# 8 APPROXIMATION

- Rounding Off Decimal Fractions
- Significant Digits in Decimals
- Approximation to Significant Digits
- Representation of Numbers

## Introduction

Rahul needs to divide a web page into three vertical parts. He wants it to look neat and thus wants the measurements to be perfect. But the web page is 640 pixels wide and $640 \div 3 = 213.3$ pixels. Is it necessary for Rahul to be so accurate? If he rounds off each part to 213 pixels, his page can be divided into 3 parts measuring 213, 214, and 213 pixels. Although the parts are not exactly equal, they are 'approximately equal to' each other, a relationship that is denoted by the symbol '≈'.

## Rounding Off Decimal Fractions

Rounding off a decimal fraction to n decimal places:

1. If the digit in the (n + 1)th decimal place is less than 5, then the digit in the (n + 1)th decimal place, and all the digits after it, are simply omitted.
2. If the digit in the (n + 1)th decimal place is 5 or more than 5, the nth digit is increased by 1 and the digit in the (n + 1)th decimal place, and all the digits after it, are omitted.

***Example 1:*** Convert $\frac{9}{7}$ into a decimal fraction and round off your answer to:

(i) 1 decimal place
(ii) 3 decimal places
(iii) 5 decimal places
(iv) a whole number

$\frac{9}{7} = 1.2857142\ldots$

(i) In order to round off the answer to the tenths or 1 decimal place, we consider the (1 + 1)th decimal place digit. As $8 > 5$, the digit in the tenths place is increased by 1 and all the digits after it are omitted.

Thus $\frac{9}{7} \approx 1.3$

(ii) As the (3 + 1)th decimal place digit is 7 and $7 > 5$, the digit in the thousandths place or 5 is increased by 1 and all the digits after it are omitted.

Thus $\frac{9}{7} \approx 1.286$

(iii) As the digit in the (5 + 1)th decimal place is 4 and $4 < 5$, the digit in the (5 + 1)th decimal place and all the digits after it are omitted.

Thus $\frac{9}{7} \approx 1.28571$

(iv) As the digit in the tenths place is 2 and $2 < 5$, it is omitted and all the digits after it are also omitted.

Thus $\frac{9}{7} \approx 1$

**Try this!**

Convert $\frac{10}{20}$ into a decimal fraction and round off your answer to 4 decimal places.

## Rounding Off to a Specified Unit

We have just learnt how to round off our answers to a specified number of decimal places. But in real life we have to use our common sense and figure out how much to round off, such that our answer makes sense.

1. Raima buys a pair of sunglasses marked at Rs 690.75 selling at a discount of 35%. The shopkeeper charges her Rs 449. Should Raima complain that she has been charged Rs 0.0125 more?
2. A lady wishing to buy a gold ring asks the jeweller the market price of gold. Would it make more sense to the lady if the jeweller quoted the price of gold in kilograms or grams?
3. A traveller consults a railway timetable to find out how much it would cost to travel from Kolkata to Mumbai. Would it help the traveller if the fare chart quoted fares in paise per metre rather than rupees per kilometre?

***Example 2:*** An ice cream vendor buys 60 ice creams for Rs 210 and wishes to earn a 40% profit. At what price should he sell each ice cream?


Cost price per ice cream = 210 ÷ 60 = Rs 3.50

Profit expected per ice cream = $3.5 \times \frac{40}{100}$
= Rs 1.40

Accurate selling price per ice cream
= Rs 3.50 + Rs 1.40 = Rs 4.90

As it would be difficult for all his customers to pay exactly Rs 4.90 per ice cream, the vendor could round off his selling price to a reasonable rate as Rs 5 per ice cream.

***Example 3:*** What is the capacity of a cubical ink-pot that is 0.04 m long?

The calculation of capacity with length in metres would give an answer in kilolitres. But the vessel in question is an ink-pot and not a tanker! Thus it would make more sense to convert the length to centimetres and find the capacity in $cm^3$ or millilitres.

0.04 m = 0.04 × 100 cm = 4 cm

Capacity of ink-pot = $4 \times 4 \times 4 = 64\ cm^3$ or 64 mℓ

## Significant Digits in Decimals

We know that 00002.3 = 2.3 = 2.30000

The significant or meaningful digits in the above numbers are only the digits 2 and 3 on either side of the decimal point that give us an idea of its value and location on the number line.

1. All non-zero digits in a decimal number are significant digits.
   (i) 5.695 has four significant digits.
   (ii) 58.2 has three significant digits.
2. The zeroes between non-zero digits in a decimal number are significant digits.
   (i) 3.01 has three significant digits, being 3, 0, and 1.
   (ii) 5.2001 has five significant digits, being 5, 2, 0, 0, and 1.
3. The zeroes to the left of the first non-zero digit are not significant digits.
   (i) 0.09 has only one significant digit, being 9.
   (ii) 0.00012 has only two significant digits, being 1 and 2.
4. The zeroes to the right of the last non-zero digit may or may not be significant.
   The condition depends on the unit of measurement or the need for approximation.

*Case I: When zeroes to the right of the last non-zero digit are significant*

***Example 4:*** Convert 1.2000 m into centimetre.

As 1.2000 m = 120 cm, in 1.2000 m there are three significant digits, being 1, 2, and 0.

***Example 5:*** Convert 3.6000000 kg into mg.

As 3.6000000 kg = 3600000 mg, in 3.6000000 kg there are 7 significant digits, being 3, 6, and five zeroes.

*Case II: When zeroes to the right of the last non-zero digit are not significant*

***Example 6:*** Express Rs 3.60000 in paise.

As Rs 3.60000 = 360 paise, Rs 3.60000 has only three significant digits, being 3, 6, and 0, the three zeroes after it being insignificant.

***Example 7:*** Given the area of the Arctic Ocean as 13079000 sq. km, express its area in thousand sq. km.

$13079000 \text{ sq. km} \times \frac{1}{1000} = 13079 \text{ thousand sq. km.}$

Thus 13079000 sq km has only 5 significant digits, being 1, 3, 0, 7, and 9, the three zeroes after it being insignificant.

## Approximation to Significant Digits

***Example 8:*** Approximate 0.0003801 to 1 significant digit.

The given number has 4 significant digits, being 3, 8, 0, and 1. To approximate it to 1 significant digit, it will have to be rounded off to its ten-thousandths place where its first significant digit is. As 8 in the hundred-thousandths place is greater than 5, $0.0003801 \approx 0.0004$.

***Example 9:*** Approximate 3.1428571 to 3 significant digits.

The given number has 8 significant digits. To approximate it to 3 significant digits it will have to be rounded off to the hundredths place. As 2 in the thousandths place is less than 5, $3.1428571 \approx 3.14$

## Representation of Numbers

Let us recall how we expressed a decimal number in expanded form in previous classes.

***Example 10:*** Write 5873.1264 in expanded form.

5873.1264 can be written as

$5000 + 800 + 70 + 3 + 0.1 + 0.02 + 0.006 + 0.0004$

or $5 \times 1000 + 8 \times 100 + 7 \times 10 + 3 \times 1 + \frac{1}{10} + \frac{2}{100} + \frac{6}{1000} + \frac{4}{10000}$

or $5 \times 10^3 + 8 \times 10^2 + 70 \times 10^1 + 3 \times 10^0 + 1 \times 10^{-1} + 2 \times 10^{-2} + 6 \times 10^{-3} + 4 \times 10^{-4}$

**Try this!**

Write 5840.183 in expanded form.

Thus, whatever be the place of a digit in the integral or decimal part of a number, its place value can be described as a multiple of a power of 10.

The distance between Earth and Pluto is 575,00,00,000 km. This is too big a number to communicate and remember.

(i) Thus, it is approximated to its significant digits and represented as '575 crore kilometres'.
Or

(ii) All the significant digits are considered and it is written in **scientific notation** as the product of a decimal number less than 10 and a power of 10, i.e., $5.8 \times 10^9$ km.

We know that a millilitre is one-thousandth part of a metre. Scientists and industrialists are very excited about 'nanotechnology' nowadays. Do you know the relationship between a nanometre and a metre?

1 nanometre = 0.000000001 of a metre

or a nanometre is a thousand-millionth part of a metre.

Such a small decimal number is again written in scientific notation as the product of a decimal number less than 10 and a power of 10.

$$1 \text{ nanometre} = 1 \times 10^{-9} \text{ m}$$

***Example 11:*** Write 0.000200100 in scientific notation.

The significant digits in the number are 2, 0, 0, and 1.

The decimal number less than 10 is 2.001 and as a product of power of 10, the scientific notation for 0.000200100 is $2.001 \times 10^{-4}$.

Try to figure out the weight of an electron if it is given in scientific notation as equal to $9.10908 \times 10^{-31}$ kg.

## Exercise 8.1

1. Approximate the following decimal numbers as indicated.

| Sl. No. | Given decimal | Rounded off to: Whole number | Tenths | Hundredths | Thousandths | Ten Thousandths |
|---|---|---|---|---|---|---|
| (i) | 1.07194 | | | | | |
| (ii) | 0.21683 | | | | | |
| (iii) | 3.21495 | | | | | |
| (iv) | 8.99999 | | | | | |
| (v) | $6.\dot{3}$ | | | | | |
| (vi) | $7.0\dot{8}$ | | | | | |
| (vii) | $3.\dot{4}\dot{6}$ | | | | | |
| (viii) | $5.58\dot{6}$ | | | | | |
| (ix) | $7.39\dot{3}$ | | | | | |
| (x) | $1.\dot{6}0\dot{6}$ | | | | | |

2. Approximate the following decimal numbers as indicated.

| Sl. No. | Given decimal | Correct up to: Whole number | 1 decimal place | 2 decimal places | 3 decimal places | 4 decimal places |
|---|---|---|---|---|---|---|
| (i) | 15.69346 | | | | | |
| (ii) | 2.00149 | | | | | |
| (iii) | 0.18375 | | | | | |
| (iv) | 175.29147 | | | | | |
| (v) | $7.\dot{5}$ | | | | | |
| (vi) | $0.1\dot{5}$ | | | | | |
| (vii) | $3.\dot{2}\dot{8}$ | | | | | |
| (viii) | $7.91\dot{8}$ | | | | | |
| (ix) | $0.\dot{5}4\dot{5}$ | | | | | |
| (x) | $0.4\dot{5}4$ | | | | | |

3. Round off 12.5% of Rs 29.30 to the nearest paise.
4. Round off $\frac{1}{6}$ of Rs 289.00 to the nearest rupee.
5. Express 4517 mg to the nearest gram.
6. Express 7.8176238 kg to the nearest gram.
7. Express 8963 mm to the nearest metre.
8. A shopkeeper wishes to earn Rs 119 selling 8 chocolate bars. How much should he sell each chocolate bar for?

9. Write the number of significant digits in the following decimal numbers.
   (i) 1.311 (ii) 22.4
   (iii) 3.07 (iv) 500.01
   (v) 0.007 (vi) 0.10001
   (vii) 3.61007 (viii) 4.020
   (ix) 7.000 (x) 0.88008

## Revision Exercise

1. Approximate 0. 0004567 to 1 significant digit.
2. Write 6245. 3178 in expanded form.
3. Round off 22.5% of Rs 58.50 to the nearest paise.
4. Express 7$\ell$ 54 m$\ell$ to the nearest litre.
5. Write 7. 000 40 23587 in scientific notation.