30 POLYGONS

- Types of Polygons
- Sum of Angles in a Polygon
- Regular Polygons

Introduction

A polygon is a closed figure formed when three or more than three non-collinear co-planar points are joined by line segments. The term comes from 'poly', meaning many, and the suffix 'gon' that comes from the Greek word 'gonia', meaning angle.

As a polygon is a closed figure,

1. all its points are shared by the line segments which form its sides.
   In Figure 30.1, ABCDE is a polygon as all points are part of two line segments each, in AB, BC, CD, DE, and EA which form its sides.

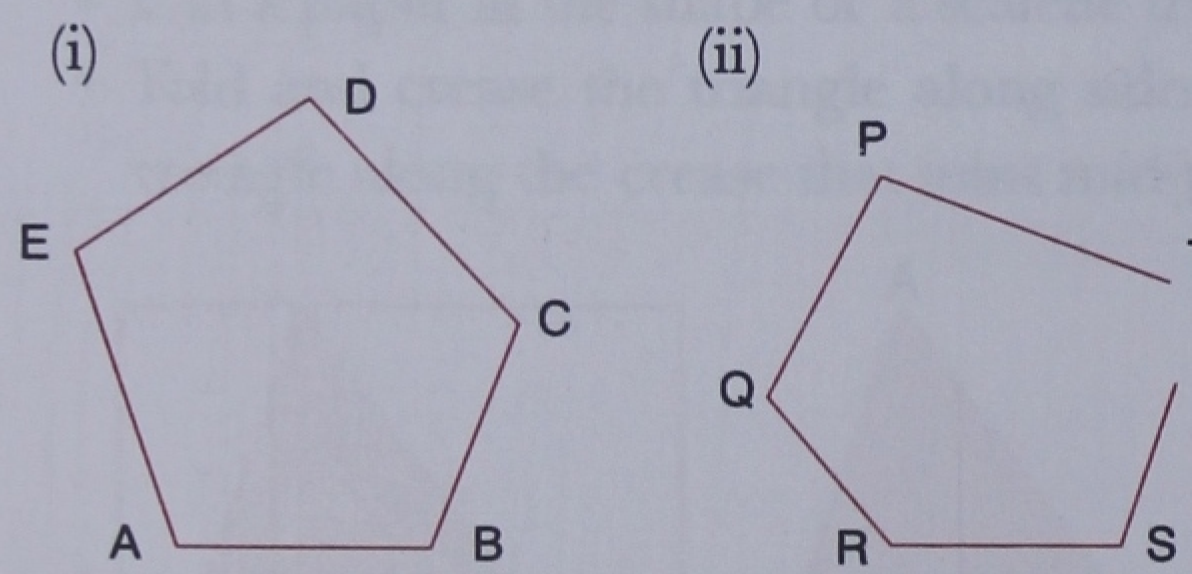


Fig. 30.1

   PQRST is not a closed figure as points T and S are part of only one line segment each. So, PQRST is not a polygon.

2. each point is shared by only two line segments which form its sides.
   In Figure 30.2, figure (ii), i.e., ABCDEFGHIJK is a polygon. In figure (i), i.e., ABCDEFGH, C is a point shared by 4 line segments that form its sides, viz. HC, CD, GC, and CB. Thus it is not a polygon. Similarly, figure (iii), i.e., ABCDEFGH is also not a polygon as two points, C and D are shared by 3 line segments each.

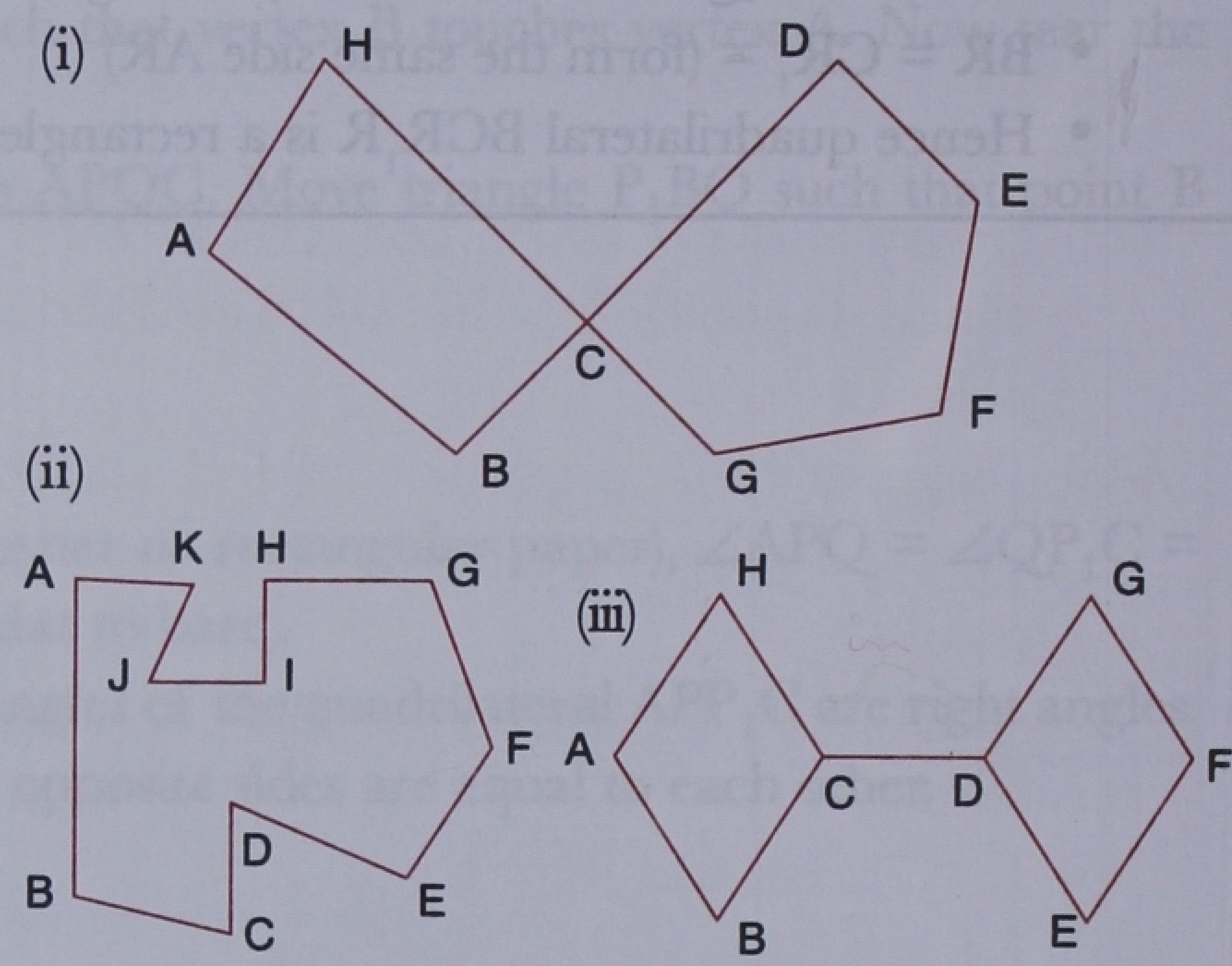


Fig. 30.2

Types of Polygons

If each of the angles in a polygon measures less than 180°, it is known as a convex polygon.

If one or more than one angle in a polygon measures more than 180°, it is known as a concave polygon.

Unless specified otherwise, a polygon, in geometry, is considered to be a convex polygon.

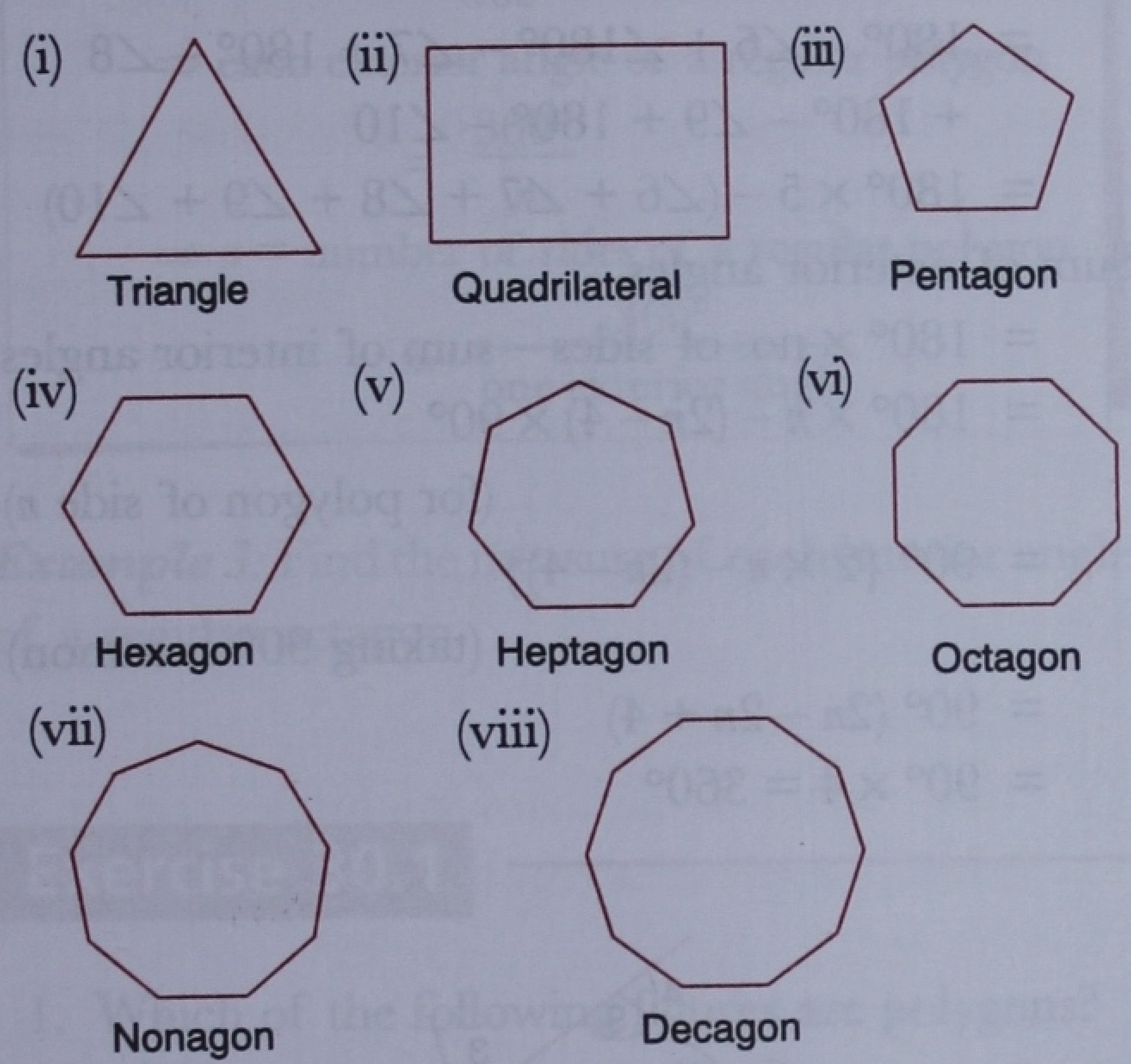
Names of Polygons

A polygon is named according to the number of angles it has, which is equal to the number of vertices as well as the number of sides.

The Greek prefixes that describe numbers are as follows:

Tri- = 3, Quadri- = 4, Penta- = 5, Hexa– = 6, Hepta- = 7, Octa- = 8, Nona- = 9, and Deca- = 10

Using these Greek prefixes, the polygons are named as follows:



We know that the number of diagonals that can be drawn from any vertex of a quadrilateral is 1, while none can be drawn from any vertex of a triangle. A diagonal connects any two **non-consecutive** vertices of a polygon.

Thus, from a vertex on a pentagon only 2 diagonals can be drawn, while from a vertex on a decagon 7 diagonals can be drawn (Figure 30.3).

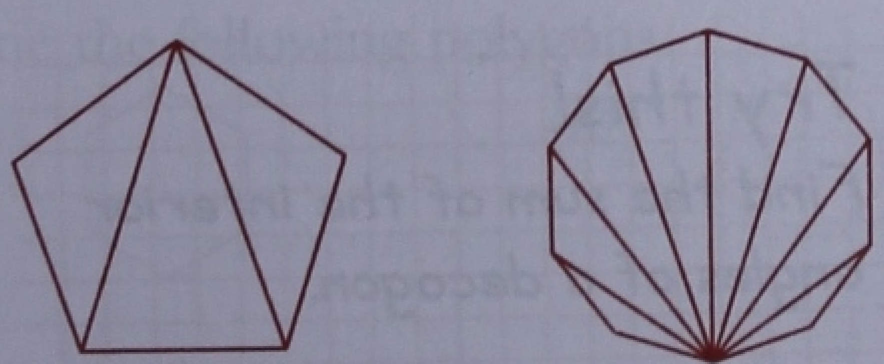


Fig. 30.3

We observe that from any one vertex of a polygon $(n - 3)$ diagonals can be drawn, where n is the number of sides in the polygon.

Sum of Angles in a Polygon

Interior Angles of a Polygon

We can see from Figure 30.4 (i), (ii), and (iii) how $(n - 3)$ diagonals drawn from any one vertex of a polygon with n sides divide the polygon into $(n - 2)$ triangles.

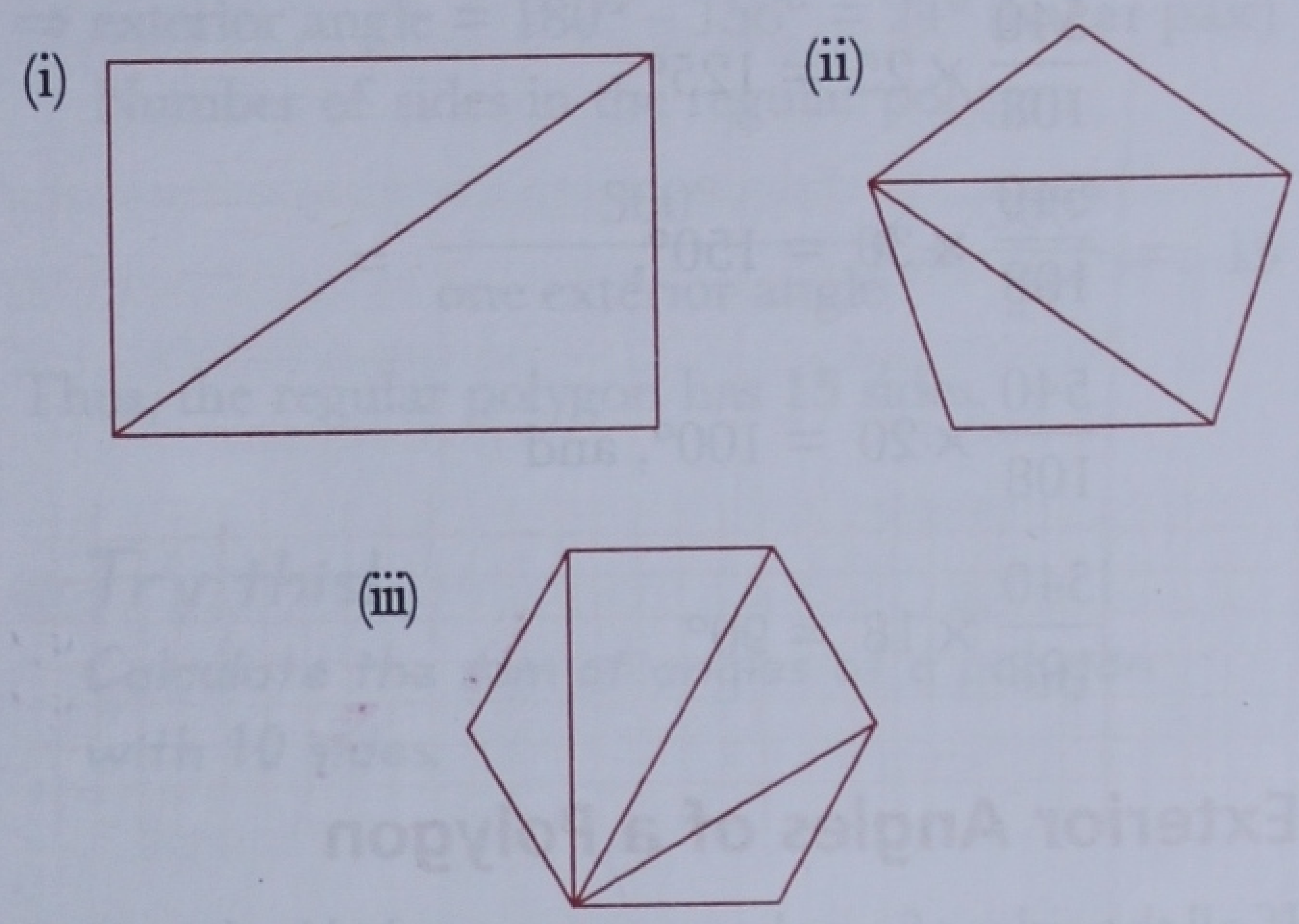


Fig. 30.4

Thus, in a decagon, number of sides $= n = 10$

number of diagonals from one vertex $= n - 3$

$= 7$

number of triangles formed $= n - 2 = 8$

But the sum of the interior angles of all triangles $= 180°$

$\therefore$ the sum of the interior angles of a polygon with n sides $= (n - 2) \times 180°$

$= (n - 2) \times 2 \times 90°$

$= (2n - 4)$ right angles

The sum of the interior angles of a polygon with n sides is equal to $(2n - 4)$ right angles.

Example 1: Find the sum of interior angles of a polygon with 14 sides.

Here $n = 14 \Rightarrow (2n - 4) = (2 \times 14 - 4)$

$= 28 - 4 = 24$

The sum of interior angles of a polygon with 14 sides = $(2n - 4)$ right angles = $24 \times 90° = 2160°$

Example 2: The interior angles of a pentagon are in the ratio 15 : 25 : 30 : 20 : 18. Find all its angles.

The sum of the interior angles of a pentagon = $(2n - 4)$ right angles.

As $n = 5$, $(2 \times 5 - 4) \times 90° = (10 - 4) \times 90°$
$= 6 \times 90° = 540°$

$15 + 25 + 30 + 20 + 18 = 108$

Thus, the angles of the pentagon are $\frac{540}{108} \times 15 = 75°$,

$\frac{540}{108} \times 25 = 125°$,

$\frac{540}{108} \times 30 = 150°$,

$\frac{540}{108} \times 20 = 100°$, and

$\frac{540}{108} \times 18 = 90°$

Exterior Angles of a Polygon

If all the sides of a polygon are extended in the same order, the exterior angles of the polygon are formed.

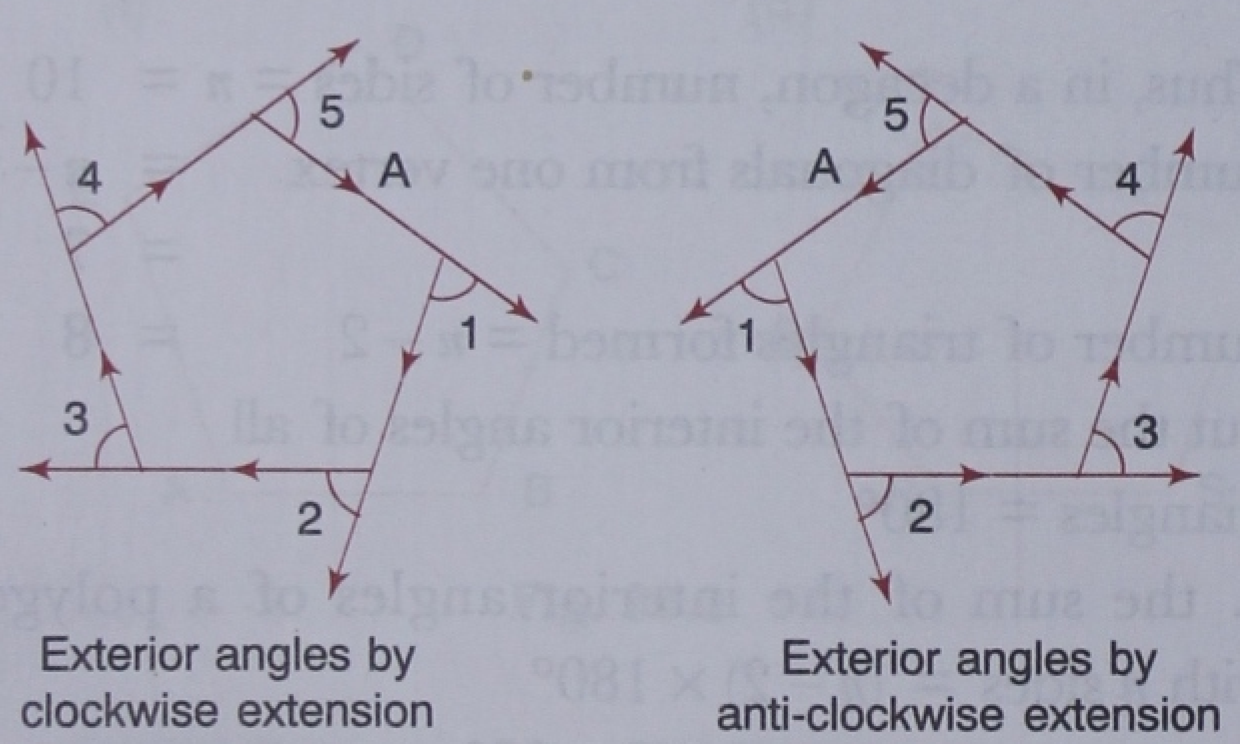


Fig. 30.5

Method I

Let us think of a man who starts walking from point A along the sides of either of the pentagons shown in Figure 30.5. After covering one side he turns by ∠1 to walk along the second side and then by ∠2 to walk along the third side. When he makes the final turn by ∠5 to return to A, he is facing exactly the same direction as he was facing when he had started off. Thus, he has turned by one complete angle or 360°.

A polygon can have any number of sides but the sum of its exterior angles will always be equal to 360° or 4 right angles.

Method II

In Figure 30.6,

$\angle 1 = 180° - \angle 6$ (linear pair)

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$

$= 180° - \angle 6 + \angle 180° - \angle 7 + 180° - \angle 8 + 180° - \angle 9 + 180° - \angle 10$

$= 180° \times 5 - (\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10)$

Sum of exterior angles

$= 180° \times$ no. of sides – sum of interior angles

$= 180° \times n - (2n - 4) \times 90°$ (for polygon of side n)

$= 90° \{2 \times n - (2n - 4)\}$ (taking 90° common)

$= 90° (2n - 2n + 4)$

$= 90° \times 4 = 360°$

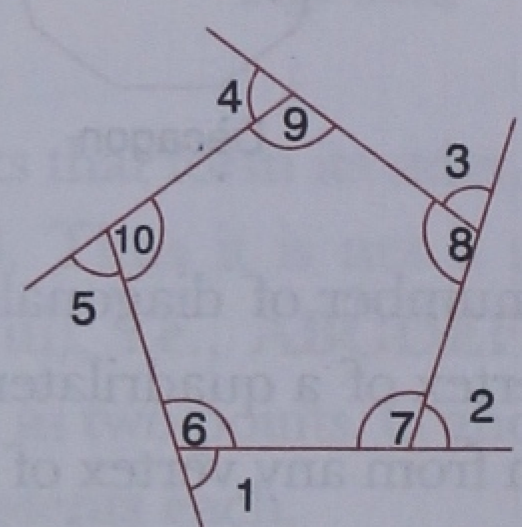


Fig. 30.6

Thus, *the sum of the exterior angles of a polygon with any number of sides always equals 360°.*

Try this!

Find the sum of the interior angles of a decagon.

Regular Polygons

A polygon in which all the angles and sides are equal in measure is known as a regular polygon.

Remember

(i) As the sum of interior angles of a polygon is $(2n - 4)$ right angles,

$\Rightarrow$ each interior angle of a regular polygon

$= \dfrac{(2n-4)}{n}$ right angles

(ii) As the sum of exterior angles of a polygon $= 360°$

$\Rightarrow$ each exterior angle of a regular polygon

$= \dfrac{360°}{n}$

$\Rightarrow n =$ number of sides of a regular polygon

$= \dfrac{360°}{\text{one exterior angle}}$

Example 3: Find the measure of each interior angle of a regular octagon.

Number of sides in an octagon $= 8 = n$

$\Rightarrow$ sum of interior angles $= (2 \times 8 - 4) \times 90°$

$= (16 - 4) \times 90° = 12 \times 90° = 1080°$

$\Rightarrow$ each interior angle of a regular octagon

$= \dfrac{1080°}{8} = 135°$

Example 4: If an interior angle of a regular polygon measures 156°, find how many sides there are in the regular polygon.

When interior angle $= 156°$

$\Rightarrow$ exterior angle $= 180° - 156° = 24°$ (linear pair)

Number of sides in the regular polygon

$= \dfrac{360°}{\text{one exterior angle}} = \dfrac{360°}{24°} = 15$

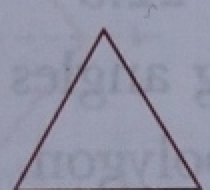
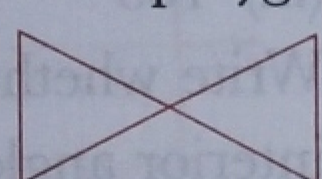
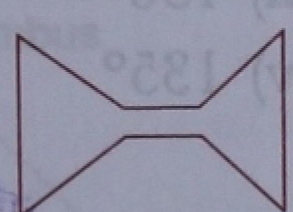
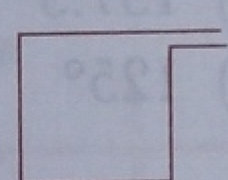
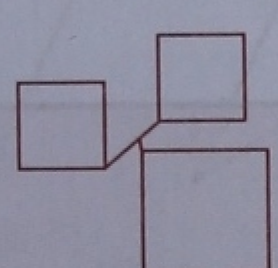
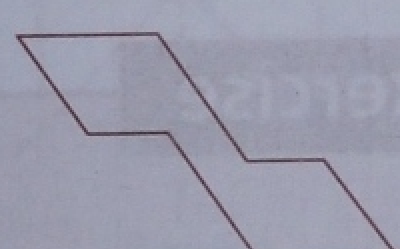
Thus, the regular polygon has 15 sides.

Try this!

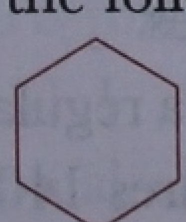
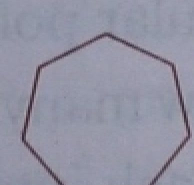
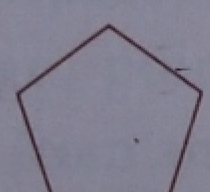
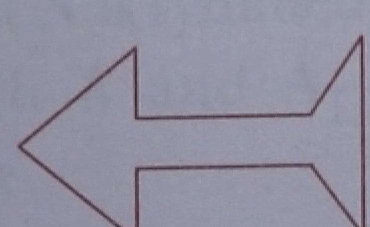
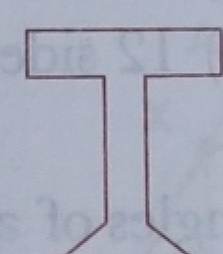
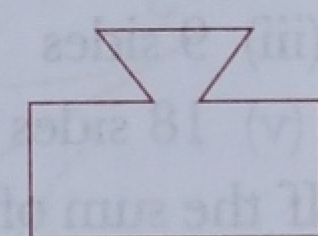
Calculate the sum of angles of a polygon with 10 sides.

Exercise 30.1

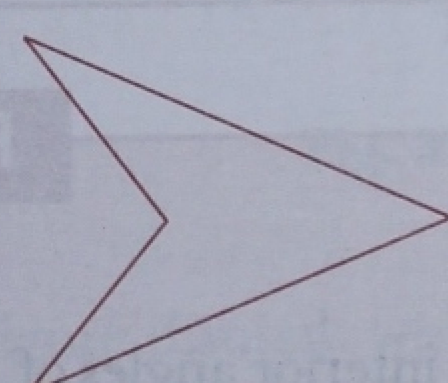
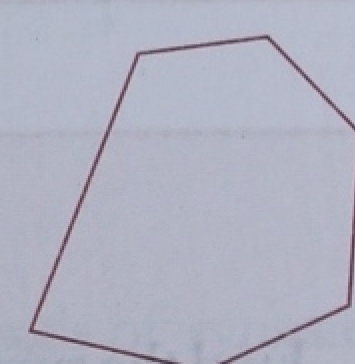
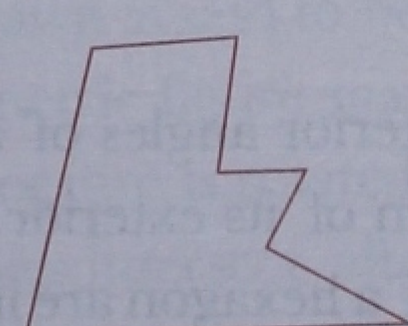
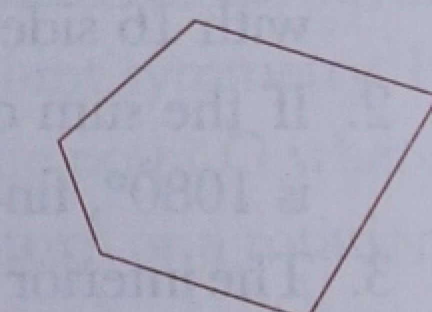
1. Which of the following figures are polygons?

(i) 
(ii) 
(iii) 
(iv) 
(v) 
(vi) 

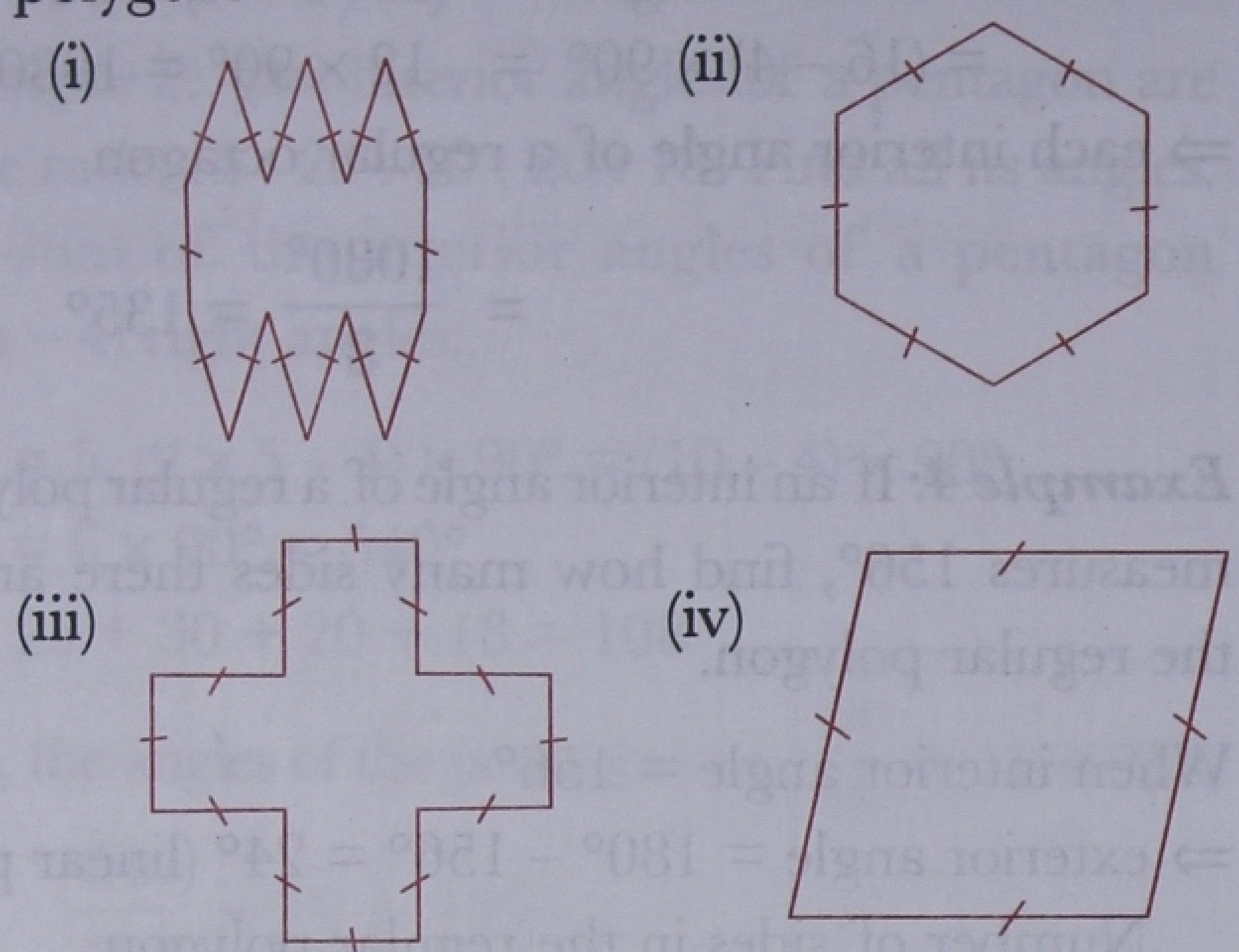
2. Name the following polygons.

(i) 
(ii) 
(iii) 
(iv) 
(v) 
(vi) 

3. Identify the type of the following polygons.

(i) 
(ii) 
(iii) 
(iv) 

4. Which of the following polygons are regular polygons?



5. How many diagonals can be drawn from one vertex in a/an:
   (i) hexagon (ii) quadrilateral
   (iii) octagon (iv) decagon
   (v) triangle
6. Drawing diagonals from one vertex, how many triangles can be formed in a:
   (i) pentagon (ii) nonagon
   (iii) hexagon (iv) quadrilateral
   (v) heptagon
7. Find the sum of the interior angles in a polygon with:
   (i) 5 sides (ii) 7 sides
   (iii) 9 sides (iv) 12 sides
   (v) 18 sides
8. If the sum of the interior angles of a polygon is 3060°, find the sum of its exterior angles.
9. What is the sum of the exterior angles of a triangle?
10. The interior angles of a pentagon are x, $2x$, $\frac{6x}{5}$, $\frac{12x}{5}$, and $\frac{12x}{5}$. Find all the angles of the pentagon.
11. The interior angles of a hexagon are in the ratio 3 : 3 : 4 : 4 : 5 : 5. Find all the angles of the hexagon.
12. Find the measure of each interior angle of a regular nonagon.
13. Find the measure of each interior angle of a regular polygon with 16 sides.
14. Find the measure of each exterior angle of a regular decagon.
15. Find the measure of each exterior angle of a regular polygon with 18 sides.
16. How many sides are there in a regular polygon if each exterior angle measures 60°?
17. How many sides are there in a regular polygon if each interior angle measures 165°?
18. Is it possible to have a regular polygon in which each exterior angle measures 50°? Give reasons for your answer.
19. Write whether the following angles can be an exterior angle of a regular polygon.
   (i) 40° (ii) 75°
   (iii) 105° (iv) 22.5°
20. Write whether the following angles can be an interior angle of a regular polygon.
   (i) 157.5° (ii) 156°
   (iii) 125° (iv) 135°

Revision Exercise

1. Find the sum of the interior angles of a polygon with 16 sides.
2. If the sum of the interior angles of a polygon is 1080°, find the sum of its exterior angles.
3. The interior angles of a hexagon are in the ratio 5 : 5 : 6 : 6 : 7 : 7. Find all the angles of the hexagon.
4. Find the measure each interior angle of a regular polygon with 14 sides.
5. How many sides are there in a regular polygon if each interior angle measures 140°