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## Decimals

A fraction with some power of 10 as the denominator is called a **decimal fraction**.

$\frac{3}{10}$ ,  $\frac{17}{100}$  and  $\frac{317}{1000}$  are some decimal fractions.  $\frac{3}{10}$  can be written as .3 or 0.3.

We call 0.3 a **decimal** or **decimal number**. Similarly,  $\frac{17}{100}$  and  $\frac{317}{1000}$  can be written as the decimal numbers 0.17 and 0.317 respectively.

For writing decimals, the positional system of writing numbers can be extended beyond the units place, to include places for tenths, hundredths, thousandths, ten thousandths, etc.

The **place-value chart** for decimal numbers is as follows.

<b>Place</b>	...	Ten thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten thousandths	...
<b>Value</b>	...	10000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	...

Consider the decimal number 17352.9876. It would be expanded as

$$1 \times 10000 + 7 \times 1000 + 3 \times 100 + 5 \times 10 + 2 \times 1 + 9 \times \frac{1}{10} + 8 \times \frac{1}{100} + 7 \times \frac{1}{1000} + 6 \times \frac{1}{10000}$$

We read 17352.9876 as “seventeen thousand three hundred and fifty-two point nine eight seven six”. 17352 is called its **integral part** and .9876 is called its **decimal part**.

### Conversion of fractions into decimals

**Method** First convert the fraction into a fraction with a power of 10 as the denominator. Count the number of zeros in the denominator. Then counting from the right of the numerator, place the decimal point after as many digits as the number of zeros in the denominator.

**Examples** (i)  $\frac{47}{100} = 0.47$  (also written as .47).

(ii)  $\frac{13}{125} = \frac{13 \times 8}{125 \times 8} = \frac{104}{1000} = 0.104$ .

(iii)  $1\frac{4}{5} = \frac{9}{5} = \frac{9 \times 2}{5 \times 2} = \frac{18}{10} = 1.8$ .



Conversion of decimals into ordinary fractions

**Method** In the numerator, write the given number leaving out the decimal point. In the denominator, write 1 followed by as many zeros as the number of digits in the decimal part. Reduce the fraction to its simplest form.

**Examples** (i)  $13.4 = \frac{134}{10} = \frac{67}{5}$ .

(ii)  $23.87 = \frac{2387}{100}$ .

(iii)  $352.9171 = \frac{3529171}{10000}$ .

(iv)  $0.78929 = \frac{78929}{100000}$ .

Fundamental operations on decimals

You are already familiar with the four fundamental operations (addition, subtraction, multiplication and division) on decimals.

EXAMPLE

(i) Find  $1.735 + 2.9872 + 12.3 - 3.95$ .

(ii) Divide the product of 1.21 and 0.056 by 0.28.

**Solution**

$$\begin{array}{r} \text{(i)} \quad 1.7350 \\ \quad 2.9872 \\ + 12.3000 \\ \hline \quad 17.0222 \\ \quad - 3.9500 \\ \hline \quad 13.0722 \end{array}$$

(ii) First, we find the product of 1.21 and 0.056.

$$\begin{array}{r} 121 \\ \times 56 \\ \hline 726 \\ 605 \\ \hline 6776 \end{array}$$

$$\therefore 1.21 \times 0.056 = 0.06776.$$

Now, we divide 0.06776 by 0.28.

$$\begin{array}{r} 28 \overline{) 6.776} \quad (0.242) \\ \underline{- 56} \\ 117 \\ \underline{- 112} \\ 56 \\ \underline{- 56} \\ \times \end{array}$$

$$\therefore 0.06776 \div 0.28 = 0.242.$$

Hence, the required quotient = 0.242.

**Simplification**

An expression involving decimals is simplified by following the rule of BODMAS.

EXAMPLE

**Simplify the following.**

(i)  $(4.001 + 24.9 + 93.93 + 0.682 - 3) \div 0.03$       (ii)  $\left( \frac{14.28 \times 3.5}{17} + \frac{3.42}{0.018} \right)$  of 0.3

**Solution**

$$\begin{array}{r} \text{(i)} \quad 4.001 \\ \quad 24.900 \\ \quad 93.930 \\ + \quad 0.682 \\ \hline \quad 123.513 \\ \quad - 3.000 \\ \hline \quad 120.513 \end{array}$$

$$120.513 \div 0.03 = \frac{12051.3}{3} = 4017.1.$$

$$\therefore 120.513 \div 0.03 = 4017.1.$$

Hence, the given expression = 4017.1.



$$(ii) \begin{array}{r} 1428 \\ \times 35 \\ \hline 7140 \\ 4284 \phantom{0} \\ \hline 49980 \end{array}$$

$\therefore 14.28 \times 3.5 = 49.98.$

$$\begin{array}{r} 17 \overline{) 49.98} \text{ (2.94)} \\ \underline{- 34} \\ 159 \\ \underline{- 153} \\ 68 \\ \underline{- 68} \\ 0 \end{array}$$

$$\therefore \frac{14.28 \times 3.5}{17} = \frac{49.98}{17} = 2.94$$

$$\begin{array}{r} 18 \overline{) 3420} \text{ (190)} \\ \underline{- 18} \\ 162 \\ \underline{- 162} \\ 0 \end{array}$$

$$\therefore \frac{3.42}{0.018} = \frac{3420}{18} = 190.$$

Hence, the given expression =  $(2.94 + 190) \times 0.3 = 192.94 \times 0.3 = 57.882.$

### Recurring decimals

$\frac{1}{2} = 0.5$ ,  $\frac{1}{4} = 0.25$  and  $\frac{3}{8} = 0.375$ . Such decimals are called **terminating decimals** as the dividend is exactly divisible by the divisor.

Now, consider the fraction  $\frac{5}{9}$ .

$$\begin{array}{r} 9 \overline{) 5.0} \text{ (0.555...)} \\ \underline{- 45} \\ 50 \\ \underline{- 45} \\ 50 \\ \underline{- 45} \\ 5... \end{array}$$

The process of division is endless and we get a succession of 5s in the quotient.

The number 0.555... is called a **recurring decimal** and is written as  $0.\dot{5}$  or  $0.\overline{5}$  in short. In general, a decimal in which a digit or a finite group of digits recurs endlessly (that is, the division never gets completed) is called a **recurring decimal**. In such decimals, either a bar is placed over the recurring group of digits or a dot is placed over each of the first and last digits of the group.

**Examples** (i)  $\frac{4}{11} = 0.363636\dots$  is written as  $0.\dot{3}6$  or  $0.\overline{36}$  as the digits 3 and 6 are repeated endlessly.

(ii)  $\frac{41}{333} = 0.123123\dots$  is written as  $0.\dot{1}2\dot{3}$  or  $0.\overline{123}$ .

Here, the group of digits 123 recurs repeatedly in the quotient. So, we place a dot each above the first and last digits of the group or a bar over the group.

(iii)  $\frac{2}{7} = 0.285714285714\dots$  is written as  $0.\dot{2}8571\dot{4}$  or  $0.\overline{285714}$ .

### Conversion of recurring decimals into fractions

#### EXAMPLE

Convert the following recurring decimals into fractions.

(i) 0.3

(ii) 0.25

(iii)  $0.\overline{25}$



- Solution**
- (i) Let  $x = 0.\dot{3} = 0.3333\dots$  ... (1)  
 Multiplying both sides by 10, we get  
 $10x = 3.3333\dots$  ... (2)  
 Subtracting (1) from (2), we get  
 $10x - x = 3$  or  $9x = 3$  or  $x = \frac{3}{9} = \frac{1}{3}$ .
- Hence,  $0.\dot{3} = \frac{3}{9} = \frac{1}{3}$ .
- (ii) Let  $x = 0.2\dot{5} = 0.2555\dots$  ... (1)  
 Multiplying both sides by 10, we get  
 $10x = 2.5555\dots$  ... (2)  
 Subtracting (1) from (2), we get  
 $10x - x = 2.3$  or  $9x = 2.3$  or  $x = \frac{2.3}{9} = \frac{23}{90}$ .
- Hence,  $0.2\dot{5} = \frac{23}{90}$ .
- (iii) Let  $x = 0.\overline{25} = 0.252525\dots$  ... (1)  
 Multiplying both sides by 100, we get  
 $100x = 25.252525\dots$  ... (2)  
 Subtracting (1) from (2), we get  
 $99x = 25 \Rightarrow x = \frac{25}{99}$ .
- Hence,  $0.\overline{25} = \frac{25}{99}$ .

## EXERCISE

## 4

1. Write the following as decimals.

(i)  $5 \times 1000 + 6 \times 10 + 2 \times 1 + \frac{2}{10} + \frac{3}{1000}$

(ii)  $\frac{9}{10}$

(iii)  $\frac{37}{10}$

(iv)  $\frac{197}{100}$

(v)  $\frac{7}{8}$

(vi)  $7\frac{3}{20}$

(vii)  $16\frac{7}{25}$

2. Write the following decimals as fractions in their simplest form.

(i) 0.6

(ii) 0.24

(iii) 2.008

(iv) 37.25

(v) 123.497

3. Write the following numbers in ascending order.

(i) 0.52, 0.502, 0.522, 0.5202

(ii) 2.135, 2.1035, 2.1305, 2.0135

4. Find the following.

(i)  $90 - (15.26 + 73.5 + 0.748)$

(ii)  $138.4597 - 68.00009 + 43.57 - 85$

(iii)  $3.54 \times 12.253 \times 0.5$

(iv)  $(2.5 \times 2.5 \times 2.4 + 2.46) \div 1.8$

(v)  $77.4 + 73.96 \div 8.6$

(vi) 0.2 of  $\left( 4.59 \times 1.8 \div 3.6 + 5.4 \times \frac{1}{9} - 0.2 \right)$

(vii)  $4.2 - [3.57 - 0.3 \text{ of } \{2.58 - \overline{1.2 - 0.3}\}]$



5. Convert the following recurring decimals into fractions.

(i)  $0.\dot{4}$

(ii)  $0.3\dot{5}$

(iii)  $0.\overline{67}$

(iv)  $0.\dot{4}2\dot{3}$

### ANSWERS

1. (i) 5062.203 (ii) 0.9 (iii) 3.7 (iv) 1.97 (v) 0.875 (vi) 7.15 (vii) 16.28

2. (i)  $\frac{3}{5}$  (ii)  $\frac{6}{25}$  (iii)  $\frac{251}{125}$  (iv)  $\frac{149}{4}$  (v)  $\frac{123497}{1000}$

3. (i) 0.502, 0.52, 0.5202, 0.522 (ii) 2.0135, 2.1035, 2.1305, 2.135

4. (i) 0.492 (ii) 29.02961 (iii) 21.68781 (iv) 9.7 (v) 86 (vi) 0.539 (vii) 1.134

5. (i)  $\frac{4}{9}$  (ii)  $\frac{16}{45}$  (iii)  $\frac{67}{99}$  (iv)  $\frac{47}{111}$

