1

Perimeter and Area

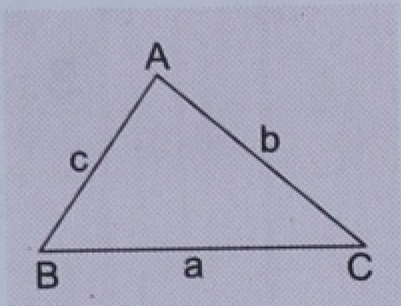
You are already familiar with the terms perimeter and area and know how to calculate the perimeters and areas of squares and rectangles. Let us revise what you have learnt in your previous class and learn some new concepts. To begin with, let us recall what perimeter and area mean.

1. The perimeter (P) of a closed plane curve is the total length of the curve. The unit of perimeter is the unit of length.
2. The area (A) of a closed plane curve is the region enclosed by the plane curve. The area of a plane figure is measured in square units, such as square cm (cm^2) and square m (m^2).

Triangles

In this section we will learn to calculate the perimeter and area of a triangle.

Perimeter of a triangle



The perimeter of a triangle ABC

= the sum of the lengths of its sides

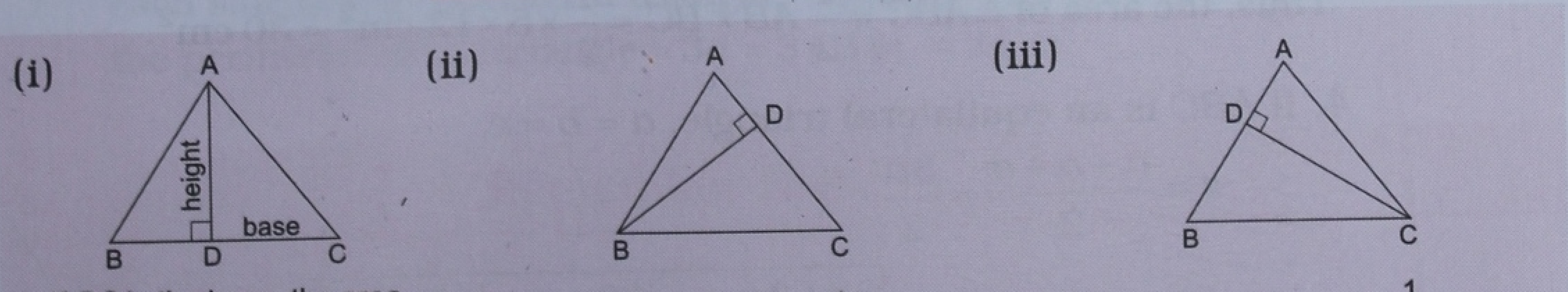
$= BC + CA + AB = a + b + c.$

Example The perimeter of a $\triangle ABC$ in which $AB = 3.2$ cm, $BC = 5$ cm and $CA = 7.3$ cm is the sum $(3.2 + 5 + 7.3)$ cm = 15.5 cm.

Area of a triangle

1. The area of a triangle is equal to half the product of its base and the corresponding height (or altitude).

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$



If BC is the base, the area of $\triangle ABC = \frac{1}{2} BC \times AD$.

If AC is the base, area $= \frac{1}{2} AC \times BD$.

If AB is the base, area $= \frac{1}{2} AB \times CD$.

If BC is taken as the base of the triangle ABC, shown in the figure, the perpendicular AD drawn from A to the base BC is called the height of the triangle. Any side of a triangle may be taken as the base.

Example If the base of a triangle is 12 cm and its height is 8 cm then its area

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 8 \text{ cm}^2 = 48 \text{ cm}^2.$$

2. We can also find the area of a triangle by Heron's formula. According to this formula, if a, b and c are the three sides of a triangle then

$$\text{area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $2s$ = the perimeter of the triangle $= a + b + c$.

EXAMPLE **Find the area of a triangle of sides 26 cm, 28 cm and 30 cm.**

Solution Here, $a = 26$ cm, $b = 28$ cm and $c = 30$ cm.

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{26 \text{ cm} + 28 \text{ cm} + 30 \text{ cm}}{2} = 42 \text{ cm}.$$

Thus, the area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-26)(42-28)(42-30)} \text{ cm}^2$$

$$= \sqrt{42 \times 16 \times 14 \times 12} \text{ cm}^2$$

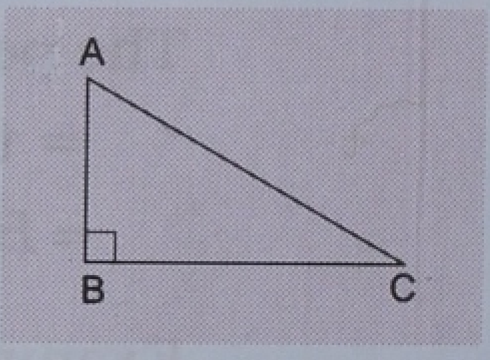
$$= \sqrt{(14 \times 3) \times 4^2 \times 14 \times (4 \times 3)} \text{ cm}^2$$

$$= \sqrt{14^2 \times 3^2 \times 4^2 \times 2^2} \text{ cm}^2$$

$$= 14 \times 3 \times 4 \times 2 \text{ cm}^2 = 336 \text{ cm}^2.$$

3. If ABC is a right-angled triangle in which $\angle B = 90°$,

its area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} BC \times AB.$



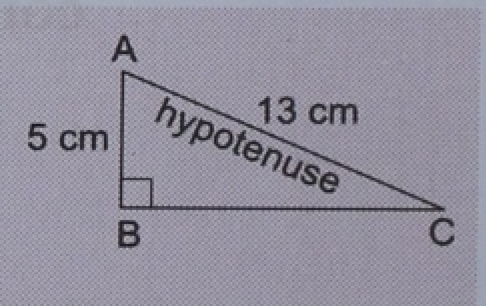
$$\text{Area of a right-angled triangle} = \frac{1}{2} \times \text{product of sides containing the right angle}$$

EXAMPLE **Find the area of $\triangle ABC$ if $AB = 5$ cm, $AC = 13$ cm and $\angle B = 90°$.**

Solution Using Pythagoras' theorem,

$$AB^2 + BC^2 = AC^2$$

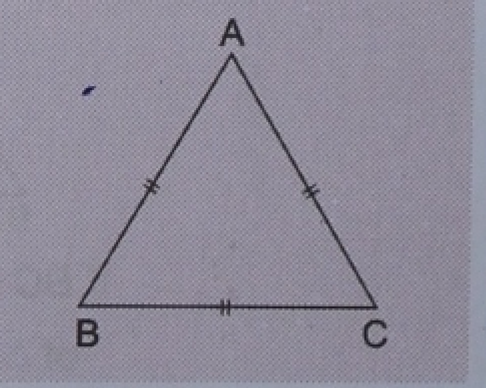
$$\Rightarrow \quad BC = \sqrt{AC^2 - AB^2} = \sqrt{169 - 25} \text{ cm} = \sqrt{144} \text{ cm} = 12 \text{ cm}.$$



Thus, the area of $\triangle ABC = \frac{1}{2} AB \times BC = \frac{1}{2} \times 5 \times 12 \text{ cm}^2 = 30 \text{ cm}^2.$

4. If ABC is an equilateral triangle, $a = b = c$.

$$\therefore \quad s = \frac{a+a+a}{2} = \frac{3a}{2}.$$



$$\therefore \quad \text{area of } \triangle ABC = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)}$$

$$= \sqrt{\frac{3a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}} = \frac{\sqrt{3}}{4} a^2.$$

Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

EXAMPLE **Find the area of an equilateral triangle of side 6 cm, correct to three places of decimal.**

Solution The side (a) of the equilateral triangle = 6 cm.

$\therefore$ the area of the triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{1.732}{4} \times 6^2 \text{ cm}^2$

$= 1.732 \times 9 \text{ cm}^2 = 15.588 \text{ cm}^2.$

Solved Examples

EXAMPLE 1 **Find the area of a triangle of sides 29 cm, 36 cm and 25 cm. Also, find the length of the perpendicular from the vertex opposite the side of length 36 cm.**

Solution Let the triangle be ABC and let $a = 29$ cm, $b = 36$ cm and $c = 25$ cm.

Then, $s = \frac{a+b+c}{2} = \frac{29 \text{ cm} + 36 \text{ cm} + 25 \text{ cm}}{2} = 45$ cm.

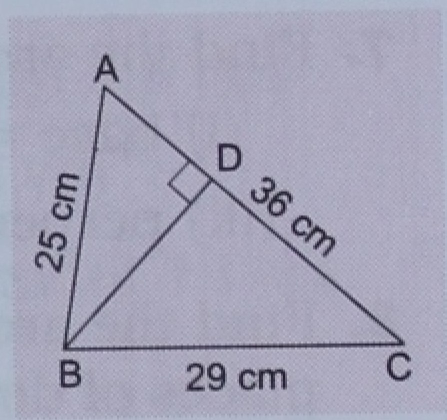
Thus, the area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{45 \times (45-29) \times (45-36) \times (45-25)} \text{ cm}^2$

$= \sqrt{45 \times 16 \times 9 \times 20} \text{ cm}^2$

$= \sqrt{(3^2 \times 5) \times 4^2 \times 3^2 \times (5 \times 2^2)} \text{ cm}^2$

$= \sqrt{3^2 \times 3^2 \times 5^2 \times 4^2 \times 2^2} \text{ cm}^2 = 360 \text{ cm}^2.$



Let BD be the perpendicular on AC from the point B.

Then, the area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} AC \times BD$

$\Rightarrow \frac{1}{2} AC \times BD = 360 \text{ cm}^2 \Rightarrow \frac{1}{2} \times 36 \times BD = 360 \text{ cm} \Rightarrow BD = \frac{360}{18} \text{ cm} = 20 \text{ cm}.$

Hence, the length of the perpendicular on AC from the point B = 20 cm.

EXAMPLE 2 **Find the perimeter of an equilateral triangle of area $16\sqrt{3}$ m^2.**

Solution Let each side of the triangle = a m. Then its area $= \frac{\sqrt{3}}{4}a^2 \text{ m}^2$.

From the question, $\frac{\sqrt{3}}{4}a^2 = 16\sqrt{3} \Rightarrow \frac{a^2}{4} = 16 \Rightarrow a^2 = 64 \Rightarrow a = 8.$

$\therefore$ each side of the equilateral triangle = 8 m.

$\therefore$ the perimeter of the triangle $= 3a = 3 \times 8$ m = 24 m.

Remember These

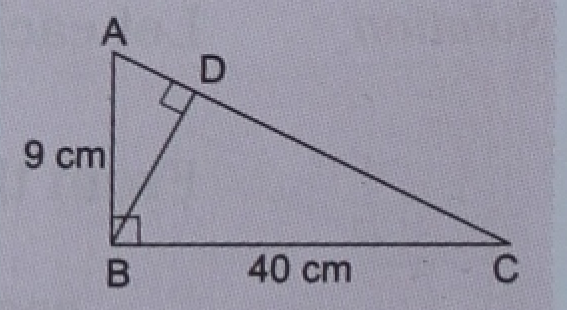
1. The area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$.
2. If a, b and c are the sides of a triangle and $2s$ is its perimeter then the area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$.

3. The area of a right-angled triangle = $\frac{1}{2}$ the product of the sides containing the right angle.

4. The area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$.

EXERCISE 1A

1. Find the area of a triangle in which the base is 8 cm and the height 11 cm.
2. If one side of a triangle is 24.5 cm and the corresponding altitude is 12 cm, find its area.
3. The area of a triangle is 216 cm^2. One side of the triangle measures 24 cm. Find the distance of the corresponding vertex from the side.
4. The area of a triangle is 196 cm^2. Find the length of the base if the corresponding height is 14 cm.
5. Find the area of a triangle if its sides are
   (i) 21 cm, 17 cm, 10 cm (ii) 17 cm, 25 cm, 26 cm
   (iii) 52 cm, 60 cm, 56 cm (iv) 42 cm, 39 cm, 45 cm
6. Find the area of a triangle of sides 13 cm, 14 cm and 15 cm. Also, find the length of the perpendicular from the vertex opposite the side of length 14 cm.
7. Find the area of a right-angled triangle if its
   (i) base = 24 cm, perpendicular = 7 cm (ii) base = 6 cm, hypotenuse = 10 cm
   (iii) perpendicular = 60 cm, hypotenuse = 61 cm
8. Find the area of an equilateral triangle of side (i) 4 cm, (ii) 5 cm, (iii) 7 cm, correct to two places of decimal. (Use $\sqrt{3} = 1.732$.)
9. If the area of an equilateral triangle is $36\sqrt{3}$ cm^2, find the length of its side and its perimeter.
10. If the area of an equilateral triangle is $25\sqrt{3}$ m^2, find its perimeter and altitude correct to three significant figures.
11. If the ratio of the base and the hypotenuse of a right-angled triangle is 4 : 5 and the perimeter of the triangle is 48 cm, find its area.
12. In the figure, *ABC* is a right-angled triangle. Find
    (i) the area of $\triangle ABC$
    (ii) the length of *AC*
    (iii) the length of *BD*, correct to two places of decimal



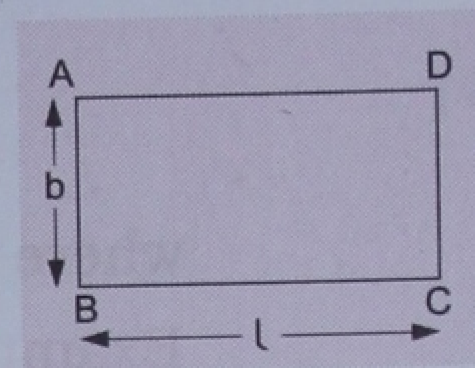
ANSWERS

1. 44 cm^2 2. 147 cm^2 3. 18 cm 4. 28 cm
5. (i) 84 cm^2 (ii) 204 cm^2 (iii) 1344 cm^2 (iv) 756 cm^2 6. 84 cm^2, 12 cm
7. (i) 84 cm^2 (ii) 24 cm^2 (iii) 330 cm^2 8. (i) 6.93 cm^2 (ii) 10.83 cm^2 (iii) 21.22 cm^2
9. 12 cm, 36 cm 10. 30 m, 8.66 m 11. 96 cm^2
12. (i) 180 cm^2 (ii) 41 cm (iii) 8.78 cm

Rectangle and square

Let us recall how to calculate the perimeters and areas of rectangles and squares.

Perimeter of a rectangle



The perimeter of a rectangle $ABCD$

= the sum of the lengths of its sides

$= 2 \times \text{length} + 2 \times \text{breadth}$ [$\because$ opposite sides are equal]

$= 2\,(\text{length} + \text{breadth})$

$$P = 2(l+b),\quad l = \frac{P}{2} - b,\quad b = \frac{P}{2} - l$$

where P = perimeter, l = length, b = breadth.

Example The perimeter of a rectangular field of length 78.24 m and breadth 56.26 m = 2(78.24 m + 56.26 m) = 2 × 134.50 m = 269 m.

Area of a rectangle

The area (A) of a rectangle is equal to the product of its length (l) and breadth (b).

$$A = l \times b,\quad l = \frac{A}{b}\quad \text{and}\quad b = \frac{A}{l}$$

Examples (i) The area of a rectangle of length 12 cm and breadth 8.5 cm

$= \text{length} \times \text{breadth} = 12\text{ cm} \times 8.5\text{ cm} = 102\text{ cm}^2$.

(ii) The length of a rectangle of area 224 cm^2 and breadth 14 cm

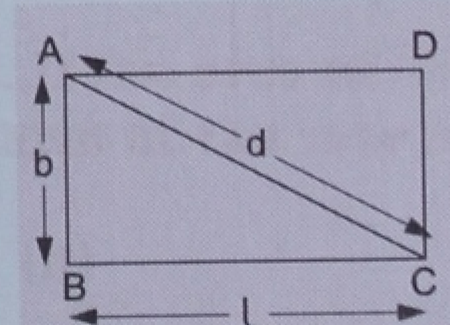
$$= \frac{\text{area}}{\text{breadth}} = \frac{224\text{ cm}^2}{14\text{ cm}} = 16\text{ cm}.$$

(iii) The breadth of a rectangle of area 2.5 m^2 and length 6 m 25 cm

$$= \frac{\text{area}}{\text{breadth}} = \frac{2.5\text{ m}^2}{6.25\text{ m}} = \frac{2.5 \times 100 \times 100\text{ cm}^2}{6.25 \times 100\text{ cm}}$$

$$= \frac{250 \times 100\text{ cm}^2}{625\text{ cm}} = 40\text{ cm}.$$

Diagonal of a rectangle



Let d be the length of the diagonal AC of a rectangle $ABCD$ of length $= l$ and breadth $= b$.

Then, in the right-angled triangle ABC,

$$AC^2 = BC^2 + AB^2 \Rightarrow d^2 = l^2 + b^2 \Rightarrow d = \sqrt{l^2 + b^2}.$$

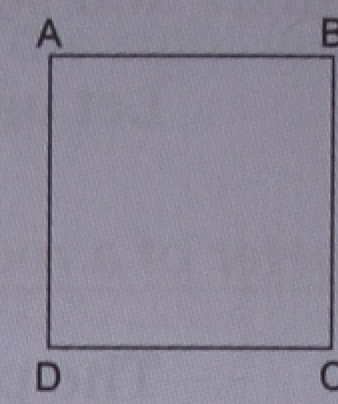
Since the diagonals of a rectangle are equal,

$$\text{diagonal} = \sqrt{(\text{length})^2 + (\text{breadth})^2}$$

Example The diagonal of a rectangle of length 12 cm and beadth 5 cm

$$= \sqrt{l^2 + b^2} = \sqrt{12^2 + 5^2}\text{ cm} = \sqrt{169}\text{ cm} = 13\text{ cm}.$$

Perimeter of a square



The perimeter of a square $ABCD$

$= AB + BC + CD + DA = 4AB$ [∵ all the sides are equal]

$= 4 \times$ length of a side.

$$P = 4a \quad \text{or} \quad a = \frac{P}{4}$$

where $P =$ perimeter, $a =$ length of a side.

Example The perimeter of a square of side 36.25 m $= 4 \times 36.25$ m $= 145$ m.

Area of a square

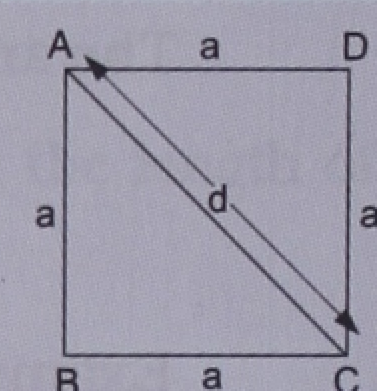
If the area of a square be A and the length of its side be a then

$$A = a^2, \quad a = \sqrt{A}$$

Examples (i) The area of a square of side 4.5 cm $= (4.5 \text{ cm})^2 = 20.25 \text{ cm}^2$.

(ii) The length of a side of a square of area 441 m^2 $= \sqrt{441}$ m $= 21$ m.

Diagonal of a square



Let d be the length of the diagonal AC of the square $ABCD$.
Then, in the right-angled triangle ABC,

$AC^2 = AB^2 + BC^2 \Rightarrow d^2 = a^2 + a^2 = 2a^2 \Rightarrow d = a\sqrt{2}$.

Since the diagonals of a square are equal,

$$\text{length of a square's diagonal} = \sqrt{2} \times \text{side}$$

Example The length of the diagonal of a square of side 6 cm

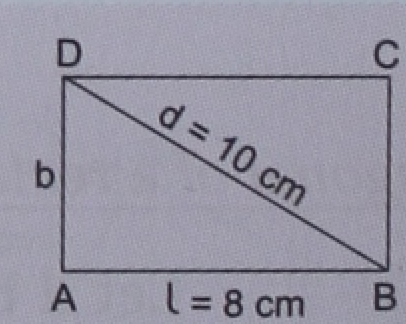
$= \sqrt{2} \times 6$ cm $= 6 \times 1.414$ cm $= 8.484$ cm.

Solved Examples

EXAMPLE 1 **If the diagonal and length of a rectangle are 10 cm and 8 cm respectively, find its (i) breadth (ii) perimeter and (iii) area.**

Solution Given, $d = 10$ cm and $l = 8$ cm.

Let the breadth $= b$.



(i) In $\triangle ABD$, $d^2 = l^2 + b^2 \Rightarrow b^2 = d^2 - l^2$

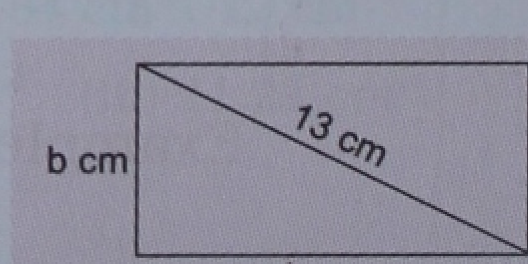
$\Rightarrow b = \sqrt{10^2 - 8^2}$ cm $= \sqrt{100 - 64}$ cm $= \sqrt{36}$ cm $= 6$ cm.

So, the breadth of the rectangle = 6 cm.

(ii) The perimeter of the rectangle $= 2(l + b) = 2(8 + 6)$ cm $= 28$ cm.

(iii) The area of the rectangle $= l \times b = 8 \times 6 \text{ cm}^2 = 48 \text{ cm}^2$.

EXAMPLE 2 **If the area of a rectangle is 60 cm^2 and its diagonal is 13 cm, find its length, breadth and perimeter.**



Solution Let the rectangle's length $= l$ cm and breadth $= b$ cm.

Given, its area $= 60 \text{ cm}^2 = l \times b \text{ cm}^2 \Rightarrow lb = 60$ ... (1)

Also, its diagonal $= 13$ cm $\Rightarrow \sqrt{l^2 + b^2} = 13$.

Squaring both sides, $l^2 + b^2 = 169$. ... (2)

Now, $l + b = \sqrt{l^2 + b^2 + 2lb} = \sqrt{169 + 2 \times 60} = \sqrt{289} = 17$.

$\therefore \quad l + b = 17$ ... (3)

Again, $l - b = \sqrt{l^2 + b^2 - 2lb} = \sqrt{169 - 2 \times 60} = \sqrt{49} = 7$.

$\therefore \quad l - b = 7$ ... (4)

Adding (3) and (4), we get $2l = 24 \Rightarrow l = 12$.

$\therefore \quad \text{breadth} = \frac{\text{area}}{\text{length}} = \frac{60 \text{ cm}^2}{12 \text{ cm}} = 5 \text{ cm}$.

Hence, the length of the rectangle = 12 cm and the breadth of the rectangle = 5 cm.

Also, the perimeter $= 2(l + b) = 2(12 + 5)$ cm = 34 cm.

EXAMPLE 3 **If the area of a rectangle is 84 cm^2 and the difference of its length and breadth is 5 cm, find its length, breadth and perimeter.**

Solution Let the rectangle's length $= l$ cm and breadth $= b$ cm.

Given that its area = 84 cm^2 $\Rightarrow lb = 84$. ... (1)

Again, given that $l - b = 5$ ... (2)

$\Rightarrow \quad l + b = \sqrt{(l-b)^2 + 4lb} = \sqrt{5^2 + 4 \times 84} = \sqrt{25 + 336} = \sqrt{361} = 19$.

$\therefore \quad l + b = 19$ ... (3)

Adding (2) and (3), we have $2l = 24 \Rightarrow l = 12$.

$\therefore$ the length of the rectangle = 12 cm.

The breadth of the rectangle $= \frac{\text{area}}{\text{length}} = \frac{84}{12}$ cm = 7 cm.

The perimeter of the rectangle $= 2(l + b) = 2(12 + 7)$ cm = 38 cm.

EXAMPLE 4 **If the diagonal of a square is 8 cm, find (i) the length of a side, (ii) the perimeter and (iii) the area of the square.**

Solution Here, the diagonal $(d) = 8$ cm. If the length of a side $= a$ cm then $d = \sqrt{2}a$.

(i) $\therefore$ the length (a) of a side $= \frac{d}{\sqrt{2}} = \frac{8 \text{ cm}}{\sqrt{2}} = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \text{ cm} = \frac{8\sqrt{2}}{2} \text{ cm} = 4\sqrt{2}$ cm

$= 4 \times 1.41$ cm $= 5.64$ cm (correct to two decimals).

(ii) The perimeter of the square $= 4a = 4 \times 5.64$ cm $= 22.56$ cm.

(iii) The area of the square $= a^2 = (4\sqrt{2})^2 \text{ cm}^2 = 32 \text{ cm}^2$.

EXAMPLE 5 **The ratio of the length and breadth of a rectangular farm is 4 : 3. If the area of the farm is 1452 m^2, find the cost of fencing the boundary of the farm at the rate of ₹ 12 per metre.**

Solution Here, the ratio of the length and breadth = 4 : 3.

Let the length $= 4x$ and the breadth $= 3x$.

Then the area of the farm $= 4x \times 3x = 1452 \text{ m}^2$ (given)

$\Rightarrow \quad 12x^2 = 1452 \text{ m}^2 \Rightarrow x^2 = \frac{1452}{12} \text{ m}^2 = 121 \text{ m}^2 \Rightarrow x = \sqrt{121} \text{ m} = 11$ m.

So, the length $= 4x = 4 \times 11$ m = 44 m

and the breadth $= 3x = 3 \times 11$ m = 33 m.

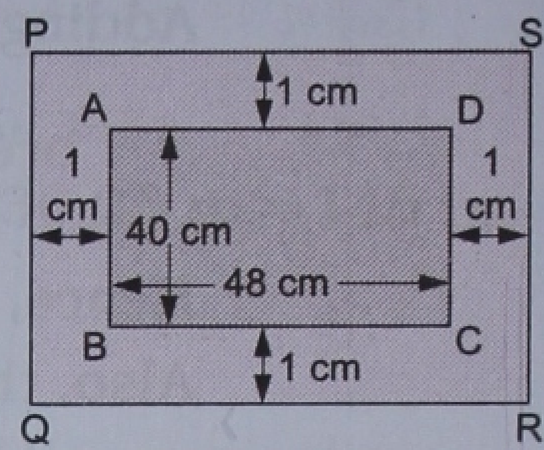
The boundary of the farm = the perimeter of the rectangular farm

$= 2(44 + 33)$ m = 154 m.

∴ the cost of fencing the farm = ₹ 12×154 = ₹ 1848.

EXAMPLE 6 **Find the cost of a 1-cm-wide frame for a 48 cm by 40 cm picture at the rate of ₹ 2 per cm^2.**

Solution In the figure, *ABCD* represents the picture and *PQRS* represents the outer edge of the frame.



The width of the frame = 1 cm and *BC* = 48 cm.

∴ $QR = (48 + 1 + 1)$ cm = 50 cm.

Similarly, $PQ = (40 + 1 + 1)$ cm = 42 cm.

∴ the area of the rectangle $PQRS = 50 \times 42 \text{ cm}^2 = 2100 \text{ cm}^2$

and the area of the rectangle $ABCD = 48 \times 40 \text{ cm}^2 = 1920 \text{ cm}^2$.

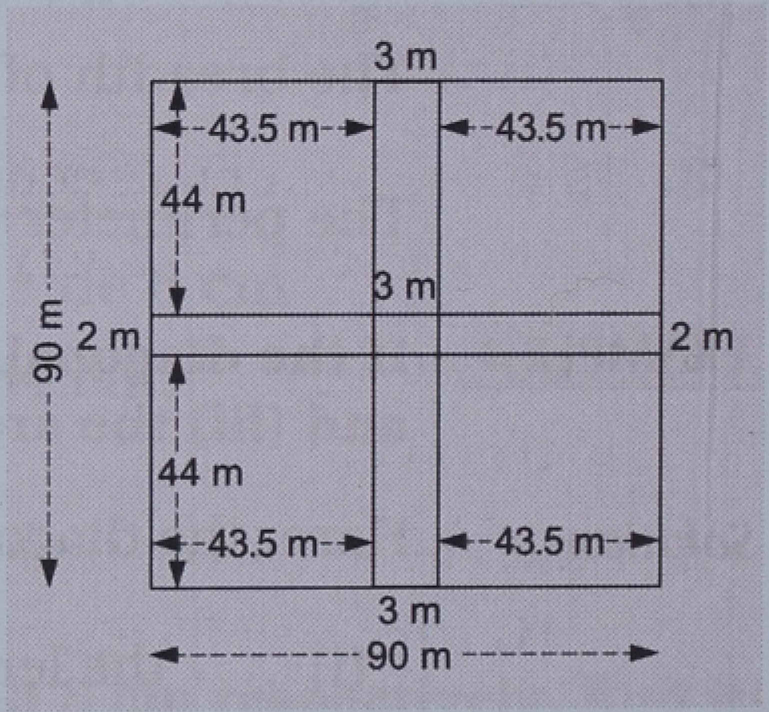
∴ the area of the frame = the area of the rectangle *PQRS* – the area of the rectangle *ABCD*

$= (2100 - 1920) \text{ cm}^2 = 180 \text{ cm}^2$.

∴ the cost of the frame = $180 \times$ ₹ 2 = ₹ 360.

EXAMPLE 7 **A square garden, each side of which is 90 m, is traversed centrally by two paths at right angles to one another, and is thus divided into 4 beds. If the paths are 3 m and 2 m wide, find (i) the area of each bed and (ii) the total area of the two paths.**

Solution The width of each bed $= \dfrac{90 \text{ m} - 3 \text{ m}}{2} = 43.5$ m.



The length of each bed $= \dfrac{90 \text{ m} - 2 \text{ m}}{2} = 44$ m.

(i) ∴ the area of each (rectangular) bed

$= 43.5 \text{ m} \times 44 \text{ m} = 1914 \text{ m}^2$.

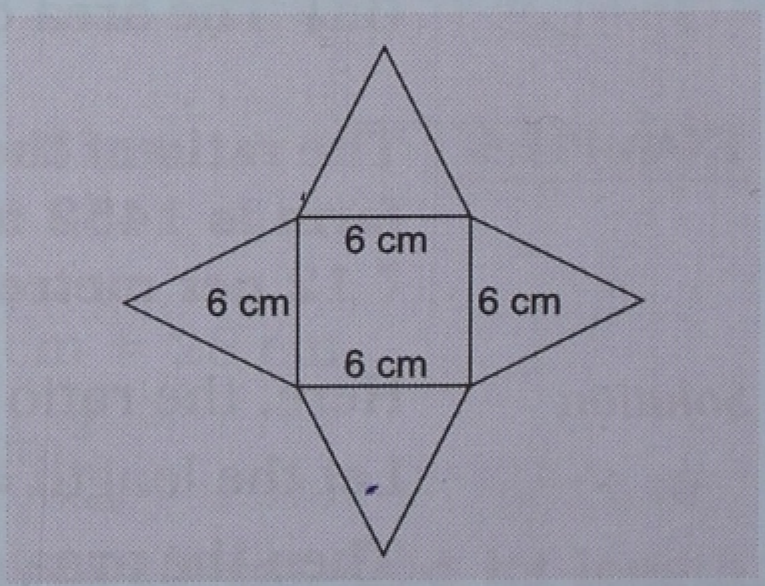
(ii) ∴ the total area of the paths

= the area of the garden – 4 × the area of each bed

$= 90 \times 90 \text{ m}^2 - 4 \times 1914 \text{ m}^2$

$= (8100 - 7656) \text{ m}^2 = 444 \text{ m}^2$.

EXAMPLE 8 **Four equilateal triangles are constructed on all the sides of a square, as shown in the figure. If the side of the square is 6 cm, find the total area and the perimeter of the figure.**



Solution The area of the square $= (6 \text{ cm})^2 = 36 \text{ cm}^2$.

The area of a triangle $= \dfrac{\sqrt{3}}{4} \times (6 \text{ cm})^2 = 9\sqrt{3} \text{ cm}^2$.

∴ the total area of the figure = the area of the square + 4 × the area of a triangle

$= 36 \text{ cm}^2 + 4 \times 9\sqrt{3} \text{ cm}^2 = (36 + 36 \times 1.73) \text{ cm}^2$

$= (36 + 62.28) \text{ cm}^2 = 98.28 \text{ cm}^2$.

The perimeter of the figure = 8 × a side of a triangle = 8 × 6 cm = 48 cm.

Remember These

1. For a rectangle:

(i) $P = 2(l+b)$, $l = \frac{P}{2} - b$, $b = \frac{P}{2} - l$, (ii) $A = l \times b$, $l = \frac{A}{b}$, $b = \frac{A}{l}$,

(iii) $d = \sqrt{l^2+b^2}$, $l = \sqrt{d^2-b^2}$, $b = \sqrt{d^2-l^2}$,

where P = perimeter, l = length, b = breadth, A = area and d = diagonal.

2. For a square:

(i) $P = 4a$, $a = \frac{P}{4}$, (ii) $A = a^2$, $a = \sqrt{A}$, (iii) $d = a\sqrt{2}$, $a = \frac{d}{\sqrt{2}}$,

where P = perimeter, a = side, A = area and d = diagonal.

EXERCISE 1B

1. Find the perimeter of a rectangle of
(i) length = 24 cm, breadth = 16 cm (ii) length = 75.3 cm, breadth = 56.2 cm
(iii) length = 1 m 26 cm, breadth = 88 cm

2. Find the perimeter of a square of side
(i) 5 cm (ii) 1.6 cm (iii) 15 m 50 cm

3. Find the area of a rectangle of dimensions
(i) 24 cm × 12.5 cm (ii) 18.5 cm × 13.5 cm (iii) 1 m 25 cm × 2 m

4. Find the area of a square of side
(i) 6 cm (ii) 2.3 cm (iii) 1 m 50 cm

5. Find the length of a rectangle of
(i) area = 570 cm^2, breadth = 15 cm (ii) area = 756 cm^2, breadth = 21 cm
(iii) perimeter = 94 m, breadth = 18 m

6. Find the length of a side of a square if its
(i) perimeter = 38 cm (ii) area = 196 cm^2 (iii) area = 6.25 m^2

7. Find the length of a diagonal of a rectangle of
(i) length = 60 cm, breadth = 11 cm (ii) area = 12 m^2, length = 4 m
(iii) breadth = 8 m, area = 128 m^2 (iv) perimeter = 34 cm, length = 12 cm

8. Find the length of a diagonal of a square if its
(i) side = 6 cm (ii) perimeter = 40 cm (iii) area = 144 m^2

9. Find the perimeter and area of a rectangle, if its
(i) diagonal = 17 cm, length = 15 cm (ii) diagonal = 61 cm, breadth = 11 cm

10. Find the perimeter and area of a square if its diagonal is
(i) $11\sqrt{2}$ cm (ii) $20\sqrt{2}$ cm (iii) 20 m

11. If the area of a rectangle is 168 cm^2 and its diagonal is 25 cm, find its length, breadth and perimeter.

12. If the area of a rectangle is 360 cm^2 and the difference of its length and breadth is 31 cm, find its length, breadth and perimeter.

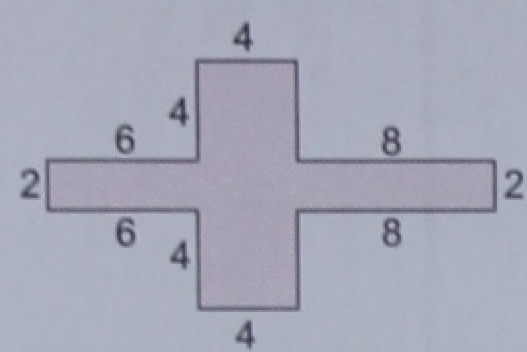
13. If the length of a rectangular farm and its breadth are in the ratio 5 : 4 and the area of the farm is 13520 m^2, find the cost of fencing the farm at the rate of ₹ 15 per metre.

14. If the length of a rectangular hall and its breadth are in the ratio 9 : 5 and its perimeter is 140 m, find the cost of flooring the hall with rectangular tiles of size 25 cm by 20 cm at the rate of ₹ 500 per hundred tiles.

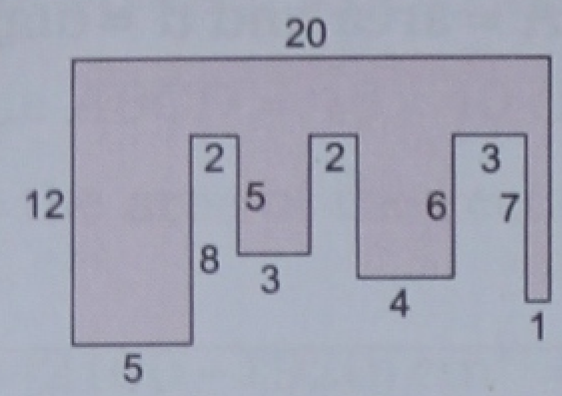
15. A square garden of side 20 m is traversed centrally by two paths at right angles to one another, and is thus divided into 4 beds. If the paths are 3 m and 2 m wide, find (i) the area of each bed, (ii) the total area of the paths.

16. Find the area of each figure. The dimensions are given in cm.

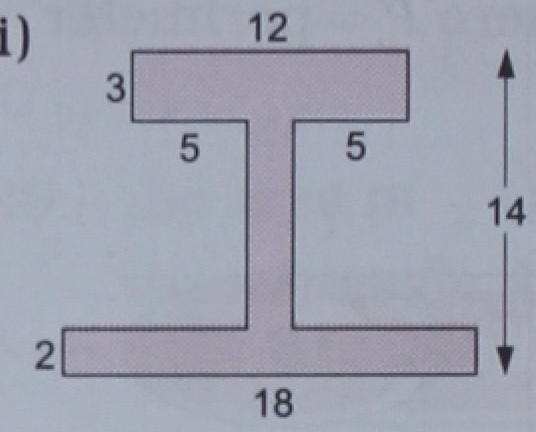
(i)



(ii)

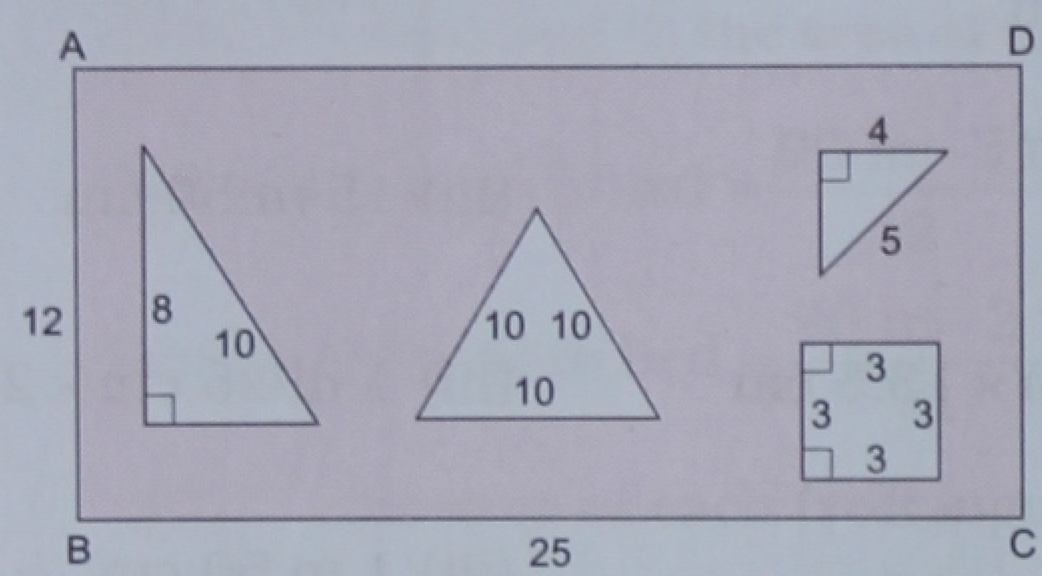


(iii)

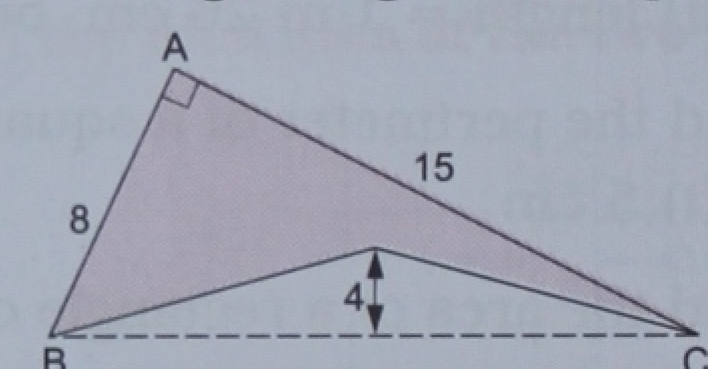


17. Find the area of the shaded regon in each of the following in which the dimensions are given in cm.

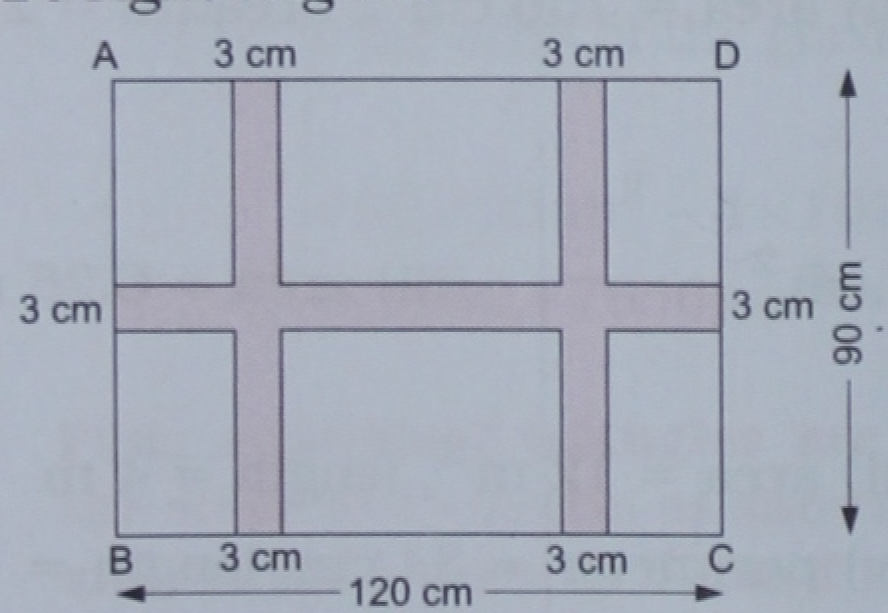
(i) In the figure, *ABCD* is a rectangle.



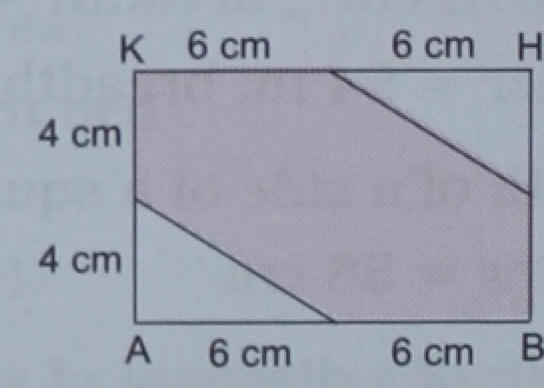
(ii) *ABC* is a right-angled triangle at *A*.



(iii) *ABCD* is a rectangle and all the corners are right-angled.



(iv) *ABHK* is a rectangle.



ANSWERS

1. (i) 80 cm (ii) 2 m 63 cm (iii) 4 m 28 cm

2. (i) 20 cm (ii) 6.4 cm (iii) 62 m

3. (i) 300 cm^2 (ii) 249.75 cm^2 (iii) 2.5 m^2

4. (i) 36 cm^2 (ii) 5.29 cm^2 (iii) 2.25 m^2

5. (i) 38 cm (ii) 36 cm (iii) 29 m

6. (i) 9.5 cm (ii) 14 cm (iii) 2.5 m

7. (i) 61 cm (ii) 5 m (iii) $8\sqrt{5}$ m (iv) 13 cm

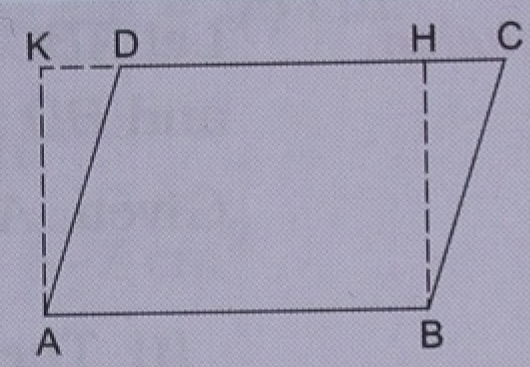
8. (i) $6\sqrt{2}$ cm (ii) $10\sqrt{2}$ cm (iii) $12\sqrt{2}$ m

9. (i) 46 cm, 120 cm^2 (ii) 142 cm, 660 cm^2
10. (i) 44 cm, 121 cm^2 (ii) 80 cm, 400 cm^2 (iii) $40\sqrt{2}$ m, 200 m^2 **11.** 24 cm, 7 cm, 62 cm
12. 40 cm, 9 cm, 98 cm **13.** ₹ 7020 **14.** ₹ 112500 **15.** (i) 76.5 m^2 (ii) 94 m^2
16. (i) 68 cm^2 (ii) 166 cm^2 (iii) 90 cm^2 **17.** (i) 217.75 cm^2 (ii) 26 cm^2 (iii) 882 cm^2 (iv) 72 cm^2

Parallelogram, Rhombus and Trapezium

In this section, we will learn to calculate the areas of a parallelogram, a rhombus and a trapezium.

Area of a parallelogram

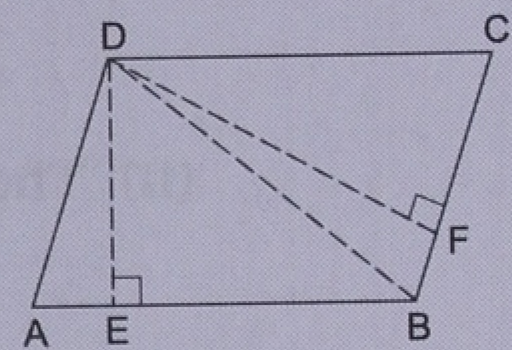


Let $ABCD$ be a parallelogram. Let us draw the rectangle $ABHK$ on the same base and between the same parallels as the parallelogram $ABCD$.

Then, the area of the parallelogram $ABCD$

$= $ the area of the rectangle $ABHK$

$= AB \times BH = $ base $\times$ height.



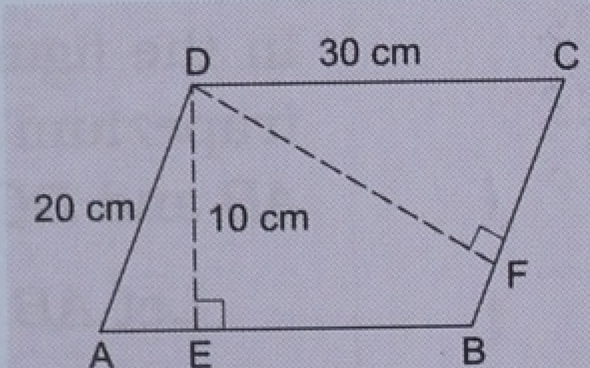
Note We can take any of the sides as the base and multiply it by the corresponding height. For example, if we take BC as the base, the corresponding height will be DF.

The area of a parallelogram = base × height

Also, since each diagonal of a parallelogram divides it into two congruent triangles,

the area of parallelogram $ABCD = 2 \times$ area $(\triangle ABD) = 2 \times$ area of $(\triangle CBD)$.

EXAMPLE **A parallelogram has adjacent sides of 20 cm and 30 cm. If the distance between the longer sides is 10 cm, find (i) the area of the parallelogram and (ii) the distance between the shorter sides.**



Solution Let $ABCD$ be the parallelogram.

Here, $AB = DC = 30$ cm, $AD = BC = 20$ cm

and distance between AB and DC, that is, $DE = 10$ cm.

(i) Area of the parallelogram = base $AB \times$ height DE

$= 30 \times 10 \text{ cm}^2 = 300 \text{ cm}^2$.

(ii) Let DF, the distance between the shorter sides AD and $BC = x$ cm.

Then the area of the parallelogram = base $BC \times$ height DF

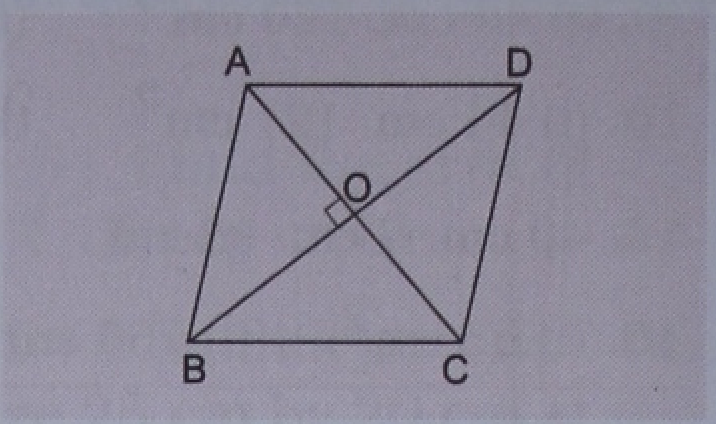
$\Rightarrow \quad 300 = 20x \quad \Rightarrow \quad x = \dfrac{300}{20} = 15.$

Hence, the distance between the shorter sides = 15 cm.

Area of a rhombus

Let $ABCD$ be a rhombus, with its diagonals AC and BD bisecting each other at right angles at the point O.

Since $ABCD$ is also a parallelogram, its area



$$= 2 \times \text{area}\,(\triangle ABC) = 2 \times \frac{1}{2} AC \times OB = AC \times OB$$

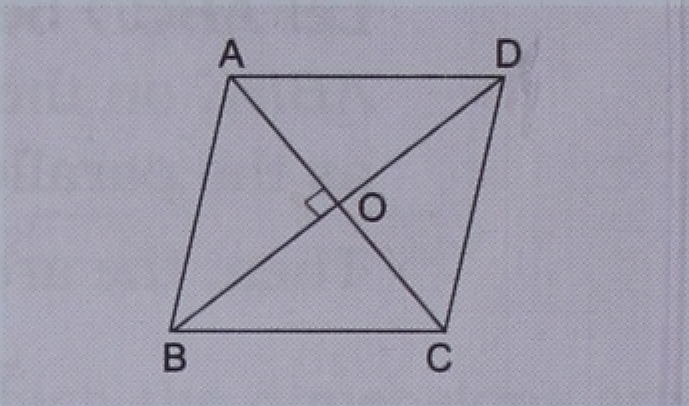
$$= \frac{1}{2} AC \times BD \qquad \left[\because\ OB = \frac{1}{2} BD\right]$$

$$= \frac{1}{2} \times \text{product of its diagonals.}$$

The area of a rhombus $= \frac{1}{2} \times$ product of its diagonals

EXAMPLE **The diagonals of a rhombus are 10 cm and 24 cm. Find (i) its area, (ii) the length of a side and (iii) its perimeter.**

Solution Let $ABCD$ be the given rhombus, with its diagonals AC and BD intersecting at the point O.



Given, $AC = 10$ cm and $BD = 24$ cm.

(i) The area of the rhombus $= \frac{1}{2} \times$ product of diagonals

$$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 10 \times 24 \text{ cm}^2 = 120 \text{ cm}^2.$$

(ii) The diagonals of a rhombus bisect each other at right angles.

$\therefore\ OA = \frac{1}{2} AC = 5$ cm, $OB = \frac{1}{2} BD = 12$ cm and $\angle AOB = 90°$.

In the right-angled triangle OAB, $AB^2 = OA^2 + OB^2$

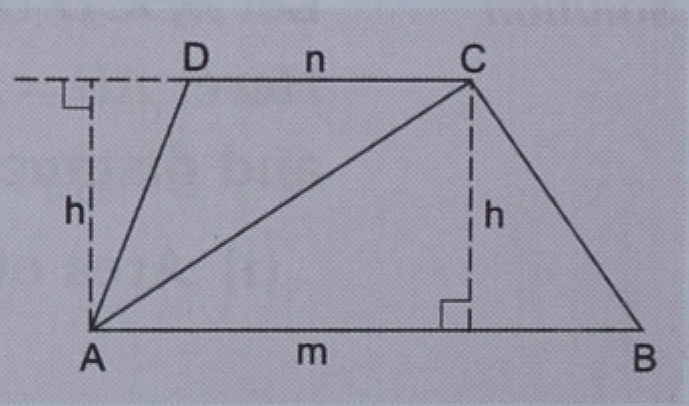
$\Rightarrow\ AB = \sqrt{OA^2 + OB^2} = \sqrt{5^2 + 12^2}$ cm $= \sqrt{169}$ cm $=$ 13 cm.

So, the length of a side of the rhombus = 13 cm.

(iii) Since the sides are equal, the perimeter of the rhombus $= 4 \times 13$ cm = 52 cm.

Area of a trapezium

In the figure, AB and DC are the parallel sides of the trapezium $ABCD$. The perpendicular distance between AB and DC, that is, the height of the trapezium is h.



Let $AB = m$ and $DC = n$.

Then the area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} mh$

and the area of $\triangle ACD = \frac{1}{2} nh$.

$\therefore$ the area of the trapezium $ABCD$ = the area of $\triangle ABC$ + the area of $\triangle ACD$

$$= \frac{1}{2} mh + \frac{1}{2} nh = \frac{1}{2}(m+n)h.$$

The area of a trapezium $= \frac{1}{2} \times$ (sum of its parallel sides) $\times$ height

Example The parallel sides of a trapezium are 5 cm and 8 cm and the distance between them is 4 cm. Its area $= \frac{1}{2}(5+8)\times 4\ \text{cm}^2 = 26\ \text{cm}^2$.

Solved Examples

EXAMPLE 1 **The area of a rhombus is equal to the area of a triangle of sides 50 cm, 48 cm and 14 cm. If one of the diagonals of the rhombus is 24 cm, find the length of the other diagonal.**

Solution The area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$.

Here, $a = 50$ cm, $b = 48$ cm and $c = 14$ cm. So, $s = \frac{50+48+14}{2}$ cm = 56 cm.

$\therefore$ the area of the triangle $= \sqrt{56(56-50)(56-48)(56-14)}\ \text{cm}^2$

$= \sqrt{56\cdot 6\cdot 8\cdot 42}\ \text{cm}^2 = \sqrt{8\cdot 7\cdot 6\cdot 8\cdot 6\cdot 7}\ \text{cm}^2$

$= 8\cdot 6\cdot 7\ \text{cm}^2 = 336\ \text{cm}^2$.

If the other diagonal of the rhombus $= d$ cm

then the area of the rhombus $= \frac{1}{2}\times 24\times d\ \text{cm}^2 = 12d\ \text{cm}^2$.

$\therefore$ $12d = 336$ (from the question) $\Rightarrow$ $d = \frac{336}{12} = 28$.

$\therefore$ the length of the other diagonal = 28 cm.

EXAMPLE 2 **The parallel sides of a trapezium are 6 cm and 8 cm. If its area is 35 cm^2, find the height of the trapezium.**

Solution Let the height of the trapezium $= x$ cm. Then the area of the trapezium

$= \frac{1}{2}\times$ (sum of parallel sides) $\times$ height

$= \frac{1}{2}(6+8)x\ \text{cm}^2 = 35\ \text{cm}^2$ (given)

$\Rightarrow\ 7x = 35 \Rightarrow x = \frac{35}{7} = 5$.

Hence, the height of the trapezium = 5 cm.

EXAMPLE 3 **Find the length of one of the parallel sides of a trapezium when the other parallel side is 16 cm and the height and the area of the trapezium are 9 cm and 180 cm^2 respectively.**

Solution Let the length of the unknown parallel side $= x$ cm.

The area of the trapezium $= \frac{1}{2}\times$ (sum of parallel sides) $\times$ height

$\Rightarrow\ 180 = \frac{1}{2}(x+16)\times 9 \Rightarrow 9(x+16) = 2\times 180$

$\Rightarrow \quad x+16=\frac{2\times 180}{9}=40 \Rightarrow x=40-16=24.$

Hence, the length of the unknown side = 24 cm.

EXAMPLE 4 **The two parallel sides of a trapezium are in the ratio 5 : 7 and the distance between them is 11 cm. If the area of the trapezium is 396 cm^2, find the lengths of the parallel sides.**

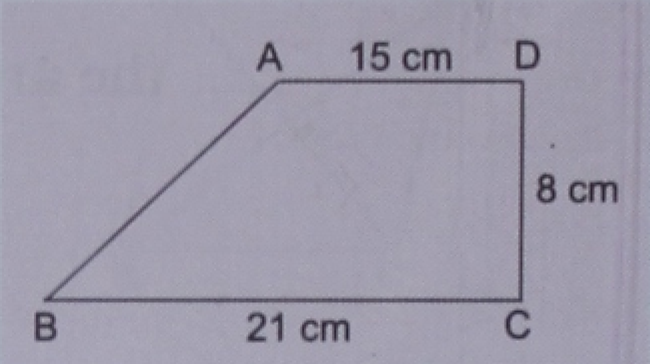
Solution Let the lengths of the two parallel sides of the trapezium be $5x$ cm and $7x$ cm.

The area of the trapezium $=\frac{1}{2}\times$ (sum of parallel sides) $\times$ height

$\Rightarrow \quad 396=\frac{1}{2}(5x+7x)\times 11 \Rightarrow 66x=396 \Rightarrow x=\frac{396}{66}=6.$

Thus, the lengths of the parallel sides are 5×6 cm, i.e., 30 cm and 7×6 cm, i.e., 42 cm.

EXAMPLE 5 **In the adjoining figure, *ABCD* is a quadrilateral in which *DC* is perpendicular to both *AD* and *BC*. Find the area of the quadrilateral.**



Solution *DC* is perpendicular to both *AD* and *BC*

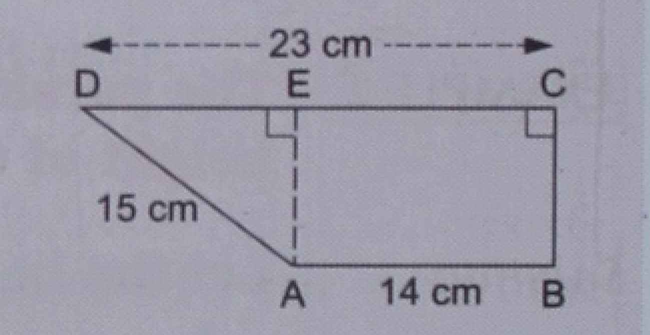
$\Rightarrow$ *AD* and *BC* are parallel $\Rightarrow$ *ABCD* is a trapezium.

Also, $\angle BCD=90°$, so, *DC* is the height of the trapezium.

$\therefore$ the area of the quadrilateral *ABCD* = the area of the trapezium *ABCD*

$=\frac{1}{2}$ (sum of parallel sides) $\times$ height $=\frac{1}{2}(15\text{ cm}+21\text{ cm})\times 8\text{ cm}=144\text{ cm}^2.$

EXAMPLE 6 **In the adjoining figure of a trapezium, $AB \parallel DC$ and $\angle BCD=90°$. Find the area of the trapezium.**



Solution Draw the perpendicular *AE* from the point *A* on *CD*.

Then, $EC=AB=14$ cm and $DE=DC-EC=23\text{ cm}-14\text{ cm}=9$ cm.

In the right-angled triangle *AED*, $AD^2=AE^2+DE^2$

$\Rightarrow \quad AE=\sqrt{AD^2-DE^2}=\sqrt{15^2-9^2}\text{ cm}=\sqrt{225-81}\text{ cm}=\sqrt{144}\text{ cm}=12\text{ cm}.$

So, the height of the trapezium = 12 cm.

$\therefore$ the area of the trapezium $=\frac{1}{2}(23+14)\times 12\text{ cm}^2=222\text{ cm}^2.$

Remember These

1. The area of a parallelogram = base $\times$ height.
2. The area of a rhombus $=\frac{1}{2}\times$ product of its diagonals.
3. The area of a trapezium $=\frac{1}{2}\times$ (sum of parallel sides) $\times$ height.

EXERCISE 1C

1. Find the area of a parallelogram if its
   (i) base = 12 cm, height = 8.5 cm (ii) base = 25 cm, height = 14 cm

2. A parallelogram has sides of 25 cm and 32 cm. If the distance between the longer sides is 15 cm, find the distance between the shorter sides.

3. The diagonals of a rhombus are 30 cm and 16 cm. Find its area, the length of a side and the perimeter of the rhombus.

4. Each side of a rhombus is 61 cm and one diagonal is 22 cm. Find (i) the length of the other diagonal and (ii) the area of the rhombus.

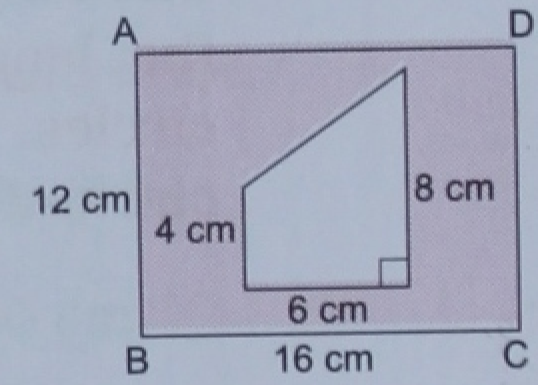
5. The base of a triangle is 24 cm and the corresponding altitude is 16 cm. The area of a rhombus is equal to the area of the triangle. If one of the diagonals of the rhombus is 20 cm, find the length of the other diagonal.

6. Find the area of a trapezium when
   (i) its parallel sides are 16 cm and 22 cm and the perpendicular distance between them is 11 cm,
   (ii) its parallel sides are 42 cm and 24 cm and the perpendicular distance between them is 15 cm.

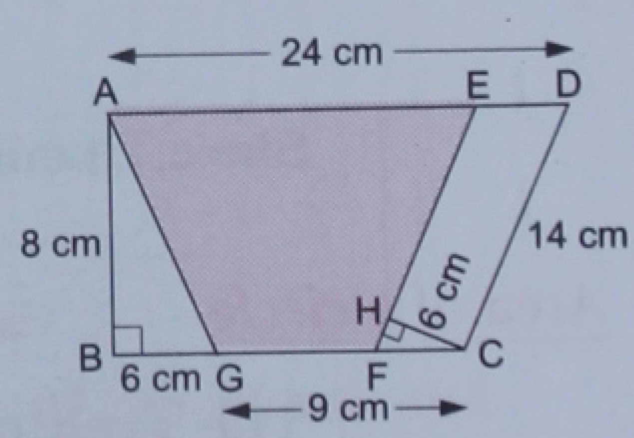
7. (i) If one of the parallel sides of a trapezium is 48 cm, the distance between the parallel sides is 25 cm and the area of the trapezium is 900 cm^2, find the length of the other parallel side.
   (ii) If the parallel sides of a trapezium are 22 cm and 38 cm and its area is 585 cm^2, find the height of the trapezium.

8. The parallel sides of a trapezium are in the ratio 7 : 11 and the distance between them is 35 cm. If the area of the trapezium is 1260 cm^2, find the lengths of the parallel sides.

9. *ABCD* is a rectangle. Calculate the shaded area.

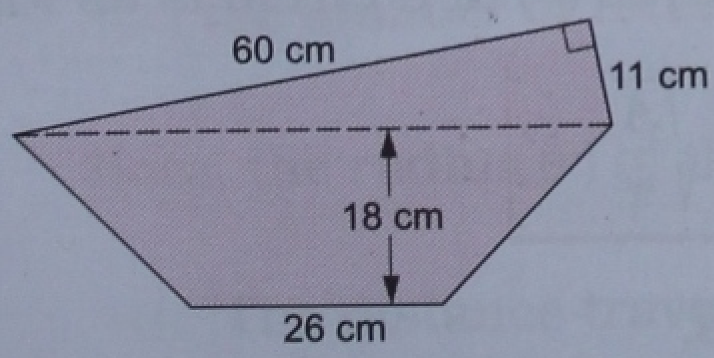


10. *ABCD* is a trapezium, *CDEF* is a parallelogram. Find the shaded area.

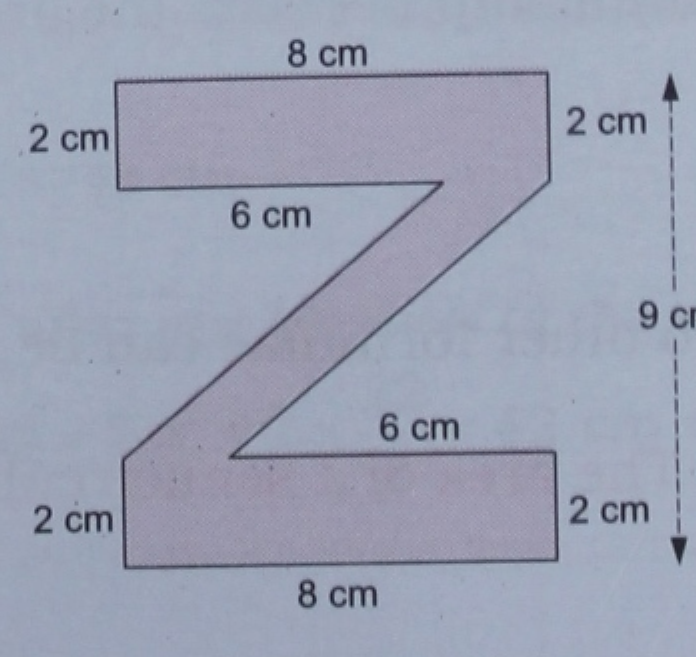


11. Find the area of each of the following shapes.

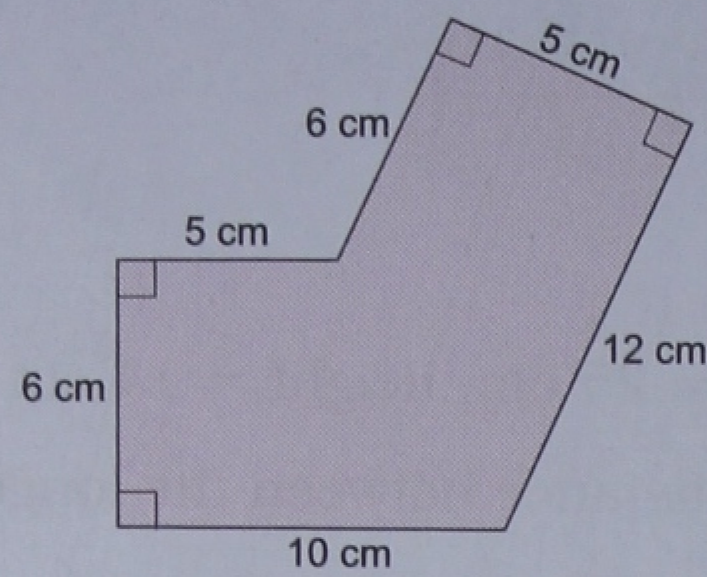
(i)



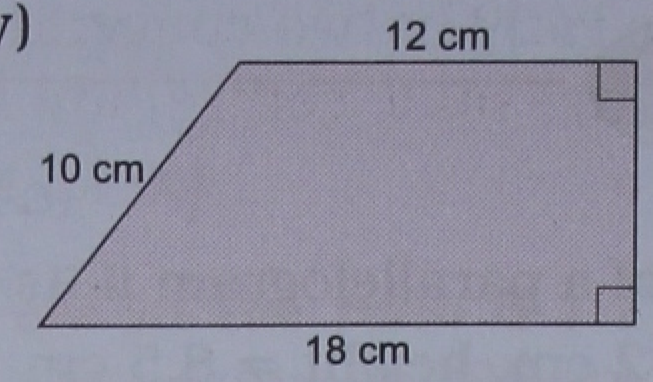
(ii)



(iii)



(iv)

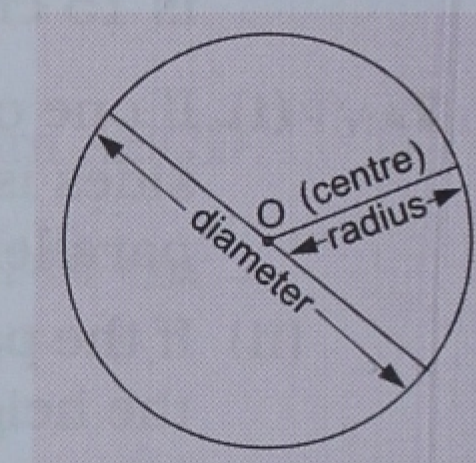


ANSWERS

1. (i) 102 cm^2 (ii) 350 cm^2	**2.** 19.2 cm	**3.** 240 cm^2, 17 cm, 68 cm
4. 120 cm, 1320 cm^2	**5.** 19.2 cm	**6.** (i) 209 cm^2 (ii) 495 cm^2
7. (i) 24 cm (ii) 19.5 cm	**8.** 28 cm, 44 cm	**9.** 156 cm^2
10. 48 cm^2	**11.** (i) 1113 cm^2 (ii) 42 cm^2 (iii) 90 cm^2 (iv) 120 cm^2	

Circle

A circle is the set of all points in a plane that are at a constant distance from a fixed point called the centre of the circle. The constant distance is called the radius of the circle. And a diameter of a circle is twice its radius.



Circumference of a circle

The length of the entire arc of a circle is called its circumference. The ratio of the length of the circumference (c) to the diameter (d) is the same for all circles. We represent this ratio by π (a Greek letter, pronounced pi). Thus, circumference (c) : diameter (d) = π. In symbols,

$$\frac{c}{d} = \pi \quad \text{or} \quad c = \pi d, \text{ where } \pi = \frac{22}{7} \quad \text{or} \quad 3.1416 \text{ approximately}$$

Since, diameter = 2 × radius, $c = 2\pi r$

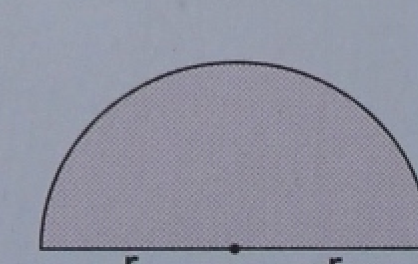
Area of a circle

The area of a circle is the measure of the region bounded by the circle. The relationship between the area (A) and the radius (r) of a circle is as follows.

$$A = \pi r^2, \quad r = \sqrt{\frac{A}{\pi}}$$

Two other formulae can be derived from this.

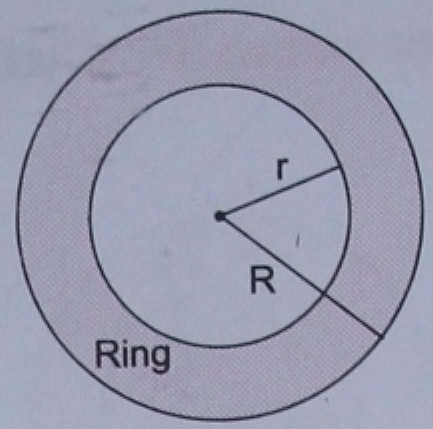
1. The area of a semicircular region of radius $(r) = \frac{1}{2}\pi r^2$.



2. If the radii of two concentric circles (that is, circles with the same centre) are r and R, $r < R$ then

the area of the ring (or annulus)

$$= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r).$$



Solved Examples

EXAMPLE 1 **Find the circumference and the area of a circle of (i) radius = 21 cm and (ii) diameter = 14 cm.**

Solution (i) The circumference of the circle $= 2\pi r = 2 \times \frac{22}{7} \times 21$ cm = 132 cm.

The area of the circle $= \pi r^2 = \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 1386 \text{ cm}^2$.

(ii) The diameter $(d) = 14$ cm, so the radius $= \frac{d}{2} = 7$ cm.

$\therefore$ the circumference of the circle $= 2\pi r = 2 \times \frac{22}{7} \times 7$ cm = 44 cm

and the area of the circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$.

EXAMPLE 2 **If the circumference of a circle is 132 cm, find its radius and area.**

Solution Here, $c = 132$ cm. Let r = the radius of the circle.

Now, $c = 2\pi r \Rightarrow r = \frac{c}{2\pi} = \frac{132}{2 \times \frac{22}{7}} \text{ cm} = \frac{132 \times 7}{2 \times 22} \text{ cm} = 21 \text{ cm}$.

$\therefore$ the area of the circle $= \pi r^2 = \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 1386 \text{ cm}^2$.

EXAMPLE 3 **If the area of a circle is 15400 cm^2, find its radius and circumference.**

Solution Here, $A = 15400 \text{ cm}^2$. Let r = the radius of the circle.

Then $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{15400}{\frac{22}{7}}} \text{ cm} = \sqrt{\frac{15400}{22} \times 7} \text{ cm} = \sqrt{700 \times 7} \text{ cm} = 70 \text{ cm}$.

$\therefore$ the circumference, $c = 2\pi r = 2 \times \frac{22}{7} \times 70$ cm = 440 cm.

EXAMPLE 4 **If the wheel of a bicycle has a diameter of 84 cm, calculate**

(i) the distance travelled by the cyclist after one rotation of the wheel, and

(ii) the number of rotations of the wheel if the cyclist covers a distance of 1.65 km.

Solution Here, the radius (r) of the wheel $= \frac{84}{2}$ cm = 42 cm.

(i) The distance travelled by the cyclist after one rotation of the wheel

= the circumference of the wheel $= 2\pi r = 2 \times \frac{22}{7} \times 42$ cm = 264 cm.

(ii) The cyclist covers 264 cm in 1 rotation of the wheel.

$\therefore$ the cyclist covers 1 cm in $\frac{1}{264}$ of a rotation of the wheel.

$\therefore$ the cyclist will cover 1.65 km, that is, 1650×100 cm in

$$\frac{1650 \times 100}{264} \text{ rotations} = 625 \text{ rotations.}$$

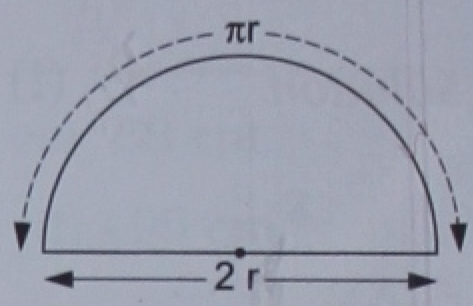
$\therefore$ the number of rotations = 625.

EXAMPLE 5 **A piece of wire 72 cm long is bent into the form of a semicircular arc bounded by its diameter. Find the radius.**

Solution Let the radius of the circle = r cm.

Then, the semicircular arc = $\frac{1}{2}$ the circumference of the circle

$= \frac{1}{2} \times 2\pi r$ cm $= \pi r$ cm.



Given, the semicircular arc + the diameter = 72 cm

$\Rightarrow \pi r + 2r = 72 \Rightarrow \frac{22}{7}r + 2r = 72 \Rightarrow \frac{36}{7}r = 72 \Rightarrow r = \frac{7}{36} \times 72 = 14.$

Hence, the radius = 14 cm.

EXAMPLE 6 **A metal wire is bent in the shape of a square of area 484 cm^2. If the same wire is bent to form a circle, find the area enclosed.**

Solution A side of the square $= \sqrt{\text{area}} = \sqrt{484}$ cm = 22 cm.

$\therefore$ the perimeter of the square $= 4 \times 22$ cm = 88 cm.

$\Rightarrow$ the length of the wire = 88 cm.

Let the radius of the circle formed by the wire = r cm.

Then, the circumference of the circle = length of the wire = 88 cm

$\Rightarrow 2\pi r = 88 \Rightarrow r = \frac{88}{2\pi} = \frac{88}{2 \times \frac{22}{7}} = \frac{44 \times 7}{22} = 14.$

$\therefore$ the radius of the circle = 14 cm.

$\therefore$ the area enclosed = the area of the circle $= \pi r^2 \text{ cm}^2 = \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= 616 \text{ cm}^2.$

EXAMPLE 7 **A wire is in the shape of a circle of area 3850 cm^2. If the same wire is bent in the form of a square, find the area of the square.**

Solution Let the radius of the circle = r cm.

Then its area $= \pi r^2 \text{ cm}^2 = 3850 \text{ cm}^2$ (given)

$\Rightarrow \pi r^2 = 3850 \Rightarrow r^2 = \frac{3850}{\pi} = \frac{3850}{22} \times 7 = 1225.$

$\therefore r = \sqrt{1225} = 35.$

$\therefore$ the circumference of the circle $= 2\pi r$ cm $= 2 \times \frac{22}{7} \times 35$ cm = 220 cm.

$\Rightarrow$ the length of the wire = 220 cm $\Rightarrow$ the perimeter of the square = 220 cm.

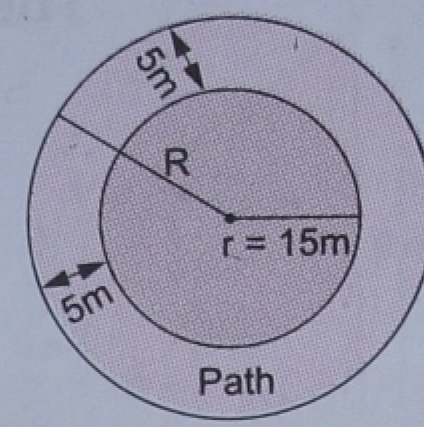
Let the length of a side of the square = a cm. So, its perimeter = $4a$ cm.

Then, $4a = 220 \Rightarrow a = 55.$

Hence, the area of the square $= 55^2 \text{ cm}^2 = 3025 \text{ cm}^2.$

EXAMPLE 8 **A circular park of diameter 30 m is surrounded by a circular path of uniform width 5 m. Find the area of the path.**

Solution Let the radius of the park = r. Then $r = \frac{30}{2}$ m = 15 m.



Given, the path is 5 m wide.

$\therefore$ the radius (R) of the outer circle = r + 5 m = 15 m + 5 m = 20 m.

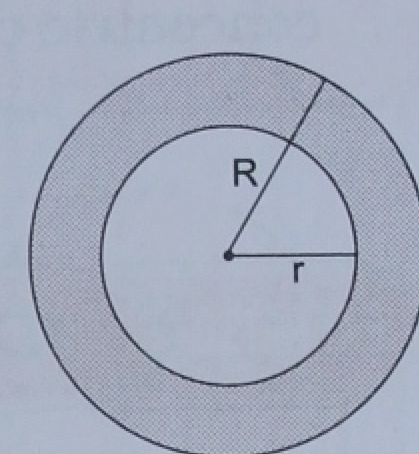
$\therefore$ the area of the path = the area of the outer circle – the area of the park

$$= \pi R^2 - \pi r^2 = \pi(R+r)(R-r) = \frac{22}{7} \times (20+15) \times (20-15)\ \text{m}^2$$

$$= \frac{22}{7} \times 35 \times 5\ \text{m}^2 = 550\ \text{m}^2.$$

EXAMPLE 9 **The area of the ring between two concentric circles is 990 cm^2. Find the radii of the two circles if their sum is 35 cm.**

Solution Let the radii of the outer circle and the inner circle be R cm and r cm respectively.



Given that $R + r = 35$. ... (1)

Now, the area of the ring = $\pi(R^2 - r^2)$ cm^2 = 990 cm^2 (given)

$\Rightarrow$ $\pi(R+r)(R-r) = 990$ $\Rightarrow$ $35\pi(R-r) = 990$ [using (1)]

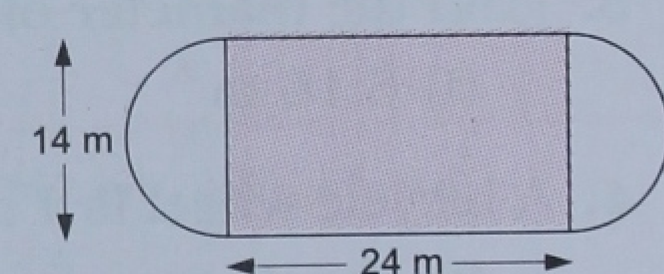
$\Rightarrow$ $R - r = \frac{990}{35\pi} = \frac{990}{35 \times 22} \times 7 = 9.$ $\therefore$ $R - r = 9$ ... (2)

Adding (1) and (2), $2R = 44 \Rightarrow R = 22$.

From (1), $r = 35 - R = 35 - 22 = 13$.

Hence, the radii of the two circles = 22 cm and 13 cm.

EXAMPLE 10 **A garden is made up of a rectangular patch of grass and two semicircular ponds as shown in the figure. If the length and width of the rectangular patch are 24 m and 14 m respectively then calculate (i) the perimeter of the garden and (ii) the total area of the garden.**



Solution (i) The length of the semicircular boundaries of the ponds

$$= \frac{2\pi r}{2} + \frac{2\pi r}{2} = \pi r + \pi r = 2\pi r = 2 \times \frac{22}{7} \times \frac{14}{2}\ \text{m} = 44\ \text{m}.$$

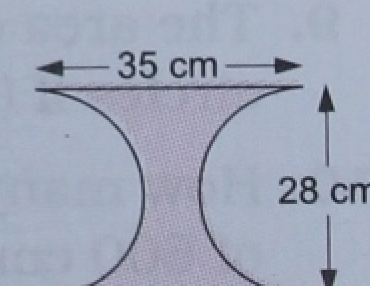
$\therefore$ the perimeter of the garden = 44 m + 2 × 24 m = 92 m.

(ii) The area of the ponds $= 2 \times \frac{1}{2}\pi r^2 = \frac{22}{7} \times \left(\frac{14}{2}\right)^2\ \text{m}^2 = 154\ \text{m}^2.$

The area of the patch = 24×14 m^2 = 336 m^2.

Hence, the total area of the garden = $(154 + 336)$ m^2 = 490 m^2.

EXAMPLE 11 **A rectangle of length 35 cm and width 28 cm has a semicircle cut out from each of its two sides, as shown. Find the area of the shaded part.**



Solution The area of the rectangle $= 35 \times 28 \text{ cm}^2 = 980 \text{ cm}^2$.

The area of one semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{28}{2}\right)^2 \text{ cm}^2 = 308 \text{ cm}^2$.

$\therefore$ the area of the shaded part = the area of the rectangle − 2 × area of a semicircle

$= 980 \text{ cm}^2 - 2 \times 308 \text{ cm}^2 = 364 \text{ cm}^2$.

Remember These

1. For a circle of diameter $= d$, radius $= r$, circumference $= c$ and area $= A$:

(i) $d = 2r, r = \frac{d}{2}$ (ii) $c = 2\pi r$ (or $d\pi$), $r = \frac{c}{2\pi}, d = \frac{c}{\pi}$

(iii) $A = \pi r^2, r = \sqrt{\frac{A}{\pi}}$ (iv) area of a semicircle $= \frac{1}{2}\pi r^2$

2. The area of ring $= \pi(R+r)(R-r)$, where R and r are the radii of the bigger and smaller concentric circles respectively.

EXERCISE 1D

1. Find the circumference and area of a circle of

(i) radius = 14 cm (ii) radius = 2.1 m

(iii) diameter = 35 cm (iv) diameter = 77 cm

2. Find the radius of a circle and its area if its circumference is

(i) 176 cm (ii) 11 m (iii) 16π cm

3. Find the diameter of a circle and its circumference if its area is

(i) 6.16 m^2 (ii) 394.24 cm^2 (iii) $25\pi \text{ cm}^2$

4. A bicycle wheel is 1 m 26 cm in diameter. Calculate

(i) the distance travelled in one rotation of the wheel, and

(ii) the number of rotations of the wheel needed to cover 1.98 km.

5. A piece of wire 54 cm long is bent into the shape of a semicircular arc bounded by its diameter. Find its radius.

6. A wire is bent in the form of a square of area 1936 cm^2. If the same wire is bent to form a circle, find the radius of the circle and the area enclosed.

7. A wire is in a circular shape of area 5544 cm^2. If the same wire is bent in the shape of a square, find the length of a side of the square and its area.

8. (i) A circular pond of diameter 28 m is surrounded by a path 7 m wide. Find the area of the path.

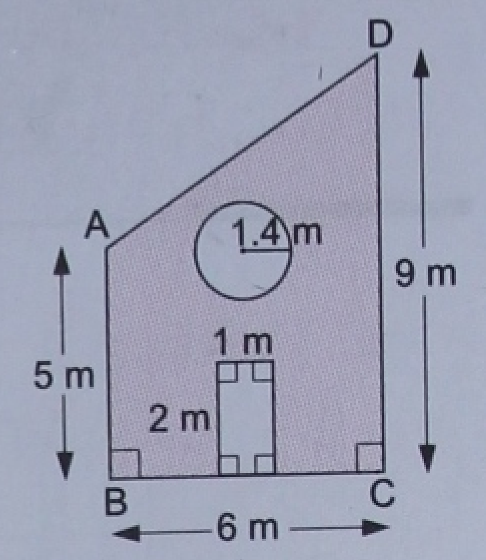
(ii) Find the area of a circular path 2 m wide surrounding a circular plot of radius 20 m.

9. The area of the ring between two concentric circles is 3168 cm^2. Find the radii of the two circles if (i) their sum is 42 cm, and (ii) their difference is 28 cm.

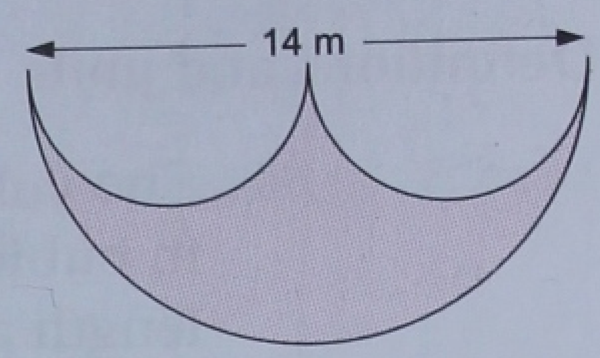
10. How many plants can be planted in a circular garden of circumference 26.40 m, if a space of 600 cm^2 is to be allowed for each plant?

11. A circle of circumference 88 cm is cut out of a square sheet of paper of side 30 cm. What is the area of the paper left?

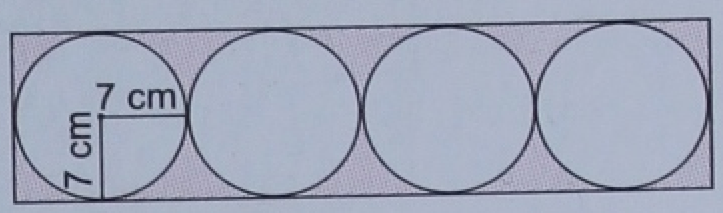
12. (i) If *ABCD* is a trapezium, find the area of the shaded part.



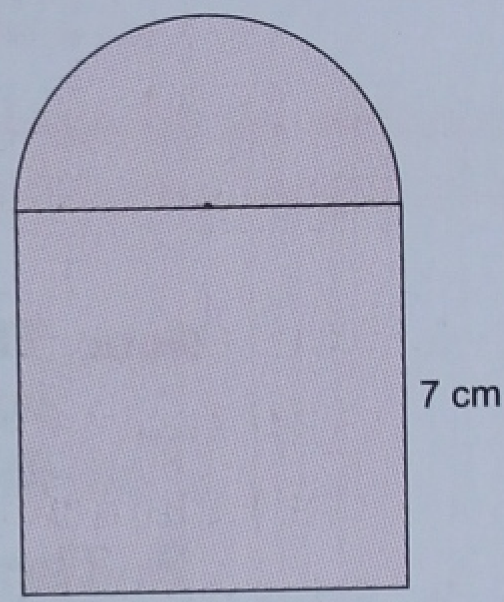
(ii) Two semicircles of equal radii are cut out of a semicircular piece of cardboard. Find the area of the shaded part.



(iii) The rectangle encloses four circles. The radius of each circle is 7 cm. Calculate the area of (a) the rectangle and (b) the shaded part.



(iv) Find the area of the shaded portion formed by a semicircle above a square.



ANSWERS

1. (i) 88 cm, 616 cm^2 (ii) 13.2 m, 13.86 m^2 (iii) 110 cm, 962.5 cm^2 (iv) 242 cm, 4658.5 cm^2

2. (i) 28 cm, 2464 cm^2 (ii) 1.75 m, 9.625 m^2 (iii) 8 cm, 64π cm^2

3. (i) 2.8 m, 8.8 m (ii) 22.4 cm, 70.4 cm (iii) 10 cm, 10π cm **4.** (i) 3 m 96 cm (ii) 500

5. 10.5 cm **6.** 28 cm, 2464 cm^2 **7.** 66 cm, 4356 cm^2

8. (i) 770 m^2 (ii) 264 m^2 **9.** (i) 33 cm, 9 cm (ii) 32 cm, 4 cm **10.** 924

11. 284 cm^2

12. (i) 33.84 m^2 (ii) 38.5 m^2 (iii) (a) 784 cm^2 (b) 168 cm^2 (iv) 68.25 cm^2

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