27 CONSTRUCTION OF TRIANGLES

- Construction of Triangles

This chapter deals with the steps of construction of various types of triangles.

Construction of Triangles

Scalene Triangles

I. Given SSS.

Construct ΔABC, given $\mathbf{S}_{AB}$ = 3 cm, $\mathbf{S}_{CA}$ = 4 cm, and $\mathbf{S}_{BC}$ = 5 cm.

Steps:

1. Draw AB = 3 cm.
2. From A draw arc *a* with radius 4 cm.
3. From B draw arc *b* with radius 5 cm.
4. Join AC and BC to get the required ΔABC (Figure 27.1).

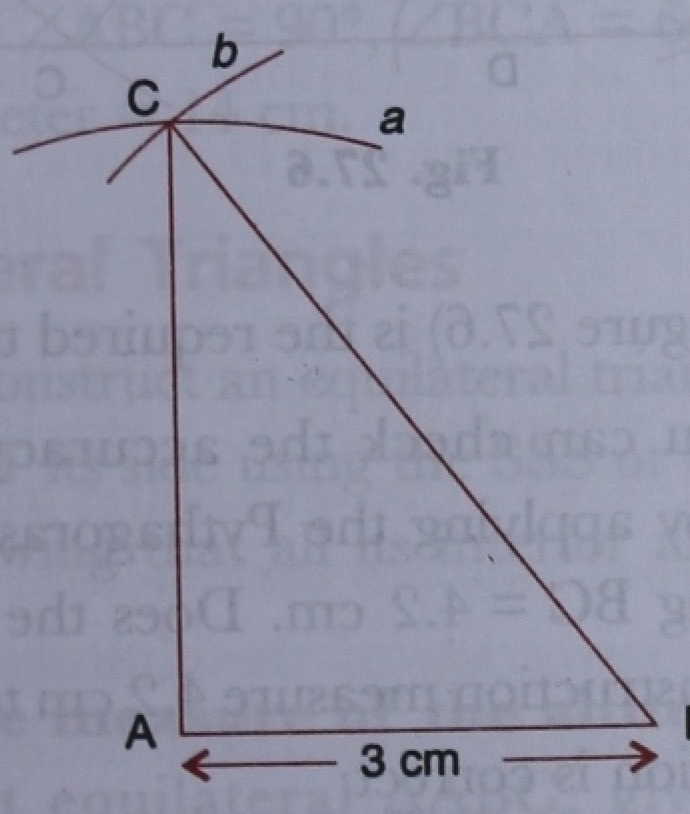


Fig. 27.1

II. Given SAS.

Construct ΔABC, given $\mathbf{S}_{AB}$ = 4 cm, included Angle ∠ABC = 60°, and $\mathbf{S}_{BC}$ = 3.5 cm.

Steps:

1. Draw BC = 3.5 cm.
2. Construct ∠XBC = 60° at point B.
3. From B measure off BA = 4 cm with arc *b*.
4. Connect points A and C.

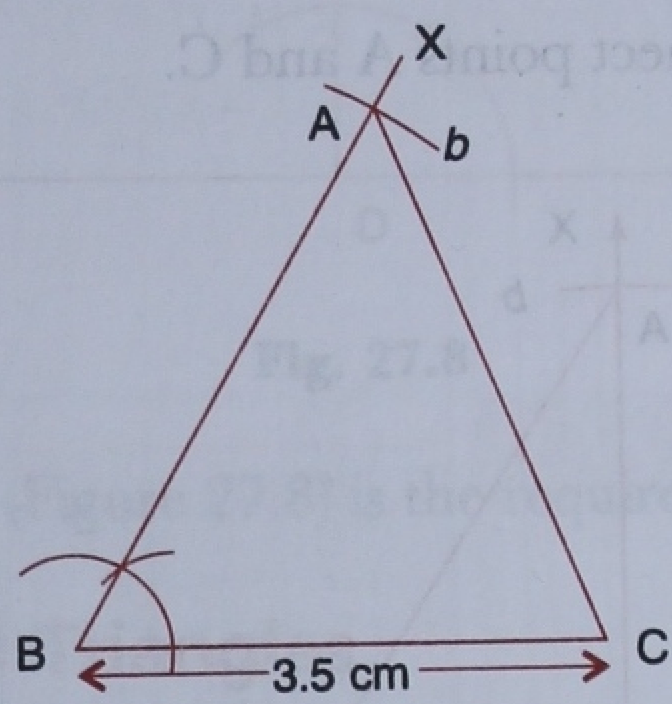


Fig. 27.2

ΔABC (Figure 27.2) is the required triangle.

III. Given ASA.

Construct ΔABC, given Angle ∠ABC = 45°, $\mathbf{S}_{BC}$ = 4 cm, and Angle ∠BCA = 30°.

Steps:

1. Draw BC = 4 cm.
2. At B construct a 90° angle and bisect it to form ∠XBC = 45°.
3. At C construct a 60° angle and bisect it to form ∠BCY = 30°.
4. Mark the point of intersection of rays BX and CY as point A.

   ΔABC (Figure 27.3) is the required triangle.

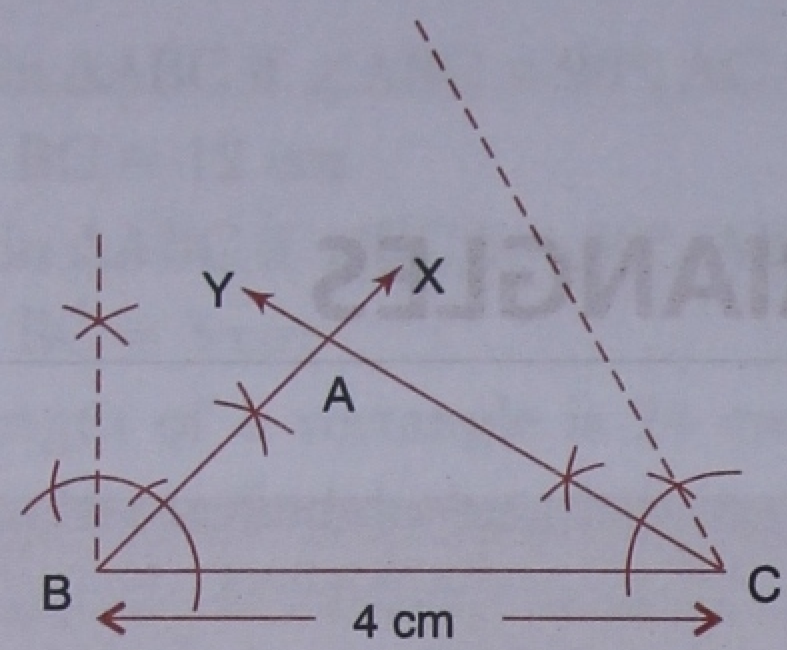


Fig. 27.3

IV. Given RHS.

Construct ΔABC, given **R**ight angle ∠ABC = 90°, **H**ypotenuse CA = 6 cm, and $\mathbf{S}_{AB}$ = 5 cm.

Steps:

1. Construct ∠XBC = 90°.
2. From B measure off BA = 5 cm with arc *b*.
3. From A measure off hypotenuse AC = 6 cm with arc *a*.
4. Connect points A and C.

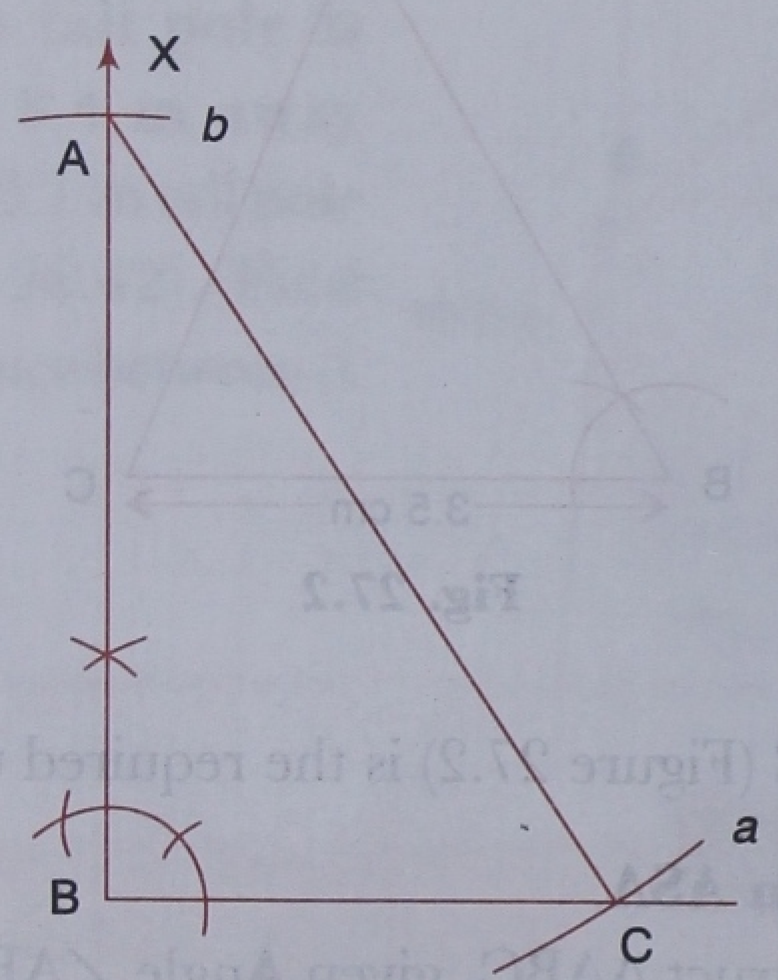


Fig. 27.4

ΔABC (Figure 27.4) is the required triangle.

V. Given the measures of two sides and the altitude on the third side.

Construct ΔABC, given AB = 3.9 cm, CA = 4.5 cm, and altitude AD = 3.6 cm.

Draw a rough sketch (Figure 27.5) of ΔABC with the given measurements to help you plan the construction.

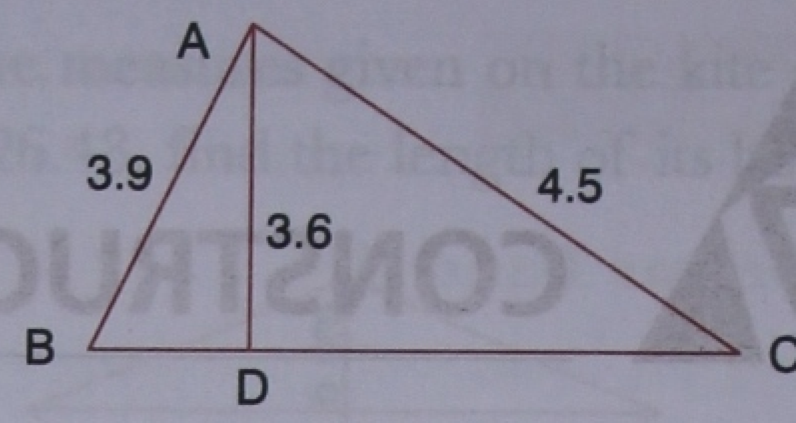


Fig. 27.5

We begin the construction by drawing the altitude first.

Steps:

1. Draw a line PQ and mark a point D on it.
2. At D construct ∠XDQ = 90°.
3. From D mark off DA = 3.6 cm with arc *d*.
4. From A mark off AB = 3.9 cm with arc *a*, that intersects PD at point B, and AC = 4.5 cm with arc a_1, that intersects DQ at point C.
5. Join points A with B and A with C.
6. We have ΔABC where AB = 3.9 cm, CA = 4.5 cm, and altitude AD = 3.6 cm.

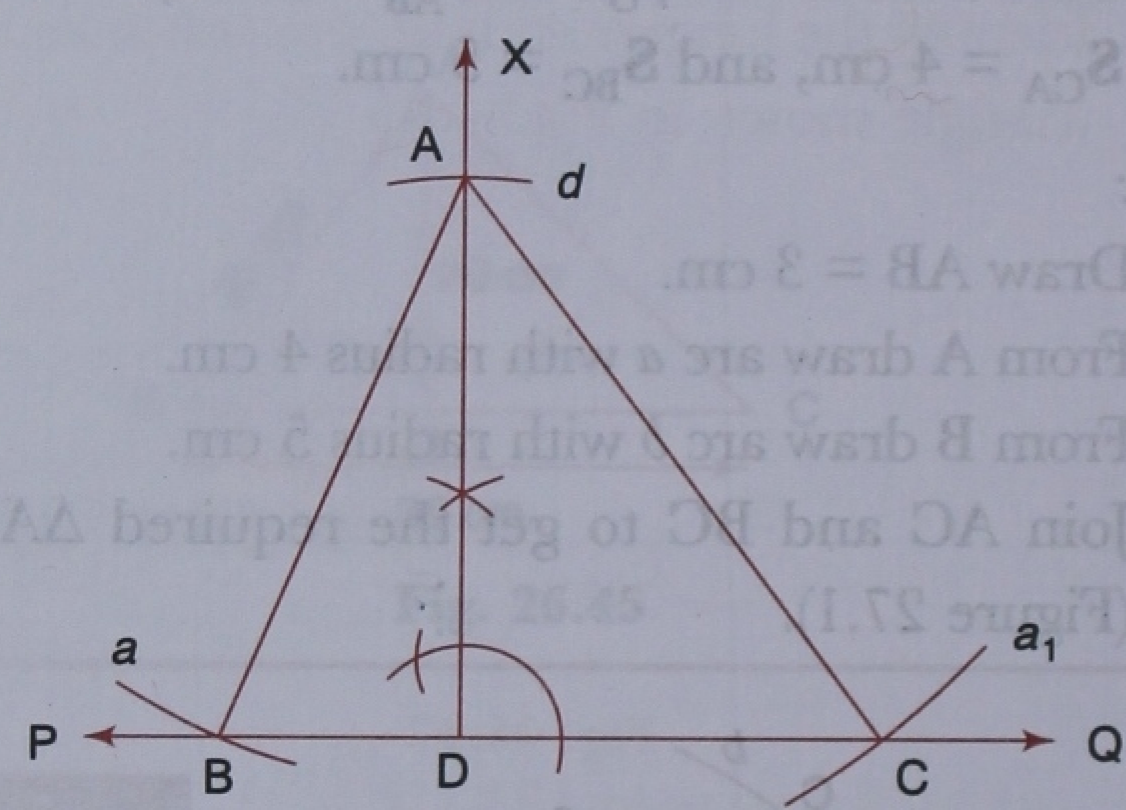


Fig. 27.6

ΔABC (Figure 27.6) is the required triangle.

CHECK: You can check the accuracy of your construction by applying the Pythagoras' theorem and calculating BC = 4.2 cm. Does the length of BC in your construction measure 4.2 cm too? If yes, your construction is correct.

VI. Given the measure of both base angles and the perimeter.

Construct ΔABC, given ∠ABC = 90°, ∠BCA = 60°, and its perimeter = 14 cm.

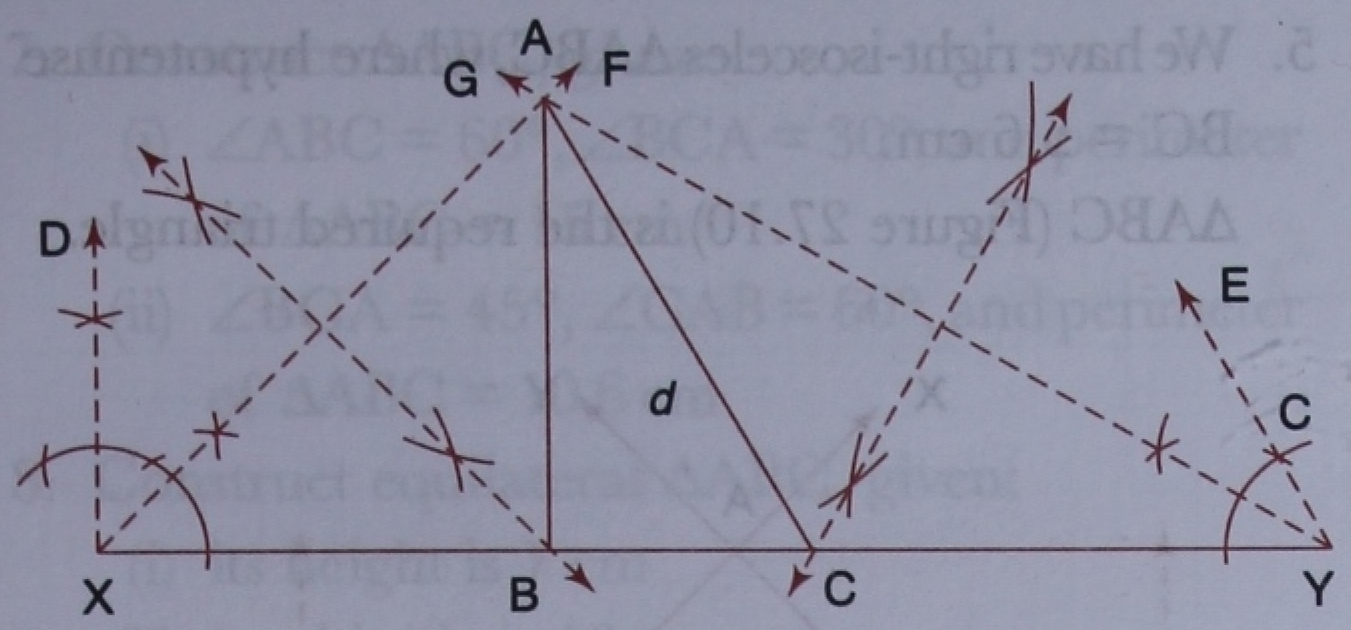


Fig. 27.7

Steps:

1. Draw XY = 14 cm.
2. At X construct ∠DXY = 90°.
3. At Y construct ∠XYE = 60°.
4. Bisect ∠DXY with ray XF and ∠XYE with ray YG.
5. Mark the point of intersection of rays XF and YG as point A.
6. Construct a perpendicular bisector of line segment XA and mark the point of intersection of the perpendicular bisector and XY as point B.
7. Construct a perpendicular bisector of line segment YA and mark the point of intersection of the perpendicular bisector and XY as point C.
8. Join A with points B and C.
9. ΔABC (Figure 27.7) is the required triangle, where ∠ABC = 90°, ∠BCA = 60°, and whose perimeter = 14 cm.

Equilateral Triangles

You can construct an equilateral triangle, given the measure of its side using the SSS or SAS and ASA rules, knowing that all its interior angles measure 60°.

Given the measure of the altitude.

Construct equilateral ΔABC, given its height = 5 cm.

Steps:

1. Draw a line PQ and mark a point D on it.
2. Construct ∠XDQ = 90°.
3. From D mark off DA = 5 cm with arc *d*.
4. Construct ∠DAM = 30° and mark the point of intersection of ray AM and PD as point B.
5. Construct ∠DAN = 30° and mark the point of intersection of ray AN and DQ as point C.
6. Join point A with points B and C. As ∠DAM (30°) + ∠DAN (30°) = ∠CAB (60°), we have equilateral ΔABC whose altitude is 5 cm.

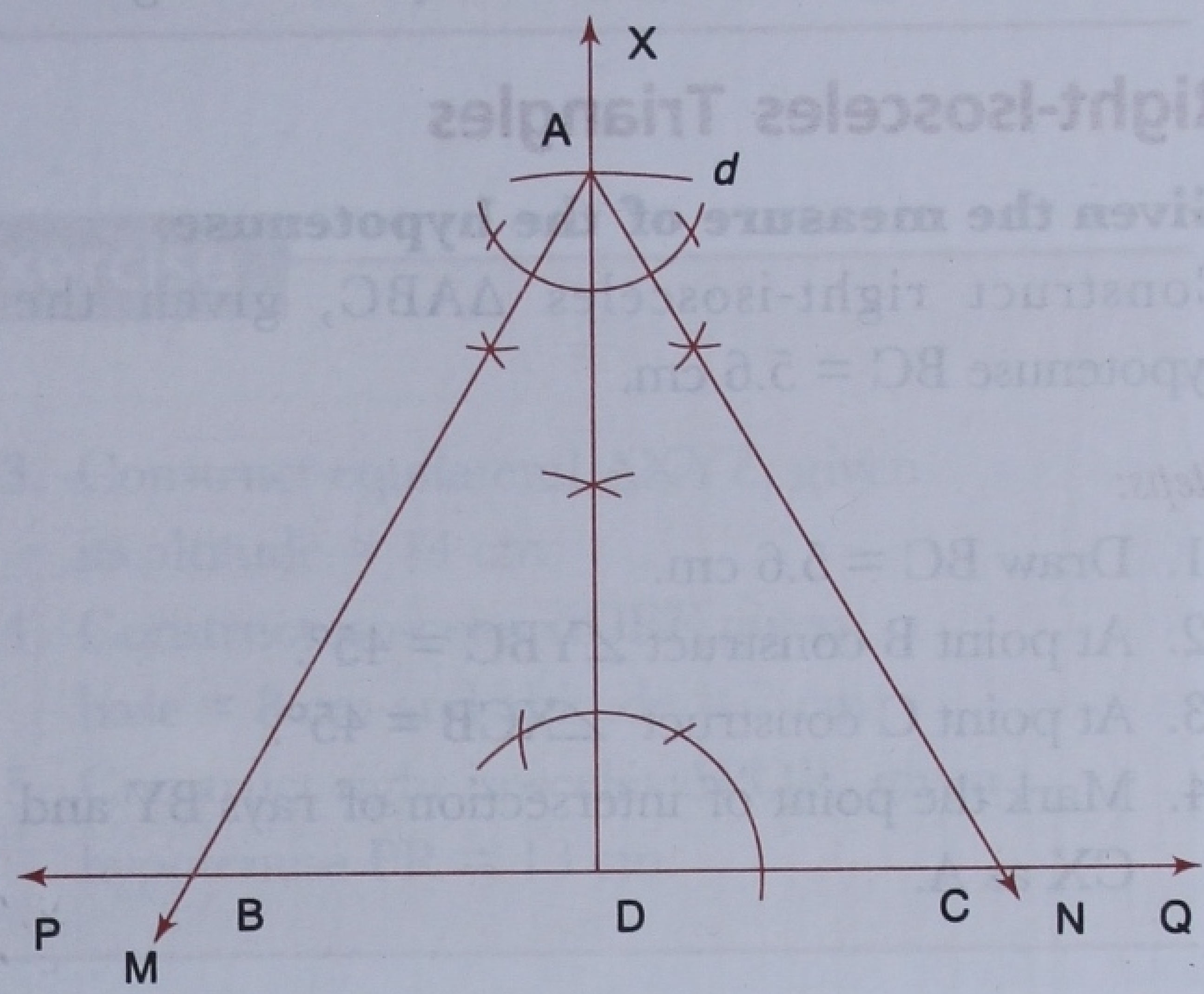


Fig. 27.8

ΔABC (Figure 27.8) is the required triangle.

Isosceles Triangles

You can construct an isosceles triangle given the measure of its base and one base angle by ASA, knowing that its base angles are equal.

Given the measure of altitude and base.

Construct isosceles ΔABC, given base BC = 3.9 cm and altitude AD = 2.6 cm.

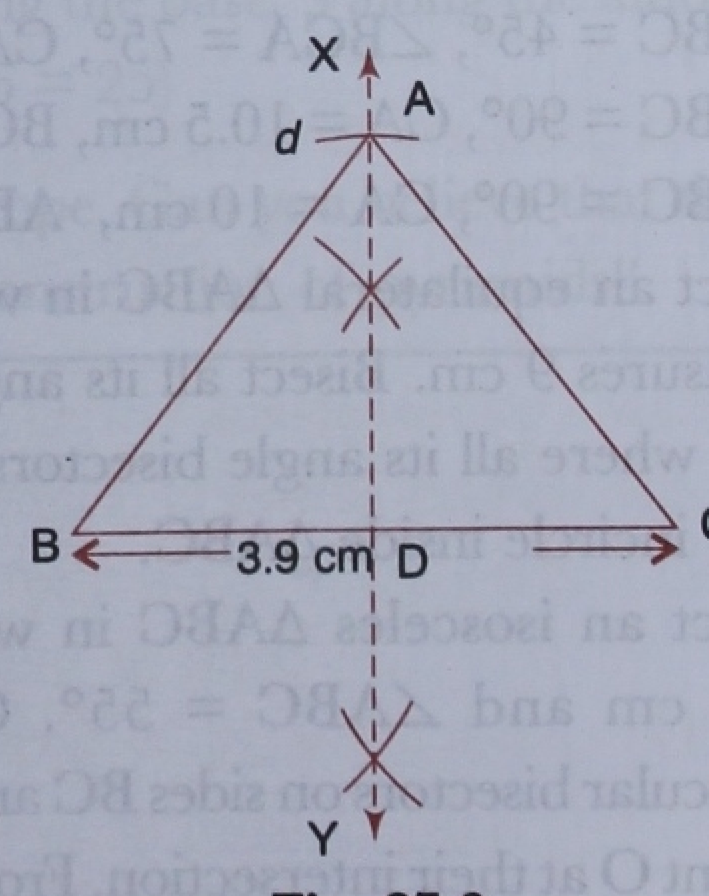


Fig. 27.9

Steps:

1. Draw BC = 3.9 cm and construct perpendicular bisector XY that intersects it at point D.
2. From D mark off DA = 2.6 cm with arc *d*.
3. Join point A with points B and C.
4. We have isosceles ΔABC, where BC = 3.9 cm whose altitude AD = 2.6 cm.
   ΔABC (Figure 27.9) is the required triangle.

Right-Isosceles Triangles

Given the measure of the hypotenuse.

Construct right-isosceles ΔABC, given the hypotenuse BC = 5.6 cm.

Steps:

1. Draw BC = 5.6 cm.
2. At point B construct ∠YBC = 45°.
3. At point C construct ∠XCB = 45°.
4. Mark the point of intersection of rays BY and CX as A.
5. We have right-isosceles ΔABC, where hypotenuse BC = 5.6 cm.
   ΔABC (Figure 27.10) is the required triangle.

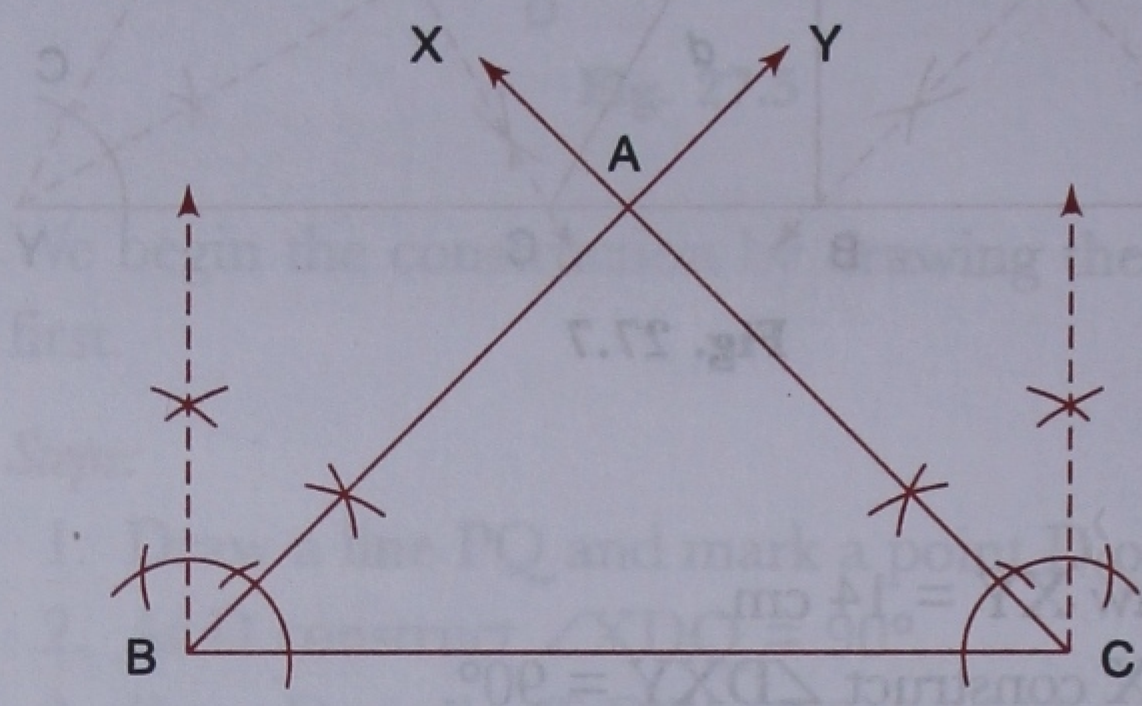


Fig. 27.10

Try this!

Construct ΔABC, given AB = 5 cm, BC = 4 cm, CA = 3 cm.

Exercise 27.1

1. Construct ΔABC, given the following measures.
   (i) AB = 7 cm, BC = 6 cm, CA = 4 cm
   (ii) AB = 9.5 cm, BC = 7.5 cm, CA = 11.5 cm
   (iii) AB = 8 cm, BC = 9 cm, ∠ABC = 60°
   (iv) CA = 8.5 cm, AB = 6 cm, ∠CAB = 45°
   (v) ∠CAB = 30°, ∠ABC = 60°, AB = 9 cm
   (vi) ∠BCA = 120°, ∠CAB = 30°, CA = 5 cm
   (vii) ∠CAB = 75°, ∠BCA = 45°, AB = 7 cm
   (viii) ∠ABC = 45°, ∠BCA = 75°, CA = 9 cm
   (ix) ∠ABC = 90°, CA = 10.5 cm, BC = 8.4 cm
   (x) ∠ABC = 90°, CA = 10 cm, AB = 8 cm
2. Construct an equilateral ΔABC in which each side measures 9 cm. Bisect all its angles. From point O, where all its angle bisectors intersect, draw the incircle inside ΔABC.
3. Construct an isosceles ΔABC in which base BC = 5 cm and ∠ABC = 55°. Construct perpendicular bisectors on sides BC and CA and mark point O at their intersection. From O draw the circumcircle passing through vertices A, B, and C.
4. Construct ΔABC, given AB = 9 cm, BC = 12 cm and CA = 15 cm. Bisect the three sides marking D, E, and F as the mid-points of AB, BC, and CA respectively. Connect D, E, and F, with C, A, and B respectively and mark the point of intersection of the medians as centroid G.
5. Construct ΔABC, given AB = 7 cm, BC = 6 cm, and ∠ABC = 45°. Use set squares to draw two altitudes from any two vertices and mark the point of intersection of the altitudes as point O. Does the altitude drawn from the third vertex also pass through O, the orthocentre of ΔABC?
6. Construct ΔABC, given:
   (i) AB = 7.6 cm, CA = 10 cm, and altitude, AD = 6 cm
   (ii) BC = 9 cm, CA = 8 cm, and altitude CD = 6

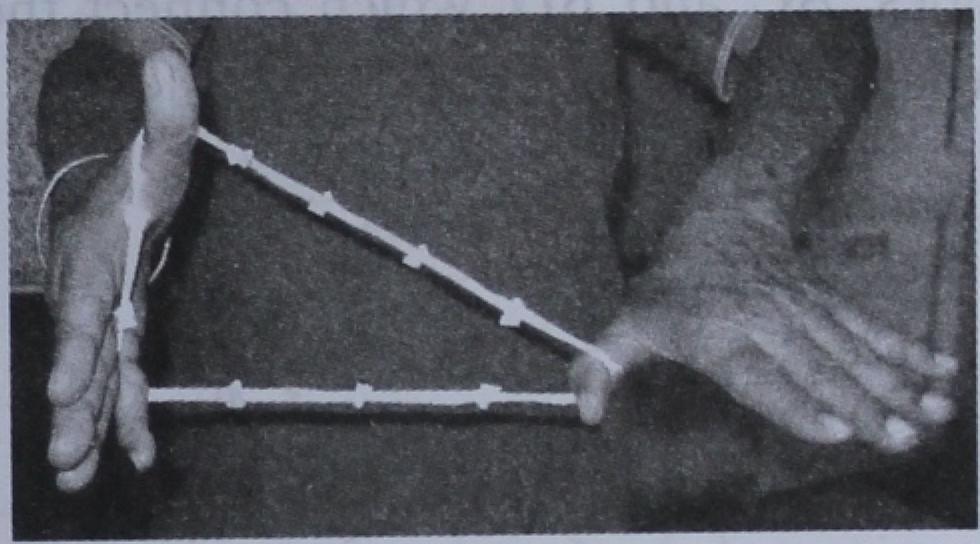
7. Construct ΔABC, given:
   (i) ∠ABC = 60°, ∠BCA = 30°, and perimeter of ΔABC = 15 cm
   (ii) ∠BCA = 45°, ∠CAB = 60°, and perimeter of ΔABC = 10.6 cm
8. Construct equilateral ΔABC, given:
   (i) its height is 7 cm
   (ii) its altitude is 10 cm
9. Construct isosceles ΔABC, given:
   (i) Base = 9 cm and altitude = 6 cm
   (ii) Base = 7 cm and altitude = 6 cm
10. Construct right-isosceles ΔABC, given:
   (i) Hypotenuse BC = 11 cm
   (ii) Hypotenuse CA = 16.2 cm

Revision Exercise

1. Construct ΔABC, given:
   (i) AB = 8 cm, BC = 7 cm, CA = 5 cm
   (ii) AB = 5 cm, CA = 7.5 cm, ∠CAB = 60°
2. Construct ΔPQR, given ∠PQR = 45°, ∠QRP = 30°, and perimeter of ΔPQR = 12 cm.
3. Construct equilateral ΔXYZ, given its altitude = 14 cm
4. Construct isosceles ΔDEF, given base = 8 cm and altitude = 7 cm.
5. Construct right isosceles ΔPQR, given hypotenuse PR = 13 cm.

Project: To construct a right-angled triangle with just a rope.

Required:
(i) a rope, about 30 cm in length
(ii) Knowledge of the Pythagoras Theorem!



Steps:
(i) Fold the rope in half and tie a knot at the fold.
(ii) Fold the halves and tie knots at the folds.
(iii) In this way, tie 12 knots at equal distance from each other.
(iv) Tie the loose ends together. We have a loop with 12 equidistant knots.
(v) Hold the loop between your fingers as shown – 4 knots along the base, 3 along the altitude and 5 knots along the hypotenuse. Note that $3^2 + 4^2 = 5^2$ (9 + 16 = 25)

We have a right angled triangle, or a set-square shape with just a rope. Can you believe that the ancient Egyptians used this technique to measure perpendiculars in the construction of pyramids!!