RELATIONS AND MAPPINGS

40.1 INTRODUCTION

Relation	In our daily life, we come across many *statements* which show some connection between two objects. For example, (i) Meeta *is sister of* Ankur, shows connection between two persons (ii) 7 *is greater than* 2, shows connection between two numbers (iii) Line AB *is perpendicular to* line CD, shows connection between two lines, etc. Such statements which show some connection (or association or correspondence) between two objects give rise to the concept of *relation.* ***A relation means an association of two objects based on some properties possessed by them.***

The letter R is generally used to represent a relation.

Re-consider the example given above, in which,

(i) the first statement shows a relation between two persons and the relation R = "*is sister of*".

(ii) the second statement shows a relation between two numbers and the relation R = "*is greater than*" and so on.

40.2 REPRESENTATION OF A RELATION

1. Roster form (as the set of ordered pairs)

For example

If A = {1, 3, 4, 7, 9, 10, 16}, B = {0, 1, 2, 3, 4, 5} and the relation R from A to B "*is square of*", then R = { (1, 1), (4, 2), (9, 3), (16, 4) }.

1. Here the relation R is from set A to set B so the first component of each ordered pair is taken from set A and the second component from set B such that, the first component ***is the square*** of the second component.
2. If the first component as well as the second component of each ordered pair are taken from set A only, then the relation is called a ***relation in set A.***

   Similarly, a relation in set B means, the first component as well as the second component of each ordered pair are from set B itself.
3. The set of first components of all the ordered pairs is called the ***domain*** and the set of second components is called the ***range*** of the relation.

   Thus, in the example given above, Domain = {1, 4, 9, 16} and Range = {1, 2, 3, 4}

2. Set-Builder Form

Let a relation R from set A to set B means "*is greater than*" ; then it can be expressed as :

$$R = \{ (x, y) : x \in A, y \in B \text{ and } x > y \}$$

Therefore, in a set builder form, the relation from set A to set B, is written in the form

$$\{ (x, y) : x \in A, y \in B \text{ and } x \text{ .... } y \},$$

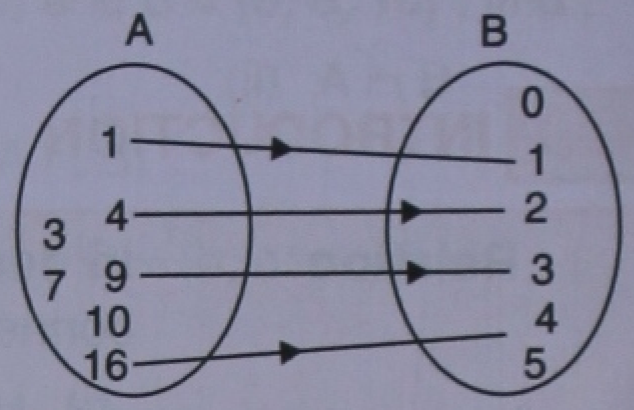
the blank is to be replaced by the rule which associates x and y.

3. By arrow diagrams

For a relation from set A to set B, arrows are drawn to indicate the pairing, which satisfy the given relation.

1. The arrow heads should indicate the direction from A to B.
2. If the relation R is from set B to set A, the arrow heads should indicate the direction from B to A.

In the example given above, the arrow diagram will be of the form as shown alongside :



TEST YOURSELF

1. Let A = {2, 5, 7, 6}; write the set of all possible ordered pairs satisfying the given relation in set A :

(a) R_1 = '*is less than*' = ...

(b) R_2 = '*is greater than*' = ...

(c) R_3 = '*is equal to*' = ...

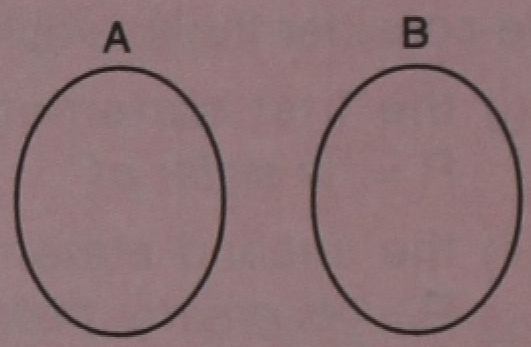
2. If A = {0, 1, 2, 3, 4, 5, 6, 7} and B = {1, 2, 3, 4, 5, 6}; write the relation R from set A to set B; where R = '*is 2 less than*' = ...

Also write :

(a) the domain of relation R = ...

(b) the range of relation R = ...

Represent the relation R using arrow diagram



3. Let A = {7, 8, 9} and B = {5, 6, 7, 8, 9} and a relation R from A to B such that

R = {(x, y) : x ∈ A, y ∈ B x ≤ y}; then R = ...

EXERCISE 40 (A)

1. Let A = {1, 2, 3, 4} and B = {a, b, c}. State, which of the followings are relations from A to B.
   (i) { (1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (3, 1) }
   (ii) { (a, b), (a, c), (b, a), (b, c), (c, a) }
   (iii) { (1, a), (1, b), (2, b), (3, c), (4, c) }
   (iv) { (a, 1), (b, 2), (c, 3), (b, 3), (b, 4) }
2. Let A = {a, b, c} and B = {5, 7, 9}. State, which of the followings are relations from B to A.
   (i) { (a, 5), (a, 7), (b, 7), (c, 9) }
   (ii) { (5, 7), (9, 9), (7, 5) }
   (iii) { (5, a), (5, b), (5, c) }
   (iv) { (5, b), (7, c), (7, a), (9, b) }
3. Given ordered pairs : (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6), (6, 7), (8, 4), (8, 5), (8, 6), (8, 8).

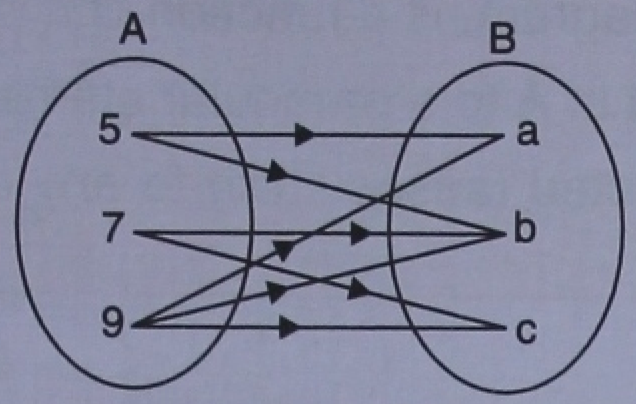
   Use these ordered pairs to find the following relations :

   (i) R_1 = "is less than" (ii) R_2 = "is equal to"
   (iii) R_3 = "is one less than"
   (iv) R_4 = "is greater than"

   In each case, write the domain and the range of the relation.
4. Let A = {6, 7, 8, 10, 12, 13} and B = {5, 7, 9, 11, 13, 15} and the relation R from A to B means, "is greater than". Find R. Also, draw a suitable diagram to represent this relation.
5. Let P = {3, 4, 5, 6} and Q = {3, 4, 5, 6, 7}. Find the following relations from Q to P.
   (i) R_1 = "is two less than"
   (ii) R_2 = "is one more than"

   In each case, draw an arrow diagram to represent the relation.
6. Given A = {8, 9, 10, 12}, B = {2, 3, 4, 5} and the relation R from A to B means : "is multiple of" :
   (i) Find R
   (ii) Write the domain and range of relation R
7. Given A = {2, 3, 4, 5}, B = {4, 5, 6, 7, 8} and the relation from A to B means, "is a factor of". Represent the relation;
   (i) in roster form (ii) by an arrow diagram.

8. Write the relation, represented by the arrow diagram given below, in roster form. Also, write the domain and range of the relation.



9. Given A = {4, 5, 6, 7}, B = {4, 6, 8} and a relation R from A to B such that :

   R = { (x, y) : x ∈ A, y ∈ B and x ≥ y}. Find R.

10. Given A = {2, 4, 6, 8, 10}, B = {5, 3, 2, 1, 0} and a relation R from A to B such that R = { (x, y) : x ∈ A, y ∈ B and x + y = 7}. Find R.

40.3 MAPPING OR FUNCTION

Mapping or function is a special type of relation.

Let A and B be two sets such that A = {a_1, a_2, a_3} and B = {b_1, b_2, b_3, b_4}.

If by some rule, each element of set A is associated with a unique element of set B, say a_1 is associated with b_1, a_2 is associated with b_2 and a_3 is associated with b_3, then the collection {(a_1, b_1), (a_2, b_2), (a_3, b_3)} of such associations is called function from A into B.

If this function is denoted by f, then we write :

f : A → B and is read as "f is a function from A to B".

The word 'mapping' is often used as synonym for 'function'.

The set A is called the ***domain*** and the set B is called ***co-domain*** or ***range*** of the function f.

Necessary conditions for mapping (function) :

For a function f from set A and set B ;

Every element of set A should be associated to a unique element of set B.

i.e. (i) there should not be any element in A which is not associated with any element of B

and (ii) no element of A should be associated with two or more elements of B.

1. A function is a special type of relation ; so every function is a relation but the converse is not always true.
2. A relation from A to B, represented in roster form, is a function (mapping) if :
   (a) each element of A is associated with unique element of B ;
   (b) no two ordered pairs have the same first component *i.e.* the first components of all the ordered pairs are different.

Example 1 :

State, giving reason, whether each of the following relations from A to B is a function or not.

(i) If A = {1, 2, 3} and B = {4, 5, 6} ; then R = { (1, 4), (2, 6), (3, 6) }.

(ii) If A = {5, 7, 9} and B = {2, 4} ; then R = { (5, 2), (5, 4), (7, 4), (9, 4) }.

(iii) If A = {a, *l*, m, n} and B = {x, y, z} ; then R = { (a, x), (*l*, y), (m, z) }.

Solution :

(i) **The relation R** = { (1, 4), (2, 6), (3, 6) } **is a function**, as each element in A has been associated with a unique element in B and no two ordered pairs have their first components the same.

(ii) **The relation R** = { (5, 2), (5, 4), (7, 4), (9, 4) } **is not a function**, as the element 5 ∈ A has been associated to two elements 2 and 4 in B.

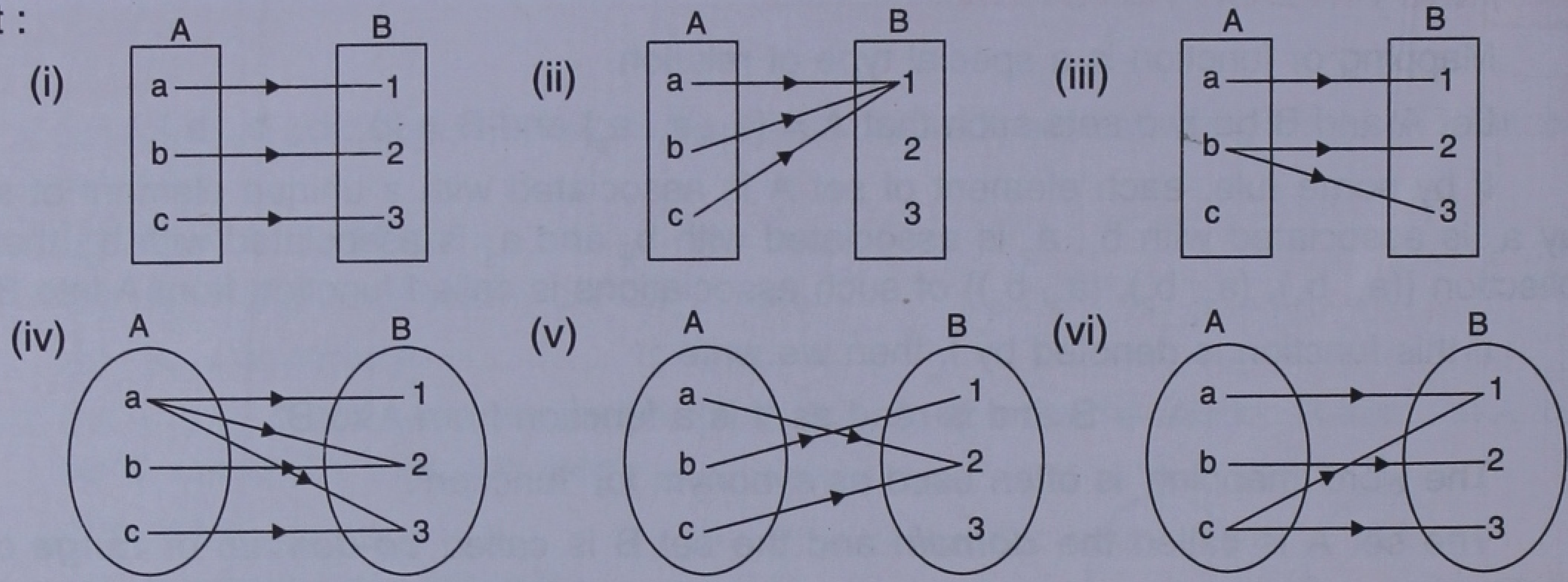
For a relation to be a function, the second component of the ordered pairs may repeat, but the first component cannot repeat.

(iii) **The relation R = { (a, x), (l, y), (m, z) } is not a function**, as the element n ∈ A is not associated with any element in B.

> A relation from A to B represented by an arrow diagram, is a function, if :
> (a) one and only one arrow connects an element in A to a particular element in B.
> (b) there is no element in A, which is not connected (associated) to any element in B.

Example 2 :

State, giving reason, whether each of the following arrow diagrams represent a function or not :



Solutions :

(i) **The given arrow diagram represents a function**, as each element in A is associated (connected) to a unique element in B.

(ii) **It represents a function** for the same reason as given in (i).

(iii) **The given arrow diagram does not represent** a function, as element b ∈ A is connected to two elements in B and also the element c ∈ A is not connected to any element in B.

(iv) **It does not represent a function** as element a ∈ A is connected to three elements in B.

(v) **It represents a function** for the same reason as given in (i).

(vi) **It does not represent a function**, as element c ∈ A is connected to two elements in B.

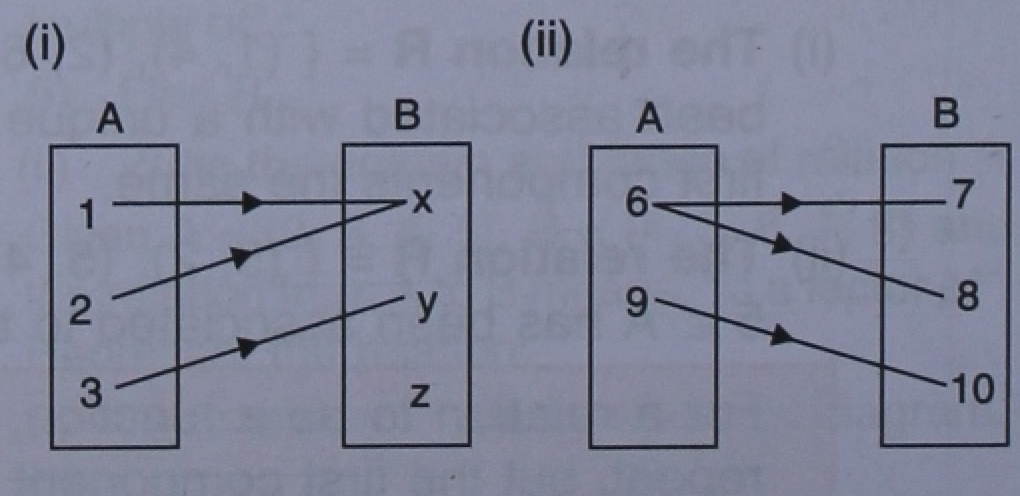
EXERCISE 40 (B)

1. State, which of the following relations are functions ? Give reason.
   (i) { (3, 7), (4, 7), (5, 7), (7, 7), (8, 7) }
   (ii) { (2, 3), (2, 4), (2, 5), (2, 6), (2, 7) }
   (iii) { $(3, \frac{1}{3})$, $(4, \frac{1}{4})$, $(5, \frac{1}{5})$, ... }
   (iv) { (a, b), (b, c), (c, d), (d, e) }

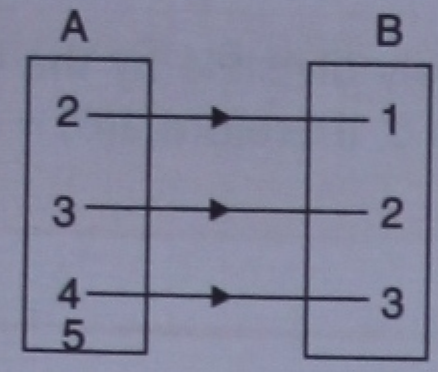
2. State, giving reason, which of the relations from A to B are functions :
   (i) If A = {a, b, c} and B = {x, y}, then R = { (a, x), (b, y), (c, x) }
   (ii) If A = {a, b, c} and B = {x, y}, then R = { (a, x), (c, x), (b, y), (a, y) }
   (iii) If A = {2, 4, 6} and B = {8, 9}, then R = { (2, 8), (2, 9), (4, 8), (4, 9), (6, 8) }
   (iv) If A = {2, 3} and B = {2, 3, 4}; then { (2, 2), (3, 3), (3, 4) }.

3. Given A = {3, 4, 5, 6} and B = {8, 9}. State, giving reason, whether { (3, 8), (4, 9), (5, 8) } is a mapping from A to B or not.

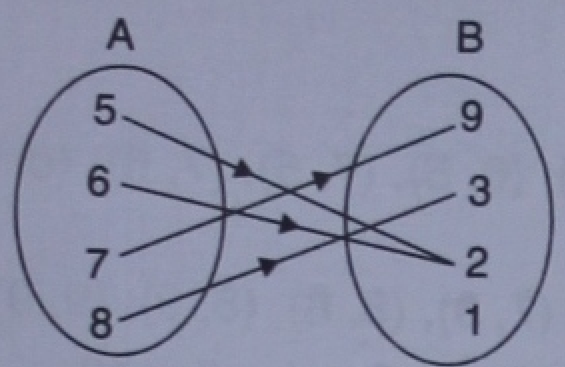
4. State, giving reason, which of the following arrow diagrams represent a function :



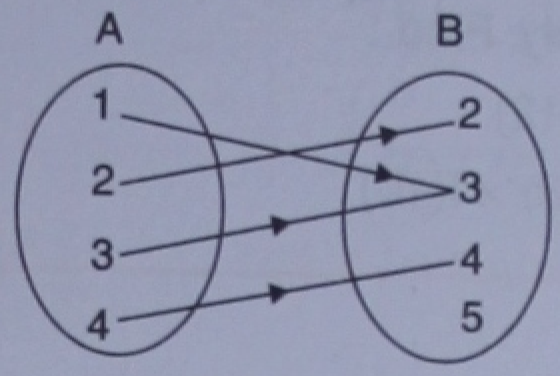
(iii)



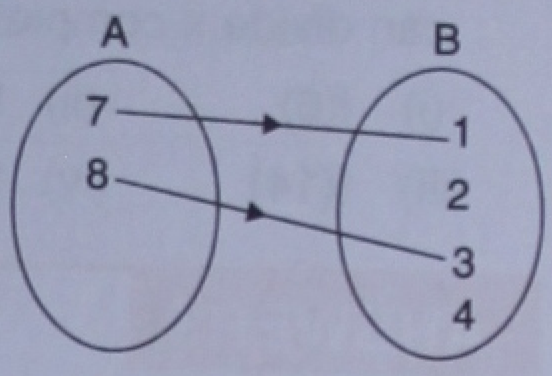
(iv)



(v)



(vi)



5. Given P = {2, 4, 6, 8} and Q = {3, 7}. State, whether { (3, 2), (7, 6) } is a function (mapping) from Q to P or not.

40.4 VALUE OF A FUNCTION

Let f be a function and (a, b) is in f, then we write f(a) = b; where f(a) is called the value of the function at a.

For a function from set A into or onto set B, usually, x is used to denote the elements of A and y is used to denote the elements of B; so that the function is exhibited in the manner :

$$y = f(x)$$

Also, a function $f : x \to 2x + 3$ means $f(x) = 2x + 3$.

The value of this function at 2 is $f(2) = 2 \times 2 + 3 = 7$

TEST YOURSELF

4. If $f(x) = 2x + 5$ and x is real number, find the value of :

(a) f(0) = ... = ... (b) f(4) = ... = ...

(c) f(2) = ... = ... and $\frac{1}{2}$ f(2) = ... = ...

(d) f(a + 1) = ... = ... = ...

(e) f(2) = ... = ..., f(–2) = ... = ... and f(2) + f(–2) = ...

(f) f(6) = ... = ..., f(3) = ... = ... and $\frac{f(6)}{f(3)}$ = ...

EXERCISE 40 (C)

1. Given $f(x) = 5x - 1$. Find :
(i) f(–3) (ii) f(3) (iii) f(0)
Is f(–3) + f(3) = f(0) ?

2. Given $f(x) = \frac{2x-1}{x+1}$. Find :
(i) f(2) (ii) f(a) (iii) f(4) – f(3)

3. Let f be a function such that $f : x \to 2x^2 + 3$.
Find the value of :
(i) f(1) (ii) f(2)
(iii) f(3) (iv) f(–2)
Is f(1) + f(2) = f(3) ? Is f(2) + f(–2) = 0 ?

4. Given $f(x) = 4x + 2$. Find :
(i) f(2) (ii) f(a + 1)
(iii) a, if f(a + 1) = f(2)

5. Let $f(x) = x^2 - 1$, $x \in R$. Find :
(i) f(3) – f(2) (ii) $\frac{f(4)}{f(5)}$ (iii) $\frac{1}{2}$ f(5)

6. A function f is defined by $f(x) = 2x^3 - 3x$, $x \in R$. What is the value of :
(i) f(4) (ii) $\frac{1}{2}$ f(2)

7. Given $f(x) = 1 - 2x$; find x, if f(x) is 15.

8. Given $f(x) = 7x + 2$; find x, if f(x) = 30.

9. Given A = {2, 3, 5} and B = {6, 10, 14, 18} ; find A × B. Also find the relation R such that :

$$R = \{ (x, y) \in A \times B, x < y \text{ and } \frac{y}{x} \in N \}.$$

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10. Given f(x) = sum of all natural numbers which can divide x completely. Find :
(i) f(6) (ii) f(12)
(iii) f(14) (iv) $f(3) \times f(10)$

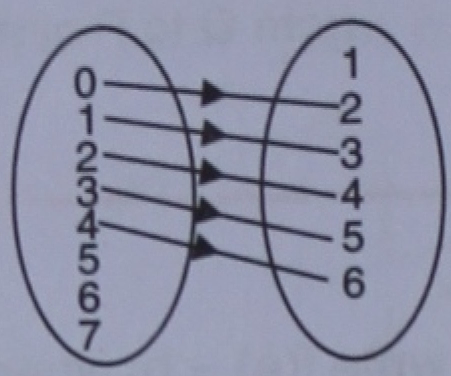
Since, 6 is completely divisible by the natural numbers 1, 2, 3 and 6, therefore, f(6) = 1 + 2 + 3 + 6 = 12.

ANSWERS

TEST YOURSELF

1. (a) {(2, 5), (2, 7), (2, 6), (5, 7), (5, 6), (6, 7)} (b) {(5, 2), (6, 2), (7, 2), (6, 5), (7, 5), (7, 6)} (c) {(2, 2), (5, 5), (6, 6), (7, 7)}

2. R = {(0, 2), (1, 3), (2, 4), (3, 5), (4, 6)}
(a) {0, 1, 2, 3, 4}
(b) {2, 3, 4, 5, 6}

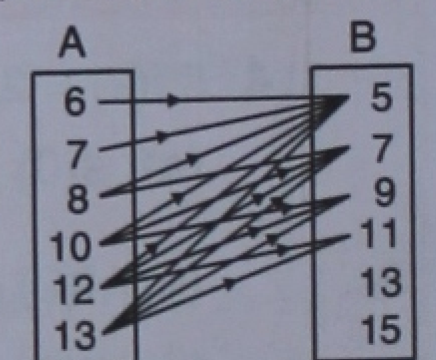


3. {(7, 7), (7, 8), (7, 9), (8, 8), (8, 9), (9, 9)}

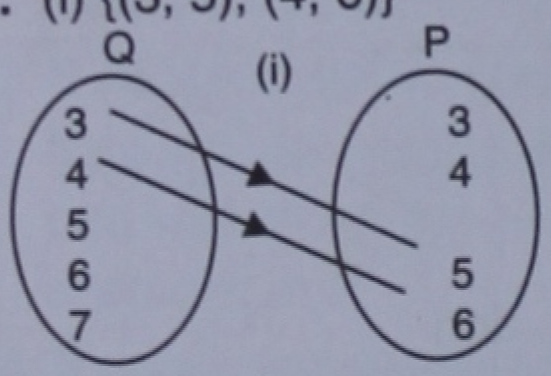
4. (a) $2 \times 0 + 5 = 5$ (b) $2 \times 4 + 5 = 13$
(c) $2 \times 2 + 5 = 9$ and $\frac{1}{2}f(2) = \frac{1}{2} \times 9 = 4\frac{1}{2}$
(d) $2 \times (a + 1) + 5 = 2a + 2 + 5 = 2a + 7$
(e) $f(2) = 2 \times 2 + 5 = 9$, $f(-2) = 2 \times -2 + 5 = 1$ and $f(2) + f(-2) = 9 + 1 = 10$
(f) $2 \times 6 + 5 = 17$, $2 \times 3 + 5 = 11$ and $\frac{f(6)}{f(3)} = \frac{17}{11} = 1\frac{6}{11}$

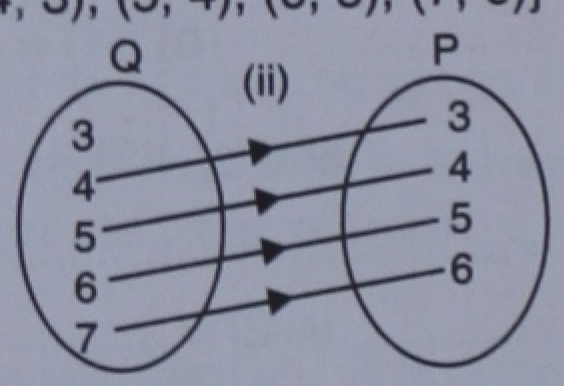
EXERCISE 40(A)

1. Only (iii) **2.** (iii) and (iv) **3.** (i) {(5, 6), (6, 7)} Domain = {5, 6} Range = {6, 7} (ii) {(5, 5), (6, 6), (8, 8)} Domain = {5, 6, 8} Range = {5, 6, 8} (iii) {(5, 6), (6, 7)} Domain = {5, 6} Range = {6, 7} (iv) {(5, 4), (6, 4), (6, 5), (8, 4), (8, 5), (8, 6)}; Domain = {5, 6, 8} Range = {4, 5, 6} **4.** {(6, 5), (7, 5), (8, 5), (8, 7), (10, 5), (10, 7), (10, 9), (12, 5), (12, 7), (12, 9), (12, 11), (13, 5), (13, 7), (13, 9), (13, 11)}



5. (i) {(3, 5), (4, 6)} (ii) {(4, 3), (5, 4), (6, 5), (7, 6)}

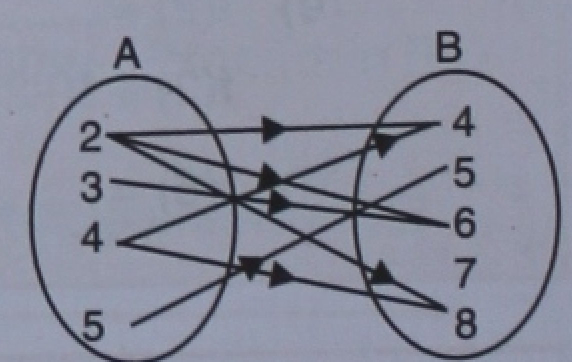




6. (i) {(8, 2), (8, 4), (9, 3), (10, 2), (10, 5), (12, 2), (12, 3), (12, 4)}
(ii) Domain = {8, 9, 10, 12} Range = {2, 3, 4, 5}

7. (i) {(2, 4), (2, 6), (2, 8), (3, 6), (4, 4), (4, 8), (5, 5)}

(ii)



8. {(5, a), (5, b), (7, b), (7, c), (9, a), (9, b), (9, c)}
Domain = {5, 7, 9} Range = {a, b, c} **9.** {(4, 4), (5, 4), (6, 4), (6, 6), (7, 4), (7, 6)} **10.** {(2, 5), (4, 3), (6, 1)}

EXERCISE 40(B)

1. (i) Function, since no two-ordered pairs have same first component (ii) Not a function, the first component is repeating (iii) Function, since no two-ordered pairs have same first component, (iv) Function **2.** Only (i) since each element in A has its unique image in B **3.** It is not a mapping as element 6 in A does not have its image in B **4.** (i), (iv), (v) and (vi). Since, each element in A has its unique image in B **5.** It is a function

EXERCISE 40(C)

1. (i) –16 (ii) 14 (iii) –1; No **2.** (i) 1 (ii) $\frac{2a-1}{a+1}$ (iii) $\frac{3}{20}$ **3.** (i) 5 (ii) 11 (iii) 21 (iv) 11; No; No **4.** (i) 10 (ii) 4a + 6 (iii) 1 **5.** (i) 5 (ii) $\frac{5}{8}$ (iii) 12 **6.** (i) 116 (ii) 5 **7.** –7 **8.** 4 **9.** A × B = {(2, 6), (2, 10), (2, 14), (2, 18), (3, 6), (3, 10), (3, 14), (3, 18), (5, 6), (5, 10), (5, 14), (5, 18)} and R = {(2, 6), (2, 10), (2, 14), (2, 18), (3, 6), (3, 18), (5, 10)} **10.** (ii) 28 (iii) 24 (iv) 72