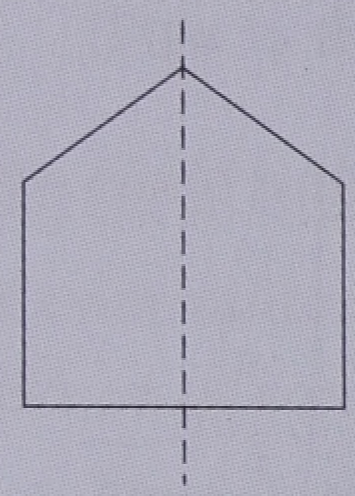
8

Symmetry, Reflection and Rotation

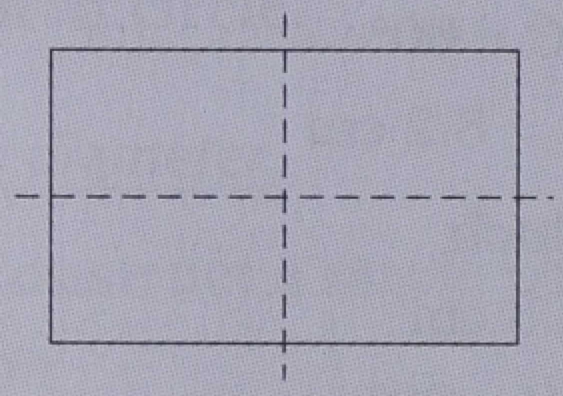
Linear Symmetry

A body is said to be symmetrical or exhibit symmetry when different parts of it matches exactly in shape and size. When this symmetry occurs across a line drawn through the middle of a figure, or when a straight line divides a plane figure into two parts that are identical in size and shape, we say that the figure has linear symmetry.

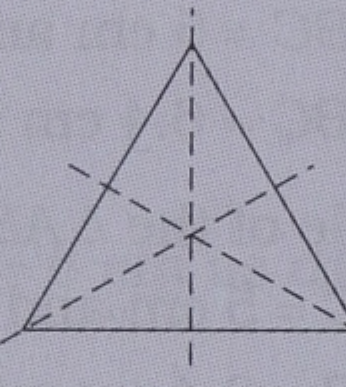
In the figure, the line AB divides the mask into two identical halves. Such a figure is called a linear symmetric figure and the line AB is called the line (or the axis) of symmetry or the mirror line. A figure may have one or more lines of symmetry, as shown below.



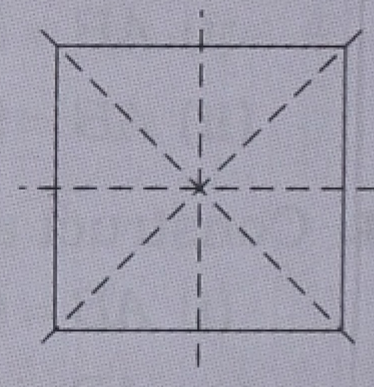
This pentagon has 1 line of symmetry



A rectangle has 2 lines of symmetry

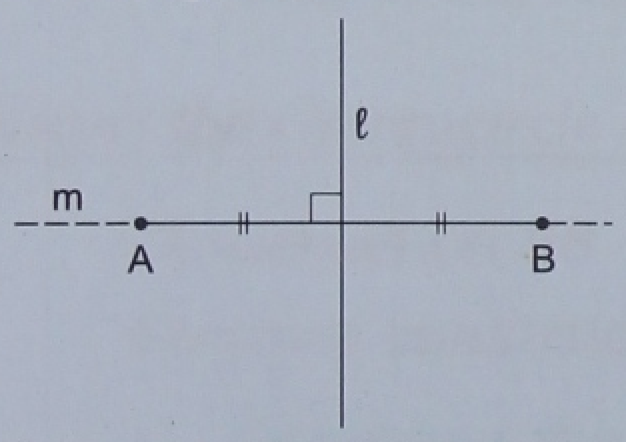
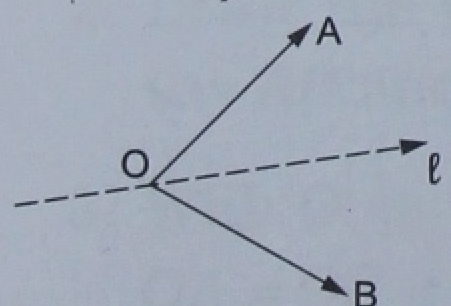


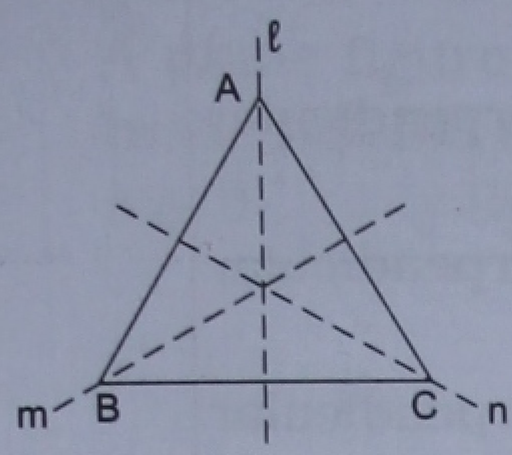
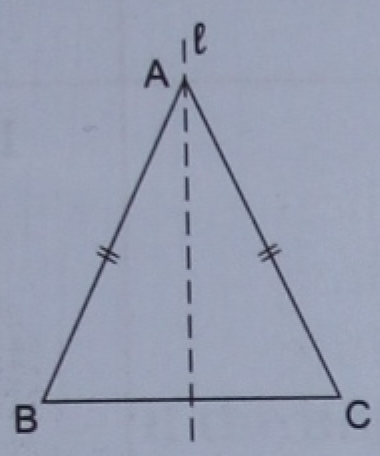
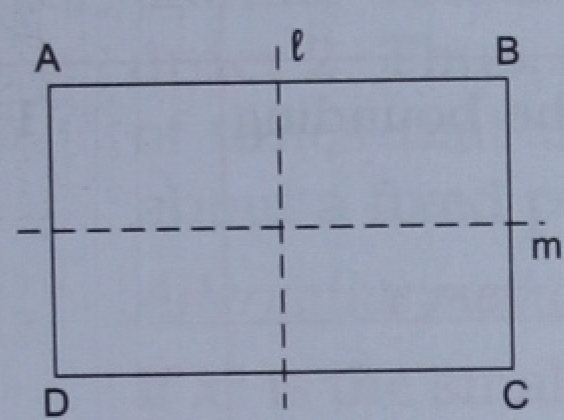
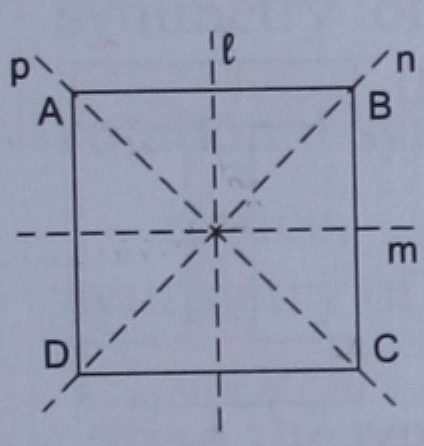
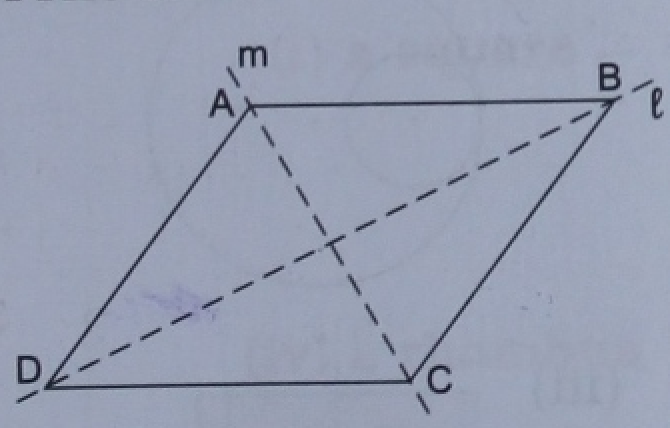
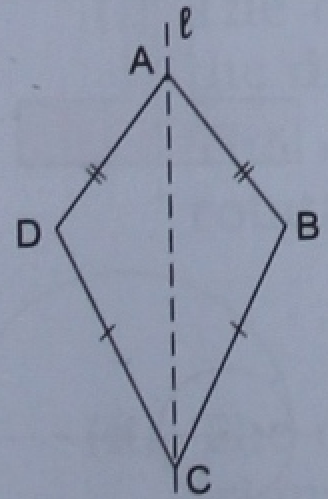
An equilateral triangle has 3 lines of symmetry

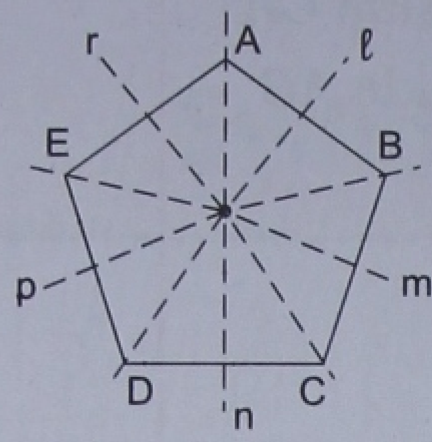
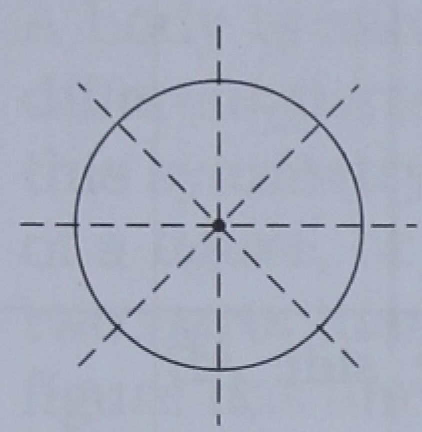
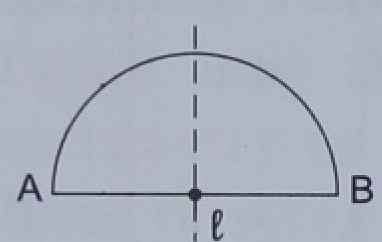


A square has 4 lines of symmetry

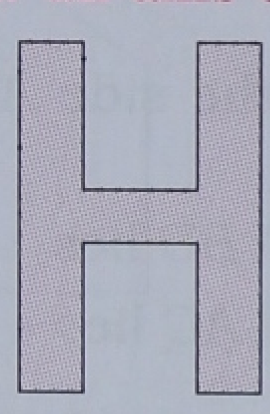
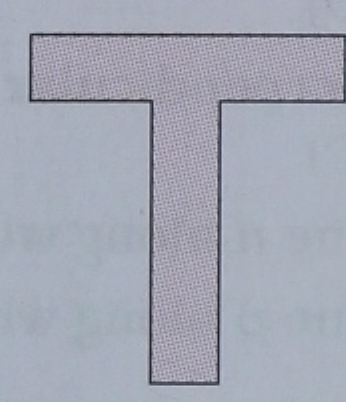
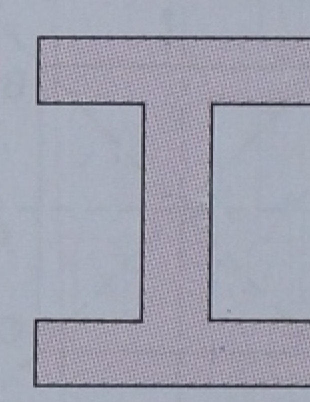
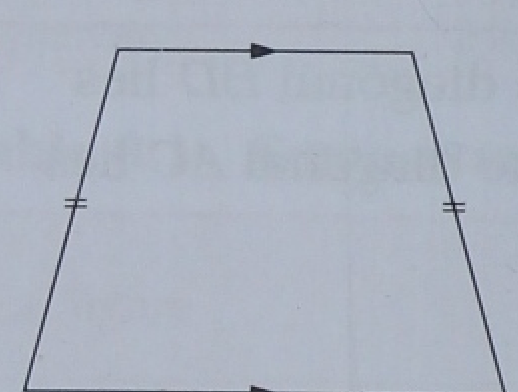
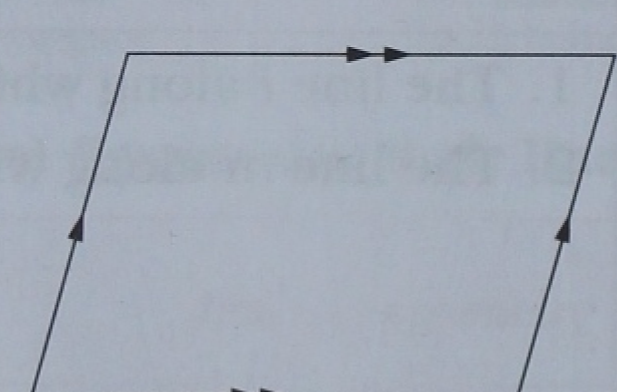
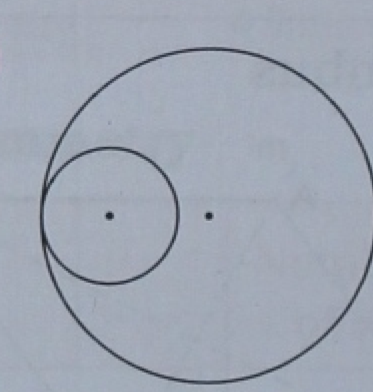
Table 8.1 Some symmetrical figures and their lines of symmetry

Geometrical figure	Lines of symmetry	Number of lines of symmetry
1. A line segment 	1. The perpendicular bisector l of the line segment AB 2. The line m on which the line segment AB lies	2
2. An angle 	The line l bisecting the $\angle AOB$	1

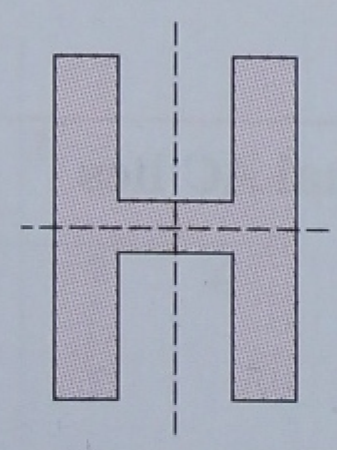
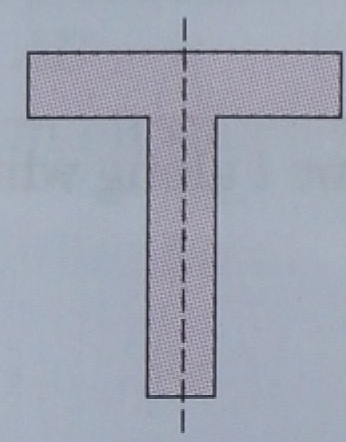
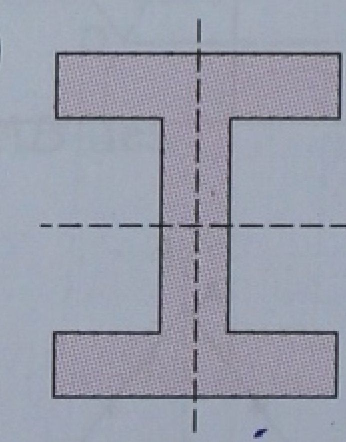
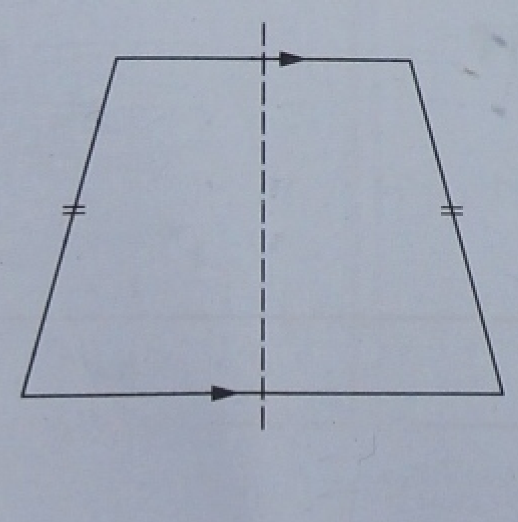
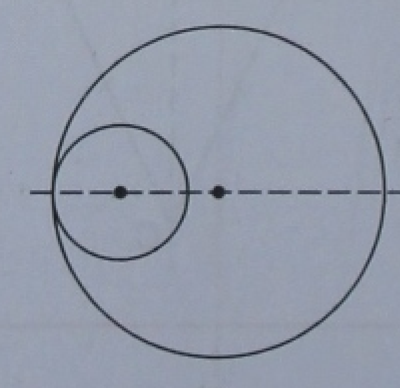
3. An equilateral triangle 	1. The perpendicular bisector *l* of the side *BC* 2. The perpendicular bisector *m* of the side *CA* 3. The perpendicular bisector *n* of the side *AB*	3
4. An isosceles triangle 	The perpendicular bisector *l* of the side *BC*	1
5. A rectangle 	1. The perpendicular bisector *l* of the side *AB* (or *CD*) 2. The perpendicular bisector *m* of the side *AD* (or *BC*)	2
6. A square 	1. The perpendicular bisector *l* of the side *AB* (or *CD*) 2. The perpendicular bisector *m* of the side *AD* (or *BC*) 3. The line *n* along which the diagonal *BD* lies 4. The line *p* along which the diagonal *AC* lies	4
7. A rhombus 	1. The line *l* along which the diagonal *BD* lies 2. The line *m* along which the diagonal *AC* lies	2
8. A kite 	The line *l* along which the diagonal *AC* lies	1

9. A regular pentagon 	1. The line l along which the perpendicular bisector of AB lies 2. The line m along which the perpendicular bisector of BC lies 3. The line n along which the perpendicular bisector of CD lies 4. The line p along which the perpendicular bisector of DE lies 5. The line r along which the perpendicular bisector of EA lies	5
10. A circle 	Each line through the centre	Infinite
11. A semicircle 	The perpendicular bisector l of the bounding diameter AB	1

EXAMPLE **Draw an axis of symmetry for the following (if possible).**

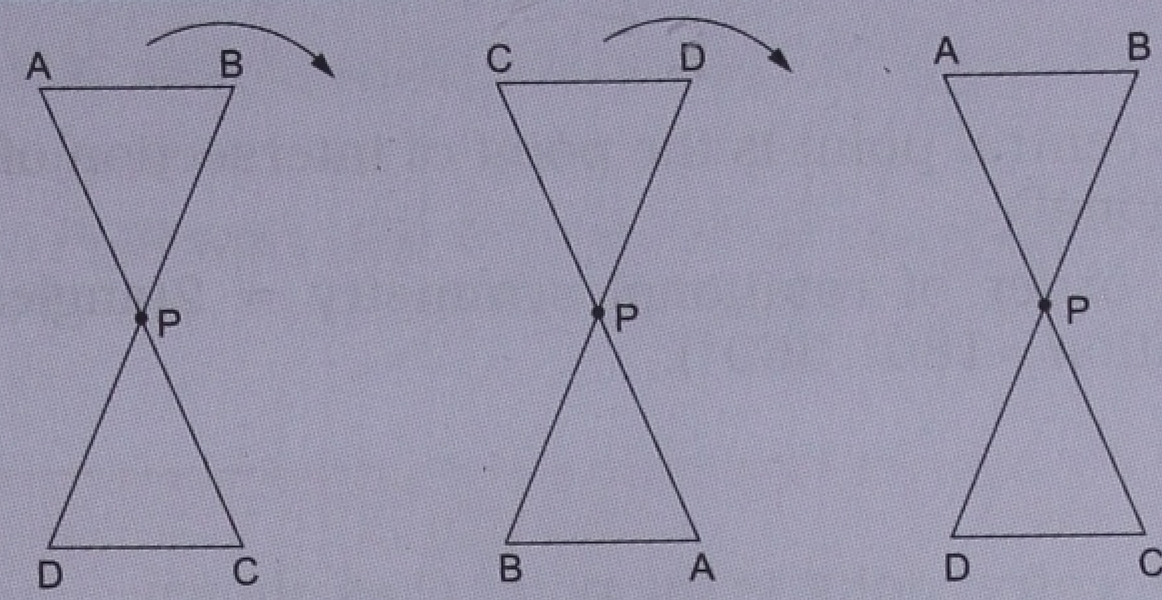
(i) 
(ii) 
(iii) 
(iv) 
(v) 
(vi) 

Solution

(i) 
(ii) 
(iii) 
(iv) 
(v) A parallelogram has no axis of symmetry.
(vi) 

Rotational Symmetry

A plane figure is said to have rotational symmetry if it remains the same after being rotated about a central point through an angle less than 360°. Needless to say that any figure will coincide with itself when rotated through 360°.

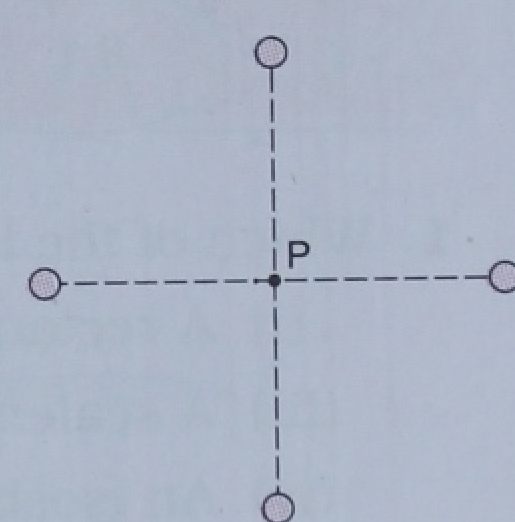


Angle of rotation = 180°   Angle of rotation = 360°

If the adjoining figure is rotated about its central point P through 360°, it will coincide with its outline twice once after a rotation of 180° and them after another rotation of 180°. We, therefore say that it has rotational symmetry of order 2. Thus, the order of rotational symmetry of a figure is the highest number of times that the figure coincides with its outline when rotated through 360° about a fixed point called its central point.

Alternative method

If $x°$ is the smallest angle of rotation that makes a figure coincide with its outline then the order of rotational symmetry of the figure $= \frac{360°}{x}$. Thus, the order of rotational symmetry of the adjoining figure is 4.



Hence, the adjoining figure has rotational symmetry of order 4.

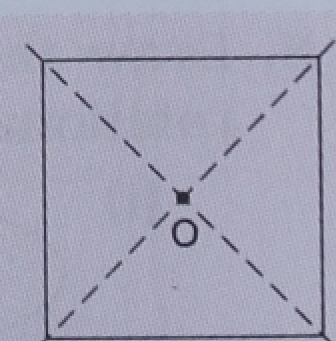
EXAMPLE **Find the central point and the order of rotational symmetry of**

(i) a square **(ii) an equilateral triangle** **(iii)** H

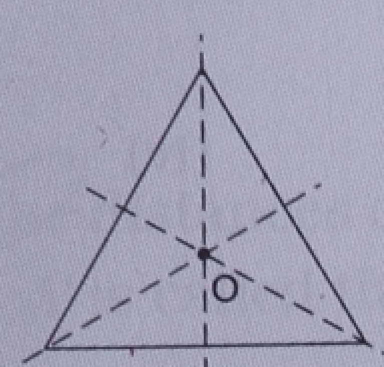
(iv) a rhombus **(v)** S **(vi)** N

Solution

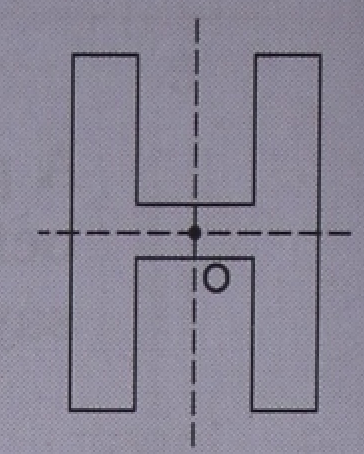
(i) The central point O is the point of intersection of the diagonals of the square.
The order of rotational symmetry = 4(angles of rotation = 90°, 180°, 270°, 360°).



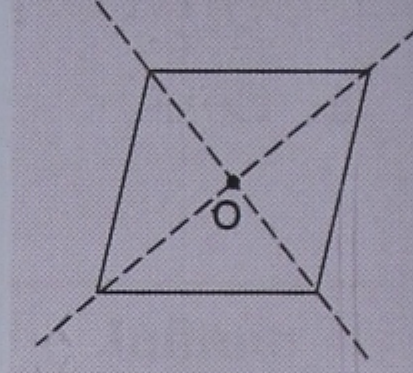
(ii) The central point is the centroid of the equilateral triangle.
The order of rotational symmetry = 3(angles of rotation = 120°, 240°, 360°).



(iii) The central point is the point O.
The order of rotational symmetry = 2(angles of rotation = 180°, 360°).

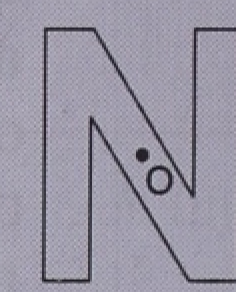


(iv) The central point is the point of intersection of the diagonals.
The order of rotational symmetry = 2(angles of rotation = 180°, 360°).



(v) The central point is the point O as shown.
The order of rotational symmetry = 2 (angles of rotation = 180°, 360°).

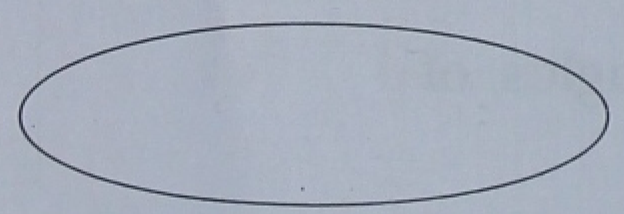
(vi) The central point is the point O as shown.
The order of rotational symmetry = 2 (angles of rotation = 180°, 360°).



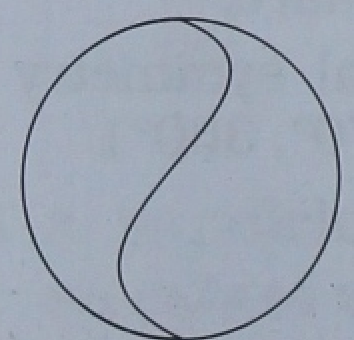
EXERCISE 8A

1. Which of the following statements are true?
   - (i) A rectangle has 4 lines of symmetry.
   - (ii) A square has 4 lines of symmetry.
   - (iii) A scalene triangle has no line of symmetry.
   - (iv) An isosceles triangle has three lines of symmetry.
   - (v) An equilateral triangle has three lines of symmetry.
   - (vi) The letter 'A' has 1 line of symmetry.
   - (vii) The letter 'D' has 2 lines of symmetry.
   - (viii) The letter 'M' has 2 lines of symmetry.
   - (ix) A circle has infinite axes of symmetry.
   - (x) A rhombus has no line of symmetry.
   - (xi) A rhombus has no rotational symmetry.
   - (xii) All regular polygons of n sides have rotational symmetry of order n.
2. Which of the letters ABCDEFGHIJKLMNOPQRSTUVWXYZ have only
   - (i) one axis of symmetry
   - (ii) two axes of symmetry
   - (iii) three axes of symmetry
   - (iv) four or more axes of symmetry
3. Draw the lines of symmetry (if any) of the following shapes. Does any of them have rotational symmetry?

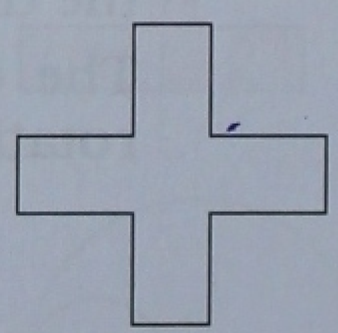
(i)



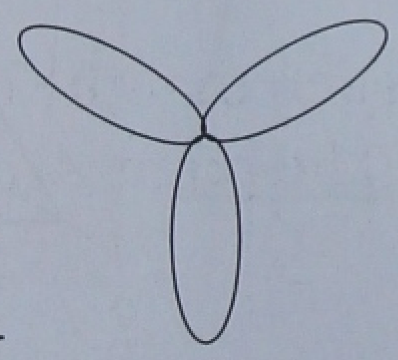
(ii)



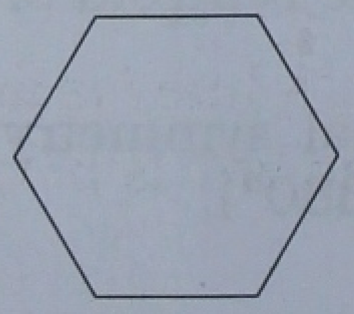
(iii)



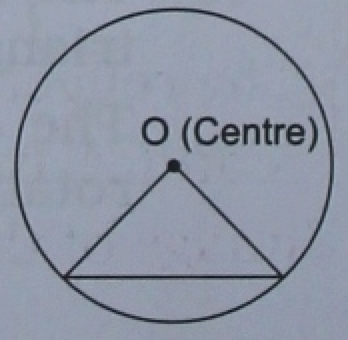
(iv)



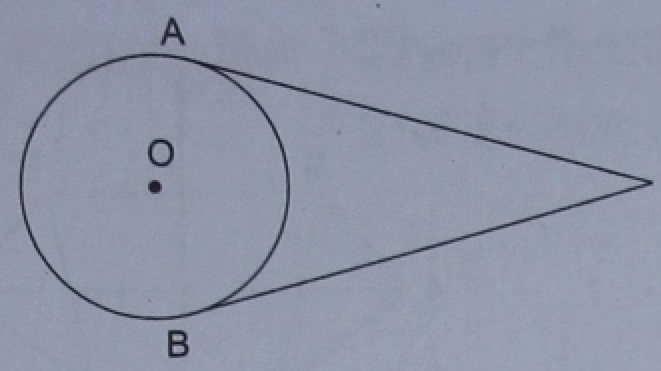
(v)



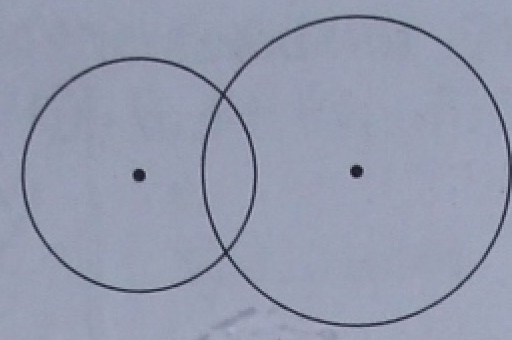
(vi)



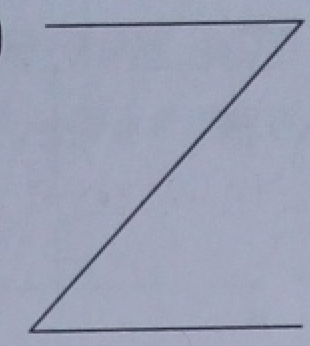
(vii)



(viii)



(ix)



4. Construct an isosceles $\triangle ABC$ with base $BC = 6$ cm and vertical $\angle A = 30°$. Draw its line(s) of symmetry (if any).

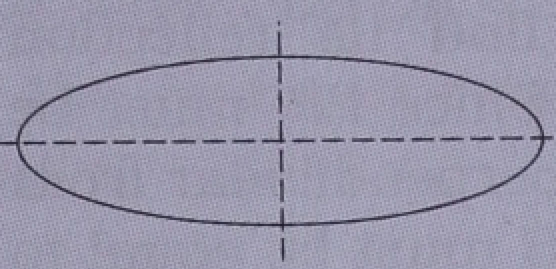
5. Construct an equilateral $\triangle PQR$ with side 4.5 cm. Draw its lines of symmetry (if any).

ANSWERS

1. (ii), (iii), (v), (vi), (ix), (xii)

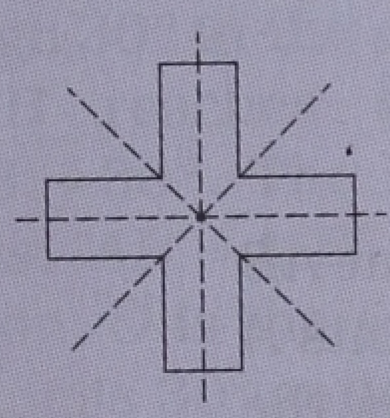
2. (i) ABCDEKMTUVWY (ii) HIX (iii) none (iv) O

3. (i) Two lines of symmetry

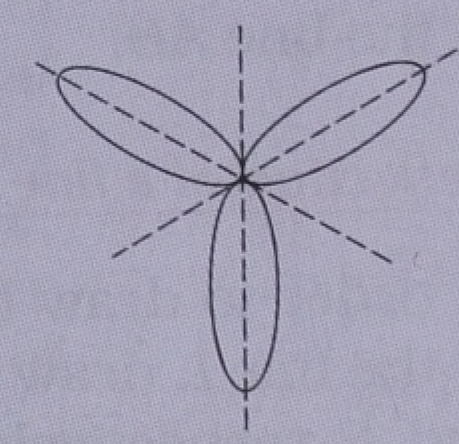


(ii) No line of symmetry, rotational symmetry of order 2

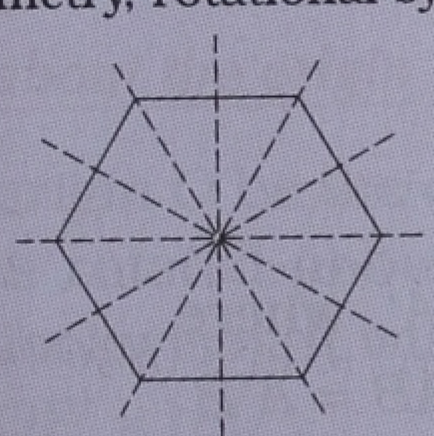
(iii) Four lines of symmetry, rotational symmetry of order 4



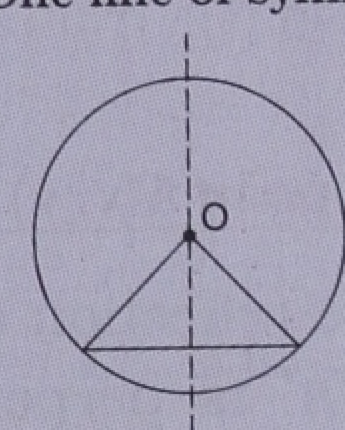
(iv) Three lines of symmetry, rotational symmetry of order 3



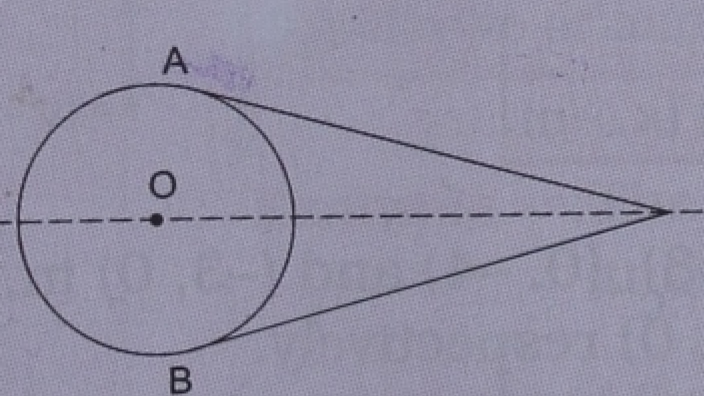
(v) Six lines of symmetry, rotational symmetry of order 6



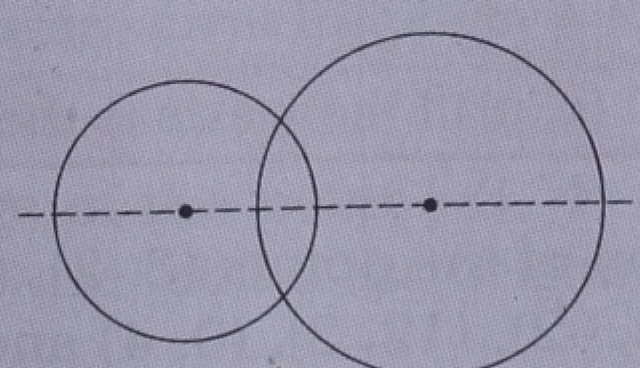
(vi) One line of symmetry



(vii) One line of symmetry



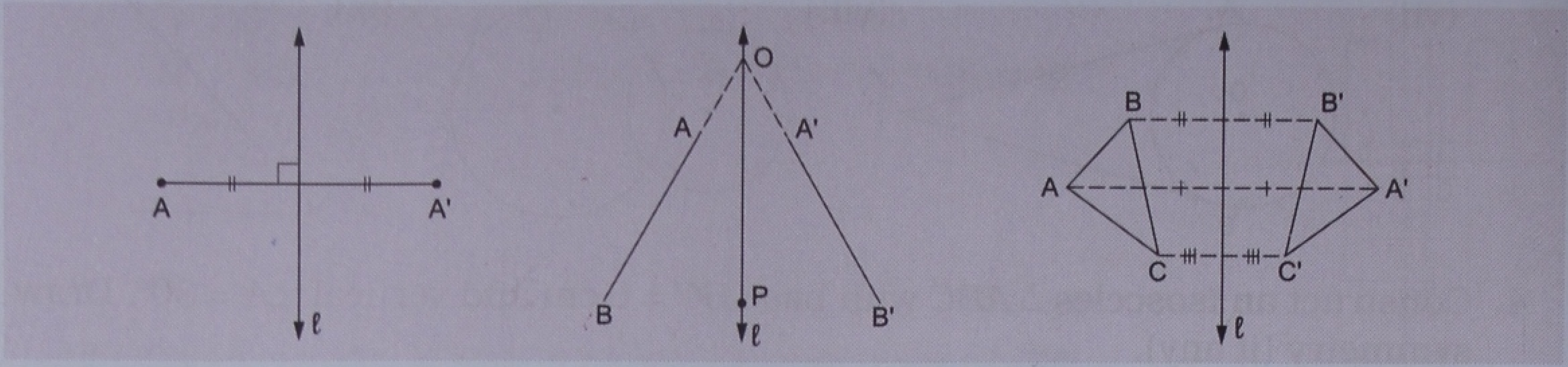
(viii) One line of symmetry



(ix) No line of symmetry, rotational symmetry of order 2

Reflection

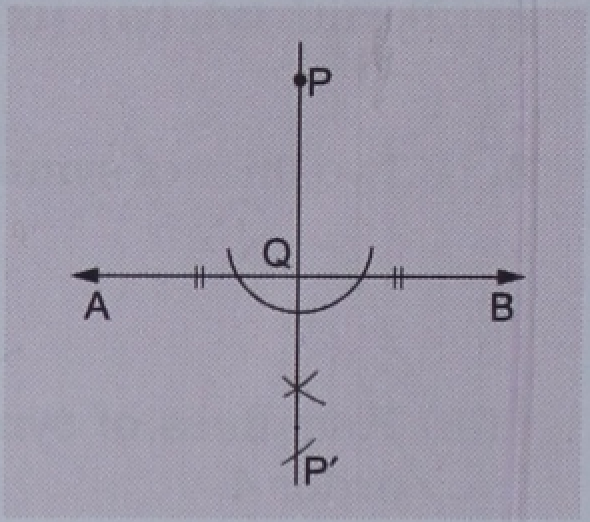
The reflection of a point A by a line l is another point A' such that the distances of A and A' from l are equal and AA' is perpendicular to l. The point A' is called the image of the point A.



Similarly, the reflection of the line segment AB by the line l is the line segment $A'B'$. Note that $AB = A'B'$. Also, AB and $A'B'$ are equally inclined to the line l, that is, $\angle BOP = \angle B'OP$. The reflection of $\triangle ABC$ by the line l is $\triangle A'B'C'$ and $\triangle ABC \cong \triangle A'B'C'$.

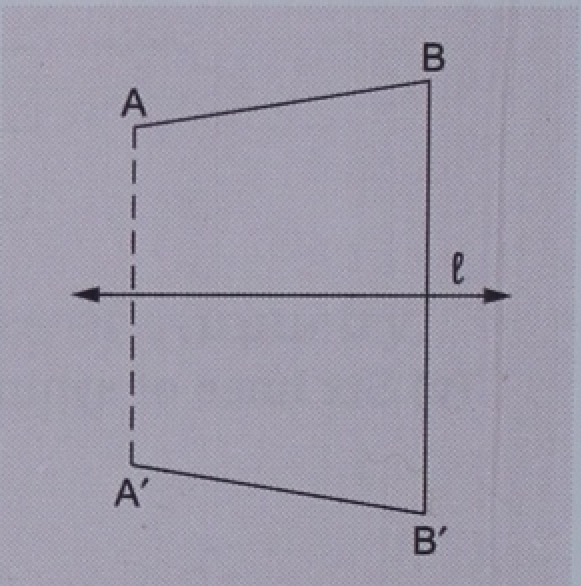
To draw the reflection of a point in a line

Let P be a point. We have to find its reflection (or image) P' in (by) the line AB. Since PP' must be perpendicular to AB, we draw the perpendicular from P to AB. The distance of P from AB must be equal to the distance of P' from AB. So, we cut off $QP' = QP$ from the perpendicular. Then, P' is the reflection of P in the line AB.



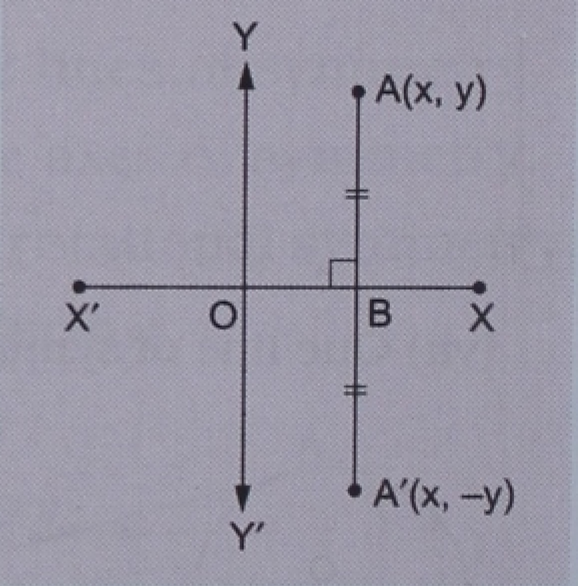
To draw the reflection of a line segment in a line

In order to draw the reflection of the line segment AB in the line l, draw the images A' and B' of the points A and B respectively. Join A' and B'. Then $A'B'$ is the image of AB in the line l.



Reflection of a point $A(x, y)$ in the x-axis

The reflection of the point A in the x-axis is A', where ABA' is perpendicular to the x-axis and $AB = A'B$. Thus, if the coordinates of A are (x, y) then the coordinates of $A' = (x, -y)$.

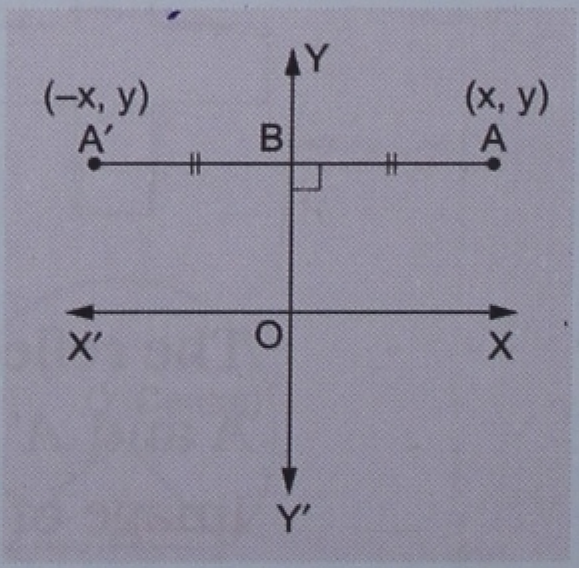


The reflection of the point (x, y) in the x-axis is the point $(x, -y)$

Examples The reflection of the points (3, 2), (12, –3), (0, –4) and (–3, 0) in the x-axis are (3, –2), (12, 3), (0, 4) and (–3, 0) respectively.

Reflection of a point in the y-axis

The reflection of the point A in the y-axis is A' where ABA' is perpendicular to the y-axis and $AB = A'B$. So, if A has the coordinates (x, y) then the coordinates of $A' = (-x, y)$.

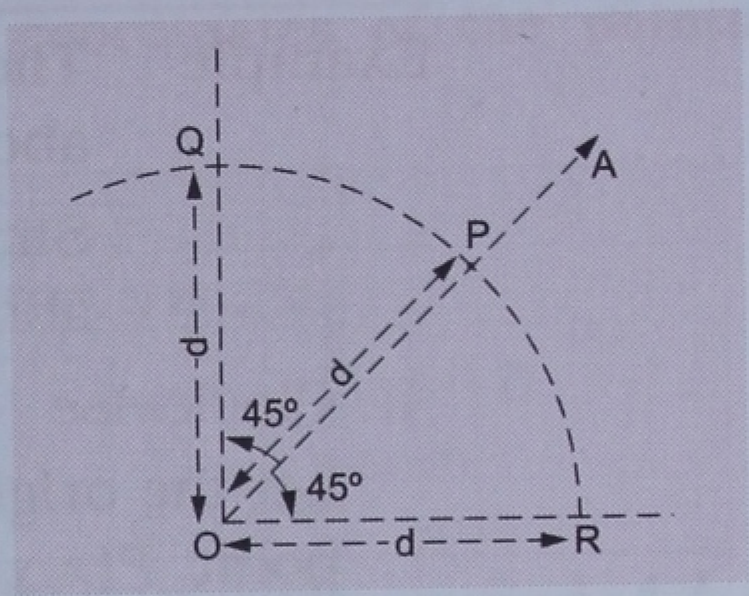


The reflection of the point (x, y) in the y-axis is the point $(-x, y)$

Examples The reflection of the points (4, 7), (2, 0), (–3, –7) and (0, –3) in the y-axis are (–4, 7), (–2, 0), (3, –7) and (0, –3) respectively.

Rotation

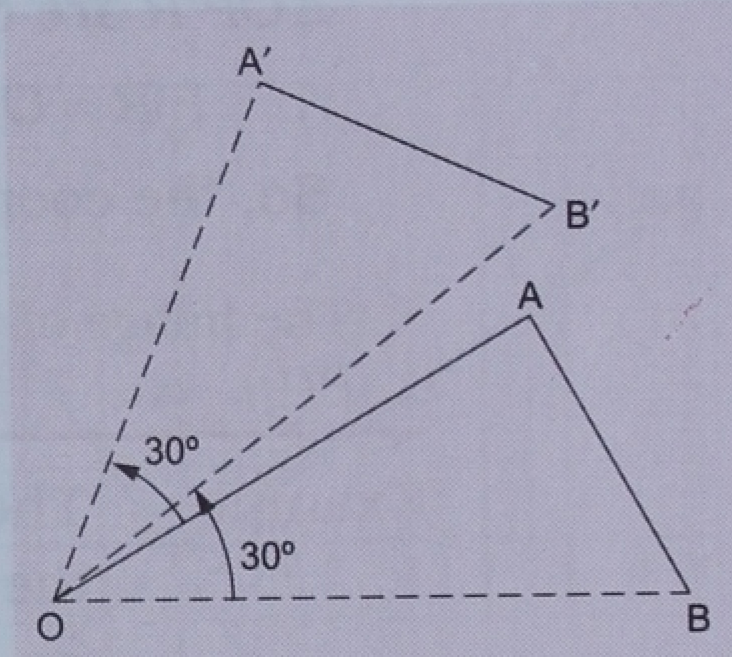
Let us take a point O. Let P be a point in the plane of O along the line OA such that $OP = d$. Now, if we rotate the line OA about O through an angle of 45° (say) in the anticlockwise direction, the point P reaches the point Q such that $\angle POQ = 45°$ and $OQ = d$. We say that the point P has been rotated about O through 45° in the anticlockwise direction. Similarly, we can rotate P about O through 45° in the clockwise direction. In that case, P will reach R such that $\angle POR = 45°$ and $OR = d$.



We call O the centre of rotation and the angle through which the line is rotated, the angle of rotation.

Rotation of a line segment about a point O

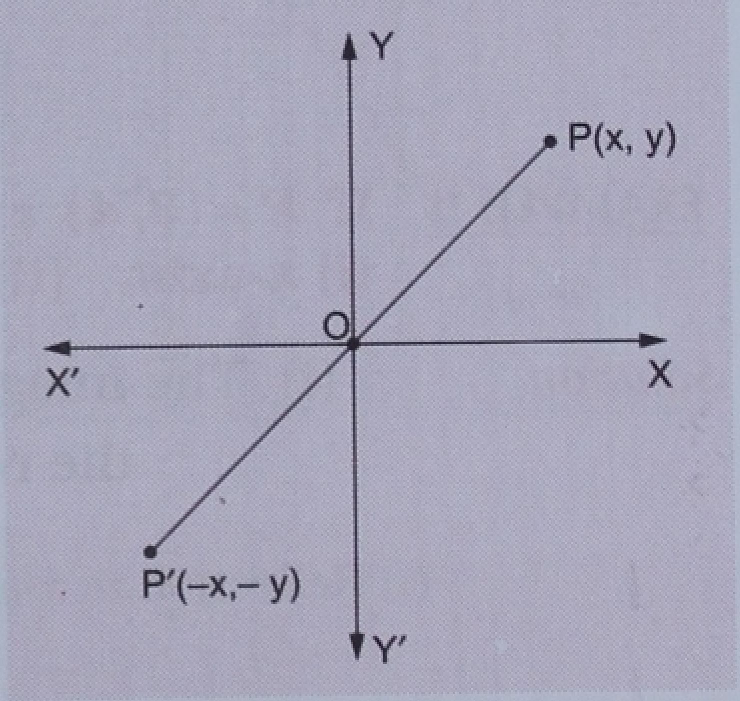
Consider the line segment AB. On rotation about O through 30° in the anticlockwise sense, A reaches A' and B reaches B', that is, $\angle AOA' = 30°$ and $\angle BOB' = 30°$. The length of AB remains the same, that is, $AB = A'B'$. $A'B'$ is the image of AB after rotation. O is the centre of rotation and the angle of rotation is 30° anticlockwise



Rotation of a point about the origin through 180° (half tunes)

Let the origin O be the centre of rotation. If the point $P(x, y)$ is rotated about O through 180° (clockwise or anticlockwise), it will reach P' such that $OP = OP'$ and $\angle POP' = 180°$.

The image of the point $P(x, y)$ for a rotation of 180° (clockwise or anticlockwise) about the origin is $P'(-x, -y)$.



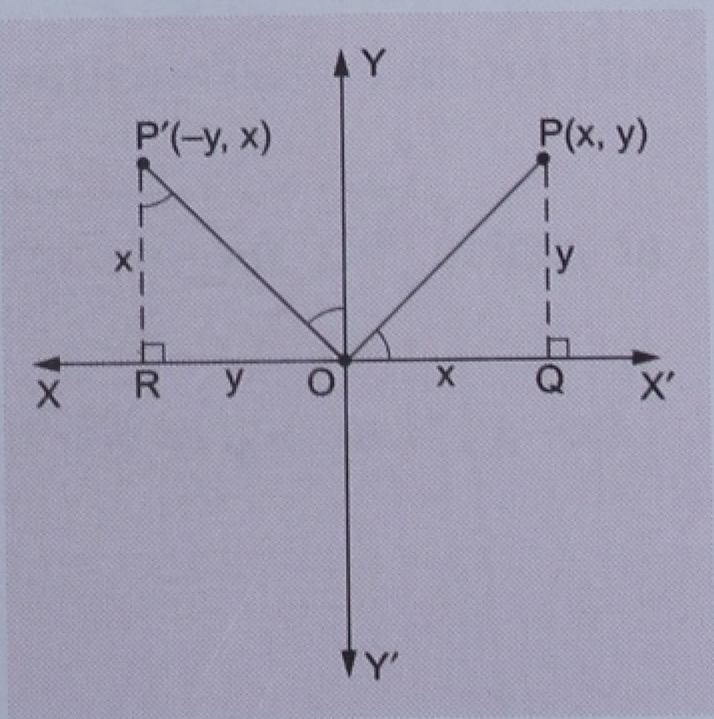
Example The image of the point (–3, 4) for a rotation of 180° (clockwise or anticlockwise) about the origin is (3, –4).

Rotation of a point about the origin through 90°

I. Anticlockwise

Let the origin O be the centre of rotation. If the point $P(x, y)$ is rotated about O through 90° (anticlockwise), it will reach P' such that $OP = OP'$ and $\angle POP' = 90°$.

Draw two perpendiculars PQ and $P'R$ on the x-axis from P and P' respectively. Then $\triangle OPQ$ and $\triangle P'OR$ are congruent.



$\therefore$ $P'R = OQ$ and $OR = PQ$.

So, the coordinates of $P' = (-y, x)$.

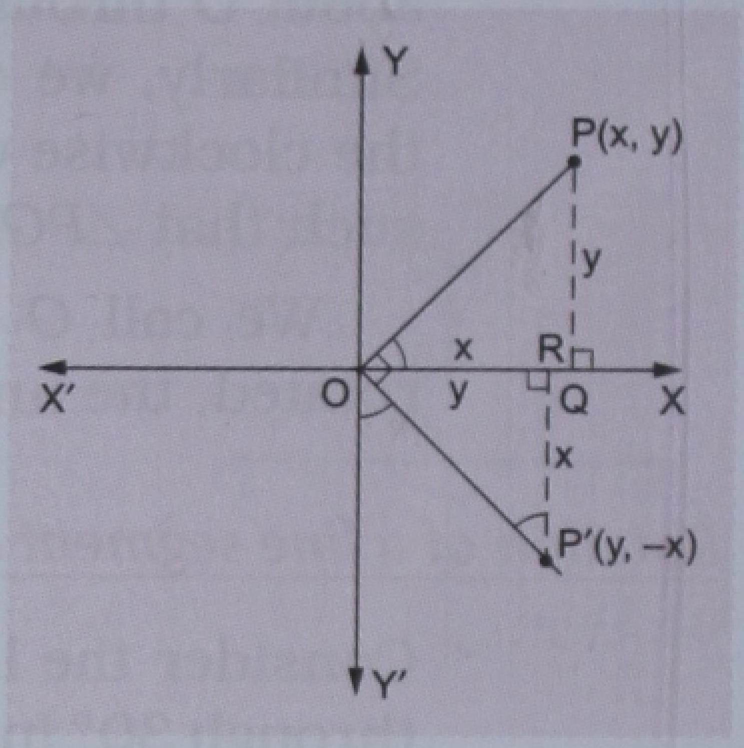
The image of the point $P(x, y)$ for a rotation of 90° (anticlockwise) about the origin is $P'(-y, x)$.

Example The image of the point (4, 3) for a rotation of 90° (anticlockwise) about the origin is (–3, 4).

Similarly, the image of (–2, –5) for a rotation of 90° (anticlockwise) about the origin is (5, –2).

II. Clockwise

Let the origin O be the centre of rotation. If the point $P(x, y)$ is rotated about O through 90° (clockwise), it will reach P' such that $OP = OP'$ and $\angle POP' = 90°$.



From P and P', draw two perpendiculars PQ and $P'R$ respectively on the x-axis. Then $\triangle OPQ$ and $\triangle OP'R$ are congruent.

$\therefore$ $P'R = OQ$ and $OR = PQ$.

So, the coordinates of $P' = (y, -x)$.

The image of the point $P(x, y)$ for a rotation of 90° (clockwise) about the origin is $P'(y, -x)$.

Example The image of a point (8, –3) for a rotation of 90° (clockwise) about the origin is (–3, –8).

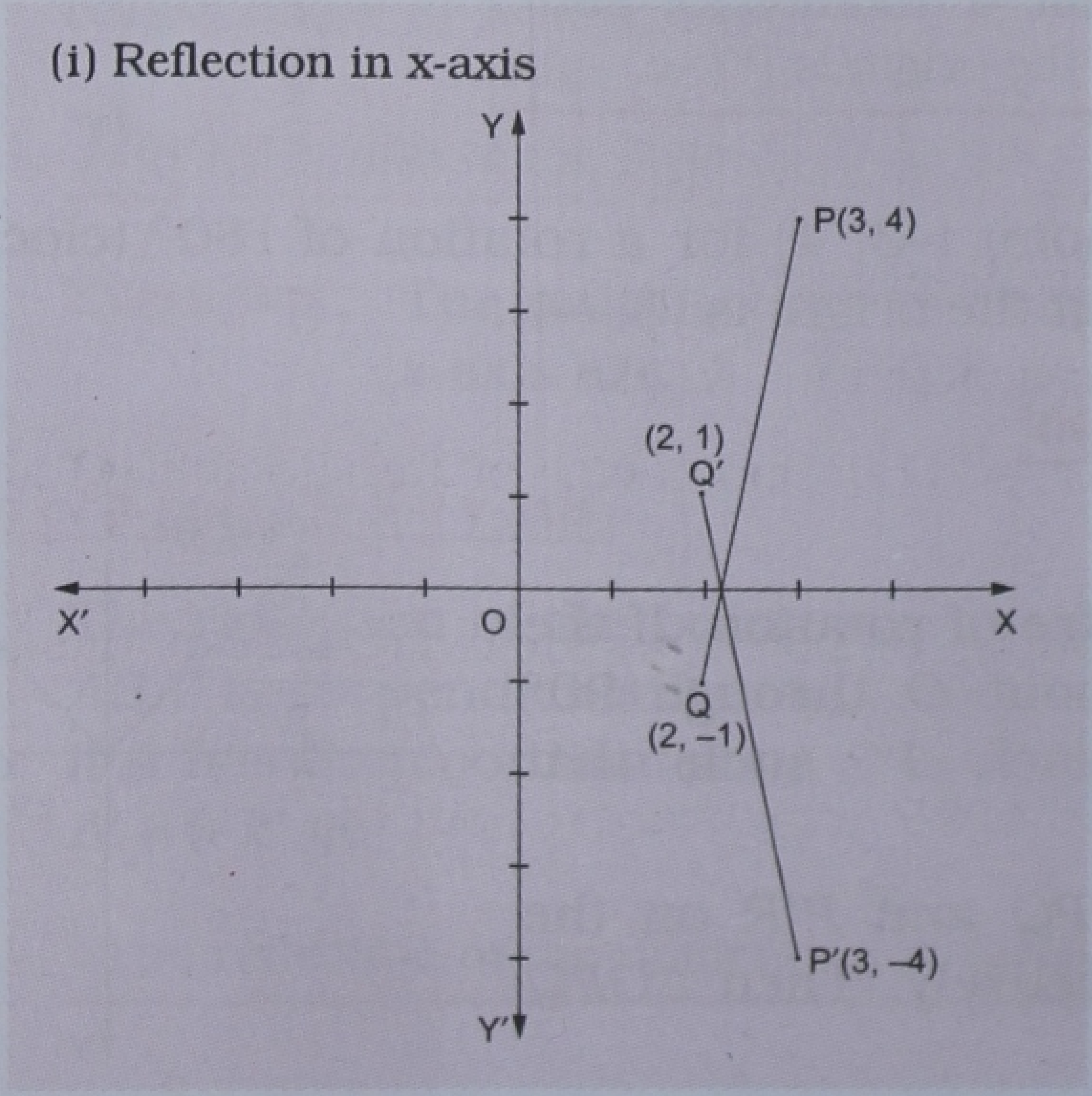
Similarly, the image of (–5, –9) is (–9, 5).

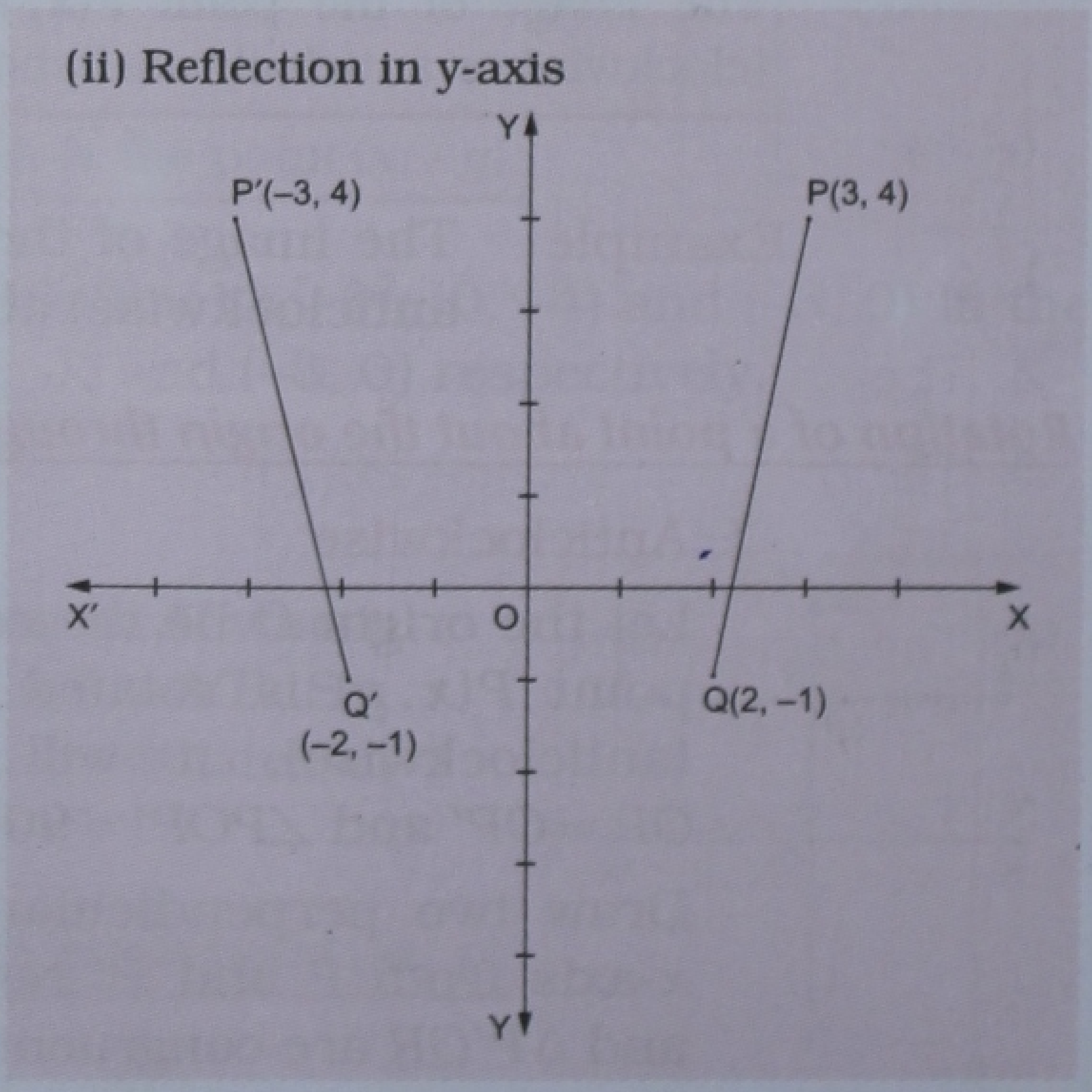
Solved Examples

EXAMPLE 1 **If $P = (3, 4)$ and $Q = (2, -1)$, plot the reflection of the line segment PQ in the (i) x-axis, (ii) y-axis.**

Solution (i) The image of $P = (3, 4)$ in the x-axis is $P'(3, -4)$ and that of $Q(2, -1)$ is $Q'(2, 1)$.

$\therefore$ the reflection of the line segment PQ is the line segment $P'Q'$.





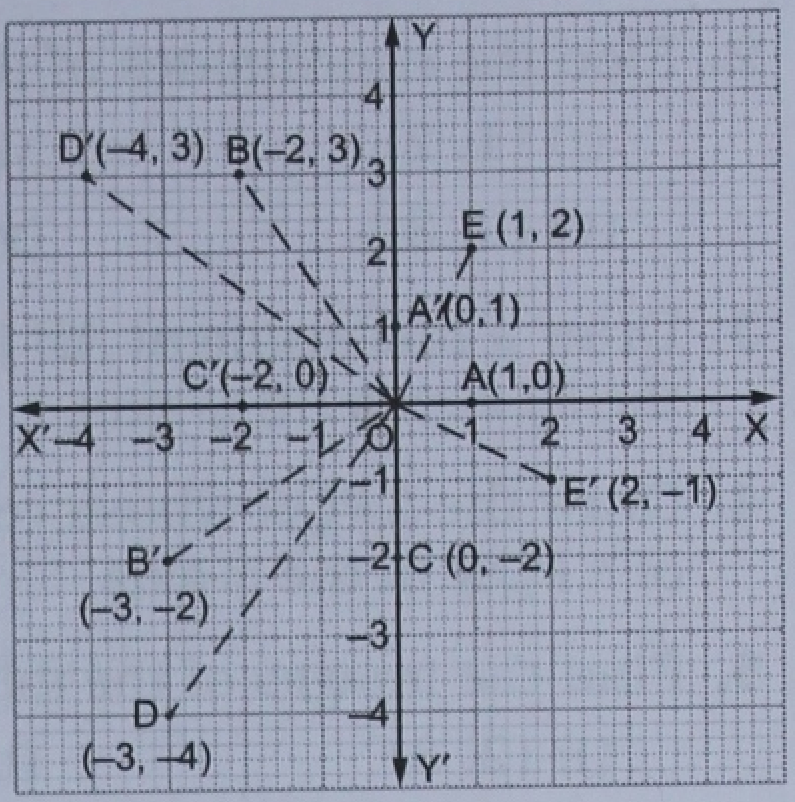
(ii) The image of $P(3, 4)$ in the y-axis is $P'(-3, 4)$ and that $Q(2, -1)$ is $Q'(-2, -1)$. So, the reflection of the line segment PQ is the line segment $P'Q'$.

EXAMPLE 2 **Plot the points $A(1, 0)$, $B(-2, 3)$, $C(0, -2)$, $D(-3, -4)$ and $E(1, 2)$ on a graph paper. Rotate the points A and B through 90° anticlockwise and the points C, D and E through 90° clockwise about the origin. Find the coordinates of the points obtained.**

Solution The points A, B, C, D and E are plotted as shown.

Draw $\angle AOA' = \angle BOB' = 90°$ (anticlockwise) and $\angle COC' = \angle DOD' = \angle EOE' = 90°$ (clockwise) such that $OA = OA'$, $OB = OB'$, $OC = OC'$, $OD = OD'$ and $OE = OE'$.

The coordinates of A', B', C', D' and E' are $(0, 1)$, $(-3, -2)$, $(-2, 0)$, $(-4, 3)$ and $(2, -1)$ respectively.

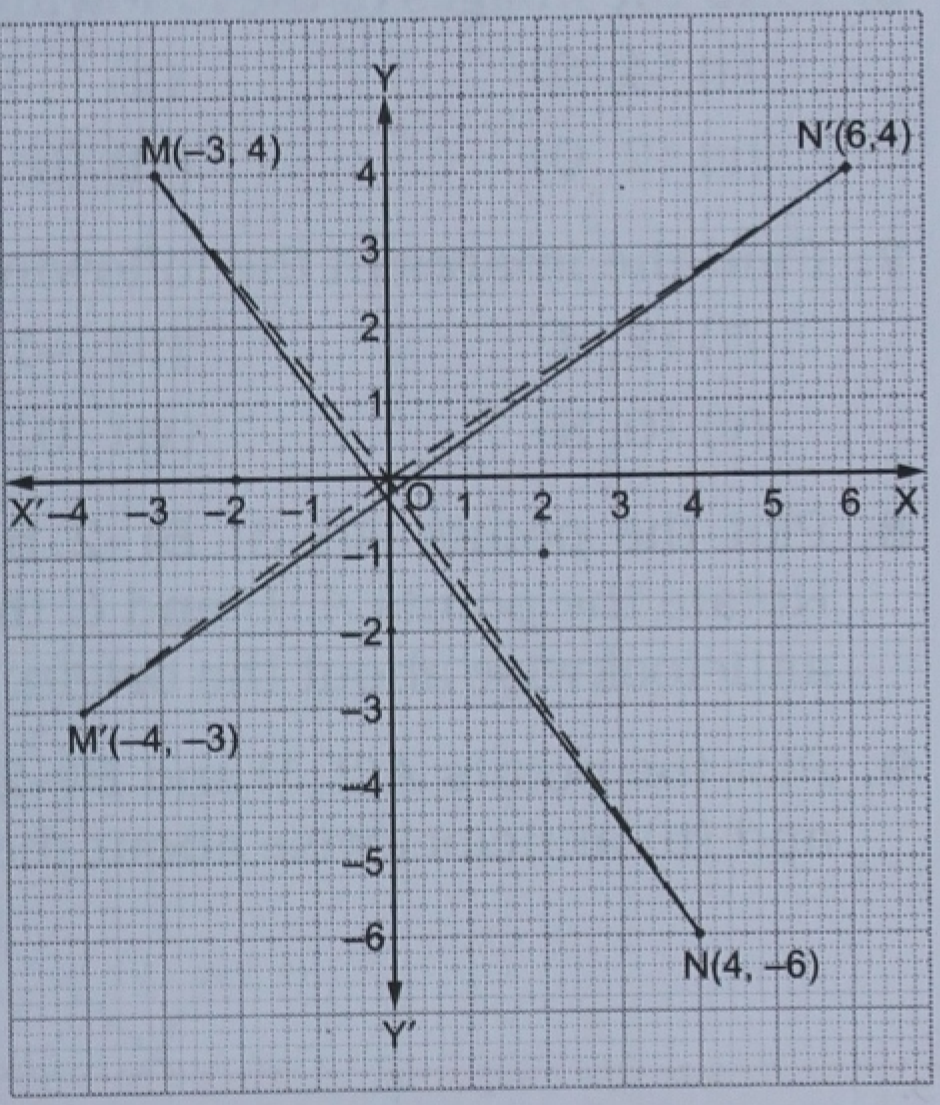


EXAMPLE 3 **Plot the points $M(-3, 4)$ and $N(4, -6)$ on a graph paper. Rotate MN through 90° (anticlockwise) about the origin to the position $M'N'$. Write the coordinates of M' and N'.**

Solution The points $M(-3, 4)$ and $N(4, -6)$ are plotted as shown.

Draw $\angle MOM' = 90°$ and $\angle NON' = 90°$ in anticlockwise direction such that $OM = OM'$ and $ON = ON'$.

The coordinates of $M' = (-4, -3)$ and the coordinates of $N' = (6, 4)$.



Remember These

1. The reflection of a point (x, y) in the x-axis is point $(x, -y)$
2. The reflection of a point (x, y) in the y-axis is point $(-x, y)$
3. The image of a point $P(x, y)$ for a rotation of 180° (clockwise or anticlockwise) about the origin is $P'(-x, -y)$.
4. The image of a point $P(x, y)$ for a rotation of 90° (anticlockwise) about the origin is $P'(-y, x)$.
5. The image of a point $P(x, y)$ for a rotation of 90° (clockwise) about the origin is $P'(y, -x)$.

EXERCISE 8B

1. Find the coordinates of the images of the following points in the *x*-axis.

(i) (2, 3) (ii) (–2, 7) (iii) (3, –5) (iv) (–4, –6)
(v) (0, 2) (vi) (3, 0) (vii) (–3, 0) (viii) (0, –5)

2. Find the coordinates of the images of the following points in the *y*-axis.

(i) (1, 2) (ii) (3, –1) (iii) (–1, 4) (iv) (–6, –3)
(v) (0, 3) (vi) (0, –4) (vii) (3, 0) (viii) (–1, 0)

3. Draw a line segment *AB* joining the points *A*(4, 5) and *B*(–2, 3). Plot the reflection of *AB* in the (i) *x*-axis (ii) *y*-axis.

4. Draw the $\triangle PQR$ whose vertices are (4, 0), (2, 5) and (1, –5) respectively. Plot the reflection of $\triangle PQR$ in the *y*-axis.

5. Find the images of the following points for a rotation about the origin through 180°.

(i) (2, 3) (ii) (–3, 6) (iii) (3, 0) (iv) (0, –7)

6. Find the images of the following points for a rotation of 90° (anticlockwise) about the origin.

(i) (4, 6) (ii) (–3, 7) (iii) (4, –8) (iv) (–1, –2)
(v) (2, 0) (vi) (0, –4)

7. Find the images of the following points for a rotation of 90° (clockwise) about the origin.

(i) (2, 4) (ii) (–1, 0) (iii) (2, –7) (iv) (–3, –9)
(v) (0, 5) (vi) (0, –7)

8. Plot the points *A*(1, 3), *B*(–2, 5), *C*(3, –6), *D*(2, –3), *E*(5, 0) and *F*(0, –4) on a graph paper. Rotate the points *A*, *B* and *C* through 90° clockwise and the points *D*, *E* and *F* through 90° anticlockwise about the origin. Find the coordinates of the points obtained.

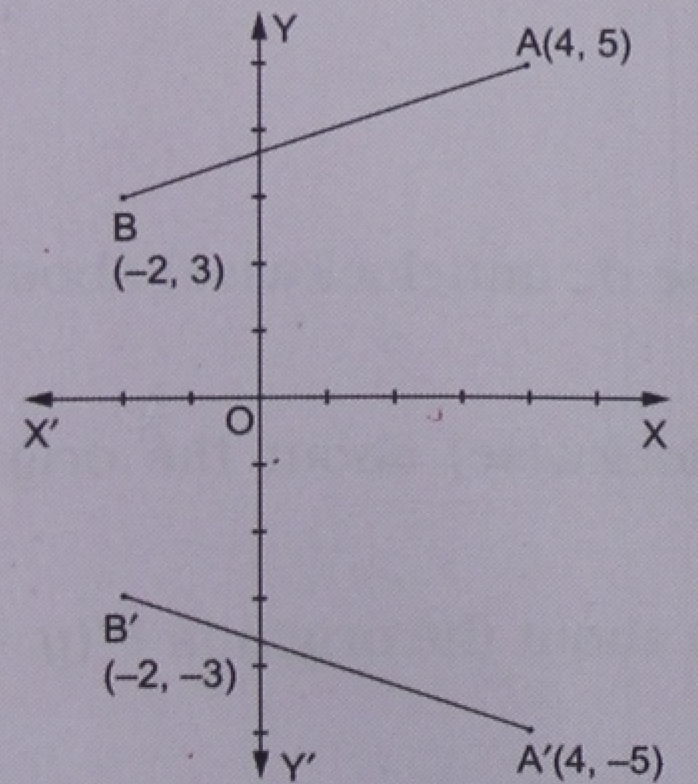
9. Plot the points *A*(2, 1) and *B*(–2, –4) on a graph paper. Rotate *AB* through 90° (i) clockwise and (ii) anticlockwise about the origin. Find the coordinates of the points obtained.

ANSWERS

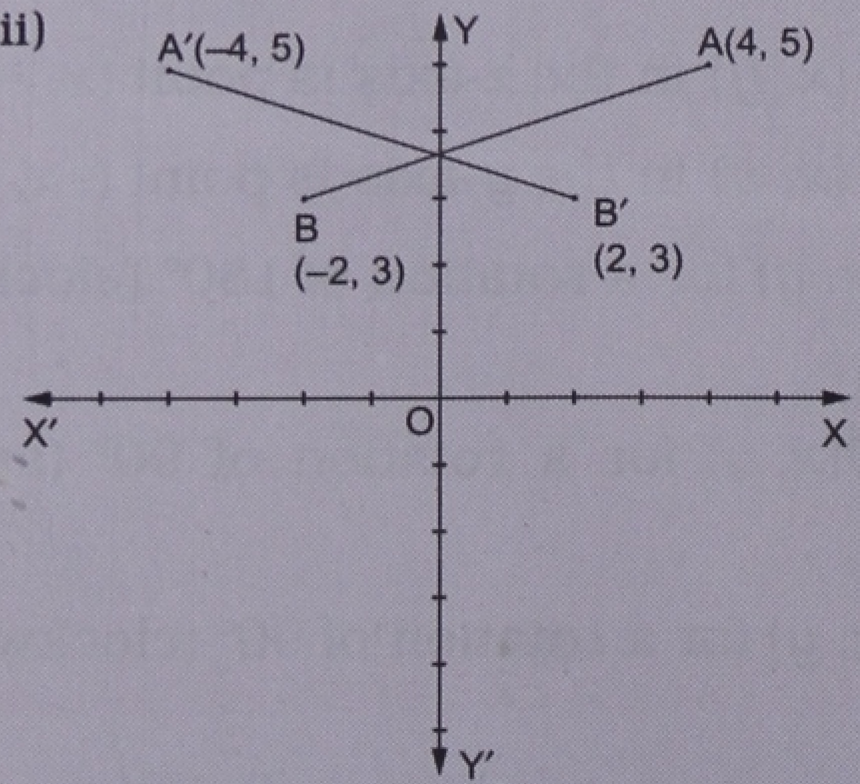
1. (i) (2, –3) (ii) (–2, –7) (iii) (3, 5) (iv) (–4, 6) (v) (0, –2) (vi) (3, 0) (vii) (–3, 0) (viii) (0, 5)

2. (i) (–1, 2) (ii) (–3, –1) (iii) (1, 4) (iv) (6, –3) (v) (0, 3) (vi) (0, –4) (vii) (–3, 0) (viii) (1, 0)

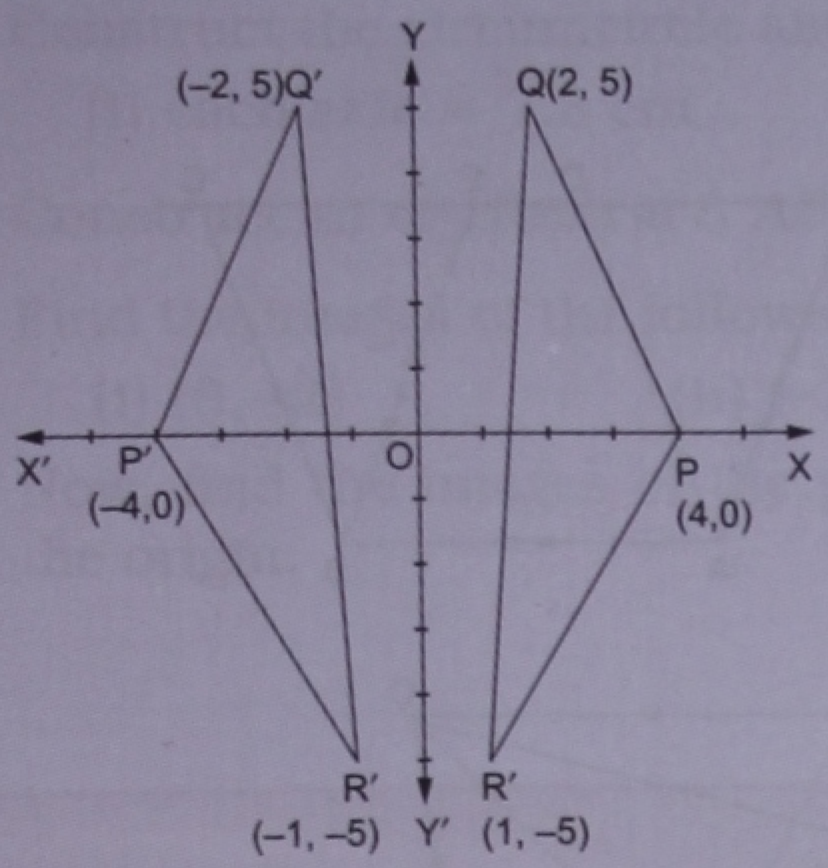
3. (i)



(ii)



4.

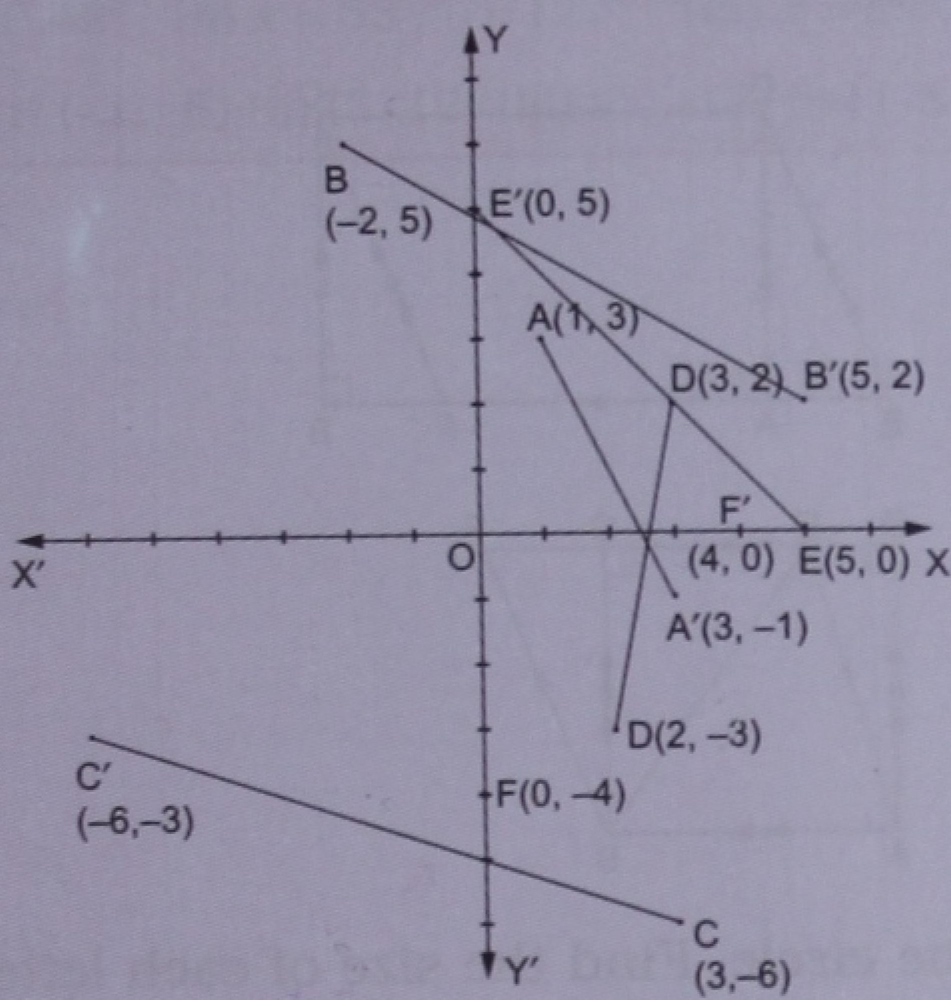


5. (i) (–2, –3) (ii) (3, –6) (iii) (–3, 0) (iv) (0, 7)

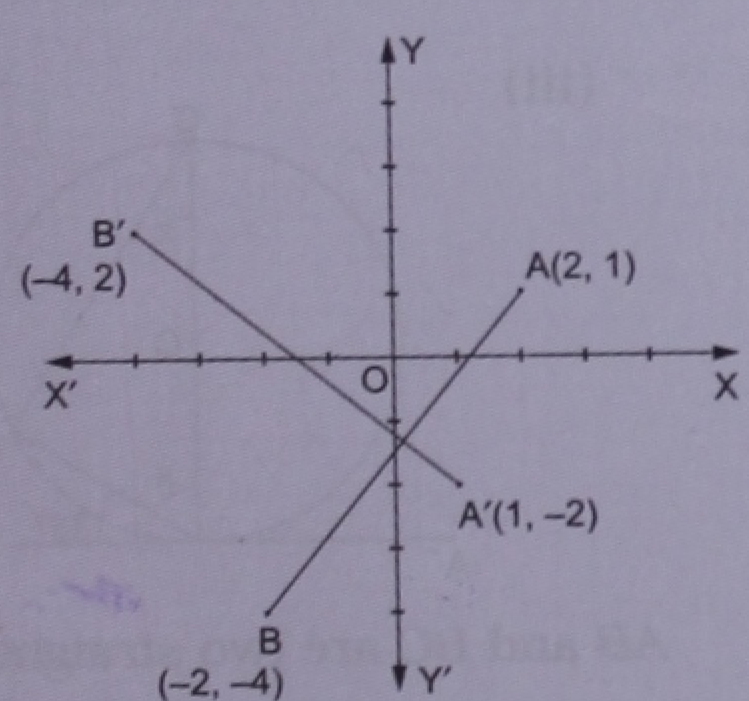
6. (i) (–6, 4) (ii) (–7, –3) (iii) (8, 4) (iv) (2, –1) (v) (0, 2) (vi) (4, 0)

7. (i) (4, –2) (ii) (0, 1) (iii) (–7, –2) (iv) (–9, 3) (v) (5, 0) (vi) (–7, 0)

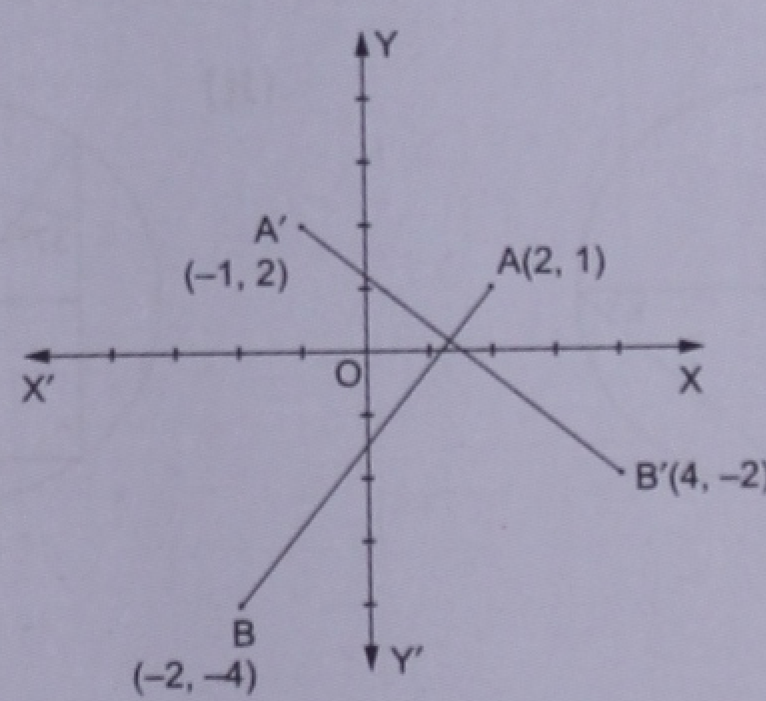
8.



9. (i)



(ii)



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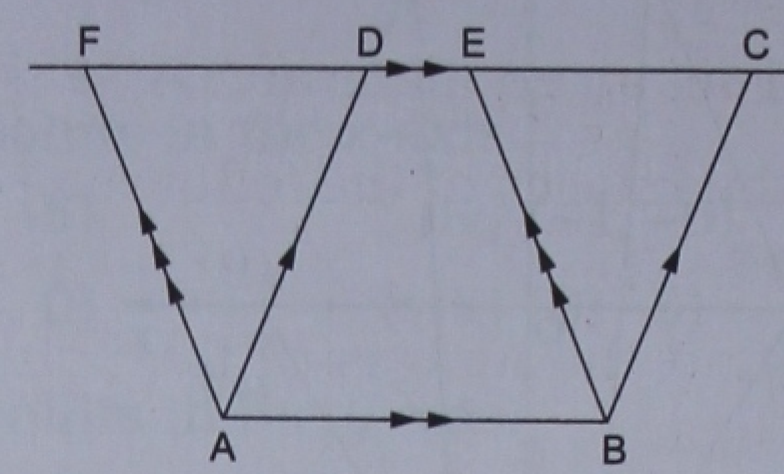
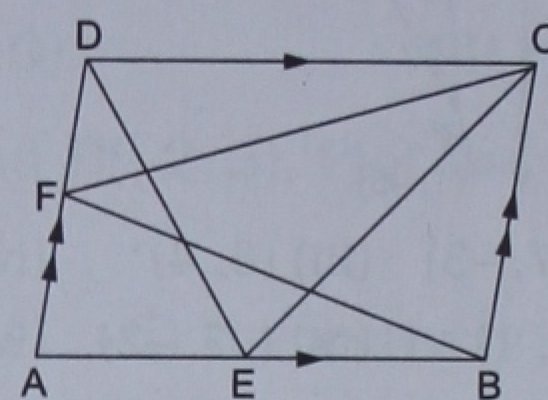
Revision Exercise 3

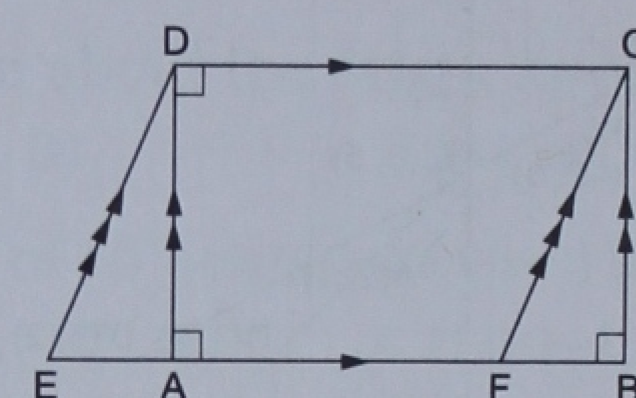
1. The area of the parallelogram *ABEF* is 165 cm^2. Find the area of the parallelogram *ABCD*. Also, prove that the area of $\triangle ADF$ = the area of $\triangle BCE$.



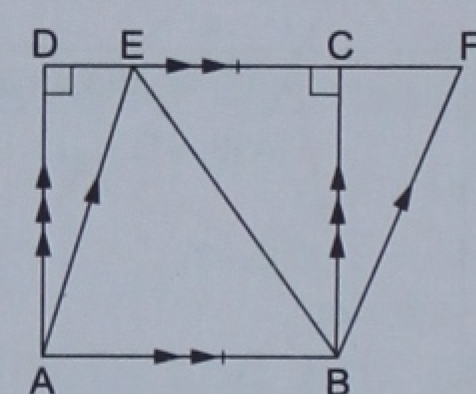
2. Prove that the area of $\triangle BFC$ = the area of $\triangle CED$.



3. The area of the parallelogram *CDEF* is 1440 cm^2. If *AB* = 40 cm, find *BC*.

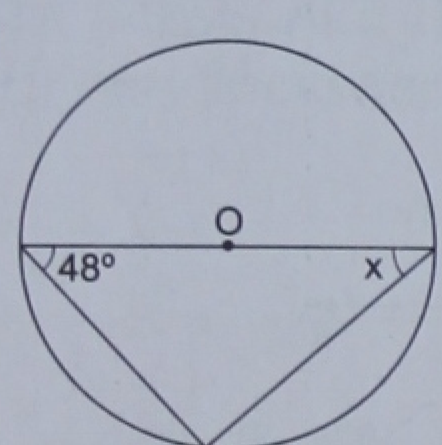


4. The area of the square *ABCD* is 324 cm^2.
Find (i) the area of $\triangle ABE$
(ii) the height of the parallelogram *ABFE*.

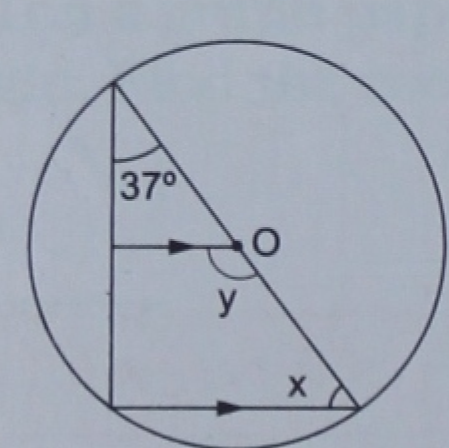


5. In each of the following figures, *O* is the centre of the circle. Find the size of each lettered angle.

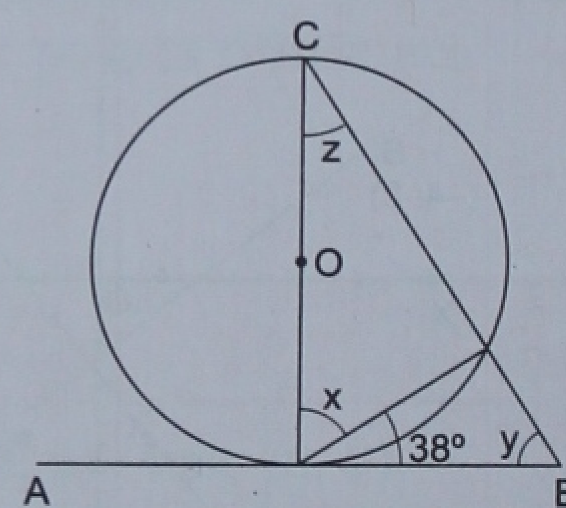
(i)



(ii)



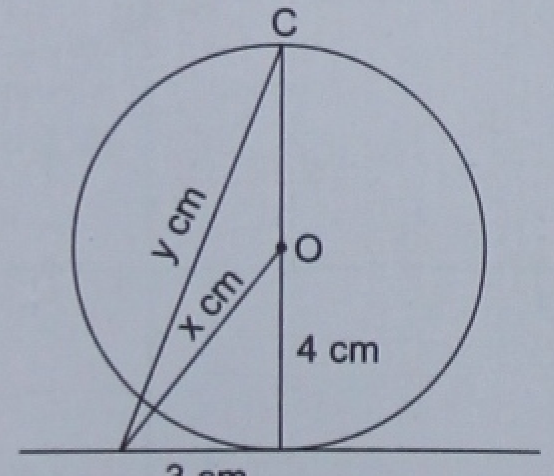
(iii)



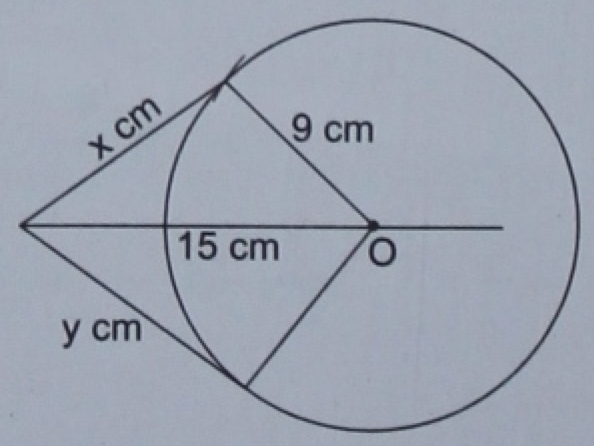
AB and *BC* are two straight lines.

6. In each of the following figures, *O* is the centre of the circle. Find the values of *x* and *y*.

(i)



(ii)



7. Construct a circle of radius
(i) 8.3 cm (ii) 6.5 cm

8. Construct the circumcircle and the incircle of $\triangle ABC$ in which

(i) each side = 7.3 cm (ii) $AB = 3.8$ cm, $BC = 5.6$ cm, $AC = 7.5$ cm

9. Construct an equilateral $\triangle ABC$ with side 6.1 cm. Draw its line of symmetry (if any).

10. Find the images of the following points for a rotation of 90° (clockwise) about the origin.

(i) (6, –2) (ii) (–12, 3) (iii) (17, 5) (iv) (0, –3)

Now, find the images of the points obtained for a rotation of 180° (anticlockwise) about the origin.

ANSWERS

1. 165 cm^2 **3.** 36 cm **4.** (i) 162 cm^2 (ii) 18 cm

5. (i) 42° (ii) $x = 53°$, $y = 127°$ (iii) $x = 52°$, $y = 52°$, $z = 38°$ **6.** (i) $x = 5$, $y = \sqrt{73}$ (ii) $x = 12$, $y = 12$

10. (i) (–2, –6) (ii) (3, 12) (iii) (5, –17) (iv) (–3, 0); images are (i) (2, 6) (ii) (–3, –12) (iii) (–5, 17) (iv) (3, 0)

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