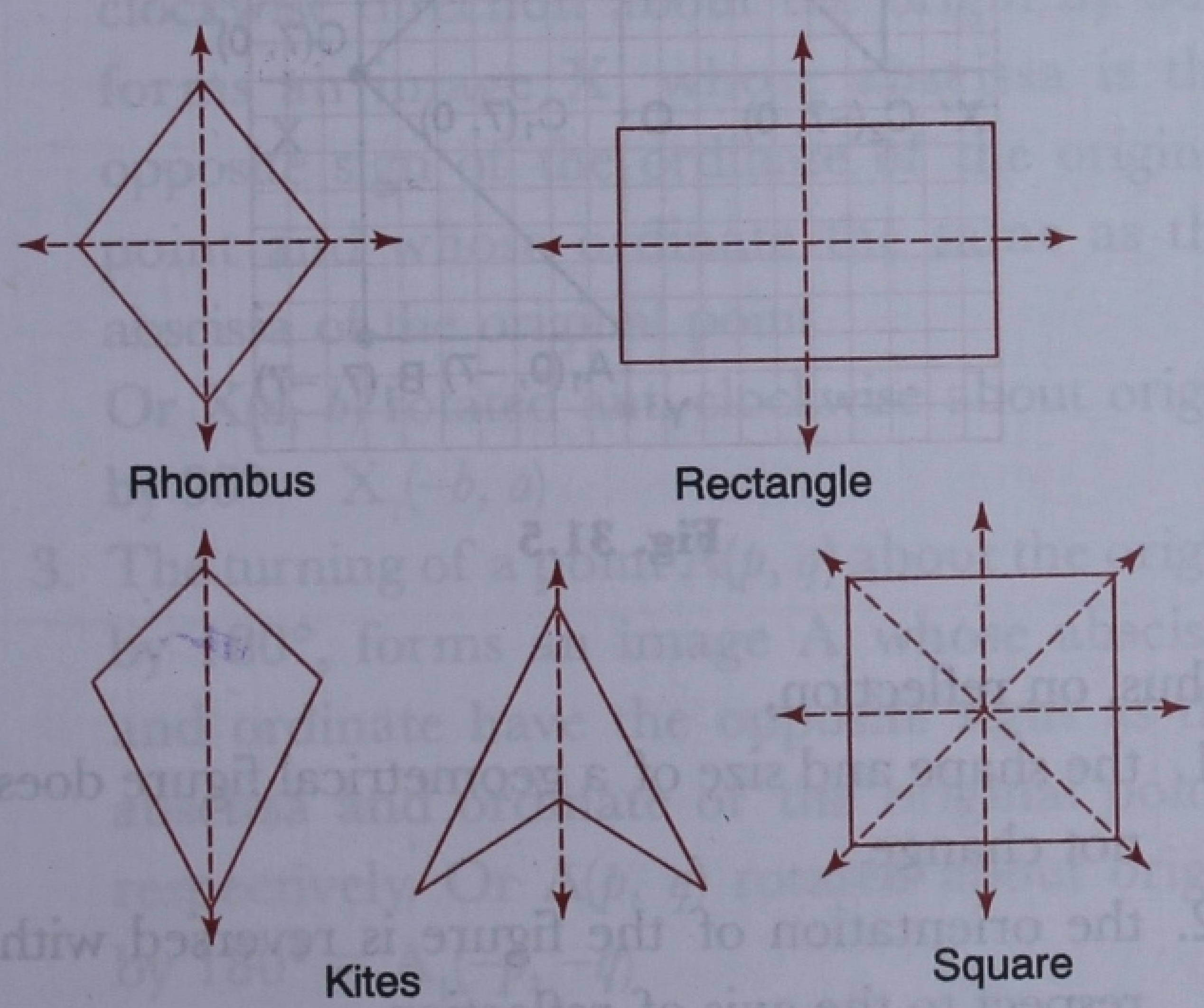
31 SYMMETRY, REFLECTION, AND ROTATION

- Symmetry
- Reflection
- Rotation

Symmetry

Symmetry refers to the exact match in size and shape between two halves, parts of sides of an object or a figure.

An axis of symmetry divides a plane figure into two congruent halves.
A rhombus is symmetrical about its diagonals.
A rectangle is symmetrical about the two axes joining the mid-points of opposite sides.
A kite is symmetrical about its vertical diagonal.
A square is symmetrical about its two diagonals and about the two axes joining the mid-points of opposite sides.



A line segment is symmetrical about its perpendicular bisector. In Figure 31.1, all points on line segment PQ are symmetrical about perpendicular bisector XY, with their respective images. Point A is symmetrical to its image, point A_1, as both A and A_1 are equidistant from any point on XY.

An angle is symmetrical about its bisector. In Figure 31.2, all points on arm OA are symmetrical about angle bisector OC, with their respective images. Point X is symmetrical to its image, point X_1, as both X and X_1 are equidistant from any point on ray OC.

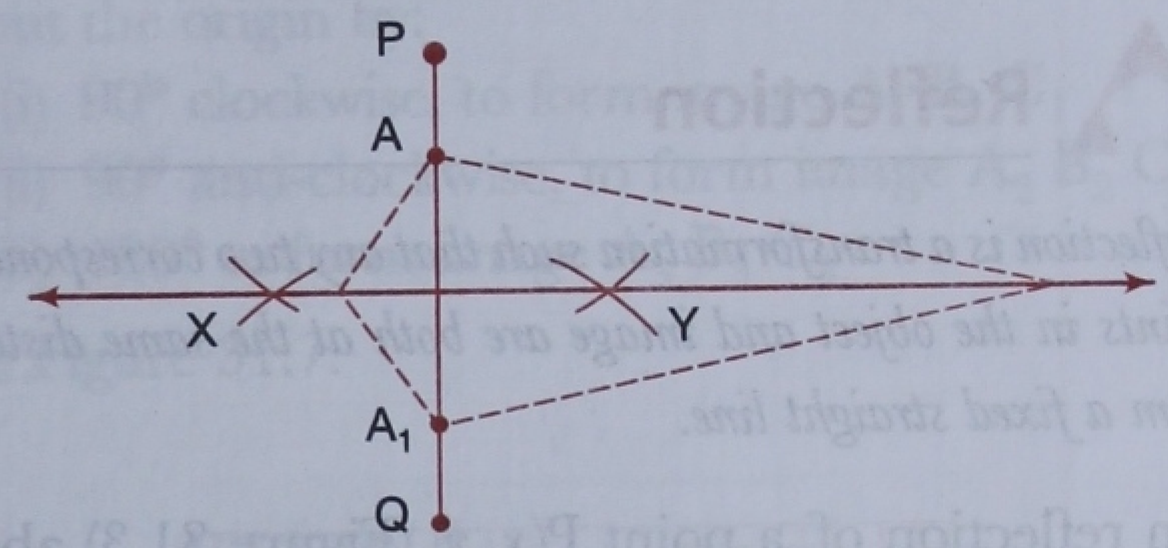


Fig. 31.1

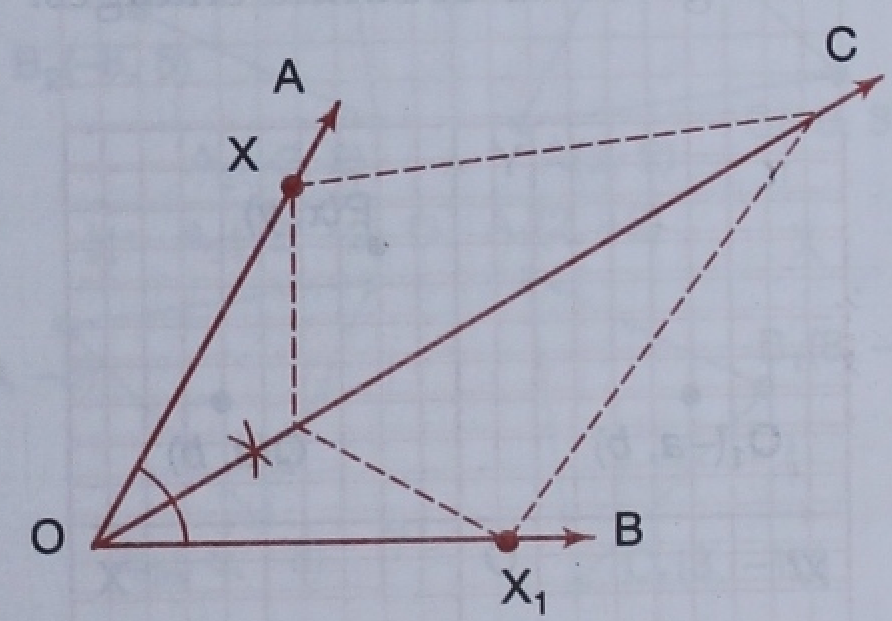
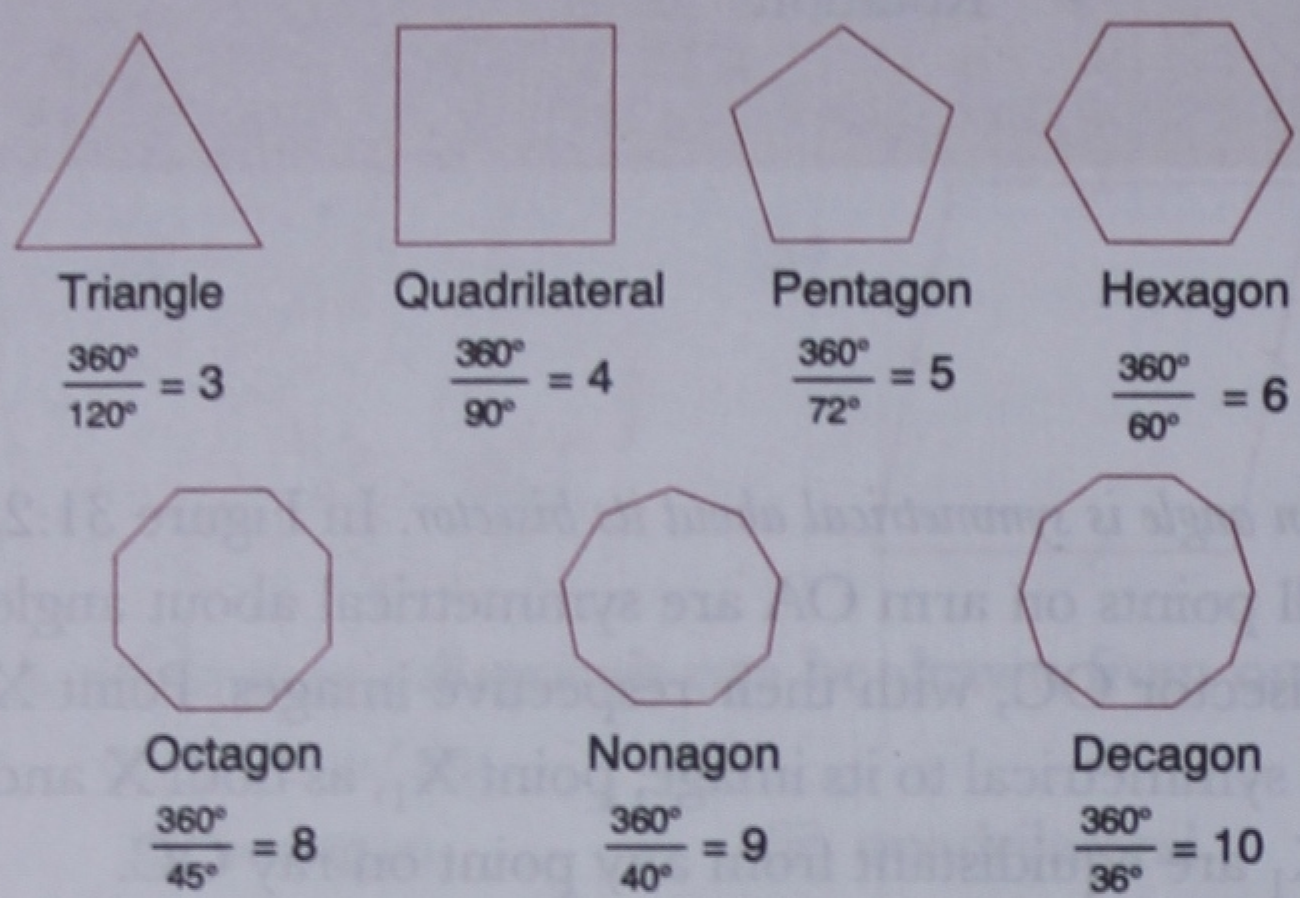


Fig. 31.2

A plane figure through which no axis of symmetry can be drawn is said to be **asymmetric**. However an asymmetric figure may have point symmetry. If a parallelogram is rotated about the point O where its diagonals intersect, after half-a-turn or a rotation of 180°, it will look exactly the same in size, shape, and orientation.

Rotational Order of Regular Polygons

The number of times a figure looks the same in a complete rotation of 360° is known as the **rotational order** of the figure.



Reflection

Reflection is a transformation such that any two corresponding points in the object and image are both at the same distance from a fixed straight line.

On reflection of a point P(x, y) (Figure 31.3) about the X axis, the abscissa of its image P_1 remains the same, but the sign of its ordinate changes.

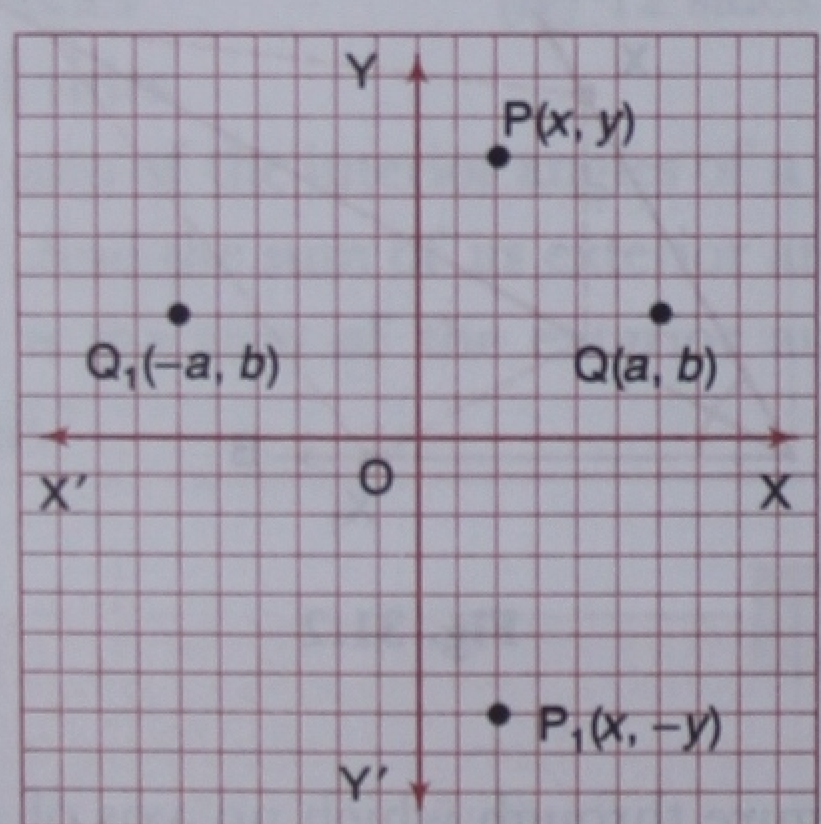


Fig. 31.3

On reflection of a point Q(a, b) about the Y axis, the ordinate of its image Q_1 remains the same, but the sign of its abscissa changes.

Example 1: Reflection of line segment P(3, 7) Q(7, 3) in Figure 31.4 about the X and Y axes to form images P_1Q_1 and P_2Q_2 respectively.

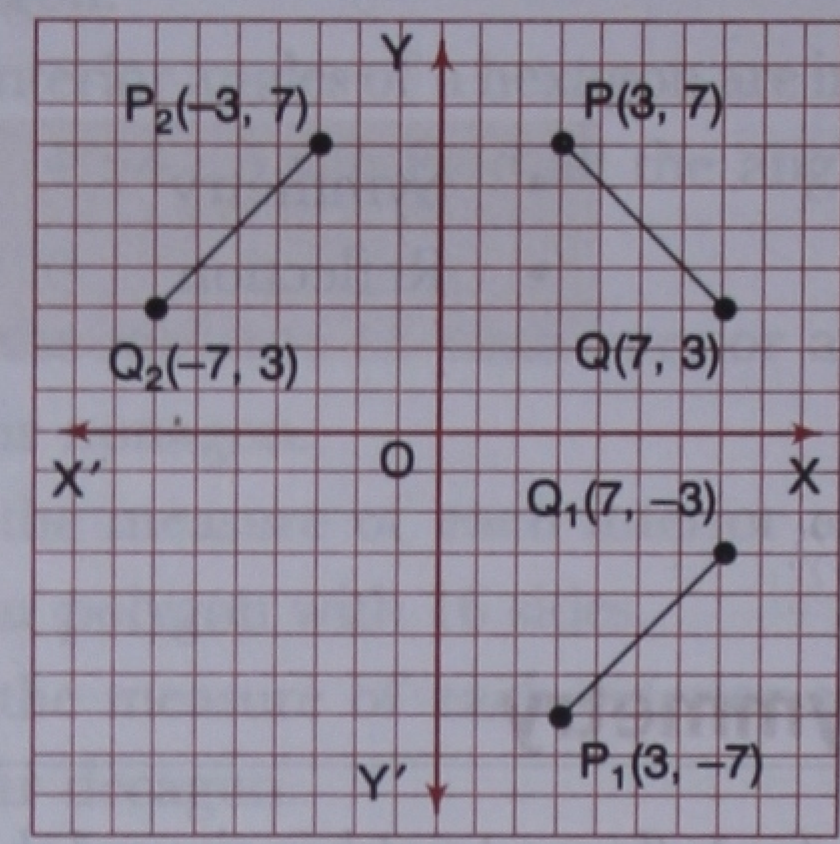


Fig. 31.4

Example 2: Reflection of ΔA(0, 7) B(7, 7) C(7, 0) in Figure 31.5 about the X and Y axes to form images $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$ respectively.

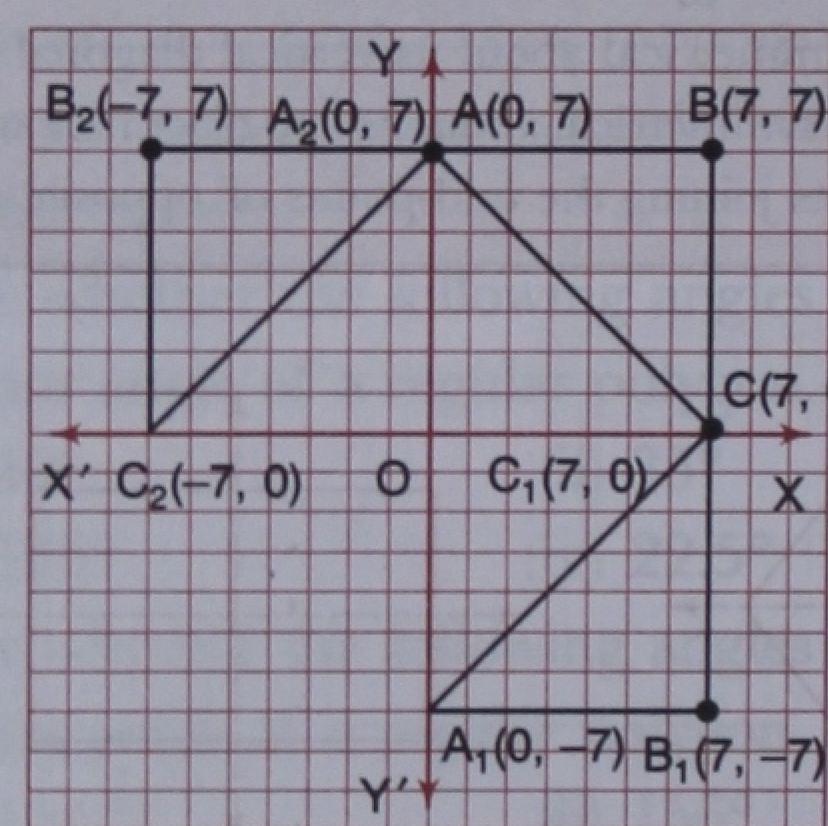


Fig. 31.5

Thus, on reflection,

1. the shape and size of a geometrical figure does not change.
2. the orientation of the figure is reversed with respect to the axis of reflection.
3. the position of a point on the axis of reflection does not change.
4. the image of a line perpendicular to the axis of reflection will also be perpendicular to the axis of reflection.

5. the image of a line parallel to the axis of reflection will also be parallel to the axis of reflection.

Try this!
Plot a point (4, 8) and find its image when it is reflected about the X axis.

Rotation

Rotation is a transformation about a fixed point such that every point in the object turns through the same angle relative to the fixed point.

1. The turning of a point P(x, y) in a clockwise direction about the origin by 90°, forms an image P_1 whose abscissa is the same as the oridinate of the original point and whose ordinate is the opposite sign of the abscissa of the original point.
   Or P(x, y) rotated clockwise about origin by 90° = $P_1(y, -x)$
2. The turning of a point X(a, b) in an anti-clockwise direction about the origin by 90°, forms an image X_1 whose abscissa is the opposite sign of the ordinate of the original point and whose ordinate the same as the abscissa of the original point.
   Or X(a, b) rotated anti-clockwise about origin by 90° = $X_1(-b, a)$
3. The turning of a point A(p, q) about the origin by 180°, forms an image A_1 whose abscissa and ordinate have the opposite signs as the abscissa and ordinate of the original points respectively. Or A(p, q) rotated about origin by 180° = $A_1(-p, -q)$

Example 3: Rotation of line segment P(2, 9) Q (8, 3) about the origin by:

(i) 90° clockwise, to form image P_1Q_1
(ii) 90° anti-clockwise, to form image P_2Q_2
(iii) 180° to form image P_3Q_3

See Figure 31.6.

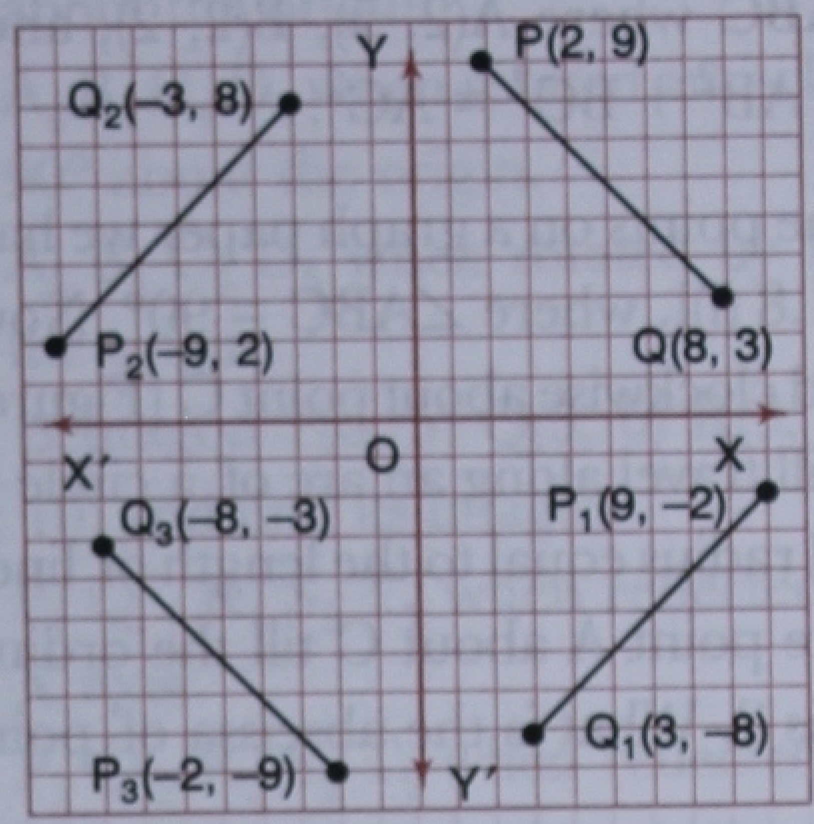


Fig. 31.6

Example 4: Rotation of ΔA(2, 2) B(5, 8) C(10, 3) about the origin by:

(i) 90° clockwise, to form image A_1 B_1 C_1
(ii) 90° anti-clockwise, to form image A_2 B_2 C_2
(iii) 180° to form image A_3 B_3 C_3

See Figure 31.7.

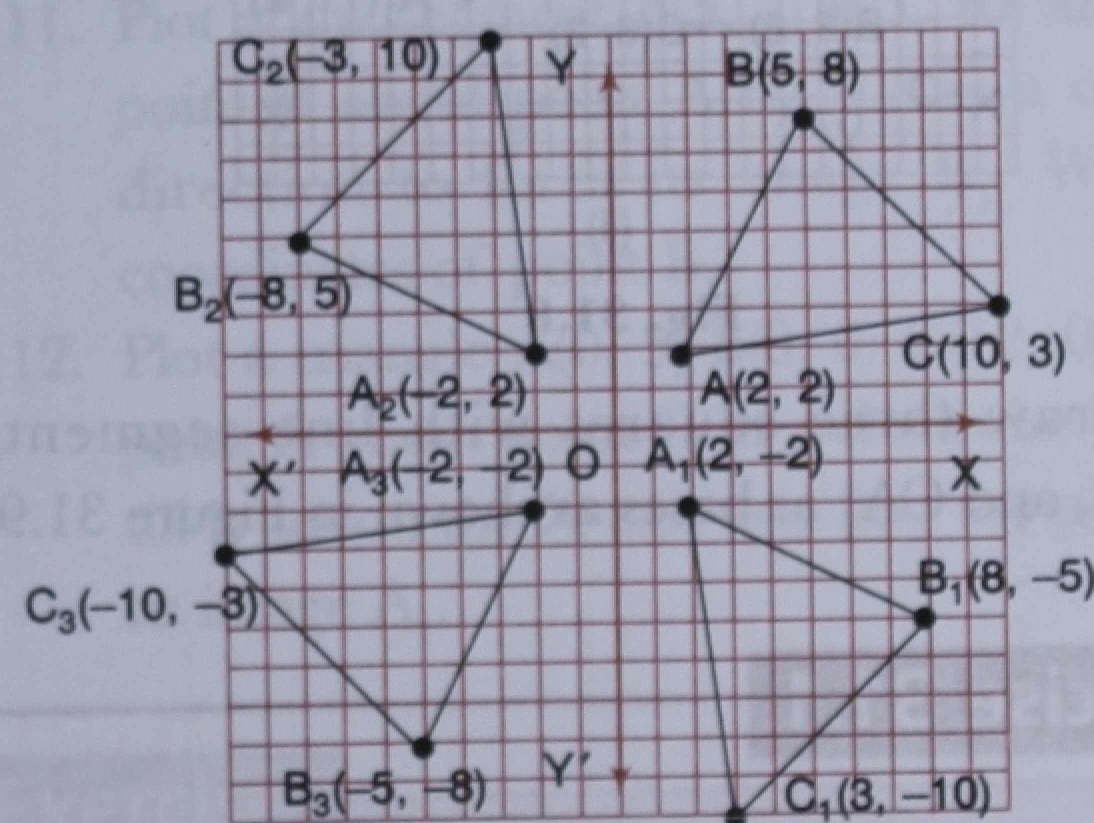


Fig. 31.7

Try this!
Plot a point (4, 8) and find its image when it is rotated about the origin through 180°.

Proof of Pythagoras' Theorem by Rotation

Given ΔABC where A(2, 5), B(2, 2), and C(6, 2), show that $AB^2 + BC^2 = AC^2$.

Plotting the points on a graph paper we have ΔABC [Figure 31.8 (i)], where $\angle ABC = 90°$. Now if point A is rotated clockwise about point C [Figure 31.8(ii)], point A will travel along an arc of a circle with C as centre and radius equal to the length of line segment AC. Rotate point A about C till the ordinate of its image A_1 is 2. What is the abscissa of point A_1?

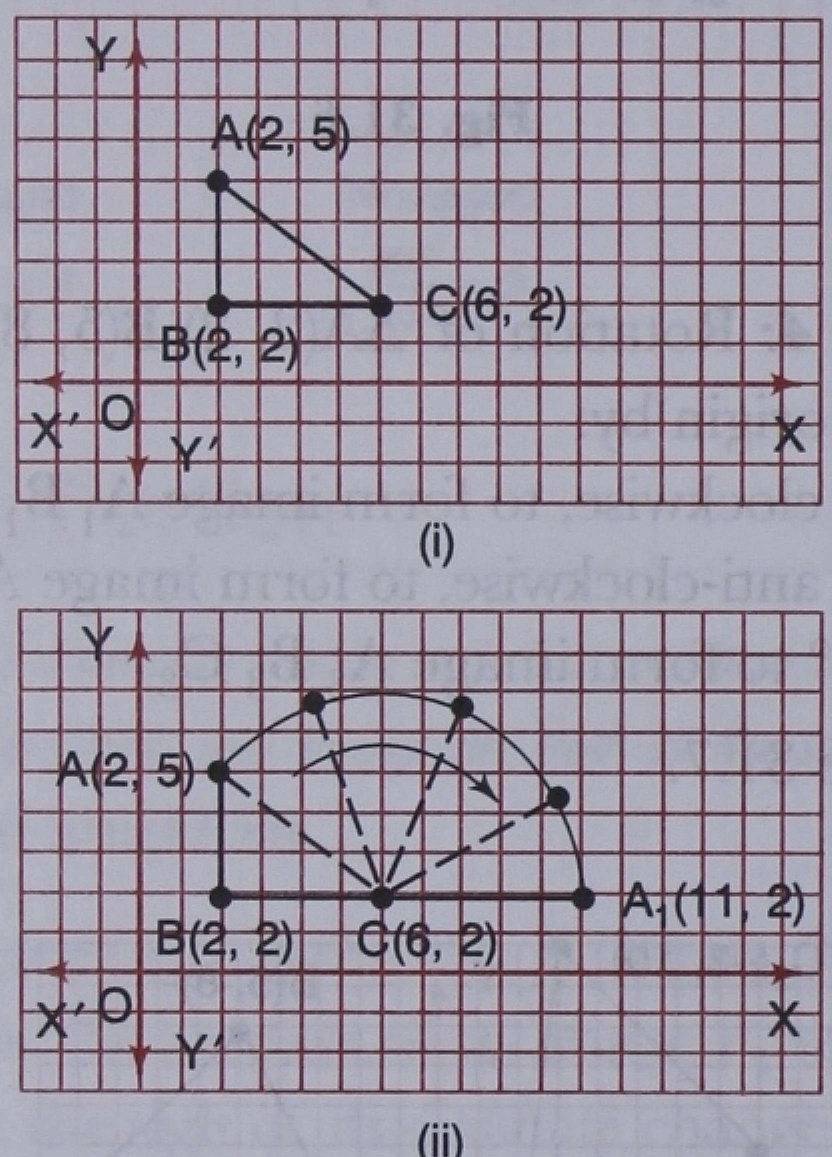


Fig. 31.8

Now draw three squares with line segments AB, BC, and CA_1 as bases as shown in Figure 31.9.

AB is a vertical line segment whose length measures ordinate – ordinate = 5 – 2 = 3 units

$\Rightarrow$ Area of square ABED = $AB^2 = 3 \times 3$
$= 9 \text{ unit}^2$

BC is a horizontal line segment whose length measures abscissa – abscissa = 6 – 2 = 4 units

$\Rightarrow$ Area of square BCGF = $BC^2 = 4 \times 4$
$= 16 \text{ unit}^2$

CA_1 is a horizontal line segment whose length measures abscissa – abscissa = 11 – 6 = 5 units

$\Rightarrow$ Area of square CA_1IH= $CA_1^2 = 5 \times 5$
$= 25 \text{ unit}^2$

But $9 \text{ unit}^2 + 16 \text{ unit}^2 = 25 \text{ unit}^2$

$\Rightarrow \quad AB^2 + BC^2 = CA_1^2$

But $CA_1 = CA$ (as a point and its image on rotation are equidistant from their point of symmetry)

$\Rightarrow \quad AB^2 + BC^2 = CA^2$ Q.E.D

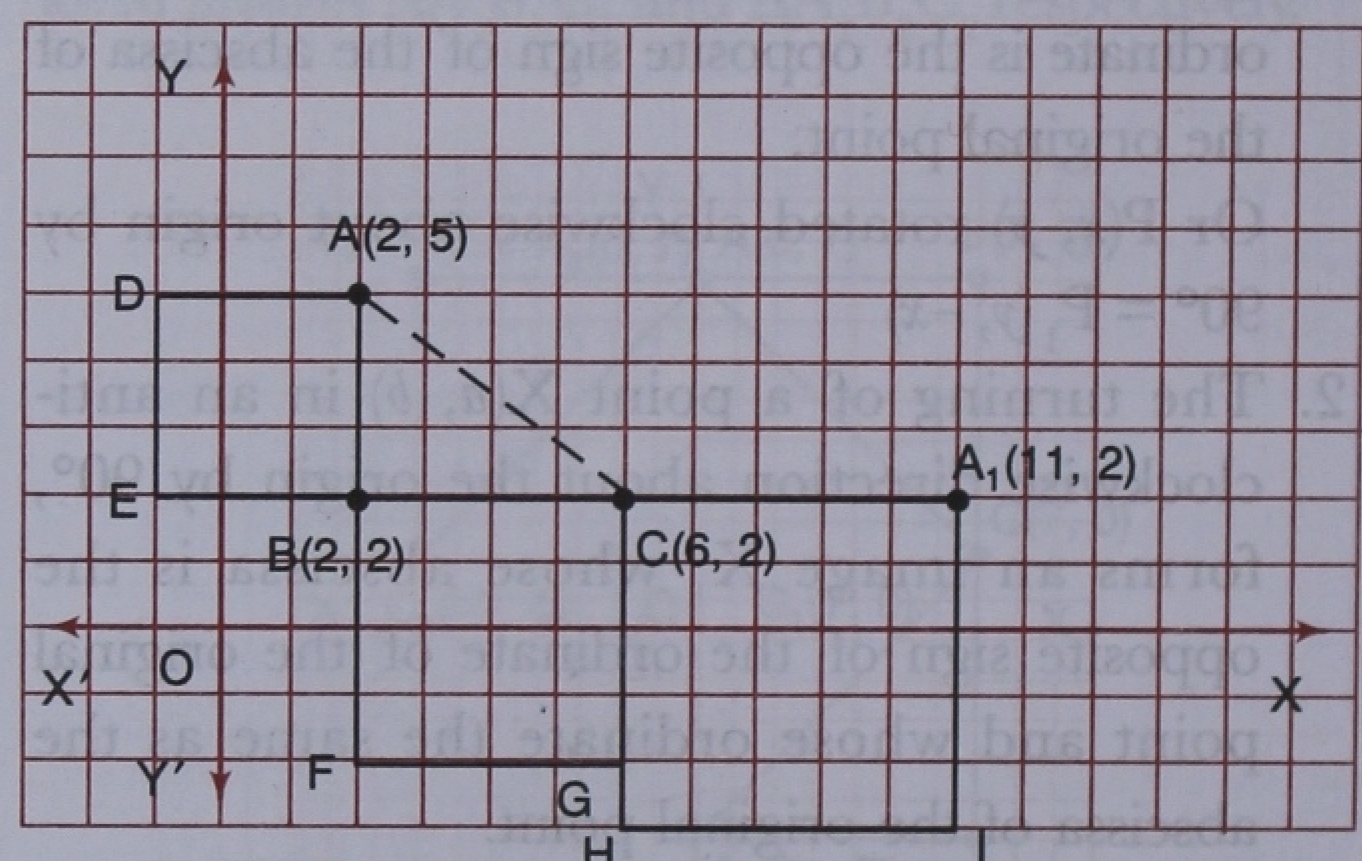


Fig. 31.9

Exercise 31.1

1. How many axes of symmetry can be drawn through:
   (i) an oval
   (ii) the English letter C
   (iii) an isosceles trapezium
   (iv) a regular pentagon
2. Identify any three letters of the English alphabet through which no line of symmetry can be drawn.
3. What is the order of rotational symmetry of:
   (i) an oval
   (ii) an isosceles triangle
   (iii) the English letter J
   (iv) a circle
4. Plot a point P(–3, 8) and find its image:
   (i) P_1 when reflected about the X axis.
   (ii) P_2 when reflected about the Y axis.
   (iii) P_3 when rotated clockwise by 90° about the origin.
   (iv) P_4 when rotated anti-clockwise by 90° about the origin.
   (v) P_5 when rotated by 180° about the origin.

5. Plot line segment A(1, 9) B(12, 2) and find its image:
   (i) $A_1 B_1$ when reflected about the X axis.
   (ii) $A_2 B_2$ when reflected about the Y axis.
   (iii) $A_3 B_3$ when rotated clockwise by 90° about the origin.
   (iv) $A_4 B_4$ when rotated anti-clockwise by 90° about the origin.
   (v) $A_5 B_5$ when rotated by 180° about the origin.
6. Plot line segment A(–6, 9) B(6, 9) and find its image:
   (i) $A_1 B_1$ when reflected about the X axis.
   (ii) $A_2 B_2$ when reflected about the Y axis.
   (iii) $A_3 B_3$ when rotated clockwise by 90° about the origin.
   (iv) $A_4 B_4$ when rotated anti-clockwise by 90° about the origin.
   (v) $A_5 B_5$ when rotated by 180° about the origin.
7. Plot triangle P(3, 1) Q(7, 9) R(11, 2) and find its image:
   (i) $P_1 Q_1 R_1$ when reflected about the X axis.
   (ii) $P_2 Q_2 R_2$ when reflected about the Y axis.
   (iii) $P_3 Q_3 R_3$ when rotated clockwise by 90° about the origin.
   (iv) $P_4 Q_4 R_4$ when rotated anti-clockwise by 90° about the origin.
   (v) $P_5 Q_5 R_5$ when rotated by 180° about the origin.
8. Plot quadrilateral A(1, 9) B(3, 1) C(9, 1) D(14, 12) and find its image:
   (i) $A_1 B_1 C_1 D_1$ when reflected about the X axis.
   (ii) $A_2 B_2 C_2 D_2$ when reflected about the Y axis.
   (iii) $A_3 B_3 C_3 D_3$ when rotated clockwise by 90° about the origin.
   (iv) $A_4 B_4 C_4 D_4$ when rotated anti-clockwise by 90° about the origin.
   (v) $A_5 B_5 C_5 D_5$ when rotated by 180° about the origin.
9. Plot pentagon P(0, 5) Q(3, 0) R(9, 0) S(12, 6) T(7, 10) and find its image:
   (i) $P_1 Q_1 R_1 S_1 T_1$ when reflected about the X axis.
   (ii) $P_2 Q_2 R_2 S_2 T_2$ when reflected about the Y axis.
   (iii) $P_3 Q_3 R_3 S_3 T_3$ when rotated clockwise by 90° about the origin.
   (iv) $P_4 Q_4 R_4 S_4 T_4$ when rotated anti-clockwise by 90° about the origin.
   (v) $P_5 Q_5 R_5 S_5 T_5$ when rotated by 180° about the origin.
10. If point P(5, 5) in the first quadrant of a graph is rotated about the origin by 180°, find the axis about which it is reflected.
11. Plot a line segment A(5, 2) B(5, 10) and rotate point B about point A by 90° in a clockwise direction to form its image B_1. Write the coordinates of point B_1.
12. Plot a triangle A(0, 5) B(0, 0) C(12, 0). Rotate point A about point C in a clockwise direction till it lies on the X-axis and find the position of its image A_1.

Revision Exercise

1. How many axes of symmetry can be drawn through:
   (i) an equilateral triangle (ii) a regular octagon
   (iii) the English letter H. (iv) a square
2. What is the order of rotational symmetry of:
   (i) a rectangle
   (ii) an isosceles trapezium
   (iii) the English Letter R.
   (iv) a rhombus .
3. Plot line segment A (–3, 7) B (3, 7) and find its image:
   (i) $A_1 B_1$ when reflected about the X-axis.
   (ii) $A_2 B_2$ when reflected about the Y-axis.
   (iii) $A_3 B_3$ when rotated clockwise by 90° about the origin.
   (iv) $A_4 B_4$ when rotated anti-clockwise by 90° about the origin.
4. Plot a line segment P (6, 3) Q (6, 15) and rotate point Q about point P by 90° in a clockwise direction to form its image B_1. Write the co-ordinates of point Q_1.