3^3}{5^3}\right)^{-1} = \left[\left(\frac{3}{5}\right)^3\right]^{-1}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x-1}{2}} = \left(\frac{3}{5}\right)^{-3}$

Comparing, we get

$\frac{x-1}{2} = -3 \Rightarrow x - 1 = -6$

$\Rightarrow x = -6 + 1 = -5$

$\therefore x = -5$ **Ans.**

Q. 30. $9 \times 3^x = (27)^{2x-5}$

Sol. $9 \times 3^x = (27)^{2x-5}$

$\Rightarrow 3^2 \times 3^x = (3^3)^{2x-5}$

$\Rightarrow 3^{x+2} = 3^{6x-15}$

Comparing, we get

$x + 2 = 6x - 15$

$\Rightarrow -6x + x = -15 - 2$

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$\Rightarrow \quad -5x = -17$

$\Rightarrow x = \frac{-17}{-5} = \frac{17}{5} = 3\frac{2}{5}$ **Ans.**

31. $2^{(5x-1)} = 4 \times 2^{(3x+1)}$

Sol. $2^{(5x-1)} = 4 \times 2^{(3x+1)}$

$\Rightarrow 2^{5x-1} = 2^2 \times 2^{(3x+1)} = 2^{3x+1+2}$

$\Rightarrow 2^{5x-1} = 2^{3x+3}$

Comparing, we get

$5x - 1 = 3x + 3$

$\Rightarrow \quad 5x - 3x = 3 + 1$

$\Rightarrow \quad 2x = 4$

$\therefore \quad x = \frac{4}{2} = 2$ **Ans.**

Q. 32. $5^{x-3} \times 3^{2x-8} = 225$

Sol. $5^{x-3} \times 3^{2x-8} = 225$

$\Rightarrow \quad 5^x \times 5^{-3} \times 3^{2x} \times 3^{-8} = 225$

$\Rightarrow 5^x \times \frac{1}{5^3} \times (3^2)^x \times \frac{1}{3^8} = 225$

$\Rightarrow 5^x \times (9)^x \times \frac{1}{5^3} \times \frac{1}{3^8} = 225$

$\Rightarrow \quad (45)^x = 225 \times 5^3 \times 3^8$

$= 3 \times 3 \times 5 \times 5 \times 5^3 \times 3^8$

$= 5^{1+1+3} \times 3^{8+1+1} = 5^5 \times 3^{10}$

$= (5^5) \times (3^2)^5 = (5)^5 \times (9)^5$

$\Rightarrow \quad 45^x = (5 \times 9)^5 = (45)^5$

Comparing, we get

$x = 5$ **Ans.**

Q. 33. If $2^x = 3^y = 12^z$, show that :

$$\frac{1}{z} = \frac{1}{y} + \frac{2}{x}.$$

Sol. Let $2^x = 3^y = 12^z = k$, then

$2^x = k \quad \Rightarrow \quad 2 = k^{1/x}$

$3^y = k \quad \Rightarrow \quad 3 = k^{1/y}$

$12^z = k \quad \Rightarrow 12 = k^{1/z}$

Now $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow \quad k^{\frac{1}{z}} = (k^{\frac{1}{x}})^2 \times k^{\frac{1}{y}}$

$\Rightarrow \quad k^{\frac{1}{z}} = k^{\frac{2}{x}} \times k^{\frac{1}{y}}$

$\Rightarrow \quad k^{\frac{1}{z}} = k^{\frac{2}{x}+\frac{1}{y}}$

Comparing, we get

$$\frac{1}{z} = \frac{2}{x} + \frac{1}{y}$$

Hence proved.

Q. 34. If $2^x = 3^y = 6^{-z}$ show that :

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0.$$

Sol. Let $2^x = 3^y = 6^{-z} = k$, then

$2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{-\frac{1}{z}}$

Now $2 \times 3 = 6$

$\Rightarrow \quad k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{\frac{-1}{z}}$

$\Rightarrow \quad k^{\frac{1}{x}+\frac{1}{y}} = k^{-\frac{1}{z}}$

Comparing, we get

$\frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$

$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Hence proved.

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