9 SQUARES AND SQUARE ROOTS

- Square of a Number
- Square Root of a Number
- Square Roots of Perfect Squares
- Square Roots of Non-Perfect Squares

Square of a Number

The square of a number is the number multiplied by itself. The number is called the **base** and as it is multiplied by itself only once, its **power** or **index** is 2.

$a^2 = a \times a$, base = a, power = 2

Example 1: $18^2 = 18 \times 18 = 324$

Example 2: $291^2 = 291 \times 291 = 84681$

Example 3: $\left(\frac{5}{7}\right)^2 = \frac{5}{7} \; \frac{5}{7} = \frac{25}{49}$

Exampls 4: $\left(1\frac{3}{5}\right)^2 = \frac{8}{5} \times \frac{8}{5} = \frac{64}{25} = 2\frac{14}{25}$

Example 5: $(0.79)^2 = 0.79 \times 0.79 = 0.6241$

Example 6: $(1.3)^2 = 1.3 \times 1.3 = 1.69$

Try this!

1. 5^2 =
2. $1\frac{4}{5}^2$ =

Observe from the above examples that

- the square of an even number is an even number.
- the square of an odd number is an odd number.
- the square of a proper fraction is less than the proper fraction.
- the square of an improper fraction is greater than the improper fraction.

Numbers which can be expressed as the square of two exact rational numbers are known as perfect squares.

Example 7: Is 3528 a perfect square?

Express 3528 as a product of its prime factors.

$$3528 = 2\times\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$$

Pairing off the factors we find one factor left unpaired.

$$3528 = 2 \times 42 \times 42$$

Thus, 3528 is not a perfect square.

Example 8: What is the smallest number by which 3528 should be divided to make it a perfect square?

$$3528 = 2 \times 42 \times 42$$

If 3528 is divided by 2, then $1764 = 42 \times 42$, which is a perfect square.

Square Root of a Number

When a number 'a' multiplied by itself gives a certain product a^2, *a is known as the square root of* a^2.

$\sqrt{a^2} = \sqrt{a \times a} = a$, where the radical sign '$\sqrt{\ }$' denotes square root.

Example 9: $\sqrt{81} = \sqrt{9 \times 9} = 9$

Example 10: $\sqrt{400} = \sqrt{20 \times 20} = 20$

Example 11: $\sqrt{\frac{9}{16}} = \sqrt{\frac{3}{4} \times \frac{3}{4}} = \frac{3}{4}$

Example 12: $\sqrt{1\frac{21}{100}} = \sqrt{\frac{121}{100}} = \sqrt{\frac{11}{10} \times \frac{11}{10}} = 1\frac{1}{10}$

Example 13: $\sqrt{0.49} = \sqrt{0.7 \times 0.7} = 0.7$

Example 14: $\sqrt{6.25} = \sqrt{2.5 \times 2.5} = 2.5$

The square roots in all the examples above are exact rational numbers.

Try this!

1. $\sqrt{225}$
2. $\sqrt{1024}$

Square Roots of Perfect Squares by Prime Factorisation Method

Step 1: Obtain the prime factorisation of the given number.

Step 2: Make pairs of identical factors.

Step 3: Find the product of one factor taken from each pair obtained above.

Example 15: Find the square root of 7056.

$$\sqrt{7056} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}}$$
$$= 2 \times 2 \times 3 \times 7$$
$$= 84$$

Example 16: Find the square root of $\sqrt{3\frac{109}{225}}$.

$$\sqrt{3\frac{109}{225}} = \sqrt{\frac{784}{225}} = \sqrt{\frac{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}}{\underline{3 \times 3} \times \underline{5 \times 5}}}$$
$$= \frac{2 \times 2 \times 7}{3 \times 5} = \frac{28}{15} = 1\frac{13}{15}$$

Example 17: Find the square root of 6.76.

$$\sqrt{6.76} = \sqrt{\frac{676}{100}} = \sqrt{\frac{\underline{2 \times 2} \times \underline{13 \times 13}}{\underline{2 \times 2} \times \underline{5 \times 5}}}$$
$$= \frac{2 \times 13}{2 \times 5} = \frac{26}{10} = 2.6$$

Exercise 9.1

1. Find the value of the following.
   (i) 14^2 (ii) 27^2
   (iii) 118^2 (iv) $\left(\frac{1}{5}\right)^2$
   (v) $\left(\frac{3}{8}\right)^2$ (vi) $\left(1\frac{2}{5}\right)^2$
   (vii) $\left(2\frac{4}{5}\right)^2$ (viii) $(0.1)^2$
   (ix) $(5.8)^2$ (x) $(3.05)^2$
2. Which of the following perfect squares will have even square roots and which will have odd square roots?
   (i) 441 (ii) 2916
   (iii) 3969 (iv) 21609
   (v) 389376
3. Which of the following numbers are perfect squares?
   (i) 128 (ii) 256
   (iii) 2187 (iv) 6561
   (v) 6084
4. Find the perfect square obtained by multiplying each of the following numbers by the smallest possible number.
   (i) 882 (ii) 405
   (iii) 567 (iv) 1690
   (v) 7776
5. Divide the following numbers by the smallest possible number to make each one a perfect square.
   (i) 1152 (ii) 2187
   (iii) 3267 (iv) 1536
   (v) 10140

6. Find the square roots of the following natural numbers by the prime factorisation method.
   (i) 225 (ii) 1296
   (iii) 1764 (iv) 2401
   (v) 2916 (vi) 5184
   (vii) 7225 (viii) 10816
   (ix) 14400 (x) 13456
7. Find the square roots of the following fractions by the prime factorisation method.
   (i) $\frac{4}{9}$ (ii) $\frac{100}{121}$
   (iii) $\frac{49}{64}$ (iv) $\frac{484}{625}$
   (v) $\frac{729}{900}$ (vi) $1\frac{9}{16}$
   (vii) $1\frac{11}{25}$ (viii) $1\frac{32}{49}$
   (ix) $10\frac{9}{16}$ (x) $8\frac{1}{36}$
8. Find the square roots of the following decimals by the prime factorisation method.
   (i) 0.0001 (ii) 1.44
   (iii) 3.24 (iv) 7.29
   (v) 10.24 (vi) 30.25
   (vii) 0.0049 (viii) 0.1296
   (ix) 15.21 (x) 0.2025

Square Roots of Perfect Squares by Division Method

In case of large numbers, it is not easy to obtain their prime factorisation. In such cases their square roots are found by a special type of division process.

Example 18: Find the square root of 101124.

Step 1: Pair the digits in periods starting from right to left

i.e., $101124 = \overline{10}\ \overline{11}\ \overline{24}$

Step 2: Consider the first pair (i.e. 10) as the dividend. Determine the largest number, the square of which is less than or equal to 10. Such a number is 3. Write 3 in the quotient as well as in the divisor. Subtract 3 × 3 from 10. The remainder is 1.

Step 3: Bring down the next pair of digits (i.e. 11). The dividend is 111. Double the quotient obtained in step 2 (i.e. 3). The result (6) is the first part of the next divisor. Place the largest possible digit on the right of 6 such that the product of this largest digit with the complete divisor does not exceed 111. Such a digit is 1 as 61 × 1 = 61. Write 1 in the quotient too. The square root obtained till now is 31.

Step 4: Now 31 × 2 = 62 is the first part of the new divisor. The new dividend formed by combining the remainder from step 3 (i.e. 50) and the next pair of digits brought down (i.e. 24) is 5024. Determine, by trial and error, the last digit of the new divisor, such that the product of the complete divisor and this last digit does not exceed 5024. The last digit is 8. Thus, the new divisor is 628, which when multiplied by 8, gives 5024. The remainder is 0.

$101124 = \overline{10}\ \overline{11}\ \overline{24}$ (Step 1)

	318	
3	$\overline{10}\ \overline{11}\ \overline{24}$	
	9	(Step 2)
61	111	
	61	(Step 3)
628	5024	
	5024	(Step 4)
	×	

Thus, $\sqrt{101124} = 318$

Example 19: Find the square root of 464.8336.

Pair the digits in periods, beginning from the decimal point and moving outwards. A digit left unpaired is considered as a period. A decimal point is placed in the quotient (the square root) as soon as the pair after the decimal point in the dividend is brought down.

```
           21 . 56
      ______________
  2   | 4 64 . 83 36
      | 4
      |_____________
 41   |   64
      |   41
      |_____________
 425  |   2383          (Decimal point
      |   2125           is placed in the
      |_____________     quotient at this
 4306 |   25836          stage.)
      |   25836
      |_____________
              ×
```

Thus, $\sqrt{464.8336} = 21.56$

Example 20: Find the square root of $1\frac{720}{961}$.

$$\sqrt{1\frac{720}{961}} = \sqrt{\frac{1681}{961}} = \frac{\sqrt{1681}}{\sqrt{961}}$$

```
        41                  31
     _________           _______
  4 | 16 81          3  | 9 61
    | 16                | 9
    |________           |_______
 81 |    81         61  |   61
    |    81             |   61
    |________           |_______
        ×                   ×
```

Thus $\sqrt{\frac{1681}{961}} = \frac{41}{31}$

$\therefore \quad \sqrt{1\frac{720}{961}} = 1\frac{10}{31}$

Square Roots of Non-Perfect Squares by Division Method

On applying the division method to the square root of a non-perfect square, we will be left with a remainder.

Example 21: What is the smallest number that must be subtracted from 6098 in order to get a perfect square?

Let us try to find the square root of 6098.

```
          78
       ________
   7  | 60 98
      | 49
      |________
 148  | 11 98
      | 11 84
      |________
          14
```

As we are left with 14 as the remainder, if 14 is subtracted from 6098, we would get 6098 – 14 = 6084, a perfect square, as $78^2 = 6084$.

As 6098 = 6098.0000… we can keep bringing down pairs of 0's in the dividend and go on dividing the remainders. However, we can find the square root up to a few decimal places to obtain an approximate square root of 6098 as shown below.

```
                78.08969…
             ______________________
        7   | 6098.00 00 00 00 00…
            | 49
            |______________________
      148   | 1198
            | 1184
            |______________________
     1560   |   1400
            |   0000
            |______________________
    15608   |   140000
            |   124864
            |______________________
   156169   |    1513600
            |    1405521
            |______________________
  1561786   |     10807900
            |      9370716
            |______________________
 15617929   |     143718400
            |     140561361
            |______________________
                    3157039
```

As the digit in the 5th decimal place is 9, the square root can be approximated to 78.0897 such that $(78.0897)^2 \approx 6098$.

Example 22: Find the square root of 7, correct up to 2 decimal places.

In order to approximate the square root of 7 up to 2 decimal places, we have to calculate the square root up to 3 decimal places.

	2.645
2	$7.\overline{00}\,\overline{00}\,\overline{00}$
	4
46	300
	276
524	2400
	2096
5285	30400
	26425
	3975

As the digit in the third decimal place is 5, the square root of 7 is approximated to 2.65, correct up to 2 decimal places such that $(2.65)^2 \approx 7$.

Example 23: Find the square root of $1\frac{4}{5}$, correct up to 3 decimal places.

In order to approximate the square root of $1\frac{4}{5}$ up to 3 decimal places, we have to calculate the square root up to 4 decimal places.

$$\sqrt{1\frac{4}{5}} = \sqrt{1.8}$$

	1.3416
1	$1.\overline{80}\,\overline{00}\,\overline{00}\,\overline{00}$
	1
23	80
	69
264	1100
	1056
2681	4400
	2681
26826	171900
	160956
	10944

As the digit in the fourth decimal place is 6 and 6 > 5, the square root of $1\frac{4}{5}$ is approximated to 1.342, correct up to 3 decimal places such that

$$(1.342)^2 \approx 1.8 \text{ or } 1\frac{4}{5}$$

Example 24: What is the smallest number that must be added to 7190 to get a perfect square?

Let us try to find the square root of 7190.

	84
8	$\overline{71}\,\overline{90}$
	64
164	790
	656
	134

The remainder indicates that $84^2 < 7190 < 85^2$. As we have to add to 7190 in order to get a perfect square we have to add the difference of 85^2 and 7190 or 7225 – 7190 = 35.

Thus if 35 is added to 7190, we get a perfect square or 7225 or 85^2.

Example 25: Find the greatest and the smallest 6-digit numbers that are perfect squares.

The greatest and the smallest 6-digit natural numbers are 999999 and 100000, respectively.

(i)

	999
9	$\overline{99}\,\overline{99}\,\overline{99}$
	81
189	1899
	1701
1989	19899
	17901
	1998

(ii)

	316
3	$\overline{10}\,\overline{00}\,\overline{00}$
	9
61	100
	61
626	3900
	3756
	144

(i) 999999 – 1998 (remainder) = 998001 = 999^2. Thus, 998001 is the greatest 6-digit perfect square.

(ii) The remainder indicates that $316^2 < 100000 < 317^2$. Thus $317^2 = 100489$ is the smallest 6-digit perfect square as $316^2 = 99856$; the perfect square preceding it is a 5-digit number.

Exercise 9.2

1. Find the square roots of the following numbers by the division method.
   (i) 4624 (ii) 11664
   (iii) 16641 (iv) 44521
   (v) 426409 (vi) 550564
   (vii) 840889 (viii) 917764
   (ix) 1014049 (x) 1560001
2. Find the square roots of the following decimal fractions by the division method.
   (i) 34.81 (ii) 44.89
   (iii) 5.4756 (iv) 1.3225
   (v) 4.9284 (vi) 11.2896
   (vii) 4.5796 (viii) 36.2404
   (ix) 1.304164 (x) 4.609609
3. Find the square roots of the following common fractions by the division method.
   (i) $\frac{121}{289}$ (ii) $1\frac{168}{361}$
   (iii) $\frac{676}{729}$ (iv) $\frac{1681}{2209}$
   (v) $\frac{5041}{7921}$
4. Find the square roots of the following numbers, correct up to 3 decimal places.
   (i) 2 (ii) 3
   (iii) 5 (iv) 11
   (v) 13 (vi) 1458
   (vii) 27196 (viii) 14502
   (ix) 1828 (x) 146923
   (xi) 1.6 (xii) 1.09
   (xiii) 2.87 (xiv) 3.99
   (xv) 21.654 (xvi) $\frac{1}{5}$
   (xvii) $1\frac{3}{4}$ (xviii) $\frac{7}{20}$
   (xix) $\frac{7}{8}$ (xx) $1\frac{7}{16}$
5. Find the smallest number that needs to be subtracted from 66671 in order to get a perfect square.
6. Find the smallest number that needs to be subtracted from 1051149 in order to get a perfect square.
7. Find the smallest number that needs to be added to 321270 in order to get a perfect square.
8. Find the smallest number that needs to be added to 1485155 in order to get a perfect square.
9. Find the smallest and greatest 5-digit numbers that are perfect squares.
10. The area of a square is 2.815684 cm^2. Find its length.
11. A farmer has 21126 seedlings, which he intends to plant in an equal number of rows and columns. What is the maximum number of rows of seedlings that can be planted? How many seedlings will be left unplanted?
12. Chairs need to be laid out in an equal number of rows and columns. If there are 1817 chairs in the stadium, find at least how many more chairs need to be brought in.

Revision Exercise

1. Find the value of the following
   (i) 29^2 (ii) $7\frac{4}{5}^{2}\ \frac{2304}{2704}$
   (iii) $(3.6)^2$ (iv) $(7.1)^2$
2. Find the square roots of the following natural numbers by the prime factorization method.
   (i) 7056 (ii) (15876)

(iii) $\frac{16900}{22500}$ (iv) 3.61

3. Find the square roots of the following numbere by the division method.
   (i) 16384 (ii) 92416
   (iii) 0.7569 (iv) $\frac{2304}{2704}$

4. Find the smallest number by which 1260 must be multiplied so that product be comes a perfect square.

5. Find the greatest and the smallest 5 - digit numbers that are perfect squares.