8 Archimedes' Principle
[Including Density and Relative Density]

DENSITY

Density of a substance is defined as the mass per unit volume of that substance.

$$\therefore \text{ Density} = \frac{\text{Mass}}{\text{Volume}}$$

UNITS

In M.K.S. (*i.e.* **S.I.**) **system,** the unit of **mass** is **kg**, the unit of **volume** is **m³** and the unit of **density** is **kg m⁻³**.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{1\text{ kg}}{1\text{ m}^3} = 1\text{ kg m}^{-3}$$

In C.G.S. system, the unit of **mass** is **g** (gram), the unit of **volume** is **cm³** and the unit of **density** is **g cm⁻³**.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{1\text{ g}}{1\text{ cm}^3} = 1\text{ g cm}^{-3}$$

The density of water at 4°C is 1 g cm^{-3}. This means, at 4°C, the mass of 1 cm^3 of water is 1 g.

The density of mercury is $13{\cdot}6 \times 10^3$ kg m^{-3}. This means, the mass of 1 m^3 of mercury is $13{\cdot}6 \times 10^3$ kg.

Also, note that :

$$\boxed{x\text{ g cm}^{-3} = x \times 1000\text{ kg m}^{-3}}$$

i.e. (i) $1\text{ g cm}^{-3} = 1000\text{ kg m}^{-3}$

(ii) $7{\cdot}8\text{ g cm}^{-3} = 7{\cdot}8 \times 10^3\text{ kg m}^{-3}$

Conversely,

$$\boxed{x\text{ kg m}^{-3} = \frac{x}{1000} \times \text{g cm}^{-3}}$$

i.e. (i) $13{\cdot}6 \times 10^3\text{ kg m}^{-3} = \frac{13{\cdot}6 \times 10^3}{1000}\text{ g cm}^{-3}$

$= 13{\cdot}6\text{ g cm}^{-3}$

(ii) $800\text{ kg m}^{-3} = \frac{800}{1000}\text{ g cm}^{-3}$

$= 0{\cdot}8\text{ g cm}^{-3}$

RELATIVE DENSITY

Relative density (R.D.) of a substance is defined as the ratio of density of the substance to the density of water at 4°C. Relative density is also known as **specific gravity** of the substance.

$$\therefore \text{ R.D.} = \frac{\text{Density of substance}}{\text{Density of water at 4°C}}$$

$$= \frac{\text{Mass of unit volume of substance}}{\text{Mass of unit volume of water at 4°C}} \times \frac{V}{V}$$

$$= \frac{\text{Mass of certain volume of substance}}{\text{Mass of same volume of water at 4°C}}$$

$\therefore$ R.D. can also be defined as the ratio of mass of a substance to the mass of same volume of water at 4°C.

Since R.D. is the ratio of two densities, it has no unit.

RELATION BETWEEN DENSITY AND RELATIVE DENSITY

We know density of water at 4°C is 1 g cm^{-3}.

$\therefore$ R.D. of a substance

$$= \frac{\text{Density of the substance}}{\text{Density of water at 4°C}}$$

$$= \frac{\text{Density of the substance in g cm}^{-3}}{1\ \text{g cm}^{-3}}$$

Thus, if the density of a substance is 3·6 g cm^{-3}, its R.D. is 3·6. Conversely, if R.D. of a substance is 1·4, its density is 1·4 g cm^{-3}.

If relative density of a substance is less than one it will float in water. For example, a piece of ice, which has a relative density of about 0·9, will float on water.

A substance with a relative density greater than 1 will sink in water.

Uses

(1) Relative density is often used by geologists and mineralogists to determine the mineral content of a rock or other sample.

(2) Gemologists use it as an aid for identification of gemstones.

Note : The relative density of a liquid can be measured using a hydrometer.

Example 1 :

A piece of iron of volume 30 cm^3 has a mass of 234 g. Find :

(i) the density of iron

(ii) relative density (R.D.) of iron

Solution :

Since, mass of iron = 234 g and its volume = 30 cm^3.

(i) **Density of iron** $= \frac{\text{Mass}}{\text{Volume}} = \frac{234\ \text{g}}{30\ \text{cm}^3} = \mathbf{7{\cdot}8\ g\ cm^{-3}}$

(ii) Since, density of iron = 7·8 g cm^{-3}

$\therefore$ **Its R.D. = 7·8**

Example 2 :

10 cm^3 of silver weighs 103 g. Find its :

(i) density in C.G.S.

(ii) density in kg m^{-3}

(iii) relative density

Solution :

(i) Since, mass = 103 g and volume = 10 cm^3

$\therefore$ **Density in C.G.S.** $= \frac{103\ \text{g}}{10\ \text{cm}^3} = \mathbf{10{\cdot}3\ g\ cm^{-3}}$

(ii) **Density in kg m^{-3}** = Density in C.G.S. × 1000 = 10·3 × 1000 kg m^{-3} = **10300 kg m^{-3}**

(iii) $\because$ Density in C.G.S. = 10·3 g cm^{-3}

$\therefore$ **R.D. = 10·3**

Example 3 :

The relative density of a substance is 2·7 and its volume is 100 cm^3. Find its mass.

Solution :

Since, R.D. = 2·7, therefore density = 2·7 g cm^{-3}

Now, $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$\Rightarrow$ $\text{Mass} = \text{Volume} \times \text{Density}$

$\Rightarrow$ $\text{Mass} = 100\ cm^3 \times 2{\cdot}7\ g\ cm^{-3}$

$= \mathbf{270\ g}$

Example 4 :

A stone of mass 6000 kg has a volume of 3 m^3. Find its :

(i) density in kg m^{-3}

(ii) density in g cm^{-3}

(iii) relative density

Solution :

(i) $$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{6000\ \text{kg}}{3\ m^3} = 2000\ kg\ m^{-3}$$

(ii) $$\text{Density in g cm}^{-3} = \frac{\text{Density in kg m}^{-3}}{1000} = \frac{2000}{1000}\ g\ cm^{-3} = 2\ g\ cm^{-3}$$

(iii) Since, density in C.G.S. = 2 g cm^{-3}

$\therefore$ **Relative density = 2**

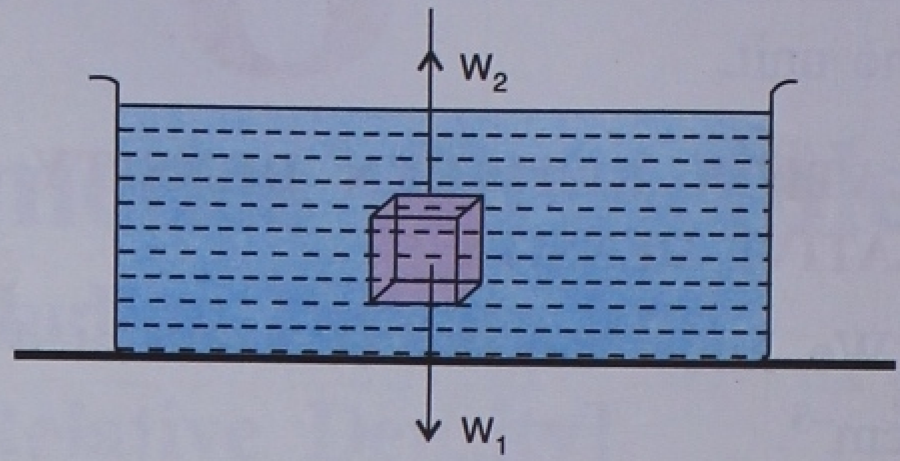
BUOYANT FORCE/UPTHRUST

Whenever a body is immersed partially or completely inside a fluid, it experiences an upward force. This upward force is called the **buoyant force** or **upthrust**, whereas the property of the fluids to apply upward force on the body immersed in it is called **buoyancy**. It is due to the buoyant force (upthrust) that a body submerged partially or wholly in a fluid appears to lose its weight *i.e.*, appears to be lighter.

When a body is immersed in a liquid, two forces act on it.

1. W_1 = Weight of the body which acts vertically downward
= True weight of the body

2. W_2 = Upthrust
= Buoyant force
= Upward force applied by the liquid on the body



Due to upthrust, the body appears to be lighter and its weight appears to be its true weight minus the buoyant force.

Apparent weight of the body

= Its true weight – buoyant force

= $W_1 - W_2$

We must note that a body experiences an upthrust in gas too. *For example*, a balloon filled with hydrogen gas moves in upward direction due to the upthrust of air present underneath it.

Do You Know ?

It is due to the 'upward force' (buoyant force or upthrust) exerted by water a ship floats in water, although it is made of iron.

FACTORS ON WHICH THE BUOYANT FORCE DEPENDS

The buoyant force primarily depends on two factors :

(a) The volume of the body immersed, *i.e.*, volume of the fluid displaced.

(b) The density of the fluid.

To understand these two factors, let us do the following simple experiments.

(1) Take a hollow metal cube and attach it to the hook of a spring balance.

Note the weight (W_1) shown on the scale of the spring balance.

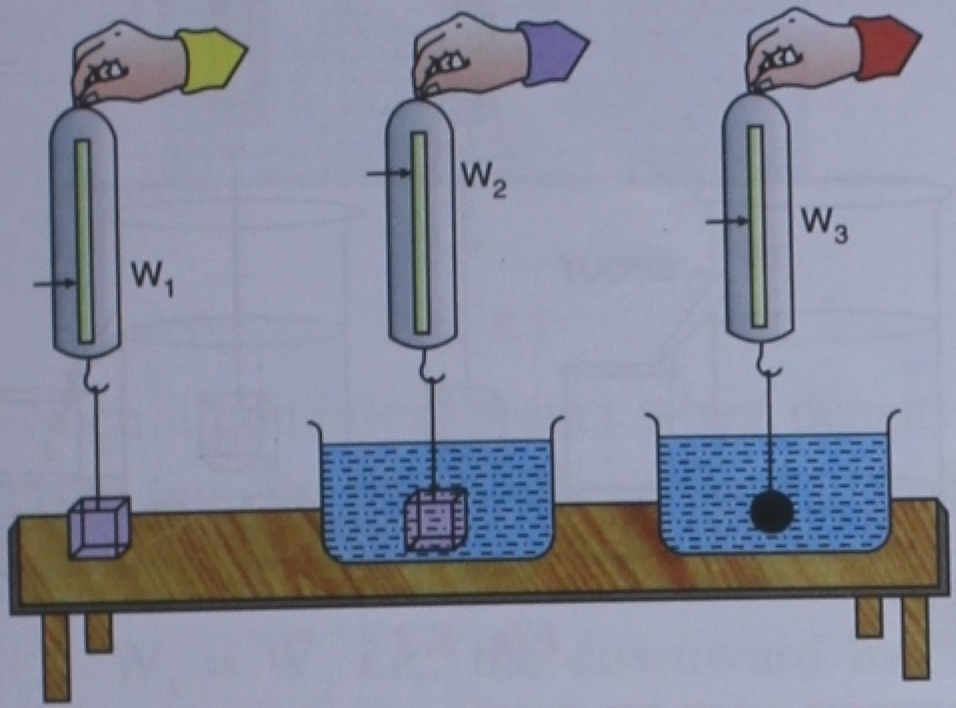


Fig. 8.1 More volume means more upthrust

Immerse the cube, hanging on the spring balance, into water in a beaker and note the reading of the weight (W_2) shown on the scale of the spring balance.

Now take the cube out and hammer it several times, to make it a solid piece. You will notice that it has become smaller by volume after you have hammered it. Now again, put it back into water and note the weight (W_3) in the spring balance. With smaller volume, you will notice that the weight shown is more than the second case when the volume was bigger, *i.e.*, W_3 is more than W_2.

Therefore we conclude, the more is the volume of the body immersed, more is the upthrust because more volume of the body immersed means more amount (volume) of liquid is displaced. This is the reason why a solid iron nail submerges in water but the same nail when beaten into a thin sheet and given some shape, floats in water.

(2) Take a metal cube and suspend it to the hook of the spring balance with the help of a thread. Record the weight (W_1) of the metal cube as shown by the scale of the balance.

Immerse the cube into water in a beaker and record the new weight (W_2) of the cube.

Add salt to the water. Adding salt increases the density of water. Now immerse the object again and note the weight (W_3) shown by the spring balance.

You will notice that weight (W_2) recorded in the second case will be more than in the third case (W_3). It means that the buoyant force in the third case is more than the second case. Hence, the denser is the liquid, the more is the buoyant force.

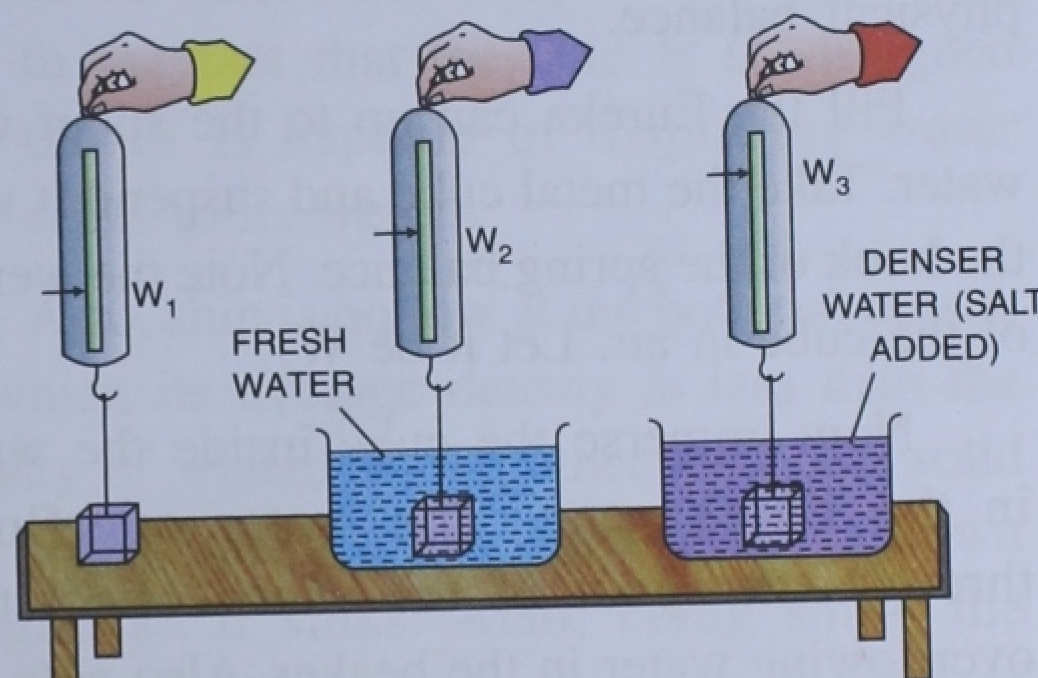


Fig. 8.2 Denser the liquid, more is the upthrust

Note :

$W_1 - W_2$ = Buoyant force when cube is immersed in water and

$W_1 - W_3$ = Buoyant force when cube is immersed in salt solution.

ARCHIMEDES' PRINCIPLE

It states that when a body is partially or completely immersed in a fluid, there is an apparent loss in its weight due to upthrust and this apparent loss in weight is equal to the weight of the fluid displaced by the immersed part of the body.

As shown above,

W_1 = Weight of cube in air

W_2 = Weight of cube in water

$\therefore\ W_1 - W_2$ = Apparent loss in weight of cube in water

and $W_1 - W_3$ = Apparent loss in weight of cube in salt solution.

TO VERIFY ARCHIMEDES' PRINCIPLE

The apparatus you need to verify Archimedes' principle are a Eureka can, a beaker, a spring balance, a metal cube and a physical balance.

Fill the Eureka can up to the spout with water. Take the metal cube and suspend it with the hook of the spring balance. Note the weight of the cube in air. Let it be W_1.

Now immerse the cube inside the water in the Eureka can. The water overflows through the spout of the can. Collect this overflowing water in the beaker. Also note the reading in the spring balance. This will give you the weight of the cube inside water. Let it be W_2. This W_2 will be less than W_1 due to upthrust. Loss in weight due to upthrust will be $(W_1–W_2)$.

Measure the weight of water collected in the beaker, which infact is the weight of water displaced by the cube.

On comparing, we find that loss in weight $(W_1 - W_2)$ is equal to the weight of water displaced by the cube.

Hence the Archimedes' principle is verified.

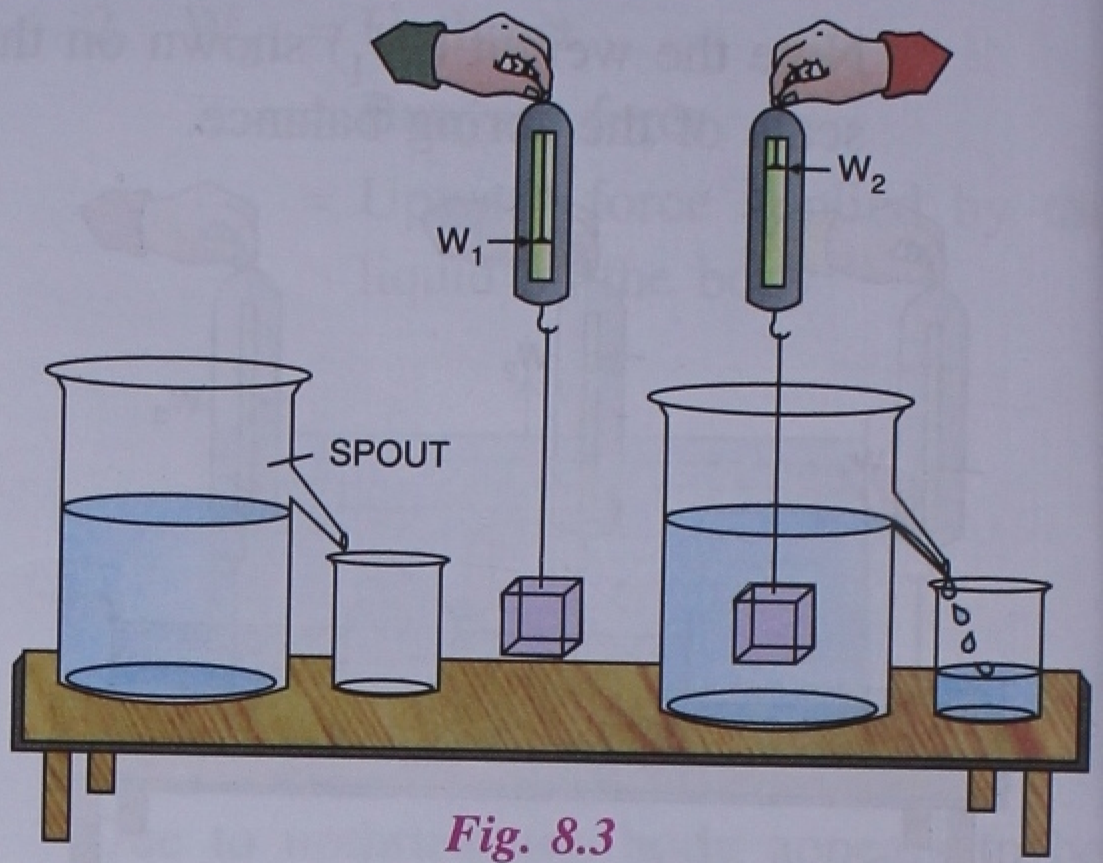


Fig. 8.3

SINKING AND FLOATATION OF BODIES IN A LIQUID

Whenever we place a body in a liquid, two forces act on it :

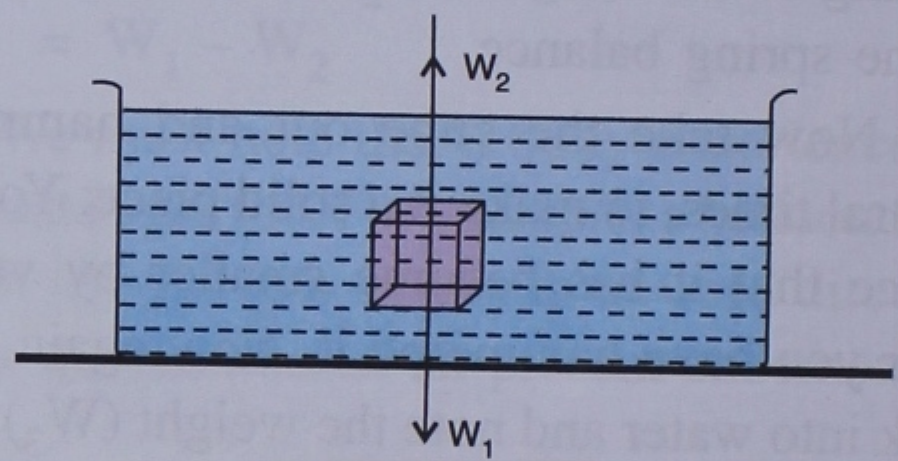


Fig. 8.4

(a) Weight of the body W_1 acting vertically downwards passing through the centre of gravity of the body.

(b) Buoyant force or upthrust W_2 acting vertically upwards and passing through the centre of gravity of the displaced liquid.

The centre of gravity of the displaced liquid is called **centre of buoyancy**.

The following are the three possibilities if we consider W_1 and W_2.

1. $W_1 > W_2$ *i.e.* the downward force is more than the upward force. In this case, the body will sink and the apparent weight of the body will be $(W_1–W_2)$.

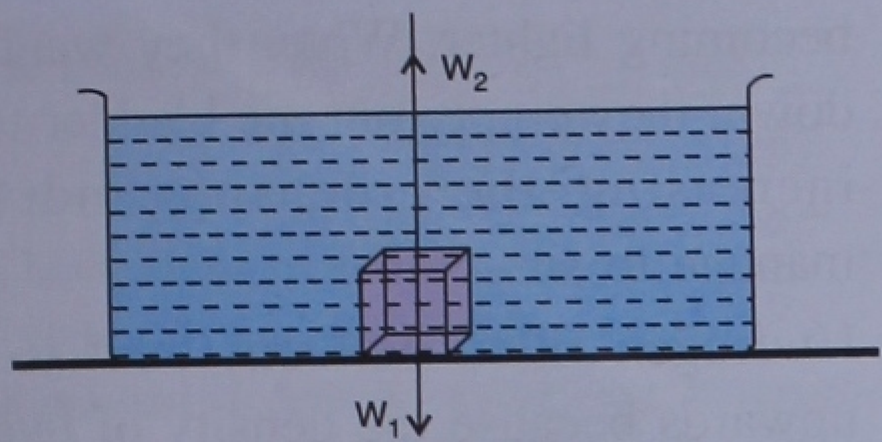


Fig. 8.5

Such a condition occurs when density of the body immersed is greater than the density of the liquid.

2. $W_1 = W_2$ *i.e.*, the downward force is equal to the upward force. In this case, the body will just float under the surface and the apparent weight of the body will be zero.

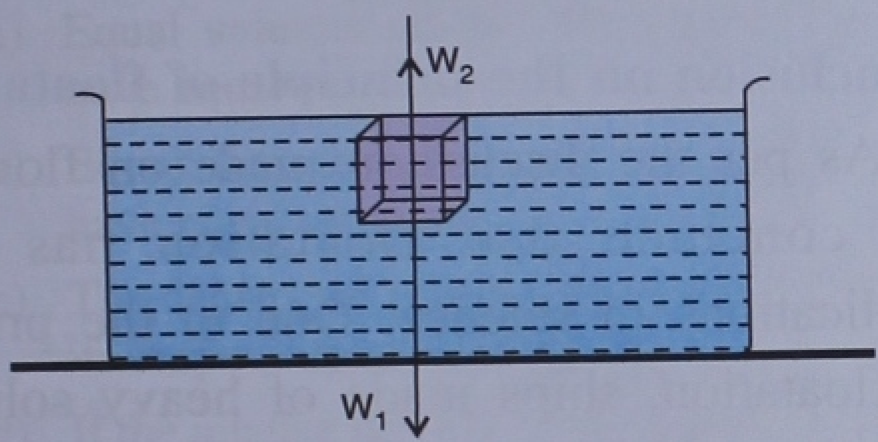


Fig. 8.6

Such a condition occurs when density of the body immersed is equal to the density of the liquid.

3. $W_1 < W_2$ *i.e.*, the downward force is less than the upward force. In this case, the body will remain in a floating condition with some of the volume above the liquid. Here also, the apparent weight is zero.

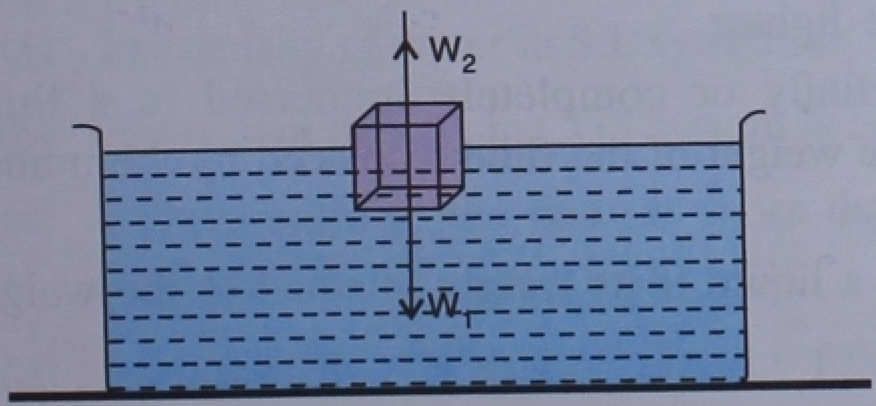


Fig. 8.7 Conditions of floatation

Such a condition occurs when density of the body is less than the density of liquid.

PRINCIPLE OF FLOATATION

A body floats in a liquid if the weight of the body is equal to the weight of the liquid displaced by it.

This is known as the principle of floatation.

APPLICATIONS OF PRINCIPLE OF FLOATATION

1. A nail made of iron sinks but a ship made of the same material can float. This is due to the fact that the ship is so designed that it is capable of displacing water equal to its weight.

Also, ship is hollow at the bottom because of which its average density is less than the density of water. On the other hand, the solid nail is unable to displace water equal to its own weight, so it sinks. Also, being solid, the density of nail is more than the density of water.

2. A specially designed ship called a submarine can float or submerge in water as desired. It has **ballast tanks** at its basement. When the submarine is to be submerged, these tanks are filled with sea water. As a result, the weight of the submarine becomes more than the weight of the water displaced by it and therefore, it submerges.

If the submarine has to come up, the tanks are emptied out, so the weight of the submarine decreases in comparison to the weight of the water displaced and the submarine rises up to the surface.

3. Large masses of ice known as **icebergs** can float on sea water. The density of ice is 0·917 g cm^{-3} while that of sea water is 1·02 g cm^{-3}. Therefore, the icebergs can float on water. Most of the part of a floating iceberg remains under water. It is estimated that 11 parts out of a volume of 12, remains under water. This can cause damage to the approaching ships.
4. The human body, with a little effort can remain floating on water. The density of water is comparable to the average density of the human body. Swimming in sea water is easier because sea water has a higher density than fresh water.
5. Some species of fish have an organ called **air bladder** or **swim bladder**. When fish want to rise, they fill the bladder with gases diffused from their own body, thus becoming lighter. When they want to go down, they empty out the bladder thereby increasing their density and hence manage to sink.
6. Hydrogen filled balloons tend to move upwards because the density of hydrogen is about 1/4th of the air. The upward force is more than the downward force (weight) of the balloon and hence it rises upwards till the density of both (balloon and air) becomes the same.
7. Lactometers, which are used to determine the purity of milk and hydrometers used for determining density of liquids, are based on this principle.

Conclusion on the principle of floatation

As per the discussion made on floatation, we conclude that floatation has many applications in science. Due to the principle of floatation, ships made of heavy solid like iron floats on water.

RECAPITULATION

- Density is defined as mass per unit volume of the substance *i.e.* Density (D) = $\frac{\text{Mass (m)}}{\text{Volume(v)}}$.
- The unit of density is kg m^{-3} in S.I. system and g cm^{-3} in CGS system. Also, 1 g cm^{-3} = 1000 kg m^{-3}.
- Relative density is the ratio of density of substance to the density of water at 4°C.
- Relative density is also the ratio of the mass of certain volume of substance to the mass of same volume of water at 4°C.
- Buoyant force or upthrust is experienced by a body whenever it is partially or completely immersed in a fluid. Because of this upthrust, the body appears to be lighter.
- Archimedes' principle states that when a body is partially or completely immersed in a fluid, it experiences an upward buoyant force which is equal to the weight of the fluid displaced by the immersed part of the body.
- According to the principle of floatation, a body floats in a liquid if its weight is equal to the weight of the liquid displaced.
- R.D. of a body is a unitless quantity.

- The buoyant force (upthrust) is equal to the weight of the liquid displaced.
- The buoyant force is more, if (i) the volume of the submerged part of the body is more *i.e.* the volume of the liquid displaced is more and (ii) the density of the liquid is more.
- A body sinks in a liquid if its density is more than the density of the liquid.
- A body floats in the liquid completely submerged, if the densities of both are same. But floats partially submerged if density of the body is less than the density of the liquid.
- The apparent weight of a floating body is zero.
- The property of buoyancy is shown by gases also.
- A gas filled balloon floating in air shows that the principle of floatation and Archimedes' principle are obeyed in gases also.

TEST YOURSELF

A. Short Answer Questions

1. Write *true* or *false* for each statement. Rewrite the false statement correctly.
   (a) Equal volumes of two different substances have equal masses.
   (b) Equal masses of two different substances have equal volumes.
   (c) The density of a copper piece will change if its shape is changed, keeping the mass same.
   (d) If the relative density of alcohol is 0·85, then it will occupy less volume than an equal amount (mass) of water.
   (e) The density of iron depends on its mass, volume and its weight.
   (f) Objects appear heavier when immersed in liquid.
   (g) Iron placed on mercury will sink.
   (h) Buoyant force is experienced when objects are immersed in fluids.
   (i) Buoyant force acts in all directions.

2. Fill in the blanks :
   (a) The density of water in S.I. system is ....
   (b) The unit of density in CGS system is ....
   (c) If the weight of the body is more than the buoyant force, then the body will ....
   (d) A submarine can sink or float by filling or emptying its ....
   (e) One kg of salt will occupy more .... than one kg of iron.
   (f) .... has no units.
   (g) A brick appears to be .... in water than in air.
   (h) The relative density of water is ....
   (i) An iceberg is dangerous for floating ships because it floats with only .... of its volume above water.
   (j) 2·7 g cm^{-3} of density will be equal to .... kg m^{-3}.

3. Match the following :

Column A	Column B
(a) Weight	(i) has no units
(b) Buoyant force	(ii) less than actual weight
(c) Apparent weight	(iii) effective force due to gravity
(d) Relative density	(iv) effective force due to liquid displaced

4. Tick the most appropriate answer :
   (a) Density has the S.I. unit as
   (i) g cm^{-3} (ii) g m^{-3}
   (iii) kg cm^{-3} (iv) kg m^{-3}
   (b) Density of water is 1 g cm^{-3}. Its density in S.I. system is
   (i) 10 kg m^{-3} (ii) 100 kg m^{-3}
   (iii) 1000 kg m^{-3} (iv) none of these

(c) Density of iron in MKS system is 7800 kg m^{-3}. Its density in CGS system will be

(i) 7·8 g cm^{-3} (ii) 78 g cm^{-3}

(iii) 0·78 g cm^{-3} (iv) 780 g cm^{-3}

(d) To obtain the density of a solid, we must know its

(i) mass and area

(ii) mass and volume

(iii) only mass

(iv) none of these

(e) For a substance, the ratio of mass to the volume is known as

(i) relative density

(ii) refractive index

(iii) density

(iv) none of these

(f) The unit of relative density is

(i) kg m^{-3} (ii) kg cm^{-3}

(iii) g m^{-3} (iv) none of these

(g) The upward force experienced by a body immersed partially or completely in a fluid is known as

(i) upthrust (ii) down thrust

(iii) horizontal force (iv) vertical force

(h) Upthrust experienced by a body is equal to

(i) mass of the body

(ii) apparent loss of weight of the body

(iii) weight of the body

(iv) none of these

(i) If the weight of the body is less than the upthrust of the liquid in which it is submerged, then the body will

(i) float

(ii) sink

(iii) float and then sink

(iv) none of these

(j) An ice cube floating on water has a fraction above water. This amount is

(i) 1/12th of total volume

(ii) 11/12 of the total volume

(iii) total volume

(iv) 1/9th of the total volume

5. Answer the following :

(a) Define relative density.

(b) Define buoyant force. Name the two factors on which the buoyant force depends.

(c) Why is it easier to lift a bucket full of water under the surface of water ?

(d) Why a person can swim more easily in sea water compared to river water ?

(e) What will a fish do to submerge in water ?

(f) How does a submarine submerge in water ?

(g) Why does a nail made of iron sink in water?

(h) Define density. State its S.I. units.

(i) State the principle of floatation.

B. Long Answer Questions

1. Demonstrate by an experiment that a body immersed in a liquid appears to be lighter.
2. Will the buoyant force be more or less when the body displaces greater volume of liquid ? Give reason.
3. Show by an experiment that a body immersed in a liquid of greater density experiences greater upthrust.
4. State the Archimedes' principle. Describe an experiment to verify it.
5. What is the relation between the density of the body and the density of the liquid in which the body :

(a) sinks

(b) floats completely submerged

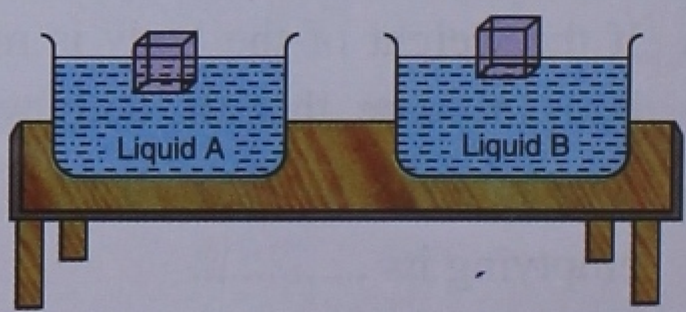
(c) floats when partially submerged.

Ans. (a) density of body is greater

(b) densities of both are same

(c) density of the body is less.

6. The following diagram shows two identical bodies floating over two different liquids A and B.



(a) Which liquid has greater density.

(b) In which liquid will the body experience greater upthrust (buoyant force).

Ans. (a) liquid B (b) same in both

7. Explain :

(a) A piece of cork, when left inside water, comes to the surface.

(b) An iron needle sinks in water but a ship made of iron floats.

(c) A piece of ice floats on water.

(d) A piece of wood left under water, comes to the surface.

(e) It is easier to swim in sea water than in river water.

(f) Icebergs floating in the sea are dangerous for the ships.

(g) It is easier to lift a stone in water than in air.

C. Numerical Problems

1. (a) A substance has a mass of 200 g and has a volume of 45 cm^3. Find the density.

(b) A cube of a substance has a mass of 180 g and has a volume of 200 cm^3. Find the density in CGS and MKS systems.

(c) The volume of a metal cube is 135 cm^3. If its density is 13·6 g cm^{-3}, find the mass.

(d) An empty bottle weighs 25 g. When filled completely with water, it weighs 46·8 g. When filled completely with a liquid, it weighs 52·4 g. Find :

(i) weight of water taken

(ii) weight of liquid taken

(iii) relation between the volumes of water and the liquid displaced.

(iv) R.D. of the liquid.

(e) Calculate the volume of a cube of mass 200 kg if its density is 800 kg m^{-3}.

Ans. (a) $4\frac{4}{9}$ g cm^{-3} (b) 0·9 g cm^{-3} and 900 kg m^{-3} (c) 1·836 kg (d) (i) 21·8 g (ii) 27·4 g (iii) both are equal (iv) 1·26 (e) 0·25 m^3.

2. Density of liquid A is 1·2 g cm^{-3}, density of another liquid B is 5·6 g cm^{-3}. State, with reason, whether a body of density 3·7 g cm^{-3} will float on liquid A or on liquid B. **Ans.** On liquid B

3. A body weighs 300 gf in air and 260 gf in water. Find :

(a) apparent loss in the weight of the body.

(b) the buoyant force on the body.

Ans. (a) 40 gf (b) 40 gf