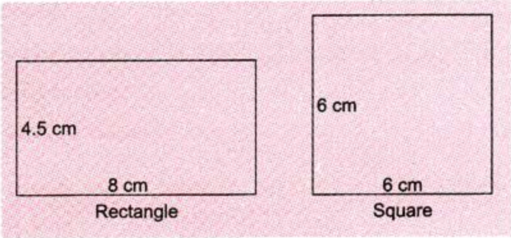
6

Areas of Geometrical Figures

The area of a closed bounded figure is the measure of the surface enclosed by its boundary. In this chapter, we will study the relations between the areas of some geometrical figures.

Two geometrical figures are called equal if they have equal areas. Such figures need not be congruent, however. For example, the rectangle and the square in the figure have equal areas but they are not congruent.

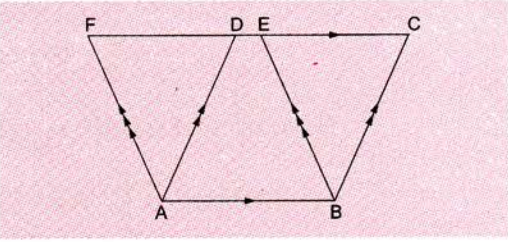


THEOREM 1 **Parallelograms on the same base and between the same parallels are equal in area.**

In the figure, the parallelograms *ABCD* and *ABEF* are on the same base *AB* and between the same parallels *AB* and *FC*.

$\therefore$ area of the parallelogram *ABCD*

= area of parallelogram *ABEF*.



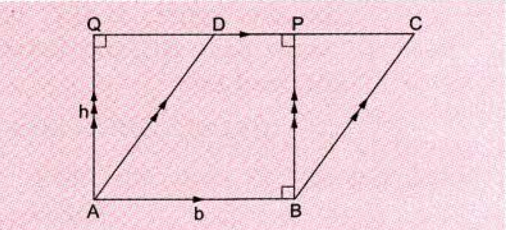
THEOREM 2 **A parallelogram is equal in area to a rectangle on the same base and between the same parallels.**

In the adjoining figure, the parallelogram *ABCD* and the rectangle *ABPQ* are on the same base *AB* and between the same parallel *AB* and *QC*.

$\therefore$ area of parallelogram *ABCD*

$= \text{area of rectangle } ABPQ = AB \times AQ$

$= \text{base} \times \text{height} = b \times h.$



Thus, the area of a parallelogram is the product of the base (b) and the perpendicular height (h).

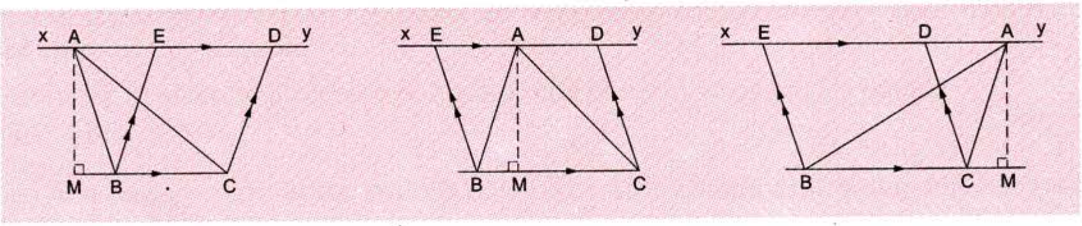
For a parallelogram,

$\text{Area } (A) = \text{base} \times \text{height} = b \times h \qquad \text{Base } (b) = \dfrac{\text{area } (A)}{\text{height } (h)} \qquad \text{Height } (h) = \dfrac{\text{area } (A)}{\text{base } (b)}$

Note Theorem 2 follows directly from Theorem 1 as every rectangle is a parallelogram too.

THEOREM 3 **The area of a triangle is half that of a parallelogram on the same base and between the same parallels.**

In each of the following figures, $\triangle ABC$ and parallelogram $BCDE$ are on the same base BC and between the same parallels BC and XY.

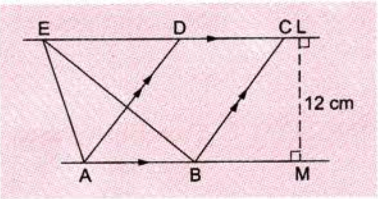


In each case, area of $\triangle ABC = \frac{1}{2} \times$ area of parallelogram $BCDE$

$$= \frac{1}{2} BC \times AM \; [\because \text{ area of parallelogram} = \text{base} \times \text{height}]$$

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height (or altitude)}$$

EXAMPLE **In the figure, the area of the parallelogram *ABCD* is 264 cm^2. Find the area of $\triangle$ *EAB*. Also, find *DC* if the height *LM* = 12 cm.**



Solution Area of $\triangle EAB = \frac{1}{2} \times$ area of parallelogram $ABCD$

$$= \frac{1}{2} \times 264 \text{ cm}^2 = 132 \text{ cm}^2.$$

Now, area of $\triangle EAB = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AB \times LM = \frac{1}{2} \times AB \times 12$ cm

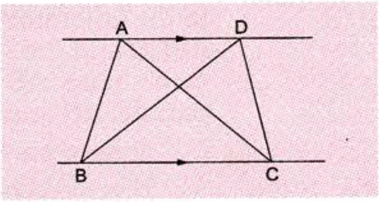
$\Rightarrow \; 132 \text{ cm}^2 = \frac{1}{2} \times AB \times 12 \text{ cm} \Rightarrow AB = \frac{132}{6} \text{ cm} = 22$ cm.

Thus, $DC = AB = 22$ cm. [Opposite sides of a parallelogram are equal.]

THEOREM 4 **Triangles on the same base and between the same parallels are equal in area.**

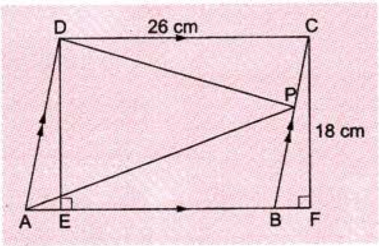
In the figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC and between the same parallels AD and BC.

$\therefore$ area of $\triangle ABC$ = area of $\triangle DBC$.



Solved Examples

EXAMPLE 1 **In the adjoining figure, find the areas of the parallelogram *ABCD* and the triangle *PAD*.**



Solution The area of the rectangle $EFCD = DC \times CF$

$$= 26 \times 18 \text{ cm}^2.$$

Since the parallelogram $ABCD$ and the rectangle $EFCD$ are on the same base DC and between the same parallels DC and AF.

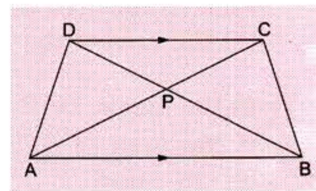
$\therefore$ the area of the parallelogram $ABCD$ = the area of the rectangle $EFCD$

$$= 26 \times 18 \text{ cm}^2.$$

Now, $\triangle PAD$ and parallelogram $ABCD$ are on the same base AD and between the same parallels AD and BC.

$\therefore$ area of $\triangle PAD = \frac{1}{2} \times$ area of parallelogram $ABCD = \frac{1}{2} \times 26 \times 18 \text{ cm}^2 = 234 \text{ cm}^2$.

EXAMPLE 2 **In the adjoining figure, *ABCD* is a trapezium. Prove that the area of $\triangle PAD$ = the area of $\triangle PBC$.**



Solution $\triangle ABD$ and $\triangle ABC$ are on the same base AB and between the same parallels AB and DC.

$\therefore$ the area of $\triangle ABD$ = the area of $\triangle ABC$.

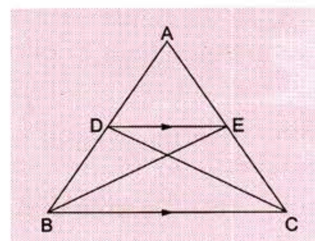
Subtracting the area of $\triangle ABP$ from both sides,

the area of $\triangle ABD$ – the area of $\triangle ABP$ = the area of $\triangle ABC$ – the area of $\triangle ABP$.

$\Rightarrow$ the area of $\triangle PAD$ = the area of $\triangle PBC$.

EXAMPLE 3 **In the adjoining figure, $DE \parallel BC$. Prove that (i) the area of $\triangle BED$ = the area of $\triangle CED$**

(ii) the area of $\triangle ABE$ = the area of $\triangle ACD$



Solution (i) $\triangle BED$ and $\triangle CED$ are on the same base DE and between the same parallels DE and BC.

$\therefore$ the area of $\triangle BED$ = the area of $\triangle CED$.

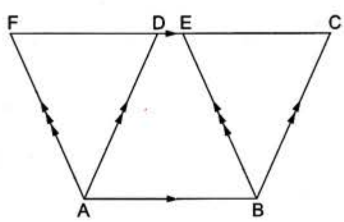
(ii) Adding the area of $\triangle ADE$ to both sides,

the area of $\triangle BED$ + the area of $\triangle ADE$ = the area of $\triangle CED$ + the area of $\triangle ADE$.

$\therefore$ the area of $\triangle ABE$ = the area of $\triangle ACD$.

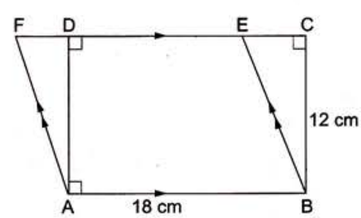
EXERCISE 6

1.



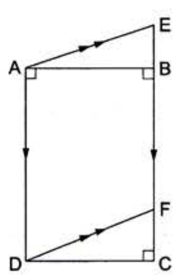
The area of the parallelogram $ABCD = 130 \text{ cm}^2$. Find the area of the parallelogram $ABEF$.

2.



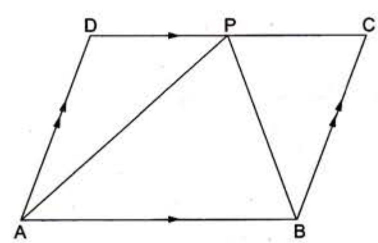
Find the area of the parallelogran $ABEF$.

3.



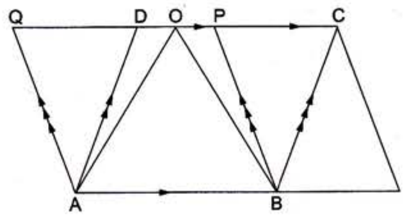
The area of the parallelogram $ADFE$ is 275 cm^2 and AD = 12.5 cm. Find DC.

4.



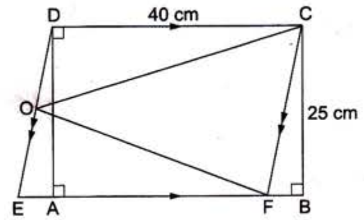
The area of $\triangle ABP$ is 125 cm^2. Find the area of the parallelogram $ABCD$.

5.



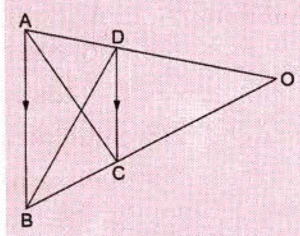
The area of the parallelogram *ABPQ* is 415 cm^2. Find (i) the area of the parallelogram *ABCD* and (ii) the area of the triangle *OAB*.

6.



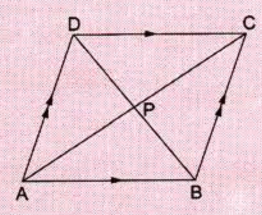
ABCD is a rectangle. Find (i) the area of the parallelogram *CDEF* and (ii) the area of $\triangle OFC$.

7. In the figure, *OAB* is a triangle in which $AB \parallel DC$.
If the area of $\triangle CAD$ is 70 cm^2 and the area of $\triangle ODC$ is 86 cm^2, find



(i) the area of $\triangle DBC$ (ii) the area of $\triangle OAC$
(iii) the area of $\triangle ODB$

8. In the figure, *ABCD* is a parallelogram. Prove that



(i) the area of $\triangle ABC = \frac{1}{2} \times$ area of the parallelogram *ABCD*

(ii) the area of $\triangle ABD = \frac{1}{2} \times$ area of the parallelogram *ABCD*

(iii) the area of $\triangle ABC$ = the area of $\triangle ABD$

(iv) the area of $\triangle PAB$ + the area of $\triangle PCD = \frac{1}{2} \times$ area of the parallelogram *ABCD*.

[Hint: Draw a line *EF* through *P* parallel to *AB* or *DC*.
Then the area of $\triangle PAB = \frac{1}{2} \times$ area of the parallelogram *ABFE*
and the area of $\triangle PCD = \frac{1}{2} \times$ area of parallelogram *DCFE*.]

ANSWERS

1. 130 cm^2 **2.** 216 cm^2 **3.** 22 cm **4.** 250 cm^2

5. (i) 415 cm^2 (ii) 207.5 cm^2 **6.** (i) 1000 cm^2 (ii) 500 cm^2

7. (i) 70 cm^2 (ii) 156 cm^2 (iii) 156 cm^2

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