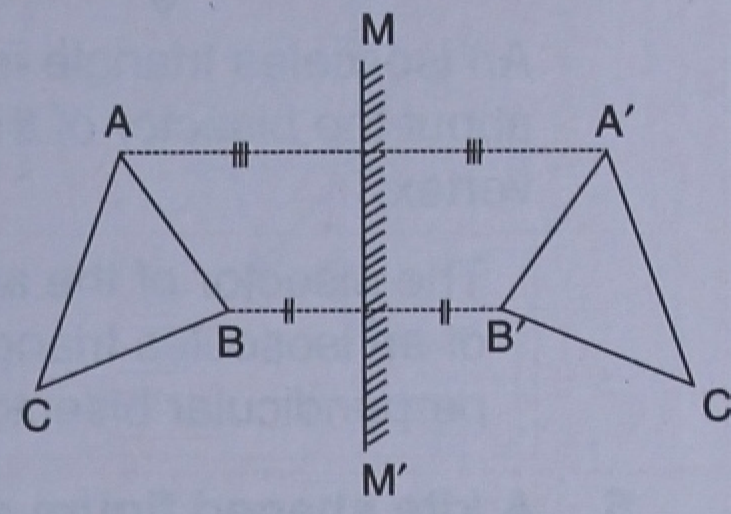
SYMMETRY, REFLECTION AND ROTATION

31.1 SYMMETRY (linear symmetry)

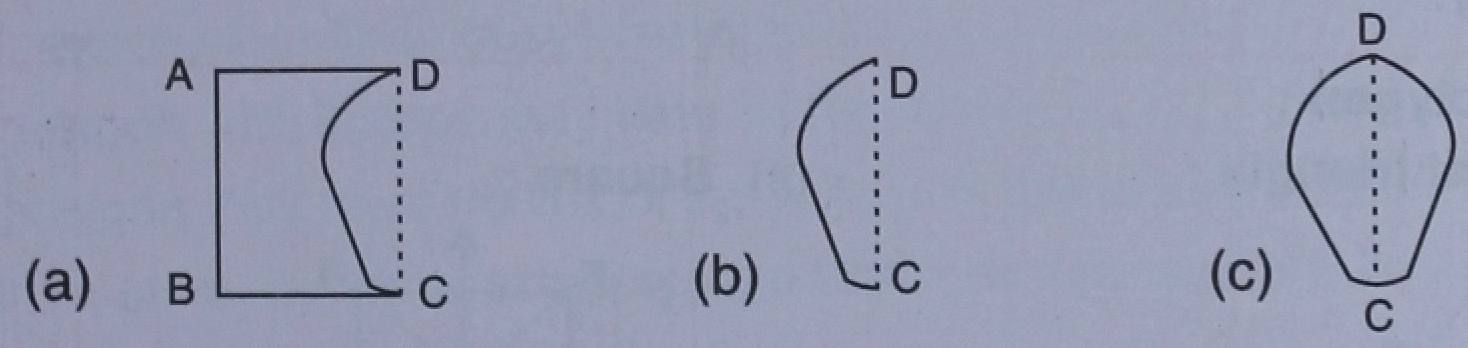
1. Consider a plane mirror MM′ and a triangle ABC placed before the mirror. As shown in the adjoining diagram, the image of triangle ABC in the mirror is triangle A′B′C′. Clearly, the images of vertices A, B and C are A′, B′ and C′ respectively. The images of sides AB, BC and AC are A′B′, B′C′ and A′C′ respectively. Also, the image triangle A′B′C′ is congruent to the object triangle ABC.



Now, if the whole figure (including the object triangle ABC, the image triangle A′B′C′ and the mirror MM′) is folded about the mirror line MM′ ; the two parts of the figure exactly coincide *i.e.* A coincides with A′, B with B′, and C with C′, similarly side AB with side A′B′ and so on. Clearly, the complete figure is identical on both the sides of the line MM′. So, it (the whole figure) is said to be **symmetrical** about the mirror line MM′.

The **line M M′**, about which the figure is symmetrical, is called the **line of symmetry**.

2. Fold a rectangular piece of paper as shown in figure (a). Now cut a piece of any pattern from the folded side of the paper as shown in figure (b).



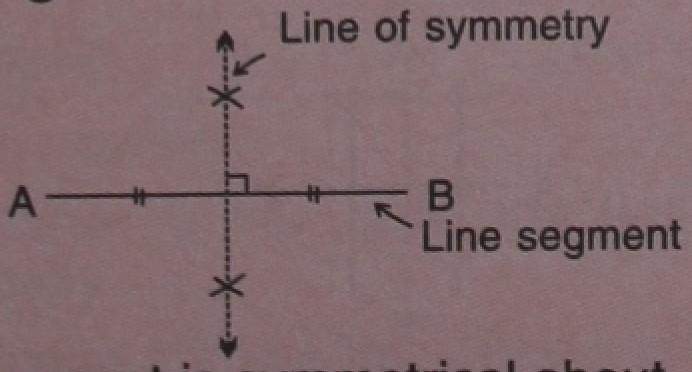
On unfolding the cutting, a design as shown in figure (c) is obtained. It is clear from the figure that the design obtained is identical on both the sides of the crease of the paper, as shown by the dotted line CD. If the figure (c) is folded again about the line CD in it, the two parts of the figure will exactly coincide. So, we say that the figure (c) is symmetrical about the line CD in it. Here, line **CD is the line of symmetry.**

1. A plane figure is said to have symmetry (or linear symmetry) if on folding the figure about a line on it, the two parts of the figure exactly coincide.
2. The line, about which the figure is symmetrical is called a **line of symmetry** or **an axis of symmetry** or simply a **mirror line.**

31.2 EXAMPLES

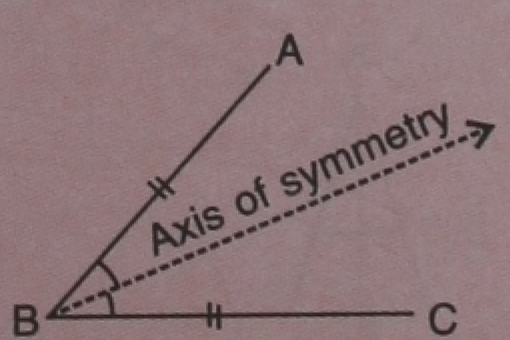
In each of the following figures, the dotted lines are the lines of symmetry.

1. A line segment :



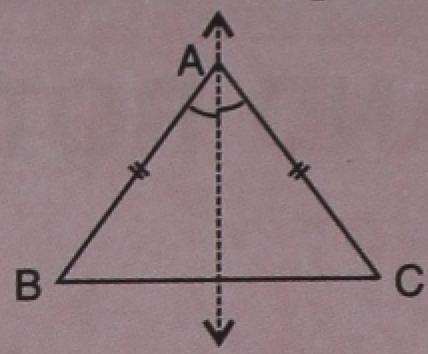
A line segment is symmetrical about its perpendicular bisector

2. An angle :



An angle with equal arms is symmetrical about its bisector.

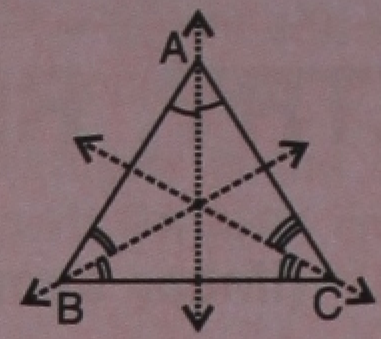
3. An isosceles triangle :



An isosceles triangle is symmetrical about the bisector of the angle of vertex.

The bisector of the angle of vertex of an isosceles triangle is also the perpendicular bisector of its base.

4. An equilaterial triangle :

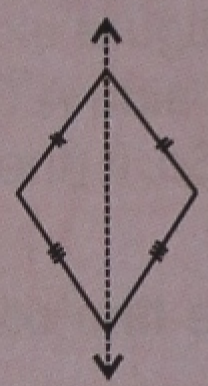


An equilateral triangle is symmetrical about the bisector of each angle of vertex, so it has **three lines of symmetry.**

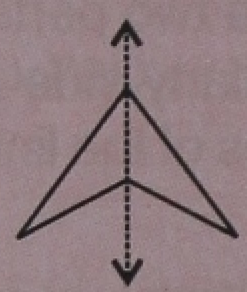
In this case also, each angle bisector is the perpendicular bisector of the opposite side.

5. A kite shaped figure and an arrow head :

(i)



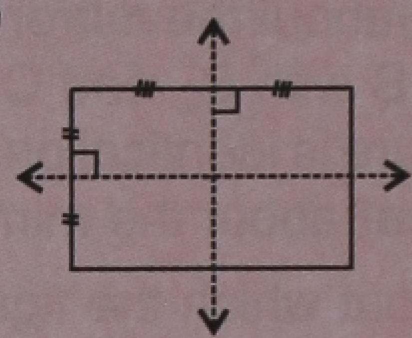
(ii)



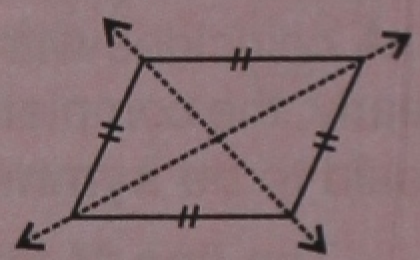
Each of these figures has **one line of symmetry.**

6. A rectangle and a rhombus :

(i)



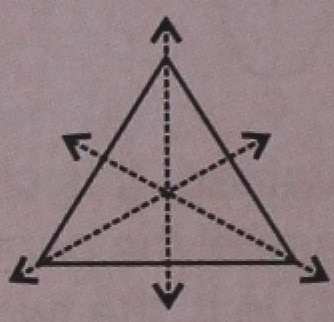
(ii)



Each figure has **two lines of symmetry.**

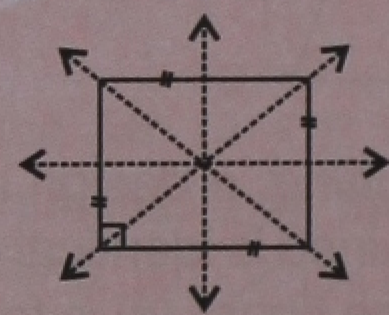
7. A regular polygon :

(i) **Equilateral triangle :**



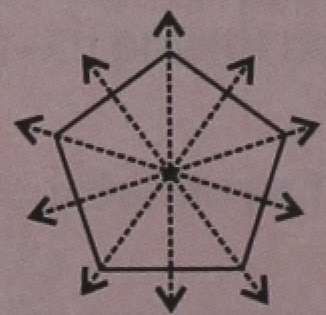
[**Three** lines of symmetry]

(ii) **Square :**



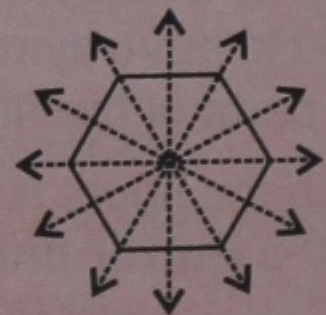
[**Four** lines of symmetry]

(iii) **Regular pentagon :**



[**Five** lines of symmetry]

(iv) **Regular hexagon :**

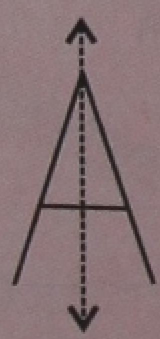


[**Six** lines of symmetry]

In case of a regular polygon : The number of lines of symmetry = No. of sides in the polygon.

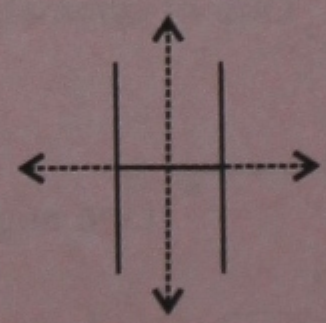
8. Letters of English alphabet :

(i)



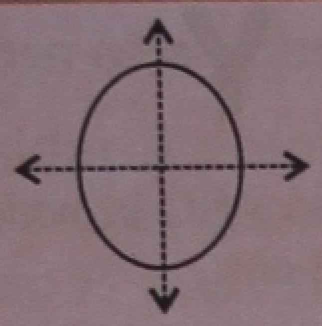
[Only **one** line of symmetry]

(ii)



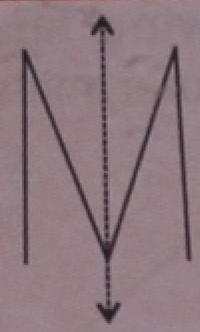
[**Two** lines of symmetry]

(iii)



[**Two** lines of symmetry]

(iv)

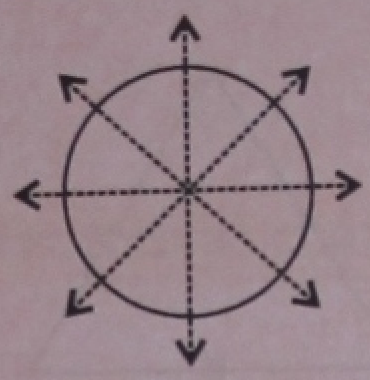


[Only **one** line of symmetry]

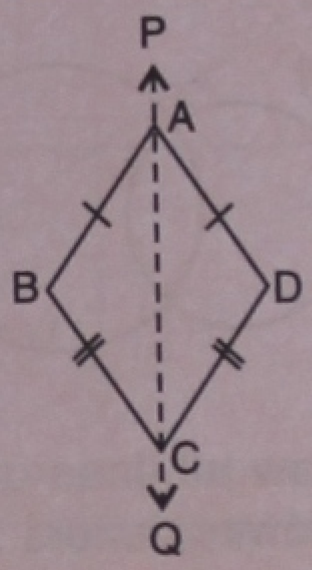
9. A circle :

Any line which passes through the centre is a line of symmetry.

A circle has an infinite number of lines of symmetry.



TEST YOURSELF



1. If on folding the given figure about the line PQ; the two parts of the figure exactly coincide; then PQ is called .... and the whole figure is said to be ....

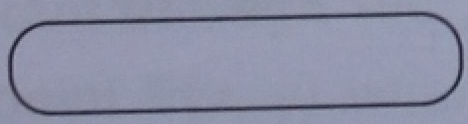
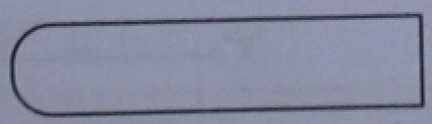
2. A quadrialteral with :
   (a) only one line of symmetry is ....
   (b) exactly two lines of symmetry is ....
   (c) exactly three lines of symmetry is ....
   (d) exactly four lines of symmetry is ....

3. A polygon with 8 sides may have at the most .... lines of symmetry.

4. A polygon may have no line of symmetry. Is this statement true ? ....
   If true, state names of two polygons which do not have any line of symmetry.
   1. .... 2. ....

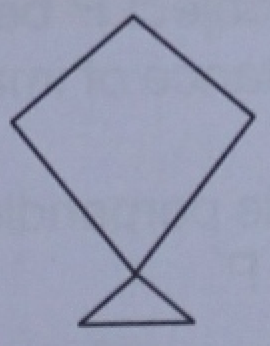
5. Draw three capital letters of English alphabet which have exactly two lines of symmetry.
   1. .... 2. .... 3. ....

EXERCISE 31 (A)

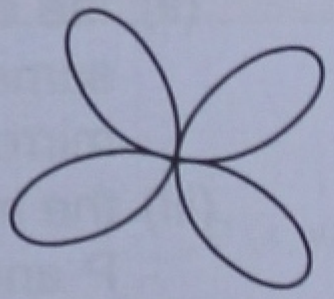
1. State, whether **true** or **false** :
   (i) The letter B has one line of symmetry.
   (ii) The letter F has no line of symmetry.
   (iii) The letter O has only two lines of symmetry.
   (iv) The figure  has no line of symmetry.
   (v) The letter N has one line of symmetry.
   (vi) The figure  has one line of symmetry.
   (vii) The letter D has only one line of symmetry
   (viii) A scalene triangle has three lines of symmetry.

2. If possible, draw the largest number of lines of symmetry in each case :

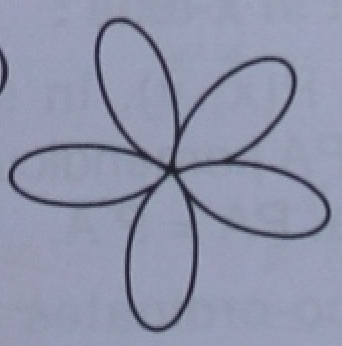
(i)



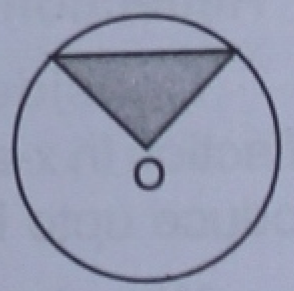
(ii)



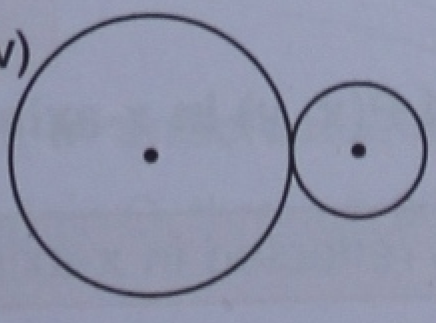
(iii)



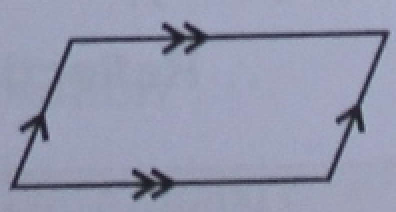
(iv)



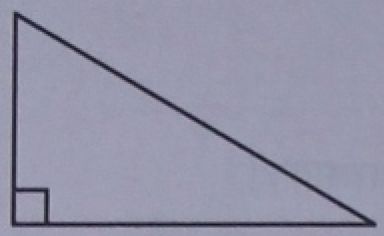
(v)

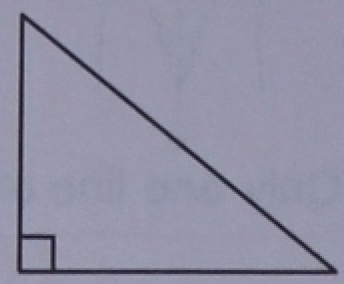


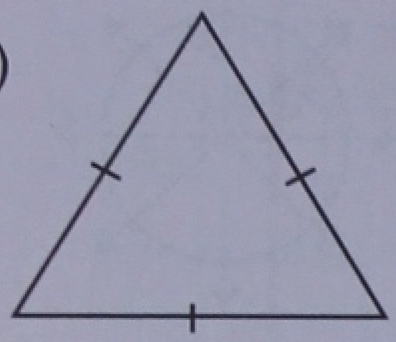
(vi)

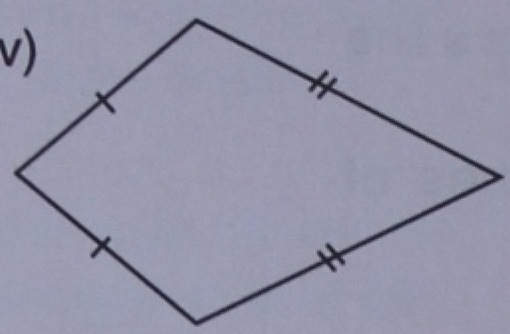


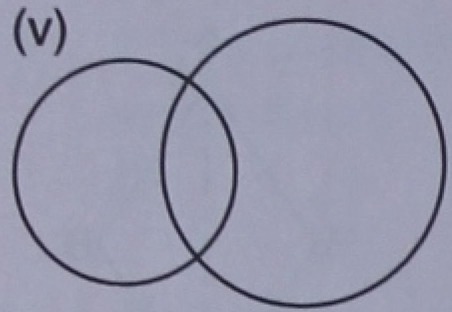
3. Examine each of the following figures, carefully, and then draw lines of symmetry if possible :

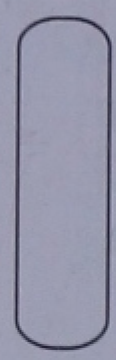
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

4. Draw all lines of symmetry for each of the following letters :

(i) **C** (ii) **E** (iii) **F**

(iv) **K** (v) **X** (vi) **V**

(vii) **M** (viii) **Y**

5. Construct a triangle ABC in which AB = AC = 5 cm and BC = 6 cm.

   Draw all its lines of symmetry.

6. Construct a triangle PQR in which : QR = 4·6 cm. $\angle Q = \angle R = 50°$. Draw all its lines of symmetry.

7. Construct a triangle XYZ in which : XY = YZ = ZX = 4·5 cm. Draw all its lines of symmetry.

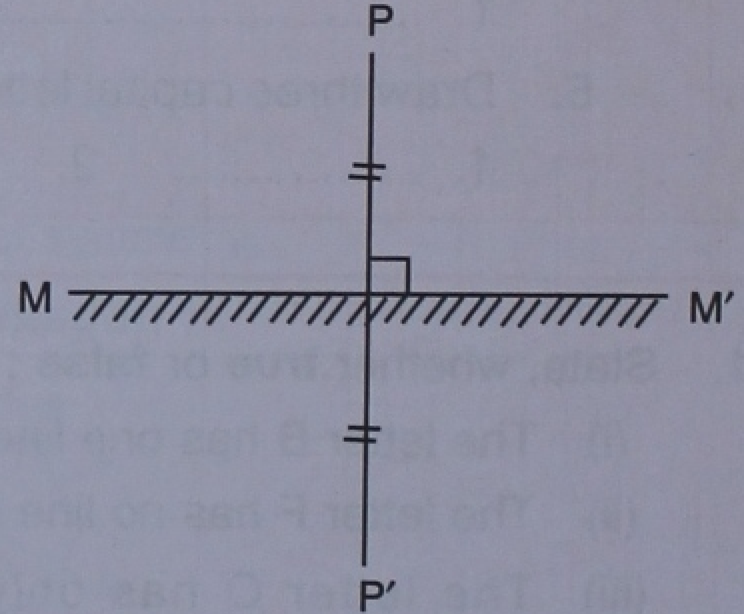
8. Construct a triangle ABC in which : AB = BC = 4 cm and $\angle ABC = 60°$. Draw all its lines of symmetry.

9. Construct a triangle PQR in which : PQ = QR = 4·2 cm and $\angle PQR = 90°$. Draw all its lines of symmetry.

10. Mark two points A and B 6·4 cm. apart. Construct the lines of symmetry so that the points A and B are symmetric with respect to this line.

11. Mark two points P and Q 5·3 cm. apart. Construct the perpendicular bisector of line segment PQ. Are the points P and Q symmetric with respect to the perpendicular bisector drawn ?

31.3 REFLECTION



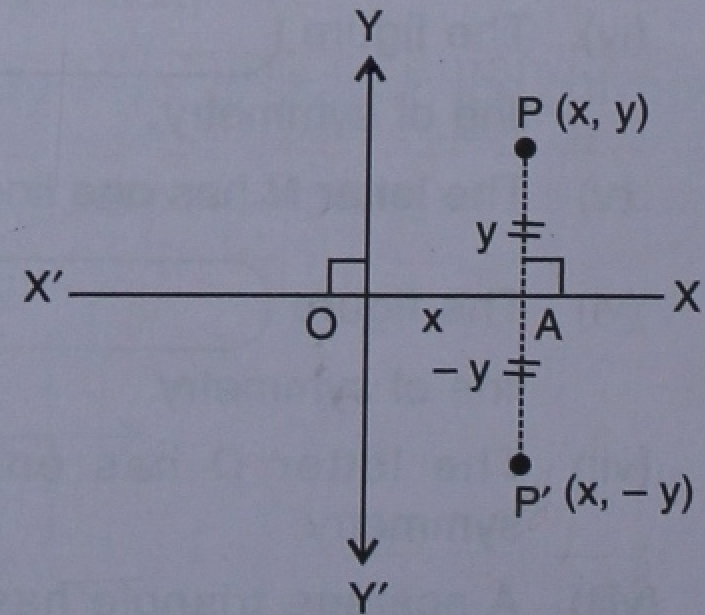
Consider a plane mirror M M′ and an object P before it. The image of object P will be formed at point P′ such that :

(i) the size of image P′ is same as the size of object P.

(ii) the distance of object P before the mirror is same as the distance of image P′ behind the mirror.

(iii) the mirror M M′ is perpendicular bisector of the line segment P P′, joining the object P and its image P′.

1. Reflection of a point in x-axis :



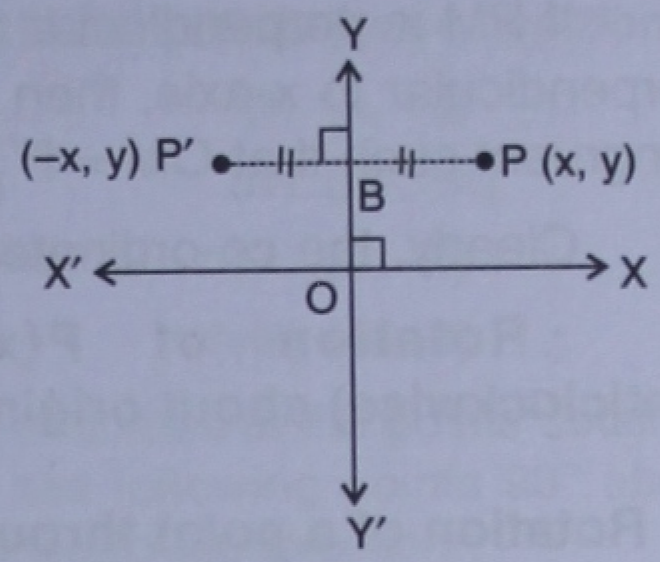
Consider a point P(x, y). In order to get its reflection in x-axis; draw PA perpendicular to x-axis and produce upto P′ such that PA = P′A.

We observe the co-ordinates of image point P′ = (x, – y)

$\therefore$ **Reflection of P(x, y) in x-axis = P′(x, – y).**

When a point is reflected in x-axis, the sign of its y-co-ordinate (ordinate) changes.

2. Reflection of a point in y-axis :



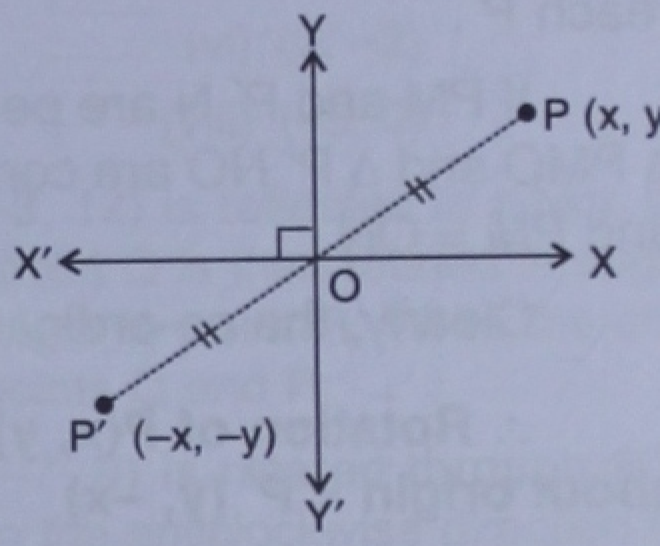
Consider a point P(x, y). In order to get its reflection in y-axis, draw PB perpendicular to y-axis and produce upto P′ such that PB = P′B.

We observe the co-ordinates of image point P′ = (–x, y)

∴ **Reflection of P(x, y) in y-axis = P′(–x, y).**

When a point is reflected in y-axis, the sign of its x-co-ordinate (abscissa) changes.

3. Reflection of a point in origin :



Consider a point P(x, y). In order to get its reflection in origin O, join PO and produce upto point P′ such that PO = P′O.

We observe the co-ordinates of image point P′ = (–x, –y)

∴ **Reflection of P(x, y) in origin = P′(–x, –y).**

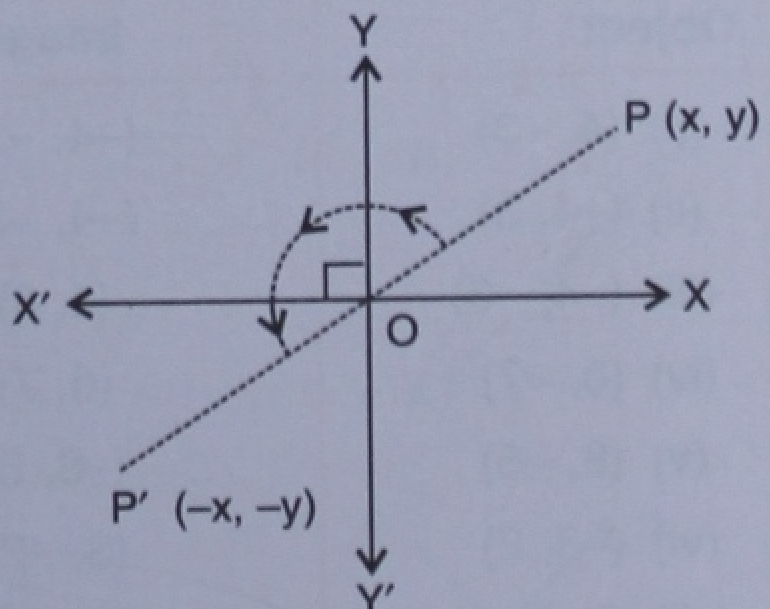
When a point is reflected in origin, signs of its x-co-ordinate (abscissa) and y-co-ordinate (ordinate) both change.

TEST YOURSELF

Point :	Reflection in x-axis	Reflection in y-axis	Reflection in origin
6. (5, 7)	....	....	....
7. (5, – 7)	....	....	....
8. (–5, 7)	....	....	....
9. (–5, –7)	....	....	....
10. (3, 0)	....	....	....
11. (0, – 6)	....	....	....
12. (0, 0)	....	....	....

31.4 ROTATION

1. Rotation of a point through 180°, about the origin :



Consider a point P(x, y) when the point P is rotated, about origin O, through an angle 180° (clockwise or anticlockwise).

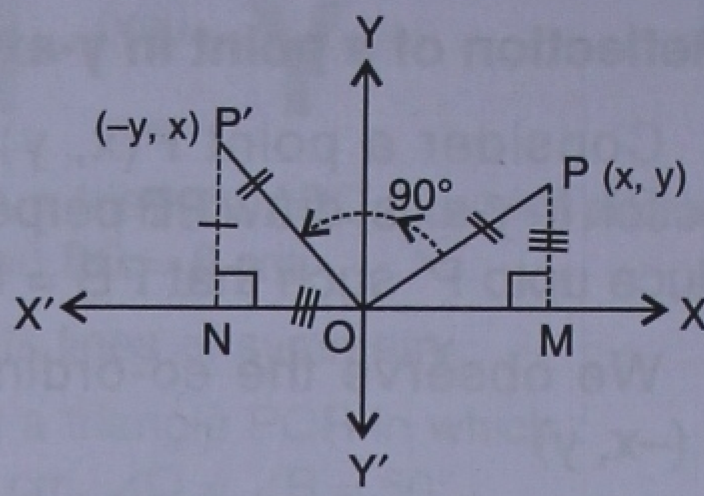
We get point P′ = (–x, –y)

∴ When a point P(x, y) is rotated through 180°, about the origin O, we get the point P′ = (–x, –y).

2. Rotation of a point through 90°, about the origin, in anticlockwise direction :

Consider a point P(x, y). On rotating P, about the origin O, through 90° in the anticlockwise direction we reach P′.

If PM is perpendicular to x-axis and P′ N is also perpendicular to x-axis, then Δ PMO and Δ P′ NO are congruent such that OM = P′ N and PM = ON.

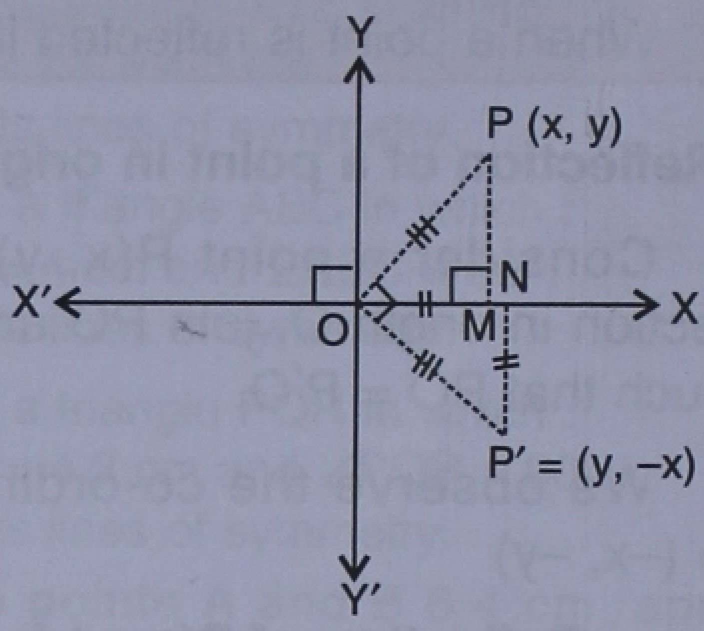


Clearly, the co-ordinates of P′ = (–y, x)

∴ **Rotation of P(x, y) through 90° (anticlockwise) about origin = P′ (– y, x).**

3. Rotation of a point through 90°, about the origin, in clockwise direction :

Consider a point P(x, y). On rotating P, about origin O, through 90° in the clockwise direction we reach P′.



If PM and P′ N are perpendiculars to x-axis, then Δ PMO and Δ P′ NO are congruent such that OM = P′N and PM = ON.

Clearly, the co-ordinates of P′ = (y, –x)

∴ **Rotation of P(x, y) through 90° (clockwise) about origin = P′ (y, –x)**

TEST YOURSELF

	Rotation about origin through		
Point :	**180°**	**90°anticlockwise**	**90°clockwise**
13. (5, 7)	....	....	....
14. (5, – 7)	....	....	....
15. (–5, 7)	....	....	....
16. (–5, –7)	....	....	....
17. (0, 5)	....	....	....
18. (–4, 0)	....	....	....
19. (6, 0)	....	....	....

EXERCISE 31 (B)

1. In each of the following cases, write the transformations as required :

Object	Image	Transformation
(i) (4, –3)	(–4, –3)	Reflection in y-axis
(ii) (–4, 3)	(–4, –3)	....
(iii) (–4, –3)	(4, 3)	....
(iv) (0, –7)	(0, 7)	....
(v) (8, –5)	(–8, 5)	Rotation through 180° about origin
(vi) (–3, 2)	(3, –2)	....
(vii) (5, 8)	(–8, 5)	....
(viii) (–7, 4)	(4, 7)	....
(ix) (8, 0)	(0, –8)	....
(x) (3, –2)	(–3, 2)	....

2. Find the co-ordinates of the following points under reflection in x-axis :

(i) (4, 8) (ii) (3, –10)
(iii) (–2, 0) (iv) (–2, –4)

3. Find the reflection of the following points in y-axis :

(i) (9, 10) (ii) (9, 0)
(iii) (0, 9) (iv) (–9, 10)
(v) (9, –10) (vi) (–9, –10)

4. Find the reflection of the following points in origin :

(i) (5, 4) (ii) (5, –4)
(iii) (–5, 4) (iv) (–5, –4)
(v) (0, 4) (vi) (0, –4)
(vii) (–5, 0) (viii) (5, 0)

5. Find the co-ordinates of the points obtained on rotating the following points through 180° about the origin :

(i) (3, 4) (ii) (3, –4)
(iii) (–3, 4) (iv) (–3, –4)
(v) (0, 4) (vi) (0, –4)
(vii) (3, 0) (viii) (–3, 0)

6. Find the co-ordinates of the points obtained on rotating the following points through 90° about origin in the anticlockwise direction :

(i) (4, 6) (ii) (4, –6)
(iii) (–4, 6) (iv) (–4, –6)
(v) (0, 6) (vi) (0, –6)
(vii) (4, 0) (viii) (–4, 0)

7. Find the co-ordinates of the points obtained on rotating the following points 90° about origin in the clockwise direction :

(i) (5, 2) (ii) (5, –2)
(iii) (–5, 2) (iv) (–5, –2)
(v) (0, 2) (vi) (0, –2)
(vii) (5, 0) (viii) (–5, 0)

8. The point P(–3, 12) is reflected in x-axis to point Q. And point Q is then rotated through 180° about origin to point R. Write the co-ordinates of points Q and R.

9. The point P(–7, 9) is rotated through 90° about origin in the anticlockwise direction to get point Q. If Q is reflected in y-axis to point R, write the co-ordinates of Q and R.

10. The point P(–5, 15) is reflected in origin to point Q. And point Q is then rotated through 90° about origin in the clockwise direction to get point R. Write the co-ordinates of points Q and R.

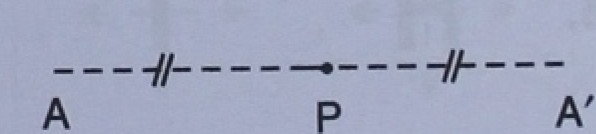
31.5 MORE ABOUT REFLECTION

1. Reflection of a point in a point

Let the reflection of point A is to be obtained in point P.

Steps :

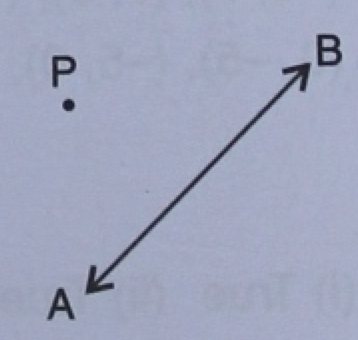
Join A and P. Produce the line-segment AP upto the point A′ so that AP = PA′.



A′ is the reflection of point A in point P.

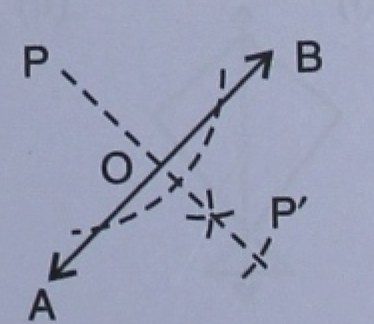
2. Reflection of a point in a line

Let the reflection of point P is to be obtained in line AB.



Steps :

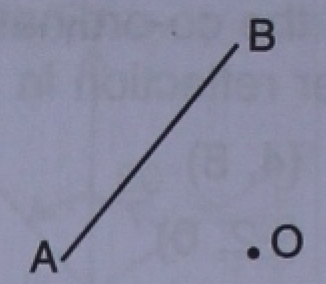
1. Through point P, draw PO perpendicular to line AB.
2. Produce PO and from the produced part of PO, cut OP′ equal to OP.



P′ is the reflection of point P in line AB.

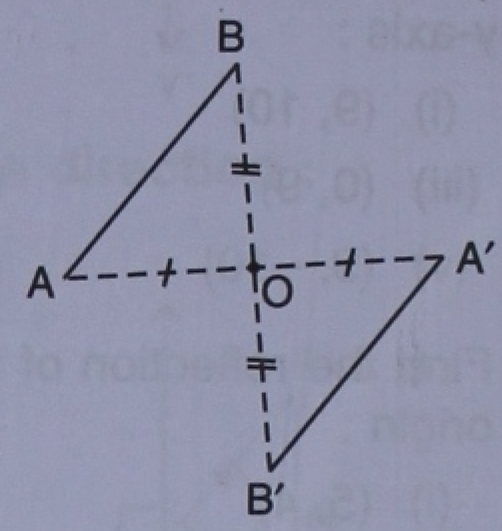
3. Reflection of a line segment in a point

Let the reflection of line segment AB is to be taken in point O.



Steps :

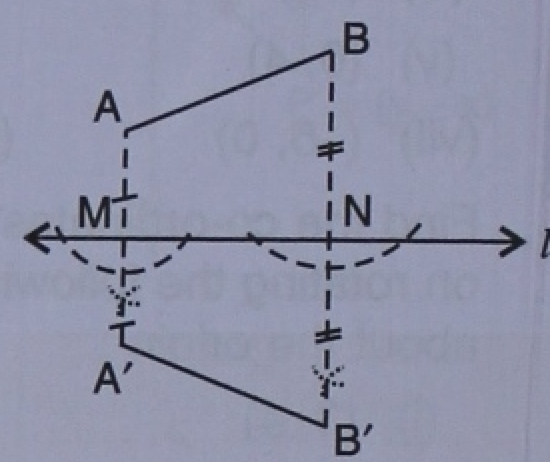
1. Join A and O.
2. Produce AO upto point A′ so that OA = OA′.
3. Join B and O.
4. Produce BO upto point B′ so that OB = OB′.
5. Join A′ and B′.



A′B′ is the reflection of line segment AB in point O such that A′B′ = AB.

4. Reflection of a line segment in a line

Let the reflection of line segment AB is to be obtained in line l.



Steps :

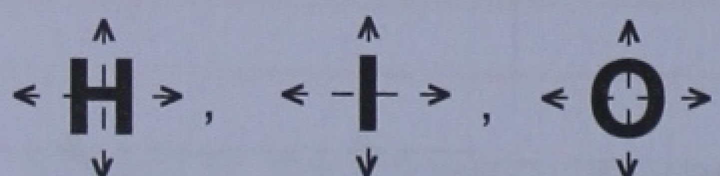
1. Through point A draw AM perpendicular to line l.
2. Produce AM upto point A′ so that AM = A′M.
3. Through point B, draw BN perpendicular to line l.
4. Produce BN upto point B′ so that BN = B′N.
5. Join A′ and B′.

Line segment A′B′ is the reflection of line segment AB in line l, such that A′B′ = AB

ANSWERS

TEST YOURSELF

1. a line of of symmetry; symmetric about line PQ **2.** (a) kite shaped figure, arrow-head (b) rectangle, rhombus (c) not-possible (d) square **3.** 8 **4.** yes; scalene triangle, trapezium

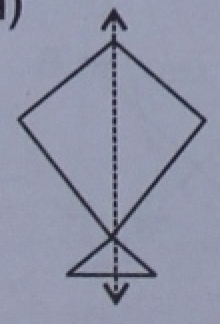
5.  **6.** (5, –7), (–5, 7), (–5, –7) **7.** (5, 7), (–5, –7), (–5, 7) **8.** (–5, –7), (5, 7), (5, –7) **9.** (–5, 7), (5, –7), (5, 7)

10. (3, 0), (–3, 0), (–3, 0) **11.** (0, 6), (0, –6), (0, 6) **12.** (0, 0), (0, 0), (0, 0) **13.** (–5, –7), (–7, 5), (7, –5) **14.** (–5, 7), (7, 5), (–7, –5) **15.** (5, –7), (–7, –5), (7, 5) **16.** (5, 7), (7, –5), (–7, 5) **17.** (0, –5), (–5, 0), (5, 0) **18.** (4, 0), (0, –4), (0, 4) **19.** (–6, 0), (0, 6), (0, –6)

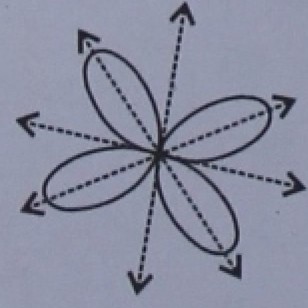
EXERCISE 31(A)

1. (i) True (ii) True (iii) False (iv) False (v) False (vi) True (vii) True (viii) False

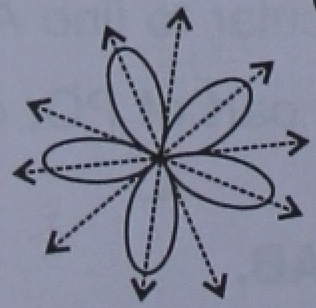
2. (i)



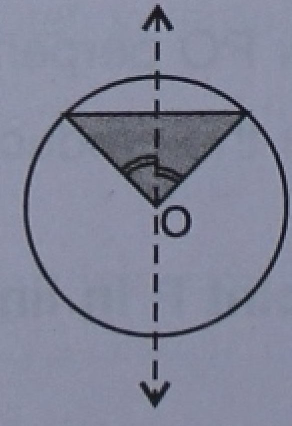
(ii)



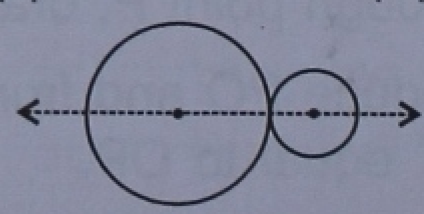
(iii)



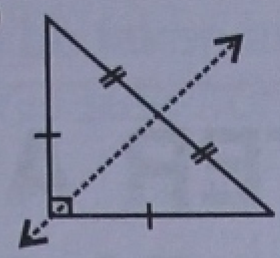
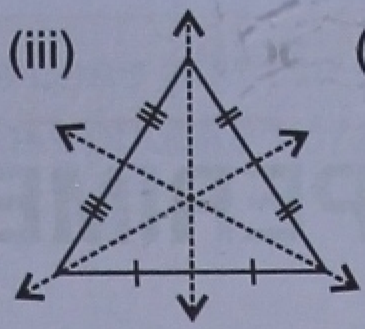
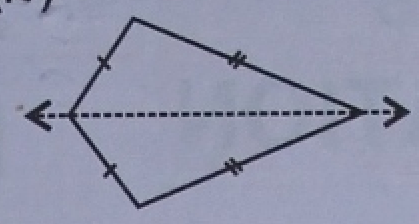
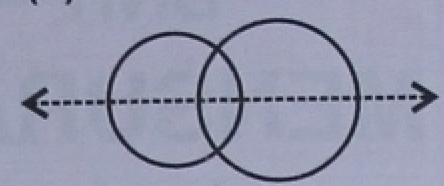
(iv)

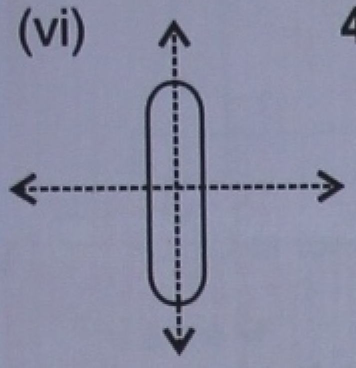


(v)

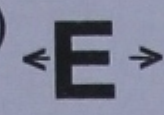


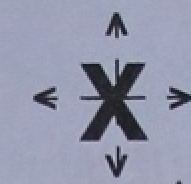
(vi) None

3. (i) No line of symmetry (ii)  (iii)  (iv)  (v) 

(vi) 

4. (i) 

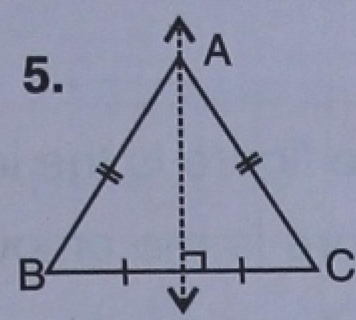
(ii) 

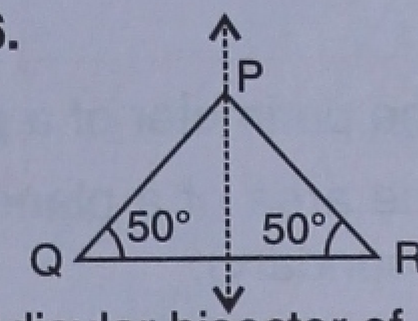
(iii) Not possible (iv) Not Possible (v) 

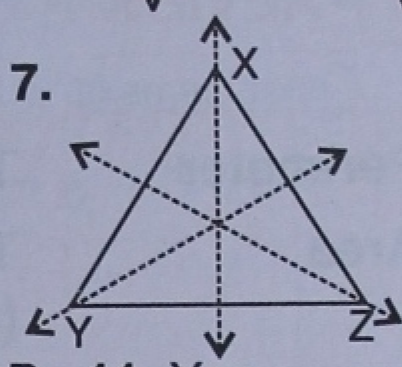
(vi) 

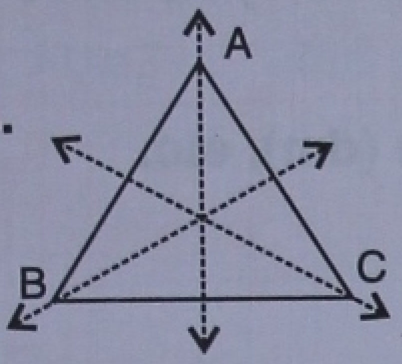
(vii) 

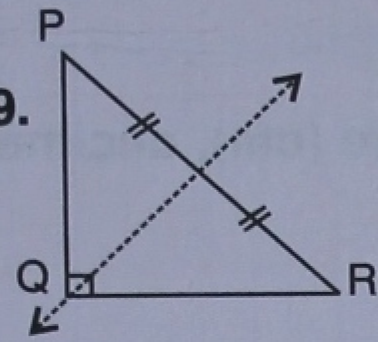
(viii) 

5. 

6. 

7. 

8. 

9. 

10. Draw perpendicular bisector of AB **11.** Yes

EXERCISE 31(B)

1. (i) Reflection in y-axis (ii) Reflection in x-axis (iii) Reflection in origin or rotation of 180° about the origin (iv) Reflection in x-axis or reflection in the origin or rotation of 180° about origin (v) Reflection in origin or rotation through 180° about the origin (vi) Reflection in origin or rotation of 180° about the origin (vii) Rotation of 90° anticlockwise about the origin (viii) Rotation of 90° clockwise about the origin (ix) Rotation of 90° clockwise about the origin (x) Reflection in origin or rotation through 180° about the origin. **2.** (i) (4, –8) (ii) (3, 10) (iii) (–2, 0) (iv) (–2, 4) **3.** (i) (–9, 10) (ii) (–9, 0) (iii) (0, 9) (iv) (9, 10) (v) (–9, –10) (vi) (9, –10) **4.** (i) (–5, – 4) (ii) (–5, 4) (iii) (5, – 4) (iv) (5, 4) (v) (0, – 4) (vi) (0, 4) (vii) (5, 0) (viii) (–5, 0) **5.** (i) (–3, – 4) (ii) (–3, 4) (iii) (3, – 4) (iv) (3, 4) (v) (0, – 4) (vi) (0, 4) (vii) (–3, 0) (viii) (3, 0) **6.** (i) (– 6, 4) (ii) (6, 4) (iii) (– 6, – 4) (iv) (6, – 4) (v) (– 6, 0) (vi) (6, 0) (vii) (0, 4) (viii) (0, – 4) **7.** (i) (2, –5) (ii) (–2, –5) (iii) (2, 5) (iv) (–2, 5) (v) (2, 0) (vi) (–2, 0) (vii) (0, –5) (viii) (0, 5) **8.** Q = (–3, –12) and R = (3, 12) **9.** Q = (–9, –7) and R = (9, –7) **10.** Q = (5, –15) and R = (–15, – 5)