33 VOLUME AND SURFACE AREA OF CUBOIDS

- Cubes and Cuboids
- Volume
- Total Surface Area
- Lateral Surface Area

Let us first recall the units of volume and their conversions.

$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$
$= 10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$

$1\ dm^3 = 1\ dm \times 1\ dm \times 1\ dm$
$= 10\ cm \times 10\ cm \times 10\ cm$
$= 1000\ cm^3 = 1000000\ mm^3 = 1\ \ell$

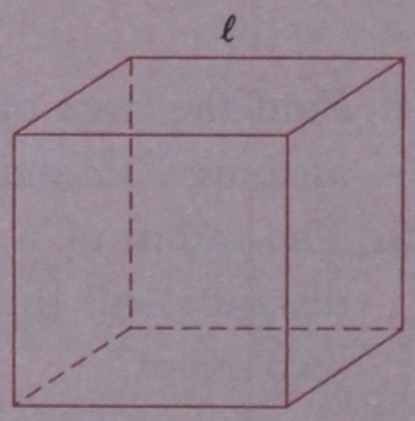
$1\ m^3 = 1\ m \times 1\ m \times 1\ m$
$= 100\ cm \times 100\ cm \times 100\ cm$
$= 1000000\ cm^3 = 1000\ \ell = 1\ k\ell$

$1\ dcm^3 = 10\ m \times 10\ m \times 10\ m = 1000\ m^3$
$= 1000000\ \ell = 1000\ k\ell$

Cubes

Formulae for a cube with length ℓ

- Area of one surface = ℓ^2
- Total surface area of a cube = $6\ell^2$
- Lateral surface area of cube = $4\ell^2$
- Volume of a cube = ℓ^3



Diagonal of a Cube

A cube has four diagonals. The opposite vertices of a cube are joined by the diagonals of the cube. In the cube shown in Figure 33.1, the four diagonals are AG, BH, FD, and EC. Consider diagonal FD. It does not lie in the same plane as any of the six surfaces of the cube.

Consider diagonal DG of square DCGH.

$DG^2 = DH^2 + HG^2 = \ell^2 + \ell^2 = 2\ell^2$

$\Rightarrow DG = \sqrt{2}\ell$

Now consider rectangle AFGD where

$AD = FG = \ell$ and

$GD = AF = \sqrt{2}\ell$

FD is a diagonal of rectangle AFGD.

$FD^2 = DG^2 + FG^2$

$\Rightarrow FD^2 = \left(\sqrt{2}\ell\right)^2 + \ell^2$

$\Rightarrow FD^2 = 2\ell^2 + \ell^2 = 3\ell^2$

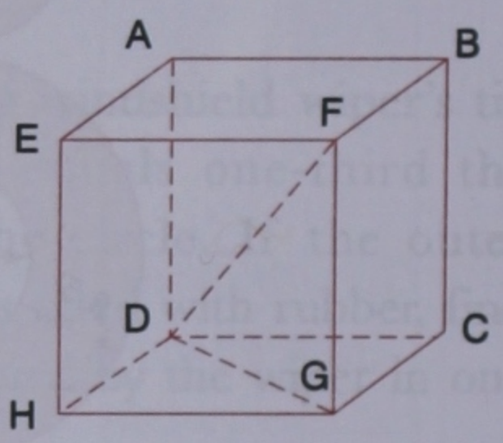


Fig. 33.1

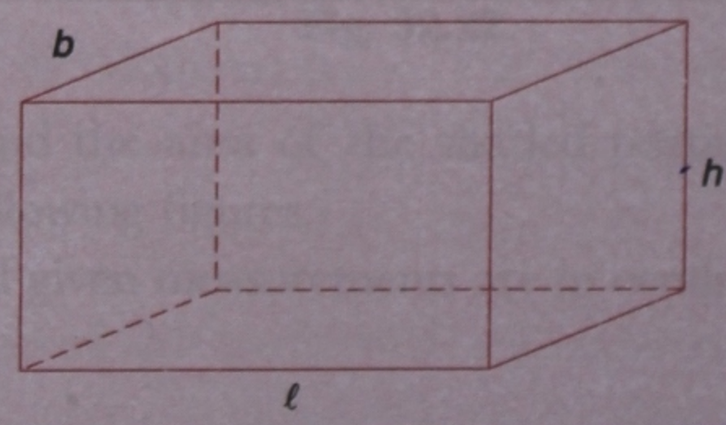
$\Rightarrow FD = \sqrt{3}\ell$

Length of the diagonal of a cube = $\sqrt{3}\ell$

Cuboids

Formulae for a cuboid with length ℓ, breadth b, and height h

- Total surface area of a cuboid = $2(\ell h + \ell b + bh)$
- Lateral surface area of a cuboid = $2h(\ell + b)$
- Volume of a cuboid = $\ell \times b \times h$



Diagonal of a Cuboid

A cuboid has four diagonals. The opposite vertices of a cuboid are joined by the diagonals of a cuboid. In the cuboid shown in Figure 33.2, the four diagonals are AG, BH, CE, and DF.

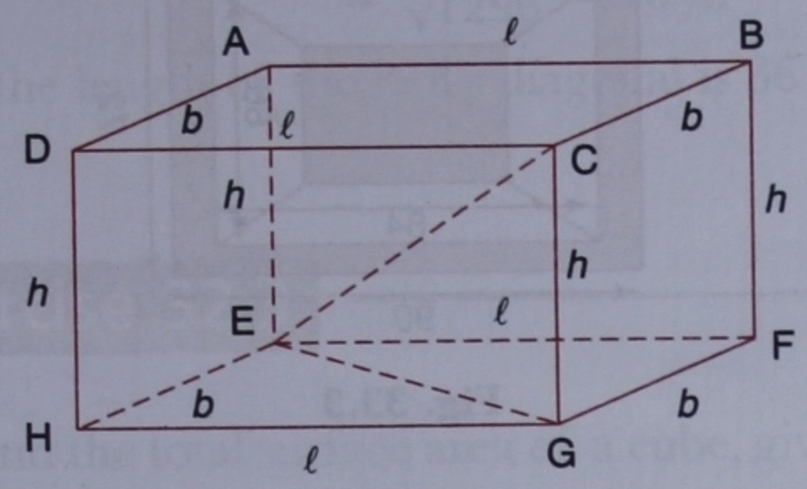


Fig. 33.2

Consider diagonal EG in rectangle EFGH.

$$EG^2 = EH^2 + HG^2 = b^2 + \ell^2$$

$$\Rightarrow \quad EG = \sqrt{b^2 + \ell^2}$$

Now consider diagonal CE in rectangle ACGE.

$$CE^2 = CG^2 + EG^2$$

$$= h^2 + \left(\sqrt{b^2 + \ell^2}\right)^2$$

$$= h^2 + b^2 + l^2$$

$$\Rightarrow \quad CE = \sqrt{\ell^2 + b^2 + h^2}$$

Length of diagonal of a cuboid $= \sqrt{\ell^2 + b^2 + h^2}$

Example 1: If the volume of a cube is 1728 cm^3, find its total surface area and lateral surface area.

Given volume of cube $= \ell^3 = 1728\ cm^3$

$$\Rightarrow \quad \ell = \sqrt[3]{1728}\ cm$$

$$= \sqrt[3]{2\times2\times2\times2\times2\times2\times3\times3\times3}\ cm$$

$$= \sqrt[3]{\underline{2\times2\times3}\times\underline{2\times2\times3}\times\underline{2\times2\times3}}\ cm$$

$$= 2\times2\times3\ cm$$

Thus, the length of the cube is 12 cm.

Total surface area of the cube $= 6\ell^2 = 6\times12^2$

$$= 6\times144$$

$$= 864\ cm^2$$

Lateral surface area of the cube $= 4\ell^2$

$$= 4\times12^2$$

$$= 4\times144$$

$$= 576\ cm^2$$

Example 2: The length, breadth, and height of a cuboid are in the ratio 7 : 4 : 3. If the total surface area of the cuboid is 5978 cm^2, find its volume.

Let the length, breadth, and height of the cuboid be $7x$ cm, $4x$ cm, and $3x$ cm, respectively.

Given total surface area $= 2(\ell b + bh + \ell h)$

$$= 5978\ cm^2$$

$$\Rightarrow \quad 2\{(7x\times4x) + (4x\times3x) + (7x\times3x)\}$$

$$= 5978\ cm^2$$

$$\Rightarrow \quad 2(28x^2 + 12x^2 + 21x^2) = 5978\ cm^2$$

$$\Rightarrow \quad 2\times61x^2 = 5978\ cm^2 \Rightarrow 122x^2 = 5978\ cm^2$$

$$\Rightarrow \quad x^2 = \frac{5978}{122} = 49\ cm^2 \Rightarrow x = \sqrt{49} = 7\ cm$$

Thus, the length $= 7\times7 = 49$ cm,

breadth $= 4\times7 = 28$ cm,

and height $= 3\times7 = 21$ cm

Volume of cuboid $= 49\ cm\times28\ cm\times21\ cm$

$$= 28812\ cm^3 = 28.812\ dm^3$$

Example 3: A conference hall is 35.5 m long, 19.4 m wide, and 6 m high. Its ceiling is covered with sound absorbing material. Find out how much it would cost to:

(i) cover its floor with a wall-to-wall carpet at Rs 78.50 per sq. m

(ii) paint its walls at Rs 47.50 per sq. m

Area of floor of hall

$$= \text{length}\times\text{breadth} = 35.5\ m\times19.4\ m$$

$$= 688.7\ m^2$$

At Rs 78.50 per sq. m, covering the entire floor with carpet would cost

$$688.7\times78.50 = \text{Rs } 54062.95$$

The lateral surface area of the hall or the area of its four walls =

$$2h(\ell + b) = 2\times6(35.5 + 19.4)\ m^2$$

$$= 12\times54.9 = 658.8\ m^2$$

Painting the walls of the hall at Rs 47.50 per m^2 would cost

$$= 47.50\times658.8 = \text{Rs } 31293.00$$

Example 4: A car mechanic, wishing to collect distilled water, set up a tray 1 metre in length and breadth on the roof top and connected a pipe to drain the tray in a cuboidal tin below that was

50 cm long, 30 cm wide, and 30 cm high. If 3 cm of rain fell during the day, what was the height of the water that collected in the mechanic's tin?

Rainfall is recorded in terms of height of water collected, irrespective of how wide or narrow a container is.

The area of the tray on the roof

$= 100 \text{ cm} \times 100 \text{ cm} = 10000 \text{ cm}^2$

Height of rainwater collected in tray = 3 cm

Thus, volume of rainwater collected

$= 3 \times 10000 = 30000 \text{ cm}^3$ or $30\ \ell$

When $30\ \ell$ of water is drained into the empty tin, volume of water in tin $= \ell \times b \times h = 30000 \text{ cm}^3$

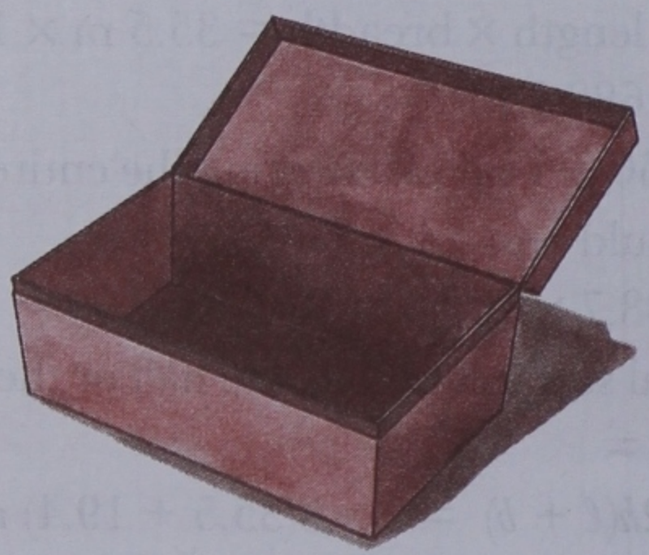
$\Rightarrow \quad 50 \text{ cm} \times 30 \text{ cm} \times h = 30000 \text{ cm}^3$

$\Rightarrow \quad h \times 1500 \text{ cm}^2 = 30000 \text{ cm}^3$

$\Rightarrow \quad h = \dfrac{30000}{1500} = 20 \text{ cm}$

Thus, the height of the water collected in the mechanic's tin = 20 cm

Example 5: The exterior of an empty wooden box measures 90 cm in length, 72 cm in breadth, and 60 cm in height. If the wood is 3 cm thick all around, find the volume of wood used to make the box. If 1 cc of wood weighs 0.09 g find the weight of the empty wooden box.



Volume of the outer cuboid

$= 90 \text{ cm} \times 72 \text{ cm} \times 60 \text{ cm} = 388800 \text{ cm}^3$

If the open box was seen from top it would look like the figure shown in Figure 33.3 As the wood is 3 cm thick on all sides, the length of the inner cuboid

$= 90 \text{ cm} - 3 \text{ cm} - 3 \text{ cm} = 84 \text{ cm}$

Breadth of the inner cuboid

$= 72 \text{ cm} - 3 \text{ cm} - 3 \text{ cm} = 66 \text{ cm}$

Similarly as the top and bottom of the box are also 3 cm thick,

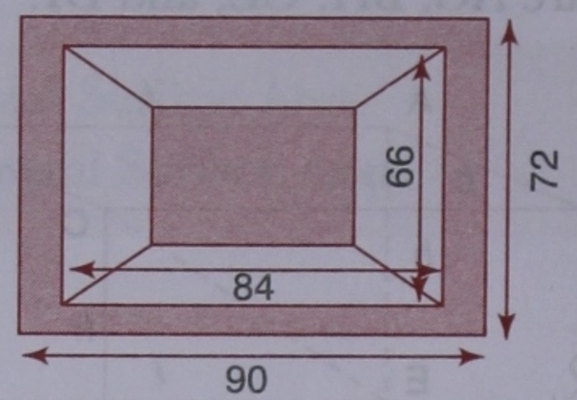


Fig. 33.3

height of the inner cuboid

$= 60 \text{ cm} - 3 \text{ cm} - 3 \text{ cm} = 54 \text{ cm}$

Thus, the volume of the inner cuboid

$= 84 \text{ cm} \times 66 \text{ cm} \times 54 \text{ cm} = 299376, \text{ cm}^3$

Volume of wood = Volume of outer cuboid

– Volume of inner cuboid

$= 388800 - 299376 = 89424 \text{ cm}^3$

Thus, 89424 cm^3 of wood was used to make the box.

Given 1 cm^3 of wood weighs 0.09 g,

89424 cm^3 of wood weighs

$89424 \times 0.09 = 8048.16$ g

Thus, the weight of the empty wooden box is 8 kg 48.16 g.

Example 6: The length, breadth, and height of a hall are in the ratio 2 : 2 : 1. If the lateral surface area of the hall is 1152 m^2, find the length of its diagonal.

Lateral surface area $= 2h(\ell + b)$

Let the length, breadth, and height of the hall be $2x$, $2x$, and x respectively.

$\Rightarrow \quad (2 \times x)(2x + 2x) = 1152 \text{ m}^2$

$\Rightarrow \quad 2x \times 4x = 1152 \text{ m}^2$

$\Rightarrow \quad 8x^2 = 1152 \text{ m}^2$

$\Rightarrow \quad x = \sqrt{\dfrac{1152}{8}} = 12 \text{ m}$

Thus, length $= 2 \times 12 = 24$ m, breadth = 24 m, and height = 12 m.

Length of diagonal $= \sqrt{\ell^2 + b^2 + h^2}$

$= \sqrt{24^2 + 24^2 + 12^2}$

$= \sqrt{576 + 576 + 144}$

$= \sqrt{1296} = 36$ m

Thus, the length of the hall's diagonal is 36 m.

Try this!

1. Find the volume of a cube, whose length measures 7 cm.
2. Find the volume of a cuboid, given its respective length, breadth and height as 5 cm, 3 cm and 2 cm.

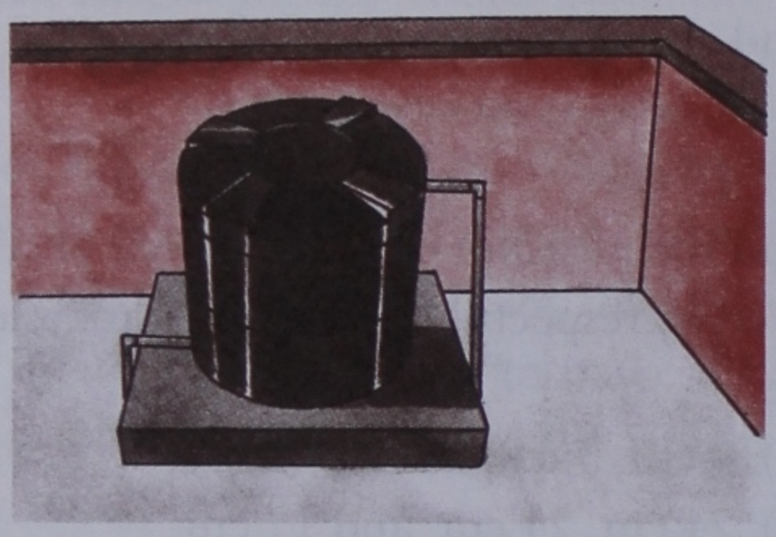
Exercise 32.3

1. Find the total surface area of a cube, given that its length is:
   (i) 6 cm (ii) 7 cm
   (iii) 8.5 cm (iv) 11.4 cm
2. Find the total surface area of a cuboid, given the length, breadth, and height as
   (i) 5 cm, 3 cm, and 2 cm
   (ii) 8 cm, 6 cm, and 5 cm
   (iii) 5.5 cm, 3.4 cm, and 2 cm
   (iv) 7.2 cm, 4.5 cm, and 2.4 cm
3. Find the lateral surface area of a cube, given that its length is:
   (i) 4 cm (ii) 9 cm
   (iii) 6.5 cm (iv) 10.7 cm
4. Find the lateral surface area of a cuboid, given its respective length, breadth, and height as:
   (i) 6 cm, 4 cm, and 2 cm
   (ii) 9 cm, 7 cm, and 5 cm
   (iii) 7.8 cm, 3.2 cm, and 5 cm
   (iv) 9.7 cm, 4.8 cm, and 3.5 cm
5. Find the volume of a cube, given that its length is:
   (i) 6 cm (ii) 9 cm
   (iii) 1.1 cm (iv) 3.5 cm
6. Find the volume of a cuboid, given its respective length, breadth, and height as:
   (i) 10 cm, 7 cm, and 4 cm
   (ii) 12 cm, 9 cm, and 3.5 cm
   (iii) 8.25 cm, 5.4 cm, and 3.2 cm
   (iv) 17.6 cm, 12.2 cm, and 7.5 cm
7. Find the length of a cube, given its total surface area is:
   (i) 150 cm^2 (ii) 486 cm^2
8. Find the length of a cube, given its lateral surface area is:
   (i) 144 cm^2 (ii) 484 cm^2
9. If the volume of a cube is 512 cm^3, find its total surface area and its lateral surface area.
10. If the volume of a cube is 2744 cm^3, find its total surface area and its lateral surface area.
11. Find the measure of the diagonal of a cube, given its total surface area is:
    (i) 18 cm^2 (ii) 54 cm^2
12. Find the measure of the diagonal of a cuboid, given its respective length, breadth, and height are:
    (i) 9 cm, 3 cm, and $\sqrt{10}$ cm
    (ii) 24 cm, 24 cm, and 12 cm
13. A room is 15.5 m long, 10.8 m wide, and 4.5 m high. Find how much it would cost to:
    (i) cover the floor with tiles at Rs 64 per m^2.
    (ii) plaster the ceiling at Rs 18 per m^2.
    (iii) paint the walls at Rs 45 per m^2.

14. A hall has 4 windows, each 4 m wide and 2 m high, 2 doors, each 2.5 m tall and 2 m wide

and 3 wall closets, each 2.5 m tall and 1 m wide. If the hall is 20 m long, 18 m wide, and 6 m high, find the cost of painting its walls at Rs 38 per m^2.

15. The area of the floor of an empty room that is 4 m high, is 108 m^2. Find the volume of air in the room.
16. The length, breadth, and height of a cuboid are in the ratio 6 : 5 : 3. If the total surface area of the cuboid is 2016 cm^2, find its volume.
17. The length, breadth, and height of a cuboid are in the ratio 8 : 5 : 3. If the lateral surface area of the cuboid is 702 cm^2, find its volume.
18. An overhead tank 5.6 m long, 2.5 m wide, and 2 m high is full of water. The entire water is drained into an empty underground reservoir 10 m long and 8 m wide. What is the height of the water in the reservoir now?



19. A gold bar that measures 20 cm in length, 10 cm in breadth and 10 cm in height is melted and recast as gold biscuits 10 cm in length, 5 cm in breadth, and 2.5 cm thick. How many gold biscuits were made from the bar of gold?
20. 8 cm of rainfall was recorded after a thundershower. How much water fell on a farmer's 5 hectare field?
21. Rainwater is collected in a tray 1.5 m long and 1 m wide and then drained into a tin 50 cm long and 40 cm wide. If the rainfall recorded was 4 cm, what was the height of the rainwater that collected in the tin?
22. The total surface area of a tank in which the length, breadth, and height are in the ratio 2 : 2 : 1 is 5.76 m^2. Find the length of the tank's diagonal.
23. The diagonal of an empty cube-shaped tank measures 5.19 m. Given $\sqrt{3} = 1.73$, how many kilolitres of water can be filled in the tank?
24. A wooden box is 140 cm long, 100 cm wide, and 80 cm high. If the wood is 5 cm thick on all sides of the box, find the volume of wood used to make the box.
25. The exterior of an empty steel safe measures 54 cm in length, 39 cm in width, and 49 cm in height. If the steel is 2 cm thick all around find:
    (i) the volume of air inside the safe.
    (ii) the volume of steel used to make the safe.
    (iii) the weight of the empty safe, given that 1 cm^3 of the steel weighs 7.5 g.

Revision Exercise

1. If the volume of a cube is 4913 cm^3, find its total surface area and its lateral surface area.
2. Find the measure of the diagonal of a cube, given its total surface area is:
    (i) 72 cm^2 (ii) 108 cm^2
3. Find the measure of the diagonal of a cuboid given its respective length, breadth and height are: 36 cm, 36 cm and 24 cm.
4. The area of the floor of an empty room that is 6 m high, is 114 m^2. Find the volume of air in the room.
6. The length, breadth and height of a room are in the ratio 5 : 5 : 3. If the lateral surface area of the room is 7260 m^2, find the length of its diagonal.