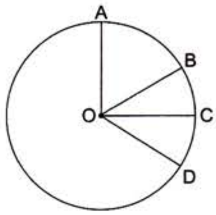
7 Circle



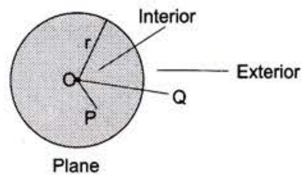
A circle is a closed plane curve consisting of all points at the same (or constant) distance from a fixed point O of the plane. The fixed point O is called the centre of the circle and the constant distance of the points of the circle from its centre is called the radius of the circle. In the figure, O is the centre of the circle and A, B, C and D are points on the circle.

$\therefore \quad OA = OB = OC = OD =$ the radius of the circle $= r$ (say)

A line segment joining the centre and a point on the circle is also known as a radius. Thus, OA, OB, OC and OD are radii (plural of radius) of the circle.

The symbol for a circle is $\odot$. Any three points on a circle can be used to name it. Thus, the circle in the figure may be called $\odot ABC$ or $\odot ABD$ or $\odot ACD$ or $\odot BCD$. A circle with centre O and radius r is denoted by $C(O, r)$.

Exterior and interior of a circle



In the figure, r is the radius of the circle which has its centre at O. Consider a point P in the plane of the circle.

If $OP < r$ then P lies inside the circle.

The set of all such points in the plane of the circle which are at a distance less than r from the centre of the circle is called the interior of the circle.

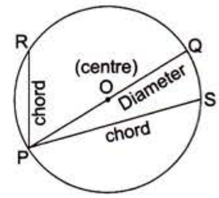
Let Q be a point in the plane of the circle.

If $OQ > r$ then Q lies outside the circle.

The set of all such points in the plane of the circle which are at a distance greater than r from the centre of the circle is called the exterior of the circle.

The interior of a circle together with the points on the circle is called a circular region.

Chord of a circle



A line segment joining two points on a circle is called a chord of the circle. In the figure, PQ, PR and PS are chords of the circle.

Diameter of a circle

A chord that passes through the centre of a circle, is called a diameter of the circle. In the figure above, PQ is a chord passing through the centre O. So, the line segment PQ is a diameter of the circle. The length of the line segment PQ is also called the diameter of the circle. In other words, the diameter is the length of the longest chord of a circle.

Now, $PQ = PO + OQ$ = radius + radius = 2 × radius. Therefore,

$$\text{radius} = \frac{\text{diameter}}{2} \qquad \text{diameter} = 2 \times \text{radius}$$

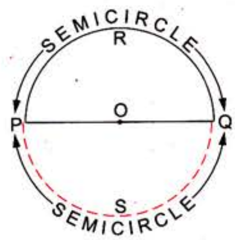
Arc of a circle

A continuous part of a circle is called an arc of the circle. In the figure, PQ is an arc. The symbol for an arc is $\frown$. Thus, $\overset{\frown}{PQ}$ denotes the arc PQ.

Circumference of a circle

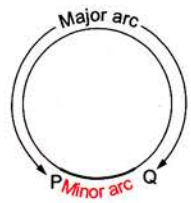
The length of the boundary of a circle is called the circumference of the circle.

Semicircle



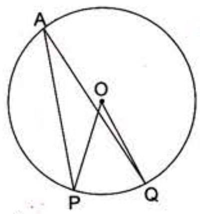
A semicircle is half a circle formed when a circle is cut along a diameter. In the figure, arcs PRQ and PSQ are two semicircles.

Minor and major arcs of a circle



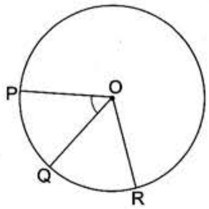
When P and Q are two points on a circle not at the ends of a diameter, the smaller part of the circle from P to Q is called the minor arc and the larger part from P to Q is called the major arc.

Angle subtended by an arc



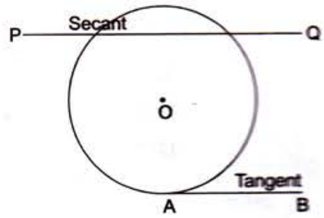
In the figure, A is a point on a circle which has its centre at O. PQ is an arc of the circle. Then $\angle PAQ$ is the angle subtended by $\overset{\frown}{PQ}$ at the point A of the circle. The $\angle POQ$ is the angle subtended by $\overset{\frown}{PQ}$ at the centre O of the circle.

Central angle

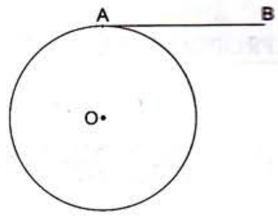


The angle subtended by any arc of a circle at the centre of the circle is called a central angle on the arc. In the figure, $\angle POQ$, $\angle QOR$ and $\angle POR$ are all central angles.

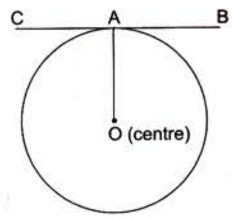
Secant and tangent



A straight line which intersects a circle at two distinct points is called a secant of the circle. In the figure, PQ is a secant of the circle with centre at O.



A straight line that intersects a circle at only one point is called a tangent to the circle at that point. In the figure, the line BA is a tangent to the circle at A. The length of the line segment BA is known as the length of the tangent from B and the point A is known as the point of contact of the tangent.



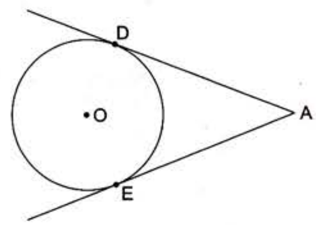
PROPERTY 1 **The tangent at any point of a circle and the radius through the point are perpendicular to each other.**

In the figure, $\angle OAB = 90°$.

PROPERTY 2 **There is one, and only one, tangent at any point of a circle.**

In the figure, since BA and CA are tangents to the circle at A, CAB is a straight line.

PROPERTY 3 **Two tangents can be drawn to a circle from a point outside the circle.**

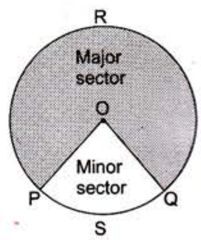


In the figure, AD and AE are two tangents to the circle from the point A outside the circle.

PROPERTY 4 **If two tangents are drawn to a circle from a point outside the circle then the tangents are equal in length.**

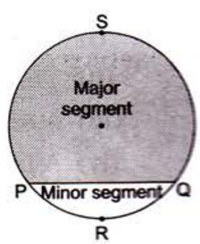
In the figure, AD and AE are two tangents to the circle drawn from A outside the circle, so $AD = AE$.

Sector of a circle



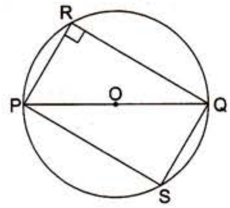
A sector of a circle is the region bounded by two radii of the circle and either of the arcs that they cut off. In the figure, OP and OQ are two radii. They cut off the minor arc PSQ and the major arc PRQ. The region bounded by OP, OQ and $\overset{\frown}{PSQ}$ is a minor sector. The region bounded by OP, OQ and $\overset{\frown}{PRQ}$ is a major sector.

Segment of a circle



The region bounded by a chord of the circle and one of the arcs cut off by the chord is called a segment of the circle. In the figure, the chord PQ cuts the circle into the minor arc PRQ and the major arc PSQ. The smaller part $PRQP$ of the circular region is called the minor segment while the bigger part $PSQP$ is called the major segment.

Angle in a semicircle

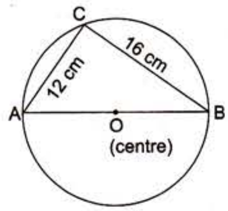


In the figure, PQ is a diameter of a circle which has its centre at O. R and S are two points on the circle. $\angle PRQ$ and $\angle PSQ$ are examples of angles in a semicircle.

PROPERTY **An angle in a semicircle is a right angle.**

In the figure, $\angle PRQ = \angle PSQ = 90°$.

EXAMPLE **In the adjoining figure, find AB and the radius of the circle.**



Solution An angle in a semicircle = 90° ⇒ $\angle ACB = 90°$.

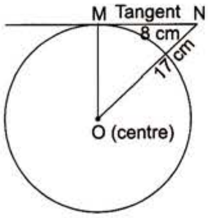
In the right-angled triangle ABC,

$$AB^2 = AC^2 + BC^2 = (12^2 + 16^2)\text{ cm}^2 = 400\text{ cm}^2$$

$\therefore$ $AB = 20$ cm. So, the radius $= \frac{1}{2} \times AB = \frac{1}{2} \times 20\text{ cm} = 10$ cm.

Solved Examples

EXAMPLE 1 **In the adjoining figure, find OM.**



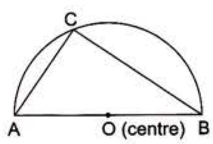
Solution MN is a tangent to the circle at M. So, $OM \perp MN$.

Now, in the right $\triangle OMN$, $OM^2 + MN^2 = ON^2$

$$\Rightarrow \quad OM = \sqrt{ON^2 - MN^2} = \sqrt{17^2 - 8^2}\text{ cm}$$

$$= \sqrt{289 - 64}\text{ cm} = \sqrt{225}\text{ cm} = 15\text{ cm}.$$

EXAMPLE 2 **In the figure, $OA = 6.5$ cm and $AC = 5$ cm. Find BC.**



Solution An angle in a semicircle = 90° ⇒ $\angle ACB = 90°$.

$AB = 2 \times OA = 2 \times 6.5\text{ cm} = 13$ cm.

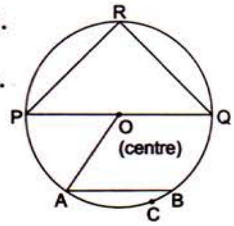
In the right-angled triangle ABC, $AB^2 = AC^2 + BC^2$

$$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2} = \sqrt{13^2 - 5^2}\text{ cm} = \sqrt{169 - 25}\text{ cm} = \sqrt{144}\text{ cm} = 12\text{ cm}.$$

EXERCISE 7A

1. Identify the false statements.
   (i) The radius of a circle is half of its diameter.
   (ii) The diameter of a circle is the length of its longest chord.
   (iii) A secant of a circle intersects a circle at three distinct points.
   (iv) A tangent to a circle can touch the circle at more than one point.
   (v) If O is the centre of the circle with diameter 4 cm and $OP = 3$ cm then the point P lies in the interior of the circle.
   (vi) An arc of a circle may be a discontinuous part of the circle.

2. Fill in the blanks on the basis of the adjoining figure.



(i) PQ is a ...... of the circle. (ii) $\overset{\frown}{ACB}$ is a ...... of the circle.

(iii) $\overset{\frown}{ARB}$ is a ...... of the circle. (iv) $\overset{\frown}{PRQ}$ is a ...... of the circle.

(v) The region bounded by OP, OA and PA is a ...... of the cirle.

(vi) The part $APRQBA$ is a ...... of the circle.

(vii) $\angle PRQ$ = ...

(viii) The maximum number of tangents that can be drawn at the point A = ...

(ix) If $OA = 8$ cm then the length of the longest chord = ...

3. The diameter of a circle with its centre at O is 11 cm. If $OA = 6$ cm, $OB = 5.5$ cm and $OC = 4.8$ cm, where are the points (i) A, (ii) B and (iii) C (interior/exterior/on circle)?

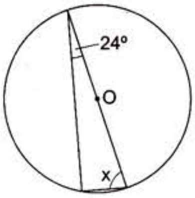
4. A tangent to a circle at the point P from A is 24 cm long. If O is the centre of the circle and $OA = 26$ cm, find the diameter of the circle.

5. Find the length of the tangent drawn to a circle of radius 11 cm from a point at a distance 61 cm from the centre.

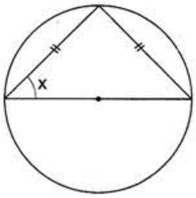
6. A circle has its centre at O and its radius is 4 cm. Two radii OA and OB are such that $\angle AOB = 60°$. Find the length of the chord AB.

7. In each of the following figures, O marks the centre of the circle. Find the size of each lettered angle.

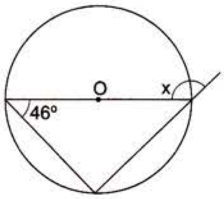
(i)



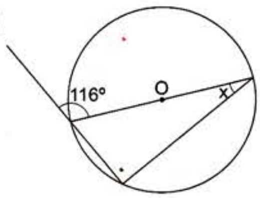
(ii)



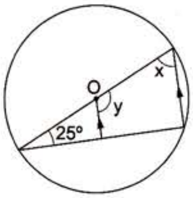
(iii)



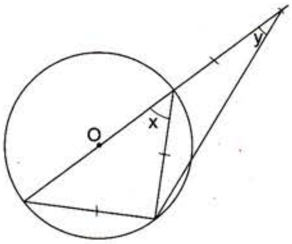
(iv)



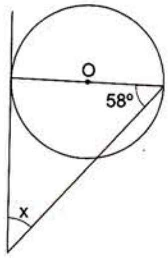
(v)



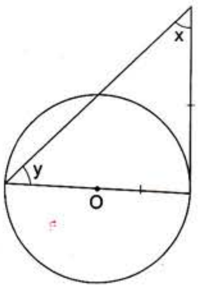
(vi)



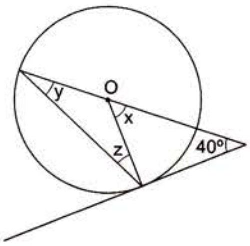
(vii)



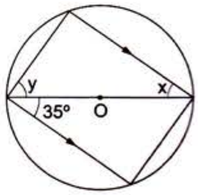
(viii)



(ix)

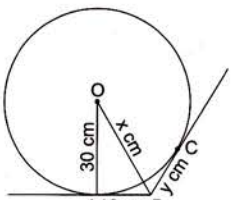


(x)

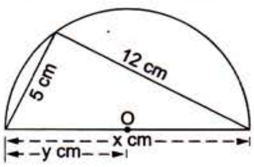


8. In each of the following figures, O is the centre of the circle. Find the values of x and y.

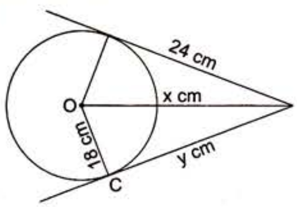
(i)



(ii)



(iii)



ANSWERS

1. (iii), (iv), (v), (vi)

2. (i) diameter (ii) minor arc (iii) major arc (iv) semicircle (v) minor sector (vi) major segment (vii) 90° (viii) 1 (ix) 16 cm

3. (i) in exterior of the circle (ii) on the circle (iii) in the interior of the circle

4. 20 **5.** 60 cm **6.** 4 cm

7. (i) 66° (ii) 45° (iii) 136° (iv) 26° (v) $x = 65°, y = 115°$ (vi) $x = 45°, y = 22.5°$ (vii) 32° (viii) $x = y = 45°$ (ix) $x = 50°, y = z = 25°$ (x) $x = 35°, y = 55°$

8. (i) $x = 34, y = 16$ (ii) $x = 13, y = 6.5$ (iii) $x = 30, y = 24$

Construction of Circles

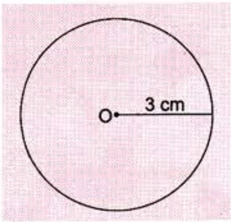
To construct a circle with a given radius

Construction 36 Construct a circle of radius 3 cm.

Steps of construction

1. Take a point O as the centre of the circle.
2. Draw an arc with O as the centre and a radius of 3 cm and continue the arc round O to complete the circle.

   This is the required circle.



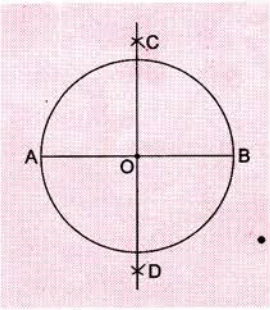
To construct a circle with a given diameter

Construction 37 Construct a circle with $AB = 6$ cm as diameter.

Steps of construction

1. Draw a line segment $AB = 6$ cm.
2. Draw the perpendicular bisector CD of AB. Let it intersect AB at O.
3. With O as the centre and radius OB (or OA), draw a circle.

   This is the required circle.



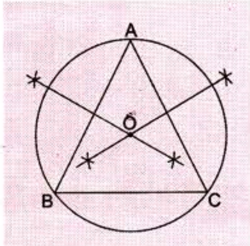
To construct the circumcircle of a given triangle

Construction 38 Construct the circumcircle of the given $\triangle ABC$.

Steps of constructon

1. Construct the $\triangle ABC$ with the given measures.
2. Draw the perpendicular bisectors of any two sides, say AB and AC. Let them meet at O.
3. With O as the centre and radius OA (or OB or OC as $OA = OB = OC$), draw a circle.

   This is the required circumcircle of $\triangle ABC$.



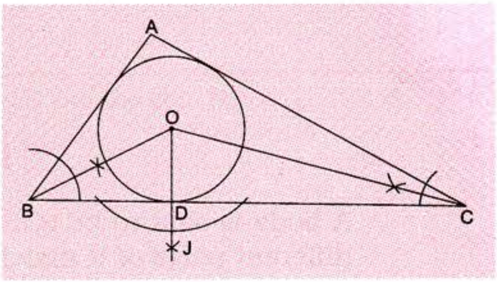
To construct the incircle of a given triangle

Construction 39 Construct the incircle of the given $\triangle ABC$.

Steps of constructon

1. Construct the $\triangle ABC$ with the given meausres.
2. Bisect any two angles of $\triangle ABC$, say $\angle B$ and $\angle C$. Let the bisectors of these angles meet at O.
3. Draw a perpendicular OJ from O on BC. Let it meet BC at D.
4. With O as the centre and radius OD, draw a circle.

   This is the required incircle of $\triangle ABC$.



EXERCISE 7B

1. Construct a circle of radius (i) 4 cm and (ii) 6 cm.

2. Construct a circle of diameter (i) 8 cm and (ii) 9 cm.

3. Construct a circumcircle of the $\triangle ABC$ in which

(i) $AB = 5.5$ cm, $BC = 5$ cm and $\angle ABC = 60°$

(ii) $AB = 9.2$ cm, $BC = 6.4$ cm and $CA = 8.2$ cm

4. Construct an incircle of the $\triangle ABC$ in which

(i) $AB = 6$ cm, $BC = 5$ cm and $AC = 8.3$ cm

(ii) $AB = 5$ cm, $BC = 6.4$ cm and $\angle ABC = 60°$

5. Construct the (i) incircle and (ii) circumcircle of the right-angled triangle ABC in which the hypotenuse = 6 cm and a side = 4 cm.

❖