6

Framing of Formula

POINTS TO REMEMBER

1. **Formula :** It is a relation between two or more variables.
2. **Framing of Formula :**
   We can frame a formula or relation between the given variables based on certain given conditions.
3. **Subject of a formula :**
   The subject of a formula is the variable which is expressed in terms of the other variables.

EXERCISE 6 (A)

Frame a formula for each of the following statements :

Q. 1. The area (A) of a rectangle is equal to the product of its length (l) and breadth (b)

Ans. $A = l.b.$

Q. 2. The circumference (C) of a circle is equal to π times its diameter (d).

Ans. $C = \pi d.$

Q. 3. The area (A) of a circular ring is π times the difference between the squares of outer radius (R) and inner radius (r).

Ans. $A = \pi (R^2 - r^2)$

Q. 4. The volume (V) of a cylinder is the product of π, the square of the base radius (r) and the height (h).

Ans. $V = \pi r^2 h.$

Q. 5. The total surface area (S) of a cone is equal to the product of π, the base radius (r) and the sum of the base radius (r) and slant height (l)

Ans. $S = \pi r (r + l)$

Q. 6. Nine-fifths of the temperature (C) in centigrade of a body increased by 32 is equal to its temperature in Fahrenheit (F).

Ans. $F = \frac{9}{5}C + 32$

Q. 7. The reciprocal of focal length (f) is equal to the sum of the reciprocals of the object disztance (u) and the image distance (v).

Ans. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Q. 8. The number of diagonals (d) that can be drawn from one vertex of an n-sided polygon to all over vertices is equal to 3 less than the number of sides of the polygon.

Ans. $d = (n - 3)$

Q. 9. The distance (s metres) when a freely falling body covers in time (t seconds) is 4·8 times the square of time (t).

Ans. $s = 4{\cdot}8\ t^2$

Q. 10. The arithmetic mean (M) of three quantities a, b, c is equal to their sum divided by the number of quantities.

Ans. $M = \frac{a + b + c}{3}$

($\because$ Here number of quantities is 3)

Q. 11. Frame the formula for finding the number of minutes (M) in x hours, y minutes and z seconds.

Ans. $M = 60x + y + \frac{z}{60}$

($\because$ 1 hour = 60 minutes and 1 minute = 60 seconds)

Q. 12. A shopkeeper marks each article at Rs. m and gives 10% discount on the marked price. If the cost price of each article be Rs. c, find the formula for profit (P).

Sol. Marked price = Rs. m

Discount = 10%

Cost price = Rs. c

Gain = P

Gain = S.P. – C.P.

$$= \frac{MP \times (100 - \text{Discount }\%)}{100} - CP$$

$$= \frac{m \times 90}{100} - c$$

$$\therefore \quad P = \frac{9}{10}m - c \textbf{ Ans.}$$

Q. 13. A man buys n articles at a total cost of Rs. C and sells them at x paise each. Obtain the formula for gain percent.

Sol. Cost price of n articles = Rs. C

Selling price of 1 article = x paise

$\therefore$ S.P. of n article = Rs. $\frac{nx}{100}$

$\therefore$ Gain = S.P. – C.P.

$$= \frac{nx}{100} - C = \frac{nx - 100C}{100}$$

$$\text{Gain \%} = \frac{nx - 100C}{100} \times \frac{100}{C}$$

$$= \left(\frac{nx - 100C}{C}\right)\% \textbf{ Ans.}$$

Q. 14. A cricketer has an average score of 68 runs per inning in x innings and average of 53 runs per inning in y innings. Find the formula for the average score (A) per inning for all the innings.

Sol. Average score in x innings = 68 runs

$\therefore$ Total runs in x innings = $68x$ runs

Similarly total runs in y innings at the rate of 53 runs per inning = $53y$

Now total number of innings = $x + y$

and total no. of runs scored = $68x + 53y$

$$\therefore \text{ Average (A)} = \frac{68x + 53y}{x + y} \textbf{ Ans.}$$

Q. 15. The hiring charges (h) for a taxi are Rs. 200 plus Rs. 12 per km for distances covered beyond 30 km. If the distance travelled be x km ($x > 30$) write the formula for the hiring charges.

Sol. Hiring charges = Rs. 200 + Rs. 12 per km. after 30 km.

Total distance travelled = x km ($x > 30$)

$\therefore$ Hiring charges = Rs. 200 + $(x - 30) \times 12$

$\therefore$ h = Rs. 200 + 12 $(x - 30)$ **Ans.**

Q. 16. A telephone subscriber has to pay Rs. 450 as quarterly rent in addition to y paise for each call exceeding 125 calls. If he makes n calls in a quarter ($n > 125$), write down the formula for the total bill (M) in rupees.

Sol. Quarterly rent = Rs. 450

Rate of per call exceeding 125 calls = y paise

No. of calls in a quarter = n

$\therefore$ Total bill (M) = Rs. 450 + $\frac{y}{100}(x - 125)$

$$= \text{Rs. } 450 + \frac{y(n - 125)}{100} \textbf{ Ans.}$$

Q. 17. The average weight of a set of p articles is a gm and the total weight of another set of q articles is b kg. Frame the formula for the average weight (A) of $(p + q)$ articles in kg.

Sol. Total weight of p articles with an average of a gm. per article = $\frac{ap}{1000}$ kg.

and total weight of q articles with our average of b kg per article = $b\ q$ kg.

$\therefore$ Total weight of $(p + q)$ articles

$$= \left(\frac{ap}{1000} + bq\right) \text{ kg}$$

$$= \frac{pa + 1000\,qb}{1000} \text{ kg.}$$

$\therefore$ Average (A)

$$= \frac{pa + 1000\,qb}{1000(p+q)} \text{ kg Ans.}$$

Q. 18. x men can do a piece of work in d days. Write the formula, for finding the number of men required (n) to complete the same work in 7 days.

Sol. In d days a work is completed by = x men

and in 1 day the work will be completed by = $x\ d$ men

$\therefore$ In 7 days, the work will be completed

by $= \frac{xd}{7}$ men

$\therefore \quad n = \frac{xd}{7}$ **Ans.**

Q. 19. 4 years ago, a boy was x years old. Find the formula for finding his age (A) after y years.

Sol. 4 years ago, the age of a boy = x years

$\therefore$ Present age = $(x + 4)$ years

and y years after, his age will be

$= (x + 4) + y$ years

$\therefore \quad A = (x + 4) + y$ years. **Ans.**

Q. 20. A man can run at m metres per second. Find the formula for finding the number of minutes (n) taken by him in covering x km.

Sol. Speed of a man = m metre per second

$\therefore$ Time taken in covering x km

$$= \frac{x \times 1000}{m \times 60} \text{ minutes}$$

$$n = \frac{50x}{3m} \quad \textbf{Ans.}$$

Q. 21. A labourer is engaged for 30 days on condition that he receives Rs. 100 for each day he works and loses Rs. 20 for each day, he is absent. Write the formula for the amount (A) received by him, if he remains absent for x days.

Sol. No. of days = 30 days

No. of days for which he remained absent = x day

$\therefore$ No. of days for which he worked

$= (30 - x)$ days

Amount receives for working

= Rs. 100 per day

and amount paid by him for remaining absent = Rs. 20 per day

Total amount receives = A

$\therefore \quad A = (30 - x) \times 100 - x \times 20$

$\Rightarrow A = 3000 - 100x - 20x$

$\Rightarrow A = 3000 - 120x$ **Ans.**

Q. 22. Each pencil costs a paise and pens cost Rs. b per dozen. Find the formula for total cost (C) in rupees of x pencils and y pens.

Sol. Cost of one pencil = a paise = Rs. $\frac{a}{100}$

Cost of one dozen pens or 12 pens

= Rs. b

$\therefore$ Cost of one pen = Rs. $\frac{b}{12}$

Now cost of x pencils and y pens = C

$$\therefore \quad C = \frac{a}{100} \times x + \frac{b}{12} \times y$$

$$\Rightarrow \quad C = \frac{ax}{100} + \frac{by}{12} \textbf{ Ans.}$$

Q. 23. A car can cover k kilometers on l litres of petrol. Write the formula for finding the distance (d) in km covered by the car on p litres of petrol.

Sol. On l litres of petrol, a car can cover

= k km

and on p litre of petrol, the car will

cover $= \frac{k}{l} \times p = \frac{pk}{l}$

$\Rightarrow \quad d = \frac{pk}{l}$ **Ans.**

EXERCISE 6 (B)

Q. 1. Given : $\frac{x}{1-x} = \frac{a}{b}$.

(*i*) Express x in terms of a and b.

(*ii*) Find the value of x, when $a = 0{\cdot}6$ and $b = 0{\cdot}4$.

Sol. (*i*) $\frac{x}{1-x} = \frac{a}{b}$

$\Rightarrow \quad xb = a - ax$

(By cross multiplication)

$\Rightarrow \quad ax + bx = a$

$\Rightarrow x(a+b) = a \Rightarrow x = \frac{a}{a+b}$

(*ii*) If $a = 0{\cdot}6$ and $b = 0{\cdot}4$, then

$x = \frac{0{\cdot}6}{0{\cdot}6 + 0{\cdot}4} = \frac{0{\cdot}6}{1{\cdot}0} = 0{\cdot}6$ **Ans.**

Q. 2. Given : $y = \frac{3x-4}{2x+5}$.

(*i*) Express x in terms of y.

(*ii*) Find the value of x when $y = 2$.

Sol. (*i*) $y = \frac{3x-4}{2x+5}$

$\Rightarrow y(2x+5) = 3x - 4$

(By cross multiplication)

$\Rightarrow \quad 2xy + 5y = 3x - 4$

$\Rightarrow \quad 2xy - 3x = -4 - 5y$

$\Rightarrow \quad 3x - 2xy = 5y + 4$

$x(3 - 2y) = (5y + 4)$

$\therefore \quad x = \frac{5y+4}{3-2y}$

(*ii*) When $y = 2$, then $x = \frac{5 \times 2 + 4}{3 - 2 \times 2}$

$\Rightarrow \quad x = \frac{10+4}{3-4} = \frac{14}{-1} = -14$ **Ans.**

Q. 3. If $\frac{5x-4y}{7x-6y} = \frac{5}{3}$, find $\frac{x}{y}$.

Sol. $\frac{5x-4y}{7x-6y} = \frac{5}{3}$

$\Rightarrow 3(5x - 4y) = 5(7x - 6y)$

(By cross multiplication)

$\Rightarrow \quad 15x - 12y = 35x - 30y$

$\Rightarrow -12y + 30y = 35x - 15x$

$\Rightarrow \quad 18y = 20x$

$\Rightarrow \quad 20x = 18y$

Dividing by y,

$\frac{20x}{y} = \frac{18y}{y} \Rightarrow 20\frac{x}{y} = 18$

$\therefore \quad \frac{x}{y} = \frac{18}{20} = \frac{9}{10}$

Hence $\frac{x}{y} = \frac{9}{10}$ **Ans.**

Q. 4. Given : $A = P\left(1 + \frac{rt}{100}\right)$.

(*i*) Express t in terms of A, P and r.

(*ii*) Find the value of t when $A = 605$, $P = 500$ and $r = 12$.

Sol. $A = P\left(1 + \frac{rt}{100}\right) \Rightarrow \frac{A}{P} = 1 + \frac{rt}{100}$

$\Rightarrow \frac{rt}{100} = \frac{A}{P} -$

(*i*) $\Rightarrow t = \frac{100}{r}\left(\frac{A}{P} - 1\right)$

(*ii*) When $A = 605$, $P = 500$ and $r = 12$, then

$t = \frac{100}{12}\left(\frac{605}{500} - 1\right) = \frac{100}{12} \times \left(\frac{605-500}{500}\right)$

$\Rightarrow t = \frac{100}{12} \times \frac{105}{500} = \frac{7}{4} = 1\frac{3}{4}$ **Ans.**

Q. 5. Given : $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

(*i*) In the above formula, make u the subject.

(*ii*) Find the value of u when $f = 6$ and $v = 10{\cdot}5$.

Sol. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$

$\Rightarrow \frac{1}{u} = \frac{v-f}{fv} \Rightarrow u = \frac{fv}{v-f}$

$\therefore \quad u = \frac{fv}{v-f}$

(*ii*) When $f = 6$, $v = 10{\cdot}5$, then

$u = \frac{6 \times 10{\cdot}5}{10{\cdot}5 - 6} = \frac{63}{4{\cdot}5} = \frac{63 \times 10}{45} = 14$ **Ans.**

Q. 6. Given : $\mathrm{I} = \frac{n\mathrm{E}}{(\mathrm{R} + nr)}$.

(*i*) Express n in terms of I, R, r and E

(*ii*) Find the value of n when $\mathrm{I} = 4$, $r = 5$, $\mathrm{E} = 25$ and $\mathrm{R} = 12{\cdot}5$

. Sol. (*i*) $\mathrm{I} = \frac{n\mathrm{E}}{\mathrm{R} + nr} \Rightarrow \mathrm{IR} + n\mathrm{I}r = n\mathrm{E}$

$n\mathrm{E} - n\mathrm{I}r = \mathrm{IR}$

$n(\mathrm{E} - \mathrm{IR}) = \mathrm{IR} \Rightarrow n = \frac{\mathrm{IR}}{\mathrm{E} - \mathrm{I}r}$

(*ii*) When $\mathrm{I} = 4$, $r = 5$, $t = 25$

and $\mathrm{R} = 12{\cdot}5$, then

$n = \frac{\mathrm{IR}}{\mathrm{E} - \mathrm{I}r} = \frac{4 \times 12{\cdot}5}{25 - 4 \times 5} = \frac{50}{25 - 20}$

$= \frac{50}{5} = 10$ **Ans.**

Q. 7. (*i*) Make 'b' the subject in the formula,

$$x = \sqrt{\frac{a-b}{a+b}}.$$

(*ii*) Find the value of b when $a = 10$ and $x = \frac{1}{2}$.

Sol. (*i*) $x = \sqrt{\frac{a-b}{a+b}}$

Squaring both sides

$x^2 = \frac{a-b}{a+b} \Rightarrow x^2a + x^2b = a - b$

$x^2b + b = a - ax^2$

$\Rightarrow b(x^2 + 1) = a(1 - x^2)$

$\Rightarrow b = \frac{a(1-x^2)}{1+x^2}$

(*ii*) When $a = 10$ and $x = \frac{1}{2}$, then

$b = \frac{10\left\{1 - \left(\frac{1}{2}\right)^2\right\}}{1 + \left(\frac{1}{2}\right)^2} = \frac{10\left(1 - \frac{1}{4}\right)}{1 + \frac{1}{4}}$

$= \frac{10 \times \frac{3}{4}}{\frac{5}{4}} = 10 \times \frac{3}{4} \times \frac{4}{5} = 6$ **Ans.**

Q. 8. Given : $x = \frac{4 - 3p}{p + 2q}$.

(*i*) Make 'p' the subject of the formula.

(*ii*) Find the value of p, when $q = \frac{1}{2}$ and $x = \frac{1}{3}$.

Sol. (*i*) $x = \frac{4 - 3p}{p + 2q}$

$\Rightarrow px + 2qx = 4 - 3p.$

$\Rightarrow px + 3p = 4 - 2qx$

$\Rightarrow p(x + 3) = 4 - 2qx$

$\therefore p = \frac{4 - 2qx}{x + 3}$

(*ii*) When $q = \frac{1}{2}$ and $x = \frac{1}{3}$, then

$p = \frac{4 - 2 \times \frac{1}{2} \times \frac{1}{3}}{\frac{1}{3} + 3} = \frac{4 - \frac{1}{3}}{3 + \frac{1}{3}} = \frac{\frac{11}{3}}{\frac{10}{3}}$

$= \frac{11}{3} \times \frac{3}{10} = \frac{11}{10} = 1\frac{1}{10}$ **Ans.**

Q. 9. Given : $\mathrm{R} = \sqrt{\frac{3\mathrm{V}}{\pi h}}$

(*i*) Make 'h' the subject of the formula.

(*ii*) Find the value of h when

$$\pi = 3\frac{1}{7},\ R = 2\frac{1}{2} \text{ and } V = 13\frac{3}{4}$$

Sol. (*i*) $R = \sqrt{\frac{3V}{\pi h}}$

Squaring both sides

$$R^2 = \frac{3V}{\pi h} \Rightarrow \pi h R^2 = 3V$$

$$h = \frac{3V}{\pi R^2}$$

(*ii*) When $\pi = 3\frac{1}{7},\ R = 2\frac{1}{2}$

and $V = 13\frac{3}{4}$, then

$$h = \frac{3V}{\pi R^2} = \frac{\frac{3 \times 55}{4}}{\frac{22}{7} \times \left(\frac{5}{2}\right)^2}$$

$$= \frac{\frac{165}{4}}{\frac{22}{7} \times \frac{25}{4}} = \frac{\frac{165}{4}}{\frac{550}{28}}$$

$$= \frac{165}{4} \times \frac{28}{550} = \frac{21}{10} = 2\frac{1}{10}$$ **Ans.**

Q. 10. Given : $v = \sqrt{u^2 + 2as}$.

(*i*) Express s in terms of v, u and a.

(*ii*) Find the value of s, when $u = 22{\cdot}5$, $v = 27{\cdot}5$ and $a = 12{\cdot}5$.

Sol. (*i*) $v = \sqrt{u^2 + 2as}$.

Squaring both sides,

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{v^2 - u^2}{2a}$$

(*ii*) $u = 22{\cdot}5,\ v = 27{\cdot}5$ and $a = 12{\cdot}5$

$$s = \frac{(27{\cdot}5)^2 - (22{\cdot}5)^2}{2 \times 12{\cdot}5}$$

$$= \frac{(27{\cdot}5 + 22{\cdot}5)\,(27{\cdot}5 - 22{\cdot}5)}{25}$$

$$= \frac{50 \times 5}{25} = 10$$ **Ans.**

Q. 11. Given : $x = \frac{m+n}{m-n}$

(*i*) Express n in terms of m and x.

(*ii*) Find the value of n when

$m = 15$, and $x = 4$.

Sol. (*i*) $x = \frac{m+n}{m-n}$

$\Rightarrow x(m-n) = m+n$

$\Rightarrow xm - xn = m + n$

$\Rightarrow xm - m = xn + n$

$\Rightarrow m(x-1) = n(x+1)$

$\Rightarrow n = \frac{m(n-1)}{x+1}$

(*ii*) When $m = 15$ and $x = 4$, then

$$n = \frac{15(4-1)}{4+1} = \frac{15 \times 3}{5} = 9$$ **Ans.**

Q. 12. Given : $r = \sqrt{x^2 + y^2}$.

(*i*) Express x in terms of y and r.

(*ii*) Find the value of x, when

$r = 17$ and $y = 8$,

Sol. (*i*) $r = \sqrt{x^2 + y^2}$

Squarring both sides,

$$r^2 = x^2 + y^2$$

$$\Rightarrow x^2 = r^2 - y^2$$

$$\Rightarrow x = \sqrt{r^2 - y^2}$$

(*ii*) When $r = 17$ and $y = 8$, then

$$x = \sqrt{(17)^2 - (8)^2}$$

$$= \sqrt{289 - 64} = \sqrt{225} = 15$$

$\therefore$ $x = 15$ **Ans.**

Q. 13. The resultant resistance R of the parallel combination of two resistances R_1 and R_2 is given by the formula :

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

(*i*) Make 'R_2' the subject of the formula.

(*ii*) Find R_2, when $R = 1{\cdot}2$ and $R_1 = 3$.

Sol. (*i*) $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$\Rightarrow \dfrac{1}{R_2} = \dfrac{1}{R} - \dfrac{1}{R_1} = \dfrac{R_1 - R}{RR_1}$

$\therefore \quad R_2 = \dfrac{RR_1}{R_1 - R}$

(*ii*) When $R = 1{\cdot}2$ and $R_1 = 3$, then

$$R_2 = \frac{1{\cdot}2 \times 3}{3 - 1{\cdot}2} = \frac{3{\cdot}6}{1{\cdot}8} = 2 \textbf{ Ans.}$$

Q. 14. The total energy possessed by a body of mass m moving vertically upwards with velocity v at a height h is given by :

$$E = \frac{1}{2}mv^2 + mgh$$

(*i*) Make 'm' the subject of the formula

(*ii*) Find m, when $E = 225$, $v = 5$, $g = 10$ and $h = 5$.

Sol. (*i*) $E = \dfrac{1}{2}mv^2 + mgh$

$\Rightarrow E = m\left(\dfrac{1}{2}v^2 + gh\right) = \dfrac{m}{2}(v^2 + 2gh)$

$\therefore \quad m = \dfrac{2E}{(v^2 + 2gh)}$

(*ii*) When $E = 225$, $v = 5$, $g = 10$ and $h = 5$, then

$$m = \frac{2E}{(v^2 + 2gh)} = \frac{2 \times 225}{(5)^2 + 2 \times 10 \times 5}$$

$$= \frac{450}{25 + 100} = \frac{450}{125} = \frac{18}{5} = 3{\cdot}6 \textbf{ Ans.}$$

Q. 15. Given : $T + mg = \dfrac{mv^2}{r}$.

(*i*) Make 'm' the subject of the formula.

(*ii*) Find m, when $T = 3{\cdot}5$, $v = 8$, $r = 5$ and $g = 10$.

Sol. (*i*) $T + mg = \dfrac{mv^2}{r}$

$\Rightarrow \quad Tr + mrg = mv^2$

$\Rightarrow \quad mv^2 - mrg = Tr$

$\Rightarrow \quad m(v^2 - rg) = Tr$

$\therefore \quad m = \dfrac{Tr}{(v^2 - rg)}$

(*ii*) When $T = 3{\cdot}5$, $v = 8$, $r = 5$ and $g = 10$, then

$$m = \frac{Tr}{(v^2 - rg)} = \frac{3{\cdot}5 \times 5}{(8)^2 - 5 \times 10}$$

$$= \frac{17{\cdot}5}{64 - 50} = \frac{17{\cdot}5}{14}$$

$$= \frac{2{\cdot}5}{2} = 1{\cdot}25 \textbf{ Ans.}$$

Q. 16. The total surface area of a cone is given by the formula : $S = \pi rl + \pi r^2$

(*i*) Make 'l', the subject of the formula.

(*ii*) Find l, when $S = 282{\cdot}6$, $r = 5$ and $\pi = 3{\cdot}14$.

Sol. (*i*) $S = \pi rl + \pi r^2$

$\Rightarrow \quad \pi rl = S - \pi r^2$

$\therefore \quad l = \dfrac{(S - \pi r^2)}{\pi r}$

(*ii*) When $S = 282{\cdot}6$, $r = 5$ and $\pi = 3{\cdot}14$, then

$$l = \frac{282{\cdot}6 - 3{\cdot}14(5)^2}{3{\cdot}14 \times 5}$$

$$= \frac{282{\cdot}6 - 3{\cdot}14 \times 25}{15{\cdot}70} = \frac{282{\cdot}6 - 78{\cdot}5}{15{\cdot}7}$$

$$= \frac{204{\cdot}1}{15{\cdot}7} = 13 \textbf{ Ans.}$$

Q. 17. The total surface area of a cylinder is given by the formula :

$$S = 2\pi rh + 2\pi r^2$$

(*i*) Make 'h' the subject of the formula.

(*ii*) Calculate h, when $S = 462$, $r = 3{\cdot}5$ and $\pi = 3\dfrac{1}{7}$.

Sol. (*i*) $S = 2\pi rh + 2\pi r^2$

$\Rightarrow 2\pi rh = S - 2\pi r^2$

$\therefore \quad h = \dfrac{S - 2\pi r^2}{2\pi r}$

(*ii*) When $S = 462$, $r = 3{\cdot}5$ and

$\pi = 3\dfrac{1}{7} = \dfrac{22}{7}$, then

$h = \dfrac{S - 2\pi r^2}{2\pi r}$

$= \dfrac{462 - 2 \times \frac{22}{7} \times (3{\cdot}5)^2}{2 \times \frac{22}{7} \times 3{\cdot}5}$

$= \dfrac{462 - 77}{22} = \dfrac{385}{22} = 17{\cdot}5$ **Ans.**

Q. 18. The volume, V cm^3 of a hollow cylindrical pipe of length l cm, outer radius R cm and inner radius r cm is given by the formula :

$V = \pi (R^2 - r^2)\, l$

(*i*) Make 'r' the subject of the formula.

(*ii*) Find r, when $V = 660$, $R = 8$, $\pi = 3\dfrac{1}{7}$ and $l = 14$.

Sol. (*i*) $V = \pi (R^2 - r^2)\, l$

$\Rightarrow \quad \dfrac{V}{\pi l} = R^2 - r^2$

$\Rightarrow \quad r^2 = R^2 - \dfrac{V}{\pi l} = \dfrac{\pi l R^2 - V}{\pi l}$

$\therefore \quad r = \sqrt{\dfrac{\pi l R^2 - V}{\pi l}}$

(*ii*) When $V = 660$, $R = 8$, $\pi = 3\dfrac{1}{7} = \dfrac{22}{7}$ and $l = 14$, then

$r = \sqrt{\dfrac{\pi l R^2 - v}{\pi l}}$

$= \sqrt{\dfrac{\frac{22}{7} \times 14 \times (8)^2 - 660}{\frac{22}{7} \times 14}}$

$= \sqrt{\dfrac{44 \times 64 - 660}{44}} = \sqrt{\dfrac{2816 - 660}{44}}$

$= \sqrt{\dfrac{2156}{44}}$

$= \sqrt{49} = 7$ cm. **Ans.**

Q. 19. The fahrenhiet temperature (F) is equal to 32 more than nine-fifth of the centrigrade temperature 'C'.

(*i*) Write the above statement in the form of a formula.

(*ii*) Make 'C' the subject in the above formula.

(*iii*) Given $F = 104°$, find C.

(*iv*) If $x°$ is the temperature at which centigrade and fahrenheit read the same, find x.

Sol. (*i*) $F = \dfrac{9}{5}C + 32$

(*ii*) $\dfrac{9}{5}C = F - 32$

$\therefore \quad C = \dfrac{5}{9}(F - 32)$

(*iii*) When $F = 104°$, then

$C = \dfrac{5}{9}(104° - 32) = \dfrac{5}{9} \times 72° = 40°$

(*iv*) Let C and F are equal at x then $C = F = x$

Now $C = \dfrac{5}{9}(F - 32)$

$\Rightarrow \quad x = \dfrac{5}{9}(x - 32)$

$9x = 5x - 160$

$\Rightarrow \quad 9x - 5x = -160$

$\Rightarrow \quad 4x = -160$

$$\Rightarrow x = \frac{-160}{4}$$

$$\therefore \quad x = -40 \quad \textbf{Ans.}$$

Q. 20. (*i*) Make 'x' the subject of the formula,

$$W = pq + \frac{1}{2} N x^2.$$

(*ii*) Find x, when $W = 49{\cdot}8$, $p = 8$, $q = 5$ and $N = 10$.

Sol. (*i*) $W = pq + \frac{1}{2} N x^2$

$$\Rightarrow \frac{1}{2} N x^2 = W - pq$$

$$\Rightarrow x^2 = \frac{2(W - pq)}{N}$$

$$\therefore \quad x = \sqrt{\frac{2(W - pq)}{N}}$$

(*ii*) When $W = 49{\cdot}8$, $p = 8$, $q = 5$ and $N = 10$, then

$$x = \sqrt{\frac{2(49{\cdot}8 - 8 \times 5)}{10}}$$

$$= \sqrt{\frac{2(49{\cdot}8 - 40)}{10}}$$

$$= \sqrt{\frac{2 \times 9{\cdot}8}{10}} = \sqrt{\frac{19{\cdot}6}{10}}$$

$$= \sqrt{1{\cdot}96} = 1{\cdot}4 \quad \textbf{Ans.}$$

Q. 21. (*i*) Make 'g' the subject of the formula,

$$T = 2\pi \sqrt{\frac{l}{g + k}}.$$

(*ii*) Find g when $\pi = 3\frac{1}{7}$, $l = 539$, $T = 44$ and $k = 1{\cdot}2$.

Sol. (*i*) $T = 2\pi \sqrt{\frac{l}{g + k}}$

$$\Rightarrow \frac{T}{2\pi} = \sqrt{\frac{l}{g + k}}$$

Squaring both sides,

$$\frac{T^2}{4\pi^2} = \frac{l}{g + k}$$

$$\Rightarrow T^2 g + T^2 k = 4 l \pi^2$$

$$\Rightarrow T^2 g = 4 l \pi^2 - T^2 k$$

$$\Rightarrow \quad g = \frac{4 l \pi^2 - T^2 k}{T^2}$$

(*ii*) When $\pi = 3\frac{1}{7} = \frac{22}{7}$, $l = 539$, $T = 44$, $k = 1{\cdot}2$

$$\therefore \ g = \frac{4 \times 539 \times \left(\frac{22}{7}\right)^2 - (44)^2 \times 1{\cdot}2}{(44)^2}$$

$$= \frac{\frac{4 \times 539 \times 484}{49} - 1936 \times 1{\cdot}2}{1936}$$

$$= \frac{21296 - 2323{\cdot}2}{1936} = \frac{18972{\cdot}8}{1936}$$

$$= 9{\cdot}8 \quad \textbf{Ans.}$$

Q. 22. Given : $\frac{1}{f} = (a - 1)\left(\frac{1}{p} + \frac{1}{q}\right)$.

(*i*) Express 'a' in terms of f, p and q.

(*ii*) Find 'a' when $p = 4$, $q = 6$ and $f = \frac{6}{5}$.

Sol. (*i*) $\frac{1}{f} = (a - 1)\left(\frac{1}{p} + \frac{1}{q}\right)$

$$\Rightarrow \frac{1}{f} = (a - 1)\left(\frac{q + p}{pq}\right)$$

$$a - 1 = \frac{1}{f} + \frac{pq}{(q + p)} = \frac{pq}{f(q + p)}$$

$$a = \frac{pq}{f(q + p)} + 1 = \frac{pq + f(q + p)}{f(q + p)}$$

$$= \frac{pq + f(p + q)}{f(p + q)}$$

(*ii*) When $p = 4$, $q = 6$ and $f = \frac{6}{5}$, then

$$a = \frac{4 \times 6 + \frac{6}{5}(4+6)}{\frac{6}{5}(6+6)} = \frac{24 \times \frac{6}{5} \times 10}{\frac{6}{5} \times 10}$$

$$= \frac{24+12}{12} = \frac{36}{12} = 3 \quad \textbf{Ans.}$$

Q. 23. If $S = 2\pi r(r+h)$ and $V = \pi r^2 h$, express S in terms of V, h and r.

Sol. $S = 2\pi r(r+h)$ ...(*i*)

$V = \pi r^2 h$ ...(*ii*)

Dividing (*i*) by (*ii*)

$$\frac{S}{V} = \frac{2\pi r(r+h)}{\pi r^2 h} = \frac{2(r+h)}{rh}$$

$$\therefore \quad S = \frac{2V(r+h)}{rh} \textbf{ Ans.}$$

Q. 24. If $V = \pi r^2 h$ and $A = (2\pi rh + 2\pi r^2)$, find V in terms of A, π and r.

Sol. $V = \pi r^2 h$ ...(*i*)

$A = 2\pi rh + 2\pi r^2$ ...(*ii*)

From (*i*) $h = \frac{V}{\pi r^2}$

Substituting the value of h in (*ii*),

$$A = 2\pi r \frac{V}{\pi r^2} + 2\pi r^2$$

$$\Rightarrow A = \frac{2V}{r} + 2\pi r^2$$

$$\Rightarrow \frac{2V}{r} = A - 2\pi r^2$$

$$\Rightarrow V = (A - 2\pi r^2)\frac{r}{2}$$

$$\Rightarrow V = \frac{1}{2} r(A - 2\pi r^2) \textbf{ Ans.}$$

Q. 25. If $x + y = m$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{t}$, find the formula for m in terms of x and t.

Sol. $x + y = m$ ...(*i*)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{t}$ ...(*ii*)

From (*i*) $y = m - x$

Substituting the value of y in (*ii*)

$$\frac{1}{x} + \frac{1}{m-x} = \frac{1}{t}$$

$$\Rightarrow \frac{1}{m-x} = \frac{1}{t} - \frac{1}{x}$$

$$\Rightarrow \frac{1}{m-x} = \frac{x-t}{tx}$$

$$\Rightarrow m - x = \frac{tx}{x-t}$$

$$\Rightarrow m = \frac{tx}{x-t} + x = \frac{tx + x^2 - tx}{x-t}$$

$$\Rightarrow m = \frac{x^2}{x-t} \textbf{ Ans.}$$