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POWERS AND ROOTS

- Cubes and Cube Roots

- Table of Squares and Cubes

Introduction

In the previous chapter we have learnt how to find square roots of numbers by prime factorisation as well as division method.

We have learnt how to find cube roots of perfect cubes by prime factorisation method in our previous class.

Cubes and Cube Roots

When a number is multiplied by its square, it is said to be cubed.

Example 1: $5 \times 5^2 = 5 \times 5 \times 5 = 5^3 = 125$

Example 2: $18 \times 18^2 = 18 \times 18 \times 18 = 18^3 = 5832$

When a number 'a' multiplied by its square gives a certain product a^3 , the number a is known as the cube root of the product a^3 .

$\sqrt[3]{a^3} = \sqrt[3]{a \times a \times a} = a$ where the sign ' $\sqrt[3]{\quad}$ ' denotes cube root.

Example 3: Find the cube root of 27000.

Prime factorisation of 27000
 $= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Triples of three identical factors
 $= \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Product of one factor from each triplet gives the cube root.

$$\begin{aligned}\sqrt[3]{27000} &= \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}} \\ &= 2 \times 3 \times 5 = 30\end{aligned}$$

Thus, the cube root of 27000 is 30.

Example 4: Find the cube root of 250047.

Prime factorisation of 250047
 $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

Triples of three identical factors
 $= \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

Product of one factor from each triplet gives the cube root.

$$\begin{aligned}\sqrt[3]{250047} &= \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}} \\ &= 3 \times 3 \times 7 = 63\end{aligned}$$

Thus, the cube root of 250047 is 63.

Try this!

Find the cube root of 117649.

Table of Squares and Cubes

In this chapter, a table showing the squares, cubes, square roots, and cube roots of natural numbers up to 50 is given. A table like this is an 'aid' to make calculations easier just the way an electronic calculator is. It is a 'ready reckoner' that can be consulted whenever calculations involve indices 2, 3, $\frac{1}{2}$, or $\frac{1}{3}$.

This is the table of powers and roots.

Number	Square	Cube	Square Root	Cube Root
n	n^2	n^3	\sqrt{n}	$\sqrt[3]{n}$
1	1	1	1.000	1.000
2	4	8	1.414	1.260
3	9	27	1.732	1.442
4	16	64	2.000	1.587
5	25	125	2.236	1.710
6	36	216	2.449	1.817
7	49	343	2.646	1.913
8	64	512	2.828	2.000
9	81	729	3.000	2.080
10	100	1000	3.162	2.154
11	121	1331	3.317	2.224
12	144	1728	3.464	2.289
13	169	2197	3.606	2.351
14	196	2744	3.742	2.410
15	225	3375	3.873	2.466
16	256	4096	4.000	2.520
17	289	4913	4.123	2.571
18	324	5832	4.243	2.621
19	361	6859	4.359	2.668
20	400	8000	4.472	2.714
21	441	9261	4.583	2.759
22	484	10648	4.690	2.802
23	529	12167	4.796	2.844
24	576	13824	4.899	2.884
25	625	15625	5.000	2.924
26	676	17576	5.099	2.962
27	729	19683	5.196	3.000
28	784	21952	5.292	3.037
29	841	24389	5.385	3.072
30	900	27000	5.477	3.107
31	961	29791	5.568	3.141
32	1024	32768	5.657	3.175
33	1089	35937	5.745	3.208
34	1156	39304	5.831	3.240
35	1225	42875	5.916	3.271
36	1296	46656	6.000	3.302
37	1369	50653	6.083	3.332
38	1444	54872	6.164	3.362
39	1521	59319	6.245	3.391
40	1600	64000	6.325	3.420
41	1681	68921	6.403	3.448
42	1764	74088	6.481	3.476
43	1849	79507	6.557	3.503
44	1936	85184	6.633	3.530
45	2025	91125	6.708	3.557
46	2116	97336	6.782	3.583
47	2209	103823	6.856	3.609
48	2304	110592	6.928	3.634
49	2401	117649	7.000	3.659
50	2500	125000	7.071	3.684

Using the Table

Example 5: Evaluate $\sqrt{38} - \sqrt[3]{38}$.

Run your finger down the column for natural numbers 'n' and stop at 38. Move right along the row of 38 and stop at the column for the square root of n, correct up to 3 decimal places. Read the value of $\sqrt{38}$. Similarly, read the value of $\sqrt[3]{38}$ under the column for $\sqrt[3]{n}$.

$$\sqrt{38} - \sqrt[3]{38} = 6.164 - 3.362 = 2.802$$

Example 6: Evaluate $2\sqrt{19} + 3\sqrt[3]{15}$.

From the table,

$$\sqrt{19} = 4.359 \text{ and } \sqrt[3]{15} = 2.466$$

$$\begin{aligned} \text{Then } 2 \times 4.359 + 3 \times 2.466 \\ = 8.718 + 7.398 = 16.116 \end{aligned}$$

Example 7: Use the table to verify

$$45^3 - 20^3 = (45 - 20)(45^2 + 45 \times 20 + 20^2)$$

Reading the values of the squares and cubes from the given table,

$$91125 - 8000 = 25(2025 + 900 + 400)$$

$$\Rightarrow 83125 = 25 \times 3325$$

$$\Rightarrow 83125 = 83125$$

Thus the given statement is verified.

Example 8: Evaluate $\sqrt[3]{930}$, correct up to 3 decimal places.

$$\sqrt[3]{930} = \sqrt[3]{31 \times 30}$$

$$= \sqrt[3]{31} \times \sqrt[3]{30}$$

$$= 3.141 \times 3.107$$

$$= 9.759087$$

$$= 9.759 \text{ correct up to 3 decimal places}$$

Exercise 10.1

1. Read the values of the following from the table.

(i) 23^2

(ii) 49^2

(iii) 28^2

(iv) 39^2

(v) 47^2

(vi) 19^3

(vii) 27^3

(viii) 36^3

(ix) 41^3

(x) 48^3

(xi) $\sqrt{15}$

(xii) $\sqrt{26}$

(xiii) $\sqrt{29}$

(xiv) $\sqrt{37}$

(xv) $\sqrt{47}$

(xvi) $\sqrt[3]{13}$

(xvii) $\sqrt[3]{31}$

(xviii) $\sqrt[3]{22}$

(xix) $\sqrt[3]{50}$

(xx) $\sqrt[3]{34}$

2. Use the table to verify

$$35^3 + 15^3 = (35 + 15)(35^2 - 35 \times 15 + 15^2)$$

3. Evaluate the following using the table.

(i) $19^2 + 10^3$

(ii) $45^3 - 30^3$

(iii) $12^3 + 9^3 - 11^3$

(iv) $14^3 + 15^3 - 16^3$

(v) $\sqrt{12} + \sqrt{15}$

(vi) $\sqrt{27} - \sqrt{7}$

(vii) $\sqrt[3]{43} + \sqrt[3]{34}$

(viii) $\sqrt[3]{37} - \sqrt[3]{3}$

(ix) $\sqrt{2} + \sqrt[3]{2} - \sqrt[3]{3}$

(x) $\sqrt[3]{49} + \sqrt[3]{50} - \sqrt{50}$

(xi) $\sqrt{57}$

(xii) $\sqrt{430}$

(xiii) $\sqrt[3]{99}$

(xiv) $\sqrt[3]{196}$

Revision Exercise

1. Read the values of the following from the table.

(i) 26^2

(ii) 43^3

(iii) 32^2

(iv) 23^3

(v) $\sqrt{41}$

(vi) $\sqrt{33}$

(vii) $\sqrt[3]{38}$

(viii) $\sqrt[3]{46}$

2. Use the table to verify.

(a) $28^2 - 21^2 = (28 + 21)(28 - 21)$

(b) $48^3 - 23^3 = (48 - 23)(48^2 + 48 \times 23 + 23^2)$

3. Evaluate the following using the table.

(i) $21^2 + 19^2$

(ii) $49^2 - 32^2$

(iii) $16^3 + 8^3 - 12^3$

(iv) $18^3 + 19^3 - 20^3$

(v) $\sqrt{33} + \sqrt{35}$

(vi) $\sqrt[3]{45} - \sqrt[3]{2}$

(vii) $\sqrt{16} - \sqrt{14}$

(viii) $\sqrt[3]{50} + \sqrt[3]{49} - \sqrt[3]{30}$