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Volume and Surface Area of Cuboids

Definitions and units

1. The volume of a solid figure is the space occupied by it. Volume is measured in cubic units. The common units of volume and the corresponding units of length are given in the following table.

Table 2.1 Units of length and volume

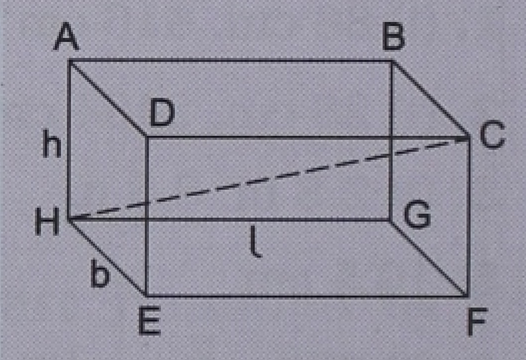
Unit of length	Unit of volume
mm	cubic mm (mm^3)
cm	cubic cm (cm^3)
m	cubic m (m^3)

Note The litre (L) is a unit commonly used for measuring the capacity of vessels or the volume of a liquid.
1 L = 1 cubic decimetre (dm^3) = 1000 cm^3, 1 mL = 1 cm^3 or 1 cc

2. The surface area of a solid is the sum of the areas of the plane or curved faces of the solid. It is measured in square units, such as the square centimetre (cm^2) and square metre (m^2).

Cuboid

A cuboid is a solid figure bounded by six rectangular faces. The adjacent faces are mutually perpendicular and the opposite faces have the same dimensions. A cuboid has eight vertices (*A, B, C, D, E, F, G, H*) and 12 edges (*AB, BC, CD, DA, EF, FG, GH, HE, AH, DE, CF, BG*).



A cuboid

The volume of a cuboid is the product of its length, breadth and height. Denoting the volume, length, breadth and height by *V, l, b* and *h* respectively, we have

$$V = l \times b \times h, \quad l = \frac{V}{bh}, \quad b = \frac{V}{lh} \quad \text{and} \quad h = \frac{V}{lb}$$

The surface area of a cuboid is the sum of the surface areas of its six rectangular faces, which works out to the following.

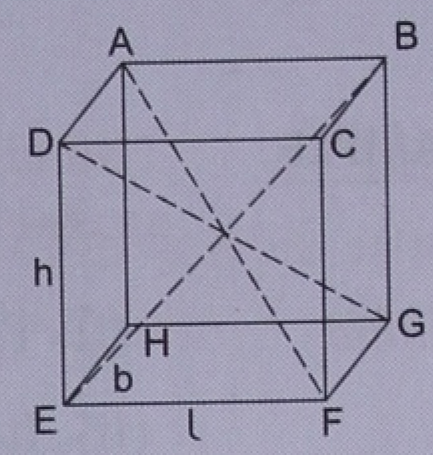
$$\text{The surface area of a cuboid} = 2(lb + bh + hl)$$

The lateral surface area or the area of the four walls of a cuboid works out to.

The area of the four walls $= 2(l+b)h =$ perimeter of the floor × height

Diagonal of a cuboid

A diagonal of a cuboid is a line segment joining two vertices which are not on the same face. A cuboid has four diagonals (also called principal diagonals), namely *HC*, *AF*, *BE* and *DG*. All these diagonals are equal in length. Let us find the length of *HC*.



A cuboid has 4 diagonals.

In the rectangle *HFCA*, *HC* is the diagonal.

$\therefore \quad HC^2 = HF^2 + CF^2.$

Now, *HF* is the diagonal of the rectangle *EFGH*.

$\therefore \quad HF^2 = EF^2 + EH^2 = l^2 + b^2.$

So, $HC^2 = l^2 + b^2 + CF^2 = l^2 + b^2 + h^2.$

$\therefore \quad HC = \sqrt{l^2 + b^2 + h^2}.$

The length of a diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$

EXAMPLE **The dimensions of a cuboid are 10 cm by 9.5 cm by 8 cm. Find (i) its volume, (ii) its surface area, (iii) the surface area of the four walls, and (iv) the length of a diagonal.**

Solution Here, $l = 10$ cm, $b = 9.5$ cm, $h = 8$ cm.

(i) The volume of the cuboid $= l \times b \times h = 10 \times 9.5 \times 8 \text{ cm}^3 = 760 \text{ cm}^3$.

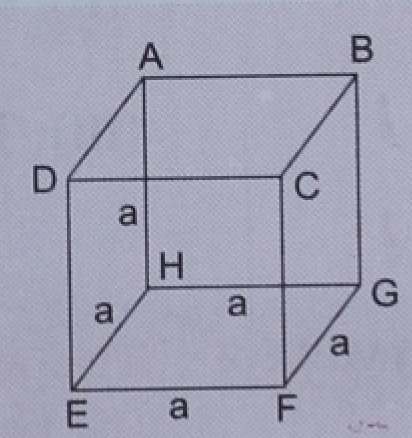
(ii) Its surface area $= 2(lb + bh + hl) = 2(10 \times 9.5 + 9.5 \times 8 + 8 \times 10) \text{ cm}^2$
$= 2 \times 251 \text{ cm}^2 = 502 \text{ cm}^2$.

(iii) The surface area of the four walls $= 2(l+b)h = 2(10+9.5) \times 8 \text{ cm}^2 = 312 \text{ cm}^2$.

(iv) The length of a diagonal $= \sqrt{l^2 + b^2 + h^2} = \sqrt{10^2 + 9.5^2 + 8^2}$ cm
$= \sqrt{254.25}$ cm = 15.9 cm (approximately).

Cube

A cube is a solid bounded by six square faces. Its adjacent faces are perpendicular to each other and all its 12 edges are equal in length. So, a cube is a cuboid in which length = breadth = height.



A cube

The voume of a cube is the cube of the length of its side. Denoting the volume of a cube by *V* and its length by *a*,

$$V = a^3 \quad \text{and} \quad a = \sqrt[3]{V}$$

The surface area of a cube is the sum of the areas of its six square faces or $6 \times (\text{length of an edge})^2$. Denoting the surface area by S,

$$S = 6a^2 \quad \text{and} \quad a = \sqrt{\frac{1}{6}S}$$

The area of the four walls (lateral surface area) = $4 \times (\text{length of an edge})^2 = 4a^2$

Since a cube is a special cuboid in which $l = b = h = a$, the length of a diagonal of a cube $= \sqrt{l^2 + b^2 + h^2} = \sqrt{3a^2} = \sqrt{3}a$.

The length of a diagonal $= \sqrt{3}\,a$

EXAMPLE **The side of a cube is 6 cm. Find (i) its volume, (ii) its surface area, (iii) its lateral surface area, and (iv) the length of its diagonal.**

Solution (i) The volume of the cube = $(\text{length of a side})^3 = (6 \text{ cm})^3 = 216 \text{ cm}^3$.

(ii) Its surface area $= 6 \times (\text{length of a side})^2 = 6 \times (6 \text{ cm})^2 = 216 \text{ cm}^2$.

(iii) The lateral surface area of the cube $= 4 \times (\text{length of a side})^2 = 4 \times (6 \text{ cm})^2$
$= 144 \text{ cm}^2$.

(iv) The length of a diagonal $= \sqrt{3} \times$ length of a side $= 6\sqrt{3}$ cm.

Solved Examples

EXAMPLE 1 **The dimensions of a cuboid are in the ratio 4 : 5 : 6 and its surface area is 5328 m^2. Find (i) its length, breadth and height, (ii) its volume and (iii) the length of a diagonal.**

Solution Given that length : breadth : height = 4 : 5 : 6.

Let $\frac{l}{4} = \frac{b}{5} = \frac{h}{6} = x$ m (say) or $l = 4x$ m, $b = 5x$ m and $h = 6x$ m.

Then the surface area of the cuboid $= 2(lb + bh + hl)$

$= 2(4x \times 5x + 5x \times 6x + 6x \times 4x) \text{ m}^2$

$= 2(20x^2 + 30x^2 + 24x^2) \text{ m}^2 = 148x^2 \text{ m}^2$.

Given, $148x^2 = 5328$ or $x^2 = \frac{5328}{148} = 36$ or $x = 6$.

(i) $\therefore$ length $= 4 \times 6$ m = 24 m, breadth $= 5 \times 6$ m = 30 m, height $= 6 \times 6$ m = 36 m.

(ii) The volume of the cuboid $= l \times b \times h = 24 \times 30 \times 36 \text{ m}^2 = 25920 \text{ m}^2$.

(iii) The length of a diagonal $= \sqrt{l^2 + b^2 + h^2} = \sqrt{24^2 + 30^2 + 36^2}$ m
$= \sqrt{2772}$ m $= 6\sqrt{77}$ m.

EXAMPLE 2 **The surface area of a cube is 294 cm^2. Find (i) the length of an edge of the cube, (ii) the volume of the cube, and (iii) the length of a diagonal.**

Solution (i) Let the length of an edge of the cube = a cm.

Then the surface area of the cube $= 6a^2 \text{ cm}^2 = 294 \text{ cm}^2$ (given).

$\therefore \quad a^2 = \frac{294}{6} = 49$ or $a = 7$.

So, the length of an edge of the cube = 7 cm.

(ii) The volume of the cube $= (7 \text{ cm})^3 = 343 \text{ cm}^3$.

(iii) The length of a diagonal $= \sqrt{3} \times a$ cm $= 1.732 \times 7$ cm = 12.124 cm.

EXAMPLE 3 **The length of a diagonal of a cube is $11\sqrt{3}$ cm. Find (i) the length of an edge of the cube, (ii) the volume of the cube, and (iii) the surface area of the cube.**

Solution (i) Let the length of each edge = a m.

Then the length of a diagonal = $\sqrt{3}a$ cm = $11\sqrt{3}$ cm (given). $\therefore$ $a = 11$.

So, the length of each edge of the cube = 11 cm.

(ii) The volume of the cube = $(11)^3$ cm^3 = 1331 cm^3.

(iii) The surface area of the cube = 6×11^2 cm^2 = 726 cm^2.

EXAMPLE 4 **A hall 8 m long, 6 m wide and 4 m high has three doors of size 1.5 m by 2 m and four windows of size 1.2 m by 1 m. Find the cost of papering the walls if the wall paper is 80 cm wide and costs ₹ 7 per metre.**

Solution Here, $l = 8$ m, $b = 6$ m and $h = 4$ m.

$\therefore$ the area of the walls of the hall = lateral surface area

$= 2(l + b)h = 2(8 + 6) \times 4 \text{ m}^2 = 112 \text{ m}^2$.

The area of three doors = $3 \times (1.5 \times 2)$ m^2 = 9 m^2.

The area of four windows = $4 \times (1.2 \times 1)$ m^2 = 4.8 m^2.

$\therefore$ the area to be papered = $[112 - (9 + 4.8)]$ m^2 = $(112 - 13.8)$ m^2 = 98.2 m^2.

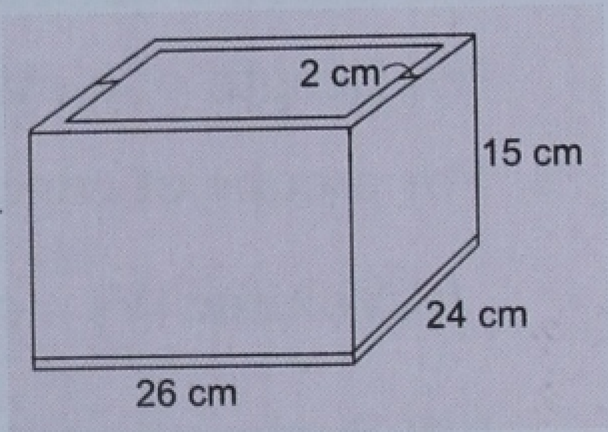
The width of the paper = 80 cm = 0.8 m.

$\therefore$ the length of paper required = $\frac{98.2}{0.8}$ m = $\frac{982}{8}$ m = 122.75 m.

The cost of papering the walls = ₹ 7×122.75 = ₹ 859.25.

EXAMPLE 5 **The external dimensions of an open wooden box are 26 cm by 24 cm by 15 cm. If the wood is 2 cm thick, find (i) the internal dimensions of the box, (ii) the capacity of the box, (iii) the volume of the wood used in making the box, (iv) the cost of making the box at the rate of ₹ 2/cm^3, and (v) the weight of the box if 1 cm^3 of wood weighs 0.75 g.**

Solution (i) The internal dimensions of the box are as follows.



The internal length = 26 cm − 2×2 cm = 22 cm.

The internal breadth = 24 cm − 2×2 cm = 20 cm.

The internal height = 15 cm − 2 cm = 13 cm

($\because$ the box is open at the top).

(ii) The capacity of the box = the internal volume of the box

$= 22 \times 20 \times 13 \text{ cm}^3 = 5720 \text{ cm}^3$.

(iii) The volume of wood = the external volume of the box − the internal volume of the box

$= (26 \times 24 \times 15 - 5720) \text{ cm}^3$

$= (9360 - 5720) \text{ cm}^3 = 3640 \text{ cm}^3$.

(iv) The cost of making the box = ₹ 2×3640 = ₹ 7280.

(v) The weight of the box = the weight of the wood

$= 3640 \times 0.75$ g = 2730 g = 2 kg 730 g.

EXAMPLE 6 **The dimensions of a cuboid are equal to half the dimensions of another cuboid. Find the ratio of the (i) volumes, (ii) surface areas of the two cuboids.**

Solution Let the dimensions of the bigger cuboid be length = l, breadth = b, height = h.

Then those of the smaller cuboid are length = $\frac{1}{2}l$, breadth = $\frac{1}{2}b$, height = $\frac{1}{2}h$.

(i) The volume of the bigger cuboid = lbh.

The volume of the smaller cuboid = $\frac{1}{2}l \times \frac{1}{2}b \times \frac{1}{2}h = \frac{1}{8}lbh$.

$$\therefore \quad \frac{\text{volume of the smaller cuboid}}{\text{volume of the bigger cuboid}} = \frac{\frac{1}{8}lbh}{lbh} = \frac{1}{8} = 1:8.$$

(ii) The surface area of the bigger cuboid = $2(lb + bh + hl)$.

The surface area of the smaller cuboid = $2\left(\frac{1}{2}l \times \frac{1}{2}b + \frac{1}{2}b \times \frac{1}{2}h + \frac{1}{2}h \times \frac{1}{2}l\right)$

$= 2(lb + bh + hl) \times \frac{1}{4}$.

$$\therefore \quad \frac{\text{surface area of the smaller cuboid}}{\text{surface area of the bigger cuboid}} = \frac{2(lb + bh + hl) \times \frac{1}{4}}{2(lb + bh + hl)} = \frac{1}{4} = 1:4.$$

Remember These

1. For a cuboid of length l, breadth b and height h:

(i) Volume $(V) = l \times b \times h$, $l = \frac{V}{b \times h}$, $b = \frac{V}{l \times h}$, $h = \frac{V}{l \times b}$

(ii) Surface area $(S) = 2(lb + bh + hl)$

(iii) Surface area of the four walls (lateral surface area) = $2(l + b)h$

(iv) Length of a diagonal = $\sqrt{l^2 + b^2 + h^2}$

2. For a cube of edge a:

(i) Volume $(V) = a^3$, $a = \sqrt[3]{V}$ (ii) Surface area $(S) = 6a^2$, $a = \sqrt{\frac{1}{6}S}$

(iii) Surface area of the four walls = $4a^2$ (iv) Length of a diagonal = $\sqrt{3}a$

EXERCISE 2

1. Find the volume, surface area, lateral surface area and length of a diagonal of a cuboid with dimensions:

(i) 8 cm by 6 cm by 5 cm (ii) 12 cm by 9 cm by 8 cm (iii) 16 cm by 12 cm by 10 cm

2. Find the volume, surface area, lateral surface area and length of a diagonal of a cube of edge:

(i) 7 cm (ii) 10 cm (iii) 4 cm

3. The length and breadth of a cuboid are 14 cm and 8 cm respectively. If its volume is 672 cm^3, find (i) the height, and (ii) the surface area of the cuboid.

4. The length, breadth and height of a cuboid are in the ratio 4 : 3 : 2.
   (i) If its surface area is 2548 m^2, find its volume and the length of a diagonal.
   (ii) If its volume is 3000 cm^3, find its surface area and lateral surface area.

5. The surface area of a cube is 216 cm^2. Find (i) the length of an edge and (ii) the volume of the cube.

6. The length of a diagonal of a cube is $9\sqrt{3}$ cm. Find its volume and surface area.

7. How many bricks, each 25 cm by 15 cm by 8 cm, are required for a wall 32 m long, 3 m high and 40 cm thick?

8. A room is 6 m long, 5 m broad and 4 m high. It has two doors, each measuring 1.5 m by 2 m, and four windows, each measuring 1 m by 1 m. Find the cost of distempering the walls at ₹ 25/m^2.

9. The external dimensions of a closed cuboidal vessel are 95 cm by 86 cm by 56 cm and the thickness of the material is 3 cm. How many litres will the vessel hold? (1 litre = 1000 cm^3)

10. The external dimensions of an open wooden box are 45 cm by 39 cm by 24 cm. If the wood is 2.5 cm thick, find (i) the capacity of the box, (ii) the volume of the wood, (iii) the cost of making the box at the rate of ₹ 1 per cm^3, and (iv) the weight of the box if 1 cm^3 of the wood weighs 0.65 g.

11. Find the volume of the wood required to make an open box of internal dimensions 20 cm by 12.5 cm by 9.5 cm, the wood being 1.5 cm thick.

12. A room contains 750.4 m^3 of air and its height is 4 m. Find the area of the floor.

13. Find the ratio of the volumes of two cubes if the length of an edge of the bigger cube is thrice the length of an edge of the smaller cube.

ANSWERS

1. (i) 240 cm^3, 236 cm^2, 140 cm^2, $5\sqrt{5}$ cm (ii) 864 cm^3, 552 cm^2, 336 cm^2, 17 cm (iii) 1920 cm^3, 944 cm^2, 560 cm^2, $10\sqrt{5}$ cm

2. (i) 343 cm^3, 294 cm^2, 196 cm^2, $7\sqrt{3}$ cm (ii) 1000 cm^3, 600 cm^2, 400 cm^2, $10\sqrt{3}$ cm (iii) 64 cm^3, 96 cm^2, 64 cm^2, $4\sqrt{3}$ cm

3. (i) 6 cm (ii) 488 cm^2

4. (i) 8232 m^3, $7\sqrt{29}$ m (ii) 1300 cm^2, 700 cm^2

5. (i) 6 cm (ii) 216 cm^3

6. 729 cm^3, 486 cm^2

7. 12800

8. ₹ 1950

9. 356 L

10. (i) 29240 cm^3 (ii) 12880 cm^3 (iii) ₹ 12880 (iv) 8 kg 372 kg

11. 1546.5 cm^3

12. 187.6 m^2

13. 27 : 1

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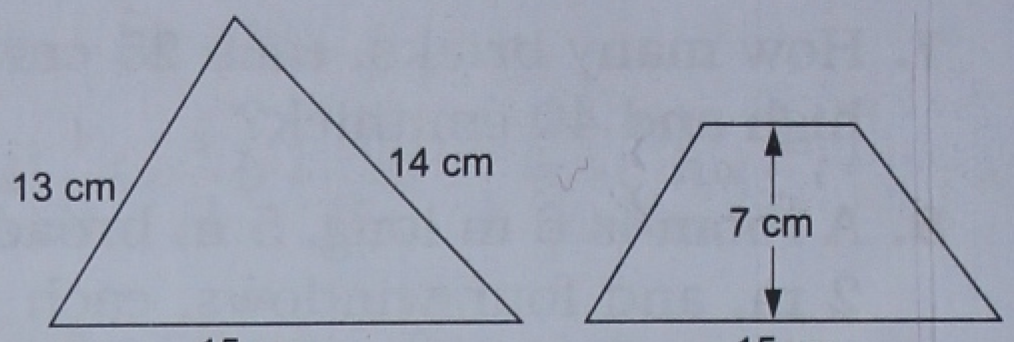
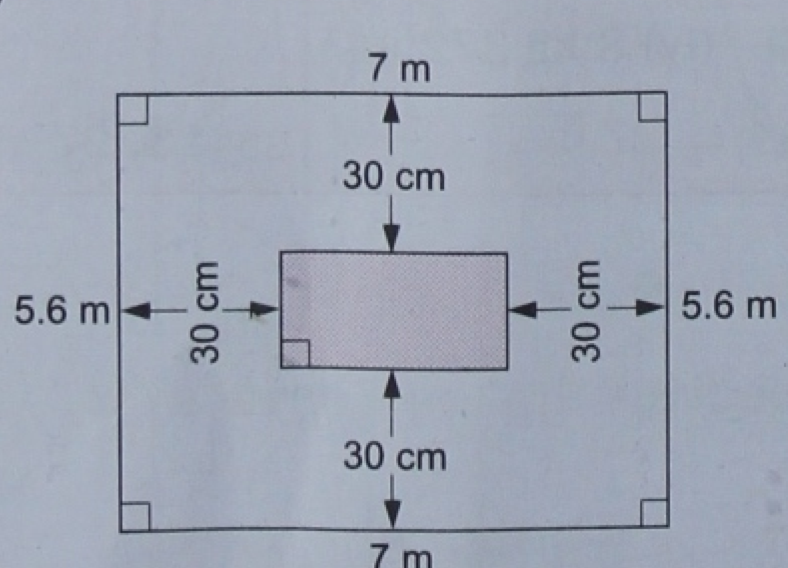
Revision Exercise

1. The ratio of the base of a triangle to the corresponding altitude is 4 : 3 and its area is 1350 cm^2. Find the length of the altitude.
2. A square and a rectangle have the same perimeter of 80 cm. If the difference between their areas is 100 cm^2, find the sides of the rectangle.
3. A path of uniform width 2 m surrounds a lawn of dimensions 10 m $\times$ 8 m. Find the area of the path.
4. In the adjoining figure, a triangle and a trapezium have an equal base and an equal area. Find the length of the other parallel side of the trapezium.



5. The areas of two concentric circles are 154 cm^2 and 616 cm^2. Find the width of the ring between the circles.
6. The perimeters of a circle and a square are equal. Find the ratio of their areas.
7. The sum of the areas of two circles is equal to the area of a third circle of diameter 30 cm. If the diameter of one of the two circles is 18 cm, find the diameter of the second circle.
8. The cost of fencing a rectangular park, 68 m long, at the rate of ₹ 7/m is ₹ 1610. Find the width of the park.
9. A room has a carpet 13 m by 9 m in the middle, leaving a margin of 1.5 m wide all around. Find the area of the floor.
10. The height of a wall is 5 times its width and its length is 8 times its height. If the volume of the wall is 12.8 m^3, find its length.
11. The dimensions of a room are 10 m by 8 m by 3.3 m. If a space of 3 m^3 is needed to store a carton, how many such cartons may be stored in the room?
12. The area of the base of a rectangular water tank is 6500 cm^2. If the volume of the water in the tank is 2.6 m^3, find the depth of water.
13. The edges of three cubes are 3 cm, 4 cm and 5 cm. The cubes are melted and recast into a single cube. Find the length of a diagonal of the cube.
14. Find the area of the shaded part.

(i)



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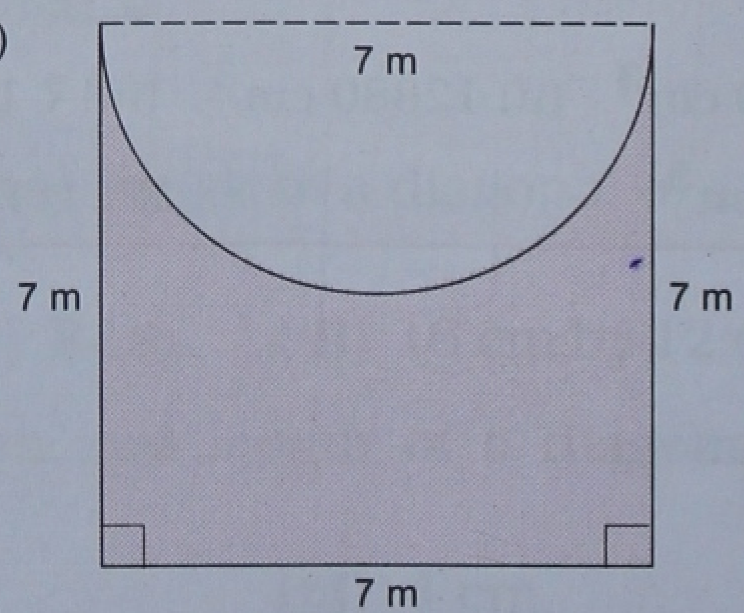

ANSWERS

1. 45 cm	**2.** 30 cm, 10 cm	**3.** 88 m^2	**4.** 9 cm
5. 7 cm	**6.** 4 : π	**7.** 24 cm	**8.** 47 m
9. 192 m^2	**10.** 16 m	**11.** 88	**12.** 4 m
13. $6\sqrt{3}$ cm	**14.** (i) 32 m^2 (ii) 29.75 m^2		

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