CHAPTER 33

VOLUME AND SURFACE AREA
(Cuboid and Cube)

33.1 INTRODUCTION

Volume	The space occupied by a body (solid) is called its volume.
Surface area	The sum of areas of all the faces of a body is called its surface area.

Units of length	Unit of volume	Unit of surface-area
m (metre)	m^3 (cubic metre)	m^2 (square metre)
cm	cm^3	cm^2
mm	mm^3	mm^2

Also,

$1\ m^3 = 100 \times 100 \times 100\ cm^3 = 1000000\ cm^3$ and $1\ cm^3 = \frac{1}{100\times100\times100}\ m^3$

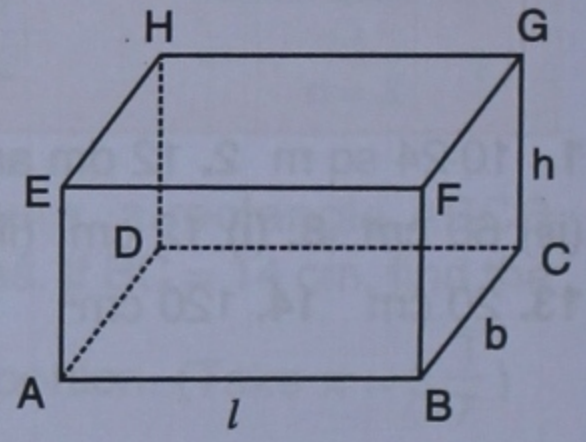
$1\ cm^3 = 10 \times 10 \times 10\ mm^3 = 1000\ mm^3$ and $1\ mm^3 = \frac{1}{1000}\ cm^3$

In general, the volume of a liquid or a gas is measured in litres, such that

$1\ m^3 = 1000$ litre and 1 litre = 1000 cm^3 (c.c. or millilitre)

33.2 CUBOID (a rectangular solid)

A **cuboid** is a solid bounded by six rectangular faces.



1. **Volume of a cuboid**
   = its length × breadth × height
   = $l \times b \times h$

2. **Total surface area of a cuboid = Area of six rectangular faces**

Since, Area of ABCD + Area of EFGH = $2(l \times b)$ [Opposite faces are equal]

Area of BCGF + Area of ADHE = $2(b \times h)$ [Opposite faces are equal]

and Area of ABFE + Area of DCGH = $2(h \times l)$ [Opposite faces are equal]

$\therefore$ **Total surface area of cuboid = $2(l \times b + b \times h + h \times l)$**

33.3 CUBE

A **cube** is a rectangular solid whose *each face* is a *square*.

In other words, a cube is a cuboid whose, length = breadth = height = a (say)

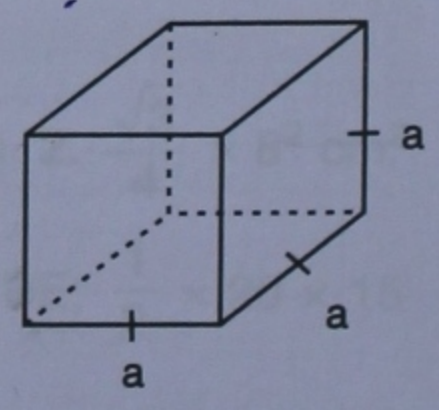
1. Since volume of a cuboid = $l \times b \times h$

   $\therefore$ **Volume of a cube** = $a \times a \times a$

   = $\mathbf{a^3}$ = (its edge)^3

2. **Total surface area of a cube** = $2\ (a \times a + a \times a + a \times a)$

   = $\mathbf{6a^2}$ = 6 (edge)^2



Example 1 :

The length, breadth and height of a cuboid are in the ratio 6 : 5 : 4. If its volume is 15,000 cm^3; find : (i) its dimensions (ii) its surface area.

Solution :

Dimension means : Its length, breadth and height.

(i) Given : Length : breadth : height = 6 : 5 : 4

$\Rightarrow$ If length = 6x cm, breadth = 5x cm and height = 4x cm

$\therefore$ Length $\times$ breadth $\times$ height = volume

$\Rightarrow$ $6x \times 5x \times 4x = 15{,}000$

$\Rightarrow$ $x^3 = \dfrac{15{,}000}{6 \times 5 \times 4} = 125 = 5 \times 5 \times 5 = 5^3$

$\therefore$ $x = 5$

i.e. **length** = 6x cm = 6 $\times$ 5 cm = **30 cm**

breadth = 5x cm = 5 $\times$ 5 cm = **25 cm**

and, **height** = 4x cm = 4 $\times$ 5 cm = **20 cm** **(Ans.)**

(ii) **Surface area of the cuboid** = $2(l \times b + b \times h + h \times l)$

$= 2(30 \times 25 + 25 \times 20 + 20 \times 30)\ cm^2$

$= 2(750 + 500 + 600)\ cm^2 =$ **3700 cm^2** **(Ans.)**

Example 2 :

The total surface area of a cube is 294 cm^2, find its volume.

Solution :

Since total surface area of cube = $6 \times (side)^2$

$\Rightarrow$ $6 \times (side)^2 = 294$

$\Rightarrow$ side = 7 cm

$\therefore$ **volume** = $(side)^3 = (7\ cm)^3 =$ **343 cm^3** **(Ans.)**

Example 3 :

A rectangular solid of metal has dimensions 50 cm, 64 cm and 72 cm. It is melted and recast into identical cubes each with edge 4 cm, find the number of cubes formed.

Solution :

$\therefore$ Volume of rectangular solid melted = its length $\times$ breadth $\times$ height

$= 50 \times 64 \times 72\ cm^3$

And, volume of each cube formed = $(its\ edge)^3$

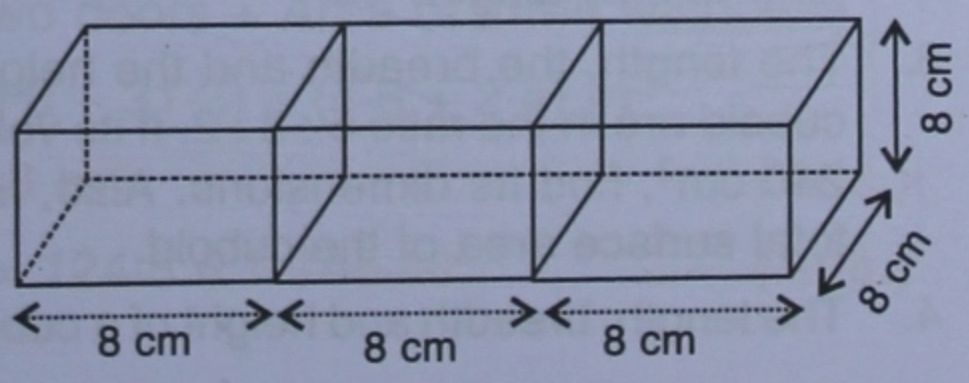
$= (4)^3\ cm^3 = 4 \times 4 \times 4\ cm^3$

$\therefore$ **Number of cubes formed** = $\dfrac{\text{Volume of solid melted}}{\text{Volume of each cube}}$

$= \dfrac{50 \times 64 \times 72}{4 \times 4 \times 4} =$ **3600** **(Ans.)**

Example 4 :

Three cubes, each of edge 8 cm, are joined as shown alongside. Find the total surface area and the volume of the cuboid.



Solution :

Since, length (l) of the resulting cuboid = 3×8 cm = 24 cm,
its breadth (b) = 8 cm and its height (h) = 8 cm

Total surface area = $2(l \times b + b \times h + h \times l)$
= $2(24 \times 8 + 8 \times 8 + 8 \times 24)$ cm^2 = **896 cm^2** **(Ans.)**

Volume = $l \times b \times h$
= $24 \times 8 \times 8$ cm^3 = **1536 cm^3** **(Ans.)**

TEST YOURSELF

1. 1 m = ... cm, 1 m^2 = ... × ..... cm^2 = ... cm^2 and 1 m^3 = ... × ..... × ..... cm^3 = ... cm^3.
2. 1 m^3 = ... litre and 1 litre = ... cm^3.
3. A cube is always a ..... ; but a cuboid is not necessarily a .....
4. The volume of a cube with side a cm is numerically equal to its surface area; then ..... = ... and a = ...
5. Each edge of a cube is 8 cm; area of each face of the cube = ... × ..... cm^2 = ... cm^2 and total surface area of the cube is ..... = ...
6. Each edge of a cube is doubled, then its total surface area becomes ..... times and its volume becomes ..... times.
7. A solid cuboid (36 cm × 3 cm × x cm) has the same volume as a solid cube of edge 6 cm; then ..... = ... and x = ... = ...
8. A cubical container, with each edge 10 cm; is full of water. This water is transferred to an empty rectangular container with length 20 cm and breadth 5 cm. If the height of water in the rectangular container is x cm, then $10 \times 10 \times 10$ = ... and x = ...

EXERCISE 33 (A)

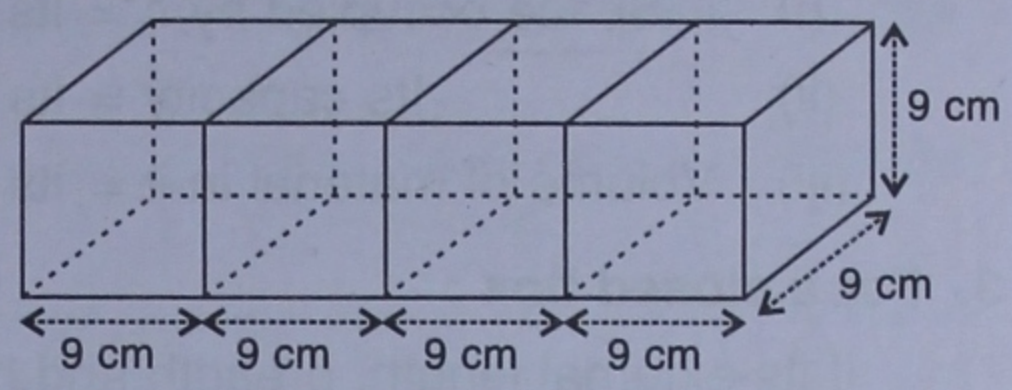
1. Find the volume and the total surface area of a cuboid, whose :
   (i) length = 15 cm, breadth = 10 cm and height = 8 cm
   (ii) l = 3·5 m, b = 2·6 m and h = 90 cm.
2. (i) The volume of a cuboid is 3456 cm^3. If its length = 24 cm and breadth = 18 cm, find its height.
   (ii) The volume of a cuboid is 7·68 m^3. If its length = 3·2 m and height = 1·0 m; find its breadth.
   (iii) The breadth and height of a rectangular solid are 1·20 m and 80 cm respectively. If the volume of the cuboid is 1·92 m^3, find its length.
3. The length, the breadth and the height of a cuboid are in the ratio 5 : 3 : 2. If its volume is 240 cm^3; find its dimensions. Also, find the total surface area of the cuboid.
4. The length, breadth and height of a cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm^2, find its dimensions. Also, find the volume of the cuboid.
5. Find the volume and total surface area of a cube whose each edge is :
   (i) 8 cm (ii) 2 m 40 cm.
6. Find the length of each edge of a cube, if its volume is :
   (i) 216 cm^3 (ii) 1·728 m^3
7. The total surface area of a cube is 216 cm^2. Find its volume.
8. A solid cuboid of metal has dimensions 24 cm, 18 cm and 4 cm. Find its volume.
9. A wall 9 m long, 6 m high and 20 cm thick, is to be constructed, using bricks of dimensions 30 cm, 15 cm and 10 cm. How many bricks will be required ?

$$\text{No. of bricks} = \frac{\text{Volume of wall}}{\text{Volume of one brick}}$$

10. A solid cube of edge 14 cm is melted down and recasted into smaller and equal cubes each of edge 2 cm, find the number of smaller cubes obtained.

11. A closed box is cuboid in shape with length = 40 cm, breadth = 30 cm and height = 50 cm. It is made of thin metal sheet. Find the cost of metal sheet required to make 20 such boxes, if 1 m^2 of metal sheet costs ₹ 45.

12. Four cubes, each of edge 9 cm, are joined as shown below :



Write the dimensions of the resulting cuboid obtained. Also, find the total surface area and the volume of the resulting cuboid.

33.4 APPLICATION

1. For a room :

Every room has four walls; two walls along its length and two walls along its width.

$\therefore$ (i) Area of each wall along the length = $l \times h$

and, (ii) Area of each wall along the width = $b \times h$

$\therefore$ **Area of 4 walls of the room** = $2 \times l \times h + 2 \times b \times h$

$= \mathbf{2(l + b) \times h}$

This area includes the area of doors and windows.

Also, (iii) **The area of roof = the area of floor = $l \times b$**

Example 5 :

The internal length, breadth and height of a rectangular room are 6 m, 5·2 m and 4·5 m respectively. It has two doors each of 1·2 m by 2 m and three windows each of 1 m by 80 cm. Find the total internal area of the room to be whitewashed.

Also, find the cost of whitewashing the room (excluding the doors and windows) at the rate of ₹ 6 per m^2.

Solution :

For the room, its l = 6 m, b = 5·2 m and h = 4·5 m

$\therefore$ Area of its four walls = $2(l + b)h$

$= 2(6 + 5{\cdot}2) \times 4{\cdot}5\ m^2 = 100{\cdot}8\ m^2$

Area of its roof = $l \times b = 6 \times 5{\cdot}2\ m^2 = 31{\cdot}2\ m^2$

Since, area of one door = $1{\cdot}2 \times 2\ m^2 = 2{\cdot}4\ m^2$

$\therefore$ area of two doors = $2 \times 2{\cdot}4\ m^2 = 4{\cdot}8\ m^2$

Also, area of each window = $1 \times 0{\cdot}80\ m^2 = 0{\cdot}80\ m^2$ [80 cm = 0·80 m]

$\therefore$ Area of three windows = $3 \times 0{\cdot}80\ m^2 = 2{\cdot}40\ m^2$

$\therefore$ **Total internal area of the room to be whitewashed**

= (Area of four walls + Area of roof) – (Area of two doors + Area of three windows)

= $(100{\cdot}8 + 31{\cdot}2) - (4{\cdot}8 + 2{\cdot}4)\ m^2$

= **124·8 m^2** **(Ans.)**

Cost of whitewashing = ₹ 6 × 124·8 = **₹ 748·80** **(Ans.)**

2. For a box :

(i) Space occupied by it = its external volume

(ii) Its capacity = its internal volume

(iii) Volume of material in it = its external volume – its internal volume.

3. For a closed box :

If its external length, breadth and height are l, b and h respectively, and its walls are x unit thick throughout, then :

(i) Its **internal length** = External length – twice the thickness of walls
$= l - 2x$

(ii) Its **internal breadth** = $b - 2x$ and

(iii) Its **internal height** = $h - 2x$

Conversely, if the internal dimensions of a box are l, b and h respectively and its sides (walls) are x unit thick everywhere, then its external dimensions are $l + 2x$, $b + 2x$ and $h + 2x$ respectively.

Example 6 :

The external length, breadth and height of a closed wooden box are 30 cm, 18 cm and 20 cm respectively. If the walls of the box are 1·5 cm thick, find :

(i) capacity of the box;

(ii) volume of the wood used in making the box;

and (iii) weight of the box; if 1 cm^3 of the wood weighs 0·80 g.

Solution :

Given, external length of the box = 30 cm

external breadth of the box = 18 cm

and, external height of the box = 20 cm

$\therefore$ External volume of the box = $30 \times 18 \times 20\ cm^3$
$= 10{,}800\ cm^3$

Since, the walls of the box are 1·5 cm thick throughout;

$\therefore$ Internal length of the box = $(30 - 2 \times 1{\cdot}5)$ cm = 27 cm

internal breadth of the box = $(18 - 2 \times 1{\cdot}5)$ cm = 15 cm

and, internal height of the box = $(20 - 2 \times 1{\cdot}5)$ cm = 17 cm

$\therefore$ Internal volume of the box = $27 \times 15 \times 17\ cm^3$;
$= 6{,}885\ cm^3$

(i) **Capacity of the box** = its internal volume
$= \mathbf{6{,}885\ cm^3}$ **(Ans.)**

(ii) **Volume of the wood used** = External volume – Internal volume
$= 10{,}800\ cm^3 - 6{,}885\ cm^3$
$= \mathbf{3{,}915\ cm^3}$ **(Ans.)**

(iii) Since, 1 cm^3 of wood weighs 0·80 g

$\therefore$ **Weight of the box** = $3{,}915 \times 0{\cdot}80$ g
$= 3132$ g = **3·132 kg** **(Ans.)**

TEST YOURSELF

9. The perimeter of a floor of a rectangular room is 18 m and its height is 3·5 m; then the area of its four-walls = ... × ..... m^2 = ...

10. The area of the floor of a rectangular room is 18 m^2 and its height is 3·5 m, then the space (internal volume of the room) is ..... × ..... m^3 = ...

11. The external dimensions of a rectangular hollow box are 16 cm × 15 cm × 10 cm and its internal dimensions are 12 cm × 10 cm × 8 cm. The space occupied by the box = ... = ...; the capacity of the box = ... = ... and the volume of material used to make this box = ... − ..... = ...

12. Three solid metal cubes with edges 3 cm, 5 cm and 4 cm are melted to form a single solid cube of edge x, then x^3 = ... + ... + ... = ... + ... + ... = ... $\Rightarrow$ x = ... cm.

EXERCISE 33 (B)

1. How many persons can be accommodated in a big-hall of dimensions 40 m, 25 m and 15 m, assuming that each person requires 5 m^3 of air ?

$$\text{No. of persons} = \frac{\text{Volume of hall}}{\text{Volume of air required for each person}}$$

2. The dimensions of a class-room are, length = 15 m, breadth = 12 m and height = 7·5 m. Find, how many children can be accommodated in this class-room, assuming 3·6 m^3 of air is needed for each child.

3. The length, breadth and height of a room are 6 m, 5·4 m and 4 m respectively. Find the area of : (i) its four-walls (ii) its roof

4. A room 5 m long, 4·5 m wide and 3·6 m high has one door 1·5 m by 2·4 m and two windows, each 1 m by 0·75 m. Find :
   (i) the area of its walls, excluding doors and windows.
   (ii) the cost of distempering its walls at the rate of ₹ 4·50 per m^2.
   (iii) the cost of painting its roof at the rate of ₹ 9 per m^2.

5. The dining-hall of a hotel is 75 m long, 60 m broad and 16 m high. It has five-doors 4 m by 3 m each and four windows 3 m by 1·6 m each. Find the cost of :
   (i) papering its walls at the rate of ₹ 12 per m^2;
   (ii) carpeting its floor at the rate of ₹ 25 per m^2.

6. Find the volume of wood required to make a closed box of external dimensions 80 cm, 75 cm and 60 cm, the thickness of walls of the box being 2 cm throughout.

7. A closed box measures 66 cm, 36 cm and 21 cm from outside. If its walls are made of metal-sheet, 0·5 cm thick; find :
   (i) the capacity of the box;
   (ii) volume of metal-sheet and
   (iii) weight of the box, if 1 cm^3 of metal weighs 3·6 g.

8. The internal length, breadth and height of a closed box are 1 m, 80 cm and 25 cm respectively. If its sides are made of 2·5 cm thick wood; find :
   (i) the capacity of the box
   (ii) the volume of wood used to make the box.

9. Find the area of metal-sheet required to make an open tank of length = 10 m, breadth = 7·5 m and depth = 3·8 m.

> The area of metal sheet = Area of 4 walls of the tank + area of its base = $2(l + b)\, h + l \times b$

10. A tank 30 m long, 24 m wide and 4·5 m deep is to be made. It is open from the top. Find the cost of iron-sheet required, at the rate of ₹ 65 per m^2, to make the tank.

EXERCISE 33 (C)

1. The edges of three solid cubes are 6 cm, 8 cm and 10 cm. These cubes are melted and recast into a single cube. Find the edge of the resulting cube.

2. Three solid cubes of edges 6 cm, 10 cm and x cm are melted to form a single cube of edge 12 cm, find the value of x.

3. The length of the diagonal of a cube is $8\sqrt{3}$ cm.

Find its :

(i) edge (ii) total surface area

(iii) volume

Diagonal of a cube = edge $\times\sqrt{3}$

4. A cube of edge 6 cm and a cuboid with dimensions 4 cm × x cm × 15 cm are equal in volume. Find :
   (i) the value of x.
   (ii) total surface area of the cuboid.
   (iii) total surface area of the cube.
   (iv) which of these two has greater surface and by how much ?

5. The capacity of a rectangular tank is 5·2 m^3 and the area of its base is $2{\cdot}6 \times 10^4$ cm^2; find its height (depth).

6. The height of a rectangular solid is 5 times its width and its length is 8 times its height. If the volume of the wall is 102·4 cm^3, find its length.

7. The ratio between the lengths of the edges of two cubes are in the ratio 3 : 2. Find the ratio between their :
   (i) total surface area
   (ii) volume.

8. The length, breadth and height of a cuboid (rectangular solid) are 4 : 3 : 2.
   (i) If its surface are is 2548 cm^2, find its volume.
   (ii) If its volume is 3000 m^3, find its surface area.

ANSWERS

TEST YOURSELF

1. 100; 100 × 100 : 10,000; 100 × 100 × 100; 1000000 **2.** 1000; 1000 **3.** cuboid; cube **4.** $a^3 = 6a^2$; 6

5. $8 \times 8\ cm^2 = 64$; $6 \times 64\ cm^2 = 384\ cm^2$ **6.** four; eight **7.** $36 \times 3 \times x = 6 \times 6 \times 6$, $x = \frac{6 \times 6 \times 6}{36 \times 3}$ cm = 2 cm

8. $20 \times 5 \times x$; $\frac{10 \times 10 \times 10}{20 \times 5}$ cm ; 10 cm **9.** 18 × 3·5; 63 m^2 **10.** 18 × 3·5; 63 m^3 **11.** $16 \times 15 \times 10\ cm^3$ $= 2400\ cm^3$; $12 \times 10 \times 8\ cm^3 = 960\ cm^3$; $2400\ cm^3 - 960\ cm^3 = 1440\ cm^3$ **12.** $3^3 + 5^3 + 4^3$ $= 27 + 125 + 64 = 216$; 6

EXERCISE 33(A)

1. (i) 1200 cm^3 ; 700 cm^2 (ii) 8·19 m^3 ; 29·18 m^2 **2.** (i) 8 cm (ii) 2·4 m (iii) 2 m **3.** 10 cm, 6 cm and 4 cm ; 248 cm^2 **4.** 12 cm, 10 cm and 6 cm ; 720 cm^3 **5.** (i) 512 cm^3 ; 384 cm^2 (ii) 13·824 m^3 ; 34·56 m^2 **6.** (i) 6 cm (ii) 1·2 m **7.** 216 cm^3 **8.** 1728 cm^3 **9.** 2400 **10.** 343 **11.** ₹ 846 **12.** length = 36 cm, breadth = 9 cm and height = 9 cm. Total surface area = 1458 cm^2; Volume = 2916 cm^3.

EXERCISE 33(B)

1. 3000 **2.** 375 **3.** (i) 91·2 m^2 (ii) 32·4 m^2 **4.** (i) 63·3 m^2 (ii) ₹ 284.85 (iii) ₹ 202.50 **5.** (i) ₹ 50889.60 (ii) ₹ 1,12,500 **6.** 57824 cm^3 **7.** (i) 45500 cm^3 (ii) 4396 cm^3 (iii) 15825·6 g **8.** (i) 0·2 m^3 (ii) 0·06775 m^3 **9.** 208 m^2 **10.** ₹ 78390

EXERCISE 33(C)

1. 12 cm **2.** x = 8 **3.** (i) 8 cm (ii) 384 cm^2 (iii) 512 cm^3 **4.** (i) x = 3·6 (ii) 256·8 cm^2 (iii) 216 cm^3 (iv) cuboid, by 40·8 cm^3 **5.** 2 m **6.** 32 cm **7.** (i) 9 : 4 (ii) 27 : 8 **8.** (i) 8232 cm^2 (ii) 1300 m^2