26 TRIANGLES

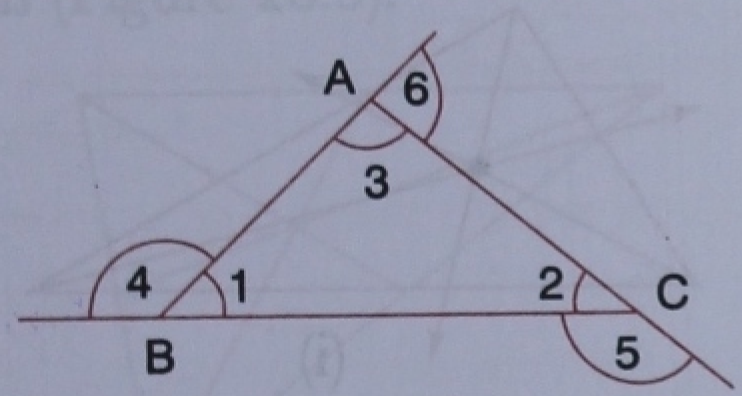
- Elements of a Triangle
- Properties of Triangles
- Terms Associated with Triangles
- Congruency of Triangles

Triangles

Three non-collinear co-planar points A, B, and C joined by three line segments AB, BC, and CA form ΔABC.

Elements of a Triangle

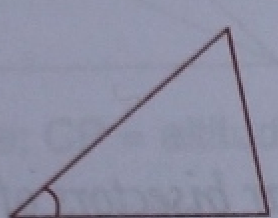
The six elements of a triangle are its three sides and three angles. A, B, C are the vertices of ΔABC given below:

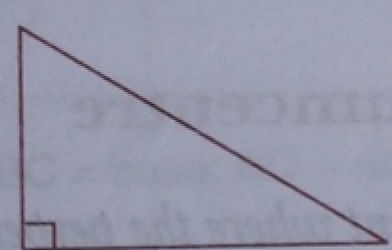


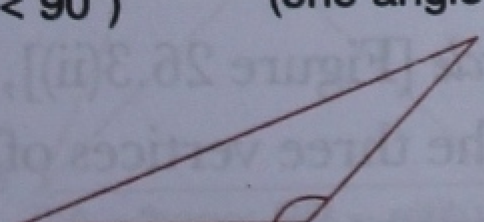
∠1, ∠2, ∠3 are its three interior angles; AB, BC, and CA are its three sides; and ∠4, ∠5, ∠6 are its three exterior angles formed when the sides of the triangle are produced in that order.

Types of Triangles

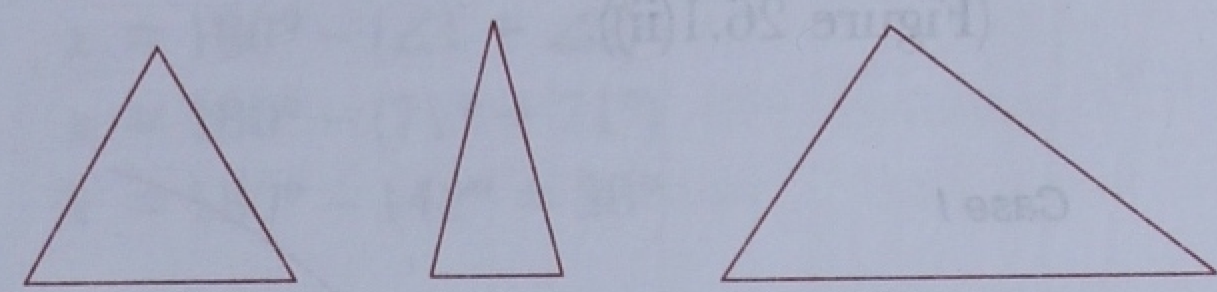
According to the magnitude of its interior angles, a triangle may be acute-angled, right-angled, or obtuse-angled.


Acute-angled
(all angles < 90°)


Right-angled
(one angle = 90°)


Obtuse-angled
(one angle > 90°)

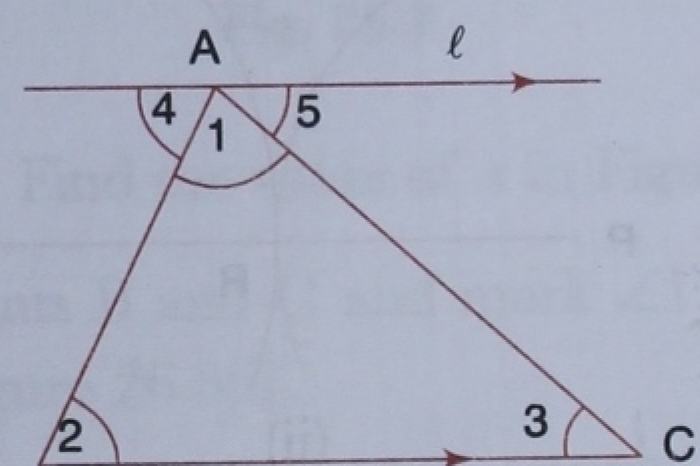
According to the measure of its sides, a triangle may be equilateral, isosceles, or scalene.


Equilateral (three sides equal) | Isosceles (two sides equal) | Scalene (no sides equal)

Properties of Triangles

1. *To prove:* The sum of the interior angles of a triangle is 180°.

   Proof: Draw ΔABC and ℓ || BC such that ℓ passes through vertex A. Mark ∠1, ∠2, ∠3, ∠4, and ∠5 as shown below:



Now ∠2 = ∠4 (alternate interior angles)

and ∠3 = ∠5 (alternate interior angles)

But ∠4 + ∠1 + ∠5 = 180° (straight angle)

⇒ ∠2 + ∠1 + ∠3 = 180°

Thus, **the sum of the interior angles of a triangle is 180°.** Q.E.D.

Try this!

If the sum of two angles of a triangle is 56°, find the third angle. What kind of angle will it be?

2. *To prove:* The sum of the length of two sides is greater than the length of the third side of a triangle.

Proof:

Case I: Construct ΔABC, given AB = 6 cm, BC = 3 cm. and CA = 2 cm.
The arcs for BC = 3 cm and CA = 2 cm do not intersect at any point (Figure 26.1(i)).

Case II: Construct ΔPQR, given PQ = 5 cm, QR = 3 cm, and RP = 2 cm.
The arcs for QR = 3 cm and RP = 2 cm do not intersect but meet each other at point R. But point R is on line segment PQ itself (Figure 26.1(ii)).

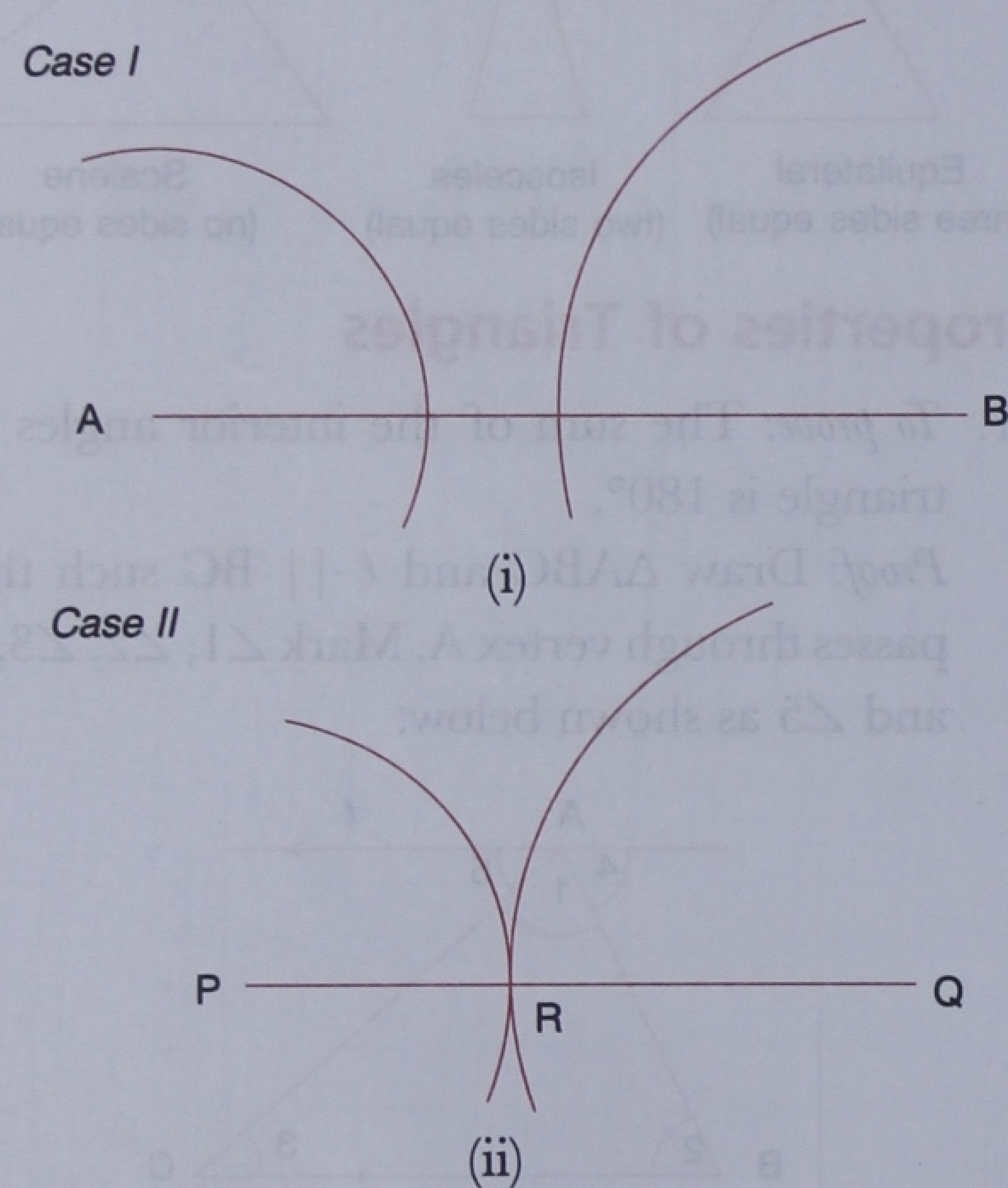


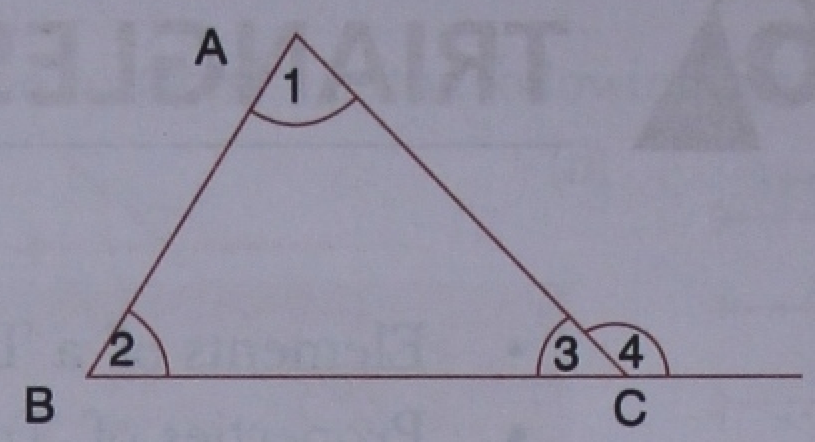
Fig. 26.1

Thus ΔABC and ΔPQR are not possible with the given measurements or ***a triangle is possible only if the sum of two sides is greater than the measure of the third side.*** Q.E.D.

3. *To prove:* The measure of an exterior angle is equal to the sum of its two opposite interior angles.
Proof: Extend side BC of ΔABC to form exterior ∠4.

Now $\angle 3 + \angle 4 = 180°$ (linear pair)

$\Rightarrow \angle 4 = 180° - \angle 3$



But $\angle 1 + \angle 2 + \angle 3 = 180°$ (from first property)

$\Rightarrow \angle 1 + \angle 2 = 180° - \angle 3$

$\Rightarrow \angle 1 + \angle 2 = \angle 4$

Or, ***exterior angle is equal to the sum of opposite interior angles.*** Q.E.D.

Some Special Terms Related to Triangles

Incentre

The point where the angle bisectors of all three interior angles of a triangle meet is known as its incentre [Figure 26.2(i)].

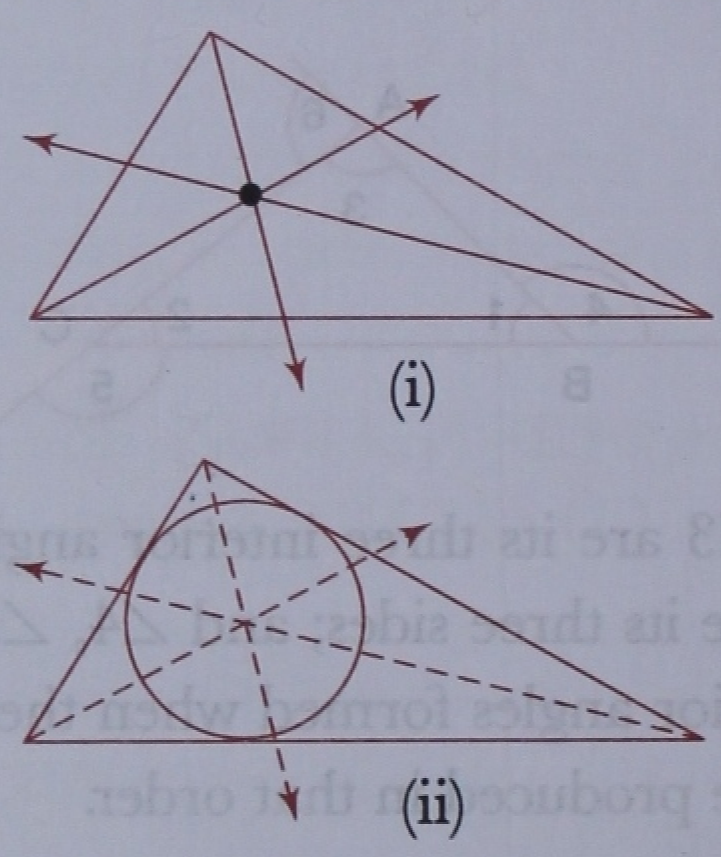


Fig. 26.2

The *incircle* [Figure 26.2(ii)], or the circle touching all the three sides of a triangle, is drawn with the incentre as its centre.

Circumcentre

The point where the perpendicular bisectors of all the sides of a triangle meet is known as its circumcentre [Figure 26.3(i)].

The *circumcircle* [Figure 26.3(ii)], or the circle passing through all the three vertices of a triangle, is drawn with the circumcentre as its centre.

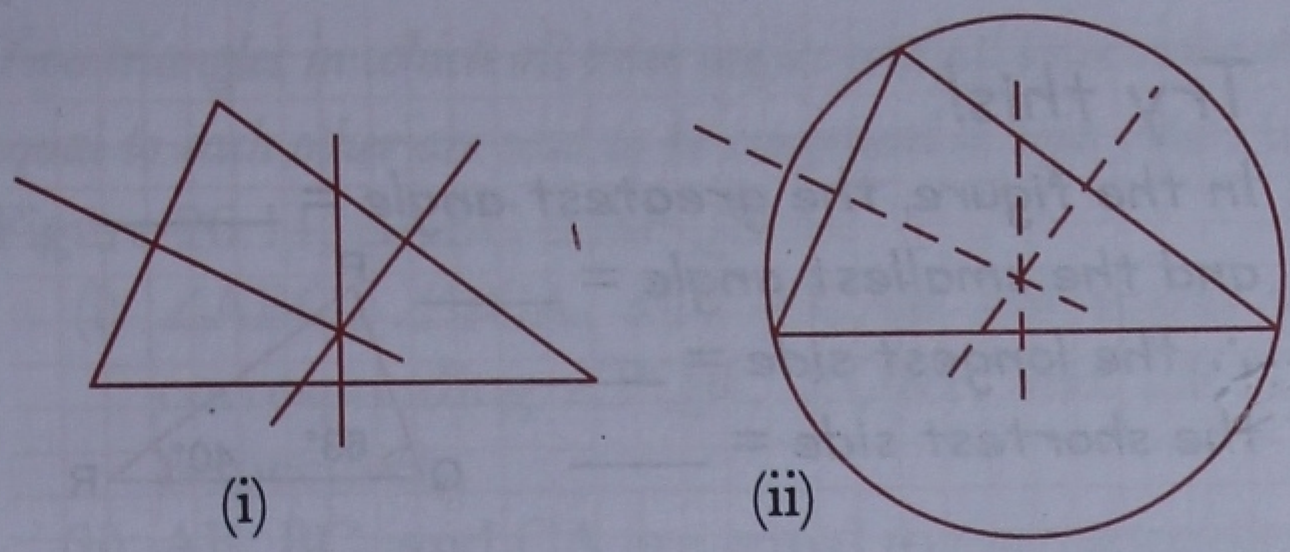


Fig. 26.3

Median

A median is a line joining the mid-point of a side of a triangle to the vertex opposite it (Figure 26.4).

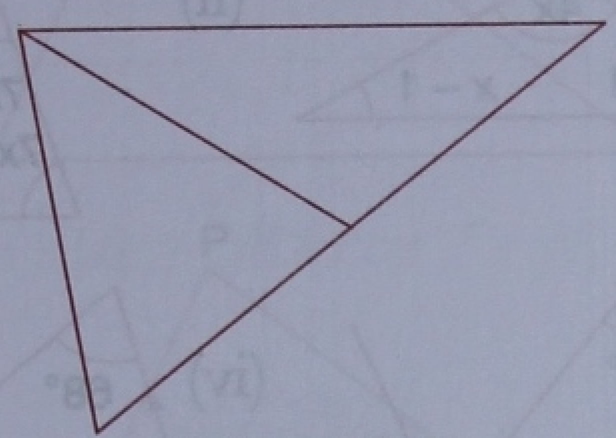


Fig. 26.4

Centroid

The centroid of a triangle is the point of intersection of its three medians (Figure 26.5).

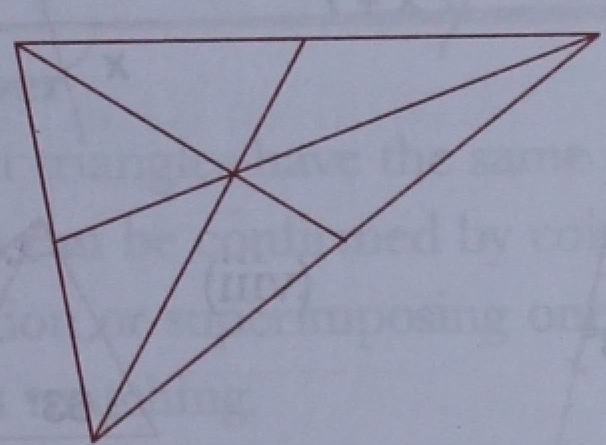
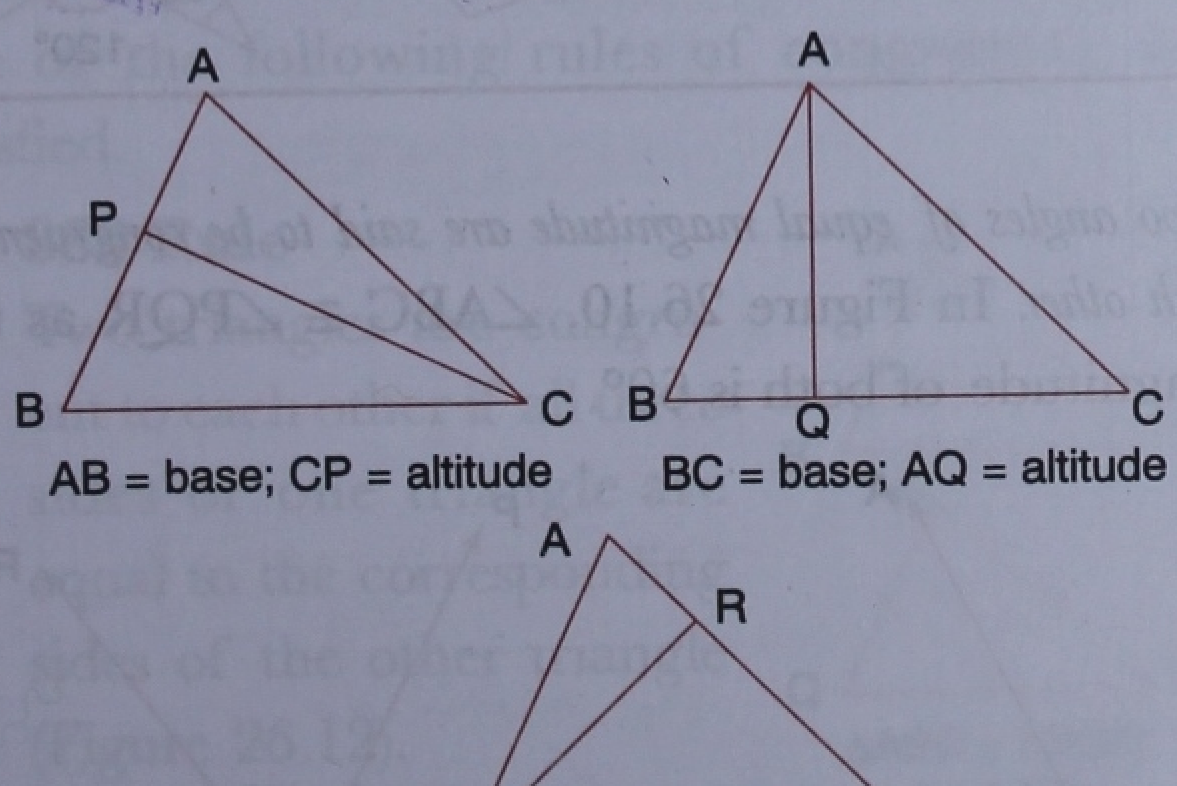


Fig. 26.5

Altitude

A perpendicular drawn from a vertex to its opposite side is called an altitude of the triangle (Figure 26.6).



AB = base; CP = altitude

BC = base; AQ = altitude

CA = base; BR = altitude

Fig. 26.6

Orthocentre

The point where the three altitudes of a triangle meet is known as its orthocentre. **The orthocentre of a scalene triangle lies in its exterior.**

Example 1: Find the value of x in Figure 26.7, given ABC is an isosceles triangle.

In the given figure $\angle 1 = 71°$

(vertically opposite angles)

As the given triangle is an isosceles triangle,

$\angle 1 = \angle 2$

$\Rightarrow \quad \angle 2 = 71°$

$x + \angle 1 + \angle 2 = 180°$

(sum of interior angles of a triangle)

$\Rightarrow \quad x = 180° - (\angle 1 + \angle 2)$

$\Rightarrow \quad x = 180° - (71° + 71°)$

$\Rightarrow \quad x = 180° - 142° = 38°$

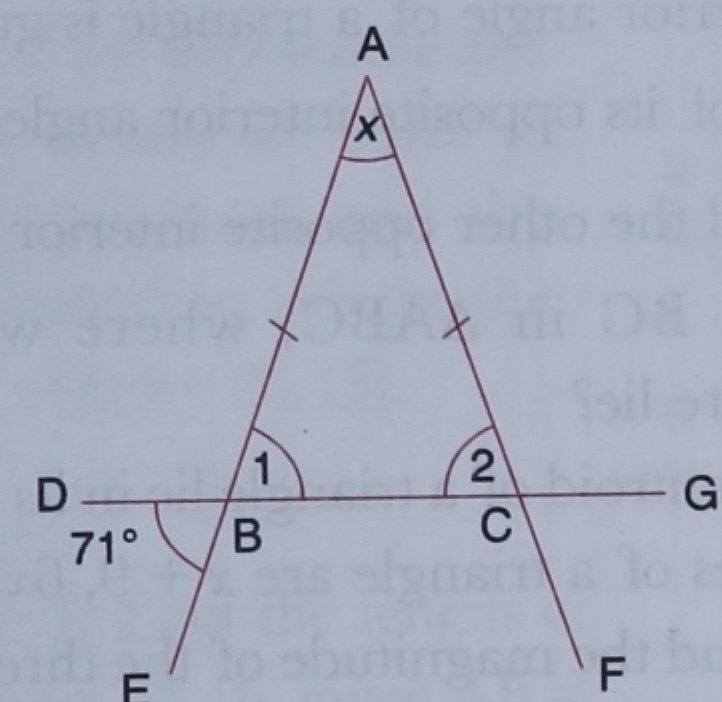


Fig. 26.7

Example 2: Find the value of x in Figure 26.8(i).

Connect points B and C and mark $\angle 1$ and $\angle 2$ as shown in Figure 26.8(ii).

In ΔDBC, $\angle 1 + \angle 2 + 124° = 180°$

$\angle 1 + \angle 2 = 180° - 124° = 56°$

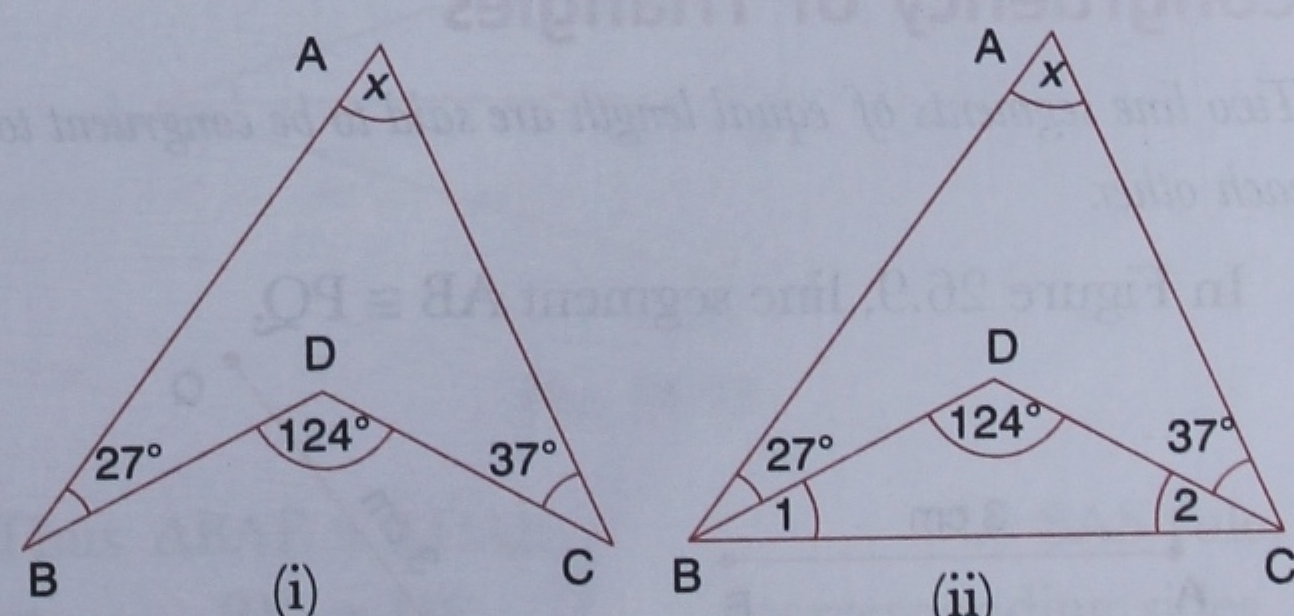


Fig. 26.8

In ΔABC, $x + (27° + \angle 1) + (37° + \angle 2) = 180°$

$\Rightarrow \quad x + 27° + 37° + \angle 1 + \angle 2 = 180°$

$\Rightarrow \quad x + 27° + 37° + 56° = 180°$

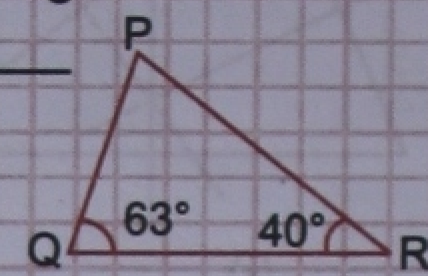
(substituting $\angle 1 + \angle 2 = 56°$ obtained earlier)

$\Rightarrow \quad x + 120° = 180°$

$\Rightarrow \quad x = 180° - 120° = 60°$

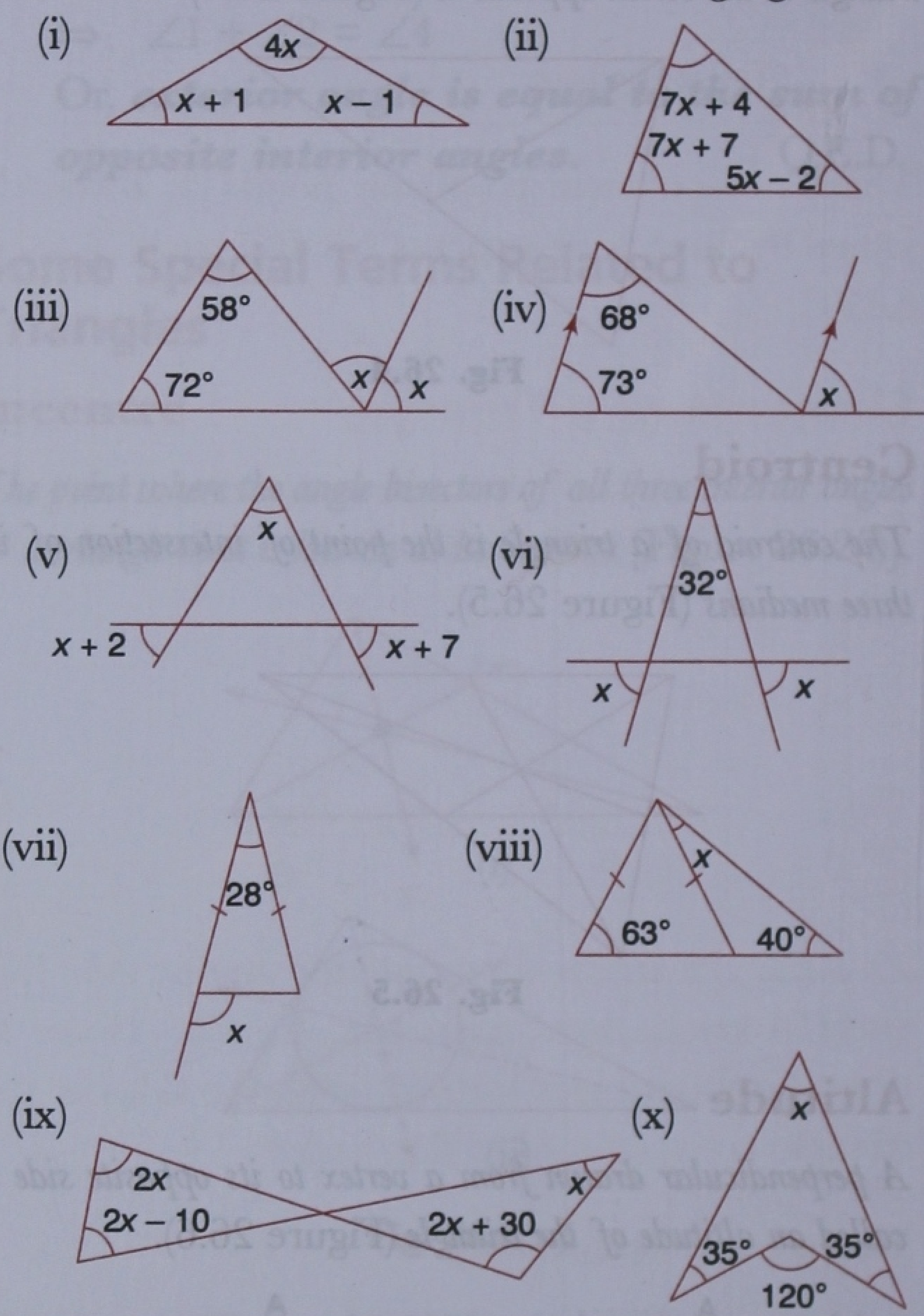
Try this!

In the figure, the greatest angle = ______
and the smallest angle = _____
∴ the longest side = _____
the shortest side = _____



Exercise 26.1

1. The angles of a triangle are in the ratio 1 : 4 : 5. Find the magnitude of the angles.
2. The other two angles in a right-angled triangle are in the ratio 3 : 7. Find the magnitude of the angles of the triangle.
3. Is a triangle with the measure of its sides in the ratio 3 : 9 : 5 possible?
4. If an exterior angle of a triangle is given as $\frac{x}{3}$ and one of its opposite interior angles is given as $\frac{x}{5}$, find the other opposite interior angle.
5. If AB ⊥ BC in ΔABC, where would its orthocentre lie?
6. Can the centroid of a triangle lie in its exterior?
7. The angles of a triangle are $x + 9$, $6x - 6$, and $9x + 1$. Find the magnitude of the three angles.
8. In an isosceles triangle, a base angle is x and the vertical angle is $x + 6$. Find the magnitude of the three angles of the triangle.
9. In an isosceles triangle, the vertical angle is $6x$ and a base angle is $4.5x$. Find the magnitude of the three angles of the triangle.
10. Find the values of x in the following figures.



Congruency of Triangles

Two line segments of equal length are said to be congruent to each other.

In Figure 26.9, line segment AB ≅ PQ.

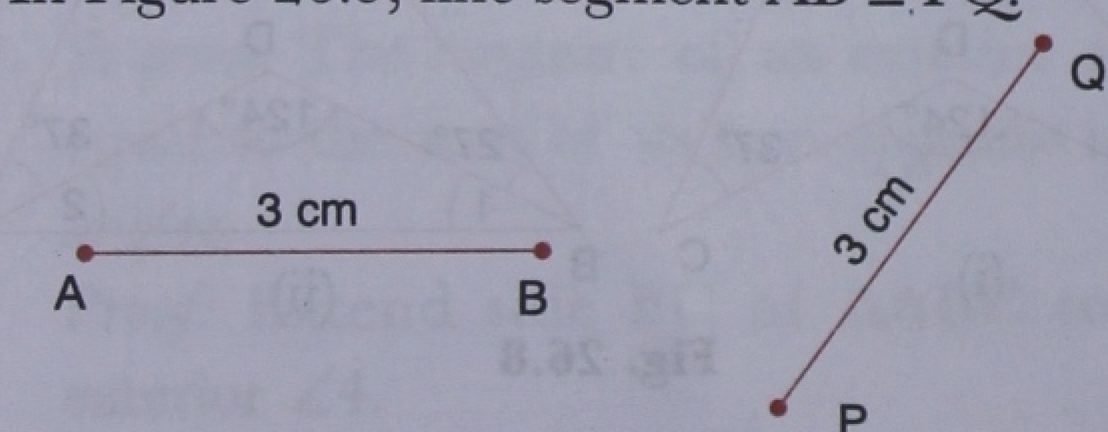


Fig. 26.9

Two angles of equal magnitude are said to be congruent to each other. In Figure 26.10, ∠ABC ≅ ∠PQR as the magnitude of both is 60°.

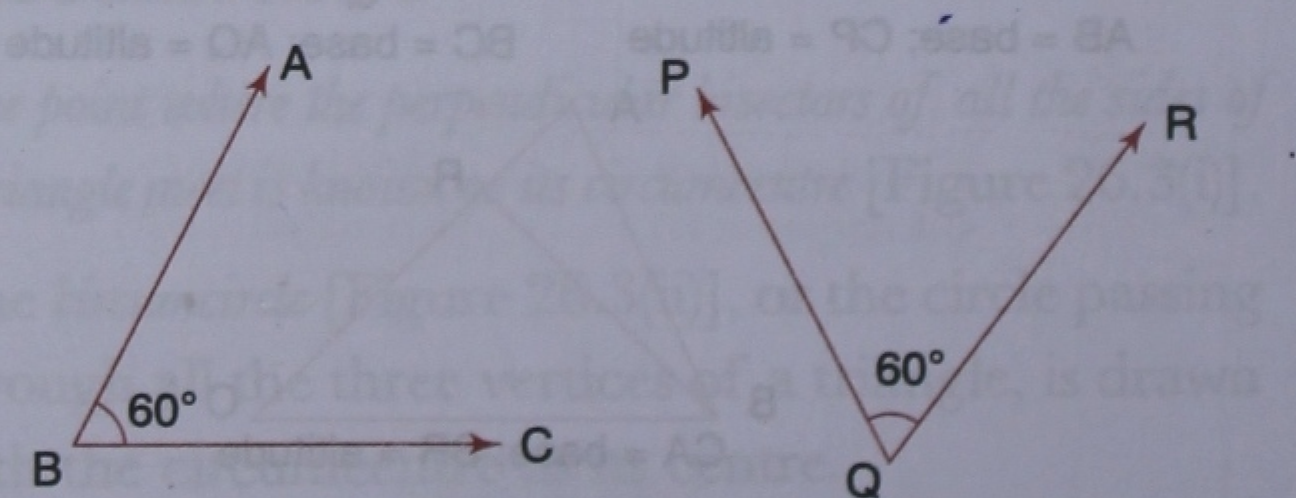


Fig. 26.10

Two triangles in which all three angles and all three sides are equal to each other are said to be congruent to each other. In Figure 26.11, $\Delta ABC \cong \Delta PQR$ as:

(i) $\angle ABC$, $\angle BCA$, and $\angle CAB$ are equal to corresponding $\angle PQR$, $\angle QRP$, and $\angle RPQ$ respectively.

(ii) AB, BC, and CA are equal to corresponding sides PQ, QR, and RP respectively.

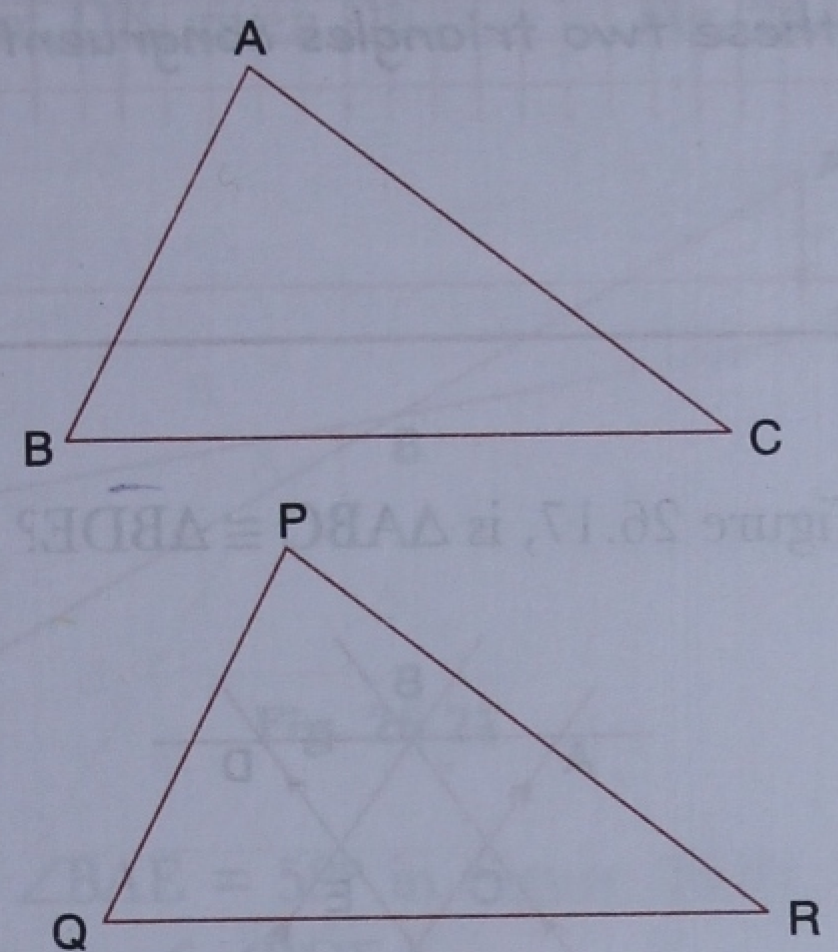


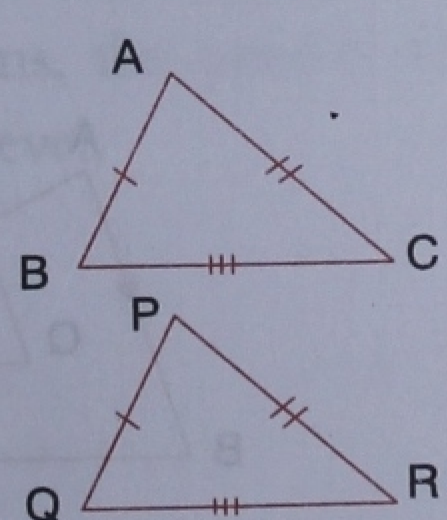
Fig. 26.11

> **Remember**
> Congruent triangles have the same shape and size which can be confirmed by coincident superposition or superimposing one on the other for perfect matching.
> Similar triangles are the same in shape but not in size. The corresponding angles of similar triangles are all equal to each other.

Two triangles will be congruent to each other if any one of the following rules of congruency are satisfied.

1. SSS Rule

Two triangles are congruent to each other if all three sides of one triangle are equal to the corresponding sides of the other triangle (Figure 26.12).

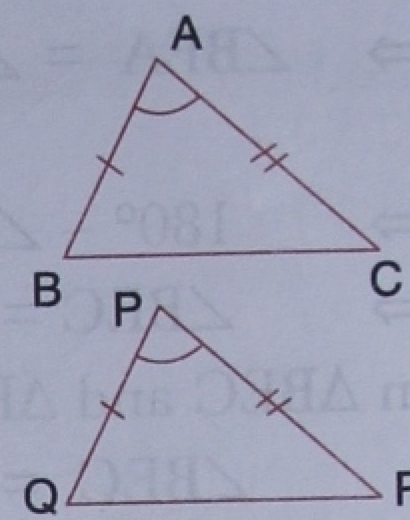


$\Delta ABC \cong \Delta PQR$ by SSS

Fig. 26.12

2. SAS Rule

Two triangles are congruent to each other if two sides and the included angle in one are equal to two sides and the included angle in the other (Figure 26.13).

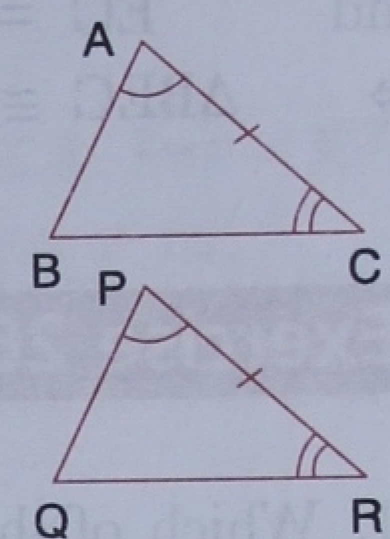


$\Delta ABC \cong \Delta PQR$ by SAS

Fig. 26.13

3. ASA Rule

Two triangles are congruent to each other if two angles and the included side in one are equal to two angles and the included side in the other (Figure 26.14).

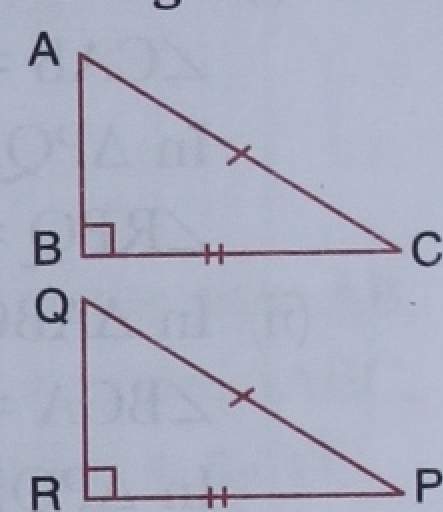


$\Delta ABC \cong \Delta PQR$ by ASA

Fig. 26.14

4. RHS Rule

Two right-angled triangles are congruent to each other if the hypotenuse and a side in one are equal to the hypotenuse and the corresponding side in the other (Figure 26.15).



$\Delta ABC \cong \Delta PQR$ by RHS

Fig. 26.15

Example 3: Find the length of DC in Figure 26.16, given AB = AD, $\angle BAE = \angle DAE$, and BC = 5 cm.

In ΔBAE and ΔDAE,

$$AB = AD \quad \text{(given)}$$
$$\angle BAE = \angle DAE \quad \text{(given)}$$
$$AE = AE \quad \text{(same side)}$$

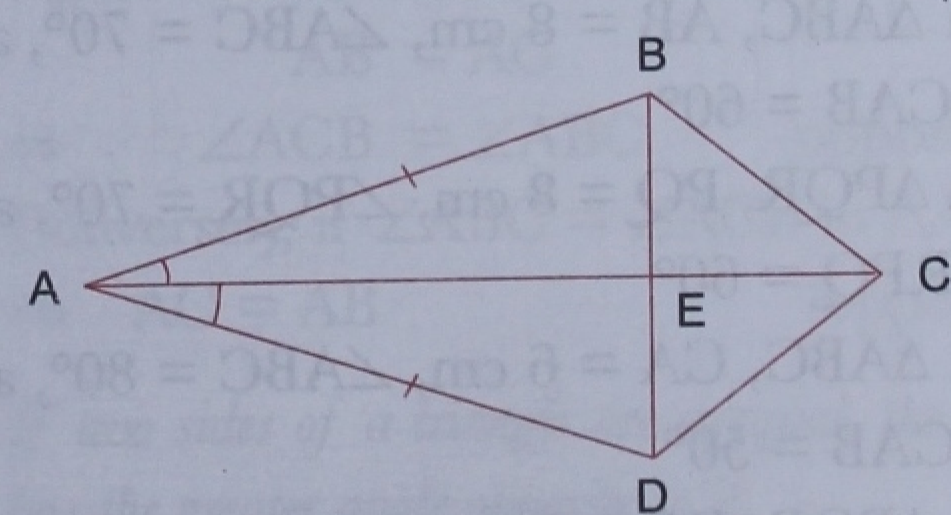


Fig. 26.16

Thus $\Delta BAE \cong \Delta DAE$ (by SAS rule)

$\Rightarrow$ BE = DE (corresponding sides of congruent triangles)

⇒ ∠BEA = ∠DEA (corresponding angles of congruent triangles)

⇒ 180° – ∠BEA = 180° – ∠DEA

⇒ ∠BEC = ∠DEC (linear pairs)

In ΔBEC and ΔDEC

∠BEC = ∠DEC (Proved)

BE = DE (corresponding sides of triangles)

and EC = EC (same side)

⇒ ΔBEC ≅ ΔDEC (by SAS rule)

⇒ BC = DC (corresponding sides of congruent triangles)

As BC = 5 cm

⇒ DC = 5 cm

Try this!

Δ ABC has ∠A = 50° and ∠B = 90°

Δ PQR has ∠Q = 90° and ∠R = 40.

Are these two triangles congruent?

Exercise 26.2

1. Which of the following pairs of triangles are congruent and why?
   (i) In ΔABC, AB = 7 cm, CA = 8 cm, and ∠CAB = 60°
   In ΔPQR, PQ = 8 cm, RP = 7 cm, and ∠RPQ = 60°
   (ii) In ΔABC, AB = 6 cm, CA = 5 cm, and ∠BCA = 105°
   In ΔPQR, PQ = 6 cm, RP = 5 cm, and ∠PQR = 105°
   (iii) In ΔABC, AB = 6 cm, CA = 8 cm, and ∠ABC = 90°
   In ΔPQR, PQ = 6 cm, RP = 10 cm, and ∠PQR = 90°
   (iv) In ΔABC, BC = 6 cm, CA = 9 cm, and ∠ABC = 90°
   In ΔPQR, PQ = 6 cm, RP = 9 cm, and ∠PQR = 90°
   (v) In ΔABC, AB = 8 cm, ∠ABC = 70°, and ∠CAB = 60°
   In ΔPQR, PQ = 8 cm, ∠PQR = 70°, and ∠RPQ = 60°
   (vi) In ΔABC, CA = 6 cm, ∠ABC = 80°, and ∠CAB = 50°
   (vii) In ΔPQR, RP = 6 cm, ∠PQR = 80°, and ∠RPQ = 50°
2. Are two right-angled triangles congruent if the hypotenuse and base of one triangle are equal to the hypotenuse and altitude of the other?
3. In Figure 26.17, is ΔABC ≅ ΔBDE? Why?

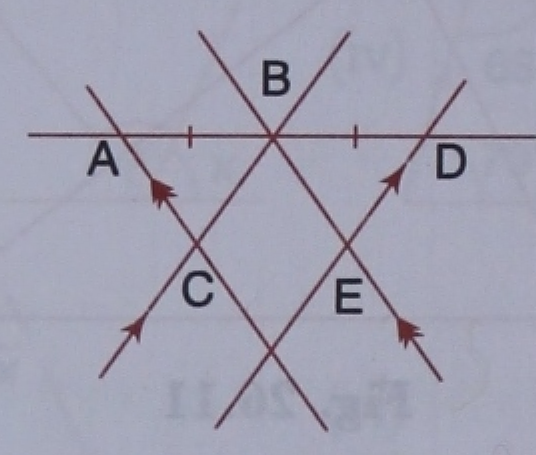


Fig. 26.11

4. From Figure 26.18, given ΔABC ≅ ΔPQR, can you say with certainty that BC || QR?

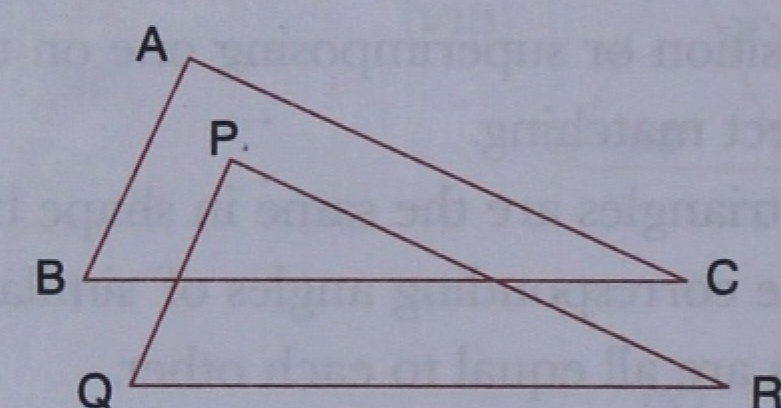


Fig. 26.18

5. From Figure 26.19, can you tell with certainty that ΔABC ≅ ΔPQR? What can you tell with certainty about the two triangles?

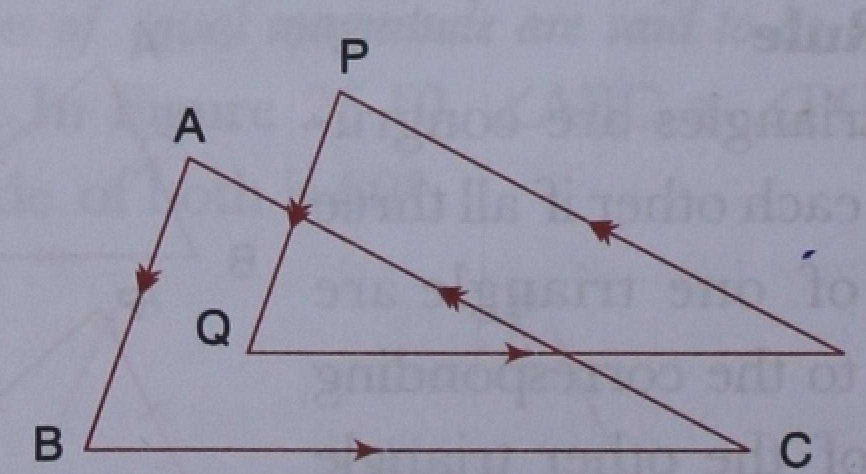


Fig. 26.19

6. Show that the diagonal of a parallelogram divides it into two congruent triangles.

7. Show that the median connecting the vertex of an isosceles triangle to the mid-point of its base, divides it into two congruent triangles.
8. In Figure 26.20, find ∠DCE, given AE = ED, ∠EBC = ∠ECB, and ∠ABE = 35°.

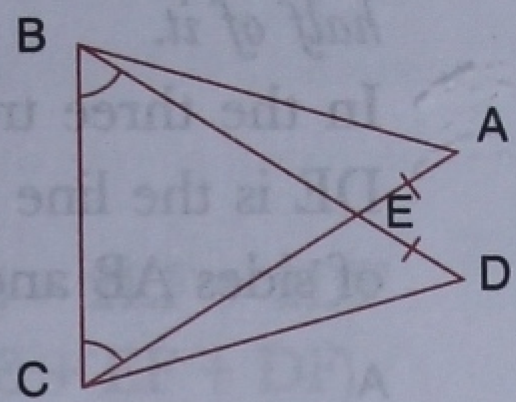


Fig. 26.20

9. In Figure 26.21, find the length of DB, given DE = 15 cm.

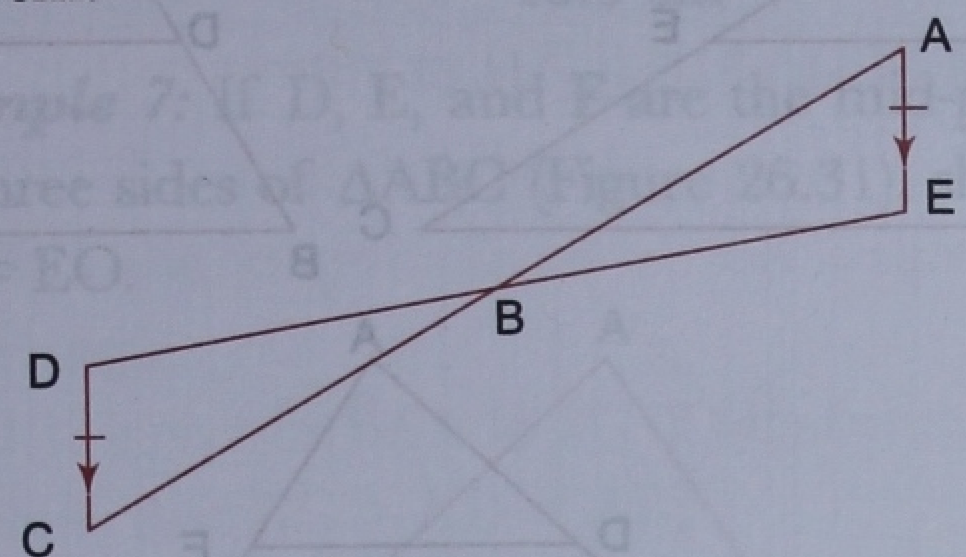


Fig. 26.21

10. Given ∠BAE = 59° in Figure 26.22, find the magnitude of ∠BCE.

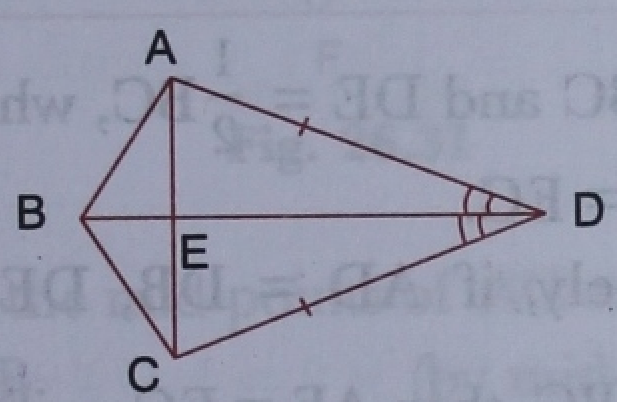


Fig. 26.22

11. Given altitudes CP = BQ in Figure 26.23, show that AB = AC.

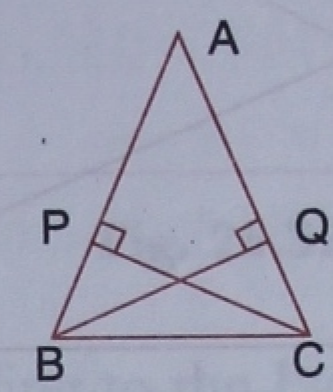


Fig. 26.23

12. In Fig. 26.24, if ∠DCA = ∠ECB, DC = EC, and AC = 12 cm, find the length of BC.

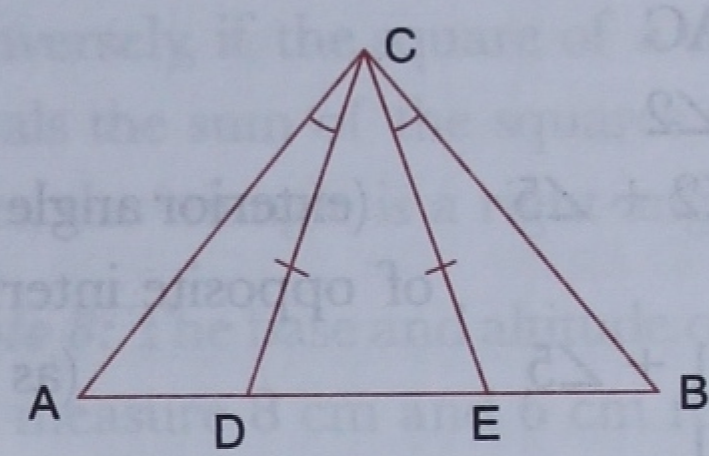


Fig. 26.24

13. X, Y, and Z are the mid-points of the sides AB, BC, and AC, respectively, of an isosceles ΔABC, where AB = AC. Show that ΔXBY ≅ ΔZYC.
14. X, Y, and Z are the mid-points of the sides AB, BC, and AC of an equilateral ΔABC respectively. Show that AY = BZ = CX.

More Theorems on Measures and Magnitudes of Triangles

A **theorem** is a statement that can be demonstrated or proved to be true using **axioms** or facts that are known to be true. Once a theorem is proved, a **corollary** or another statement can be derived from it. This section covers 4 theorems, the proofs of which will not be covered at this level.

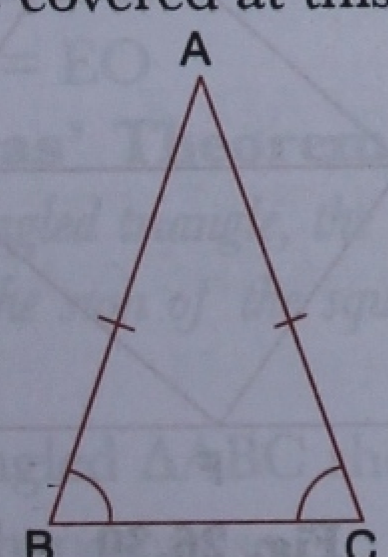


Fig. 26.25

I. *If two sides of a triangle are equal, the angles opposite to them are also equal.*

This theorem describes the property of an isosceles triangle, like ΔABC shown in Figure 26.25, where

AB = AC

⇒ ∠ACB = ∠ABC (opposite angles)

Conversely, if ∠ABC = ∠ACB in ΔABC

⇒ AC = AB (opposite sides)

II. *If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.*

This theorem describes a scalene triangle, like ΔABC in Figure 26.26, where

AC > AB

⇒ ∠ABC > ∠BCA (opposite angles)

Conversely, if $\angle CAB > \angle BCA$,

$\Rightarrow$ BC > AB

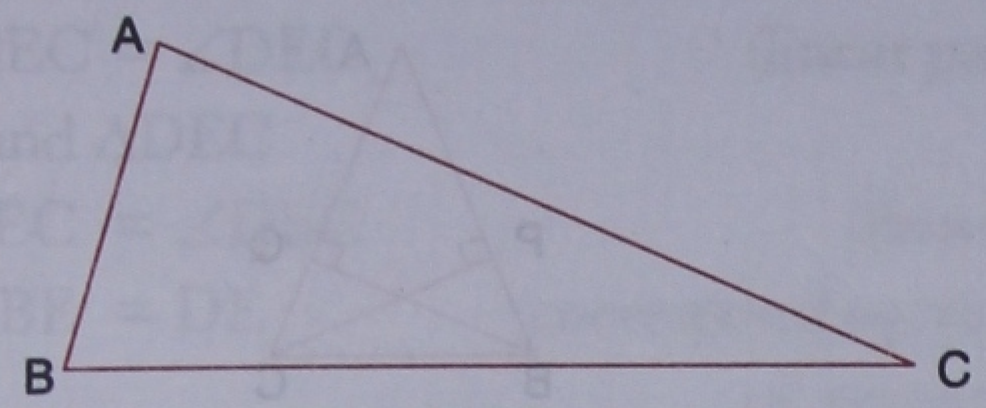


Fig. 26.26

Example 4: In ΔABC (Figure 26.27), AB = AC and P is a point on its base BC. Show that AC > AP < AB.

As AB = AC

$\Rightarrow$ $\angle 1 = \angle 2$

Now $\angle 3 = \angle 2 + \angle 5$ (exterior angle being sum of opposite interior angles)

$\Rightarrow$ $\angle 3 = \angle 1 + \angle 5$ (as $\angle 2 = \angle 1$)

$\Rightarrow$ $\angle 3 > \angle 1$

$\Rightarrow$ AB > AP (opposite sides)

Similarly $\angle 4 = \angle 1 + \angle 6$

$\Rightarrow$ $\angle 4 = \angle 2 + \angle 6$

$\Rightarrow$ $\angle 4 > \angle 2$

$\Rightarrow$ AC > AP

Thus, AC > AP < AB

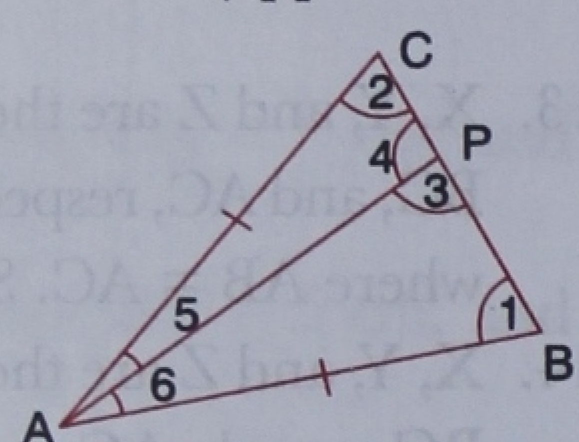


Fig. 26.27

Example 5: Identify the longest and shortest sides in ΔABC (Figure 26.28).

$\angle ABC = 180° - 100° = 80°$ (linear pair)

$\angle BCA = 180° - 140° = 40°$ (linear pair)

$\angle CAB = 180° - (80° + 40°) = 60°$

(sum of angles being 180°)

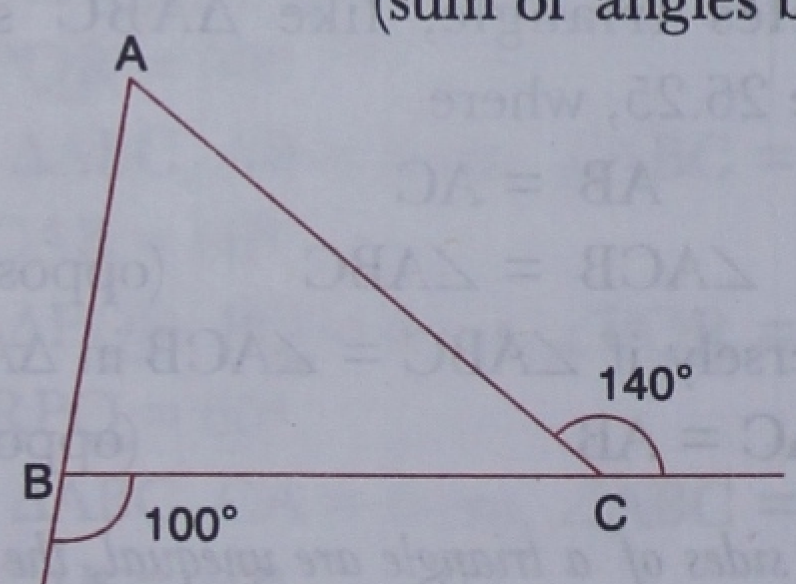


Fig. 26.28

Thus, side opposite $\angle ABC$ or AC is the longest side and the side opposite $\angle BCA$ or AB is the shortest side in ΔABC.

III. Mid-point Theorem

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

In the three triangles shown in Figure 26.29, DE is the line segment joining the mid-points of sides AB and CA.

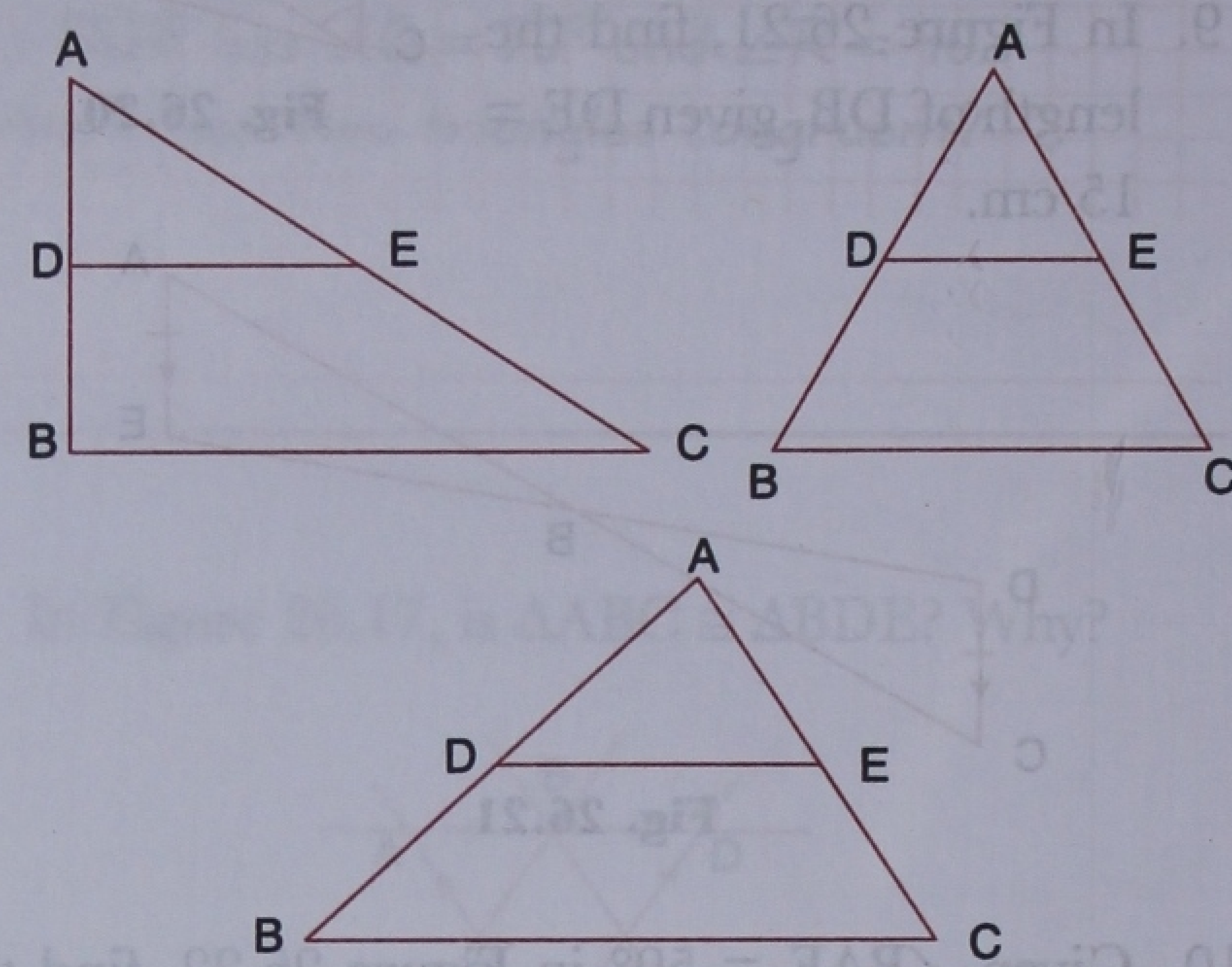


Fig. 26.29

DE || BC and DE = $\frac{1}{2}$BC, where AD = DB and AE = EC.

Conversely, if AD = DB, DE || BC, and DE = $\frac{1}{2}$ BC, then AE = EC or if a line segment is drawn from the mid-point of any one side of a triangle parallel to an adjacent side and equal to half its length, it meets the mid-point of the third side.

Example 6: In ΔABC (Figure 26.30), the vertices of ΔDEF are on the mid-points of its sides. If DE + EF + FD = 7.8 cm, find the measure of BC + AB + AC.

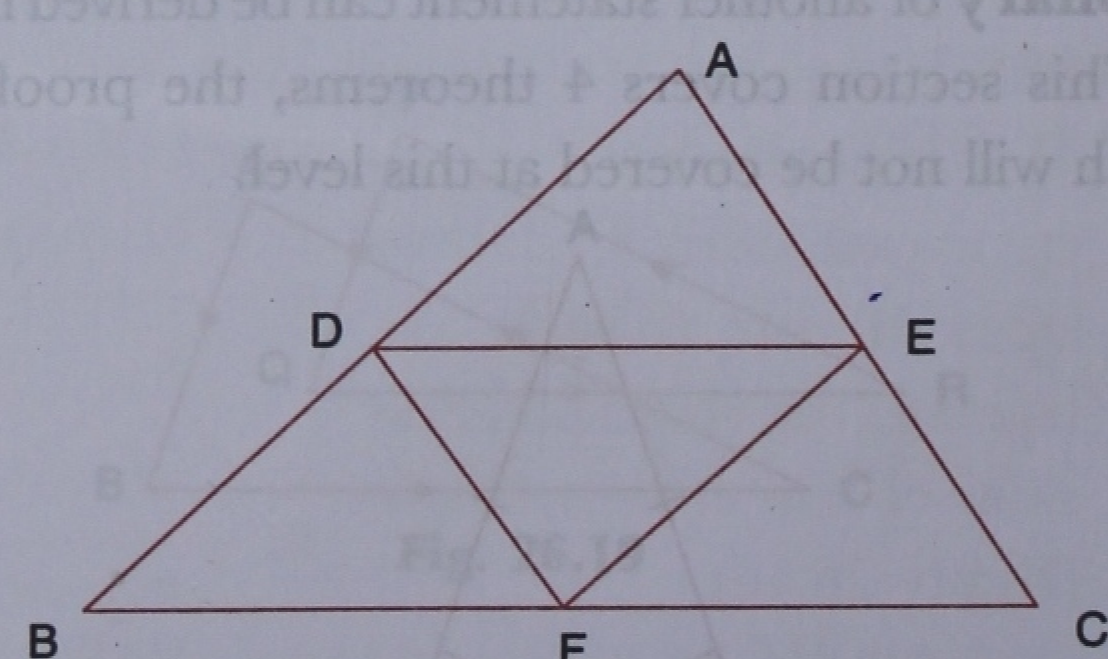


Fig. 26.30

As DE joins the mid-points of sides AB and AC,

$DE = \frac{1}{2}BC$ (by mid-point theorem)

$\Rightarrow$ BC = 2DE

Similarly AB = 2EF

and AC = 2DF

$\Rightarrow$ BC + AB + AC = 2DE + 2EF + 2DF

= 2(DE + EF + DF)

= 2 × 7.8 cm

= 15.6 cm

Example 7: If D, E, and F are the mid-points of the three sides of ΔABC (Figure 26.31), show that BO = EO.

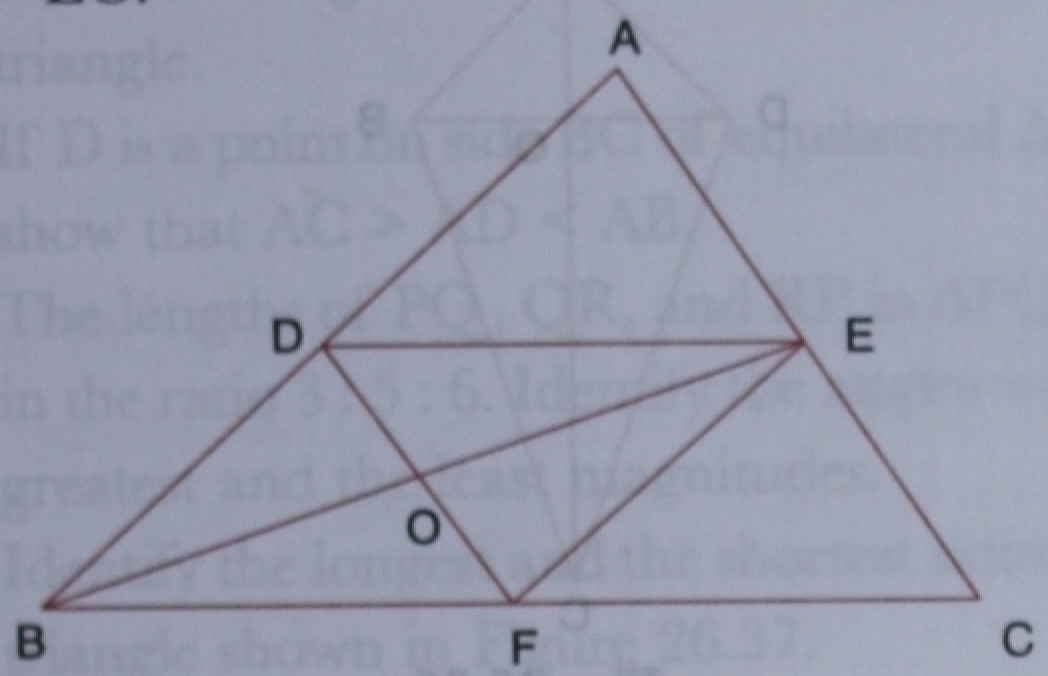


Fig. 26.31

As EF joins the mid-points of AC and BC,

EF || AB (by mid-point theorem)

As DE joins the mid-points of AB and AC,

DE || BC (by mid-point theorem)

In ΔBFO and ΔEDO

∠OBF = ∠OED (alternate interior angles)

∠OFB = ∠ODE (alternate interior angles)

As F is mid-point of BC, $BF = \frac{1}{2}BC$

As DE joints the mid-points of AB and AC,

$DE = \frac{1}{2}BC$ (by mid-point theorem)

$\Rightarrow$ BF = ED

$\Rightarrow$ ΔBFO ≅ ΔEDO (by ASA)

$\Rightarrow$ BO = EO

IV. Pythagoras' Theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the base and the altitude.

In right-angled ΔABC shown in Figure 26.32, AC is the hypotenuse.

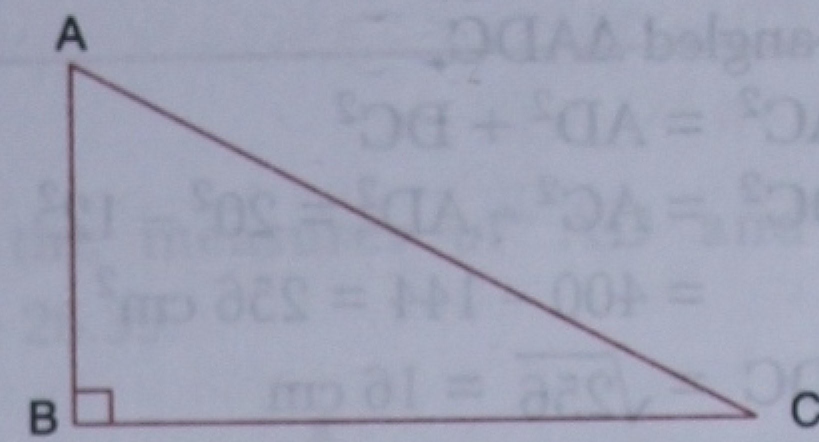


Fig. 26.32

So, according to the Pythagoras' theorem

$AC^2 = AB^2 + BC^2$

(The proof of this theorem has been demonstrated in the chapter on Symmetry, Reflection, and Rotation.)

Conversely, if the square of a side in a triangle equals the sum of the squares of its other two sides, the triangle is a right-angled triangle.

Example 8: The base and altitude of a right-angled triangle measure 8 cm and 6 cm respectively. Find the length of its hypotenuse.

In right-angled ΔABC shown in Fig. 26.33,

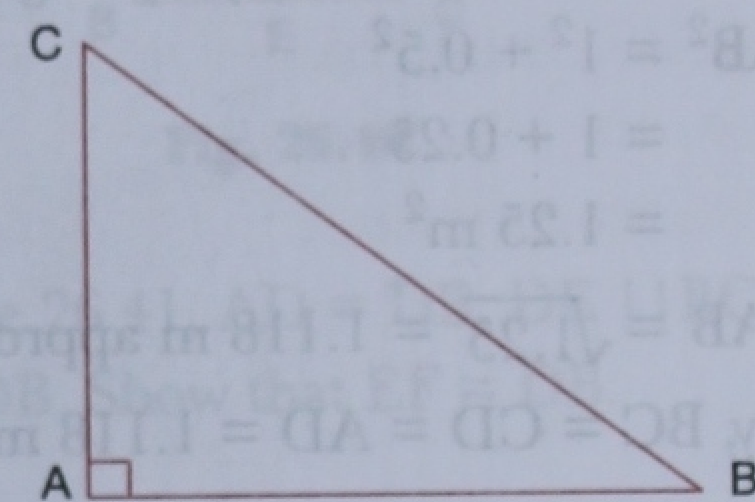


Fig. 26.33

$BC^2 = AB^2 + CA^2$ (by Pythagoras' theorem)

$\Rightarrow BC^2 = 8^2 + 6^2$

$= 64 + 36 = 100 \text{ cm}^2$

$\Rightarrow BC = \sqrt{100} = 10 \text{ cm}$

Example 9: In Figure 26.34, AD ⊥ BC, AC = 20 cm, AD = 12 cm, and BC = 25 cm. Find the length of AB.

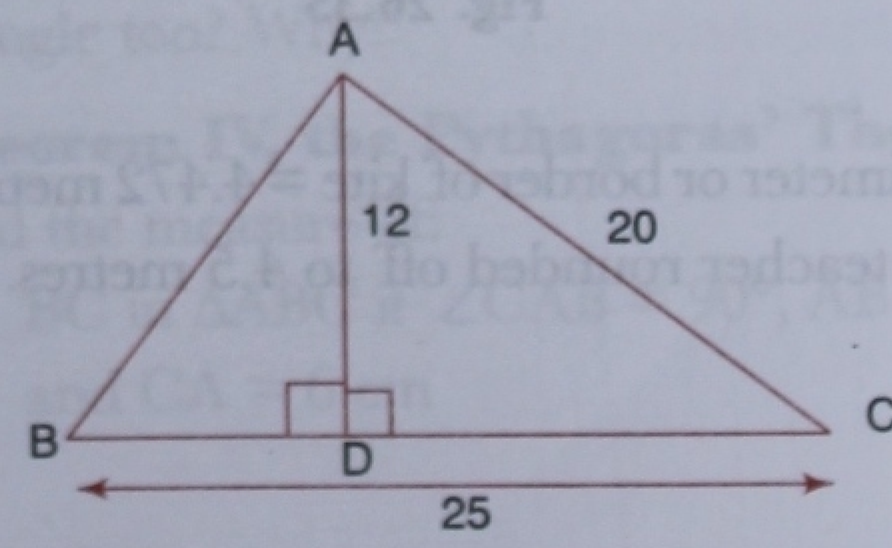


Fig. 26.34

In right-angled ΔADC,

$AC^2 = AD^2 + DC^2$

$\Rightarrow \quad DC^2 = AC^2 - AD^2 = 20^2 - 12^2$

$= 400 - 144 = 256 \text{ cm}^2$

$\Rightarrow \quad DC = \sqrt{256} = 16 \text{ cm}$

$\Rightarrow \quad BD = BC - DC = 25 \text{ cm} - 16 \text{ cm} = 9 \text{ cm}$

In right-angled ΔADB,

$AB^2 = AD^2 + BD^2 = 12^2 + 9^2$

$= 144 + 81 = 225 \text{ cm}^2$

$\Rightarrow \quad AB = \sqrt{225} = 15 \text{ cm}$

Example 10: Let us now work out how Rahul's computer teacher worked out the 'kite problem' at the beginning of this unit.

Rahul wanted to know how much tape would be needed to have a border to his 2 metre tall and 1 m wide kite. See Figure 26.35.

Now, AO = 1 m, OB = 0.5 m

$\Rightarrow \quad AB^2 = AO^2 + OB^2$ (by Pythagoras' theorem)

$\Rightarrow \quad AB^2 = 1^2 + 0.5^2$

$= 1 + 0.25$

$= 1.25 \text{ m}^2$

$\Rightarrow \quad AB = \sqrt{1.25} = 1.118$ m approximately

Similarly, BC = CD = AD = 1.118 m

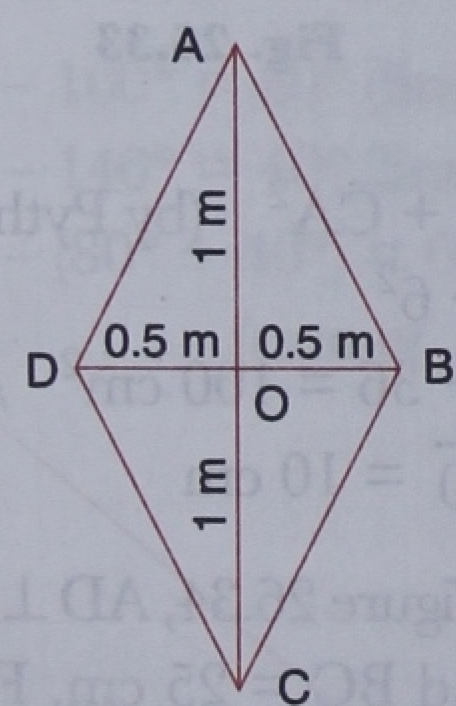


Fig. 26.35

$\Rightarrow$ Perimeter or border of kite = 4.472 metres which the teacher rounded off to 4.5 metres.

As an approximate answer was expected from the teacher, Rahul did not specify where the diagonals of the kite intersect.

Example 11: Find the perimeter of Rahul's kite if the diagonal meets at a point O (Figure 26.36) such that AO = OB = OD = 0.5 m and OC = 1.5 m.

$AB^2 = 0.5^2 + 0.5^2$

$= 0.25 + 0.25$

$= 0.5$

$\Rightarrow \quad AB = \sqrt{0.5} = 0.707$ m approximately

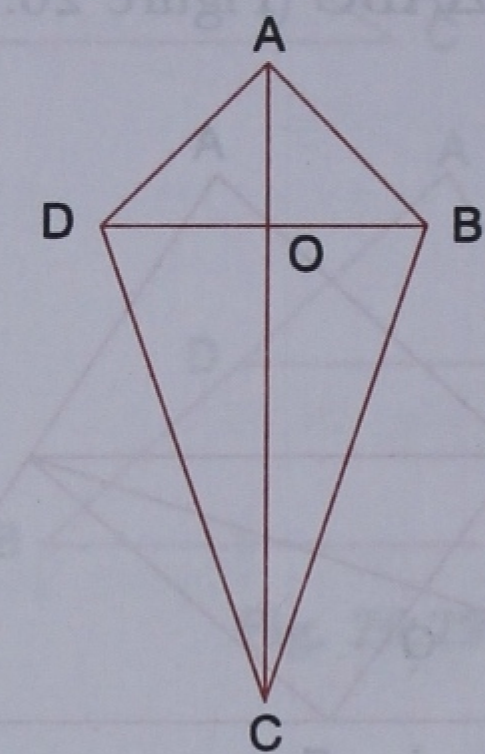


Fig. 26.36

Similarly, AD = 0.707 m

$\Rightarrow \quad AB + AD = 0.707 \times 2 = 1.414$ m

$BC^2 = 0.5^2 + 1.5^2$

$= 0.25 + 2.25$

$= 2.5$

$\Rightarrow \quad BC = \sqrt{2.5} = 1.581$ m approximately

Similarly CD = 1.581 m and

$BC + CD = 1.581 \times 2 = 3.162$ m

Thus perimeter = AB + AD + BC + CD

= 1.414 + 3.162 = 4.58 m approximately.

Try this!

In ΔABC and ΔDEF

AB = EF, AC = DF and

∠B = ∠F. Is ΔABC ≅ ΔDEF?

Exercise 26.3

On Theorems I and II

1. Identify the longest and the shortest sides in the following triangles.
   (i) ΔABC, where ∠ABC = 80°, ∠BCA = 40°
   (ii) ΔABC, where ∠ABC = 70°, ∠BCA = 60°
   (iii) ΔABC, where ∠ABC = 40°, ∠BCA = 50°
   (iv) ΔABC, where ∠ABC = 90°, ∠BCA = 60°
   (v) ΔABC, where ∠ABC = 60°, ∠BCA = 70°
2. The altitude of a right-angled triangle measures 6.23 cm while its base also measures 6.23 cm. Find the magnitude of all the angles of the triangle.
3. If D is a point on side BC of equilateral ΔABC, show that AC > AD < AB.
4. The lengths of PQ, QR, and RP in ΔPQR are in the ratio 3 : 5 : 6. Identify the angles with the greatest and the least magnitudes.
5. Identify the longest and the shortest sides in the triangle shown in Figure 26.37.

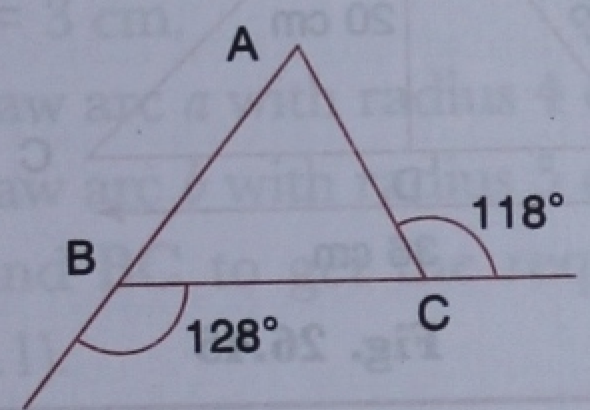


Fig. 26.37

On Theorem III, the Mid-Point Theorem

6. Find the measures of DE and ∠DEA in Figure 26.38.

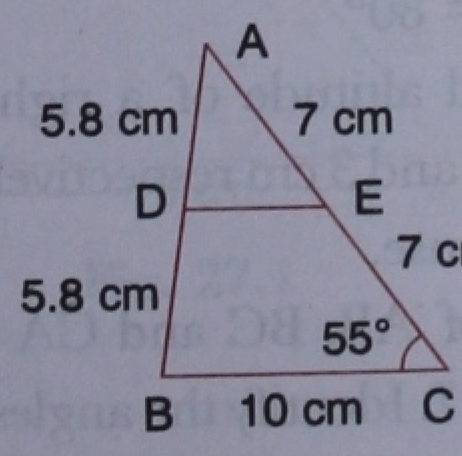


Fig. 26.38

7. Find the measures of AD and BC in Figure 26.39.

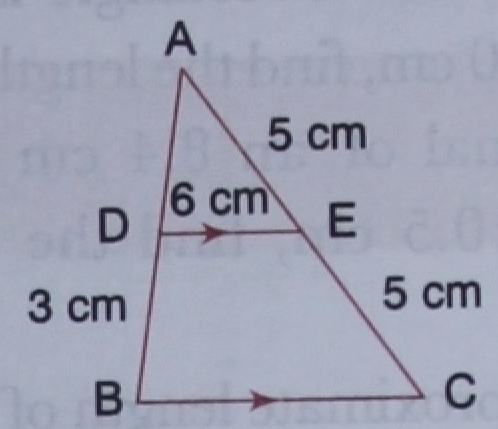


Fig. 26.39

8. In Figure 26.40, if D and E are the mid-points on sides AC and BC of isosceles ΔABC respectively and ∠B = ∠C = 71°, find ∠CDE.

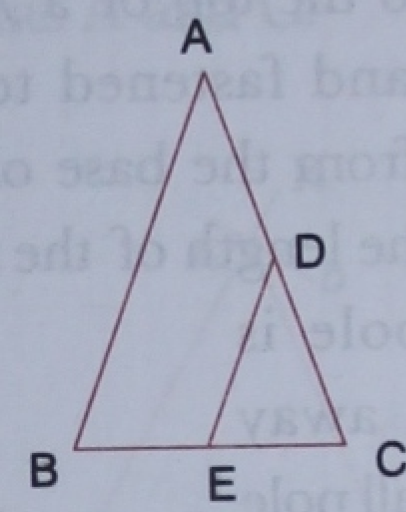


Fig. 26.40

9. In Figure 26.41, AD = DB, DE | |BC, and EF, | | DB. Show that EF = DB.

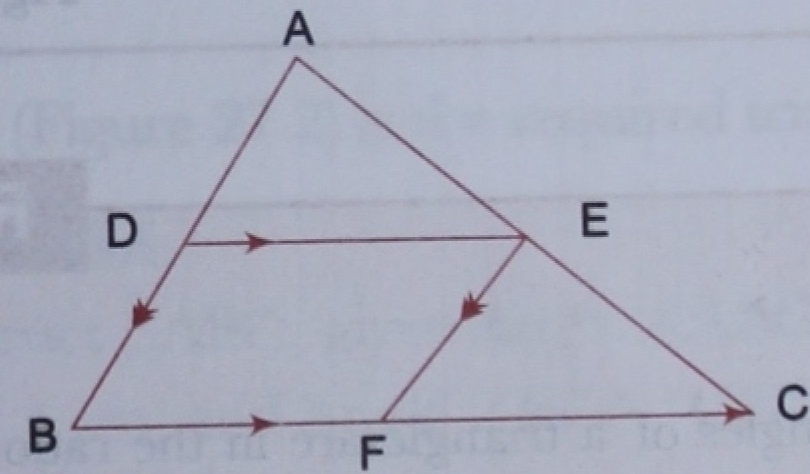


Fig. 26.41

10. If D, E, and F are the mid-points of the sides of an isosceles triangle, is ΔDEF an isosceles triangle too? Why?

On Theorem IV, the Pythagoras' Theorem

11. Find the measure of:
    (i) BC in ΔABC if ∠CAB = 90°, AB = 8 cm, and CA = 6 cm

(ii) AB in ΔABC if ∠ABC = 90°, AC = 20 cm, and BC = 12 cm

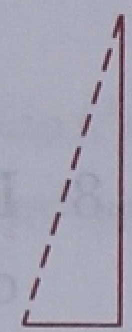
(iii) AC in ΔABC if ∠BCA = 90°, AB = 5 cm, and BC = 3 cm

12. If the length of a rectangle is 24 cm and its breadth is 10 cm, find the length of its diagonal.

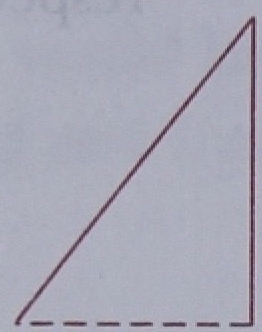
13. If a diagonal of an 8.4 cm long rectangle measures 10.5 cm, find the measure of its breadth.

14. Find the approximate length of a square whose diagonal measures 10 cm.

15. A ladder placed 1.4 m from the base of a boundary wall reaches its top. If the ladder is 5 m long, find the height of the boundary wall.



16. A wire is tied to the top of a 7.2 m high flagpole and fastened to the ground 5.4 m from the base of the flagpole. Find the length of the wire.



17. A 17 m tall pole is erected 8.4 m away from a 10.7 m tall pole (Figure 26.42). Find the distance between A and B.

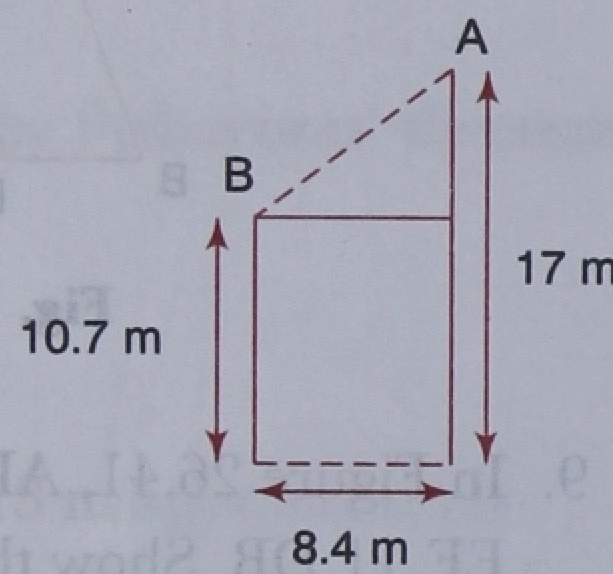


Fig. 26.42

18. From the measures given on the kite shown in Figure 26.43, find the length of its border.

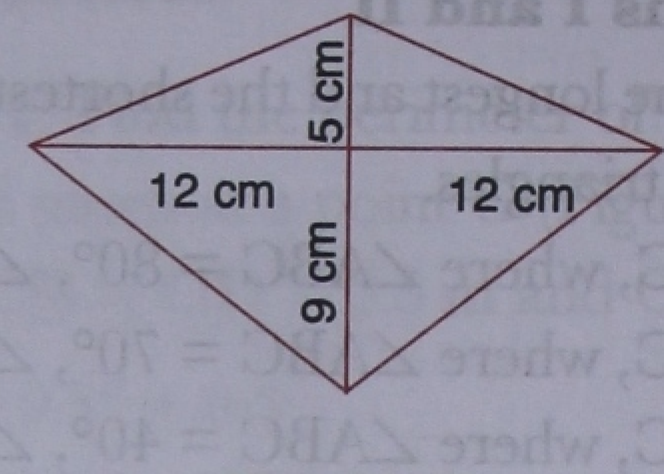


Fig. 26.43

19. Find the length of BC in Figure 26.44, given AD ⊥ BC, AB = 20 cm, AC = 34 cm and AD =16 cm.

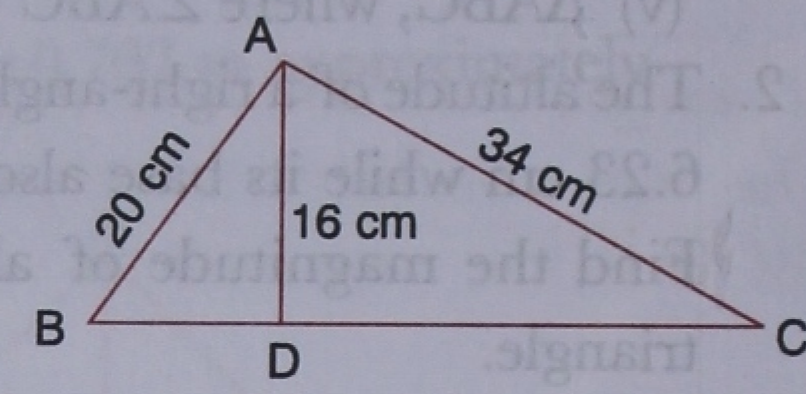


Fig. 26.44

20. Find the length of AC in Figure 26.45, given AD ⊥ BC, AB = 25 cm, AD = 20 cm, and BC = 36 cm.

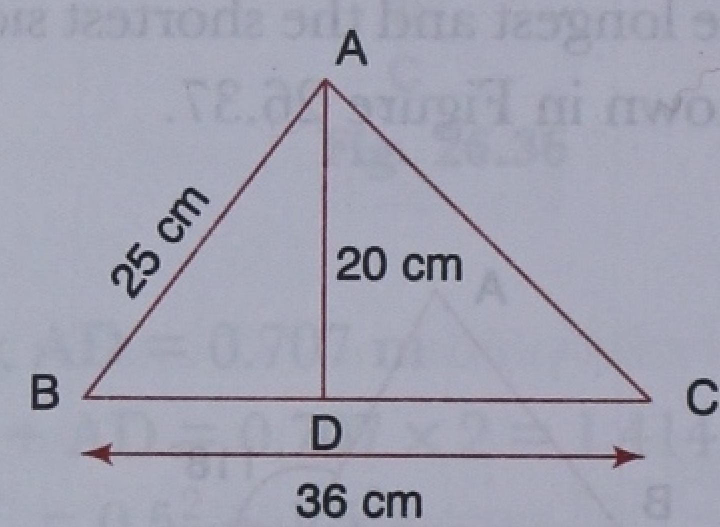


Fig. 26.45

Revision Exercise

1. The angles of a triangle are in the ratio 2 : 3 : 4. Find the magnitude of the angles.

2. If an exterior angle of a triangle is given as $\frac{x}{2}$ and one of its opposite interior angles is given as $\frac{x}{3}$, find the other opposite interior angle.

3. Which of the following pairs of triangles are congruent and why?

   (i) In Δ ABC, AB = 9 cm, CA = 7 cm and ∠CAB = 50°
   In Δ XYZ, XY = 6 cm, XZ = 5 cm and ∠YXZ = 50°

   (ii) In Δ PQR, PQ = 7cm, PR = 8 cm and ∠QRP = 80°
   In Δ DEF, DE = 7cm, DF = 8 cm and ∠EFD = 80°

4. The base and altitude of a right-angled triangle measure 5 cm and 3 cm respectively. Find the length of its hypotenuse.

5. The lengths of AB, BC and CA in Δ ABC are in the ratio 4 : 5 : 7. Identify the angles with the greatest and the least magnitudes.