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Factors and Multiples

Factors and multiples

If a given number is exactly divisible by another number, that number is called a factor of the given number, while the given number is called a multiple of the other number.

Examples (i) $10 = 2 \times 5$. So, 2 and 5 are factors of 10. Also, 10 is a multiple of 2 and 5.

(ii) $70 = 2 \times 5 \times 7$. So, 2, 5 and 7 are factors of 70. Also, 70 is a multiple of 2, 5 and 7. Further, $2 \times 5, 2 \times 7, 5 \times 7$ and $2 \times 5 \times 7$ are factors of 70.

Note
- Every number is a factor of itself and 1 is a factor of every number.
- Every number is a multiple of itself and every number is a multiple of 1.
- Every number has countless multiples.

Classification of numbers

Even numbers

Numbers that leave no remainder on being divided by 2 are called even numbers. In other words, numbers that are multiples of 2 are even numbers. For example, 0, 4, 78, 126, 5132 and 23450 are even numbers.

Note The even digits are 0, 2, 4, 6 and 8. Any number having an even digit in the units place is an even number.

Odd numbers

Numbers not exactly divisible by 2 are called odd numbers. In other words, numbers that are not multiple of 2 are called odd numbers. For example, 1, 13, 125, 4137 and 18249 are odd numbers.

Note The odd digits are 1, 3, 5, 7 and 9. Any number having an odd digit in the units place is an odd number.

Prime numbers

A number is called a prime number or prime if it has no other factors except 1 and itself. 2, 3, 5, 7, 11, 13, 17, 19 and 23 are some prime numbers. 1 is not considered a prime number. 2 is the only even prime number and it is the smallest prime number.

Composite numbers

A number that has two or more prime factors is called a composite number. 4, 6, 8, 21, 136 are some composite numbers.

Co-prime numbers

Two numbers are called co-prime or relatively prime if they do not have a common factor except 1. For example, 3 and 4; 17 and 21; and 23 and 25 are co-prime numbers.

Twin primes

Pairs of prime numbers that differ by 2 are called twin primes. 3 and 5; 5 and 7; 11 and 13; and 17 and 19 are some examples of twin primes.

Prime factorization

Writing a number as a product of prime numbers is called prime factorization.

Examples (i) $48 = 2 \times 2 \times 2 \times 2 \times 3$.

(ii) $1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$.

Method of prime factorization

Divide the number repeatedly by prime numbers 2, 3, 5, 7, ... till the quotient is 1.

Alternatively, break the number into smaller factors till all the factors are prime.

EXAMPLE **Express 840 as a product of primes.**

Solution

$$\begin{aligned}840 &= 2 \times 420\\ &= 2 \times 2 \times 210\\ &= 2 \times 2 \times 2 \times 105\\ &= 2 \times 2 \times 2 \times 3 \times 35\\ &= 2 \times 2 \times 2 \times 3 \times 5 \times 7\end{aligned}$$

Another method

2	840
2	420
2	210
3	105
5	35
7	7
	1

$\therefore\ 840 = 2 \times 2 \times 2 \times 3 \times 5 \times 7$.

Highest Common Factor (HCF)

The greatest number which is a common factor of two or more numbers is their highest common factor (HCF) or greatest common divisor (GCD).

Example The factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24.

The factors of 80 = 1, 2, 4, 5, 8, 10, 16, 20, 40, 80.

$\therefore$ the common factors of 24 and 80 = 1, 2, 4, 8 and of these, 8 is the greatest number. So, the HCF of 24 and 80 = 8.

Methods of finding HCF

There are two methods of finding the HCF of two or more numbers.

(i) By prime factorization (ii) By division

To find HCF by prime factorization

Steps 1. Express each number as a product of prime numbers.

2. Find the common factors of the numbers.

3. Find the product of the common factors. This product is the required HCF.

EXAMPLE **Find the HCF of 252, 315 and 441.**

Solution $252 = 2 \times 126 = 2 \times 2 \times 63 = 2 \times 2 \times 3 \times 21 = 2 \times 2 \times 3 \times 3 \times 7.$

$315 = 3 \times 105 = 3 \times 3 \times 35 = 3 \times 3 \times 5 \times 7.$

$441 = 3 \times 147 = 3 \times 3 \times 49 = 3 \times 3 \times 7 \times 7.$

The common factors of 252, 315 and 441 = 3, 3, 7.

$\therefore$ the HCF $= 3 \times 3 \times 7 = 63.$

To find HCF by division

Steps 1. Divide the bigger number by the smaller number.

2. If there is no remainder then the smaller number is the HCF. If there is a remainder, take it as the new divisor and take the previous divisor as the new dividend.

3. Continue Step 2 till there is no remainder. The last divisor is the required HCF.

To find the HCF of three or more numbers, take the following steps.

Steps 1. Find the HCF of any two of the numbers.

2. Find the HCF of the third number and the HCF obtained in Step 1. This is the HCF of the three numbers.

Continue similarly for more numbers.

EXAMPLE **Find the HCF of 594, 810 and 1260.**

Solution First we find the HCF of 594 and 810.

594 < 810, so, 594 is the divisor and 810 is the dividend.

```
594) 810 (1
   - 594
   ------
     216) 594 (2
        - 432
        ------
          162) 216 (1
             - 162
             ------
               54) 162 (3
                 - 162
                 ------
                     ×
```

$\therefore$ the HCF of 594 and 810 = 54.

Now we find the HCF of 54 and 1260.

```
54) 1260 (23
   - 108
   ------
      180
    - 162
    ------
      18) 54 (3
        - 54
        -----
           ×
```

$\therefore$ the HCF of 54 and 1260 = 18.

Hence, the HCF of the given numbers = 18.

Lowest Common Multiple (LCM)

The lowest common multiple (LCM) of two or more numbers is the smallest of the common multiples of those numbers.

Example The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, etc.

The multiples of 6 are 6, 12, 18, 24, 30, 36, etc.

The multiples of 8 are 8, 16, 24, 32, 40, etc.

Clearly, 24 is the smallest or the lowest common multiple of 4, 6 and 8.

So, the LCM of 4, 6 and 8 = 24.

Methods of finding LCM

There are two methods of finding the LCM of two or more numbers.

(a) By prime factorization (b) By division

To find LCM by prime factorization

Steps 1. Express each number as a product of prime factors.

2. Take each prime factor the greatest number of times it appears in any of the prime factorizations of the numbers.

3. The product of the prime factors in Step 2 is the LCM of the numbers.

EXAMPLE **Find the LCM of 75, 125 and 150.**

Solution $75 = 3 \times 25 = 3 \times 5 \times 5$, $125 = 5 \times 25 = 5 \times 5 \times 5$, $150 = 2 \times 75 = 2 \times 3 \times 25 = 2 \times 3 \times 5 \times 5$.

The greatest number of times that 2 appears as a factor of any of the numbers is one, 3 appears once, 5 appears thrice.

$\therefore$ LCM $= 2 \times 3 \times 5 \times 5 \times 5 = 750$.

To find LCM by division

Steps 1. Divide the numbers by a prime number which is a factor of at least two of the given numbers.

2. Write the quotients and carry forward the numbers which are not divisible.

3. Repeat Steps 1 and 2 till no two of the numbers have a common factor.

4. The product of the divisors of all the steps and the remaining numbers is the LCM of the given numbers.

EXAMPLE **Find the LCM of 24, 28, 35 and 75.**

Solution

2	24, 28, 35, 75
2	12, 14, 35, 75
3	6, 7, 35, 75
5	2, 7, 35, 25
7	2, 7, 7, 5
	2, 1, 1, 5

$\therefore$ LCM $= 2 \times 2 \times 3 \times 5 \times 7 \times 2 \times 5 = 4200$.

Some important properties of HCF and LCM

1. The product of two numbers is equal to the product of their HCF and LCM.

HCF × LCM = first number × second number

EXAMPLE **The product of two numbers is 1008 and their LCM is 168. Find their HCF.**

Solution HCF of two numbers $= \dfrac{\text{product of the two numbers}}{\text{LCM of the numbers}} = \dfrac{1008}{168} = 6.$

2. The HCF of two numbers cannot exceed either of the numbers.
3. The LCM of two numbers cannot be less than either of the numbers.
4. The HCF of two co-prime numbers = 1, and if the HCF of two numbers is 1, the two numbers are co-prime.
5. The LCM of two co-prime numbers = the product of the two numbers.

EXAMPLE **Find the HCF and LCM of 14 and 25.**

Solution 14 and 25 are co-prime numbers.

$\therefore$ HCF = 1 and LCM $= 14 \times 25 = 350$.

Solved Examples

EXAMPLE 1 **Find the greatest number that will divide 326, 895 and 1790 leaving the remainders 11, 13 and 5 respectively.**

Solution $326 - 11 = 315$, $895 - 13 = 882$, $1790 - 5 = 1785$.

The required number is the HCF of 315, 882 and 1785.

```
315) 882 (2
   - 630
     252) 315 (1
        - 252
           63) 252 (4
             - 252
                 ×
```

$\therefore$ the HCF of 315 and 882 = 63.

```
Now, 63) 1785 (28
       - 126
          525
        - 504
          21) 63 (3
            - 63
               ×
```

$\therefore$ the HCF of 63 and 1785 = 21.

$\therefore$ the HCF of 315, 882 and 1785 = 21.

Hence, the required number = 21.

EXAMPLE 2 **(i) Find the LCM of 24, 36, 60, 72 and 90.**
(ii) Find the smallest number which when divided by 24, 36, 60, 72 and 90 leaves the remainder 5 in each case.
(iii) Find the smallest number which when increased by 8 is exactly divisible by 24, 36, 60, 72 and 90.

Solution (i)

2	24, 36, 60, 72, 90
2	12, 18, 30, 36, 45
2	6, 9, 15, 18, 45
3	3, 9, 15, 9, 45
3	1, 3, 5, 3, 15
5	1, 1, 5, 1, 5
	1, 1, 1, 1, 1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

(ii) The required number = LCM + 5 = 360 + 5 = 365.

(iii) The required number = LCM − 8 = 360 − 8 = 352.

EXAMPLE 3 **Find the greatest number of six digits which is exactly divisible by 4, 8, 12, 15, 21 and 25.**

Solution First, we find the LCM of 4, 8, 12, 15, 21 and 25.

2	4, 8, 12, 15, 21, 25
2	2, 4, 6, 15, 21, 25
3	1, 2, 3, 15, 21, 25
5	1, 2, 1, 5, 7, 25
	1, 2, 1, 1, 7, 5

$\therefore$ LCM $= 2 \times 2 \times 3 \times 5 \times 2 \times 7 \times 5 = 4200$.

The greatest number of six digits = 999999.

Let us divide this by the LCM.

4200) 999999 (238
– 8400
15999
– 12600
33999
– 33600
399

$\therefore$ 999999 ÷ LCM leaves the remainder 399.

Hence, the required number = 999999 – remainder = 999999 – 399 = 999600.

EXAMPLE 4 **Find the smallest number of five digits which is exactly divisible by 8, 12, 14, 28 and 56.**

Solution First, we find the LCM of 8, 12, 14, 28 and 56.

2	8, 12, 14, 28, 56
2	4, 6, 7, 14, 28
2	2, 3, 7, 7, 14
7	1, 3, 7, 7, 7
	1, 3, 1, 1, 1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 7 \times 3 = 168$.

The smallest number of five digits = 10000.

Let us divide this by the LCM.

168) 10000 (59
– 840
1600
– 1512
88

So, the 59th multiple of 168 is less than 10000.

$\therefore$ the required number = 60th multiple of 168 $= 60 \times 168 = 10080$.

Alternatively

The required number = 10000 – remainder + LCM = 10000 – 88 + 168 = 10080.

EXAMPLE 5 **Five bells start ringing together. If the bells ring at intervals of 12, 18, 25, 40 and 54 seconds respectively, after what interval of time will they ring together again?**

Solution The required time (in seconds) = the LCM of 12, 18, 25, 40 and 54.

2	12, 18, 25, 40, 54
2	6, 9, 25, 20, 27
3	3, 9, 25, 10, 27
3	1, 3, 25, 10, 9
5	1, 1, 25, 10, 3
	1, 1, 5, 2, 3

$\therefore$ the required LCM $= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 2 \times 3$
$= 5400$.

Hence, the required time = 5400 s = 90 min = 1 h 30 min.

EXAMPLE 6 **The LCM of two numbers is 12 times their HCF and the difference of the LCM and the HCF is 110. If one of the numbers is 30, find the other.**

Solution Let the HCF of the two numbers = x.

$\therefore$ the LCM of the two numbers = $12x$.

Given, the LCM – the HCF = 110

$\Rightarrow$ $12x - x = 110 \Rightarrow 11x = 110.$ $\therefore$ $x = 10.$

$\therefore$ the HCF = 10 and the LCM = 120.

$\because$ the product of two numbers = HCF × LCM

$\therefore$ 30 × the other number = HCF × LCM.

$\therefore$ the other number $= \frac{\text{HCF} \times \text{LCM}}{30} = \frac{10 \times 120}{30} = 40.$

Remember These

1. Expressing a number as a product of prime factors is called prime factorization.
2. The HCF of two or more numbers is the greatest number which exactly divides all the numbers.
3. There are two methods of finding the HCF: (i) by prime factorization and (ii) by division.
4. The LCM of two or more numbers is the smallest number which can be divided exactly by all the given numbers.
5. There are two methods of finding the LCM: (i) by prime factorization and (ii) by division.
6. The product of two numbers = the LCM of the numbers × the HCF of the numbers.
7. The HCF of two co-prime numbers = 1.
8. The LCM of two co-prime numbers = the product of the two co-prime numbers.

EXERCISE 2

1. Find the HCF of the following by prime factorization.
   (i) 48, 120 and 216 (ii) 210, 280 and 525
2. Find the HCF of the following.
   (i) 216, 756 and 1260 (ii) 108, 288 and 720
   (iii) 528, 1584 and 2178 (iv) 90, 216, 720 and 1908
3. Find the greatest number that will divide
   (i) 399 and 828 leaving the remainder 9 in each case,
   (ii) 445, 721 and 942 leaving the remainders 5, 6 and 7 respectively.
4. Find the LCM of the following by prime factorization.
   (i) 16, 28, 84 (ii) 132, 165, 220 (iii) 36, 64, 96, 100

5. Find the LCM of the following.
   (i) 4, 8, 21, 30 (ii) 48, 56, 105, 225 (iii) 12, 18, 30, 48, 60
   (iv) 8, 18, 30, 24, 54 (v) 12, 18, 24, 30, 36, 42
6. Find the smallest number which when divided by 18, 28, 108 and 105 leaves the remainder 7 in each case.
7. Find the smallest number which when increased by 5 is exactly divisible by 12, 16, 18, 27 and 36.
8. Find the least number which when decreased by 2 is exactly divisible by 6, 8, 12, 18 and 45.
9. Find the greatest number of six digits which is exactly divisible by 8, 12, 16, 20 and 24.
10. Find the smallest number of six digits which is exactly divisible by 12, 16, 30, 48 and 60.
11. Five bells start ringing together. If the bells ring at intervals of 2, 4, 6, 8 and 10 seconds respectively, after what interval of time will they ring together again?
12. The traffic lights at four different crossings change at intervals of 48, 72, 90 and 108 seconds respectively. If these lights change together at 7:05 p.m., when will they change together again?
13. The LCM and HCF of two numbers are 144 and 24 respectively. Find the product of the two numbers.
14. The product of two numbers is 22176. If their HCF is 12, find their LCM.
15. The LCM and HCF of two numbers are 960 and 16 respectively. If one of the numbers is 768, find the other number.
16. The HCF of two numbers is $\frac{1}{28}$ of their LCM, and the sum of the LCM and the HCF is 116. If one of the numbers is 16, find the other.

ANSWERS

1. (i) 24 (ii) 35 **2.** (i) 36 (ii) 36 (iii) 66 (iv) 18 **3.** (i) 39 (ii) 55
4. (i) 336 (ii) 660 (iii) 14400 **5.** (i) 840 (ii) 25200 (iii) 720 (iv) 1080 (v) 2520
6. 3787 **7.** 427 **8.** 362 **9.** 999840
10. 100080 **11.** 2 minutes **12.** 7:41 p.m. **13.** 3456
14. 1848 **15.** 20 **16.** 28

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Revision Exercise 1

1. Find the largest seven digit number which is exactly divisible by 301.

2. Find the smallest eight digit number which is exactly divisible by 529.

3. Simplify the following.

(i) $3 \text{ of } [125 - 2\{48 \div (6 \times \overline{4-2})\}]$ (ii) $[12 + 12 \div \{12 \div (12 \div \overline{6-2})\}] \div 3$

4. Find the greatest number of five digits which is exactly divisible by 2, 8, 12, 15 and 20.

5. (i) Find the LCM of 66, 84, 112, 165 and 189.

(ii) Find the HCF of 396, 882 and 1404.

6. The HCF of two numbers is $\frac{1}{21}$ of their LCM and the sum of the LCM and the HCF is 308. If one of the numbers is 42, find the other.

7. The product of two numbers is 313404 and their HCF is 14. Find their LCM.

8. Seven bells start ringing together. If the bells ring at the intervals of 4, 6, 12, 15, 18, 20 and 24 seconds respectively, after what interval of time will they ring together?

9. What is the least number of steps in a staircase if Aditya goes up 2 steps at a time, or 3 steps at a time, or 4 steps at a time, there is always one step left at the top?

10. Find the smallest number which when divided by 10, 12, 14, 18, 24, 32 and 36 leaves the remainder 3.

ANSWERS

1. 9999822	**2.** 10000216	**3.** (i) 351 (ii) 5	**4.** 99960
5. (i) 166320 (ii) 18	**6.** 98	**7.** 22386	**8.** 6 minutes
9. 13	**10.** 10083		

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