1

Fundamental Concepts

Lines and Angles

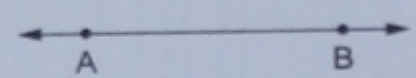
You are familiar with geometrical objects such as points, straight lines, rays, line segments, angles and parallel lines. In this chapter, we will revise what you have already learnt.

Point

A point has a position but it has no length, width or thickness. It is represented by a dot on a sheet of paper.

Line

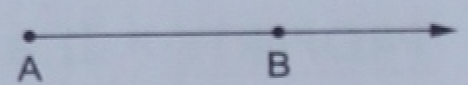
A straight line or a line is formed by a collection of points. Its basic quality is straightness. A line has position and shape, but not breadth or thickness. A line has no end points and can be extended indefinitely in both directions, so it does not have a definite length.



Line

Ray

A ray is a part of a straight line. It extends indefinitely in one direction from a fixed point. Thus, a ray has one end point and it does not have a definite length.



Ray

Line segment

A line segment is a part of a straight line between two points. It has two end points and a finite length.

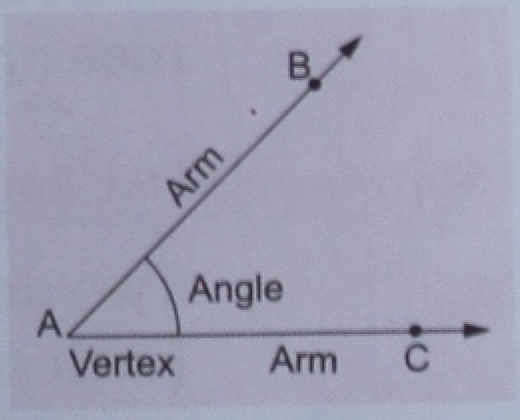
Line segment

Plane

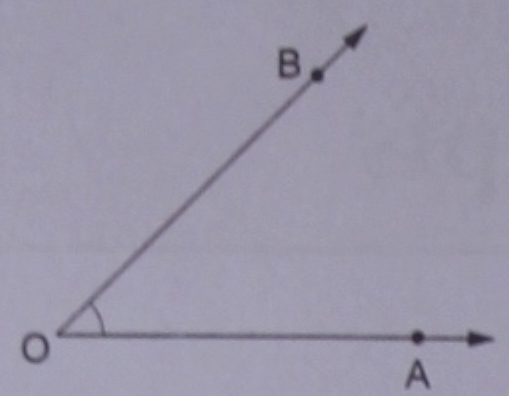
A plane is a flat surface that has length and breadth but no thickness.

Angle

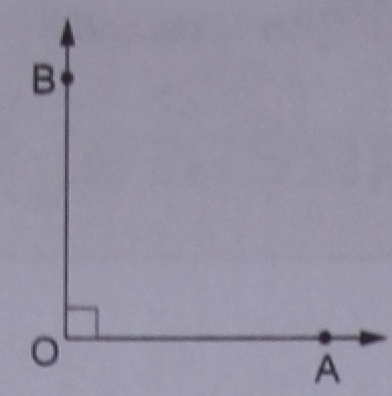
Two rays or line segments starting from the same point, form an angle. The common end point is called the vertex of the angle. The rays or line segments are called the arms of the angle. Angles are usually measured in degrees (°) and are given names in accordance with their magnitudes, as shown in the following figures.



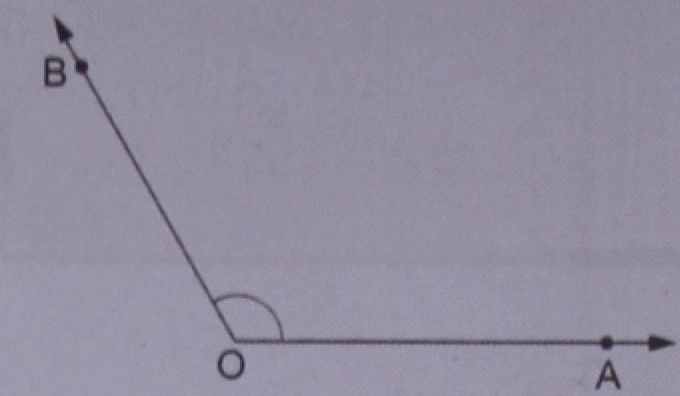
Types of Angles



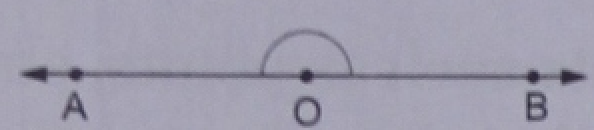
Acute angle (between 0° and 90°)



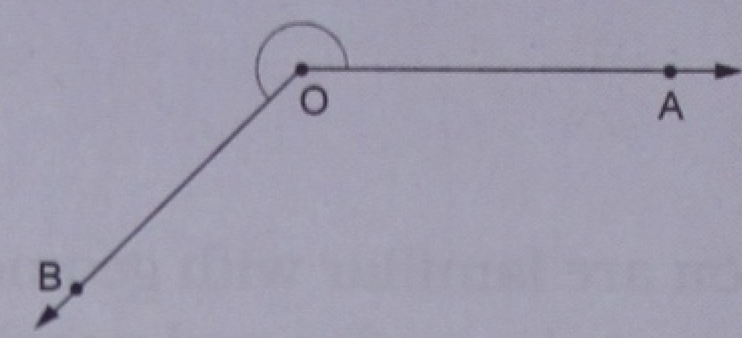
Right angle (90°)



Obtuse angle (between 90° and 180°)



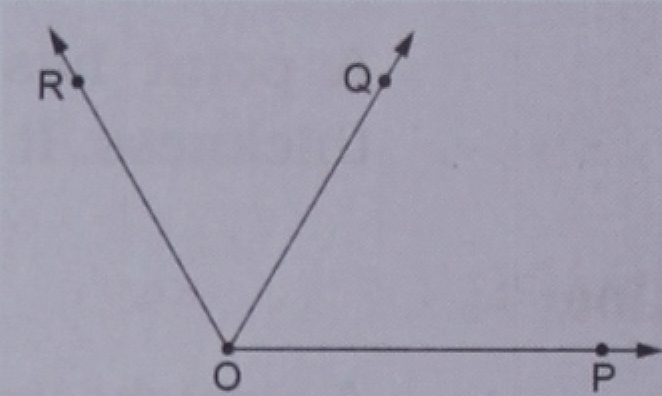
Straight angle (180°)



Reflex angle (between 180° and 360°)

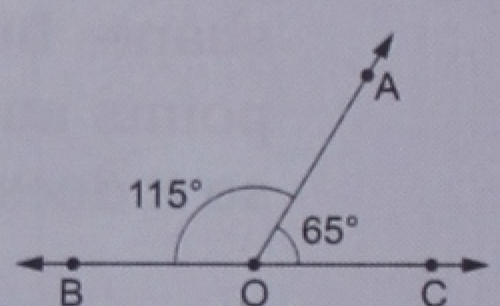
Adjacent angles

Two angles are called adjacent angles if they have the same vertex and a common arm and their other arms are on either side of the common arm. In the figure, $\angle POQ$ and $\angle QOR$ are adjacent angles, while $\angle POR$ and $\angle POQ$ are not adjacent angles.

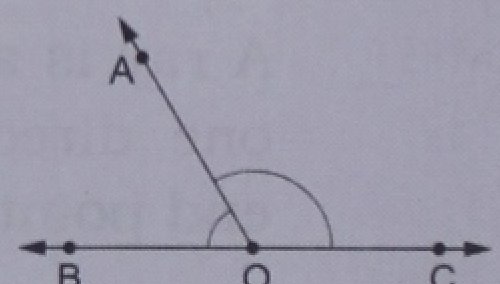


Linear pair

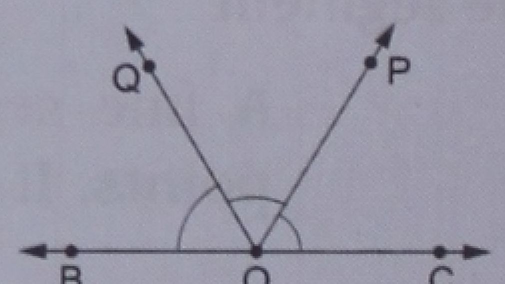
Two adjacent angles are said to form a linear pair if the sum of their measures is 180°. Here, $\angle AOB$ and $\angle AOC$ form a linear pair because $115° + 65° = 180°$.



PROPERTY If a ray OA stands on a line BC then the adjacent angles $\angle AOB$ and $\angle AOC$ form a linear pair, that is, $\angle AOB + \angle AOC = 180°$.



Also, the sum of all the angles at a point of a line on one side of it is 180°. Here, $\angle BOQ + \angle QOP + \angle POC = 180°$.



Complementary angles

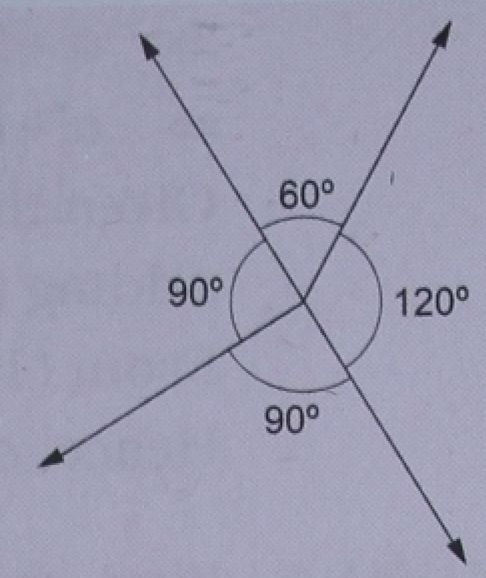
Two angles are called complementary angles if the sum of their measures is 90°. Each angle is said to be the complement of the other. For example, two angles of measures 32° and 58° are complementary angles.

Supplementary angles

Two angles are called supplementary angles if the sum of their measures is 180°. Each angle is said to be the supplement of the other. For example, angles of measures 104° and 76° are supplementary angles.

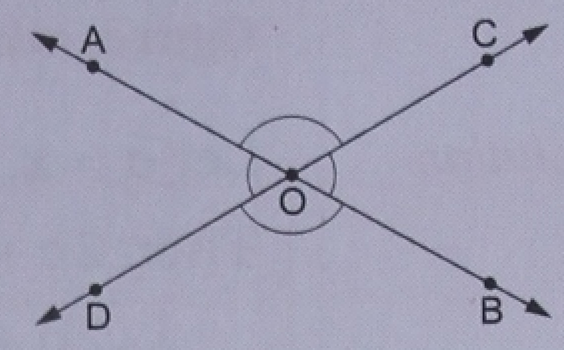
Angles at a point

The sum of all the angles at a point, making a complete rotation, is 360°. In the figure, $60°+90°+90°+120°=360°$.



Vertically opposite angles

When two straight lines *AB* and *CD* intersect at the point *O* then the pairs (i) $\angle AOD$ and $\angle COB$ and (ii) $\angle AOC$ and $\angle BOD$ are called vertically opposite angles.



PROPERTY **If two lines intersect then the vertically opposite angles so formed are equal.**

In the figure, $\angle AOD = \angle COB$ and $\angle AOC = \angle BOD$.

Solved Examples

EXAMPLE 1 **If an angle is $\frac{2}{3}$ of its supplement, find the angles.**

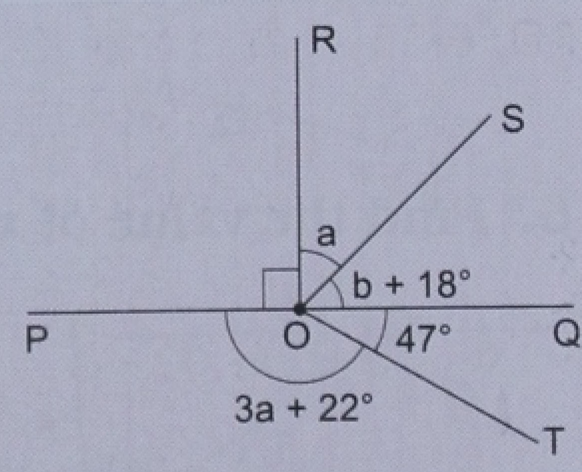
Solution Let the angle $= x°$. So, its supplement $= 180° - x°$.

Given that $x° = \frac{2}{3}(180° - x°) \Rightarrow 3x° = 2(180° - x°) \Rightarrow 3x° = 360° - 2x°$

$\Rightarrow 3x° + 2x° = 360° \Rightarrow 5x° = 360° \Rightarrow x° = \frac{360°}{5} = 72°$.

So, the angles are $72°$ and $180° - 72°$, i.e., $108°$.

EXAMPLE 2 **Find the values of *a* and *b* from the adjoining figure, where *POQ* is a straight line.**



Solution Here, *OT* stands on the line *PQ*. So, $\angle POT + \angle QOT = 180°$

$\Rightarrow 3a + 22° + 47° = 180° \Rightarrow 3a = 180° - 22° - 47° = 111°$.

$\therefore a = \frac{111°}{3} = 37°$.

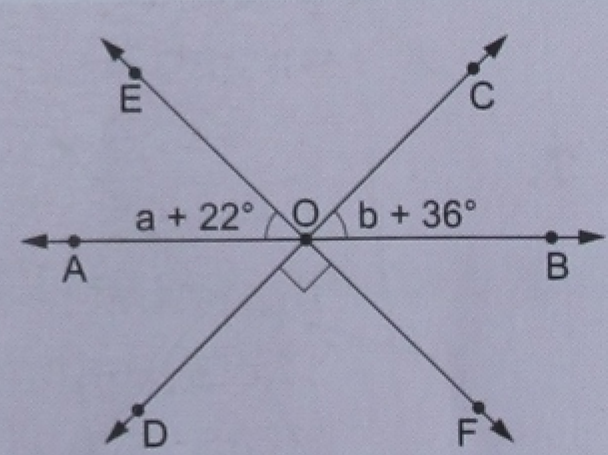
Being angles at a point on one side of the line *PQ*, $\angle POR + \angle ROS + \angle SOQ = 180°$

$\Rightarrow 90° + a + b + 18° = 180° \Rightarrow 90° + 37° + b + 18° = 180°$.

$\therefore b = 180° - 145° = 35°$.

Hence, $a = 37°$ and $b = 35°$.

EXAMPLE 3 **Find the values of *a* and *b* from the adjoining figure when $a - b = 4°$.**



Solution $\angle COE$ = vertically opposite $\angle DOF = 90°$.

$\because$ sum of the angles at a point on one side of a straight line $= 180°$, $\angle AOE + \angle COE + \angle BOC = 180°$

$\Rightarrow \quad a + 22^\circ + 90^\circ + b + 36^\circ = 180^\circ$

$\Rightarrow \quad a + b + 148^\circ = 180^\circ \Rightarrow a + b = 32^\circ$ ... (1)

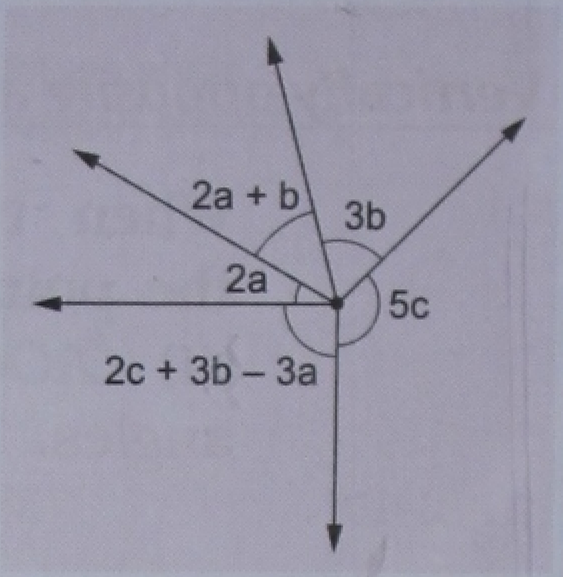
Given that $a - b = 4^\circ$ ... (2)

Adding (1) and (2), we get $2a = 36^\circ \Rightarrow a = 18^\circ$.

From (1), $b = 32^\circ - a = 32^\circ - 18^\circ = 14^\circ$.

Hence, $a = 18^\circ, b = 14^\circ$.

EXAMPLE 4 **Find the values of *a*, *b* and *c* from the adjoining figure, where $a : b : c = 1 : 2 : 3$.**



Solution Let $a = x$, $b = 2x$ and $c = 3x$.

Then $2c + 3b - 3a = 6x + 6x - 3x = 9x$, $5c = 15x$, $3b = 6x$,

$2a + b = 2x + 2x = 4x$ and $2a = 2x$.

$\because$ sum of the angles at a point $= 360^\circ$,

$$9x + 15x + 6x + 4x + 2x = 360^\circ \Rightarrow 36x = 360^\circ \Rightarrow x = \frac{360^\circ}{36} = 10^\circ.$$

Hence, $a = 10^\circ, b = 20^\circ$ and $c = 30^\circ$.

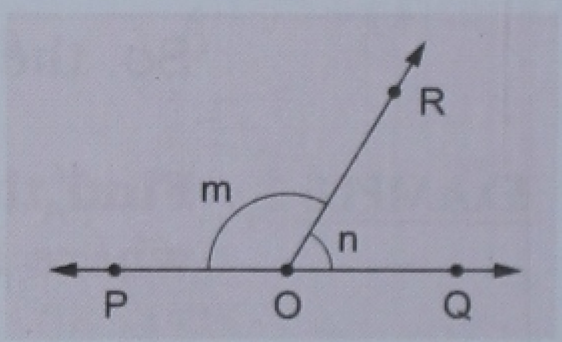
EXERCISE 1A

1. Two angles are in the ratio $4 : 5$. Find the angles if they are (i) complementary (ii) supplementary to each other.

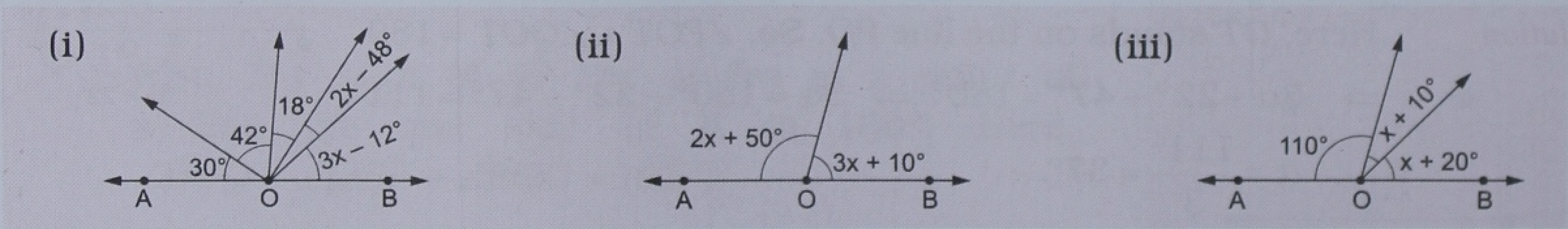
2. An angle is 30° more than twice its complement. Find the angle.

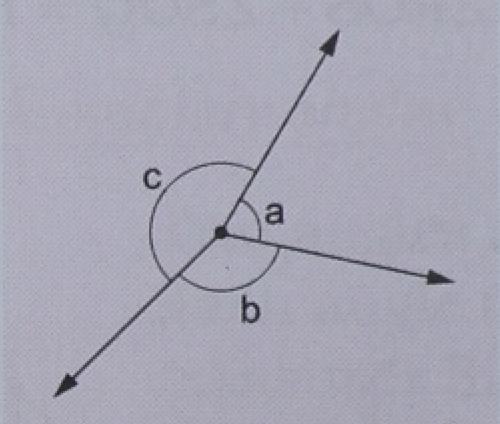
3. If the angles $(2a - 30)^\circ$ and $(b + 60)^\circ$ make a linear pair, find the values of a and b when $a - b = 30$.

4. In the adjoining figure, POQ is a straight line. Find m and n when
   (i) $m - n = 60^\circ$ (ii) $m : n = 7 : 5$

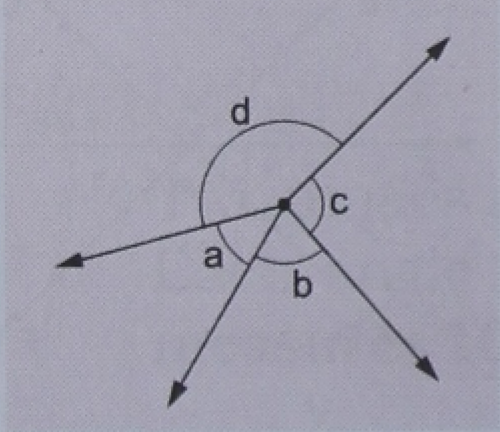


5. Find the value of x if AOB is a straight line.



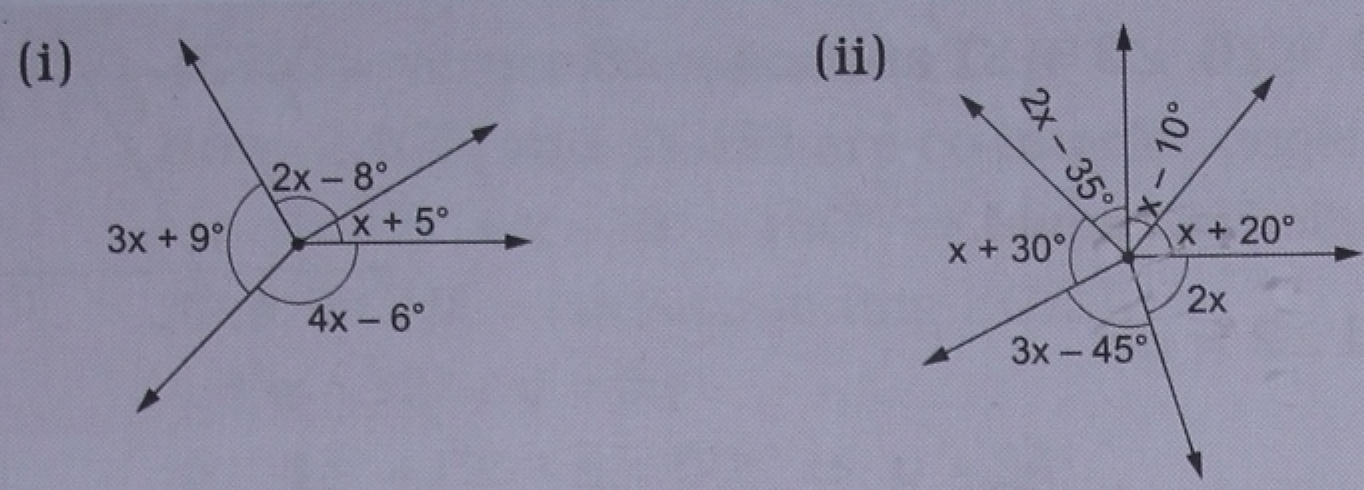
6. 

If $a : b : c = 2 : 3 : 4$, find a, b and c.

7. 

If $a : b : c : d = 2 : 3 : 5 : 8$, find a, b, c and d.

8. Find the value of x.



ANSWERS

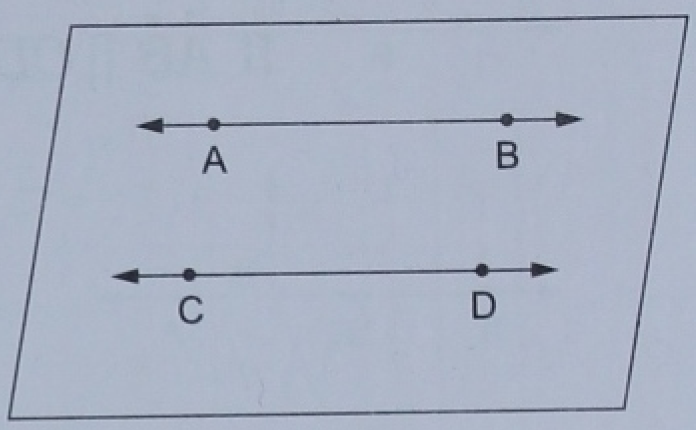
1. (i) $40°, 50°$ (ii) $80°, 100°$ **2.** $70°$ **3.** $a = 60, b = 30$

4. (i) $120°, 60°$ (ii) $105°, 75°$ **5.** (i) $30°$ (ii) $24°$ (iii) $20°$ **6.** $a = 80°, b = 120°, c = 160°$

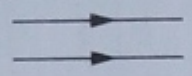
7. $a = 40°, b = 60°, c = 100°, d = 160°$ **8.** (i) $36°$ (ii) $40°$

Parallel Lines

Two straight lines lying in the same plane are said to be parallel to each other when they do not meet (if produced on both sides). Here, AB and CD are two parallel lines. We write $AB \parallel CD$ or $CD \parallel AB$.

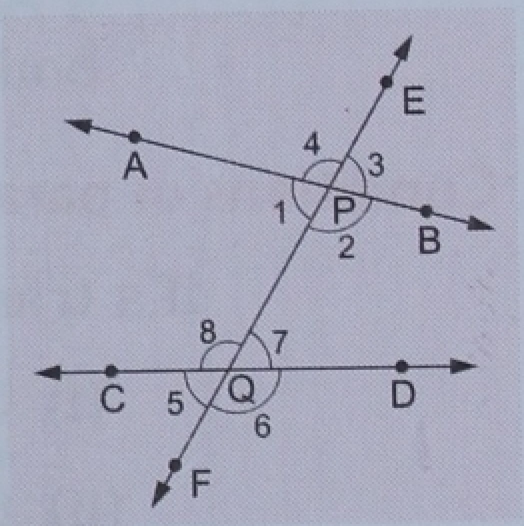


Parallel lines may be marked with arrows as shown.



Transversal

A transversal is a straight line that intersects two or more lines in a plane. In the figure, the transversal EF intersects the two lines AB and CD at the points P and Q respectively. The angles formed at the point P are $\angle APE$, $\angle BPE$, $\angle APQ$ and $\angle BPQ$. The angles formed at the point Q are $\angle CQP$, $\angle DQP$, $\angle CQF$ and $\angle DQF$.



These angles, labelled 1 to 8 for convenience, are called:

Exterior angles $\angle 4, \angle 3, \angle 5$ and $\angle 6$

Interior angles $\angle 1, \angle 2, \angle 8$ and $\angle 7$

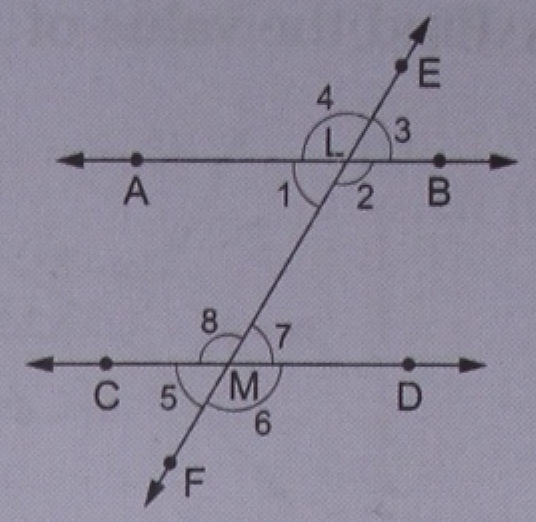
Alternate angles The pairs $\angle 1, \angle 7$ and $\angle 2, \angle 8$

Corresponding angles The pairs $\angle 1, \angle 5$; $\angle 2, \angle 6$; $\angle 3, \angle 7$; and $\angle 4, \angle 8$

Co-interior angles The pairs $\angle 1, \angle 8$ and $\angle 2, \angle 7$

Properties of parallel lines

Let the transversal EF intersect two parallel lines AB and CD at the points L and M respectively. Then the following properties hold.



PROPERTY 1 **Each pair of corresponding angles are equal.**

In the figure, $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$.

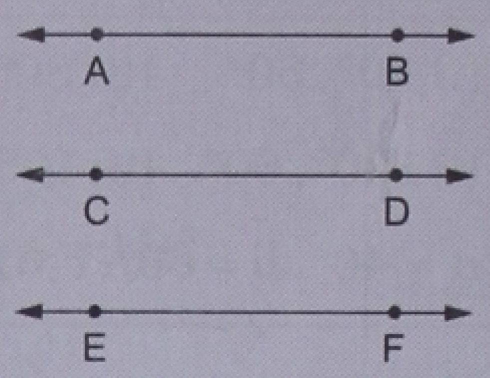
PROPERTY 2 **Each pair of alternate angles are equal.**

In the figure, $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$.

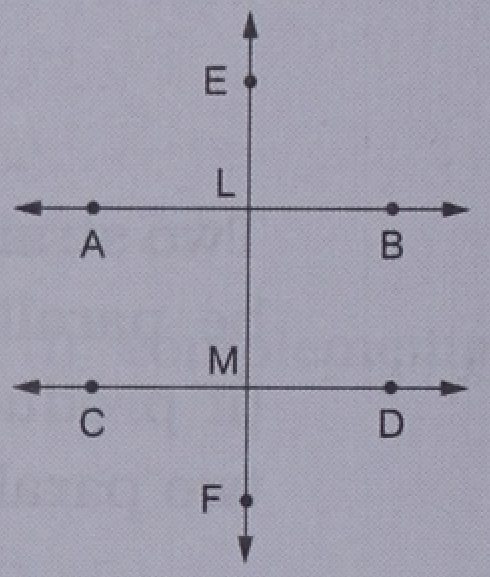
PROPERTY 3 **The interior angles on the same side of the transversal (also called co-interior angles) are supplementary.**

In the figure, $\angle 1 + \angle 8 = 180°$ and $\angle 2 + \angle 7 = 180°$.

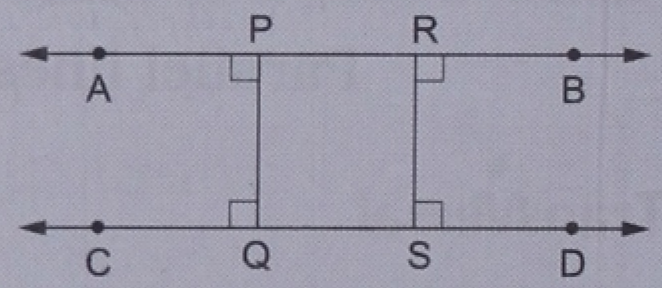
Additional properties of parallel lines



1. Lines which are parallel to the same line are parallel to each other.
   If $AB \parallel EF$ and $CD \parallel EF$ then $AB \parallel CD$.



2. If two lines are parallel, a line perpendicular to one of them is also perpendicular to the other.
   If $AB \parallel CD$ and $EF \perp AB$ then $EF \perp CD$.



3. The distance between two parallel lines is constant.
   If $AB \parallel CD$ and PQ and RS are perpendiculars to both AB and CD then $PQ = RS$.

Conditions of parallelism

If a transversal intersects two straight lines such that

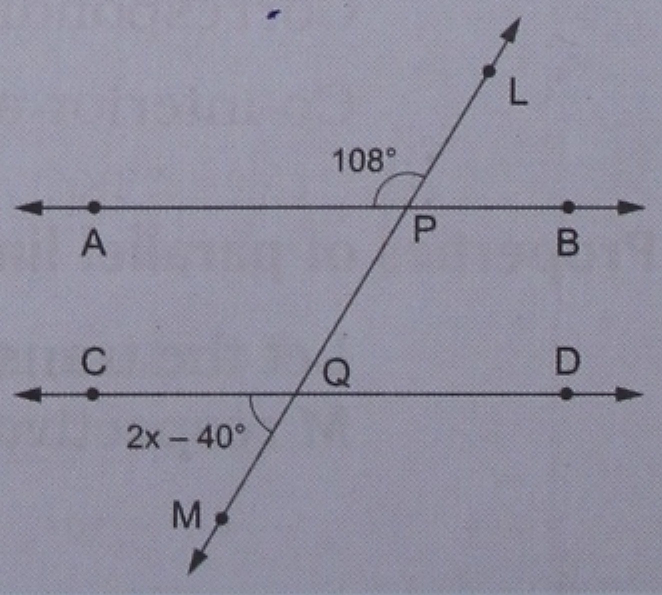
(i) the corresponding angles are equal, or

(ii) the alternate angles are equal, or

(iii) the co-interior angles are supplementary

then the two lines are parallel to each other.

Solved Examples



EXAMPLE 1 **In the adjoining figure, $AB \parallel CD$. Find x.**

Solution $\angle CQP$ and $\angle APL$ are corresponding angles.

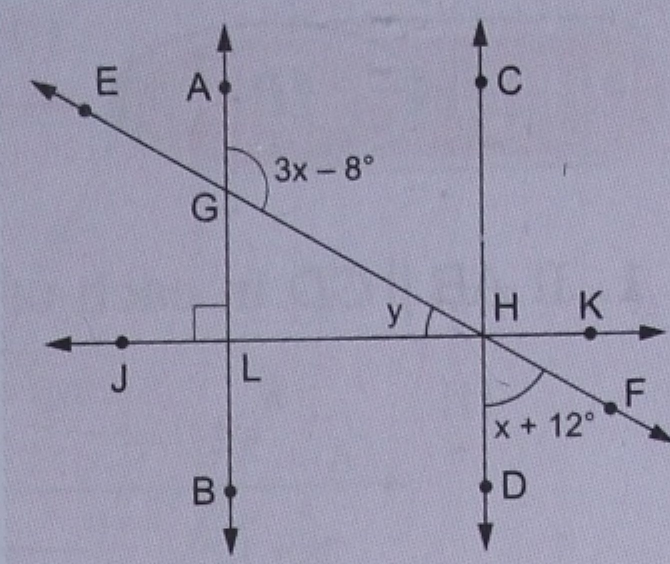
$\therefore \quad \angle CQP = \angle APL = 108°$.

Now, $\angle CQM$ and $\angle CQP$ form a linear pair.

$\therefore \quad \angle CQM + \angle CQP = 180°$

$\Rightarrow \quad 2x - 40° + 108° = 180° \Rightarrow 2x = 112° \Rightarrow x = 56°$.

EXAMPLE 2 **In the adjoining figure, $AB \parallel CD$. Find x and y.**



Solution $\angle CHG$ = vertically opposite $\angle DHF = x + 12°$.

Now, $\angle AGH$ and $\angle CHG$ are co-interior angles.

$\therefore \quad 3x - 8° + x + 12° = 180° \Rightarrow 4x = 176° \Rightarrow x = 44°$.

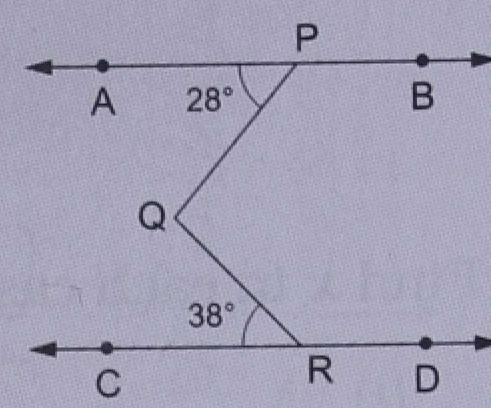
Also, $\angle CHL$ = corresponding $\angle GLJ$.

$\therefore \quad x + 12° + y = 90°$

$\Rightarrow \quad 44° + 12° + y = 90° \Rightarrow y = 34°$.

$\therefore \quad x = 44°, y = 34°$.

EXAMPLE 3 **In the adjoining figure, $AB \parallel CD$. Find $\angle PQR$ and the reflex angle PQR.**



Solution Draw a line EF through Q, parallel to AB.

As EF is parallel to AB and $AB \parallel CD$, we get $EF \parallel CD$.

$\angle PQF$ and $\angle APQ$ are alternate angles.

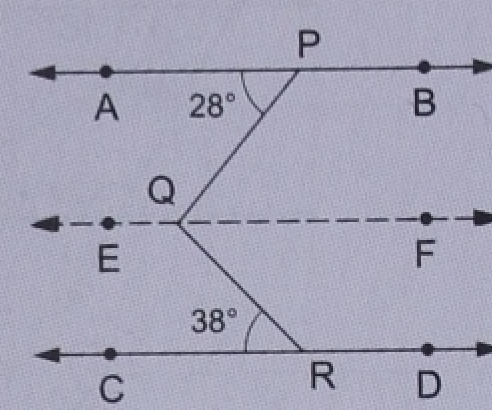
$\therefore \quad \angle PQF = \angle APQ = 28°$.

Again, $\angle RQF$ and $\angle CRQ$ are alternate angles.

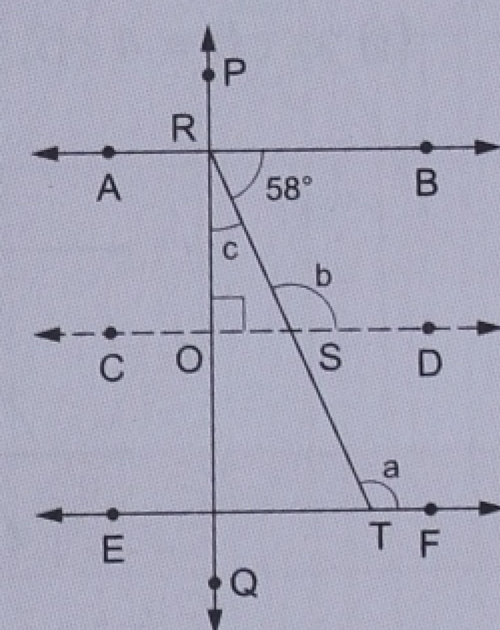
$\therefore \quad \angle RQF = \angle CRQ = 38°$.

$\therefore \quad \angle PQR = \angle PQF + \angle RQF = 28° + 38° = 66°$.

Reflex angle $PQR = 360° - \angle PQR = 360° - 66° = 294°$.



EXAMPLE 4 **In the adjoining figure, $AB \parallel CD \parallel EF$. Find a, b and c.**



Solution $\angle FTR$ and $\angle BRT$ are co-interior angles.

$\therefore \quad a + 58° = 180° \quad$ or $\quad a = 180° - 58° = 122°$.

Again, $\angle DSR$ and $\angle FTS$ are corresponding angles.

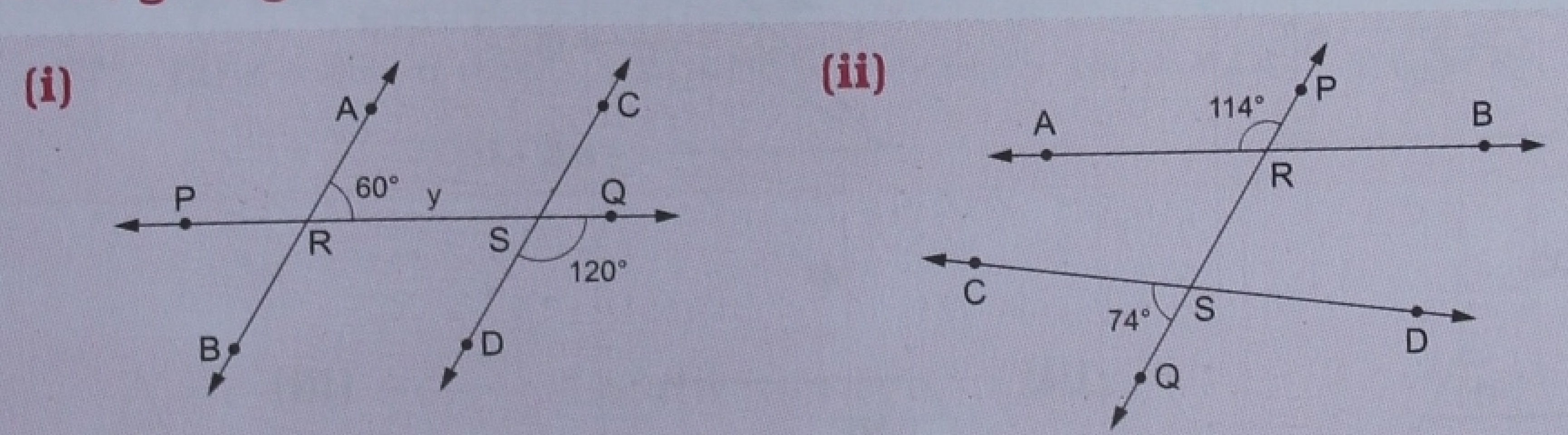
$\therefore \quad b = a \quad$ or $\quad b = 122°$.

Further, $\angle BRO$ and $\angle SOR$ are co-interior angles.

$\therefore \quad c + 58° + 90° = 180° \quad$ or $\quad c = 32°$.

Hence, $a = 122°$, $b = 122°$ and $c = 32°$.

EXAMPLE 5 **State, giving reasons, whether AB is parallel to CD.**



Solution (i) $\angle CSR$ = vertically opposite $\angle QSD = 120°$.

Now, $\angle ARS + \angle CSR = 60° + 120° = 180°$.

But these are co-interior angles. Hence, $AB \parallel CD$.

(ii) $\angle CSQ$ and $\angle CSR$ make a linear pair.

$\therefore \quad \angle CSQ + \angle CSR = 180° \Rightarrow \angle CSR = 180° - \angle CSQ = 180° - 74° = 106°$.

Given, $\angle ARP = 114°$. So, $\angle ARP \neq \angle CSR$,

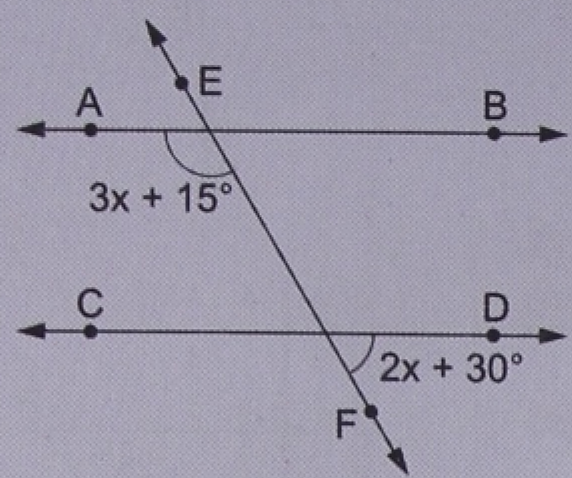
or corresponding angles are not equal.

Hence, AB is not parallel to CD.

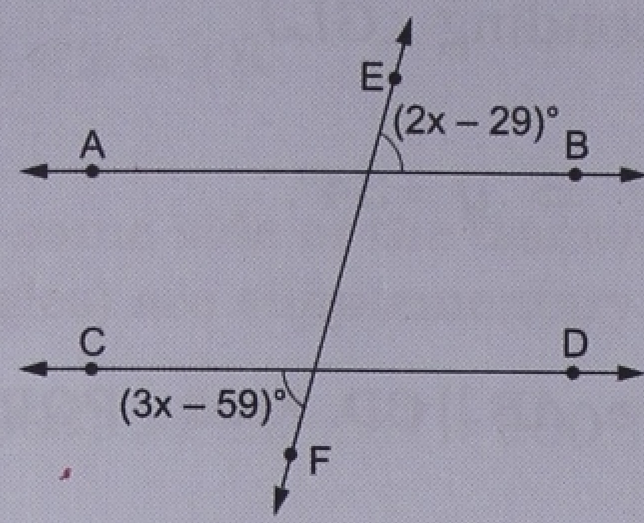
EXERCISE 1B

1. If $AB \parallel CD$ in each of the following, find x.

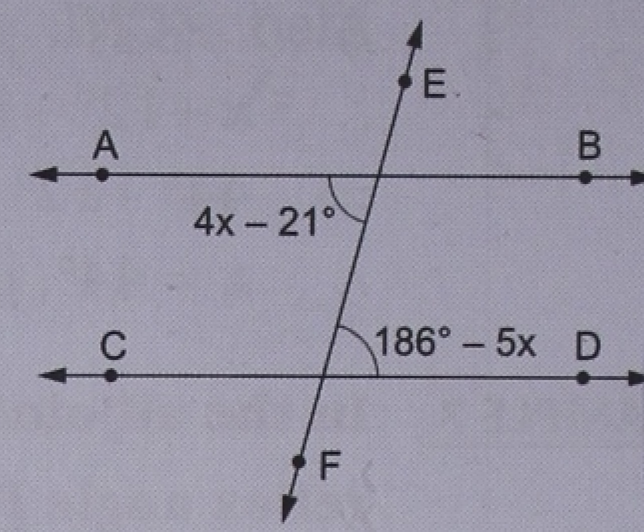
(i)



(ii)

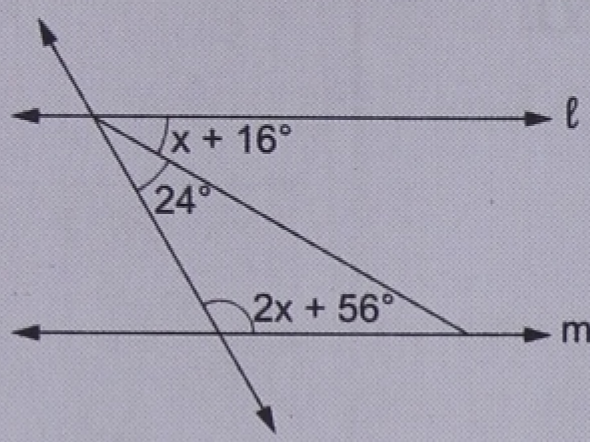


(iii)

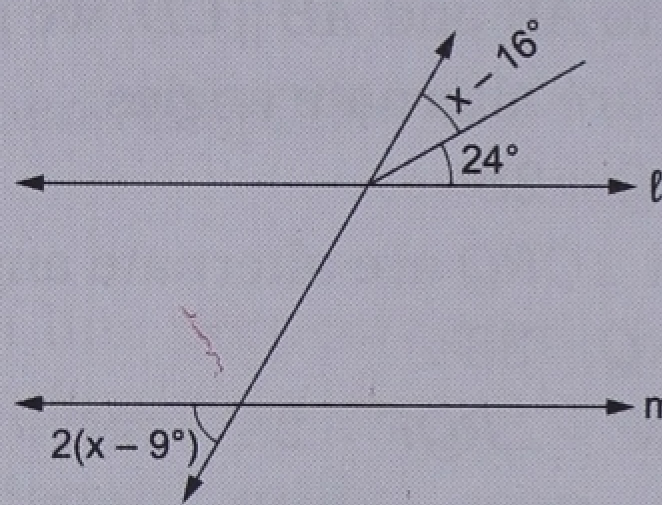


2. Find x in each case, where $l \parallel m$.

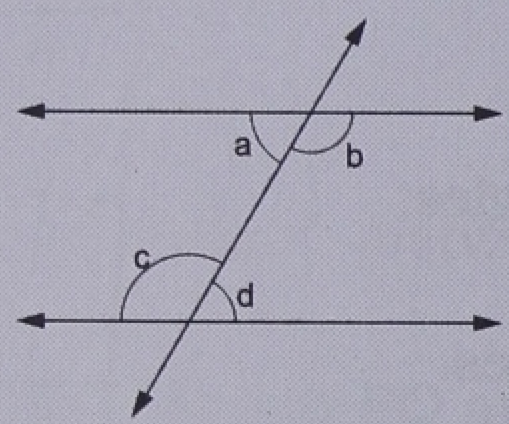
(i)

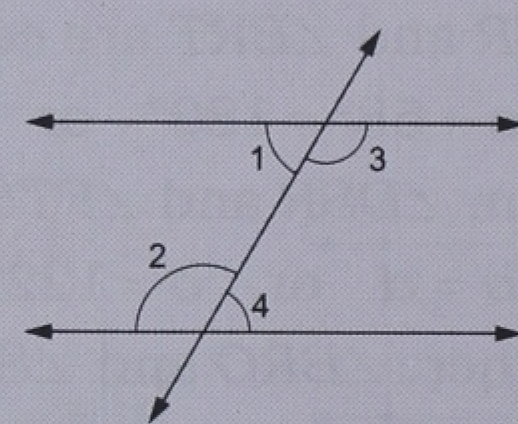


(ii)



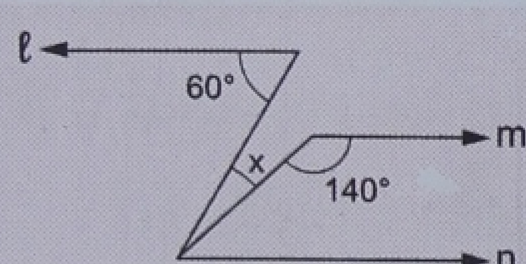
3. (i) $a : b = 4 : 5$, find the angles c and d. (ii) If $\angle 1 : \angle 2 = 1 : 2$, find $\angle 3$ and $\angle 4$.



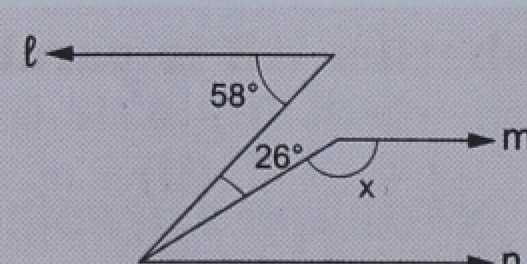


4. In the following figures, $l \parallel m \parallel n$. Find x in each case.

(i)

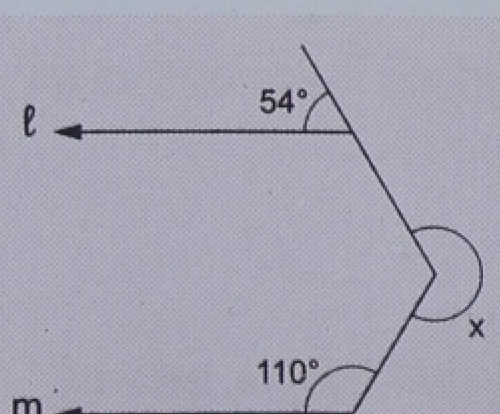


(ii)

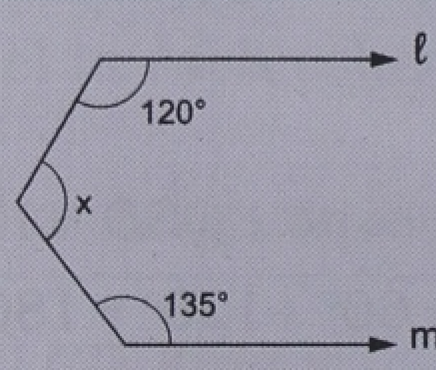


5. If $l \parallel m$, find x.

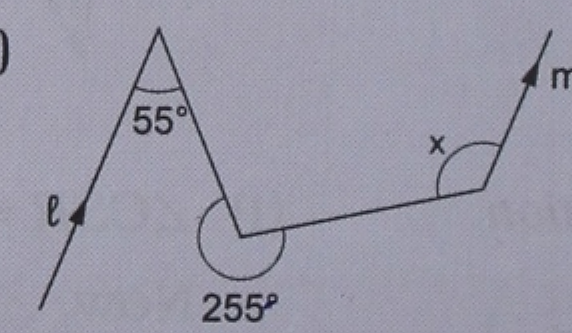
(i)



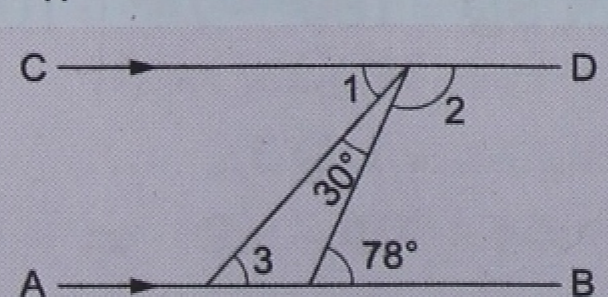
(ii)



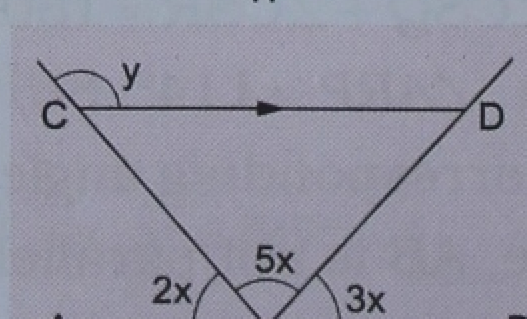
(iii)



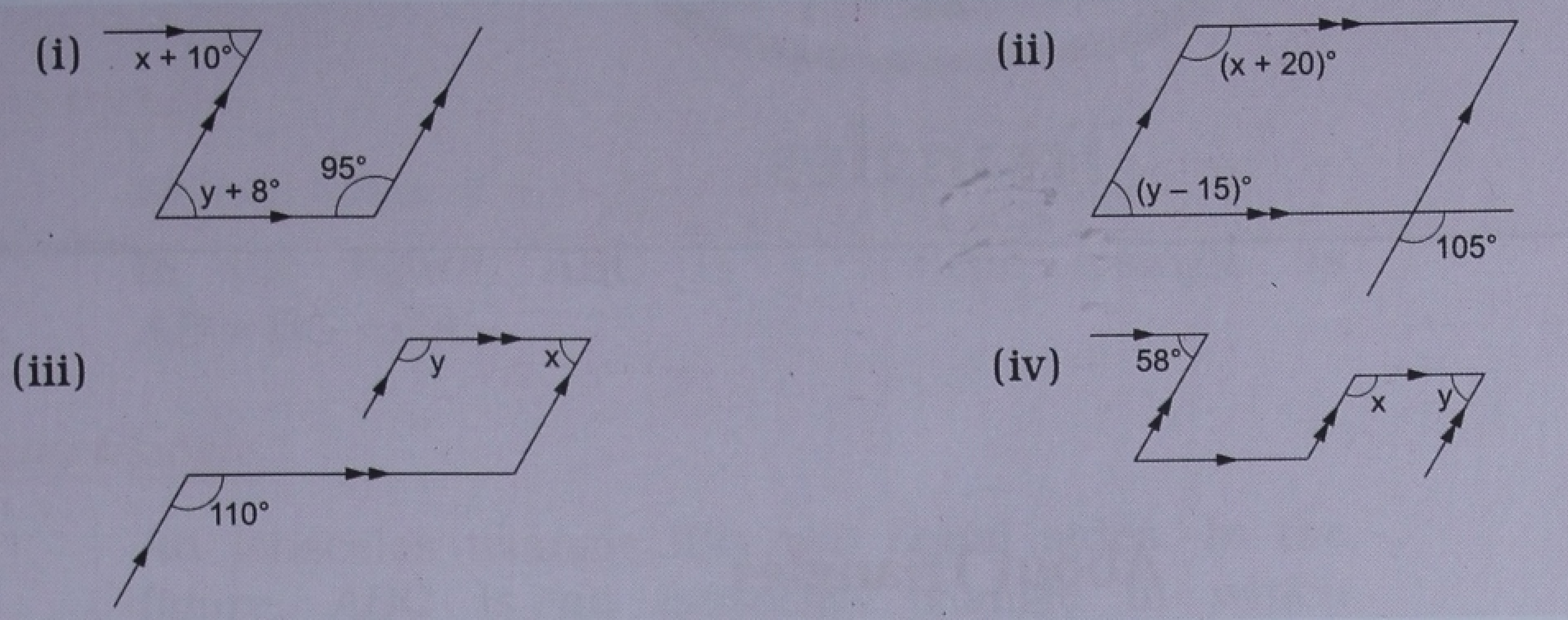
6. If $AB \parallel CD$, find $\angle 1$, $\angle 2$ and $\angle 3$.



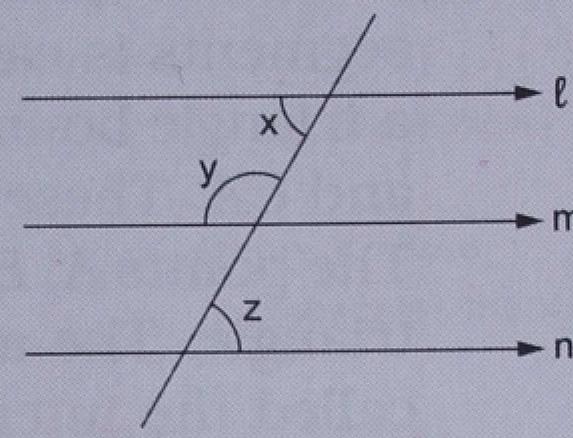
7. Given $AB \parallel CD$. Find x and y.



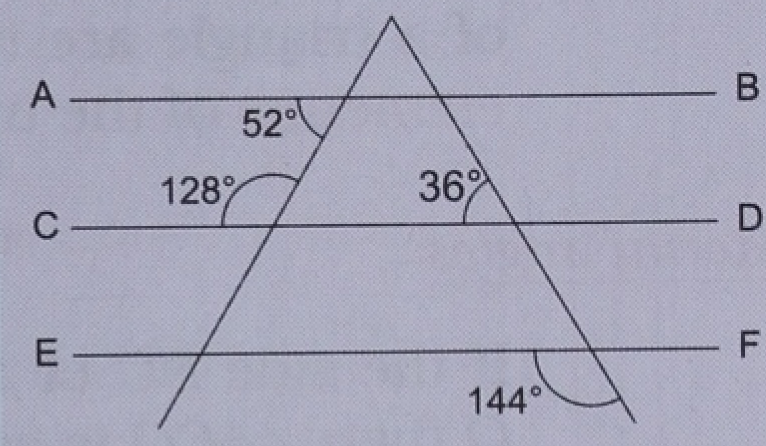
8. Find x and y. (Lines with the same number of arrows are parallel.)



9. In the adjoining figure, $l \parallel m \parallel n$ and $x : y = 2 : 3$. Find z.



10. State, giving reasons, whether AB, CD and EF are parallel.



ANSWERS

1. (i) 27° (ii) 30° (iii) 23° **2.** (i) 28° (ii) 26°

3. (i) $\angle c = 100°$, $\angle d = 80°$ (ii) $\angle 3 = 120°$, $\angle 4 = 60°$ **4.** (i) 20° (ii) 148°

5. (i) 236° (ii) 105° (iii) 130° **6.** $\angle 1 = 48°$, $\angle 2 = 102°$, $\angle 3 = 48°$ **7.** $x = 18°$, $y = 144°$

8. (i) $x = 75°, y = 77°$ (ii) $x = 85°, y = 90°$ (iii) $x = 70°, y = 110°$ (iv) $x = 122°, y = 58°$

9. 72° **10.** Yes

❖